Waves(20) 11.15

EE23BTECH11051-Rajnil Malviya

Question :-

A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of $10ms^{-1}$, (b) recedes from the platform with a speed of $10ms^{-1}$? (ii) What is the speed of sound in each case ? The speed of sound in still air can be taken as $340ms^{-1}$. This problem

requires knowledge of *Doppler Effect* , So first we will learn Doppler effect and then we will solve our problem . Before learning Doppler effect , we will also understand Sound Waves .

Symbol	Description
$y(t)$	instantaneous displacement of wave
f	frequency transmitted by source
f_r	received frequency
A	amplitude of wave
k	wave number
t	time
λ	wavelength of wave
T	time period of wave

TABLE I

Equation of Sound Wave :-

Sound Wave is transmission of energy ; sound wave depends on many parameters . A general equation of sound wave is shown below

$$y(t) = A\sin(2\pi ft - kx) \tag{1}$$

From equation 1 , equation of sound wave when whistle is blown by train is

Symbol	Description	Value
f	frequency of source	400Hz
v	velocity of sound in air	$340ms^{-1}$
v_o	velocity of observer	$0ms^{-1}$
v_s	velocity of source	$10ms^{-1}$

TABLE II

$$y(t) = A\sin(2\pi \times 400 \times t - kx)$$

$$\text{for this case } f = 400Hz$$

Physical interpretation of frequency :-

In context of sound waves , frequency represents number of cycles that occur in unit of time . In sound waves Higher frequency corresponds to high pitch and lower frequency corresponds to low pitch . Aging causes thinning of vocal cords , due this pitch become higher in old men and lower in old women .

Physical interpretation of Amplitude :-

In context of sound waves , amplitude is magnitude of sound wave . Higher the amplitude results in louder the sound and lower amplitude results in softer sound .

Physical interpretation of Wavelength :-

Wavelength of sound waves is distance between two consecutive compressions or rarefactions .

Wave number :-

Wave number(k) is parameter used to represent number of wavelengths per unit distance .

$$k = \frac{2\pi}{\lambda} \tag{2}$$

Transmitted Signal	Received Signal
Source transmits a signal with frequency	Source will receive reflection of its transmitted Signal
$A\sin(2\pi f t+\phi)$	$A\sin(2\pi f_r t+\phi)$

TABLE III

Velocity of Sound in medium :-

Velocity of sound (v) differs medium to medium , it influenced by some parameters . v in a particular medium is defined as

Symbol	Description
γ	adiabatic index or ratio of specific heats of medium
ρ	density of medium
P	pressure of medium

TABLE IV

$$v = \sqrt{\frac{\gamma.P}{\rho}} \tag{3}$$

Doppler Effect for Sound Waves :-

Doppler effect for sound wave refers to change in frequency or pitch of sound wave observed by an observer when there is a relative motion between observer and source .

General Equation of Doppler :-

Now we will see formulas for Doppler effect in different situations .

$$f' = \frac{v'}{\lambda'} \tag{4}$$

$$f = \frac{v}{\lambda} \tag{5}$$

$$f' = \frac{v \pm v_o}{\lambda \pm v_s T} \tag{6}$$

$$f = \frac{1}{T} \tag{7}$$

From equations 5 and 7 , then substituting in equation 6

$$f' = \frac{(v \pm v_o) f}{v \pm v_s} \tag{8}$$

Above equation is general equation for any case in doppler effect . Signs of equation 8 depends on velocities of both observer and source .

Effect of observer's velocity on frequency :-

If observer is moving towards source and source is stationary ($v_s = 0$), then sound will reach observer faster , so observed frequency will increase. It means

$$f' > f \tag{9}$$

From equation 8

$$f' = \frac{(v \pm v_o) f}{v} \tag{10}$$

From equations 10 and 9 ,

$$\frac{(v \pm v_o) f}{v} > f \tag{11}$$

$$v \pm v_o > v \tag{12}$$

There must be (+) sign to satisfy equation 12 . And for vice-versa case(observer is moving away) , so frequency will be less than f_n So ,

$$f' < f \tag{13}$$

From equation 10 ,

$$\frac{(v \pm v_o) f}{v} < f \tag{14}$$

$$v \pm v_o < v \tag{15}$$

There must be (-) sign to satisfy equation 15 .

Effect of source's velocity on frequency :-

If source is moving towards stationary observer($v_o = 0$), so λ' in equation 4 will compress and denominator will decrease , so f' will increase ,

$$f' > f \tag{16}$$

From equation 8

$$f' = \frac{vf}{v \pm v_s} \tag{17}$$

From equations 10 and 5

$$\frac{vf}{v \pm v_s} > f \tag{18}$$

$$\frac{v}{v \pm v_s} > 1 \tag{19}$$

There must be (-) sign to satisfy equation 19 . And for vice-versa case(source is moving away) , so wavelength will increase and denominator in equation 6 increases so f' will decrease

$$f' < f \tag{20}$$

From equation 8

$$f' = \frac{vf}{v \pm v_s} \tag{21}$$

From equations 21 and 20 ,

$$\frac{vf}{v \pm v_s} < f \tag{22}$$

$$\frac{v}{v \pm v_s} < 1 \tag{23}$$

There must be (+) sign to satisfy equation 23.

v_o and v_s directly affect equation 8 , independent of each other . We can change signs of numerator and denominator independently by analysing situation .

Let's get back to our problem solution
(i).a When the train approaches the platform (i.e., the observer at rest),

Solution :-
So here source is approaching , wavelength will decrease and frequency will increase , so we have to increase f'

$$f' = \frac{vf}{v - v_s} \tag{24}$$

On Substituting in equation 24

$$f'_a = 400\left(\frac{340}{340 - 10}\right)$$
$$f' = 412.1212$$

b. When the train recedes the platform (i.e., the observer at rest),

Solution :-

It is vice-versa of above, From Table:3

$$f' = f\left(\frac{v}{v + v_s}\right) \tag{25}$$

$$f' = 400\left(\frac{340}{340 + 10}\right)$$
$$f' = 388.5714$$

(ii) The speed of sound in each will be same.It is $340ms^{-1}$ in each case.

From equation 8 we will interchange v_o *and* v_s

$$f_r = f'\left(\frac{v \pm v_s}{v \pm v_o}\right) \tag{26}$$

For signs , we will use same logic, if someone is approaching another one , so definitely it will increase frequency(f_r) and if receding so it will decrease frequency(f_r) .

In our problem if , train approaches with $v_s 10ms^{-1}$ and if platform is taken as obstacle with $v_o = 0$,
Received frequency by platform is *f'* 412.1212 ,
Here train(source) is approaching , so v_s will try to increase f_r
On substituting in equation 26

$$f_r = f'\left(\frac{v \pm v_s}{v}\right) \tag{27}$$

To increase f_r , there must be (+) sign ;

$$f_r = 412.1212\left(\frac{340 + 10}{340}\right)$$

f_r=424.2424

So Received signal will be ,

From table Table:3

$$y_r(t) = A\sin\left(2\pi(424.2424)t - kx\right)$$

if train is receding with $10ms^{-1}$, so v_s will try to decrease f_r so we will use (-) sign in equation 26 with $v_o = 0$

$$f_r = \left(\frac{v - v_s}{v}\right)$$

f'=388.5714
on substituting in equation 26

$$f_r = 388.5714\left(\frac{340 - 10}{340}\right)$$

$f_r = 377.1428$
So received signal will be ,
From table Table:3

$$y_r(t) = A\sin\left(2\pi(377.1428)t - kx\right)$$