H.W → 3

$$\frac{\csc}{e^2} = \frac{\sin x}{c^2 \cdot 1}$$

Q.1)

(i) x^2 and x^4 in $[a,b]$ where $a>0$, $b>0$.

Ans→

let $f(x) = x^2$

$f'(x) = 2x$

$g(x) = x^4$

$g'(x) = 4x^3$

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)} \quad \text{and} \quad g(x) \neq 0$$

$$\frac{2c}{4c^3} = \frac{b^2 - a^2}{b^4 - a^4}$$

$$\frac{1}{2c^2} = \frac{1}{b^2 + a^2}$$

$$c^2 = \frac{a^2 + b^2}{2}$$

(ii) $\sin x$ and $\cos x$ in $\left[0, \frac{\pi}{2}\right]$

Ans → $f(x) = \sin x$ $g(x) = \cos x$

         $f'(x) = \cos x$ $g'(x) = -\sin x$

∴ $f'(x)$ is differentiable for   ∴ $g'(x)$ is differentiable for
   $x \in R$ $x \in R$

∴ $f(x)$ is continous on $\left[0, \frac{\pi}{2}\right]$ ∴ $g(x)$ is continous on $\left[0, \frac{\pi}{2}\right]$
  and   differentiable on $\left(0, \frac{\pi}{2}\right)$ and differentiable on $\left(0, \frac{\pi}{2}\right)$

but $g(x)$ is 0   at $x = 0$

So   CMVT   is   not   verified.

(iii) x^2 and x^3 in $[1, 2]$

Ans → $f(x) = x^2$ $g(x) = x^3$

         $f'(x) = 2x$ $g'(x) = 3x^2$

∴ $f'(x)$ is differentiable for   ∴ $g'(x)$ is differentiable for
   $x \in R$ $x \in R$

∴ $f(x)$ is continous on $[1, 2]$ ∴ $g(x)$ is continous on $[1, 2]$
  and differentiable on $\left(0, \frac{\pi}{2}\right)$ and differentiable on $\left(0, \frac{\pi}{2}\right)$

$g(x) \neq 0$ for $x \in [1, 2]$

Hence   CMVT   is   verified.

Q.2) If $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x}$, prove that c of CMVT

is the harmonic mean between a and b.

Ans➔ $f'(x) = -2x^{-3}$　　　　　　　　　$g'(x) = -1\,x^{-2}$

So　$\dfrac{f'(c)}{g'(c)} = \dfrac{f(b) - f(a)}{g(b) - g(a)}$

So　$\dfrac{-2\,c^{-3}}{-c^{-2}} = \dfrac{\frac{1}{b^2} - \frac{1}{a^2}}{\frac{1}{b} - \frac{1}{a}}$

$\dfrac{2}{c} = \dfrac{(a^2 - b^2)\,ab}{(a-b)\,a^2 b^2}$

$\dfrac{2}{c} = \dfrac{a+b}{ab}$

$\dfrac{2ab}{a+b} = c$

hence proved.