

H.W  $\rightarrow$  2

Q-1)

$$(i) f(x) = x^{\frac{2}{3}} \text{ in } [-8, 8]$$

$$\text{Ans} \rightarrow f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$$

$\therefore$  The function is differentiable for  $x \in R$

$\therefore$   $f(x)$  is continuous for  $(-8, 8)$  and differentiable for  $[-8, 8]$

$$(ii) f(x) = 2x^2 - 7x - 10 \text{ over } [2, 5] \text{ and find } c \text{ using LMVT}$$

$$\text{Ans} \rightarrow f'(x) = 4x - 7$$

$\therefore$  The function is differentiable for  $x \in R$

$\therefore$   $f(x)$  is continuous for  $(2, 5)$  and differentiable for  $[2, 5]$

$$\therefore f'(c) = \frac{f(5) - f(2)}{5 - 2}$$

$$4c - 7 = \frac{(2 \times 25 - 7 \times 5 - 10) - (2 \times 4 - 7 \times 2 - 10)}{3}$$

$$4c - 7 = \frac{(50 - 35 - 10) - (8 - 14 - 10)}{3}$$

$$4c - 7 = \frac{5 + 16}{3}$$

$$4c - 7 = 7$$

$$4c = 14$$

$$c = \frac{7}{2}$$

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Q.2) Apply Lagrange's mean value theorem for the function  $\log(x)$  in  $[a, a+h]$  & determine  $\theta$  in terms of  $a$  and  $h$  and deduce that  $0 < 1 / (\log(1+\theta)) - 1/h < 1$

$$\text{Ans} \rightarrow f(x) = \log(x) \quad [a, a+h]$$

$$f'(x) = \frac{1}{x}$$

$\therefore$  function is differentiable for  $x \in R - 0$

$\therefore$  function is differentiable for  $(a, a+h)$  and continuous for  $[a, a+h]$  if  $a \neq 0$ .

$$f(a) = f(a+h) \quad f'(c) = \frac{f(a+h) - f(a)}{a+h - a}$$

$$\theta \log a = \log(a+h)$$

$$\log a - \log(a+h) = 0 \quad \frac{1}{c} = \frac{\log(a+h) - \log(a)}{h}$$

$$\log\left(\frac{a}{a+h}\right) = 0$$

$$e^0 = \frac{a}{a+h}$$

$$\frac{1}{c} = \frac{\log\left(\frac{a+h}{a}\right)}{h}$$

$$\frac{1}{1} = \frac{a}{a+h}$$

$$\therefore c = \frac{h}{\log\left(\frac{a+h}{a}\right)}$$

$$c = a + \theta h$$

$$\therefore a + \theta h = \frac{h}{\log\left(\frac{a+h}{a}\right)}$$

$$\therefore \frac{a}{h} + \theta = \frac{1}{\log\left(\frac{a+h}{a}\right)}$$

$$\theta = \frac{1}{\log\left(\frac{a+h}{a}\right)} - \frac{a}{h}$$

$$\therefore \theta = \frac{1}{\log\left(\frac{a+h}{a}\right)} - \frac{a}{h}$$

$$\text{so } 0 < \theta < 1$$

$$\text{so } 0 < \frac{1}{\log(1+\frac{h}{a})} - \frac{a}{h} < 1$$

let  ~~$a=1$~~   $a=1$  and  $h=n$

$$\text{so } 0 < \frac{1}{\log(1+n)} - \frac{1}{n} < 1$$

Q3) show that  $\frac{h}{1+h^2} < \tan^{-1} h < h$  when  $h \neq 0$  and  $h > 0$ .

Ans →

$$\begin{aligned} \frac{h}{1+h^2} &< \tan^{-1} h < h \\ \frac{1}{1+h^2} &< \frac{\tan^{-1} h}{h} < 1 \\ \frac{1}{1+h^2} &< \frac{\tan^{-1} h - \tan^{-1} 0}{h-0} < 1 \end{aligned}$$

~~last part~~

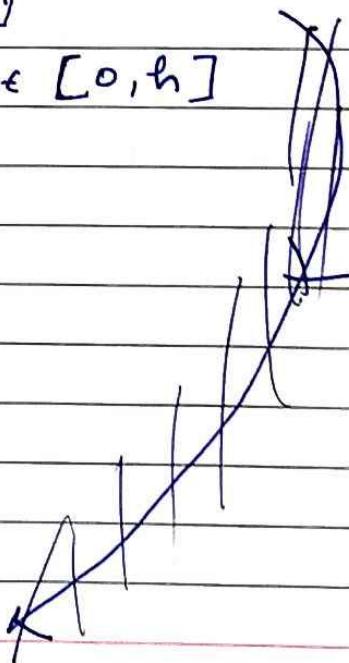
let  $f(x) = \tan^{-1} x$  for  $x \in [0, h]$

$$\text{so } f'(x) = \frac{1}{1+x^2}$$

$$f'(c) = \frac{f(h) - f(0)}{h-0}$$

$$\frac{1}{1+c^2} = \frac{\tan^{-1} h - \tan^{-1} 0}{h-0}$$

$$\text{so } \frac{1}{1+h^2} / 17$$



∴  $f'(0) > f'(c) > f'(h)$  {since it is  
a decreasing function}

∴  $\frac{1}{1+0^2} > \frac{1}{1+c^2} > \frac{1}{1+h^2}$

$$1 > \frac{\tan^{-1} h - \tan^{-1} 0}{h-0} > \frac{1}{1+h^2}$$

hence proved.