Sub → MATHS 13|1|2022

Module → 1. Calculus - I

Ex. 1

Q.1) Verify Rolle's Theorem:

i) $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$

Ans →

If a function is differentiable, then It is also continous

$\therefore f'(x) = \frac{e^x \cdot \cos x - e^x \cdot \sin x}{e^{2x}}$

$f'(x) = \frac{\cos x - \sin x}{e^x}$

$\therefore$ The function is differentiable for $x \in [0, \pi]$

$f(0) = \frac{\sin 0}{e^0} = 0$

$f(\pi) = \frac{\sin \pi}{e^\pi} = 0$

$\therefore f(0) = f(\pi)$

$\therefore$ hence rolle's Theorem is Justified.

ii) $f(x) = 1 - 3(x-1)^{2/3}$ in $0 \leq x \leq 2$

Ans → $f'(x) = 0 - 3(x-1)^{-1/3} \times \left(\frac{2}{3}\right)$

$f'(x) = -2(x-1)^{-1/3}$

at $x = 1$

~~$\therefore$ The Function is differentiable for $x \in R$~~

$f'(x) = \frac{-2}{0}$ Undefined

hence It is not differentiable and continous.

$$\therefore f(0) = 1 - 3(0-1)^{2/3}$$
$$= 1 - 3(-1)^{2/3}$$
$$= 1 - 3$$
$$= -2$$

$$\therefore f(2) = 1 - 3(2-1)^{2/3}$$
$$= 1 - 3(1)^{2/3}$$
$$= 1 - 3$$
$$= -2$$

$$\therefore f(0) = f(2)$$

hence rolle's Theorem is not Justified.

iii) $f(x) = |x|$ in $[-1, 1]$

Ans → $f(x) = -x$ for $[-1, 0]$
$f(x) = x$ for $[0, 1]$

L.H.D
$[-1, 0]$
$f(x) = -x \quad [-1, 0]$

$f'(x) = -1$

hence It is differentiable

R.H.D
$[0, 1]$
$f(x) = x \quad [0, 1]$

$f'(x) = 1$

hence It is differentiable

$\therefore$ L.H.D $\neq$ R.H.D

$\therefore f(x) = |x|$ is not continous and differentiable.

$f(-1) = |-1| = 1$
$f(1) = |1| = 1$
$\therefore f(-1) = f(1)$

hence Rolle's theorem is not Justified.

iv) $f(x) = \begin{cases} x^2+1 & , 0 \le x \le 1 \\ 3-x & , 1 \le x \le 2 \end{cases}$

Ans → (x^2+1) and $(3-x)$ is a polynomial function, hence they are differentiable.

L.H.D for $x[0,1]$

$F(x) = x^2+1$

$f'(x) = 2x$

R.H.D for $x[0,1]$

$f(x) = 3-x$

$f'(x) = -1$

$\therefore$ L.H.D $\neq$ R.H.D

Hence the function is not differenciable

$\therefore$ rolle's theorem is not Justified.

Q.2) Use Rolle's Theorem to prove that the equation $ax^2+bx = \frac{a}{3}+\frac{b}{2}$ has a root between 0 and 1.

Ans →

$$ax^2+bx = \frac{a}{3}+\frac{b}{2}$$

$$ax^2+bx-\frac{a}{3}-\frac{b}{2} = 0$$

$$6ax^2+6bx-2a-3b = 0$$

$\therefore$ let $f'(x) = 6ax^2+6bx-2a-3b$

{we are considering $f'(x)$ and not $f(x)$ because we will proved it by reverse of rolle's theorem}

$$\int f'(x)\,dx = \int (6ax^2 + 6bx - 2a - 3b)\,dx$$

$$f(x) = \frac{6ax^3}{3} + \frac{6bx^2}{2} - 2ax - 3bx + c$$

$$f(x) = 2ax^4 + 3bx^2 - 2ax - 3bx + c$$

let C be 0.

$$f(0) = 0 + 0 - 0 - 0 + 0$$

$$f(0) = 0$$

$$f(1) = 2a + \cancel{3b} - 2a - \cancel{3b}$$

$$= 0$$

$\therefore$ $f(0) = f(1)$

$\therefore$ by using rolle's theorem $0 \leq c < 1$