STAT 270 Assignment 3

Upload to Crowdmark by 5pm on Monday, June 29, 2020

Instructions: Create separate file(s) for each question (in .jpg, .pdf, or .png format) for uploading to Crowdmark.

1. (12 marks) The random variables X and Y have joint pdf given by

$$f_{XY}(x,y) = \lambda x e^{-x(y+\lambda)}, \;\; x, y \geq 0$$

   a. (4 marks) Calculate the joint cdf of X and Y.
   b. (2 marks) Verify that $f_{XY}(x,y)$ is a valid pdf.
   c. (2 marks) Calculate the marginal pdf of X.
   d. (3 marks) Calculate the conditional probability that $Y > y$ given that $X = x$.
   e. (1 mark) In which family of distributions is the distribution of Y given X?

2. (11 marks) The length of time, X, taken by a student to submit an exam (in hours) has pdf given by

$$f_X(x) = x e^{-\frac{x^2}{2}}, \;\; x \geq 0$$

   a. (4 marks) Calculate the expected time taken to submit the exam. **HINT**: Find a way to frame the problem in terms of the integral of the right half of a N(0,1) pdf.
   b. (5 marks) Calculate the median time taken to submit the exam.
   c. (2 marks) Calculate the probability that a student takes between 1 and 2 hours to submit the exam.

3. (8 marks) The random variables X and Y have joint pdf given by

$$f_{XY}(x,y) = x e^{-xy}, \;\; 1 \leq x \leq 2, y \geq 0$$

   a. (2 marks) Show that X and Y are not independent.
   HINT: $f_Y(y) = \frac{1}{y}e^{-y} - \frac{2}{y}e^{-2y} - \frac{1}{y^2}e^{-2y} + \frac{1}{y^2}e^{-y}$

   b. (6 marks) Calculate the covariance of X and Y. **HINT**: $E[Y] = \log(2)$.

4. (4 marks) Suppose that the lengths of a particular cell phone model follow a normal distribution with mean 5.6 cm and variance 0.003 cm^2.
   a. (1 mark) What is the 80th percentile of cell phone lengths?
   b. (1 mark) What percentage of cell phones have length less than 5.5 cm?
   c. (2 marks) If the manufacturer wants to keep the mean length at 5.6 cm but adjust the variance so that only 1% of the lengths are less than 5.55 cm, how small do they need to make the new variance?

5. (8 marks) I roll a fair, 6-sided die. Let X be the outcome of my roll. For a given value $X = x$, I then roll the die x more times and define Y as the number of 6's that I observe. E.g., if I roll a 3 initially (so that $X = 3$), I will then roll the die 3 more times and define Y as the number of 6's that I observe (0, 1, 2, or 3). The outcomes of all my rolls are independent.

   a. (2 marks) What is the conditional probability mass function of Y given X?
   b. (2 marks) What is the joint probability mass function of X and Y?
   c. (4 marks) What is the expected value of Y? **HINT**: You may use the fact that the expected value of a Binomial(n, p) random variable is np.