Sub → Maths

Module → 1

lecture → 05

ex → 5

Q.1) expand 5^x upto the first three non-zero terms of the series

Ans → let $f(x) = 5^x$

$\therefore$ $f(0) = 5^0 = 1$

$f'(0) = 5^x \cdot \log 5 = 5^0 \cdot \log 5 = \log 5$

$f''(0) = 5^x \cdot (\log 5)^2 = 5^0 \cdot (\log 5)^2 = (\log 5)^2$

$\therefore$ $5^x = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!}$

$= 1 + x \times \log 5 + \frac{x^2 \cdot (\log 5)^2}{2!}$

Q.2). expand $\sqrt{1+\sin x}$

Ans → let $f(x) = \sqrt{1+\sin x}$

$\therefore$ $f(0) = \sqrt{1+0} = 1$

$\therefore$ $f'(0) = \frac{1}{2} \times (1+\sin x)^{-1/2} (0+\cos x)$

$= \frac{1}{2} \frac{\cos x}{\sqrt{1+\sin x}} = \frac{1}{2} \times \frac{1}{1} = \frac{1}{2}$

$\therefore$ $f''(0) = \frac{1}{2} \left[\frac{\sqrt{1+\sin x} \cdot (-\sin x) - \cos x \times \frac{1}{2} \frac{\cos x}{\sqrt{1+\sin x}}}{1+\sin x} \right]$

$= \frac{1}{2(1+\sin x)} \times \left[-\sin x \cdot \sqrt{1+\sin x} - \frac{1}{2} \frac{\cos^2 x}{\sqrt{1+\sin x}} \right]$

$= \frac{1}{2(1)} \times \left[0 - \frac{1}{2} \right]$

$= -\frac{1}{4}$

$\therefore$ $f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) \ldots\ldots$

$f(x) = 1 + \frac{x}{2} - \frac{x^2}{8} \ldots\ldots$

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Q.3) prove that $\sin h x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$

Ans → let $f(x) = \sin h x$

$\therefore f(0) = \sin 0 = 0$

$\therefore f'(0) = \cos h x \cdot h = \cos 0 \cdot h = h$

$\therefore f''(0) = h.(-\sin h x). h = -h^2 \sin 0 = 0$

$\therefore f'''(0) = -h^2 (\cos h x) \cdot h = -h^3 \cdot \cos 0 = -h^3$

$\therefore f''''(0) = -h^3 \cdot (-\sin h x) \cdot h = h^4 \cdot \sin 0 = 0$

$\therefore f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \frac{x^4 f''''(0)}{4!} \ldots$

$\therefore \cancel{\sin h x = 0 + x \cdot h + \frac{x^2}{2}(0) + \frac{x^3}{6}(-h^3) + 0}$

$\cancel{\sin h x = x h - \frac{x^3 h^3}{6}}$

I had kept that mistake because, most of us will do this like that but $\sin h x$ is not or odinary ~~sin~~ sin function it is a hyperbolic function.

$\therefore \boxed{\sin h x = \frac{e^x - e^{-x}}{2}}$

$\therefore f(x) = \sin h x$

$f(0) = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0$

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$$f'(0) = \frac{1}{2}\frac{d}{dx}\left(e^{x} - e^{-x}\right) = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

$$= \frac{1}{2}(1+1)$$

$$= 1$$

$$f''(0) = \frac{1}{2}\left(e^{x} - e^{-x}\right) = \frac{1}{2}(1-1) = 0$$

$$f'''(0) = \frac{1}{2}\left(e^{x} + e^{-x}\right) = \frac{1+1}{2} = 1$$

$$f''''(0) = \frac{1}{2}\left(e^{x} - e^{x}\right) = 0$$

$$\therefore f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \frac{x^4 f''''(0)}{4!}$$

$$= 0 + x(1) + \frac{x^2(0)}{2!} + \frac{x^3(1)}{3!} + 0 + \cdots$$

$$= x + \frac{x^3}{3!} + \cdots$$