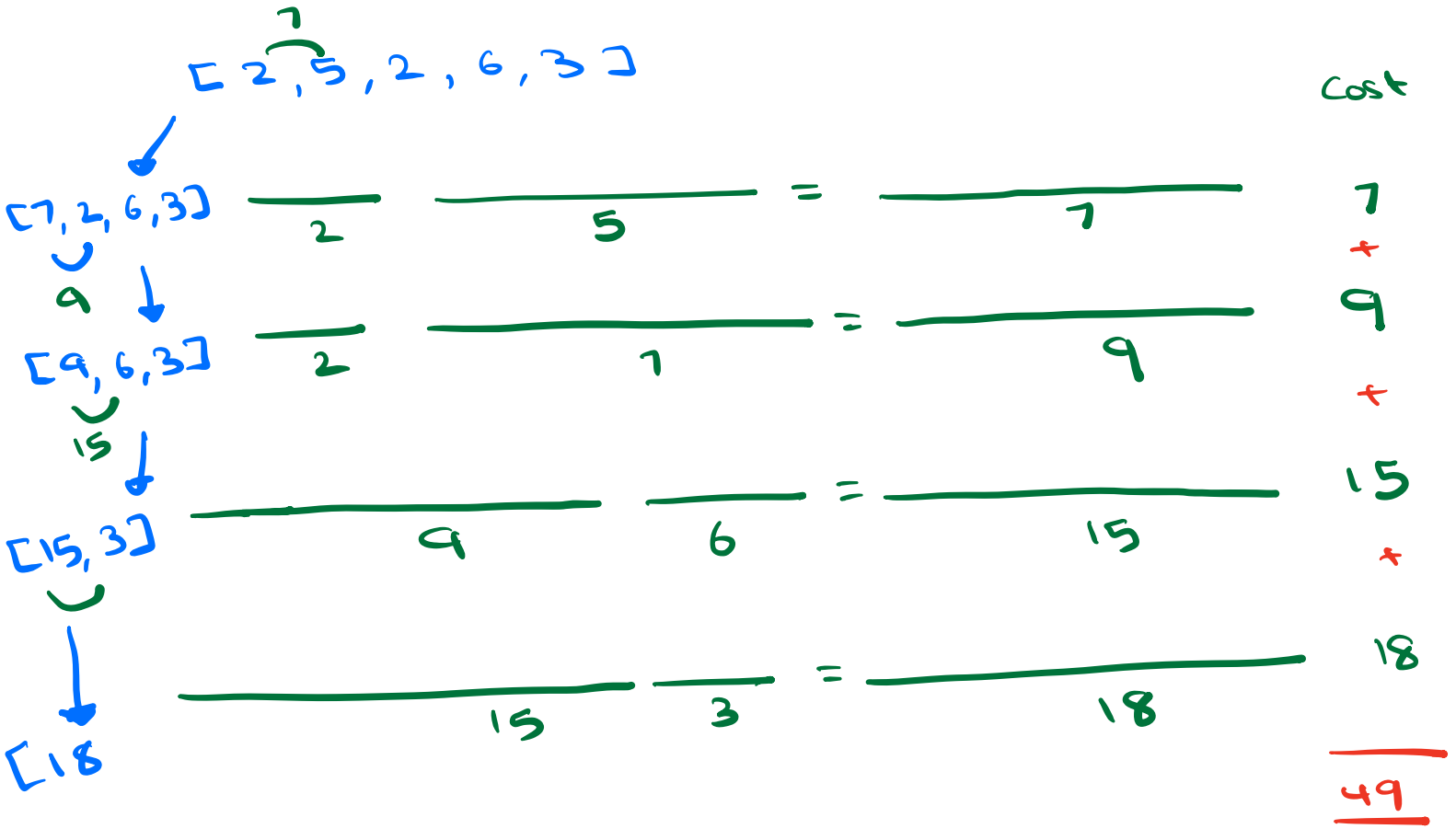
Today's Content

- Connecting the Robes
- Heap Introduction
- Insertion
- Heapify
- Extract Min
- Build Heap

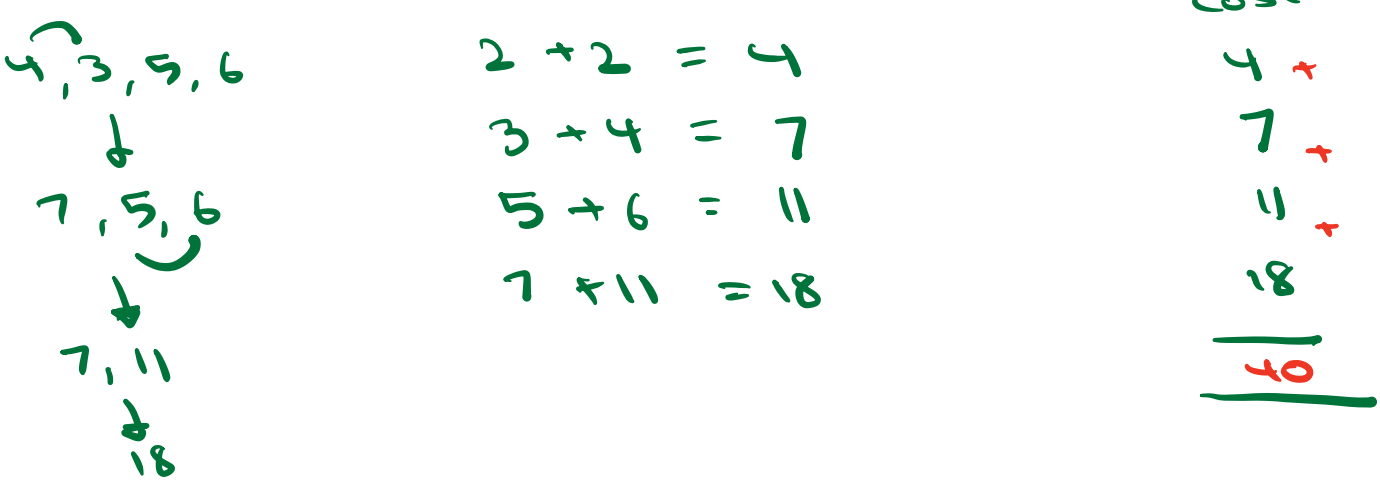
Q. Connecting the Ropes

You can connect any two ropes together, there's a cost associated to connect them = sum of length of ropes that you're connecting.

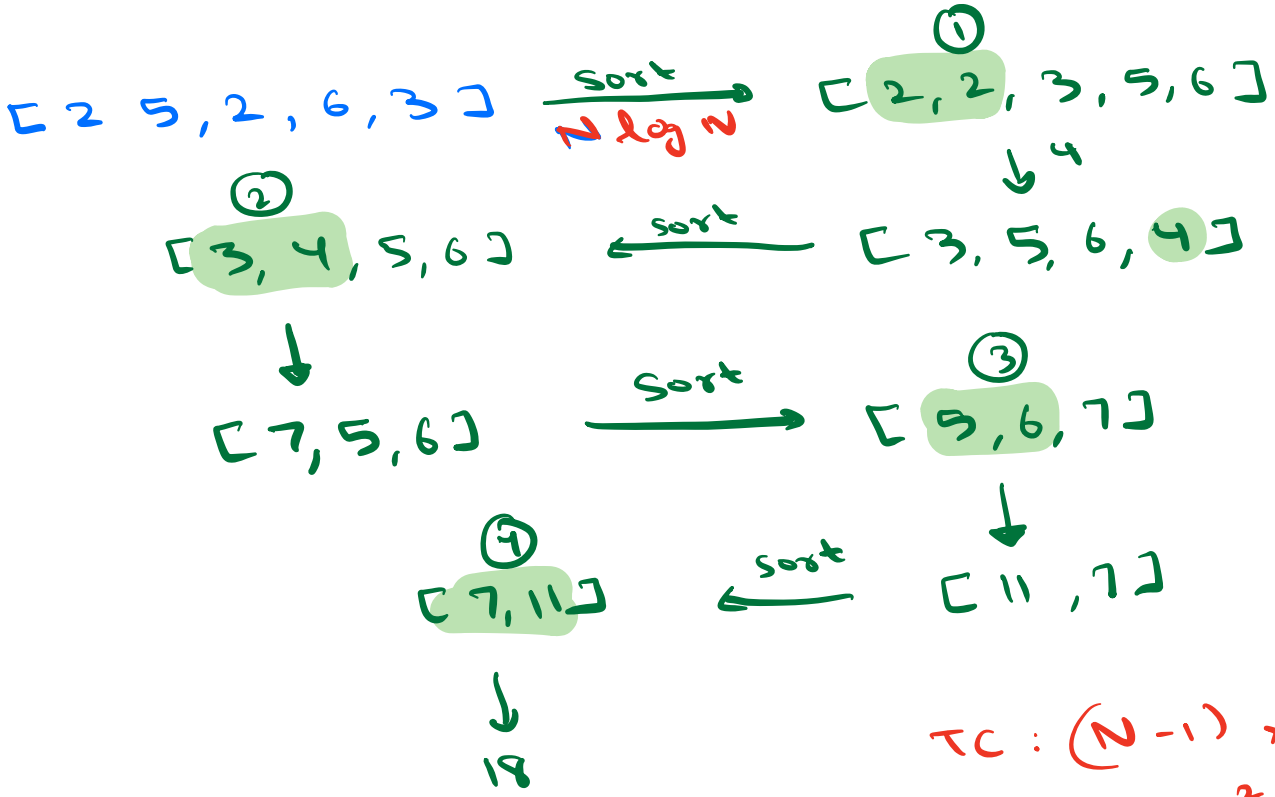
Find min. cost required to connect all ropes.



Sort [2 5, 2, 6, 3] → [2, 2, 3, 5, 6]



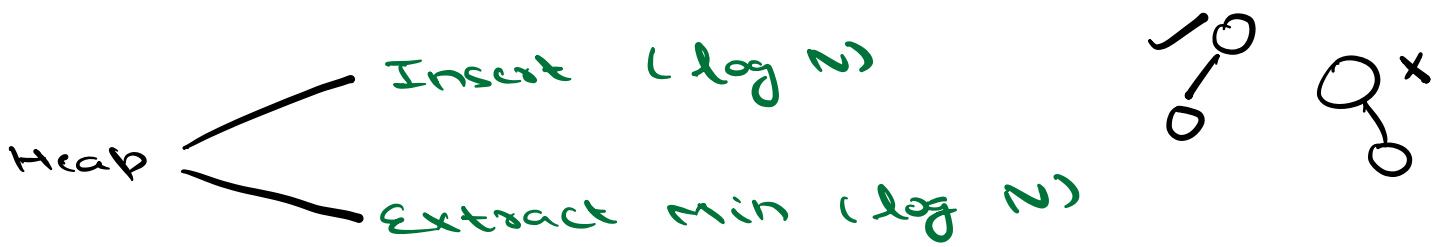
Idea: Always pick 2 smallest ropes and combine them.



TC: $(N-1) \times N \log N$

$O(N^2 \log N)$

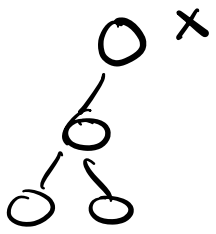
We need a DS which is optimized for finding min element



Heap DS

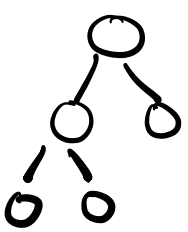
↓

Binary Tree



① Complete BT (CBT)

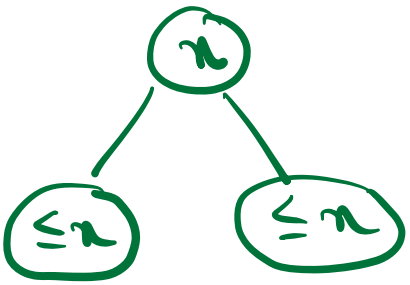
All levels are filled completely except last level, data can be filled from left to right



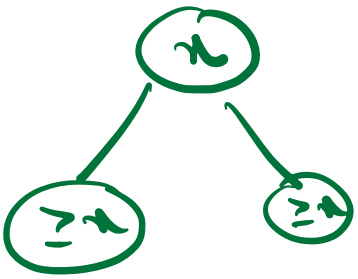
② Order of elements

↓

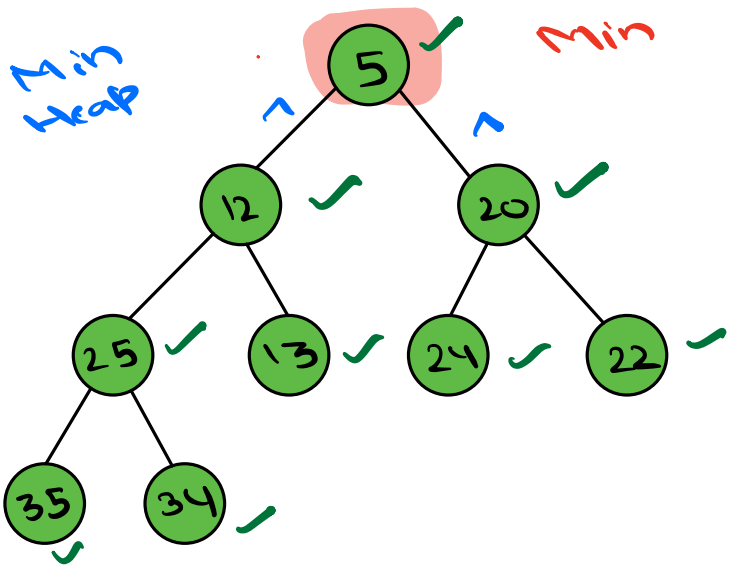
Heap Order Property [HOP]



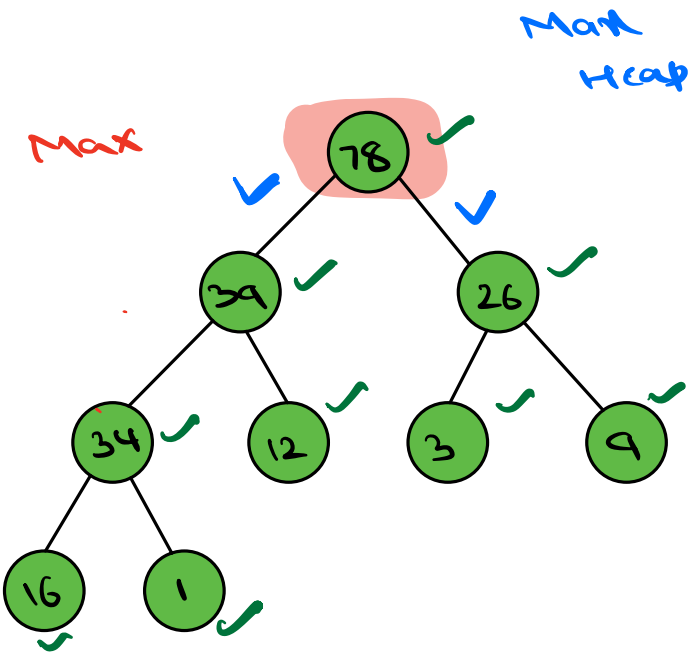
Max Heap Node ≥ Children



Min Heap Node ≤ Children



1. Complete BT
2. HOP

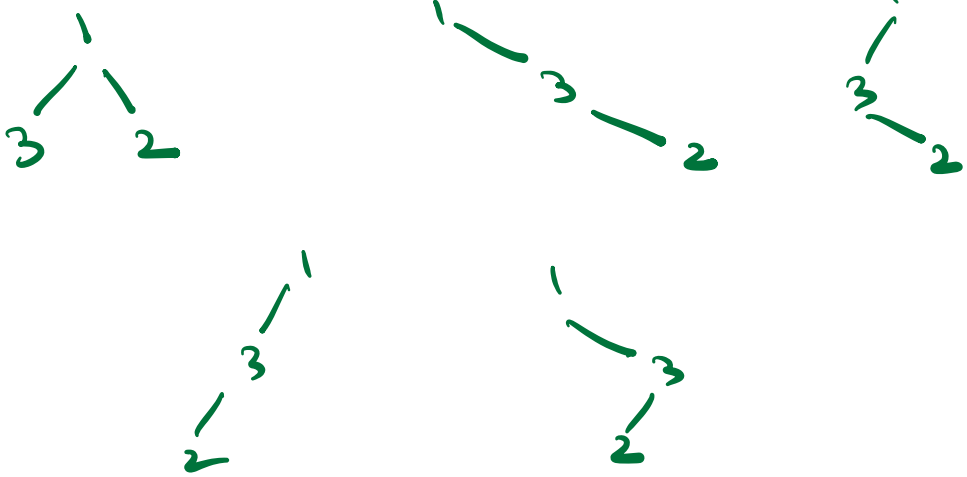


1. Complete BT
2. HOP

Min Heap → min de is at root O(1)

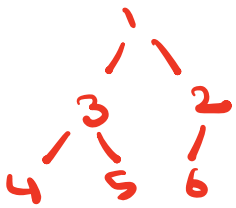
Max Heap → Max de is at root O(1)

Level order Traversal [1,3,2]



[1,3,2]

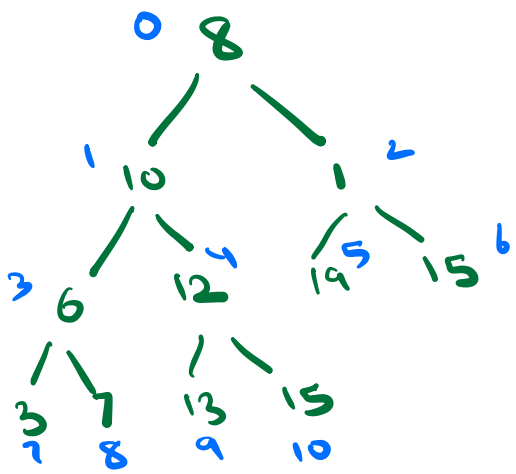
[1,3,2,4,5,6]



Visualize array as Tree (CBT)

8	10	1	6	12	19	15	3	7	13	15
0	1	2	3	4	5	6	7	8	9	10

Heap → CBT → Arr (Level order)

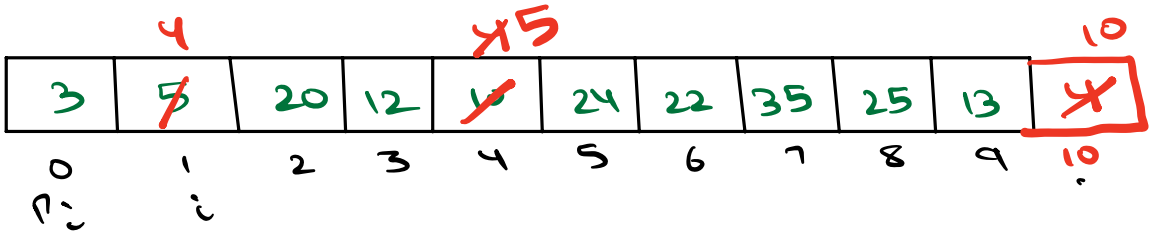


Parent	Child
0 →	1, 2 [2*0+1, 2*0+2]
3 →	7, 8 [2*3+1, 2*3+2]
4 →	9, 10 [2*4+1, 2*4+2]
i →	[2i+1, 2i+2]

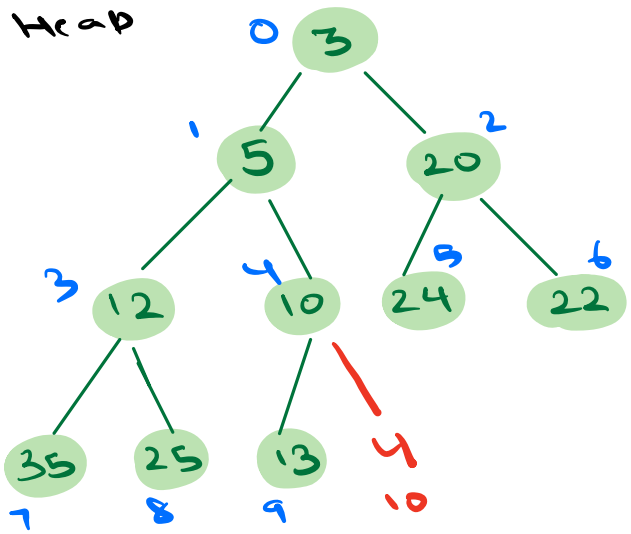
$c_i \xrightarrow{\text{par}} \frac{c_i - 1}{2}$

Insertion in minHeap

Insert 4



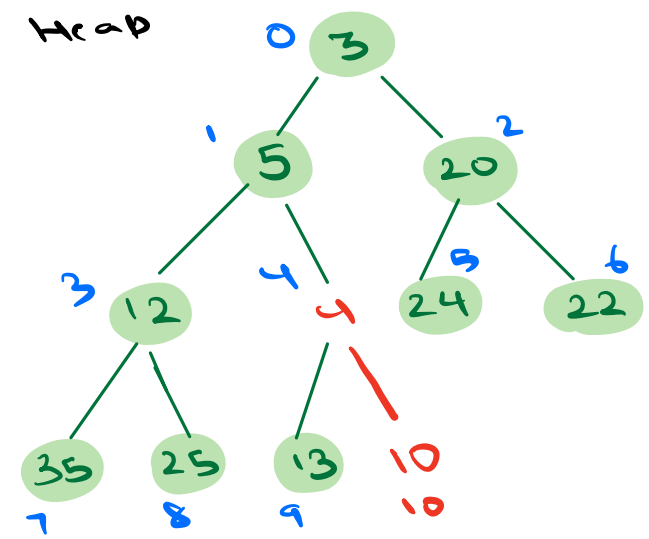
Min Heap



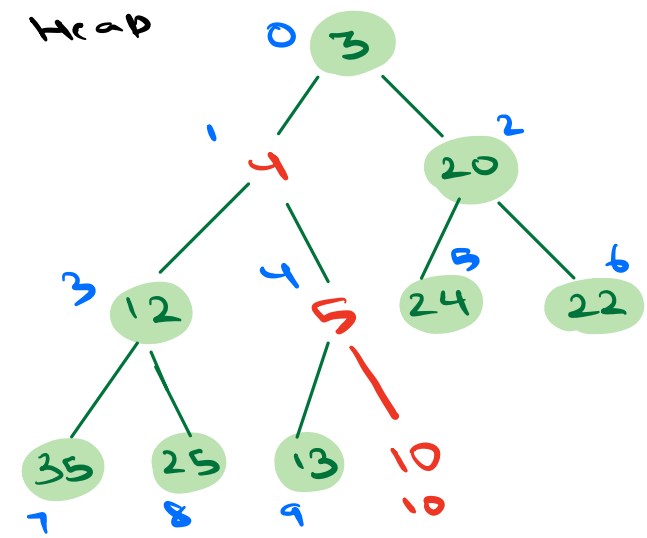
$\frac{i-1}{2}$

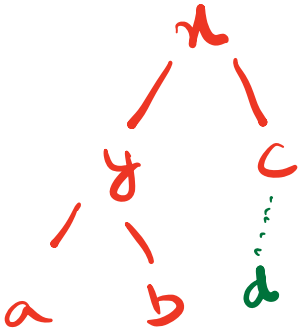
i	Pi	ar[i] > ar[Pi]
10	4	4 > 10 No, Swap
4	1	4 > 5 No, Swap
1	0	4 > 3 Yes, Break

Min Heap



Min Heap



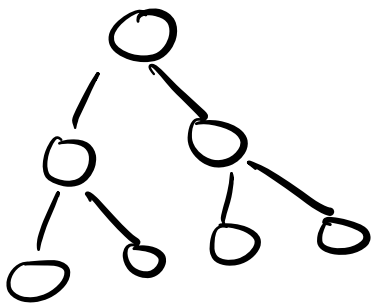


$x < y < a$
$\quad < b$
$\quad < c$

swap $d < c$

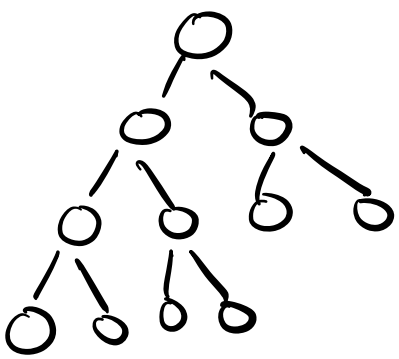
swap $d < x$

$d < x$
$\quad < c$



$N \rightarrow \log_2 N$

7 $\quad \log_2 7 = 2.\_$



11 $\quad \log_2 11 = 3.\_$

Dynamic array

```
void insert(int x, int[] heap) {
    heap.addLast(x)
    i = heap.size() - 1

    while(i > 0) {
        int Pi = (i-1)/2
        if(heap[i] < heap[Pi]) {
            swap(heap[i], heap[Pi])
            i = Pi
        }
        else {
            break
        }
    }
}
```

TC: O(Height of tree)

↓

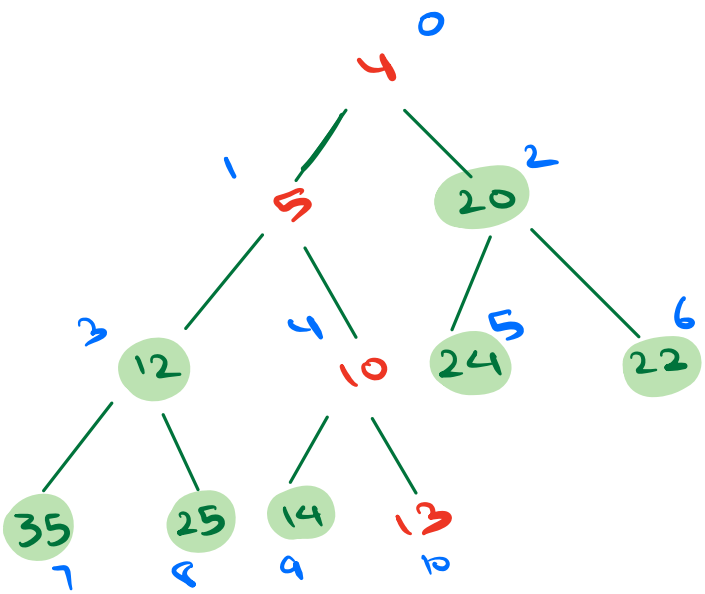
TC: O(log2N)

SC: O(1)

Heapify

Given a min heap, all elements are following heap property except for first ele, fix the heap

0	1	2	3	4	5	6	7	8	9	10
~~13~~ 4	~~4~~ 5	20	12	~~5~~ 13	24	22	35	25	14	~~13~~ 10



i	child idx	min(a[i], a[2i+1], a[2i+2])
0	1, 2	min(13, 4, 20) = 4 → idx 1 swap (0, 1)
1	3, 4	min(13, 12, 5) = 5 (idx 4) swap(1, 4)
4	9, 10	min(13, 14, 10) = 10 (idx 10) swap(4, 10)
10	21, 22	Break

N = 11 (Heap size)

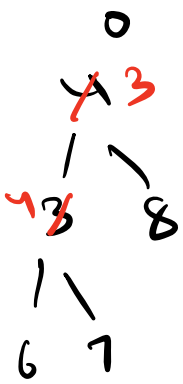
idx → 0 - 10

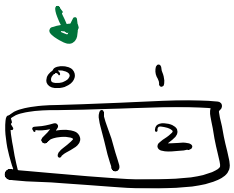
Invalid idx ≥ N

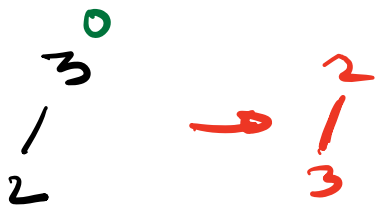
i	2i+1	2i+2
5	11 X	12 X
6	13 X	14 X

```
void heapify (heap[], N, i){
    while(2i+1 < N){
        int x = min(heap[i], heap[2i+1], heap[2i+2])
        if (x == heap[i])
            break
        else if (x == heap[2i+1]){
            swap(heap[i], heap[2i+1])
            i = 2i+1
        }
        else if (x == heap[2i+2]){
            swap(heap[i], heap[2i+2])
            i = 2i+2
        }
    }
}
```

NOTE *





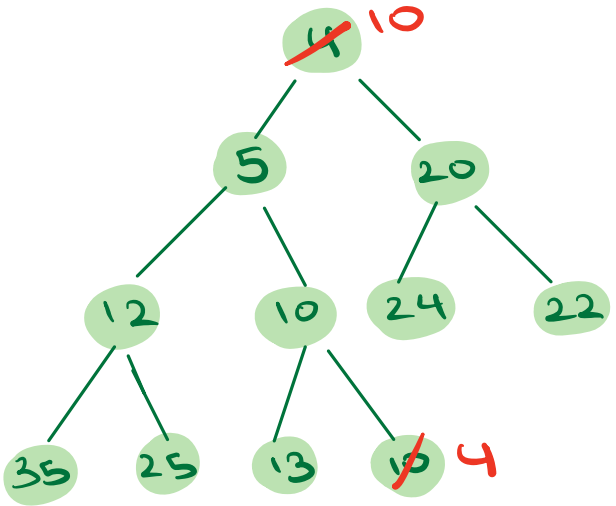


TC: O(log N)
SC: O(1)

10:55

Extract / Remove min

10										4
~~4~~	5	20	12	10	24	22	35	25	13	~~10~~
0	1	2	3	4	5	6	7	8	9	10



N = 11 idx 0-10

N-- → 10 0-9

① swap heap[0] with heap[last]

② remove last el

③ heapify with idx 0

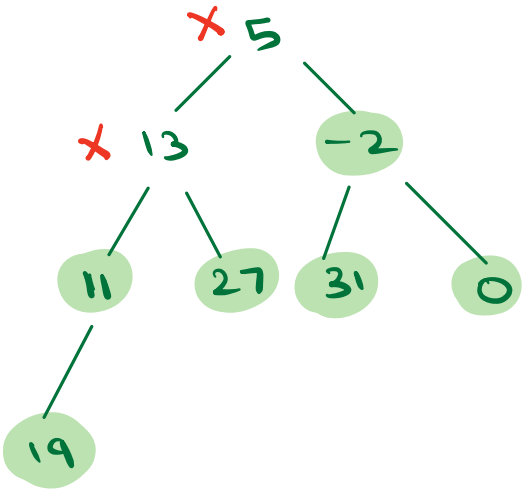
TC : O(log N)

SC : O(1)

Heap [ ] → Min el → heap[0]

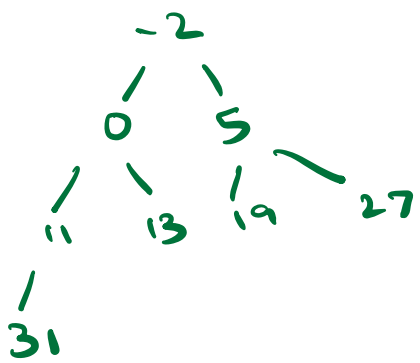
Build Heap [Min Heap]

[5 13 -2 11 27 31 0 19]



Idea 1 : Sort the arr

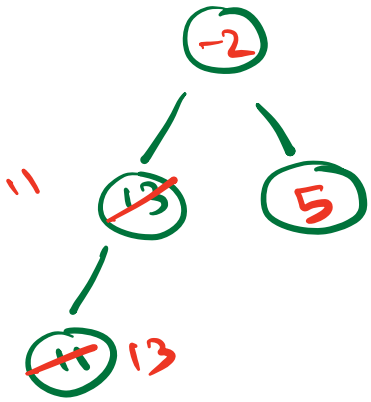
[-2 0 5 11 13 19 27 31]



TC: O(NlogN)

[5 13 -2 11 27 31 0 19]

Idea 2: Insert elements one by one in heap



heap

[5 13] -2

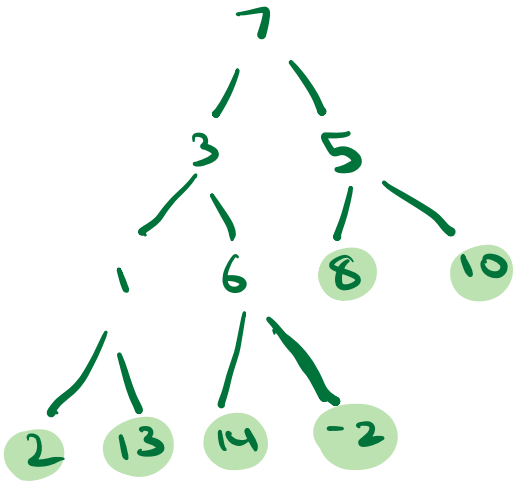
-2 5

TC: O(N log$_2$N)

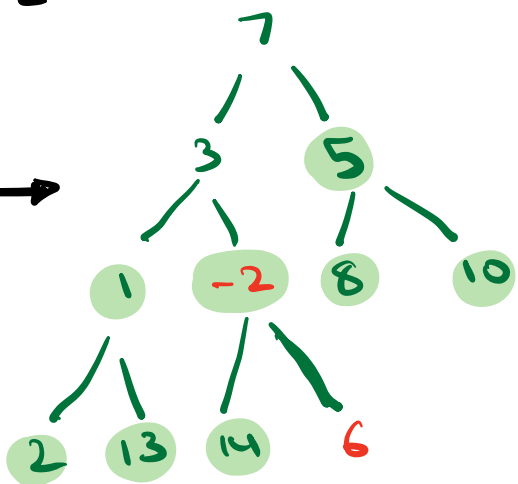
Idea 3 : [7, 3, 5, 1, 6, 8, 10, 2, 13, 14, -2]

0 1 2 3 4 5 6 7 8 9 10

Build heap from bottom to top and R→L

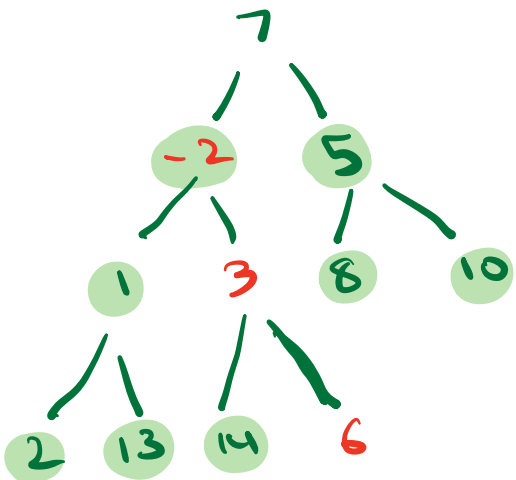


heapify (6)

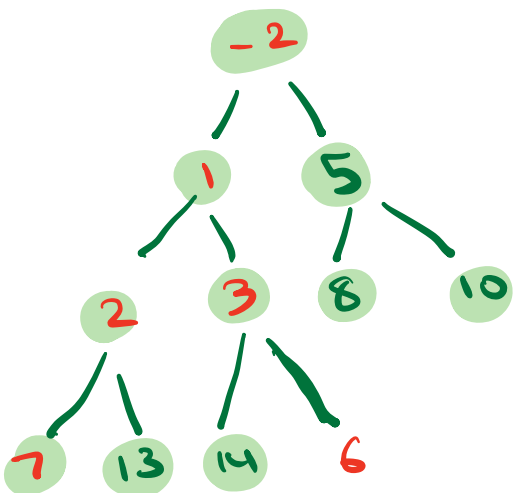


heapify (1)
heapify (5)

heapify (3)



heapify (7)



Last Non-Leaf Node

↓

Parent of Last leaf

Heap

1-based

Last leaf (ll) = N-1

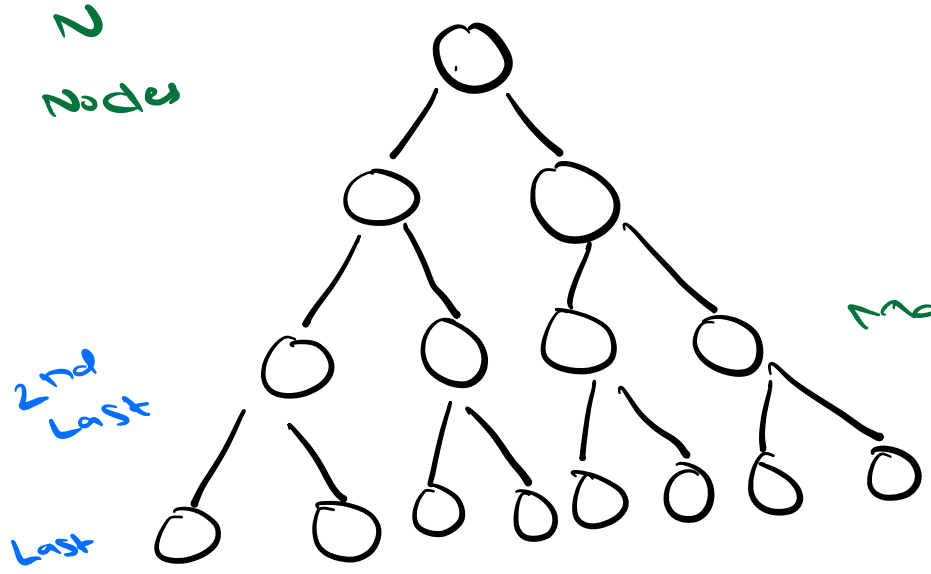
$$\text{Parent} = \frac{i-1}{2} = \frac{N-1-1}{2} = \frac{N-2}{2} = \frac{N}{2} - 1$$

$$\text{Last Non-Leaf Node} = \frac{11-2}{2} = \frac{9}{2} = 4$$

```
for (i = (N-2)/2 ; i >= 0 ; i--) {
    heapify (heap, i)
}
```

TC: O(N)

$\times \frac{N}{2} \log_2 N$



Max No. of nodes in Last level = N/2

Max no. of nodes in 2nd last level = N/4

3rd Last → N/8

Total swaps

$$= \frac{N}{2} * 0 + \frac{N}{4} * 1 + \frac{N}{8} * 2 + \frac{N}{16} * 3 + \dots$$

($\frac{N}{2} * 0 = 0$)

$$= \frac{N}{2}\left(\underbrace{\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \dots}_{\text{AGP}}\right)$$

$$= \frac{N}{2} * 2 = N$$

$$S = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \dots$$

$$\frac{S}{2} = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \dots$$

$$S/2 = \frac{1/2 + 1/4 + 1/8 + 1/16 + \dots}{}$$

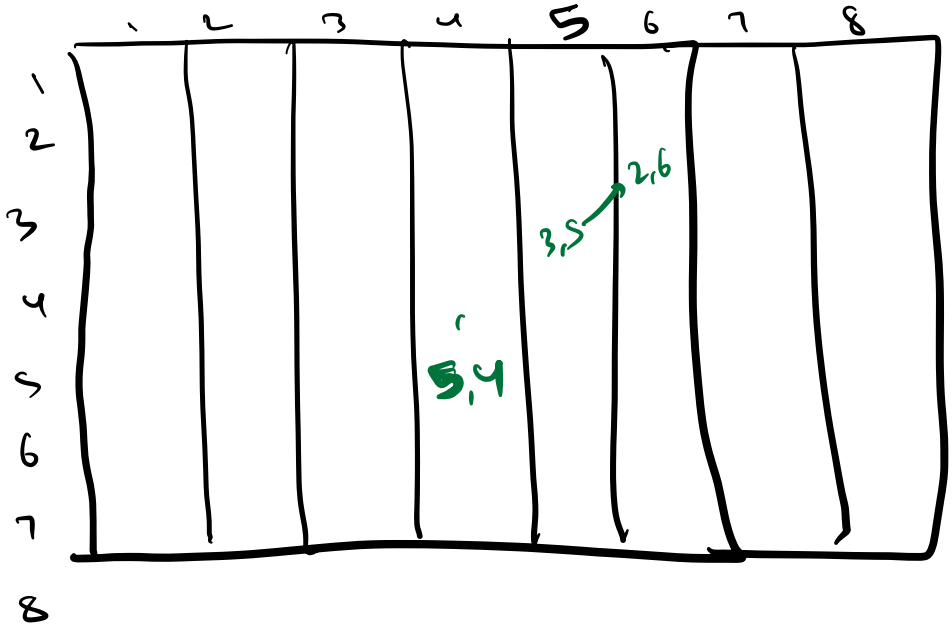
GP, $r = \frac{1}{2}$

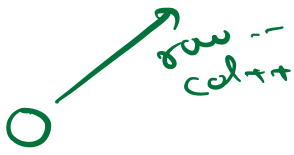
$$\text{Sum} = \frac{a}{1-r}$$ Formula for Infinite GP

$$S/2 = \frac{1/2}{1 - 1/2}$$

$$S/2 = 1 \Rightarrow S = 2$$

Doubt



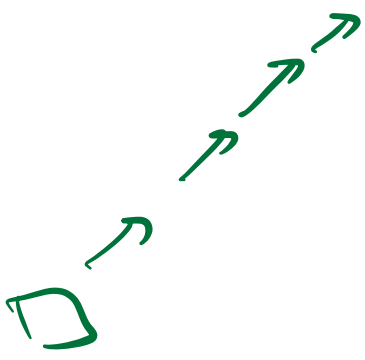


Row

4 → 1 3 steps

4 → 8 4 steps

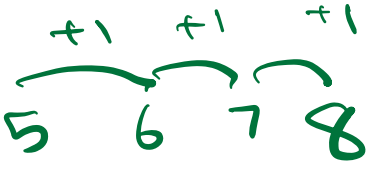
min (3,4)

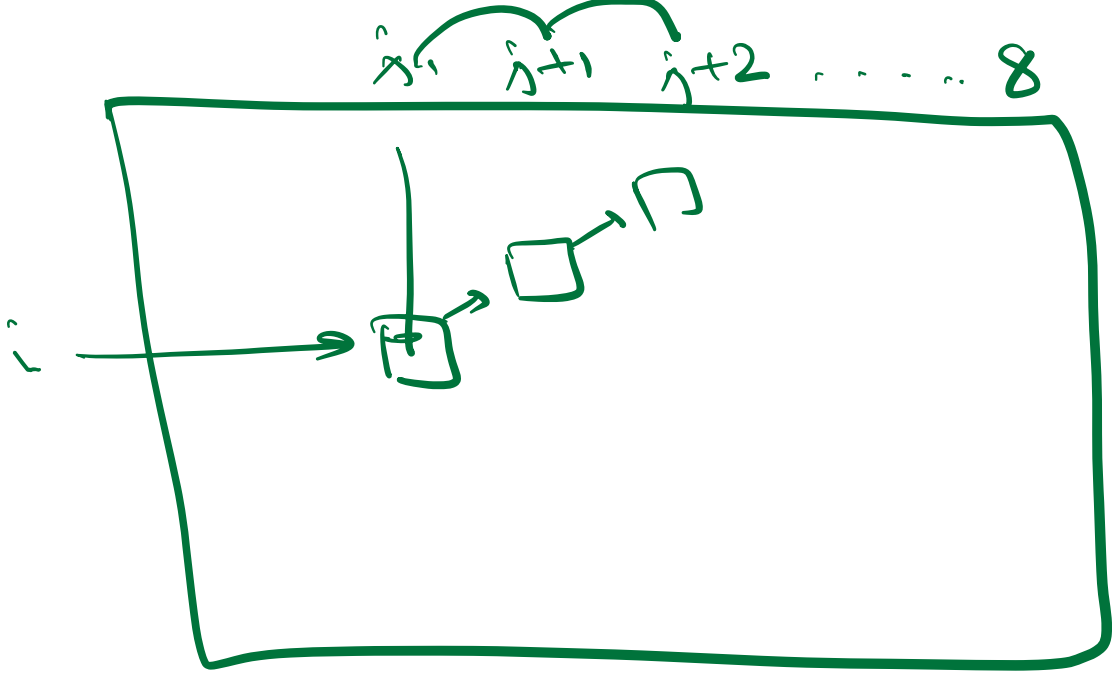


Row

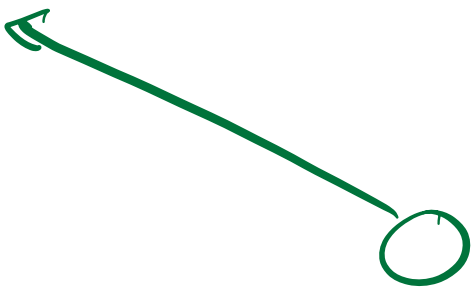
i → 1 (i-1) steps

j → 8 (8-j) steps





row--
col--



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4 → 1 3 steps

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