Module . 2 .

Subject → MATHS

Lecture → 8

ex → 8

Q.1) evaluate $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ for $u = e^{x^y}$

Ans → $\frac{\partial u}{\partial x} = \frac{\partial e^{x^y}}{\partial x} = e^{x^y} \times x^{y-1} \times y$

$\frac{\partial u}{\partial y} = \frac{\partial e^{x^y}}{\partial y} = e^{x^y} \cdot x^y \cdot \log x$

Q.2) If $u = (1-2xy+y^2)^{-\frac{1}{2}}$, then show that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$

Ans → $\frac{\partial u}{\partial x} = -\frac{1}{2}(1-2xy+y^2)^{-\frac{3}{2}} \times (0-2y+0)$

$\therefore \frac{\partial u}{\partial x} = y(1-2xy+y^2)^{-\frac{3}{2}}$

$\therefore \frac{\partial u}{\partial y} = -\frac{1}{2}(1-2xy+y^2)^{-\frac{3}{2}} \times (0-2x+2y)$

$\therefore \frac{\partial u}{\partial y} = (x-y)(1-2xy+y^2)^{-3/2}$

$\therefore x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = x\left[y(1-2xy+y^2)^{-3/2}\right] - y\left[(x-y)(1-2xy+y^2)^{-3/2}\right]$

$= y(1-2xy+y^2)^{-3/2}\left[x-(x-y)\right]$

$= y(1-2xy+y^2)^{-3/2}[+y]$

$= y^2(1-2xy+y^2)^{-3/2} = y^2u^3$

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Q.3) If $u = \log(\tan x + \tan y + \tan z)$, then show that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$

Ans → $\frac{\partial u}{\partial x} = \frac{1}{(\tan x + \tan y + \tan z)} \times (\sec^2 x)$

$\frac{\partial u}{\partial y} = \frac{1}{(\tan x + \tan y + \tan z)} \times (\sec^2 y)$

$\frac{\partial u}{\partial z} = \frac{1}{(\tan x + \tan y + \tan z)} \times (\sec^2 z)$

$\therefore \left[\sin 2x\ (\sec^2 x) + \sin 2y\ (\sec^2 y) + \sin 2z\ (\sec^2 z)\right] \times \frac{1}{(\tan x + \tan y + \tan z)}$

$\therefore \left(2 \sin x \cdot \cos x \cdot (\sec^2 x) + 2 \sin y \cdot \cos y \cdot \sec^2 y + 2 \sin z \cdot \cos z \cdot (\sec^2 z)\right) \times \frac{1}{(\tan x + \tan y + \tan z)}$

$\therefore (2\tan x + 2\tan y + 2\tan z) \times \frac{1}{\tan x + \tan y + \tan z}$

$\therefore \frac{2(\tan x + \tan y + \tan z)}{\tan x + \tan y + \tan z}$

$\therefore 2$