H.W → 4

Q.1) Expand $\tan^{-1}(x+h)$ in powers of h and hence find the value of $\tan^{-1}(1.003)$ upto 5 places of decimal.

Ans → $f(x+h) = \tan^{-1}(x+h)$

let $x+h = y$

$\therefore \quad f(y) = \tan^{-1}(y)$

let $a = x$

$\therefore \quad \cancel{f(y) \neq f(h) + (y-h)\, f'(h)/}$

$$f(y) = f(x) + (y-x)f'(x) + \frac{(y-x)^2}{2!}f''(x) + \frac{(y-x)^3}{3!}f'''(x) \; - - - - -$$

$\therefore \quad 1.003 = y$

$1 + 0.003 = y$

$\frac{1}{1+x^2}$

$x = 1$

$= \frac{-2x}{(1+x^2)^2}$

$h = 0.003$

$$\therefore \tan^{-1}(1.003) = \tan^{-1}(1) + (0.003)\left(\frac{1}{1+1}\right) + (0.003)^2\left(\frac{-2}{(1+1)^2}\right)$$

$$= \frac{\pi}{4} + \frac{0.003}{2} + \frac{0.000006(-2)}{4}$$

$= 0.78689$

Q.2) Using Taylor's theorem evaluate upto 4 places of decimals.

(i) $\sqrt{1.02}$

Ans → $y = 1.02$

$f(x+h) = \sqrt{x+h}$

$x = 1$

$h = 0.02$

let $x + h = y$, $y = 1.02$

$\therefore f(y) = f(x) + (y-h) f'(x) + \dfrac{(y-h)^2}{2!} f''(x)$

$\therefore f(x) = \sqrt{x}$

$f'(x) = \dfrac{1}{2\sqrt{x}}$

$f''(x) = \dfrac{-1}{4x^{3/2}}$

$\therefore f(1.02) = f(1) + (0.02) f'(1) + \dfrac{(0.02)^2}{2!} f''(1)$

$= 1 + 0.02 \times \dfrac{1}{2} + \dfrac{(0.02)^2}{2} \times \left(\dfrac{-1}{4}\right)$

$= 1 + 0.01 - \dfrac{0.0004}{8}$

$= 1 + 0.01 - 0.00005$

$= 1.00995$

(ii) $\sqrt{10}$

Ans $f(x+h) = \sqrt{9+1}$

$x = 9$

$h = 1$

$y = x + h = 10$

$\therefore f(y) = \sqrt{y}$

$a = 9$

$$\therefore f(y) = f(a) + (y-a) f'(a) + \frac{(y-a)^2}{2} f''(a)$$

$$\cancel{f(y) = f(1) + 9 f'(1) + \frac{8x}{2} f''(1)}$$

$$f(10) = f(9) + 1 \times f'(9) + \frac{1}{2} f''(9)$$

$$f(10) = 3 + \frac{1}{2\sqrt{9}} - \frac{1}{2} \times \frac{1}{4 \times 3^3}$$

$$f(10) = 3 + \frac{1}{6} - \frac{1}{216}$$

$$= 3 + 0.1667 - 0.00463$$

$$= 3.16207$$

3

$f(0) = \log(\cos 0) = \log(1) = 0$

$f'(0) = \frac{1}{\cos 0} \times (-\sin 0) = 0$

$f''(0) = -\sec^2 0 = -1$

$f'''(0) = -2\sec 0 \times \operatorname{cosec}$ X

Q.4) Expand $(2x^3+7x^2+x-1)$ in powers of $(x-2)$.

Ans→ $a=2$

$f(x) = 2x^3+7x^2+x-1$

$f(2) = 2(8)+7(4)+2-1 = 16+28+1 = 45$

$f'(2) = 6(2)^2+14(2)+1 = 24+28+1 = 53$

$f''(2) = 12(2)+14 = 24+14 = 38$

$f'''(2) = 12$

$f''''(2) = 0$

$\therefore f(x) = f(2) + \frac{(x-2)}{1!}f'(2) + \frac{(x-2)^2}{2!}f''(2) + \frac{(x-2)^3}{3!}f'''(2)$

$f(x) = 45 + 53(x-2) + \frac{(x-2)^2}{2} \times 38 + \frac{(x-2)^3}{6} \times 12$

$f(x) = 45 + 53x - 106 + 14(x-2)^2 + 2(x-2)^3$

2 : dont know

Q.5 Same as Q. 3

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Q.6) If $y = \sin \log(x^2 + 2x + 1)$, Expand y in ascending powers of x upto x^6.

Ans→ $y = \sin\left(\log(x+1)^2\right)$

$y = \sin\left(2\log(x+1)\right)$

let $m = x + 1$

$\therefore \quad f(m) = \sin(2\log m)$

$a = 1$

$\therefore f(m) = f(1) + (m-1) f'(1) + \dfrac{(m-1)^2}{2} f''(1) + \dfrac{(m-1)^3}{3!} f'''(1) \dots\dots$

$f(1) = 0$

$f'(1) = \cos(2\log 1) \cdot 2 \cdot \dfrac{1}{(1)} = 2$

$f''(1) = 2 \times \left(\dfrac{-\sin(2\log 1) \cdot 2\left(\frac{1}{m}\right) - \cos(2\log 1)}{m^2} \right)$

$= 2 \times \dfrac{-1}{1}$

$= -2$

$\vdots$

$f''''''(1)$

we have to find all.