Option Pricing via Monte Carlo simulations

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Monte Carlo Methods

Context

- a broad class of computational algorithms
  - repeated random sampling
- increasing utility value with increasing complexity
  - can be difficult to capture all variables with deterministic models

Principle

- essentially sample mean
- provides an unbiased estimate of the entity being estimated
  - the entity can be a deterministic one (includes the expectation of a stochastic entity)
- "The Law of Large Numbers" ensures convergence to true value

Notation

$r \triangleq$ risk free interest rate

$T \triangleq$ time to maturity

$S \triangleq$ stock Price

$\mu \triangleq$ drift of stock (expected increment)

$\sigma \triangleq$ volatality of the stock

$W \triangleq$ geometric brownian motion

Computing S_t

The stock price can be modeled as the following Ito process:

$$dS = S\mu dt + S\sigma dW(t)$$

Prerequisites

- given that this is an Ito's Process, one may choose to employ Ito's Lemma
- one may wish to formulate this as a Markov chain and then use the Euler-Maruyama approximation

Project Summary

Code

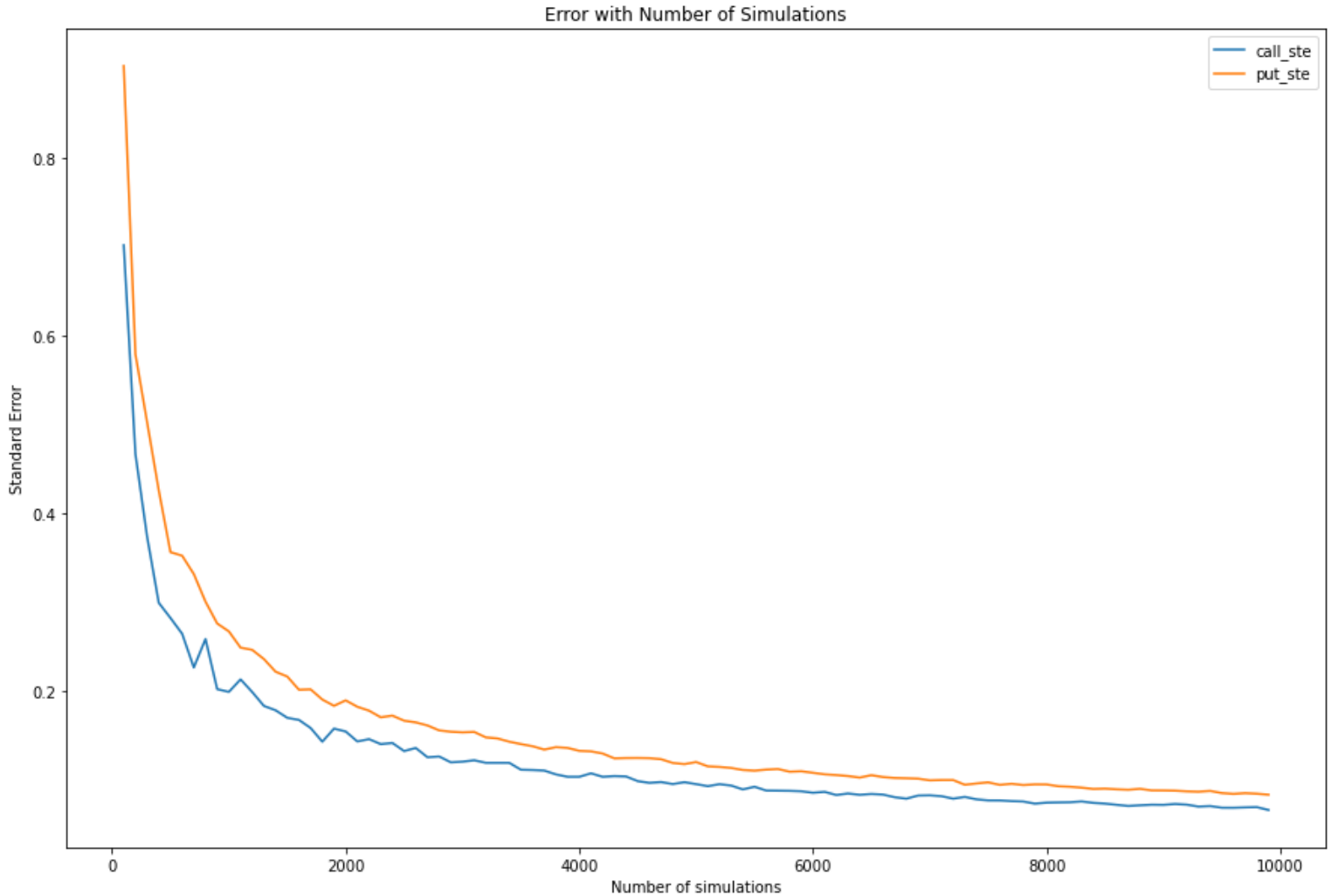
- pricer for European options
  - Black-Scholes as a baseline
  - tested antithetic variate

Experiments

- Effect of Number of Simulations
- Effect of Granularity
- Effect of Antithetic Paths

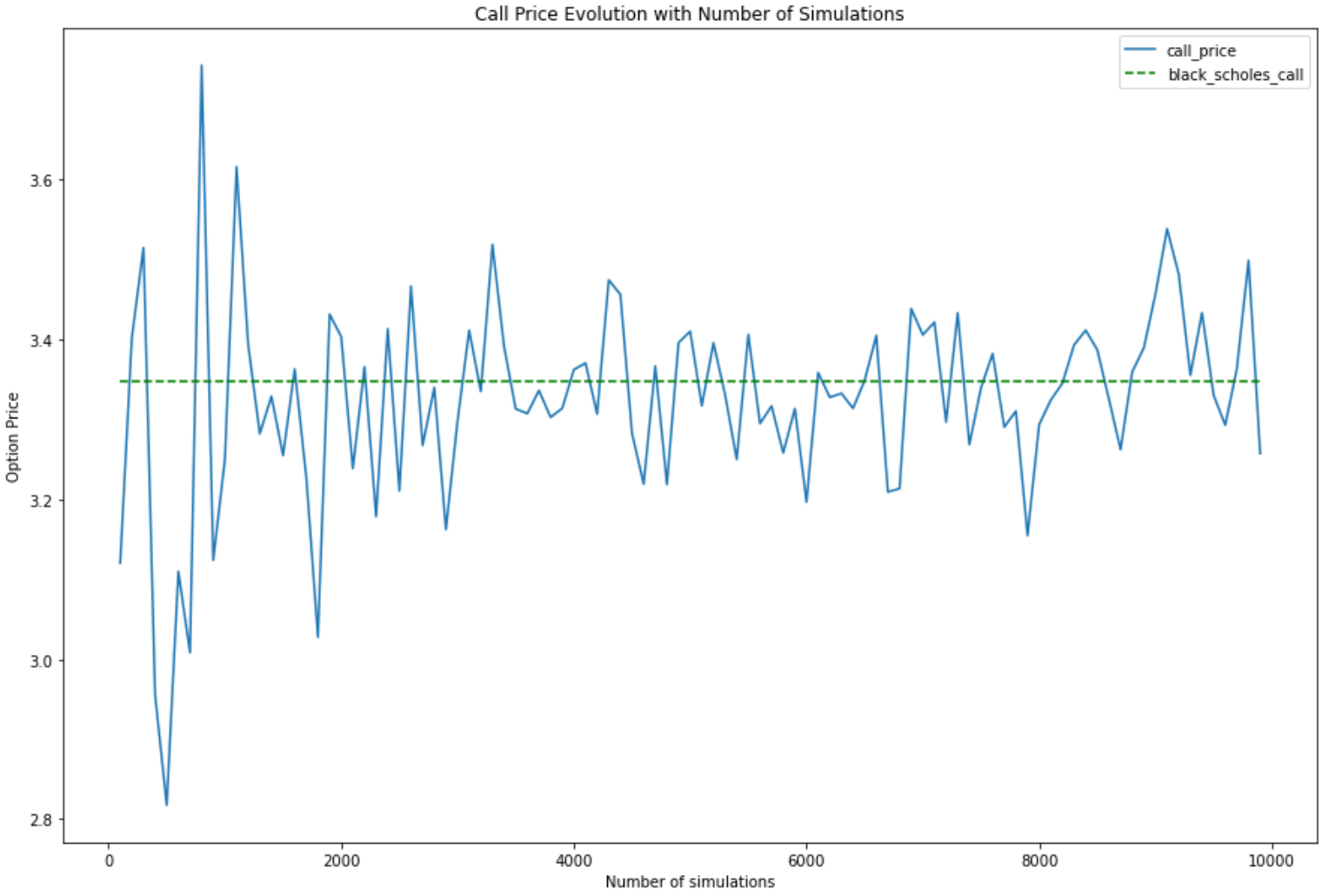
Number of simulations

Reduction in standard error



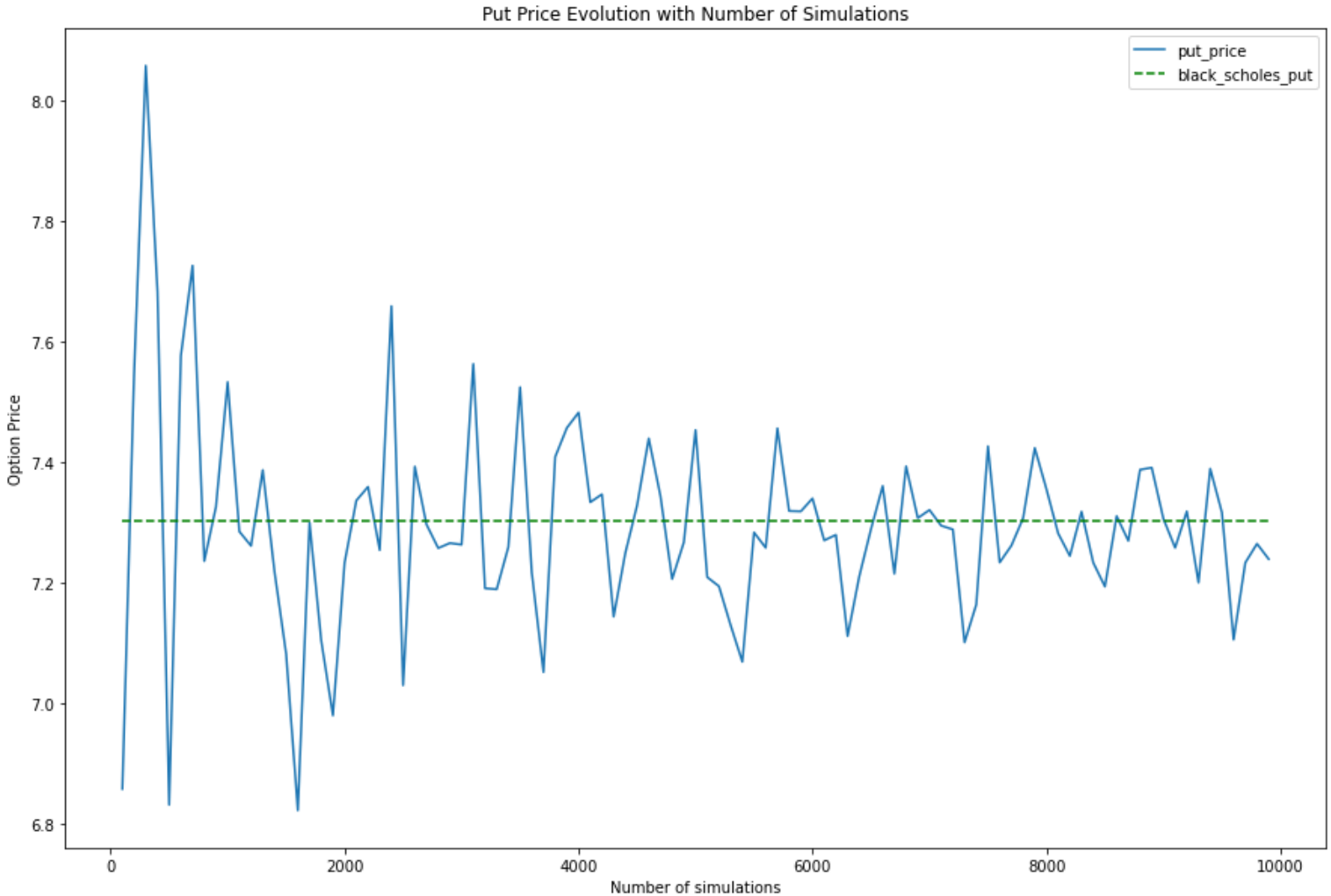
Number of simulations

Convergence towards Black-Scholes (call)



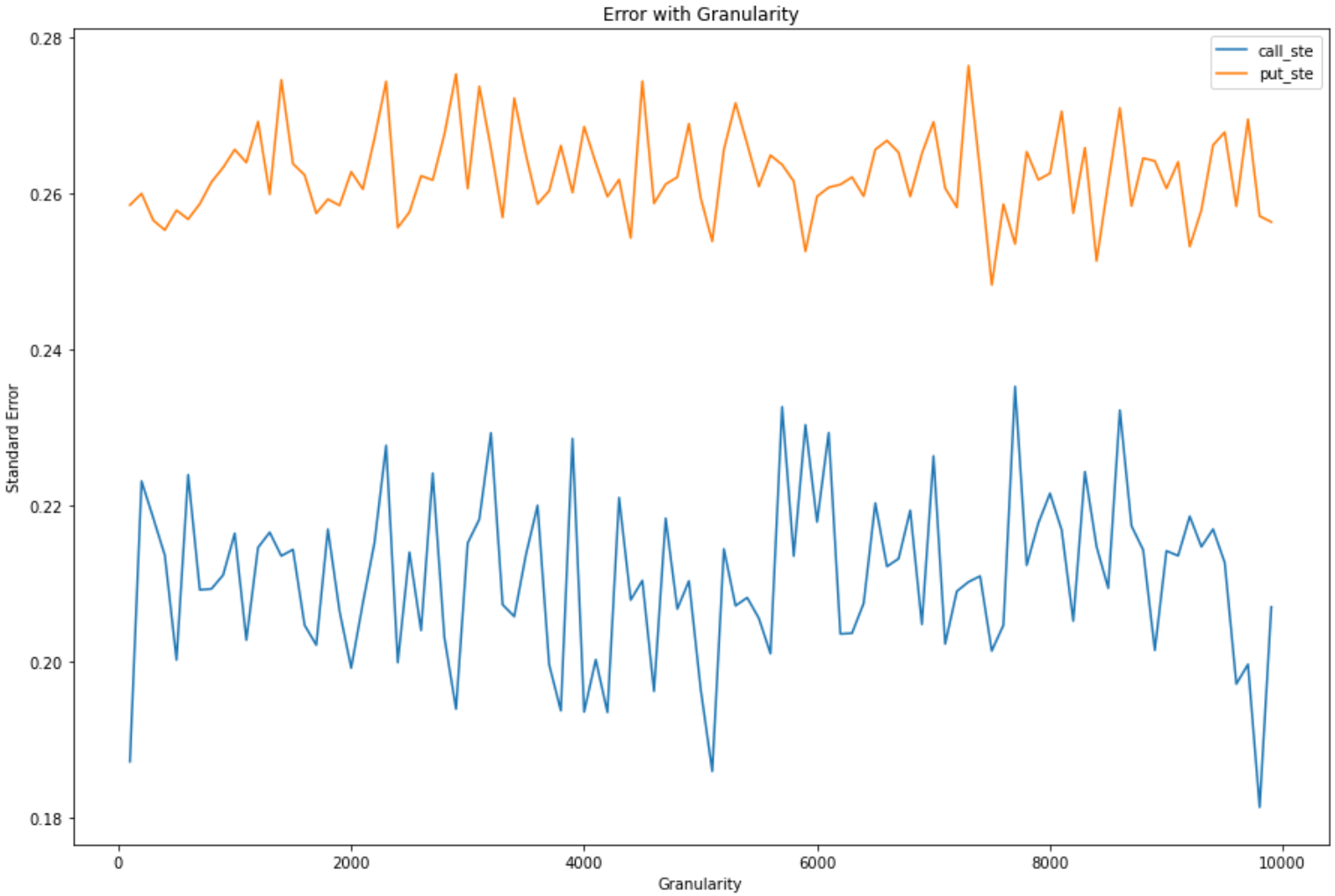
Number of simulations

Convergence towards Black-Scholes (put)



Variation with Granularity

No effect on standard error



Antithetic Variate

Principle

- for every sampling session of Brownian motion, the antithetic path (should occur with equal probability) is also considered under the estimate
  - original : $\{\epsilon_1, \epsilon_2, \cdots, \epsilon_M\}$
  - antithetic : $\{-\epsilon_1, -\epsilon_2, \cdots, -\epsilon_M\}$
- do note that this will only invert the effect of the volatility term and not the one corresponding to the drift of the stock

Advantage

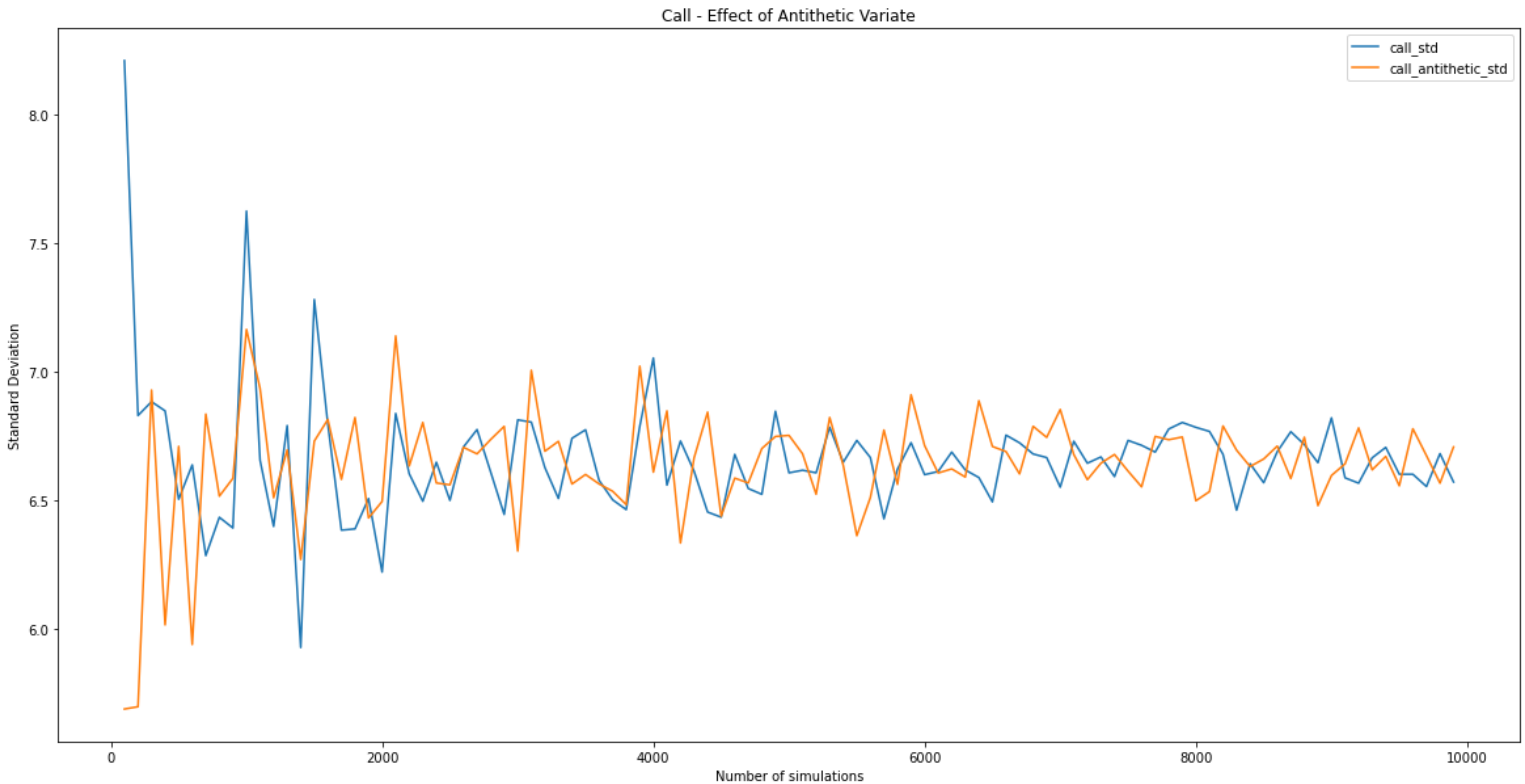
- lesser number of samples needed to gain a certain amount of usable paths
- reduces the variance of the estimate

Effect of Antithetic Paths

Lower variance for lesser simulation

- observation tapers off for large number of simulations

Call



Effect of Antithetic Paths

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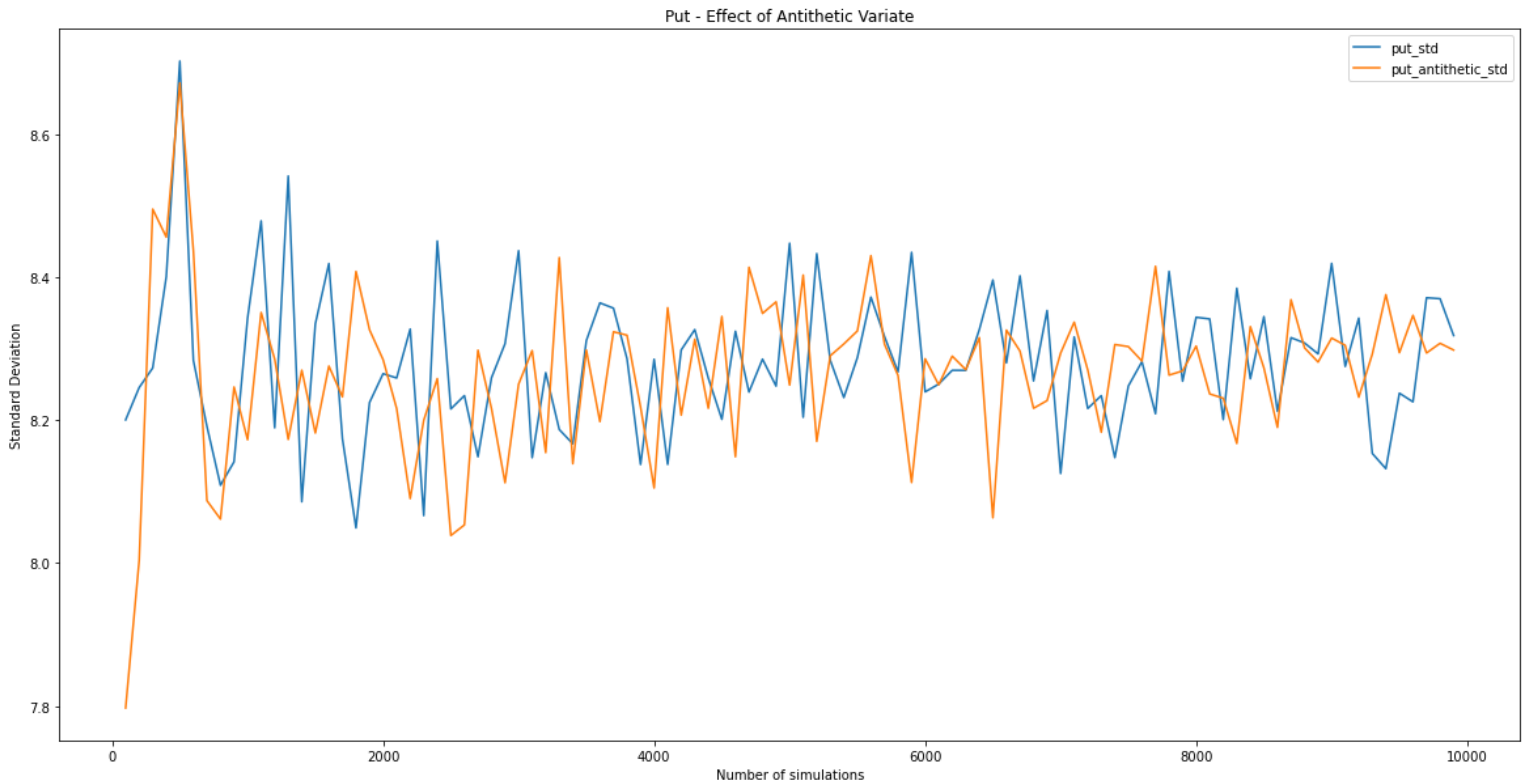
Put



Conclusion

- Closed form solutions are only available for relatively simple derivatives
- Complicated derivatives can only be bounded by closed form approaches
  - Monte Carlo pricing can be applied with ease
    - simply the expectation of the discounted value of the derivative
  - some examples being:
    - Up and Out, Down and Out, Asian, American

Issues/Aspects

- Variance reduction
  - explored one solution (Antithetic Paths) but several other variants target this issue
- Compute
  - not really an issue these days