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H.W → 2

Q.1)

(i) $f(x) = x^{2/3}$ in $[-8, 8]$

Ans → $f'(x) = \frac{2}{3} x^{-1/3}$

∴ The function is differentiable for $x \in R$

∴ $f(x)$ is continous for $(-8, 8)$ and differentiable for $[-8, 8]$

(ii) $f(x) = 2x^2 - 7x - 10$ over $[2, 5]$ and find c using LMVT

Ans → $f'(x) = 4x - 7$

∴ The function is differentiable for $x \in R$

∴ $f(x)$ is continous for $(2, 5)$ and differentiable for $[2, 5]$

∴ $f'(c) = \frac{f(5) - f(2)}{5 - 2}$

$$4c - 7 = \frac{(2 \times 25 - 7 \times 5 - 10) - (2 \times 4 - 7 \times 2 - 10)}{3}$$

$$4c - 7 = \frac{(50 - 35 - 10) - (8 - 14 - 10)}{3}$$

$$4c - 7 = \frac{5 + 16}{3}$$

$$4c - 7 = 7$$

$$4c = 14$$

$$c = 7/2$$

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Q-2) Apply Lagrange's mean value theorem for the function $\log(x)$ in $[a, a+h]$ & determine θ in terms of a and h and deduce that $0 < 1/(\log(1+x)) - 1/x < 1$

Ans → $f(x) = \log(x)$ $\quad [a, a+h]$

$$f'(x) = \frac{1}{x}$$

∴ function is differenciable for $x \in R - 0$

∴ function is differenciable for $(a, a+h)$ and continous for $[a, a+h]$ if $a \neq 0$.

$$f(a) = f(a+h)$$

$$\log a = \log(a+h)$$

$$\log a - \log(a+h) = 0$$

$$\log\left(\frac{a}{a+h}\right) = 0$$

$$e^0 = \frac{a}{a+h}$$

$$1 = \frac{a}{a+h}$$

$$f'(c) = \frac{f(a+h) - f(a)}{a+h-a}$$

$$\frac{1}{c} = \frac{\log(a+h) - \log(a)}{h}$$

$$\frac{1}{c} = \frac{\log\left(\frac{a+h}{a}\right)}{h}$$

$$\therefore c = \frac{h}{\log\left(\frac{a+h}{a}\right)}$$

$$c = a + \theta h$$

$$\therefore a + \theta h = \frac{h}{\log\left(\frac{a+h}{a}\right)}$$

$$\therefore \frac{a}{h} + \theta = \frac{1}{\log\left(\frac{a+h}{a}\right)}$$

$$\theta = \frac{1}{\log\left(\frac{a+h}{a}\right)} - \frac{a}{h}$$

$$\therefore \theta = \frac{1}{\log\left(\frac{a+h}{a}\right)} - \frac{a}{h}$$

$\therefore \quad 0 < \theta < 1$

$\therefore \quad 0 < \dfrac{1}{\log\left(1+\frac{h}{a}\right)} - \dfrac{a}{h} < 1$

let ~~h = x~~ $a = 1$ and $h = x$

$\therefore \quad 0 < \dfrac{1}{\log(1+x)} - \dfrac{1}{x} < 1$

Q.3) show that $h/(1+h^2) < \tan^{-1} h < h$ when $h \neq 0$ and $h > 0$.

Ans →

$$\boxed{\begin{aligned} \frac{h}{1+h^2} &< \tan^{-1} h < h \\ \frac{1}{1+h^2} &< \frac{\tan^{-1} h}{h} < 1 \\ \frac{1}{1+h^2} &< \frac{\tan^{-1} h - \tan^{-1} 0}{h-0} < 1 \end{aligned}}$$

~~last page~~

let $f(x) = \tan^{-1} x$ for $x \in [0, h]$

$\therefore \quad f'(x) = \dfrac{1}{1+x^2}$

$f'(c) = \dfrac{f(h) - f(0)}{h-0}$

$\dfrac{1}{1+c^2} = \dfrac{\tan^{-1} h - \tan^{-1} 0}{h-0}$

$\therefore$ ~~$\frac{1}{1+h^2}$ //~~

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$\therefore \quad f'(0) > f'(c) > f'(h)$ {since it is a decreasing function}

$\therefore \quad \dfrac{1}{1+0^2} > \dfrac{1}{1+c^2} > \dfrac{1}{1+h^2}$

$$1 > \frac{\tan^{-1} h - \tan^{-1} 0}{h - 0} > \frac{1}{1+h^2}$$

hence proved.