Progressions (7) 11.9.5

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Question :-

If a function Satisfying f($x+y$) = f(x) f(y) for all $x, y \in N$ such that f(1) =3 and $\sum_{x=1}^{n} f(x) = 120$, find the value of n .

Solution:- $x = 1$ and $y = 1$, we get

$$f(2) = f(1+1) \tag{1}$$
$$= f(1)f(1) \tag{2}$$
$$= [f(1)]^2 \tag{3}$$
$$f(3) = f(2+1) \tag{4}$$
$$= f(2)f(1) \tag{5}$$
$$= [f(1)]^3 \tag{6}$$

Using induction , we get ;

$$f(x) = [f(1)]^x \tag{7}$$
$$r = f(1) \tag{8}$$
$$= 3 \tag{9}$$

Symbol	Value	Description
$x(0)$	3	first term
r	3	common ratio
$y(n)$	120	sum of all n terms
$x(n)$	$x(0)\,r^n u(n)$	$n+1^{th}$ term

TABLE I

Refering from table ;

$$X(z) = \frac{3}{1-3z^{-1}} \quad |z| > |3| \tag{10}$$
$$Y(z) = \frac{x(0)}{(1-rz^{-1})(1-z^{-1})} \quad |z| > |r| \tag{11}$$

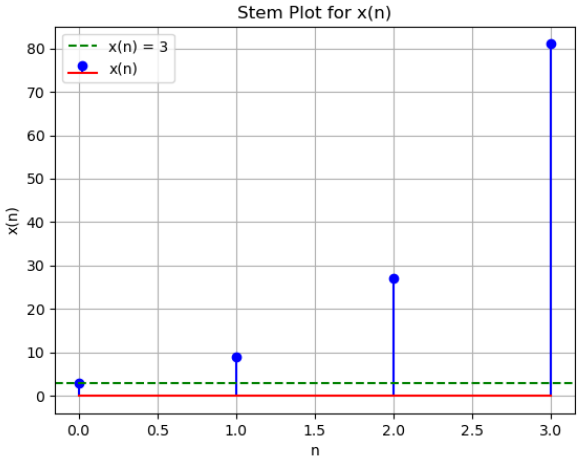
Using partial differentiation

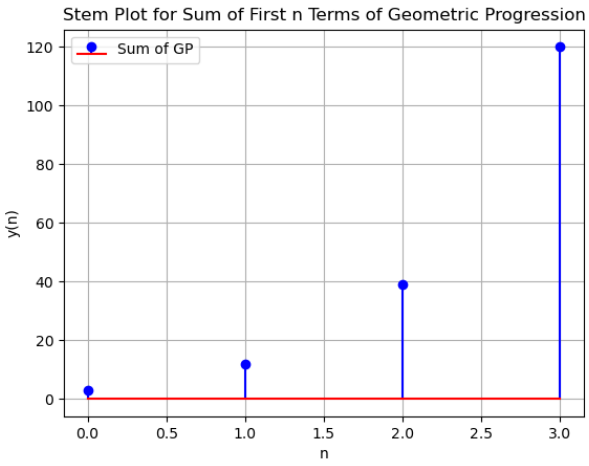
$$Y(z) = \frac{x(0)}{r-1}\left(\frac{r}{1-rz^{-1}} - \frac{1}{1-z^{-1}}\right) \quad |z| > |r| \tag{12}$$

applying inverse z transform ;

$$y(n) = x(0)\left(\frac{r^{n+1}-1}{r-1}\right)u(n) \tag{13}$$
$$\implies 120 = 3\left(\frac{3^{n+1}-1}{3-1}\right) \tag{14}$$
$$n = 3 \tag{15}$$

Ans . n take values from $n = 0$ to $n = 3$, so there are total four terms .


Stem Plot for x(n)
x(n) = 3
x(n)
x(n)
n
0
10
20
30
40
50
60
70
80
0.0
0.5
1.0
1.5
2.0
2.5
3.0


Stem Plot for Sum of First n Terms of Geometric Progression
Sum of GP
y(n)
n
0
20
40
60
80
100
120
0.0
0.5
1.0
1.5
2.0
2.5
3.0