CASE STUDY ON GOAL PROGRAMMING

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ABSTRACT

This paper presents a model for determining an optimal blend of ingredients for livestock feed by application of goal programming. Besides the standard problem of livestock feed where the requirements for basic nutrients have to be met at minimized costs and which is solved mainly by linear programming, the authors also introduce the goals of meal quality where different requirements of decision makers are modeled by goal programming.

AIM

1. To find and know more about the importance and uses of “Linear Goal Programming”.
2. To formulate a goal programming problem and solve in simplex method using software.

Introduction to goal programming:

The conventional linear programming models are based on the optimization of single objective function. However, there are situation where multiple (possibly conflicting) objectives may be more appropriate. For example, aspiring politician may promise to reduce the national debt and simultaneously offer income tax relief.

In such situations it may be impossible to find a solution that optimizes the conflicting objectives. This problem can be overcome by a technique known as GOAL PROGRAMMING in which we try to seek a COMPROMISE OR EFFICIENT SOLUTION based on the relative importance of each objective.

Goal programming, a powerful and effective methodology for the modeling, solution and analysis of problems have multiple and conflicting goals and objectives, has often been cited as being the "WORKHORSE" of multiple objective function.

History and Philosophy:

The roots of GP lie in a paper by Charnes et al. in 1955 in which they deal with executive compensation methods. Recognizing that the method could be extended to a more general class of problems- that is, any quantifiable problems having multiple objective and soft as well as rigid constraint – Charnes and Cooper later renamed the method goal programming when describing their classic two volume text *"Management Models and Industrial Applications of Linear programming."*

The two philosophical concepts that serve to best distinguish goal programming from conventional (i.e. single objectives) methods of optimization are the incorporation of flexibility in in constraint function and the adherence to the philosophy of "satisficing" as opposed to optimization.

As a consequence of the principle of satisficing, the "goodness" of any solution to a goal programming problem is represented by "*achievement function"* rather than objective function of conventional optimization.

Difference between Linear programming and goal programming:

	Linear programming LP	**Goal programming GP**
Goal and objectives	One primary - to be maximized or minimized	All objectives are ranked each with a target
Targets or constraints	Inflexible, no deviations are allowed	Flexible, deviations are acceptable, constraints can be relaxed
Objective functions	Maximize (minimize) the value of the primary goal	Minimize the sum of the undesirable deviations (weighted by their relative importance)
Theory	Optimization	Satisfaction

Computer programs	Very efficient, many packages	Inefficient, few computer packages
Applications	Many and varied	Few, but increasing

Concept of Real and Goal Constraints:

The real constraints are absolute restrictions on the decision variables, while the goal constraints are the conditions one would like to achieve but are not mandatory. For example- a real constraint is given by

$$x1 + x2 = 3$$

requires all possible values of x1 + x2 to always equal to 3. As opposed to this, a goal requiring x1+ x2 = 3 (aspiration level is 3 here) is not mandatory, and we can choose values of $x1 + x2 \geq 3$ as well as $x1 + x2 \leq 3$. In a goal constraint, positive and negative deviational variables are introduced as follows:

$$x1 + x2 + d_1^- - d_1^+ = 3$$

Note: that if d1- > 0, then x1 + x2 < 3 and if d1+ > 0 then x1 + x2 >3 and

Aspiration level referred to target level.

Positive Deviation is overachievement of aspiration or target or positive deviation of goal from target level.

Negative Deviation is underachievement of aspiration or target or negative deviation of goal from level of aspiration.

Application areas:

1. Accounting,
2. Agriculture,
3. Economics,
4. Engineering,
5. Finance,
6. Government,
7. International Context,
8. Management and Marketing and etc.

Types of Goal Programming:

1.Lexicographic Goal Programming Model or preemptive or non-Archimedean goal Programming model:

Pre-emptive goal programming is used when there are measure differences in the importance of the goals.

- ✓ The goals are listed in order of their importance.
- ✓ It begins by focusing solely on the most important goal.
- ✓ It next does the same for the second important goal (as is possible without hurting the first goal).
- ✓ It continues the following goals (as is possible without hurting the previous more important goal).

2.Weighted Goal Programming model:

Weighted goal programming is designed for problems where all the goals are quite important with only modest differences in importance that can be measured by assigning weights to the goals.

Goal Programming Formulation:

There are following steps to formulate GP-

- Identify the decision variables
- Identify the constraints and determine which ones are goal constraints.
- Formulate the non-goal (hard) constraints.
- Formulate the goal (soft) constraints.
- Formulate the objective function
- Add the non-negativity constraints
- Set the priority of goals

Goal Programming Problem (Optimization of livestock feed blend):

1.Intoduction:

Cattle feed blend is a mixture of ingredients used for animal feeding. An adequate quality of feed blend ensures the growth of livestock to meet the increasing needs for food in a continually increasing population. The increasing size, income, and living standard of the population leads to an ever higher demand for food products of animal origin, which can be provided by increasing the quantity and quality of livestock feed. The increase of quantity cannot be achieved only by larger acreage and imports, as the arable land is limited and so are the funds available for livestock feed import and animal food products import. Consequently, rational production of high quality livestock feed is an important task of any economy. Such livestock feed has to meet the nutritional requirements of livestock in order to maximize weight gain, while production of such feed has to be economical, which can be achieved only by an optimal blending of ingredients.

Optimization of ingredients blend in terms of nutritional requirements and in terms of economic criteria can be carried out by application of mathematical optimization methods. These mathematical methods can quickly and efficiently determine an optimal combination of ingredients to meet the nutritional requirements of livestock leading to a rational use of available resources and cost reduction.

Since 1951, when Waugh defined the feeding problem in mathematical form, linear programming (LP) has formed the basis of livestock ration formulations (Waugh, 1951). However, linear programming has many limitations in formulating rations in practice. Rehman and Romero (1984) found them mostly in singularity of objective function and the rigidity of the constraint set. Lara (1993) also criticizes practical applications of LP due to the restrictions placed on the decision maker's preferences through a singular objective function.

From that time many other mathematical programming methods have been used in the problem of livestock ration formulations. Rehman and Romero (1987) were first to use goal programming because they found that goal programming does not impose such rigid conditions and also allows consideration of several decision criteria. Lara and Romero (1994) used interactive multi-criteria programming (STEM method) with the intention to relax over-rigid specifications of nutrient requirements of livestock rations. Houghton and Portougal (1997) used multi-stage process for the reengineering of the production planning process in the food industry. Their production planning model is a variant of the discrete lot-sizing and scheduling problem. Glen and Tipper (2001) used linear and integer programming in agricultural planning in developing countries. Tozer and Stokes (2001) used multi-objective programming approach to reduce nutrient excretion from dairy cows through incorporation of nutrient excretion functions into a ration formulation framework. Similarly, Bailleul et al. (2001) used multi-objective optimization method and modified the traditional least-cost formulation algorithm to reduce nitrogen excretion in pig diets. Anets and Audsley (2002) presented a multiple objective linear programming model developed to consider a wide range of farming situations, which allows optimization of profit and environmental outcomes. Itoh et al. (2003) formulated the model of crop planning under uncertainty in agricultural management using linear programming and fuzzy constraints. Castrodeza et al. (2005) gave a multicriteria fractional model for feed formulation with economic, nutritional and environmental criteria. Together with the search for the lowest possible cost, they introduced some other aspects such as maximizing diet efficiency and minimizing any excess that may lead to unacceptable damage to the environment. Ghosh et al. (2005) again used goal programming technique for nutrient management by determining the optimum fertilizer combination for rice production in West Bengal. Pomar et al. (2007) developed multi-objective optimization model based on the traditional least-cost formulation program to reduce both feed cost and total phosphorus content in pig feeds. Pla (2007) presented a very interesting review article, which is a survey of different mathematical methodologies used in sow herd management. Trienekens and Zuurbier (2008) gave some quality and safety standards in the food industry, whileHan et al. (2009) gave some relationships and quality management in the Chinese pork supply chain. Finally, Niemi et al. (2010) used stochastic dynamic programming to determine the value of precision feeding technologies for grow-finish swine.

Besides the standard problem of feed where the requirements for basic nutrients have to be met at minimized costs and which is solved mainly by linear programming, the authors also introduce the goals of meal quality where different requirements of decision makers are modeled by goal programming. Consequently, the main goals of this paper are:

(a) To point to the fact that optimization of feed ingredients blend is a multi-criteria problem.

(b) To develop a multi-criteria programming model for ingredients blend optimization, including the criteria of meal quality.

(c) To apply the model of goal programming in the solution of the described problem.

The rest of the paper is organized as follows:

In Section 2 the general multi-criteria model for determination of feed ingredients blend are formulated. In Section 3 input data are given for the particular problem of determining the feed blend for pig fattening. Section 4 formulates the multi-criteria and goal programming model for solving the posted problem, and in Section 5 four scenarios due to decision making preferences are presented. In Section 6 the analyses of obtained results are considered while Section 7 presents the conclusions.

2. Multi-criteria programming model for determination of feed blend ingredients

Formulating the model for determination of feed blend ingredients we have to consider:

- blend preparation costs,
- the needs of the animals for which the blend is prepared, and
- blend quality.

Consequently, it would be ideal if the blend preparation costs were minimal, the needs of the animal completely satisfied, and the quality of feed maximal. Therefore, it can be said that the preparation of an optimal feed will require the following criteria:

1. Cost expressed in monetary units.

2. Nutrients (in percentage) needed for the maximal weight gain.

3. Water (in percentage) affecting the quality of the blend, and thus also the weight gain of the animal for which the feed is prepared.

The aim of the second criterion is to obtain a blend containing components that will maximize weight gain in animals. In our example the weight gain has to be achieved by maximizing nutrients, i.e. by favouring those kinds of feed that contain high digestibility ingredients. Higher digestibility ensures higher weight gain with a smaller blend quantity, which eventually reduces feed costs.

Table 1: Sorts of feed (PS-2).

	Feed	Price – C_{i1} (Min)	Nutrients – C_{i2} (Max)	Water – C_{i3} (Min)
H1	Barley	1.75	70	11
H2	Maize	1.75	80	12
H3	Lucerne	1.65	32	6.9
H4	Powdered Milk	6	86	8.4
H5	Fish Meal	9	69	9
H6	Soya	2.7	92	10
H7	Soya Hulls	3.5	79	11
H8	Dried Whey	9	78	6
H9	Rape Pellets	1.8	66	8
H10	Wheat	1.8	79	12
H11	Rye	1.8	75	11.4
H12	Millet	3.5	65	10
H13	Sunflower Pellets	1.8	68	7

The aim of the third criterion is to maximize the blend quality in terms of its shelf life, which is achieved by reducing the content of water. Reduced water content allows the same weight gain with a smaller blend quantity, which eventually also reduces feed costs. All the three criteria are correlated and inherently conflicted. Namely, most expensive kinds of feed are those that contain ingredients contributing most to weight gain, as well as those containing a small quantity of water. However, feeds containing the best nutrients need not also be those that contain a small quantity of water. That can be seen from the input data for the three criteria in Table 1.

All of these criteria are of economic importance because each one of them contributes to some extent to the business performance expressed in profits. Cost reduction affects business performance directly, whereas feed blend quality affects it through better weight gain in animals and better quality of the final product (meat). Better quality ensures better prices and directly contributes to the better business performance. It has to be noted, however, that there is no guarantee that the company will achieve the best business result at minimal feed cost. Also, maximizing quality of feed blend will not necessarily lead to an optimal performance. Consequently, the company will achieve optimal performance by producing feed blends that are adequate in terms of both cost and quality. The optimal acceptable levels of criteria functions have to be determined by analyzing the relation between the criteria functions levels and the total performance, which cannot be done without participation of the decision maker.

Obviously, the problem of determining an optimal blend for feed is a multi-criteria programming problem (see Anets and Audsley, 2002). If we want to solve it by MCDM, we have to start from the following:

- Criteria for determining an optimal blend are given.
- The blend has to satisfy the needs for nutrients of the given kind and category of animal.
- A certain number of feed sorts are available that can be used as blend components.

Let us introduce the following marks:

f_j → functions of optimization criteria (j = 1, 2,…, p),

m → number of different needs for nutrients in a particular kind and category of animal,

b_k → need for a nutrient of k kind in the blend unit (k = 1, 2,…, m),

n → number of available sorts of feed,

C_{ij} → I coefficient of j criterion function (i = 1, 2,…, n; j = 1, 2,…, p),

H_i → quantity (share) of a particular feed in the blend (i = 1, 2,…, n),

a_{ik} → quantity of the k nutrient per unit of the i feed (i = 1, 2,…, n; k = 1, 2,…, m).

Model 1:

The multi-criteria problem of determining an optimal blend for feed has p criteria functions, and the optimal blend is formulated from n sorts of feed. Each kind of feed contains a certain quantity of nutrients affecting its quality. In this example m nutrients are considered and for each one of them there are minimal or maximal requirements for the quantity (bk) needed in an optimal ration of feed blend. Let us now formulate a multi-criteria linear programming model (MCLP) to determine the optimal feed blend:

$$\text{Min} \sum c_{i1} H_i \qquad \text{(Cost)}$$

$$\text{Max} \sum c_{i2} H_i \qquad \text{(Nutrients)}$$

$$\text{Max} \sum c_{i3} H_i \qquad \text{(water)}$$

$$\sum a_{ik} H_i \geq (\leq) b_k \quad (k=1,2.....15)$$

$$\sum H_i = 0.97$$

$$0 \leq H_i \leq 0.15 \quad (i=1,2......13)$$

Naturally, besides the constraint of minimal needs for particular nutrients (bk) there can also be other constraints, such as for example the maximal quantity of a particular nutrient. It is also frequently required that a particular sort of feed is not included in quantities too large or too small. All the requirements will depend on the kind of animal and suggestions of nutritionists. The relation (3) is set when the blend recipe is composed, with the possibility to make the left side of this constraint less than 1, for the ingredients of additives to the blend no matter what the blend ingredients are. We will now establish the MCLP model for determining the optimal feed blend, and then we will reformulate the model into a corresponding goal programming model following the nutritionists' suggestions. The difference from some other similar papers is in taking two new criteria, which reflect the quality of the feed blend, the criterion which takes into account the maximum digestibility of feed blend, and the criterion requesting that optimal blend contains minimum share of water. We also show the way how the decision maker can make his/her decisions interactively changing the priorities of posted goals or learning about the optimal decision through the formulation of the model.

3. Input data for determination of feed blend for pig fattening (PS-2):

Our case processes the given data required to work out the optimal feed plan (feed blend) for pig fattening PS-2. The mark PS-2 represents the blend recipe for pigs of 20–50 kg, while PS-1 stands for the recipe for pigs up to 20 kg, and PS-3 for pigs of 50–100 kg, etc. The meal has to contain minimal and maximal shares of daily nutrients. Determination of maximal and minimal share of nutrients in the blend is based on scientific research. The given data are shown in Tables 1–3. The sort of feeds used to prepare the feed blend for this kind of livestock (pigs of 20–50 kg), their price per unit, and the percentage of nutrients and water per ingredient unit are shown in the Table 1. The total cost has to be minimized, the share of nutrients in the blend has to be maximized, and the share of water in the optimal meal has to be minimized. The nutrients needed in the feed used for growing pigs and the required quantities (as suggested by nutritionists) are shown in

Table 2:Need for nutrients

	Nutrients	Constraint type	Min or Max requirement -- b_k
E1	Raw protein	>=	14
E2	Pulp	<=	7

E3	Calcium—Ca	<=	0.80
E4	Phosphorus—P	>=	0.50
E5	Ash	<=	7
E6	Metionin	>=	0.50
E7	Lizin	>=	0.74
E8	Triptofan	>=	0.11
E9	Treonin	>=	0.45
E10	Izoleucin	>=	0.52
E11	Histidin	>=	0.23
E12	Valin	>=	0.46
E13	Leucin	>=	0.77
E14	Arginin	>=	0.55
E15	Fenkalanin	>=	0.54

Table 2. Some of the ingredients are required in minimal and some in maximal quantities. Table 3 is a nutrition matrix and its elements a_{ik} are the contents of a particular nutrient in the feed unit.

Table 3: Nutrition matrix (a_{ik}).

	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13
E1	11.5	8.9	17.0	33	61	38	42	12	36	13.5	12.6	11.0	42
E2	5.0	2.9	24.0	0.0	1.0	5.0	6.5	0.0	13.2	3.0	2.8	10.5	13.0
E3	0.08	0.01	1.3	1.25	7.0	0.25	0.2	0.87	0.6	0.05	0.08	0.1	0.4
E4	0.42	0.25	0.23	1.0	3.5	0.59	0.6	0.79	0.93	0.41	0.3	0.35	1.0
E5	2.5	1.5	9.6	8.0	24	4.6	6.0	9.7	7.2	2.0	1.45	4.0	7.7
E6	0.18	0.17	0.28	0.98	1.65	0.54	0.6	0.2	0.67	0.25	0.16	0.2	1.5
E7	0.53	0.22	0.73	2.6	4.3	2.4	2.7	1.1	2.12	0.4	0.4	0.4	1.7
E8	0.17	0.09	0.45	0.45	0.7	0.52	0.65	0.2	0.46	0.18	0.14	0.18	0.5
E9	0.36	0.34	0.75	1.75	2.6	1.69	1.7	0.8	1.6	0.35	0.36	0.28	1.5
E10	0.42	0.37	0.84	2.1	3.1	2.18	2.8	0.9	1.41	0.69	0.53	0.53	2.1
E11	0.23	0.19	0.35	0.86	1.93	1.01	1.1	0.2	0.95	0.17	0.27	0.18	1.0
E12	0.62	0.42	1.04	2.38	3.25	2.02	2.2	0.7	1.81	0.69	0.62	0.62	2.3
E13	0.8	1.0	1.3	3.3	4.5	2.8	3.8	1.2	2.6	1.0	0.7	0.9	2.6
E14	0.5	0.52	0.75	1.1	4.2	2.8	3.2	0.4	2.04	0.6	0.5	0.8	3.5
E15	0.62	0.44	0.91	1.58	2.8	2.1	2.1	0.4	1.41	0.78	0.62	0.62	2.2

4. Solving the problem for determination of the feed blend: Goal programming model

In agreement with the decision maker (farm owner) it was decided to try to solve the problem by formulating it into a corresponding goal programming model. The decision maker set the goal values for all the three goal functions, which were in accordance with the obtained marginal solutions. The goals were:

1. To determine the diet plan whose cost will be 1.85 monetary units (f1 = 1.85).

2. To determine the diet plan in which the share of nutrients in the feed blend will be 77 (f2 = 77).

3. To determine the diet plan in which the share of water in the feed blend will be 8.3 (f3 = 8.3).

The multi-criteria model thus includes three goal functions, 13 decision variables, and 16 constraints. Constraint (9) is the relation (3) from the starting model with the right side of constraint of 0.97. Namely, the diet plan always includes 3% of various vitamin additives disregarding the ingredients included in the optimal meal. The model also includes additional constraints on the quantity of particular sorts of feed. Namely, to make the diet plan as heterogeneous as possible the model limits the share of any ingredient to maximally 15%. Thus, if Hi is the share of i feed in the optimal blend the constraints Hi = 0.15 (i = 1, 2, …,13) are introduced.

This establishes the goal programming model in which the three goal functions are formulated into constraints (equations). Naturally, it is difficult to achieve all the three goals, therefore deviation variables are introduced, and the model goal function becomes the sum of deviation variables, which has to be minimized. All the three goal functions are formulated into constraints with both deviation variables, i.e. for each goal it is possible to obtain a solution whose value is higher or lower than the required goal value. The model becomes the goal programming problem.

Model 2:

$$\text{Min} = \sum Pj(d_j^+ + d_j^-)$$

s.t.

$$\sum c_{i1} H_i + d_1^- - d_1^+ = 1.85$$

$$\sum c_{i2} H_i + d_2^- - d_2^+ = 77$$

$$\sum c_{i3} H_i + d_3^- - d_3^+ = 8.3$$

$$\sum a_{ik} H_i \geq (\leq) b_k \qquad (k=1,2,\ldots,15)$$

$$\sum H_i = 0.97$$

$$0 \leq H_i \leq 0.15 \qquad (i=1,2,\ldots,13)$$

min = P1*d1plus + p2*d2minus + p3*d3plus;
S.t:
1.75*H1 + 1.75*H2 + 1.65*H3 + 6*H4 + 9*H5 + 2.7*H6 + 3.5*H7 + 9*H8 + 1.8*H9 + 1.8*H10 + 1.8*H11 + 3.5*H12 + 1.8*H13 + d1minus - d1plus = 1.85;
70*H1 + 80*H2 + 32*H3 + 86*H4 + 69*H5 + 92*H6 + 79*H7 + 78*H8 + 66*H9 + 79*H10 + 75*H11 + 65*H12 + 68*H13 + d2minus - d2plus = 77;
11*H1 + 12*H2 + 6.9*H3 + 8.4*H4 + 9*H5 + 10*H6 + 11*H7 + 6*H8 + 8*H9 + 12*H10 + 11.4*H11 + 10*H12 + 7*H13 + d3minus - d3plus = 8.3;
11.5*H1 + 8.9*H2 + 17*H3 + 33*H4 + 61*H5 + 38*H6 + 42*H7 + 12*H8 + 36*H9 + 13.5*H10 + 12.6*H11 + 11*H12 + 42*H13 >= 14;
5*H1 + 2.9*H2 + 25*H3 + 0*H4 + H5 + 5*H6 + 6.5*H7 + 0*H8 + 13.2*H9 + 3*H10 + 2.8*H11 + 10.5*H12 + 13*H13 <= 7;
0.08*H1 + 0.01*H2 + 1.3*H3 + 1.25*H4 + 7*H5 + 0.25*H6 + 0.2*H7 + 0.87*H8 + 0.6*H9 + 0.05*H10 + 0.08*H11 + 0.1*H12 + 0.4*H13 <= 0.8;
0.42*H1 + 0.25*H2 + 0.23*H3 + H4 + 3.5*H5 + 0.59*H6 + 0.6*H7 + 0.79*H8 + 0.93*H9 + 0.41*H10 + 0.3*H11 + 0.35*H12 + H13 >= 0.5;
2.5*H1 + 1.5*H2 + 9.6*H3 + 8*H4 + 24*H5 + 4.6*H6 + 6*H7 + 9.7*H8 + 7.2*H9 + 2*H10 + 1.45*H11 + 4*H12 + 7.7*H13 <= 7;
0.18*H1 + 0.17*H2 + 0.28*H3 + 0.98*H4 + 1.65*H5 + 0.54*H6 + 0.6*H7 + 0.2*H8 + 0.67*H9 + 0.25*H10 + 0.16*H11 + 0.2*H12 + 1.5*H13 >= 0.5;
0.53*H1 + 0.22*H2 + 0.73*H3 + 2.6*H4 + 4.3*H5 + 2.4*H6 + 2.7*H7 + 1.1*H8 + 2.12*H9 + 0.4*H10 + 0.4*H11 + 0.4*H12 + 1.7*H13 >= 0.74;
0.17*H1 + 0.09*H2 + 0.45*H3 + 0.45*H4 + 0.7*H5 + 0.52*H6 + 0.65*H7 + 0.2*H8 + 0.46*H9 + 0.18*H10 + 0.14*H11 + 0.18*H12 + 0.5*H13 >= 0.11;
0.36*H1 + 0.34*H2 + 0.75*H3 + 1.75*H4 + 2.6*H5 + 1.69*H6 + 1.7*H7 + 0.8*H8 + 1.6*H9 + 0.35*H10 + 0.36*H11 + 0.28*H12 + 1.5*H13 >= 0.45;
0.42*H1 + 0.37*H2 + 0.84*H3 + 2.1*H4 + 3.1*H5 + 2.18*H6 + 2.8*H7 + 0.9*H8 + 1.41*H9 + 0.69*H10 + 0.53*H11 + 0.53*H12 + 2.1*H13 >= 0.52;
0.23*H1 + 0.19*H2 + 0.35*H3 + 0.86*H4 + 1.93*H5 + 1.01*H6 + 1.1*H7 + 0.2*H8 + 0.95*H9 + 0.17*H10 + 0.27*H11 + 0.18*H12 + H13 >= 0.23;
0.62*H1 + 0.42*H2 + 1.04*H3 + 2.38*H4 + 3.25*H5 + 2.02*H6 + 2.2*H7 + 0.7*H8 + 1.81*H9 + 0.69*H10 + 0.62*H11 + 0.62*H12 + 2.3*H13 >= 0.46;
0.8*H1 + H2 + 1.3*H3 + 3.3*H4 + 4.5*H5 + 2.8*H6 + 3.8*H7 + 1.2*H8 + 2.6*H9 + H10 + 0.7*H11 + 0.9*H12 + 2.6*H13 >= 0.77;
0.5*H1 + 0.52*H2 + 0.75*H3 + 1.1*H4 + 4.2*H5 + 2.8*H6 + 3.2*H7 + 0.4*H8 + 2.04*H9 + 0.6*H10 + 0.5*H11 + 0.8*H12 + 3.5*H13 >= 0.55;
0.62*H1 + 0.44*H2 + 0.91*H3 + 1.58*H4 + 2.8*H5 + 2.1*H6 + 2.1*H7 + 0.4*H8 + 1.41*H9 + 0.78*H10 + 0.62*H11 + 0.62*H12 + 2.2*H13 >= 0.54;
H1 + H2 + H3 + H4 + H5 + H6 + H7 + H8 + H9 + H10 + H11 + H12 + H13 <= 0.97;
H1<=0.15; H2<=0.15; H3<=0.15; H4<=0.15; H5<=0.15; H6<=0.15; H7<=0.15; H8<=0.15; H9<=0.15; H10<=0.15; H11<=0.15; H12<=0.15; H13 <= 0.15;
All Hi>=0; i=1,2,…,13;

5. Scenarios for determination of feed blend ingredients:

Methodology and tools that are used to solve the problem:

- Method – Simplex Method.
- Software – TORA, Excel Solver, Lingo software.

In agreement with the decision maker the problem is solved in three variants (scenarios) in which all the set goals do not have equal weights. The consequence is that the objective function of the goal programming model assumes three different forms depending on the priority rank. In these scenarios the goals are ranked as follows:

Scenario A.

First priority: minimization of exceeding costs (d1+)

Second priority: minimization of nutrients shortfall (d2-).

Third priority: minimization of water excess (d3+).

In this case the model goal function assumes the following form: **Min $[P_1d_1^+ + P_2d_2^- + P_3d_3^+]$**

where P_i (i= 1,2,3) are very high positive numbers for which $P_1 >> P_2 >> P_3$. In this way the goal programming model first fulfils the first priority (i.e. tries to make the variable d_1^+ equal to zero), then the second priority (if other constraints allow it), and finally the third priority.

Min = P1*D1PLUS + P2*D2MINUS + P3*D3PLUS

Scenario A		Price – C_{i1}	Nutrients – C_{i2}	Water – C_{i3}
final Solution		(Min)	(Max)	(Min)
H1	8.24E-02	1.75	70	11
H2	0.15	1.75	80	12
H3	0	1.65	32	6.9
H4	0	6	86	8.4
H5	0	9	69	9
H6	0.1345861	2.7	92	10
H7	0	3.5	79	11
H8	0	9	78	6
H9	0.15	1.8	66	8
H10	0.15	1.8	79	12
H11	0.15	1.8	75	11.4
H12	0	3.5	65	10
H13	0.15	1.8	68	7
D1MINUS	0	1	0	0

D1PLUS	0	-1	0	0
D2MINUS	3.653377	0	1	0
D2PLUS	0	0	-1	0
D3MINUS	0	0	0	1
D3PLUS	1.511743	0	0	-1
		1.85	73.3466263	9.8117432

Scenario B.

First priority: minimization of nutrients shortfall d_2^-.

Second priority: minimization of exceeding costs d_1^+.

Third priority: minimization of water excess d_3^+.

Now the goal function is: **Min $[P_1d_2^- + P_2d_1^+ + P_3d_3^+]$**

min = P1*D2MINUS + P2*D1PLUS + P3*D3PLUS

Scenario B		Price – C_{i1}	Nutrients – C_{i2}	Water – C_{i3}
final Solution		(Min)	(Max)	(Min)
H1	4.02E-02	1.75	70	11
H2	0.15	1.75	80	12
H3	0	1.65	32	6.9
H4	6.72E-02	6	86	8.4
H5	0	9	69	9
H6	0.15	2.7	92	10
H7	0.15	3.5	79	11
H8	0	9	78	6
H9	0	1.8	66	8
H10	0.15	1.8	79	12
H11	0.15	1.8	75	11.4
H12	0	3.5	65	10
H13	0.1126057	1.8	68	7
D1MINUS	0	1	0	0
D1PLUS	0.5587333	-1	0	0
D2MINUS	0	0	1	0
D2PLUS	0	0	-1	0
D3MINUS	0	0	0	1
D3PLUS 1.954855		0	0	-1
		2.408733338	77.00000136	10.25485557

Scenario C.

First priority: minimization of water excess d_3^+.

Second priority: minimization of exceeding costs d_1^+.

Third priority: minimization of nutrients shortfall d_2^-.

In this case the function is: **Min [$P_1d_3^+ + P_2d_1^+ + P_3d_2^-$]**

min = P1*D3PLUS + P2*D1PLUS + P3*D2MINUS

Scenario C		Price – C_{i1}	Nutrients – C_{i2}	Water – C_{i3}
final Solution		(Min)	(Max)	(Min)
H1	0.00E+00	1.75	70	11
H2	0.15	1.75	80	12
H3	0.00E+00	1.65	32	6.9
H4	4.76E-02	6	86	8.4
H5	0	9	69	9
H6	0.15	2.7	92	10
H7	0	3.5	79	11
H8	0	9	78	6
H9	0.15	1.8	66	8
H10	1.50E-01	1.8	79	12
H11	4.83E-02	1.8	75	11.4
H12	0	3.5	65	10
H13	0.15	1.8	68	7
D1MINUS	0	1	0	0
D1PLUS	0	-1	0	0
D2MINUS	11.53671	0	1	0
D2PLUS	0	0	-1	0
D3MINUS	0	0	0	1
D3PLUS 1.954855		0	0	-1
		1.84999998	65.46328778	8.299999932

FINAL RESULTS:

	Feed	Scenarios		
		A	B	C
H1	Barley	8.24E-02	4.02E-02	0.00E+00
H2	Maize	0.15	0.15	0.15
H3	Lucerne	0	0	0.00E+00
H4	Powdered Milk	0	6.72E-02	4.76E-02
H5	Fish Meal	0	0	0
H6	Soya	0.134586	0.15	0.15
H7	Soya Hulls	0	0.15	0
H8	Dried Whey	0	0	0
H9	Rape Pellets	0.15	0	0.15
H10	Wheat	0.15	0.15	1.50E-01
H11	Rye	0.15	0.15	4.83E-02
H12	Millet	0	0	0
H13	Sunflower Pellets	0.15	0.112606	0.15
Cost		1.85	2.408733	1.85
Nutrients		73.34663	77	65.46329
Water		9.811743	10.25486	8.3

Table shows the obtained results for all the three scenarios.

1. In the first scenario the Cost is exactly as the required limit (1.85), there is a shortfall for the second goal of d_2^- = 77 – 73.35 = 3.65 units, and the third goal is exceeded by d_3^+ = 9.8 – 8.3 = 1.5 units.
2. In the second scenario Nutrients are exactly same as the required limit (77), the first goal is exceeded by d_1^+ = 2.4 – 1.85 = 0.55 units and third is exceeded by d_3^+ = 10.25 – 8.3 = 1.95 units.
3. In the third scenario Water is exactly same as the required limit (8.3) and the first goal is also same as the required limit (1.85) but there is a shortfall for the second goal of d_2^- = 77 – 65.46 = 12.54 units.

6.Conclusion:

The goal programming model proves to be a useful procedure in determining the optimal livestock feed blend. In a relatively simple way the decision maker is allowed to introduce into the model a number of additional requirements that are easily reformulated into mathematical form relatively easily leading to new output results. The paper shows how a standard multi-criteria problem is transformed into a goal programming model in the form of weighted and lexicographic goal programming, leaving open the issue of adequate weighing of single elements in the goal function and deviation variables. The developed model can be successfully used in solving similar problems in practice that are dependent on several criteria, e.g. feed blend for another kind of livestock of a diet plan for a larger group of people (hospitals, canteens, etc.)

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