H.W → 1

Q.1. verify Rolle's theorem for ~~f(x) = (tan x)~~

i) $f(x) = \tan x$, $0 \le x \le \pi$

Ans → $f'(x) = \sec^2 x$

$\therefore$ $f(x)$ is not differentiable for $x = \pi/2$

Hence $f(x)$ is not differentiable and continous

thus, not satisfy rolle's theorem

(ii) $f(x) = \log[(x^2+ab)/(a+b)x]$ in $[a,b]$, $a>0$ and $b>0$.

Ans → $f'(x) = \dfrac{1}{\left(\dfrac{(x^2+ab)}{(a+b)x}\right)} \times \left[\dfrac{(a+b)x \times (2x) - (x^2+ab)(a+b)}{((a+b)x)^2}\right]$

$$f'(x) = \frac{2x^2(a+b) - (x^2+ab)(a+b)}{(x^2+ab)(a+b)x}$$

$$f'(x) = \frac{2x^2 - x^2 - ab}{(x^2+ab)x}$$

$$f'(x) = \frac{x^2 - ab}{(x^2+ab)x}$$

∴ $f(x)$ is differentiable for $x \in R - \{0\}$

Since $x > 0$

∴ $f(x)$ is differentiable and continous

Hence rolle's theorem is satisfied.

Q.2) prove that the eq. $2x^3 - 3x^2 - x + 1 = 0$ has at least one root between 1 and 2.

Ans → let $f'(x) = 2x^3 - 3x^2 - x + 1$ ~~= 0~~

Integrate.

∴ $\int f'(x)\,dx = \int (2x^3 - 3x^2 - x + 1)\,dx$

$$f(x) = \frac{2x^4}{4} - \frac{3x^3}{3} - \frac{x^2}{2} + x + c$$

get $c = 0$

$$\therefore \; f(x) = \frac{x^4}{2} - x^3 - \frac{x^2}{2} + x$$

$$f(1) = \frac{1}{2} - 1 - \frac{1}{2} + 1 = 0$$

$$f(2) = \frac{16}{2} - 8 - \frac{4}{2} + 2 = 8 - 8 - 2 + 2 = 0$$

$$\therefore \; f(1) = f(2)$$

$\therefore$ by rolle's theorem $\quad a < c < b$

$1 < \text{root} < 2$

Q.3) If $f(x) = x(x+1)(x+2)(x+3)$, then show that $f'(x)$ has three real roots.

Ans →

$$f(x) = x(x+3)(x+1)(x+2)$$

$$f(x) = (x^2+3x)(x^2+3x+2)$$

$$f'(x) = (x^2+3x\cancel{+2})(2x+3) + (x^2+3x+2)(2x+3)$$

$$f'(x) = 2x^3 + 3x^2 + 6x^2 + 9x + 2x^3 + 3x^2 + 6x^2 + 9x + 4x + 6$$

$$f'(x) = 4x^3 + 18x^2 + 22x + 6$$

Since $f'(x)$ is a polynomial function

$\therefore$ $f'(x)$ is differential and continous for $x \in R$

$\therefore$ by rolle's theorem

there are 3 real roots

hence proved.

Q.4) Show that between any two roots of $e^x \cdot \cos x - 1 = 0$ there exist at least one root of $e^x \cdot \sin x - 1 = 0$.

Ans → let $f(x) = e^x \cdot \cos x - 1$

$f'(x) = e^x \cdot \cos x - e^x \cdot \sin x$

$f'(x) = e^x \cdot (\cos x - \sin x)$

$\therefore$ $f(x)$ is differentiable and continous for $x \in R$

let a and b be the roots of equation.

$\therefore$ $f(a) = e^a \cdot \cos a - 1$

$f(b) = e^b \cdot \cos b - 1$

$f(a) = f(b)$

$e^a \cdot \cos a - 1 = e^b \cdot \cos b - 1$

$e^a \cdot \cos a = e^b \cdot \cos b$

$e^x \cdot \cos x - 1 = 0$

$\cos x - e^{-x} = 0$

let $f(x) = \cos x - e^{-x}$

$f'(x) = -\sin x + e^{-x}$

let the two roots be a and b

$\therefore$ $f(a) = f(b) = 0$

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$$e^x \cdot \sin x - 1 = 0$$
$$\sin x - e^{-x} = 0$$
$$e^{-x} - \sin x = 0$$

∴ our second eq. is $f'(x)$

∴ by rolle's Theorem one root of $e^x \cdot \sin x - 1 = 0$ lies between root's of $e^x \cdot \cos x - 1 = 0$

5) Apply the rolle's theorem and find the value of c for $f(x) = x^3 - 4x$ in the Interval $[-2, 2]$

Ans →

$$f(x) = x^3 - 4x$$
$$f'(x) = 3x^2 - 4$$

$$f(-2) = -8 + 8 = 0$$
$$f(2) = 8 - 8 = 0$$

Hence rolle's theorem is applicable.

So

$$f'(c) = 0$$
$$3c^2 - 4 = 0$$
$$c^2 = \frac{4}{3}$$
$$c = \pm \frac{2}{\sqrt{3}}$$