dub → maths

Module → 1. calculus

[14|01|2022]

ex → 2

Q.1) verify lagrenge's mean value theorem for the function $f(x) = x^2 + x - 1$ in $[0,4]$

Ans → $f(x) = x^2 + x - 1$

$f'(x) = 2x + 1$

hence the function is differentiable for $x \in R$

∴ The function is differentiable for $x \in [0,4]$ and continous for $x \in (0,4)$

$$\therefore f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f'(c) = \frac{(4)^2 + 4 - 1 - (0 + 0 - 1)}{4 - 0}$$

$$2c + 1 = \frac{16 + 3 + 1}{4}$$

$$2c + 1 = 5$$

$$2c = 4$$

$$c = 2$$

$$\therefore 0 < c < 4$$

∴ Lagrenge's mean value theorem is verified

Q.2) using lagrange's mean value prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ and hence deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$

Ans → consider $f(x) = \tan^{-1} x$ in $[a, b]$ for $a, b > 0$

So $f'(x) = \dfrac{1}{1 + x^2}$ in (a, b)

So $f(x)$ is differentHable on (a, b) and continuous on $[a, b]$

So $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$$\frac{1}{1 + c^2} = \frac{\tan^{-1} b - \tan^{-1} a}{b - a}$$

So $\dfrac{b - a}{1 + b^2} < \tan^{-1} b - \tan^{-1} a < \dfrac{b - a}{1 + a^2}$

$\therefore$ $\dfrac{1}{1 + b^2} < \dfrac{\tan^{-1} b - \tan^{-1} a}{b - a} < \dfrac{1}{1 + a^2}$

$\therefore$ $\dfrac{1}{1 + b^2} < \dfrac{1}{1 + c^2} < \dfrac{1}{1 + a^2}$

$$1 + b^2 > 1 + c^2 > 1 + a^2$$
$$b^2 > c^2 > a^2$$
$$b > c > a$$

hence proved