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H.W

Q.1) Show that $\lim_{x \to 0} \log_x \sin x = 1$

Ans → $\therefore \lim_{x \to 0} \log_x \sin x$

$\therefore \lim_{x \to 0} \dfrac{\log \sin x}{\log x}$

$\log 0 = -\infty$ $\qquad \log \sin x = \log 0 = -\infty$

$\therefore \dfrac{\infty}{\infty}$ form

L-Hospital rule.

$\therefore \lim_{x \to 0} \dfrac{\frac{1}{\sin x} \times \cos x}{\frac{1}{x}} \Rightarrow \dfrac{x \cdot \cos x}{\sin x}$

$\therefore \lim_{x \to 0} \dfrac{x}{\tan x}$

$\therefore \dfrac{0}{0}$ form

L- Hospital rule

$\therefore \lim_{x \to 0} \dfrac{1}{\sec^2 x}$

$\therefore \lim_{x \to 0} \cos^2 x$

$\therefore 1$

Q.2) Evaluate $\lim\limits_{x\to 0} \tan x \cdot \log x$

Ans → $\tan 0 = 0$ $\qquad \log 0 = -\infty$

$\therefore$ $0 \times \infty$ form

$\therefore$ $\lim\limits_{x\to 0} \dfrac{\log x}{\cot x}$ now $\dfrac{\infty}{\infty}$ form

$\therefore$ L-Hospital rule

$\therefore \lim\limits_{x\to 0} \dfrac{\frac{1}{x}}{-\text{cosec}^2 x} = \lim\limits_{x\to 0} \dfrac{(-1)}{x} \sin^2 x$ $\left[\frac{0}{0} \text{ form}\right]$

$\therefore$ L-Hospital rule

$\therefore \lim\limits_{x\to 0} \dfrac{-2\sin x \cdot \cos x}{1}$

$\therefore 0$

Q.3) evaluate $\lim\limits_{x\to 0} \left[\dfrac{1}{x^2} - \cot^2 x\right]$

Ans → $\therefore$ $\infty - \infty$ form

$\therefore$ $\cancel{\lim\limits_{x\to 0} \dfrac{1}{x^2}\left[1 - (\cot x)^2\right]}$

$\therefore \lim\limits_{x\to 0} \left[\dfrac{1}{x^2} - \dfrac{1}{\tan^2 x}\right]$

$\therefore \lim\limits_{x\to 0} \left[\dfrac{\tan^2 x - x^2}{x^2 \cdot \tan^2 x}\right]$ $\left[\frac{0}{0} \text{ form}\right]$

by L-Hospital rule

$$\therefore \lim_{x \to 0} \frac{2\tan x . \sec^2 x - 2x}{2x . \tan^2 x + x^2 . 2 . \tan x . \sec^2 x}$$

$$\therefore \lim_{x \to 0} \frac{\tan x \cdot \sec^2 x - x}{x \cdot \tan^2 x + x^2 \cdot \tan x \cdot \sec^2 x}$$

OK is sblm connot be solved by L-Hospital rule

so

$$\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x \right)$$

$$\therefore \lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\tan^2 x} \right)$$

$$\therefore \lim_{x \to 0} \left(\frac{\tan^2 x - x^2}{x^2 . \tan^2 x} \right)$$

$$\therefore \lim_{x \to 0} \frac{\tan^2 x - x^2}{x^4 \cdot \left(\frac{\tan^2 x}{x^2} \right)}$$

$$\therefore \lim_{x \to 0} \frac{\tan^2 x - x^2}{x^4} \quad \left[\frac{0}{0} \text{ form} \right]$$

L-Hosipital rule

$$\therefore \lim_{x \to 0} \frac{2\tan x \cdot \sec^2 x - 2x}{4x^3} \quad \left[\frac{0}{0} \text{ form} \right]$$

L-Hosipital rule

$$\therefore \lim_{x \to 0} \frac{2 \cdot \sec^2 x \cdot \sec^2 x + 2\tan x \cdot 2 \cdot \sec x \cdot \sec x \cdot \tan x - 2}{12x^2}$$

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$\therefore \lim_{x \to 0} \dfrac{2 \cdot \sec^4 x + 4 \cdot \tan^2 x \cdot \sec^2 x - 2}{12x^2}$ $\left[\frac{0}{0} \text{ form}\right]$

L - Hospital rule

$\therefore \lim_{x \to 0} \dfrac{16 \tan x \cdot \sec^4 x + 8 \tan^3 x \cdot \sec^2 x}{24x}$ $\left[\frac{0}{0} \text{ form}\right]$

L - Hospital rule

$\therefore \lim_{x \to 0} \dfrac{16 \sec^6(x) + 24 \tan^2 x \cdot \sec^2 x + 64 \tan^2 x \cdot \sec^4 x + 40 \cdot \tan^4 x \cdot \sec^2 x}{24}$

$\therefore \dfrac{16}{24} = \dfrac{4}{6} = \dfrac{2}{3}$

Q.4) evaluate $\lim_{x \to 0} \dfrac{\sin^{-1} x - x}{x^3}$

Ans → $\cancel{\frac{0}{0} \text{ form}}$

~~L - Hospital - rule~~

$\cancel{\therefore \lim_{x \to 0} \dfrac{\frac{1}{\sqrt{1-x^2}} - 1}{3x^2}}$

$\lim_{x \to 0} \dfrac{\left\{x + \frac{1 x^3}{2 \cdot 3} + \frac{1 \cdot 3 x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} x^7 \cdots \right\} - x}{x^3}$

$\therefore \lim_{x \to 0} \dfrac{\frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3 x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5 \; x^7}{2 \cdot 4 \cdot 6 \cdot 7} \cdots}{x^3}$

$\therefore \lim_{x \to 0} \dfrac{1}{2 \cdot 3} + \dfrac{1 \cdot 3 x^2}{2 \cdot 4 \cdot 5} + \dfrac{1 \cdot 3 \cdot 5 x^4}{2 \cdot 4 \cdot 6 \cdot 7} + \cdots$

$\therefore \frac{1}{6}$

Q.5) evaluate $\lim_{x \to 0} \frac{\sin x \cdot \sin^{-1} x - x^2}{x^6}$

Ans → $\lim_{x \to 0} \frac{1}{x^5}\left(\frac{\sin x \cdot \sin^{-1} x - x}{x} \right)$

$\therefore \lim_{x \to 0} \frac{\sin^{-1} x - x}{x^5}$

$\therefore \lim_{x \to 0} \frac{\left\{ x + \frac{1}{2 \cdot 3} x^3 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} x^5 + \frac{1 \cdot 3 \cdot 5 x^7}{2 \cdot 4 \cdot 6 \cdot 7} \cdots \right\} - x}{x^5}$

$\therefore \lim_{x \to 0}$

$\therefore \lim_{x \to 0} \frac{\left\{ x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots \right\} \left\{ x + \frac{1 x^3}{2 \cdot 3} + \frac{1 \cdot 3 x^5}{2 \cdot 4 \cdot 5} \cdots \right\} - x^2}{x^6}$

$\therefore \lim_{x \to 0} \frac{\left\{ 1 - \frac{x^2}{3!} + \frac{x^4}{5!} \cdots \right\} \left\{ 1 + \frac{x^2}{2 \cdot 3} + \frac{1 \cdot 3 x^4}{2 \cdot 4 \cdot 5} \cdots \right\} - 1}{x^4}$

$\therefore \lim_{x \to 0} \frac{\left\{ \cancel{1} + \cancel{\frac{1 x^2}{2 \cdot 3}} + \frac{1 \cdot 3 x^4}{2 \cdot 4 \cdot 5} - \cancel{\frac{x^2}{3!}} - \frac{x^4}{3! \cdot 2 \cdot 3} + \frac{x^4}{5!} \cdots \right\} - \cancel{1}}{x^4}$

$\therefore \lim_{x \to 0} \left(\frac{1 \cdot 3 x^4}{2 \cdot 4 \cdot 5} - \frac{x^4}{3 \cdot 2 \cdot 2 \cdot 3} + \frac{x^4}{5!} \cdots \right) \frac{1}{x^4}$

$\therefore \frac{3}{40} - \frac{1}{36} + \frac{1}{120} \Rightarrow \frac{1}{18}$