

Today's agenda → 2d arrays dp

↳ unique paths in grid

↳ min sum path

↳ Dungeon Princess

↳ Subhash

↳ DTV graduate

↳ 5\* on codechef → ICPC regional

↳ 3yrs+ teaching exp

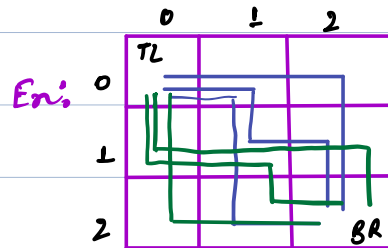
↳ youtuber → Reelcoding → Graphs (level-2) } 100K+ views

↳ SDE & instructor

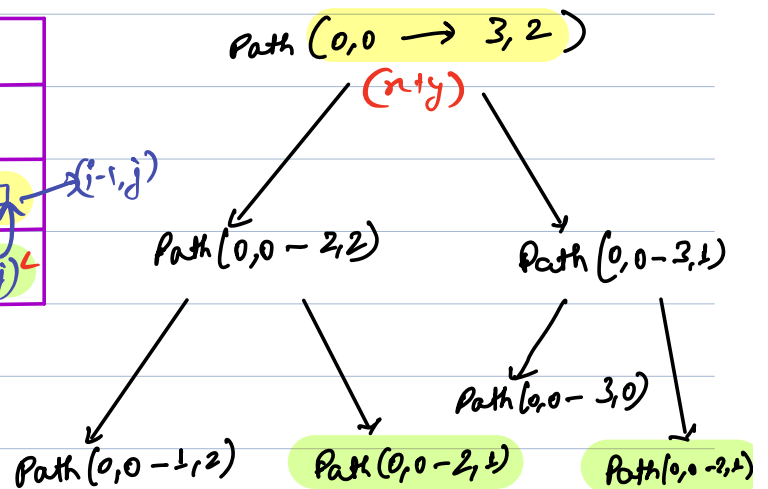
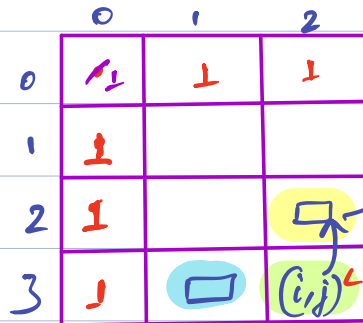
↓  
Advanced module

Q) Number of ways to go from  $(0,0) \rightarrow (N-1, M-1)$  cell.

Movement allowed: cell  $\xrightarrow{\text{1 step right}}$  or  $\downarrow \text{1 step bottom}$



Ans: 6



//Pseudo Code

int dp[N][M] = {-1}

int ways (int i, int j) {

if (i == 0 || j == 0) {return 1;}

if (dp[i][j] != -1) {return dp[i][j];}

int a = ways (i-1, j);

int b = ways (i, j-1);

dp[i][j] = a + b;

return a + b;

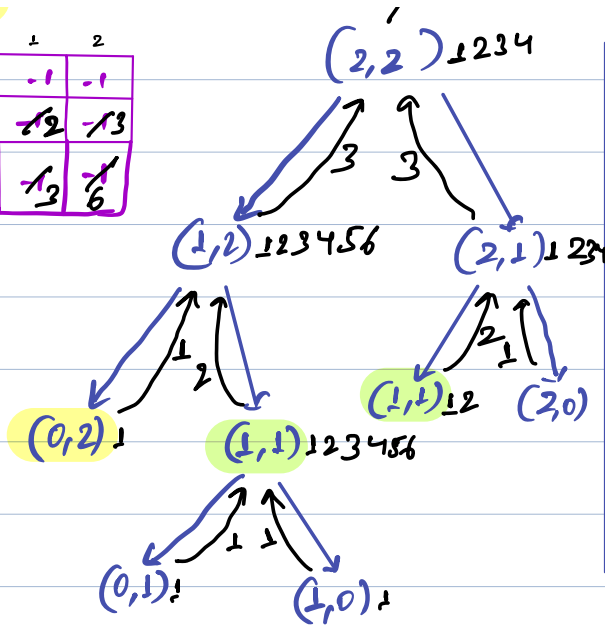
}

DRY

76

Tabulation

|   | 0  | 1  | 2  |
|---|----|----|----|
| 0 | -1 | -1 | -1 |
| 1 | -1 | 2  | 3  |
| 2 | -1 | 2  | 6  |



```

int ways (int i, int j) {
  ① if (i == 0 || j == 0) { return 1; }
  ② if (dp[i][j] != -1) { return dp[i][j]; }
  ③ int a = ways (i-1, j);
  ④ int b = ways (i, j-1);
  ⑤ dp[i][j] = a + b;
  ⑥ return a + b;
}
  
```

T.C:  $O(n \times m)$

S.C:  $O(n \times m)$  +

Stack Space

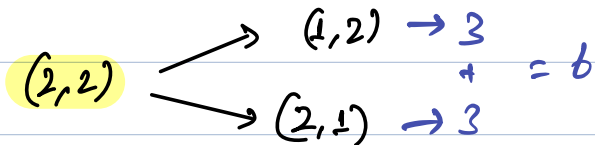
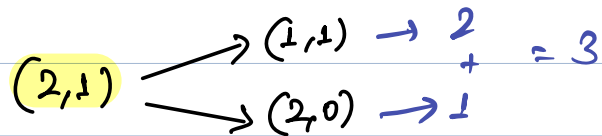
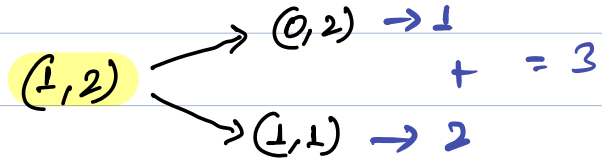
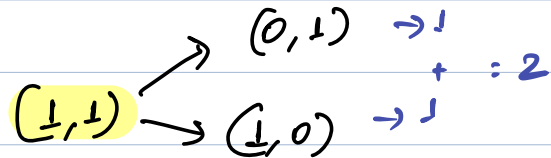
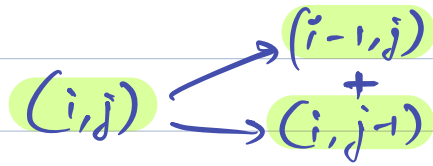
// Iterative idea

dp

|   | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 1 | 1 | 1 |
| 1 | 1 | 2 | 3 |
| 2 | 1 | 3 | 6 |

BR

Ans =  $dp[2][2] = 6$



// Pseudo code

```
int countPath ( int N, int m ) {  
    int dp[N][m];
```

```
    for (int j=0; j<m; j++) { // 1st row  
        dp[0][j] = 1  
    }
```

```
    for (int i=0; i<N; i++) { // 1st col  
        dp[i][0] = 1  
    }
```

```
    for (int i=1; i<N; i++) {  
        for (int j=1; j<m; j++) {  
            dp[i][j] = dp[i-1][j] + dp[i][j-1];  
        }  
    }
```

```
    return dp[N-1][m-1];
```

```
}
```

T.C:  $O(N*m)$     S.C:  $O(N*m)$

## Q2) Minimum Path Sum

↳ Given a 2d matrix, filled with non-negative numbers, find a path from  $(0,0) \leftrightarrow (N-1, m-1)$  which minimizes the total cost of path.  
↳ Sum of all the elements on that path.

Note: move right or down

Ex:

|   | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 2 | 3 | 5 |
| 1 | 1 | 6 | 4 |
| 2 | 9 | 2 | 7 |

Arrows indicate a path from  $(0,0)$  to  $(2,2)$ :  $(0,0) \rightarrow (1,0) \rightarrow (1,1) \rightarrow (2,1) \rightarrow (2,2)$ .

dp State:  $dp[i][j]$  = min cost to reach  $(i,j)$  from  $(0,0)$

dp expression:

$$dp[i][j] = \min(dp[i-1][j], dp[i][j-1]) + arr[i][j]$$

| arr | 0 | 1 | 2 |
|-----|---|---|---|
| 0   | 2 | 3 | 5 |
| 1   | 1 | 6 | 4 |
| 2   | 9 | 2 | 7 |

| dp | 0  | 1 | 2  |
|----|----|---|----|
| 0  | 2  | 5 | 10 |
| 1  | 3  | 9 |    |
| 2  | 12 |   |    |

Arrows indicate the calculation of  $dp[1][1]$  from  $dp[0][1]$  and  $dp[1][0]$ .

iterative

## 11Pseudo Code

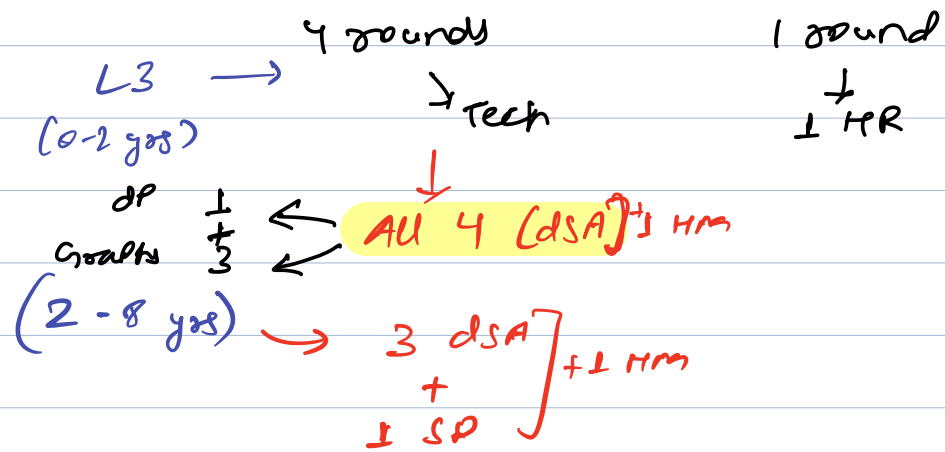
```
int minPathSum (int arr[N][M]) {  
    int dp[N][M];  
    dp[0][0] = arr[0][0];  
    for (int j=1; j<M; j++) { // 1st row  
        dp[0][j] = dp[0][j-1] + arr[0][j];  
    }  
  
    for (int i=1; i<N; i++) {  
        dp[i][0] = dp[i-1][0] + arr[i][0];  
    }  
  
    for (int i=1; i<N; i++) {  
        for (int j=1; j<M; j++) {  
            dp[i][j] = min(dp[i-1][j], dp[i][j-1]) +  
                arr[i][j];  
        }  
    }  
    return dp[N-1][M-1];  
}
```

T.C :  $O(N*M)$

S.C :  $O(N*M)$

Break till 10:30 PM





## Q) Dungeon Princess

↳ Given  $mat[N][M]$  where each cell indicates health gained.

→ Find out min health required at  $(0,0)$  so that we can reach  $[N-1, M-1]$ .

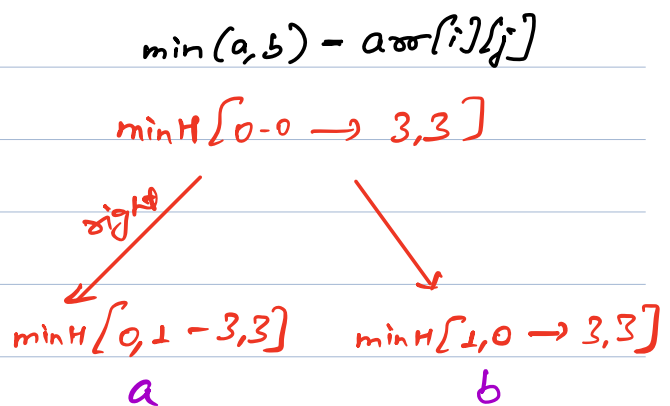
Note: ① movements: right or bottom.

② if health reaches 0, at any place you are dead.

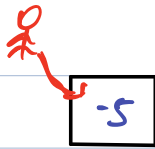
③ we are starting at  $(0,0)$ .

Ex:  $\begin{matrix} & 0 & 1 \\ 0 & -3 & -5 \\ 1 & -2 & 1 \end{matrix}$   $\begin{matrix} +4 & -5 & 6 \end{matrix}$

Ex:  $\begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & -3 & -2 & 4 & -7 \\ 1 & -6 & 5 & -4 & 6 \\ 2 & -15 & -8 & 3 & -4 \\ 3 & 7 & 4 & -2 & -7 \end{matrix}$

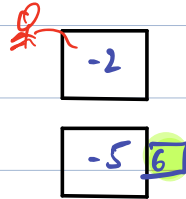


Case 1:



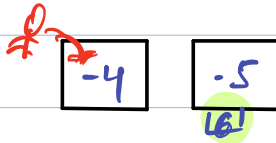
$$H + (-5) = 1 \Rightarrow H = 1 - (-5) = 6$$

Case 2:



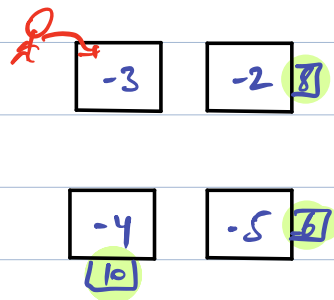
$$H + (-2) = 6 \Rightarrow H = 6 - (-2) = 8$$

Case 3:



$$H + (-4) = 6 \Rightarrow H = 6 - (-4) = 10$$

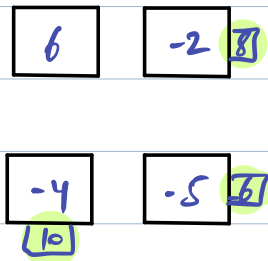
Case 4:



$$H + (-3) = 8$$

$$H = 8 - (-3) = 11$$

Case 5:

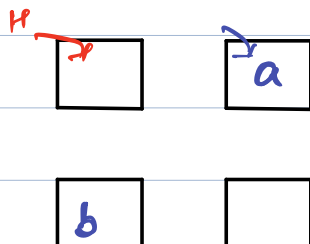


$$H + 6 = \min(8, 10)$$

$$H = 8 - 6 = 2$$

$$\min(a, b) - \text{arr}[i][j]$$

generic case:



$$H + \text{arr}[i][j] = \min(a, b)$$

$$H = \min(a, b) - \text{arr}[i][j]$$

$$H = \max(\min(a, b) - \text{arr}[i][j], 1)$$

Case 6:

|    |    |   |
|----|----|---|
| 16 | -2 | 8 |
| -4 | -5 | 6 |

$H = \min(8, 10) - 16$   
 $= \max(-8, 1) = 1$

$H + 16 = 18 \Rightarrow H = 18 - 16 = 2$   
 $H + 2 = 20 \Rightarrow H = 20 - 2 = 18$

|    |     |    |
|----|-----|----|
| 16 | 2   | 18 |
| -4 | -19 | 29 |

$$H + (-4) = 20$$

$$H = 20 - (-4) = 24$$

|   | 0   | 1  | 2  | 3  |
|---|-----|----|----|----|
| 0 | -3  | 2  | 4  | -7 |
| 1 | -6  | 5  | -4 | 6  |
| 2 | -15 | -8 | 3  | -4 |
| 3 | -7  | 4  | -2 | -7 |

// Pseudo code

```
int dp[N][M] = {-1};
```

```
int minH (int mat[N][M], int i, int j) {  
    if (i >= N || j >= M) { return INT_MAX; }  
    if (i == N-1 && j == M-1) { return mat[i][j]; }  
    if (dp[i][j] != -1) { return dp[i][j]; }
```

```
    int a = minH (mat, i+1, j);
```

```
    int b = minH (mat, i, j+1);
```

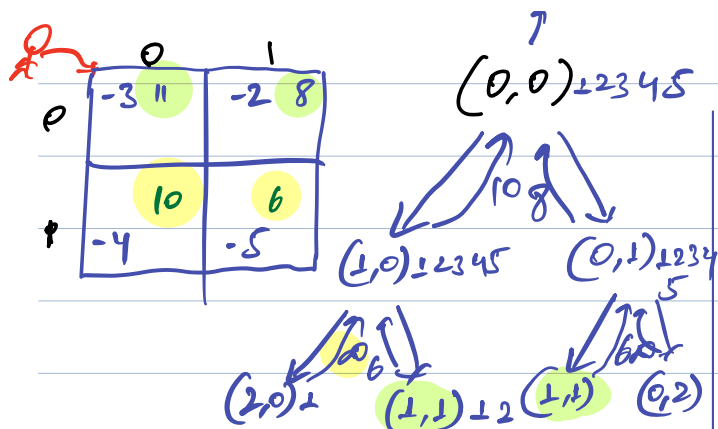
```
    dp[i][j] = min(min(a, b) - mat[i][j], 1);
```

```
    return min(min(a, b) - mat[i][j], 1);
```

}

T.C:  $O(N \times M)$

S.C:  $O(N+M)$  + Stack space



```

int minH (int mat[N][M], int i, int j) {
    ① if (i >= N || j >= M) { return INT_MAX; }
    ② if (i == N-1 && j == M-1) { return mat[i][j]; }
    ③ int a = minH(mat, i+1, j);
    ④ int b = minH(mat, i, j+1);
    ⑤ return min(a, b) - mat[i][j];
}

```

$$6 - (-4) = 1$$

$$8 - (-3) = 11$$

Doubt

arr

|   | 0   | 1  | 2  | 3  |
|---|-----|----|----|----|
| 0 | -3  | 2  | 4  | -7 |
| 1 | -6  | 5  | -4 | 6  |
| 2 | -15 | -8 | 3  | -4 |
| 3 | 7   | 4  | -2 | -7 |

dp

|   | 0 | 1  | 2  | 3  |
|---|---|----|----|----|
| 0 | 0 |    |    | 13 |
| 1 |   |    |    | 6  |
| 2 |   | 14 | 7  | 12 |
| 3 | 1 | 6  | 10 | 8  |

return  $\max(\min(a, b) - \text{mat}[i][j], 1);$

$$8 - (-2) = 10$$