Waves(20) 11.15

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Question :-

A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of $10ms^{-1}$, (b) recedes from the platform with a speed of $10ms^{-1}$? (ii) What is the speed of sound in each case ? The speed of sound in still air can be taken as $340ms^{-1}$.

This problem requires knowledge of *Doppler Effect* , So first we will learn Doppler effect and then we will solve our problem . Before learning Doppler effect , we will also understand Sound Waves .

Equation of Sound Wave :-

Sound Wave is transmission of energy ; sound wave depends on many parameters . A general equation of sound wave is shown below

$$y(t) = Asin(2\pi ft + \phi) \tag{1}$$

y(t) is instantaneous displacement of wave at time t;

A is amplitude of wave;

$$f\ is\ frequency\ of\ wave;$$

$$t\ is\ time;$$

$$\phi\ is\ phase\ angle;$$

$$f = \frac{v}{\lambda} \tag{2}$$

$$\lambda\ is\ wavelength\ of\ wave;$$

$$crest\ is\ peak(highest\ point)\ of\ wave;$$

$$trough\ is\ dip(lowest\ point)\ of\ wave;$$

$$2\pi f\ is\ called\ angular frequency;$$

On comparing our problem with equation (3) , equation for different cases are given

$equation\ of\ sound\ wave\ when\ whistle\ is\ blown\ by\ train\ is$

$$y(t) = Asin(2\pi \times 400 \times t + \phi)$$

$$\text{for this case } f = 400Hz$$

$equation\ of\ sound\ wave\ observed\ by\ observer\ when\ train\ is\ approaching\ observer$

$$y(t) = Asin(2\pi \times 412.1212 \times t + \phi)$$

$$\text{for this case } f = 412.1212Hz$$

$equation\ of\ sound\ wave\ observed\ by\ observer\ when\ train\ is\ receding\ observer$

$$y(t) = Asin(2\pi \times 388.5714 \times t + \phi)$$

$$\text{for this case } f = 388.5714Hz$$

Doppler Effect for Sound Waves :-

Doppler effect for sound wave refers to change in frequency or pitch of sound wave observed by an observer when there is a relative motion between observer and source .

Derivation of Doppler :-

To derive Doppler , we can write equation of sound as shown

using equation (2) , we get

$$y(t) = Asin(2\pi\frac{v}{\lambda}t + \phi) \tag{3}$$

$$v\ is\ speed\ of\ sound\ in\ that\ medium$$

Now we will see formulas for Doppler effect in different situations .

frequencies observed in Different cases			
Doppler Shift	Stationary Observer	Observer moving towards Source	Observer moving away from Source
Stationary Source	$f' = f$	$f' = \frac{(v+v_o)f}{v}$	$f' = \frac{(v-v_o)f}{v}$
Source moving towards Observer	$f' = \frac{vf}{v-v_s}$	$f' = \frac{(v+v_o)f}{v-v_s}$	$f' = \frac{(v-v_o)f}{v-v_S}$
Source moving away from Observer	$f' = \frac{vf}{v+v_s}$	$f' = \frac{(v+v_o)f}{v+v_s}$	$f' = \frac{(v-v_o)f}{v+v_s}$

1. Source is moving toward stationary Observer-

Now consider the relative motion in which source is moving towards observer , in that case effective wavelength λ' observed by observer will be compressed ,

$$v_s \textit{ is velocity of source}$$

$$v_o \textit{ is velocity of observer}$$

$$v_s = v_s$$

$$v_o = 0$$

$$\lambda' = \lambda - v_s T \tag{4}$$

T is time period(time taken by source wave to complete one revolution) and effective frequency f' observed by observer will be

$$f' = \frac{v}{\lambda'} \tag{5}$$

using equations (2) , we get

$$f' = \frac{v}{\lambda - v_s T} \tag{6}$$

$$f' = \frac{vf}{f(\lambda - v_s T)}$$

we know ,

$$T = \frac{1}{f} \tag{7}$$

using equation (7)

$$f' = \frac{vf}{v - v_s} \tag{8}$$

2. Source is moving away from stationary Observer- Similarly , if source is receding from observer than λ,will be increased

$$v_s = v_s$$

$$v_o = 0$$

$$\lambda' = \lambda + v_s T \tag{9}$$

using equations (5) and (9) , we get

$$f' = \frac{v}{\lambda + v_s T} \tag{10}$$

$$f' = \frac{vf}{f(\lambda + v_s T)}$$

using equation (2) and (7)

$$f' = \frac{vf}{v + v_s} \tag{11}$$

3. Observer is moving towards Stationary Source-

In this case , the velocity at which sound is approaching observer will increase .

$$v_s = 0$$

$$v_o = v_o$$

$$f' = \frac{v'}{\lambda'} \tag{12}$$

$$v' = v + v_o \tag{13}$$

But what about wavelength??

It's answer is , wavelength will be same .
Sound properties only depends on situation of source and not observer .

$$\lambda' = \lambda \tag{14}$$

using equations (13) and (14) , and substituting in equation (12)

$$f' = \frac{v + v_o}{\lambda} \tag{15}$$

using equation (2) , we get

$$f' = \frac{(v + v_o)f}{v} \tag{16}$$

4. Observer is moving away from Stationary Source-

In this case , the velocity at which sound is approaching observer will decrease .

$$v_s = 0$$

$$v_o = v_o$$

$$v' = v - v_o \tag{17}$$

In this case also , wavelength will not change.

$$\lambda' = \lambda \tag{18}$$

using equations (17) and (18) , and substituting in equation (12)

$$f' = \frac{v - v_o}{\lambda} \tag{19}$$

using equation (2) , we get

$$f' = \frac{(v - v_o)f}{v} \tag{20}$$

5. Source and Observer are both moving towards each other-

In this case , the velocity at which sound is approaching observer will increase and wavelength will compress .

$$v_s = v_s$$

$$v_o = v_o$$

$$v' = v + v_o \tag{21}$$

$$\lambda' = \lambda - v_s T \tag{22}$$

using equations (21) and (22) , and substituting in equation (12)

$$f' = \frac{v + v_o}{\lambda - v_s T} \tag{23}$$

using equation (2) , we get

$$f' = \frac{(v + v_o)f}{v - v_s} \tag{24}$$

6. Source and Observer are both moving away from each other-

In this case , the velocity at which sound is approaching observer will decrease and wavelength will stretch .

$$v_s = v_s$$

$$v_o = v_o$$

$$v' = v - v_o \tag{25}$$

$$\lambda' = \lambda + v_s T \tag{26}$$

using equations (25) and (26) , and substituting in equation (12)

$$f' = \frac{v - v_o}{\lambda + v_s T} \tag{27}$$

using equation (2) , we get

$$f' = \frac{(v - v_o)f}{v + v_s} \tag{28}$$

7. Source is moving towards Observer and Observer moving away from Source-

In this case , the velocity at which sound is approaching observer will decrease and wavelength will compress .

$$v_s = v_s$$

$$v_o = v_o$$

$$v' = v - v_o \tag{29}$$

$$\lambda' = \lambda - v_s T \tag{30}$$

using equations (29) and (30) , and substituting in equation (12)

$$f' = \frac{v - v_o}{\lambda - v_s T} \tag{31}$$

using equation (2) , we get

$$f' = \frac{(v - v_o)f}{v - v_s} \tag{32}$$

8. Source is moving away from Observer and Observer is moving towards Source-

In this case , the velocity at which sound is approaching observer will increase and wavelength will stretch.

$$v_s = v_s$$

$$v_o = v_o$$

$$v' = v + v_o \tag{33}$$

$$\lambda' = \lambda + v_s T \tag{34}$$

using equations (33) and (34) , and substituting in equation (12)

$$f' = \frac{v + v_o}{\lambda + v_s T} \tag{35}$$

using equation (2) , we get

$$f' = \frac{(v + v_o)f}{v + v_s} \tag{36}$$

9. Both Source and Observer are stationary-

If both Source and Observer are stationary , it means

$$v_s = 0$$

$$v_o = 0$$

also there will be no change in wavelength

$$\lambda' = \lambda \tag{37}$$

$$v' = v \tag{38}$$

using equation (37) and (38), and substituting in equation (12)

$$f' = \frac{v}{\lambda} \tag{39}$$

using equation (2) , we get

$$f' = f \tag{40}$$

So , Doppler effect depends on relative velocity of Observer and Source with respect to same frame and also velocity of Sound in that medium .

Let's get back to our problem solution

Solution :-

(i) a. When the train approaches the platform (i.e., the observer at rest), so this situation is similar to our **(1.)** derivation of Doppler effect , so we will use equation (8). If we compare with equation (8) which is given below

$$f' = \frac{vf}{v - v_s}$$

Symbol	Description	Value
f_o	actual frequency of source	400Hz
f'_a	frequency observed by observer when train is approaching observer	have to solve
f'_r	frequency observed by observer when train is receding observer	have to solve
v	velocity of air in that medium	$340ms^{-1}$.
v_t	velocity of source which is train	$10ms^{-1}$
v_o	velocity of observer	$0ms^{-1}$

On Substituting

$$f'_a = f_o \times \frac{v}{v - v_t} \tag{41}$$

$$f'_a = 400 \times \frac{340}{340 - 10}$$

$$f'_a = 412.1212$$

b. When the train recedes the platform (i.e., the observer at rest), it is similar to our **2.** in Doppler derivation and equation (11)

$$f' = \frac{vf}{v + v_s}$$

f'_r is frequency observed by observer when train is receding platform,
On comparing with equation (11)

$$f'_r = f'$$

substituting in equation (11)

$$f'_r = f_o \times \frac{v}{v + v_t} \tag{42}$$

$$f'_r = 400 \times \frac{340}{340 + 10}$$

$$f'_r = 388.5714$$

(ii) The speed of sound in each will be same.It is $340ms^{-1}$ in each case. We are providing a table in which various formulas of frequencies are written depending on situation .

Transmitted Signal :-

If a Source transmitting a signal with frequency f and moving with velocity $v_o ms^{-1}$, so equation of transmitting signal can be find out using equation **(1)** which is ,

$$y(t) = Asin(2\pi ft + \phi)$$

Received Signal;-

Source will receive a signal with frequency of $f_r$which is transmitted by an obstacle with frequency of f' , equation of received signal will be

$$y_r(t) = Asin(2\pi f_r t + \phi) \tag{43}$$

Now for receiving signal we have to make cases :-

What is an Obstacle?

Obstacle is basically an Observer , it can be anything like wall , mountain , etc. , it will receive the original signal with frequency of f transmitted by Source , and it will again transmit the received signal with frequency of f' to the source .And source will receive this signal with frequency of f_r.

1. If Obstacle and Source are stationary :-

$f = original\ frequency$
$f' = received\ frequency\ by\ obstacle$
$f_r = received\ frequency\ by\ source$

If both are stationary , it is similar to **9.** in Doppler derivation , So by using equation (40), Obstacle will receive f' and

$$f' = f$$

Now obstacle will transmit this signal back to source with frequency of f_r and it is again similar situation as above , so

$$f_r = f'$$

So $f_r = f$, so received signal will be

$$y_r(t) = Asin(2\pi ft + \phi) \tag{44}$$

2. If Obstacle is stationary and Source is moving towards obstacle :-

It is similar to **1.** in Doppler derivation , So by using equation (8), Obstacle will receive f' and

$$f' = \frac{vf}{v - v_s}$$

Now obstacle will transmit this signal back to source with frequency of f' , here obstacle will become source and source will become observer . Now it become same case as **3.** in Doppler derivation , on comparing with equation (16) ,

$$f' = \frac{(v + v_o)f}{v}$$

$f_r = f'(inequation16)$ $v_s = v_o inequation16)$ $f' = f(inequation16)$ On substituting in equation(16)

$$f_r = \frac{(v + v_s)f}{v - v_s}$$

so received signal will be

$$y_r(t) = Asin(2\pi\frac{(v + v_s)f}{v - v_s}t + \phi) \tag{45}$$

3. If Obstacle is stationary and Source is moving away from obstacle :-

It is similar to **2.** in Doppler derivation , So by using equation (11), Obstacle will receive f' and

$$f' = \frac{vf}{v + v_s}$$

Now obstacle will transmit this signal back to source with frequency of f' , here obstacle will become source and source will become observer . Now it become same case as **4.** in Doppler derivation , on comparing with equation (20) ,

$$f' = \frac{(v - v_o)f}{v}$$

$f_r = f'(inequation20)$ $v_s = v_o inequation20)$ $f' = f(inequation20)$ On substituting in equation(20)

$$f_r = \frac{(v - v_s)f}{v + v_s}$$

so received signal will be

$$y_r(t) = Asin(2\pi\frac{(v - v_s)f}{v + v_s}t + \phi) \tag{46}$$

4. If Source is stationary and obstacle is moving towards source :-

It is similar to **3.** in Doppler derivation , So by using equation (16), Obstacle will receive f' and

$$f' = \frac{(v + v_o)f}{v}$$

v_o is velocity of obstacle Now obstacle will transmit this signal back to source with frequency of f' , here obstacle will become source and source will become observer . Now it become same case as **1.** in Doppler derivation , on comparing with equation (8) ,

$$f' = \frac{(v + v_o)f}{v}$$

$f_r = f'(inequation 8)$ $v_s = v_o inequation 8)$ $f' = f(inequation 8)$ On substituting in equation(8)

$$f_r = \frac{(v + v_o)f}{v - v_o}$$

so received signal will be

$$y_r(t) = Asin(2\pi\frac{(v + v_o)f}{v - v_o}t + \phi) \tag{47}$$

5. If Source is stationary and obstacle is moving away from source :-

It is similar to **4.** in Doppler derivation , So by using equation (20), Obstacle will receive f' and

$$f' = \frac{(v - v_o)f}{v}$$

Now obstacle will transmit this signal back to source with frequency of f' , here obstacle will become source and source will become observer . Now it become same case as **2.** in Doppler derivation , on comparing with equation (11) ,

$$f' = \frac{(v)f}{(v + v_s)}$$

$f_r = f'(in\ equation\ 11)$ $v_o = v_s(in\ equation\ 11)$ $f' = f(in\ equation\ 11)$ On substituting in equation(11)

$$f_r = \frac{(v - v_o)f}{v + v_o}$$

so received signal will be

$$y_r(t) = Asin(2\pi\frac{(v - v_o)f}{v + v_o}t + \phi) \tag{48}$$

6. If Source and obstacle approaching each other :-

It is similar to **5.** in Doppler derivation , So by using equation (20), Obstacle will receive f' and

$$f' = \frac{(v + v_o)f}{v - v_s}$$

Now obstacle will transmit this signal back to source with frequency of f' , here obstacle will become source and source will become observer . Now it is again become same case as above $f_r = f'(in\ equation\ 20)$ $v_o = v_s(in\ equation\ 20)$ $v_s = v_o(in\ equation\ 20)$ $f' = f(in\ equation\ 11)$ On substituting in equation(20)

$$f_r = \frac{(v + v_s)(v + v_o)f}{(v - v_o)(v - v_s)}$$

so received signal will be

$$y_r(t) = Asin(2\pi\frac{(v + v_s)(v + v_o)f}{(v - v_o)(v - v_s)}t + \phi) \tag{49}$$

7. If Source and obstacle moving away from each other :-

It is similar to **6.** in Doppler derivation , So by using equation (28), Obstacle will receive f' and

$$f' = \frac{(v - v_o)f}{v + v_s}$$

Now obstacle will transmit this signal back to source with frequency of f' , here obstacle will become source and source will become observer . Now it is again become same case as above $f_r = f'(in\ equation\ 28)$ $v_o = v_s(in\ equation\ 28)$ $v_s = v_o(in\ equation\ 28)$ $f' = f(in\ equation\ 28)$ On substituting in equation(28)

$$f_r = \frac{(v - v_s)(v - v_o)f}{(v + v_o)(v + v_s)}$$

so received signal will be

$$y_r(t) = Asin(2\pi\frac{(v - v_s)(v - v_o)f}{(v + v_o)(v + v_s)}t + \phi) \tag{50}$$

8. If Source is moving towards obstacle and obstacle is moving away from source :-

It is similar to **7.** in Doppler derivation , So by using equation (32), Obstacle will receive f' and

$$f' = \frac{(v - v_o)f}{v - v_s}$$

Now obstacle will transmit this signal back to source with frequency of f' , here obstacle will become source and source will become observer . Now it become same case as **8.** in Doppler derivation , on comparing with equation (36) ,

$$f' = \frac{(v + v_o)f}{(v + v_s)}$$

$f_r = f'(in\ equation\ 36)$ $v_o = v_s(in\ equation\ 36)$ $v_s = v_o(in\ equation\ 36)$ $f' = f(in\ equation\ 36)$ On substituting in equation(36)

$$f_r = \frac{(v - v_o)(v + v_s)f}{(v + v_o)(v - v_s)}$$

so received signal will be

$$y_r(t) = Asin(2\pi\frac{(v - v_o)(v + v_s)f}{(v + v_o)(v - v_s)}t + \phi) \qquad (51)$$

9. If Source is moving away from obstacle and obstacle is moving towards source :-

It is similar to **8.** in Doppler derivation , So by using equation (36), Obstacle will receive f' and

$$f' = \frac{(v + v_o)f}{v + v_s}$$

Now obstacle will transmit this signal back to source with frequency of f' , here obstacle will become source and source will become observer . Now it become same case as **7.** in Doppler derivation , on comparing with equation (32) ,

$$f' = \frac{(v - v_o)f}{(v - v_s)}$$

$f_r = f'(in\ equation\ 32)$ $v_o = v_s(in\ equation\ 32)$ $v_s = v_o(in\ equation\ 32)$ $f' = f(in\ equation\ 32)$ On substituting in equation(32)

$$f_r = \frac{(v + v_o)(v - v_s)f}{(v - v_o)(v + v_s)}$$

so received signal will be

$$y_r(t) = Asin(2\pi\frac{(v + v_o)(v - v_s)f}{(v - v_o)(v + v_s)}t + \phi) \qquad (52)$$

In our problem **(a)** if , train approaches with $10ms^{-1}$ and if platform is taken as obstacle, then it is same as **(2.)** in Received signals section . By using equation (45)

$$y_r(t) = Asin(2\pi\frac{(v + v_s)f}{v - v_s}t + \phi)$$

$$v_s = 10$$

$$v = 340$$

$$f = 400$$

On substituting

$$y_r(t) = Asin(2\pi\frac{(340 + 10)400}{340 - 10}t + \phi)$$

$$y_r(t) = Asin(2\pi(424.2424)t + \phi)$$

if train is receding with $10ms^{-1}$, it is same as **(3.)** in Received signals section . By using equation (46)

$$y_r(t) = Asin(2\pi\frac{(v - v_s)f}{v + v_s}t + \phi)$$

On substituting data from above in equation (46)

$$y_r(t) = Asin(2\pi\frac{(340 - 10)400}{340 + 10}t + \phi)$$

$$y_r(t) = Asin(2\pi(377.1428)t + \phi)$$