Sub → maths

14/01/2022

Module. 1 → Calculus

Ex → 04

Q.1) Expand $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x-1)$ and find $f(0.99)$.

Ans → $a = 1$

$\therefore \quad f(a) = f(1) = 1 - 1 + 1 - 1 + 1 - 1 = 0$

$f'(1) = 5(1)^4 - 4(1)^3 + 3(1)^2 - 2(1) + 1 = 5 - 4 + 3 - 2 + 1 = 3$

$f''(1) = 20(1)^3 - 12(1)^2 + 6(1) - 2 = 20 - 12 + 6 - 2 = 12$

$f'''(1) = 60(1)^2 - 24(1) + 6 = 60 - 24 + 6 = 42$

$f''''(1) = 120(1) - 24 = 120 - 24 = 96$

$f'''''(1) = 120$

$f''''''(1) = 0$

$$\therefore f(x) = f(1) + (x-1)f'(1) + \frac{(x-1)^2}{2!}f''(1) + \frac{(x-1)^3}{3!}f'''(1) + \frac{(x-1)^4}{4!}f''''(1)$$

$$+ \frac{(x-1)^5}{5!}f'''''(1) + 0$$

$$f(x) = 0 + 3(x-1) + 6(x-1)^2 + 7(x-1)^3 + 4(x-1)^4 + (x-1)^5$$

$$\therefore f(0.99) = 3(-0.01) + 6(-0.01)^2 + 7(-0.01)^3 + 4(-0.01)^4 + (-0.01)^5$$

$f(0.99) = -0.03 + 0.0006 + 0$

$f(0.99) =$ ~~+1010131616~~ -0.0294

Q.2) By using Taylor's series expand $\tan^{-1}(x)$ in positive powers of $(x-1)$ up to first four non-zero terms.

Ans → $a = 1$

$$f(x) = \tan^{-1}(x)$$

So $\cancel{\tan^{-1}(x) \neq \tan^{-1}(1) + (x-1)}$

$$f'(x) = \frac{1}{1+x^2}$$

$$f''(x) = \frac{-2x}{(1+x^2)^2}$$

$$f'''(x) = \frac{(1+x^2)^2(-2) + 2x \times 2(1+x^2)(2x)}{(1+x^2)^4}$$

$$= \frac{-2(1+x^2)^2 + 8x^2(1+x^2)}{(1+x^2)^4}$$

$$= \frac{8x^2 - (1+x^2)}{(1+x^2)^3}$$

$$= \frac{7x^2 - 1}{(1+x^2)^3}$$

$$\therefore \tan^{-1}(x) = \tan^{-1}(1) + (x-1)\left(\frac{1}{1+1}\right) + \frac{(x-1)^2}{2!}\left(\frac{-2}{(2)^2}\right) + \frac{(x-1)^3}{3!}\left(\frac{7-1}{2^3}\right)$$

$$= \frac{\pi}{4} + \frac{(x-1)}{2} - \frac{(x-1)^2}{4} + \frac{(x-1)^3}{8}$$

(3.) Arrange in powers of x, by Taylor's theorem $f(x+2) = 7 + (x+2) + 3(x+2)^3 + (x+2)^4$

Ans → $f((x+2))/F|$ $\quad a = \cdot 2$

$$f(x+2) = f(2) + x f'(2) + \frac{x^2}{2!} f''(2) + \frac{x^3}{3!} f'''(2)$$

$\cancel{f(2) = 7 + 4 + 3\times(4)^3 + (4)^4 = 7+4+192+256 = 459}$

$\cancel{f'(2) = 1 + 9(2+2)^2 + 4(4)^3 = 1+144+256 = 401}$

$\cancel{f''(2) =}$

let $x + 2 = y$

$\therefore f(y) = 7 + y + 3y^3 + y^4$

$\therefore f(2) = 7 + 2 + 3\times 8 + 16 = 25 + 24 = 49$

$\therefore f'(2) = 0 + 1 + 9(2)^2 + 4(2)^3 = 1 + 36 + 32 = 69$

$\therefore f''(2) = 18(2) + 12(2)^2 = 36 + 48 = 84$

$\therefore f'''(2) = 18 + 24(2) = 18 + 48 = 66$

$\therefore f''''(2) = 24$

$$\therefore f(y) = f(2) + \frac{(y-2)}{1} f'(2) + \frac{(y-2)^2}{2!} f''(2) + \frac{(y-2)^3}{3!} f'''(2) + \frac{(y-2)^4}{4!} f''''(2)$$

$$\therefore f(y) = 49 + 69(y-2) + 42(y-2)^2 + 11(y-2)^3 + (y-2)^4$$

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$f(x+2) = 49 + 69x + 42x^2 + 11x^3 + x^4$