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Topological Black Holes in Anti-de Sitter Space

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Abstract

We consider a class of black hole solutions to Einstein's equations in d dimensions with a negative cosmological constant. These solutions have the property that the horizon is a $(d-2)$-dimensional Einstein manifold of positive, zero, or negative curvature. The mass, temperature, and entropy are calculated. Using the correspondence with conformal field theory, the phase structure of the solutions is examined, and used to determine the correct mass dependence of the Bekenstein-Hawking entropy.

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1 Introduction

Recently, a great deal of attention has been paid to a conjectured correspondence between string theory (and supergravity) defined on direct products of d-dimensional anti-de Sitter space with compact manifolds, and the large N limit of certain conformal field theories defined on the $(d-1)$-dimensional boundary of the anti-de Sitter space [1]. This followed earlier investigations of scattering from branes [2]-[6], and near-horizon brane geometry [7, 8]. The precise nature of this correspondence was explored in [9, 10].

In [10, 11], this correspondence was used to study the boundary conformal field theory at finite temperature. In such a case, one is studying the $SU(N)$ gauge theory at large N on a manifold $S^1 \times M^{d-2}$. The relevant configurations on the anti-de Sitter side of the correspondence are then black hole solutions to Einstein's equations with a negative cosmological constant. In particular, the case of $M^{d-2} = S^{d-2}$ was studied, using the black hole solutions with spherical horizon constructed in [12]. The thermodynamic properties of these solutions were discussed in [13]. Given the importance of this conjecture for understanding the large N dynamics of gauge theory, it is useful to study several examples. According to the formulation presented in [10], the partition function of the conformal field theory at finite temperature, defined on $S^1 \times M^{d-2}$, is given by summing the exponential of the supergravity action over Einstein manifolds with negative cosmological constant which have boundary $S^1 \times M^{d-2}$.

In this paper, we consider a class of static black hole solutions to Einstein's equations in d dimensions with a negative cosmological constant, which have the property that the horizon M^{d-2} is a $(d-2)$-dimensional compact Einstein space of positive, zero, or negative curvature. These solutions are obtained by a straightforward generalization of the four-dimensional ansatz presented in [14]-[16]. However, we point out that the d-dimensional generalization of this ansatz only requires the horizon to be Einstein. Among these solutions, one has a subclass consisting of black holes which are asymptotically locally anti-de Sitter; in this case, the horizon has constant curvature. We examine the horizon structure, and compute the mass, temperature, and entropy. Within the framework of the adS/CFT correspondence, these solutions allow us to study the conformal field theory at finite temperature defined on manifolds $S^1 \times M^{d-2}$. We investigate the phase structure of these solutions in the light of this correspondence, and show how this leads to a microscopic derivation of the correct mass dependence of the Bekenstein-Hawking entropy formula.

2 Construction of Black Hole Solutions

Black hole solutions to Einstein's equations with a negative cosmological constant and with spherical horizon topology were constructed in [12], and their thermodynamic properties were were investigated in [13]. In four dimensions, black holes for which the topology of the horizon is an arbitrary genus Riemann surface have been constructed in [14]-[19]; the thermodynamics of these solutions has been considered [15, 16]. This followed earlier work on the planar and toroidal case [20]-[22]. Higher-dimensional constant curvature black holes with negative cosmological constant were obtained in [23, 24].

Our goal here is to consider the generalization to d dimensions of the metric ansatz presented in [14]-[16]. In order to construct solutions to Einstein's equations with a negative cosmological constant, we begin with the metric ansatz

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 h_{ij}(x)dx^i dx^j, \tag{1}$$

where the coordinates are labelled as $x^\mu = (t, r, x^i), (i = 1, ..., (d-2))$. The metric function h_{ij} is a function of the coordinates x^i only, and we shall refer to this metric as the horizon metric. We take the horizon to be a compact orientable manifold denoted by M^{d-2}. Adopting the curvature conventions of [25], we determine the non-vanishing components of the Ricci tensor to be

$$\begin{aligned}
R_{tt} &= \frac{1}{2}ff'' + \frac{1}{2r}(d-2)ff', \\
R_{rr} &= -\frac{1}{2}\frac{f''}{f} - \frac{1}{2r}(d-2)\frac{f'}{f}, \\
R_{ij} &= R_{ij}(h) - h_{ij}[(d-3)f + rf'],
\end{aligned} \tag{2}$$

where $R_{ij}(h)$ is the Ricci tensor of the horizon metric, and $f' = df/dr$. Let us now take the function f to be of the form

$$f = k - \frac{\omega_d M}{r^{d-3}} + \frac{r^2}{l^2}, \tag{3}$$

where

$$\omega_d = \frac{16\pi G}{(d-2)\mathrm{Vol}(\mathrm{M}^{\mathrm{d}-2})}, \tag{4}$$

and $\mathrm{Vol}(\mathrm{M}^{\mathrm{d}-2}) = \int \mathrm{d}^{\mathrm{d}-2}\mathrm{x}\,\sqrt{\mathrm{h}}$. Here, k is as yet undetermined; l is a parameter with dimensions of length, and ω_d is inserted for convenience so that the parameter M has dimensions of inverse length. This metric ansatz is the d-dimensional generalization of that given in [14]-[16]. With this form of f, one can now check that the spacetime is an Einstein space with negative cosmological constant, namely

$$R_{\mu\nu} = -\frac{(d-1)}{l^2} g_{\mu\nu}, \tag{5}$$

provided the horizon is an Einstein space of the form

$$R_{ij}(h) = (d-3)k\; h_{ij}. \tag{6}$$

It is important to observe here that we have obtained a solution to Einstein's equations with a negative cosmological constant for any value of k, provided the horizon is itself Einstein. However, the horizon may be an Einstein space with positive, zero, or negative curvature. This opens up the possibility to construct black hole solutions in which the topology of the horizon is non-spherical. In particular, compact Einstein spaces of non-constant curvature exist provided $(d-2) > 3$, see [26], for example.

However, for the moment let us continue and analyse the structure of the Riemann tensor of these solutions. The non-vanishing components are given by

$$\begin{aligned}
R_{trtr} &= \frac{1}{2}f'', \quad R_{titj} = \frac{r}{2}ff'\; h_{ij}, \quad R_{rirj} = -\frac{r}{2}\frac{f'}{f}\; h_{ij}, \\
R_{ijkl} &= r^2 R_{ijkl}(h) - r^2 f[h_{ik}\; h_{jl} - h_{il}\; h_{jk}],
\end{aligned} \tag{7}$$

where $R_{ijkl}(h)$ is the Riemann tensor constructed from the horizon metric h_{ij}. We see that $M = 0$ solution is locally isometric to anti-de Sitter space (i.e., a spacetime of constant negative curvature),

$$R_{\mu\nu\rho\sigma} = -\frac{1}{l^2}(g_{\mu\rho}\, g_{\nu\sigma} - g_{\mu\sigma}\, g_{\nu\rho}), \tag{8}$$

provided that the horizon is itself a constant curvature space

$$R_{ijkl}(h) = k(h_{ik}\, h_{jl} - h_{il}\, h_{jk}). \tag{9}$$

Thus, imposing the extra requirement that the $M = 0$ solution be a constant curvature spacetime, forces the horizon to be a constant curvature space, and not simply Einstein. However, once again there is no restriction on the sign of k. Although the $M = 0$ spacetime is locally isometric to anti-de Sitter space, its topology depends on the value of the k, and hence on the topology of the horizon. In particular, we have the three possibilities of elliptic horizons ($k = 1$), flat horizons ($k = 0$), and hyperbolic horizons ($k = -1$). The hyperbolic $M = 0$ solution appeared in [23]. We note from (3) that the dominant behaviour of the metric at infinity is determined by the cosmological constant term, for any value of M. Since the $M = 0$ solution is locally isometric to anti-de Sitter space, we have a class of black hole solutions which are asymptotically locally anti-de Sitter, for all values of M.

An instructive example is to consider the situation in five dimensions. In this case, the horizon is a three-dimensional compact manifold of constant curvature, which may be either positive, negative, or zero. Now, it is known that any compact, constant curvature, 3-manifold M^3 can be expressed as a quotient space, see [27] for example,

$$M^3 = \tilde{M}^3/\Gamma, \tag{10}$$

where the universal covering space $\tilde{M}^3$ is either the 3-sphere (corresponding to $k = 1$), the torus ($k = 0$), or hyperbolic 3-space ($k = -1$), and Γ is a discrete subgroup of the isometry group of $\tilde{M}$. Thus, even in the elliptic case, we have non-spherical possibilities; for example, one may take the horizon to be a lens space.

To summarize, we have shown that the metric ansatz (1) and (3) defines an Einstein spacetime with negative cosmological constant, and parameter M, provided that the horizon is itself an Einstein space of positive, zero, or negative curvature. The subclass of these solutions which are asymptotically locally anti-de Sitter is obtained when the horizon is a constant curvature space.

3 Black Hole Thermodynamics

In this section, we discuss the properties of the solutions obtained, and in particular their interpretation as black holes, and the corresponding thermodynamics.

To implement the black hole interpretation, we wish to restrict the parameters so that the metric describes the exterior of a black hole with a non-degenerate horizon. This is achieved provided the polynomial $r^{d-3}l^2 f$ has a simple positive root r_+, such that $f(r) > 0$ for all $r > r_+$. For $k = 1$, and spherical topology, these solutions correspond to those considered in [12, 13].

We simply note here that the $k = 1$ construction is not restricted to spherical topology, but only require the horizon to be Einstein. Thus, the thermodynamic analysis of [13] applies to all the $k = 1$ solutions.

For $k = 0$, one can check that for $M > 0$ there is always a simple positive root of $r^{d-3}l^2 f$ given by

$$r_+ = (\omega_d M l^2)^{\frac{1}{d-1}}. \tag{11}$$

In addition, $f(r) > 0$ for $r > r_+$. Thus, for $k = 0$, we have black hole solutions with toroidal topology.

The analysis for the case of $k = -1$ is more involved. The four-dimensional case was studied in [14]-[19], where it was shown that these solutions do indeed have a black hole interpretation with the horizon being a Riemann surface of genus greater than one. We content ourselves here by studying the five-dimensional case. This also has particular relevance for the correspondence with conformal field theory in four dimensions, which we shall discuss. In fact, in this case, one notes that we must only solve a quadratic equation

$$(r^2)^2 - l^2 r^2 - \omega_5 M l^2 = 0. \tag{12}$$

It is convenient to define

$$r_{\text{crit}} = \frac{l}{\sqrt{2}}, \quad M_{\text{crit}} = -\frac{l^2}{4\omega_5}. \tag{13}$$

Now if $D = l^4 + 4\omega_5 M l^2 = 0$, we see that $M = M_{\text{crit}}$, and there is a double zero, with no black hole interpretation. If $D < 0$, which corresponds to $M < M_{\text{crit}}$, there is no real zero. However, for $D > 0$, i.e., $M > M_{\text{crit}}$, there are two possibilities. Firstly, for $D > 0$ and $M_{\text{crit}} < M < 0$, there are two distinct simple zeros. For $D > 0$ and $M \geq 0$, there is one simple zero. Hence, in these cases, we have non-degenerate horizons with hyperbolic geometry. The parameter M is then determined by $f(r_+) = 0$, as

$$M = \frac{r_+^2}{\omega_5}\left(-1 + \frac{r_+^2}{l^2}\right). \tag{14}$$

We see that M_{crit} corresponds to M evaluated at r_{crit}. In addition, one notes that we can write

$$r^2 l^2 f = (r^2 - r_+^2)(r^2 + r_+^2 - l^2). \tag{15}$$

Since $r_+ > r_{\text{crit}}$, we see that f is positive for all $r > r_+$. We remark that the seven-dimensional case also allows an explicit analysis, since one must only solve a cubic equation in r^2.

Our aim now is to calculate the action of these black hole solutions, and determine their thermodynamical properties. We may consider any one of the above solutions which has an acceptable horizon located at r_+. The parameter M is specified in terms of r_+ as

$$M = \frac{r_+^{d-3}}{\omega_d}\left(k + \frac{r_+^2}{l^2}\right). \tag{16}$$

The Euclidean Einstein action is proportional to the spacetime volume, namely

$$I = -\frac{1}{16\pi G}\int d^d x\, \sqrt{g}(R - 2\Lambda) = \frac{(d-1)}{8\pi G l^2}\int d^d x\, \sqrt{g}, \tag{17}$$

where the cosmological constant is $\Lambda = -\frac{(d-1)(d-2)}{2l^2}$. Since I is infinite, we proceed in the standard way and compare the action of the black hole with a convenient background [28]. We note that the boundary terms which are typically present in the action give zero contribution for the cases under consideration. Since we are working in the Euclidean formalism, we must identify the imaginary time of the solution with a period $\beta = 4\pi/f'(r_+)$, namely

$$\beta = \frac{4\pi l^2 r_+}{(d-1)r_+^2 + (d-3)kl^2}. \tag{18}$$

It is worth highlighting some of the features of these black holes which depend on the value of k. For $k = 1$, it has been shown in [13] that the inverse temperature β given by (18) has a maximum value. Hence, the black hole solutions only exist for temperatures greater than a certain minimum value. The background geometry is taken to be anti-de Sitter space with arbitrary Euclidean time period. However, for the case of $k = 0$ and $k = -1$, it is easy to check from (18) that there is no such minimum temperature; hence, the $k = 0$ and $k = -1$ black holes exist for all temperatures. For $k = 0$, the background can be taken to be the $M = 0$ solution [16, 29]. However, one does notice that for $k = -1$, the requirement of positivity of temperature enforces an inequality on the value of r_+, namely that $r_+ > r_{\text{crit}}$, where

$$r_{\text{crit}} = \left(\frac{d-3}{d-1}\right)^{\frac{1}{2}} l. \tag{19}$$

Notice also that when $r_+ = r_{\text{crit}}$, we have $M = M_{\text{crit}}$, where

$$M_{\text{crit}} = -\left(\frac{2}{d-1}\right)\left(\frac{d-3}{d-1}\right)^{\frac{d-3}{2}} \frac{l^{d-3}}{\omega_d}. \tag{20}$$

Thus, the requirement of $r_+ > r_{\text{crit}}$ is equivalent to the requirement that $M > M_{\text{crit}}$, which is needed in order to have a black hole interpretation. Since the M_{crit} spacetime can be identified with arbitrary Euclidean time period, we choose this as the background geometry [16, 29].

The background spacetime is denoted by X_1 with period β_0 and parameter M_{crit}, where M_{crit} is given by (20) for $k = -1$, and is zero for $k = 0, 1$. It should be stressed that since f has a degenerate zero for the M_{crit} spacetime with $k = -1$, the spacetime has an internal infinity [15, 16]. As a result, the Killing horizons do not have an interpretation as black hole horizons. However, for the purposes of our analysis here, we are restricting attention to the spacetime region $r \geq r_{\text{crit}}$, where r_{crit} is given by (19). The asymptotic boundary of this spacetime is then $S^1 \times M^{d-2}$. The black hole spacetime is denoted by X_2 with period β_0', and an r-integration $r \geq r_+$. To proceed further, we match β_0 and β_0' so that the geometry on the hypersurface at $r = R$ agrees. This is achieved by taking

$$\beta_0\sqrt{k - \frac{\omega_d M_{\text{crit}}}{r^{d-3}} + \frac{r^2}{l^2}} = \beta_0'\sqrt{k - \frac{\omega_d M}{r^{d-3}} + \frac{r^2}{l^2}}. \tag{21}$$

Hence, the difference in the actions is

$$I \equiv I(X_2) - I(X_1) = \frac{\mathrm{Vol(M^{d-2})}}{16\pi G l^2}\beta_0'[-r_+^{d-1} + kl^2 r_+^{d-3}] - \beta_0' M_{\text{crit}}. \tag{22}$$

Before analysing the phase structure of this action, let us first note that the energy E and entropy are given in the usual way by

$$E = \frac{\partial I}{\partial \beta_0'} = M - M_{\rm crit}, \quad S = \beta_0' E - I = \frac{\rm Vol(M^{d-2})}{4G} r_+^{d-2}. \tag{23}$$

Finally, we note that the specific heat of the $k = 0$ and $k = -1$ black holes is positive. For $k = 0$, we find

$$\frac{\partial E}{\partial T} = \frac{4\pi}{\omega_d} r_+^{d-2}, \tag{24}$$

while for $k = -1$, we have

$$\frac{\partial E}{\partial T} = \frac{4\pi r_+^{d-2}}{\omega_d} \left[\frac{(d-1)r_+^2 - (d-3)l^2}{(d-1)r_+^2 + (d-3)l^2} \right]. \tag{25}$$

In the latter case, we see that the specific heat is positive provided $r_+ > r_{\rm crit}$. We contrast this with the case of $k = 1$, where for temperatures greater than the minimum value there are two black holes, the smaller of which has negative specific heat, the larger having positive specific heat.

4 Correspondence with Conformal Field Theory

There has been a recent flurry of activity on the conjecture relating supergravity defined on anti-de Sitter spaces and conformal field theory on its boundary [1]. According to this conjecture in five dimensions, for example, Type IIB string theory defined on $adS_5 \times S^5$ is equivalent to the large N limit of $\mathcal{N} = 4$ super Yang-Mills theory with gauge group $SU(N)$ defined on S^4. For our purposes here, we wish to concentrate on one particular aspect, namely the relevance of the black hole solutions to the phase structure of the conformal field theory. Thus, we wish to study the conformal field theory at finite temperature, defined on a space $S^1 \times M^{d-2}$. In particular, it has been shown in [10, 11] that one may use the adS/CFT correspondence to give a holographic explanation of the mass dependence of the Bekenstein-Hawking entropy for the spherical black holes in anti-de Sitter space. These correspond to the $k = 1$ class with spherical horizon topology. In [29], the case of $M2$-branes wrapped around higher genus Riemann surfaces was studied, leading to information on the phase structure of the associated conformal field theory on $S^1 \times \Sigma_g$, for $g \geq 1$. Further aspects of the holography of this correspondence have been studied in [30, 31].

According to the prescription given in [10], one computes the partition function of the boundary conformal field theory on $S^1 \times M^{d-2}$ by summing over contributions from Einstein manifolds M^d with negative cosmological constant which have $S^1 \times M^{d-2}$ as their boundary. Thus, one has

$$Z_{CFT}(S^1 \times M^{d-2}) = \sum_i e^{-I(X_i)}, \tag{26}$$

where I is the supergravity action, and in general one may need to consider the contribution from several Einstein manifolds X_i which have $S^1 \times M^{d-2}$ boundary. It is then important to study the relative actions, and in this way one obtains information on the existence of a phase transition.

For the case of conformal field theory on $S^1 \times S^{d-2}$, there are two such manifolds, namely the black hole itself with compactified topology $B^2 \times S^{d-2}$, and thermal anti-de Sitter space corresponding to the background solution with topology $S^1 \times B^{d-1}$, where B^d is a d-dimensional ball with boundary S^{d-1}. It was shown in [10, 11] that for large temperatures the black hole dominated, while for small temperatures the thermal anti-de Sitter space dominated (due to the non-existence of the black holes at low temperatures). One may then compare the Bekenstein-Hawking entropy with the entropy of the conformal field theory on $S^1 \times S^{d-2}$. The limit of high temperature, $\beta_0' \to 0$, may be regarded as a high temperature system on S^{d-2}. Conformal invariance then dictates that the entropy of this system is of the order $(\beta_0'/l)^{-(d-2)}$. However, according to (18), one sees that for high temperature we have $(\beta_0'/l) \sim l/r_+$, and hence the conformal field theory entropy is of the order $(r_+/l)^{d-2}$, in agreement with the Bekenstein-Hawking formula. In this way, we obtain a microscopic understanding of the mass dependence of the entropy for large black holes with $r_+ >> l$.

We now wish to apply similar arguments to the topological black holes considered here. For the $k = 0$ case, for which $M_{\rm crit} = 0$, the phase structure of the action (22) is readily determined. We have

$$I(X_2) - I(X_1) = \frac{\mathrm{Vol}(\mathrm{M}^{\mathrm{d}-2})}{16\pi G l^2} \frac{4\pi l^2}{(d-1) r_+} [-r_+^{d-1}]. \tag{27}$$

Now, for the $k = 0$ case, we have seen that $r_+ > 0$, and thus the action is negative for all r_+. From (18), we note that low temperature corresponds to small r_+, while high temperature corresponds to large r_+. However, the action difference in (27) is negative for all values of r_+, and hence we see no evidence of a phase transition in this case. Thus, the black hole dominates over its $M_{\rm crit} = 0$ background counterpart for all temperatures, and we can once again appeal to the correspondence with conformal field theory to show that for high temperature (i.e., large r_+ black holes) the entropy does indeed scale holographically in the correct manner as r_+^{d-2}. Note also that we have seen that the toroidal black holes have positive specific heat, which is consistent with the conformal field theory correspondence.

In order to determine the phase structure for the $k = -1$ case, we should first recall that the $M_{\rm crit}$ background spacetime has an internal infinity. In the evaluation of the action, we restricted attention to the region $r \geq r_{\rm crit}$ for the background and $r \geq r_+$ for the black hole. Thus, the partition function of the corresponding conformal field theory is sensitive only to these regions. We shall adopt this as an assumption on how to treat these internal infinities. The action difference is

$$I(X_2) - I(X_1) = \frac{\mathrm{Vol}(\mathrm{M}^{\mathrm{d}-2})}{16\pi G l^2} \beta_0' [-r_+^{d-1} - l^2 r_+^{d-3}] - \beta_0' M_{\rm crit}. \tag{28}$$

We have seen that for $r_+ > r_{\rm crit}$, the temperature is positive. Thus, for all $r_+ > r_{\rm crit}$, the action is negative, tending to zero as $r_+ \to r_{\rm crit}$. As a result, the black hole again dominates over the $M_{\rm crit}$ background for all temperatures, with no phase transition occurring. Once again, we see from (18) that low temperature corresponds to small r_+, while high temperature corresponds to large r_+. Thus, using the conformal field theory correspondence, we again have a holographic explanation of the Bekenstein-Hawking entropy formula for large black holes. We stress that these conclusions rely on our assumption of how to treat the internal infinity of the background spacetime.

5 Conclusions

As indicated in [10], the conformal theory defined on $S^1 \times M^{d-2}$ takes two forms. For spinors which are periodic on S^1, we have

$$Z_1(S^1 \times M^{d-2}) = \mathrm{Tr}\ (-1)^{\mathrm{F}} \mathrm{e}^{-\beta \mathrm{H}}, \tag{29}$$

while anti-periodic spinors yield

$$Z_2(S^1 \times M^{d-2}) = \mathrm{Tr}\ \mathrm{e}^{-\beta \mathrm{H}}, \tag{30}$$

where H is the Hamiltonian. For the case of black holes with spherical topology studied in [10, 11], the thermal anti-de Sitter space background has topology $S^1 \times B^{d-1}$, and being non-simply connected contributes to both Z_1 and Z_2. However, the black holes are simply connected having topology $B^2 \times S^{d-2}$; thus, they contribute only to Z_2 where the phase transition was observed. For the black hole solutions constructed here, we see that they have compactified topology $B^2 \times M^{d-2}$, where M^{d-2} is in general non-simply connected. Thus, they will contribute to both Z_1 and Z_2, and it would be interesting to check their role in understanding the phase structure of the former.

Finally, we remark that for the hyperbolic case, the numerical coefficient in the Bekenstein-Hawking entropy is the hyperbolic volume, which is known to be a topological invariant in three-manifold theory, see for example [27, 32], and also [26] for a discussion in four dimensions. Since the correspondence with conformal field theory provided sufficient information to fix the mass dependence of the entropy, it would be interesting to see if the topological nature of the horizon volume can be used to fix the numerical coefficient.

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