H.W.

Q.1) If $u = x^3 y + e^{xy^2}$, determine $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$

Ans → $\frac{\partial u}{\partial x} = 3yx^2 + e^{xy^2} \times y^2$

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$$\frac{\partial u}{\partial y} = x^3 + e^{xy^2}(x \cdot 2 \cdot y)$$

$$= x^3 + 2e^{xy^2} \cdot x \cdot y$$

Q.2) If $u(x+y) = x^2+y^2$, then show that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$

Ans → $ux + uy = x^2 + y^2 \quad \ldots\ldots (i)$

$\therefore \frac{\partial}{\partial x}(ux + uy) = \frac{\partial}{\partial x}(x^2+y^2)$

$\therefore u + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = 2x + 0$

$\therefore \frac{x^2+y^2}{(x+y)} + (x+y)\frac{\partial u}{\partial x} = 2x$

$\therefore \boxed{\frac{\partial u}{\partial x} = \frac{2xy + x^2 - y^2}{(x+y)^2}} \quad \ldots (ii)$

$\therefore ux + uy = x^2 + y^2$

$\therefore \frac{\partial}{\partial y}(ux + uy) = \frac{\partial}{\partial y}(x^2+y^2)$

$\therefore x\frac{\partial u}{\partial y} + u + y\frac{\partial u}{\partial y} = 2y$

$\therefore \boxed{\frac{\partial u}{\partial y} = \frac{2yx - x^2 + y^2}{(x+y)^2}}$

$$L.H.S = \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2$$

$$= \left(\frac{2xy + x^2 - y^2}{(x+y)^2} - \frac{(2yx - x^2 + y^2)}{(x+y)^2}\right)^2$$

$$= \left(\frac{\cancel{2xy} + x^2 - y^2 - \cancel{2xy} + x^2 - y^2}{(x+y)^2}\right)^2$$

$$= \left(\frac{2(x^2 - y^2)}{(x+y)^2}\right)^2 = 4\frac{(x^2-y^2)^2}{(x+y)^4} = \frac{4(x+y)^2(x-y)^2}{(x+y)^4}$$

$$= \frac{4(x-y)^2}{(x+y)^2}$$

$$R.H.S = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= 4\left(1 - \left(\frac{2xy + x^2 - y^2}{(x+y)^2}\right) - \frac{(2xy - x^2 + y^2)}{(x+y)^2}\right)$$

$$\cancel{= 4\left(\frac{x + y - 2x + x^2 + y^2 - 2y + x^2 + y^2}{x+y}\right)}$$

$$\cancel{= 4\left(\frac{2x^2 + 2y^2 - x - y}{x+y}\right)}$$

$$= \frac{4}{(x+y)^2} \times \left[\cancel{x^2} + y^2 + \cancel{2xy} - 2xy - \cancel{x^2} + \cancel{y^2} - \cancel{2xy} + x^2 - \cancel{y^2}\right]$$

$$= \frac{4}{(x+y)^2} \times \left[x^2 + y^2 - 2xy\right]$$

$$= \frac{4(x^{\cancel{2}} - y^{\cancel{2}})^2}{(x+y)^2}$$

Question Incorrect hai R.H.S ka square nahi print hua hai

Q.3) If $u = \log\left(\frac{x}{y}\right)$, then find $u_x + u_y$

Ans → $u_x = \frac{\partial}{\partial x} \log\left(\frac{x}{y}\right) = \frac{\partial}{\partial x}(\log x - \log y) = 0 + \frac{1}{x}$

$u_y = \frac{\partial}{\partial y} \log\left(\frac{x}{y}\right) = \frac{\partial}{\partial y} \log\left(\frac{x}{y}\right) - \log y = -\frac{1}{y}$

∴ $u_x + u_y = \frac{1}{x} - \frac{1}{y} = \frac{y - x}{xy}$