H.W → 4

Q1) Expand $\tan^{-1}(x+h)$ in powers of h and hence find the value of $\tan^{-1}(1.003)$ upto 5 places of decimal.

Ans → $f(x+h) = \tan^{-1}(x+h)$

let $x+h = y$

$\therefore \; f(y) = \tan^{-1}(y)$

let $a = x$

$$f(y) = f(x) + (y-x)f'(x) + \frac{(y-x)^2}{2!}f''(x) + \frac{(y-x)^3}{3!}f'''(x) \text{ - - - - -}$$

$\therefore \; 1.003 = y$

$1 + 0.003 = y$

$\frac{1}{1+x^2}$

$x = 1$

$= \frac{-2x}{(1+x^2)^2}$

$h = 0.003$

$$\therefore \tan^{-1}(1.003) = \tan^{-1}(1) + (0.003)\left(\frac{1}{1+1}\right) + (0.003)^2\left(\frac{-2}{(1+1)^2}\right)$$

$$= \frac{\pi}{4} + \frac{0.003}{2} + \frac{0.000006(-2)}{4}$$

$$= 0.78689$$

Q.2> Using Taylor's theorem evaluate upto 4 places of decimals.

(i) $\sqrt{1.02}$

Ans → $y = 1.02$

$f(x+h) = \sqrt{x+h}$

$x = 1$

$h = 0.02$

let $x+h = y$, $y = 1.02$

$$\therefore f(y) = f(x) + (y-h) f'(x) + \frac{(y-h)^2}{2!} f''(x)$$

$$\therefore f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f''(x) = \frac{-1}{4x^{3/2}}$$

$$\therefore f(1.02) = f(1) + (0.02) f'(1) + \frac{(0.02)^2}{2!} f''(1)$$

$$= 1 + 0.02 \times \frac{1}{2} + \frac{(0.02)^2}{2} \times \left(\frac{-1}{4}\right)$$

$$= 1 + 0.01 - \frac{0.0004}{8}$$

$$= 1 + 0.01 - 0.00005$$

$$= 1.00995$$

(ii) $\sqrt{10}$

Ans: $f(x+h) = \sqrt{9+1}$

$x = 9$

$h = 1$

$y = x + h = 10$

$\therefore f(y) = \sqrt{y}$

$a = 9$

$$\therefore f(y) = f(a) + (y-a) f'(a) + \frac{(y-a)^2}{2} f''(a)$$

$$\cancel{f(y) = f(1) + 9 f'(1) + \frac{81}{2} f''(1)}$$

$$f(10) = f(9) + 1 \times f'(9) + \frac{1}{2} f''(9)$$

$$f(10) = 3 + \frac{1}{2\sqrt{9}} - \frac{1}{2} \times \frac{1}{4 \times 3^3}$$

$$f(10) = 3 + \frac{1}{6} - \frac{1}{216}$$

$$= 3 + 0.1667 - 0.00463$$

$$= 3.16207$$

$f(0) = \log(\cos 0) = \log(1) = 0$

$f'(0) = \frac{1}{\cos 0} \times (-\sin 0) = 0$

$f''(0) = -\sec^2 0 = -1$

$f'''(0) = -2\sec 0 \times \operatorname{cosec}$ X

2 3 don't know

Q.4) Expand $(2x^3 + 7x^2 + x - 1)$ in powers of $(x-2)$.

Ans→ $a = 2$

$f(x) = 2x^3 + 7x^2 + x - 1$

$f(2) = 2(8) + 7(4) + 2 - 1 = 16 + 28 + 1 = 45$

$f'(2) = 6(2)^2 + 14(2) + 1 = 24 + 28 + 1 = 53$

$f''(2) = 12(2) + 14 = 24 + 14 = 38$

$f'''(2) = 12$

$f''''(2) = 0$

$\therefore f(x) = f(2) + \frac{(x-2)}{1!} f'(2) + \frac{(x-2)^2}{2!} f''(2) + \frac{(x-2)^3}{3!} f'''(2)$

$f(x) = 45 + 53(x-2) + \frac{(x-2)^2}{2} \times 38 + \frac{(x-2)^3}{6} \times 12$

$f(x) = 45 + 53x - 106 + 14(x-2)^2 + 2(x-2)^3$