Sub → maths

Module → 1

Lecture → 6



ex → 6

Q.1) Show that $\log[1+\sin x] = x - \frac{x^2}{2} + \frac{x^3}{6} + \cdots$

Ans → $f(0) = \log(1) = 0$

$f'(0) = \frac{1}{1+\sin x} \times \cos x = 1$

$\therefore f''(0) = \frac{-1}{1+\sin x} = -1$

$\therefore f'''(0) = -1(-1)(1+\sin x)^{-2}(\cos x) = \frac{\cos x}{(1+\sin x)^2} = 1$

$\therefore f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \cdots$

$\therefore f(x) = 0 + x \cdot 1 + \frac{x^2}{2}(-1) + \frac{x^3}{3!}(1) \cdots$

$\therefore f(x) = x - \frac{x^2}{2} + \frac{x^3}{3!}$

Q.2) expand $\log(1+x+x^2+x^3)$ upto a term in $x^8$.

Ans → $f(x) = \log((1+x) + x(1+x))$

$= \log((1+x)(1+x))$

$= \log(1+x)^2$

$= 2\log(1+x)$

$= 2 \times \left[x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdots\right]$

$\therefore f(x) = 2x - x^2 + \frac{2x^3}{3} - \frac{x^4}{2} + \frac{2x^5}{5} - \frac{x^6}{3} + \frac{2x^7}{7} - \frac{x^8}{4} \cdots$

Q.3 is not there

Q.4) prove that $x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} \cdots$ If $y = x + \frac{x^2}{2} + \frac{x^3}{3} \cdots$

Ans → Given :- $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} \cdots\cdots$

$$y = -\left[-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \cdots\right]$$

$$y = -\log(1-x)$$

$$-y = \log(1-x)$$

$$\therefore \quad e^{-y} = 1-x$$

$$\therefore \quad x = 1 - e^{-y}$$

$$\left\{ e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots \right\}$$

$$\therefore \quad x = 1 - \left[1 - \frac{y}{1!} + \frac{y^2}{2!} - \frac{y^3}{3!} + \frac{y^4}{4!} \cdots\right]$$

$$\therefore \quad x = 1 - 1 + y - \frac{y^2}{2} + \frac{y^3}{3!} - \frac{y^4}{4!} \cdots$$

$$\therefore \quad x = y - \frac{y^2}{2} + \frac{y^3}{3!} - \frac{y^4}{4!} \cdots$$

hence proved.

Q.5). prove that $\sin^{-1} x = x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3\, x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5\, x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \cdots$

Ans → $f(x) = \sin^{-1} x$

$f(0) = \sin^{-1}(0) = 0$

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$$f'(0) = \frac{1}{\sqrt{1-x^2}} = \frac{1}{1} = 1$$

$$f''(0) = \frac{-1}{2(1-x^2)^{3/2}} \times (-2x) = \frac{x}{(1-x^2)^{3/2}} = 0$$

$$f'''(0) = \frac{(1-x^2)^{3/2}(1) - x\left(\frac{3}{2}\right)(-2x)}{(1-x^2)^3} = \frac{(1-x^2)^{3/2} + 3x^2}{(1-x^2)^3}$$

$$= \frac{1}{1} = 1$$

$$\therefore f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) \ldots$$

$$= 0 + x(1) + \frac{x^2(0)}{2!} + \frac{x^3}{3 \times 2} \ldots$$

$$= x + \frac{x^3}{2 \cdot 3} + \ldots$$