Module → 1 Sub → Maths

Lecture → 07

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Ex → 7

Q.1) evaluate $\lim\limits_{x \to \frac{1}{2}} \dfrac{\cos^2 \pi x}{e^{2x} - 2xe}$

Ans →

$$\lim_{x \to \frac{1}{2}} \frac{\cos^2 \pi x}{e^{2x} - 2xe}$$

$$\therefore \quad \frac{\cos^2 (\pi/2)}{e^{1} - e} = \frac{0}{0}$$

∴ L- Hospital rule

$$\therefore \quad \lim_{x \to \frac{1}{2}} \frac{2\cos(\pi x) \cdot (-\sin(\pi x)) \cdot \pi}{e^{2x} \cdot 2 - 2e}$$

$$\therefore \quad \lim_{x \to \frac{1}{2}} \frac{-2\pi \cdot \sin(\pi x) \cdot \cos(\pi x)}{2\,(e^{2x} - e)} \Rightarrow \frac{-\pi \cdot \sin(2\pi x)}{2\,(e^{2x} - e)}$$

L - Hospital rule

$$\therefore \quad \lim_{x \to \frac{1}{2}} \frac{-\pi}{2} \frac{[\cos(2\pi x) \cdot \cancel{2}\pi]}{[e^{2x} \cdot \cancel{2}]}$$

$$\therefore \quad \lim_{x \to \frac{1}{2}} \frac{-\pi^2}{2} \frac{\cos(2\pi x)}{e^{2x}}$$

$$\therefore \quad \frac{-\pi^2 \cos(\pi)}{2e} = \frac{\pi^2}{2e}$$

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Q.2) show that $\lim_{x \to 0} \log_{\tan x} \tan 2x = 1$

Ans →

$\therefore \lim_{x \to 0} \log_{\tan x} \tan 2x$

$\therefore \lim_{x \to 0} \frac{\log \tan 2x}{\log \tan x}$

$\therefore \frac{\infty}{\infty}$ form

$\therefore$ by L-Hospital rule

$\therefore \lim_{x \to 0} \frac{\frac{1}{\tan 2x} \times \sec^2(2x) \cdot 2}{\frac{1}{\tan x} \times \sec^2 x}$

$\therefore \lim_{x \to 0} \frac{2 \cdot \sec^2(2x) \cdot \tan x}{\tan 2x \cdot \sec^2 x}$

$\therefore \lim_{x \to 0} \frac{2\tan x \cdot \sec^2(2x)}{\left(\frac{2\tan x}{1-\tan^2 x}\right) \cdot \sec^2 x}$

$\therefore \lim_{x \to 0} \frac{2\tan x \cdot \sec^2(2x)\ (1-\tan^2 x)}{2\tan x \cdot \sec^2 x}$

$\therefore \lim_{x \to 0} \frac{\sec^2(2x)\ (1-\tan^2 x)}{\sec^2 x}$

$\therefore \frac{1\ (1-0)}{1} = 1$

hence proved

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Q.3) Show that $\lim_{x\to\infty}\left[\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+4^{\frac{1}{x}}}{4}\right]^{4x}=24$

$\therefore \lim_{x\to\infty}\left[\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+4^{\frac{1}{x}}}{4}\right]^{4x}$

$\therefore$ It is 1^{∞} form

$\therefore$ let $y=\lim_{x\to\infty}\left[\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+4^{\frac{1}{x}}}{4}\right]^{4x}$

$\therefore \log y=\lim_{x\to\infty} 4x \log\left(\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+4^{\frac{1}{x}}}{4}\right)$

$\therefore \log y=\lim_{x\to\infty}\frac{\log\left(\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+4^{\frac{1}{x}}}{4}\right)}{\left(\frac{1}{4x}\right)}$

$\therefore \frac{0}{0}$ form

L- Hospital rule

$$\therefore \log y=\lim_{x\to\infty}\frac{\frac{\cancel{4}}{\left(1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+4^{\frac{1}{x}}\right)}\times\frac{1}{\cancel{4}}\left(\frac{1^{\frac{1}{x}}\log(1)}{-x^2}+\frac{2^{\frac{1}{x}}\log(2)}{-x^2}+\frac{3^{\frac{1}{x}}\log(3)}{-x^2}+\frac{4^{\frac{1}{x}}\log(4)}{-x^2}\right)}{\frac{1}{4}\frac{(-1)}{x^2}}$$

$$\therefore \log y=\lim_{x\to\infty}\left[\frac{-\frac{1}{\cancel{x^2}}\left(0+2^{\frac{1}{x}}\log(2)+3^{\frac{1}{x}}\log(3)+4^{\frac{1}{x}}\log(4)\right)}{\frac{1}{4}\left(-\frac{1}{\cancel{x^2}}\right)}\right]$$

$\therefore \log y=\cancel{\lim_{x\to\infty}}\ 4\left(\log(2)+\log(3)+\log(4)\right)$

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$\therefore \log y = 4 \log (24)$

$y = 24^4$

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Q-4) prove that $\lim_{x \to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x} = -\frac{e}{2}$

Ans → by using $(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \dots$

$\therefore \lim_{x \to 0} \frac{\left[1 + \frac{x}{x} + \frac{\frac{1}{x}\left(\frac{1}{x}-1\right)x^2}{2!} + \dots\right] - e}{x}$

$\therefore \lim_{x \to 0} \frac{\left[1 + 1 + \frac{(1-x)}{2} + \dots\right] - e}{x}$

by using $(1+x)^{\frac{1}{x}} = e\left[1 - \frac{x}{2} + \frac{11x^2}{24} \dots\right]$

$\therefore \lim_{x \to 0} \frac{\left\{e - \frac{ex}{2} + \frac{11x^2 e}{24} \dots\right\} - e}{x}$

$\therefore \lim_{x \to 0} \frac{-\frac{ex}{2} + \frac{11x^2 e}{24} \dots}{x}$

$\therefore \lim_{x \to 0} -\frac{e}{2} + \frac{11xe}{24} \dots$

$\therefore -\frac{e}{2}$