H.W

Q.1) prove that $\sin(e^x - 1) = x + \frac{x^2}{2} - \frac{5x^4}{24} + \cdots$

Ans → let $f(x) = \sin(e^x - 1)$

$\therefore \quad f(0) = \sin(0) = 0$

$\therefore \quad f'(0) = \cos(e^x - 1)(e^x) = 1 \times 1$

$= 1$

Date ______ Page ______

$\therefore f''(0) = -\sin(e^x-1)(e^x)\cdot e^x + \cos(e^x-1)\cdot e^x$

$= -\sin(0)\cdot 1\cdot 1 + \cos(0)\cdot 1$

$= 1$

$\therefore f'''(0) = \frac{d}{dx}\left[-e^{2x}\cdot\sin(e^x-1) + e^x\cdot\cos(e^x-1)\right]$

$= -2e^{2x}\cdot\sin(e^x-1) - e^{2x}\cdot\cos(e^x-1)\cdot e^x + e^x\cdot\cos(e^x-1)$
$+ e^x\cdot(-\sin(e^x-1))e^x$

$= -2e^{2x}\cdot\sin(e^x-1) - e^{3x}\cdot\cos(e^x-1) + e^x\cdot\cos(e^x-1)$
$- e^{2x}\cdot\sin(e^x-1)$

$= -3e^{2x}\cdot\sin(e^x-1) - e^{3x}\cdot\cos(e^x-1) + e^x\cdot\cos(e^x-1)$

$= 0 - 1\cdot 1 + 1$

$= 0$

$\therefore \cancel{f''''(0) = -3\cdot 2\cdot e^{2x}\cdot\sin(e^x-1) + 2e^{2x}\cos(e^x-1)\cdot e^x} -$

$\therefore f''''(0) = -3\left[2\cdot e^{2x}\cdot\sin(e^x-1) + e^{2x}\cdot\cos(e^x-1)\cdot e^x\right] - 3\cdot e^{3x}\cdot\cos(e^x-1)$

$- e^{3x}\cdot(-\sin(e^x-1))e^x + e^x\cdot\cos(e^x-1) +$
$e^x\cdot(-\sin(e^x-1))e^x$

$= -6\cdot e^{2x}\cdot\sin(e^x-1) - 3e^{3x}\cdot\cos(e^x-1) - 3\cdot e^{3x}\cdot\cos(e^x-1)$
$+ e^{4x}\sin(e^x-1) + e^x\cdot\cos(e^x-1) - e^{2x}\cdot\sin(e^x-1)$

$= 0 - 3 - 3 + 0 + 1 - 0$

$= -5$

$$\therefore f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f''''(0) \ - - .$$

$$= 0 + x \cdot 1 + \frac{x^3}{3!} \cdot 0 + \frac{x^4}{4!}(-5) \ - - - .$$

$$= x + \frac{x^3}{6} - \frac{5x^4}{24} \ - - - - .$$

Q-2) prove that $\log(1+\tan x) = x - \frac{x^2}{2} + \frac{2x^3}{3} + - - - - .$

Ans → $f(x) = \log(1+\tan x)$

$\therefore f(0) = \log(1+0) = \log(1) = 0$

$\therefore f'(0) = \frac{1}{1+\tan x}(0+\sec^2 x) = \frac{1}{1+0}\left(\frac{1}{1}\right) = 1$

$\therefore f''(0) = (1+\tan x)(2\sec x ....$

For now I will not do this thing again and again, Just put $\frac{d}{dx}\left(\quad \right)_{x=0}$ In calculator.

~~$\therefore f''(0) = 1 - 0.1774$~~ finally Do ghanta calculator per time-pass karnay kay baad ab my kud hi karunga.

$$f''(0) = \frac{(1+\tan x)(2 \cdot \sec x \cdot \sec x \cdot \tan x) - \sec^2 x(\sec^2 x)}{(1+\tan x)^2}$$

Date ______ Page ______

$f''(0) = -1$

$\therefore f(x) = f(0) + x f'(x) + \frac{x^2}{2!} f''(x) \ldots$

$= 0 + x + \frac{x^2}{2!}(-1) \ldots$

$= x - \frac{x^2}{2!} \ldots$

Q-3). obtain the series for $\log(1+x)$ and hence find the series of $\log_e\left(\frac{1+x}{1-x}\right)$ and hence find the value of $\log_e\left(\frac{11}{9}\right)$

Ans→

let $f(x) = \log(1+x)$

$\therefore f(0) = \log(1) = 0$

$\therefore f'(0) = \frac{1}{1+x}(1) = 1$

$\therefore f''(0) = \frac{-1}{(1+x)^2} = -1$

$\therefore f'''(0) = \frac{-1 \times (-2)}{(1+x)^3} = \frac{2}{1} = 2$

$\therefore f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \ldots$

$= 0 + x(1) + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(2)$

$= x - \frac{x^2}{2} + \frac{x^3}{3} \ldots$

Date ______ Page ______

$\therefore \log\left(\frac{1+x}{1-x}\right) = \log(1+x) - \log(1-x)$

$$= \left[x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots\right] - \left[-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \cdots\right]$$

$$= 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} \cdots\right]$$

~~this is not the~~

by calculator

$\therefore \log\left(\frac{11}{9}\right) = ?$

$\log\left(\frac{1+x}{1-x}\right)$ let $x = \frac{1}{10}$.

$\therefore \log\left(\frac{1+\frac{1}{10}}{1-\frac{1}{10}}\right) = \log\left(\frac{10+1}{10-1}\right) = \log\left(\frac{11}{9}\right)$

$\therefore \log\left(\frac{11}{9}\right) = 2\left[\frac{1}{10} + \frac{1}{10^3 \times 3} + \frac{1}{10^5 \times 5} \cdots\cdots\right]$

$$= 2\left[0.1 + 0 \cdots\cdots\right]$$

$= 0.2$

{you can check with calculator}