CS5280 : THEORY A1

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Concurrency Control in Transactional Systems
Theory Assignment 1

1 EX 2.1 (pg 56)

given Σ is a set of finite alphabets, Σ^* being the set of all corresponding words. for

$$v, w \in \Sigma^*, v \leq w \iff \text{v is a prefix of w}$$

Proceeding to show that $\leq$ is a partial order over Σ^* by showing the following three properties for the corresponding relations in $\leq \subset \Sigma^* \times \Sigma^*$

for facilitating explanations, defining an infix function

$$+ : \Sigma^* \times \Sigma^* \rightarrow \Sigma^*$$

such that, for words x, y

$$x + y = append(x, y)$$

i.e. y is appended to the end of x

note that $+$ is associative but not commutative

now,note that

$$x \leq y \iff \exists z \in \Sigma^*, (y = x + z)$$

1.1 Reflexivity

Trivially, any word v is a prefix of itself as

$$v = v + \epsilon$$

where ϵ is the null word

$$\therefore (v, v) \in \leq$$

1.2 Anti-Symmetry

given $(x \leq y) \wedge (y \leq x)$

$$\exists a, b \in \Sigma^*, (y = x + a) \wedge (x = y + b)$$

$$\therefore x = (x + a) + b$$

$$\therefore x = x + (a + b)$$

$$\therefore a = b = \epsilon$$

$$\therefore x = y + b = y + \epsilon$$

$$\therefore x = y$$

1.3 Transitivity

given $(x \leq y) \wedge (y \leq z)$

$$\exists a, b \in \Sigma^*, ([y = x + a] \wedge [z = y + b])$$

$$\therefore z = (x + a) + b$$

$$\therefore z = x + (a + b)$$

$$\therefore (x, z) \in \leq$$

2 EX 3.1 (pg 120)

given

$$s = r_1(x)r_2(y)w_1(y)r_3(z)w_3(z)r_2(x)w_2(z)w_1(x)c_1c_2c_3$$

$$s' = r_3(z)w_3(z)r_2(y)r_2(x)w_2(z)r_1(x)w_1(y)w_1(x)c_3c_2c_1$$

$$D = \{x, y, z\}$$

assuming t_0 and t_∞ to be fictitious initializer and final state accounter for a schedule s $H[s] : D \rightarrow HU$ where HU is the Herbrand universe of s is given by the Herbrand semantics of the final writes of the data objects

2.1 s

2.1.1 $H[s]$

$$\begin{aligned} H[s](x) &= H_s(w_1(x)) = f_{1x}(H_s(r_1(x))) = f_{1x}(H_s(w_0(x))) \\ &= f_{1x}(f_{0x}()) \end{aligned}$$

$$\begin{aligned} H[s](y) &= H_s(w_1(y)) = f_{1y}(H_s(r_1(x))) = f_{1y}(H_s(w_0(x))) \\ &= f_{1y}(f_{0x}()) \end{aligned}$$

$$\begin{aligned} H[s](z) &= H_s(w_2(z)) = f_{2z}(H_s(r_2(x)), H_s(r_2(y))) = f_{2z}(H_s(w_0(x)), H_s(w_0(y))) \\ &= f_{2z}(f_{0x}(), f_{0y}()) \end{aligned}$$

2.1.2 $RF(s)$

$$\begin{aligned} RF(s) = \{&(t_0, x, t_1), (t_0, y, t_2), \\ &(t_0, z, t_3), (t_0, x, t_2), \\ &(t_1, x, t_\infty), (t_1, y, t_\infty), \\ &(t_2, z, t_\infty)\} \end{aligned}$$

2.1.3 $LRF(s)$

$$\begin{aligned} LRF(s) = \{&(t_0, x, t_1), (t_0, y, t_2), \\ &(t_0, x, t_2), (t_1, x, t_\infty), \\ &(t_1, y, t_\infty), (t_2, z, t_\infty)\} \end{aligned}$$

2.2 s’

2.2.1 $H[s']$

$$\begin{aligned} H[s'](x) &= H_{s'}(w_1(x)) = f_{1x}(H_{s'}(r_1(x))) = f_{1x}(H_{s'}(w_0(x))) \\ &= f_{1x}(f_{0x}()) \end{aligned}$$

$$\begin{aligned} H[s'](y) &= H_{s'}(w_1(y)) = f_{1y}(H_{s'}(r_1(x))) = f_{1y}(H_{s'}(w_0(x))) \\ &= f_{1y}(f_{0x}()) \end{aligned}$$

$$\begin{aligned} H[s'](z) &= H_{s'}(w_2(z)) = f_{2z}(H_{s'}(r_2(x)), H_{s'}(r_2(y))) = f_{2z}(H_{s'}(w_0(x)), H_{s'}(w_0(y))) \\ &= f_{2z}(f_{0x}(), f_{0y}()) \end{aligned}$$

2.2.2 $RF(s')$

$$\begin{aligned} RF(s') = \{&(t_0, z, t_3), (t_0, y, t_2)\\ &(t_0, x, t_2), (t_0, x, t_1)\\ &(t_1, x, t_\infty), (t_1, y, t_\infty)\\ &(t_2, z, t_\infty)\} \end{aligned}$$

2.2.3 $LRF(s')$

$$\begin{aligned} RF(s') = \{&(t_0, y, t_2), (t_0, x, t_2),\\ &(t_0, x, t_1), (t_1, x, t_\infty),\\ &(t_1, y, t_\infty), (t_2, z, t_\infty)\} \end{aligned}$$

3 EX 3.2 (pg 120)

given

$$s = r_1(x)r_3(x)w_3(y)w_2(x)c_3r_4(y)w_4(x)c_2r_5(x)c_4w_5(z)w_1(z)c_1c_5$$

and the information that the writes corresponding to transactions t_3 and t_4 are copiers.

Computing the Semantics for s (referred to as H'(s) as these are not exactly Herbrand semantics because we now have information regarding the application layer)

$$D = \{x, y, z\}$$
$$H'[s] : D \to HU$$

where HU is the Herbrand universe generated by collating the Herbrand

Semantics of the individual $t_i \in trans(s)$

$$\begin{aligned}
H'[s](x) &= H_s(w_4(x)) = f_{4x}(H_s(r_4(y))) = f_{4x}(H_s(w_3(y))) = f_{4x}(f_{3y}(H_s(r_3(x)))) \\
&= f_{4x}(f_{3y}(H_s(w_0(x)))) = f_{4x}(f_{3y}(f_{0x}())) \\
&= f_{4x}(f_{0x}()) \\
&= f_{0x}()
\end{aligned}$$

$$\begin{aligned}
H'[s](y) &= H_s(w_3(y)) = f_{3y}(H_s(r_3(x))) = f_{3y}(H_s(w_0(x))) = f_{3y}(f_{0x}()) \\
&= f_{0x}()
\end{aligned}$$

$$\begin{aligned}
H'[s](z) &= H_s(w_1(z)) = f_{1z}(H_s(r_1(x))) = f_{1z}(H_s(w_0(x))) \\
&= f_{1z}(f_{0x}())
\end{aligned}$$