$$h_k^{(1)} = \frac{1}{\sqrt{2\sinh\left(\frac{\pi\omega}{a}\right)}}\left(e^{\pi\omega/2a} g_k^{(1)} + e^{-\pi\omega/2a} g_{-k}^{(2)*}\right)$$

$$h_k^{(2)} = \frac{1}{\sqrt{2\sinh\left(\frac{\pi\omega}{a}\right)}}\left(e^{\pi\omega/2a} g_k^{(2)} + e^{-\pi\omega/2a} g_{-k}^{(1)*}\right)$$

Unruh modes!! → Some linear combination of +ve and -ve frequency Rindler modes!!

Imp

We will see that Unruh modes can be written as some linear combination of +ve frequency Minkowskian modes (bcz of our choice of branch cut!!)

s^2

~~$h_k^{(1,2)} = \omega(-t+x)$~~

$h_k^{(1)} \sim e^{\pi\omega/a}(t-x)^{i\omega/a}$ ← There will be some factors of ω!!

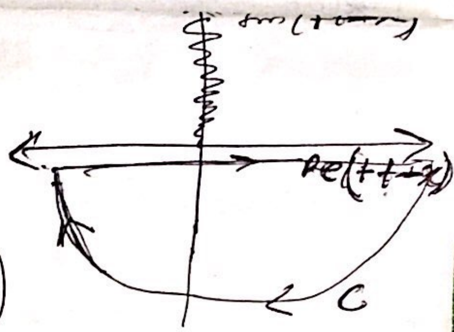
$h_k^{(2)} \sim (t+x)^{i\omega/a}$ ← There will be some factors of ω!!

Now, consider that t, x can be complex!!

After introducing the branch cut on my complex $(t-x)$ plane, the func. is analytic and bounded everywhere else!!

We choose this to be in the UHP → we will see why!!

$h_k^{(1)}$ mode is analytic and bounded in LHP



Let's evaluate :- $I = \oint h_k^{(1)} e^{-i\omega' z} dz$

$$= 2\pi i \times \text{Res}(h_k^{(1)}) = 0$$

$\boxed{\omega' > 0}$ So that the integrand vanishes at the big contour C

bcz $e^{-i\omega' z}$ exponentially decrease for $\text{Im}(z) < 0$

$$\Rightarrow \int_{-\infty}^{\infty} h_k^{(1)} e^{-i\omega' z} dz \to 0$$

$$+ \int_C h_k^{(1)} e^{-i\omega' z} dz = 0$$

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} h_k^{(1)} e^{-i\omega'(z)} dz = 0}$$

(1) $(t+x)$ is real on this line !!

To enforce this; we would require

$$\boxed{h_k^{(1)}(x) = \int_0^{\infty} \chi(\omega) e^{-i\omega z} d\omega} \quad \text{s.t.} \quad \boxed{\omega > 0}$$

$+i\omega z$

Some linear combination of +ve frequency Minkowskian modes !! (right-moving)

How is this enforced by (I) ?

~~How is~~ Evaluate this on the contour C again

$-3 \ldots 3$

bcz then

$$\int_0^{\infty} d\omega \int_{-\infty}^{\infty} \chi(\omega)\, e^{-i(\hat{\omega}+\hat{\omega}')z} dz$$

$$\rightsquigarrow \sum_{\omega} \int_{-\infty}^{\infty} dz\, \chi(\omega) \left\{ \cos\left((\omega+\omega')z\right) - i \sin\left((\omega+\omega')z\right) \right\}$$