Module → 1. Calculus

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Q.1) If $f(x) = \sqrt{x}$ and $g(x) = 1/\sqrt{x}$, then prove that c of CMVT is geometric mean between 'a' and 'b'

Ans → $f(x) = \sqrt{x}$ $\quad g(x) = \frac{1}{\sqrt{x}}$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad g'(x) = \frac{-1}{2} x^{-3/2}$

$f(x)$ is differentiable for $x \in R - \{0\}$ $\quad g(x)$ is differentiable for $x - R - \{0\}$

∴ $f(x)$ is continous on $[a,b]$ and differentiable on (a,b) $\quad$ ∴ $g(x)$ is continous on $[a,b]$ and differentiable on (a,b)

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

$$\frac{1}{2\sqrt{c}} \times \frac{2c^{3/2}}{(-1)} = \frac{\sqrt{b} - \sqrt{a}}{\frac{1}{\sqrt{b}} - \frac{1}{\sqrt{a}}}$$

$$-c = \frac{(\sqrt{b} - \sqrt{a})\sqrt{a}\cdot\sqrt{b}}{(\sqrt{a} - \sqrt{b})}$$

$$c = \sqrt{a}\cdot\sqrt{b}$$

$$c = a \cdot b$$

hence proved.

Q.2) using appropriate MVT prove that $\frac{\sin b - \sin a}{e^b - e^a} = \frac{\cos c}{e^c}$ for $a < c < b$. Hence deduce that $e^c \cdot \sin x = (e^x - 1)\cos c$.

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Ans→ here RMVT, LMVT or CMVT ~~which~~ be can choose what to use.

let $f(x) = \sin x$ $\quad g(x) = e^x$

$f'(x) = \cos x$ $\quad g'(x) = e^x$

$$\therefore \frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

$$\frac{\cos c}{e^c} = \frac{\sin b - \sin a}{e^b - e^a}$$

hence proved.