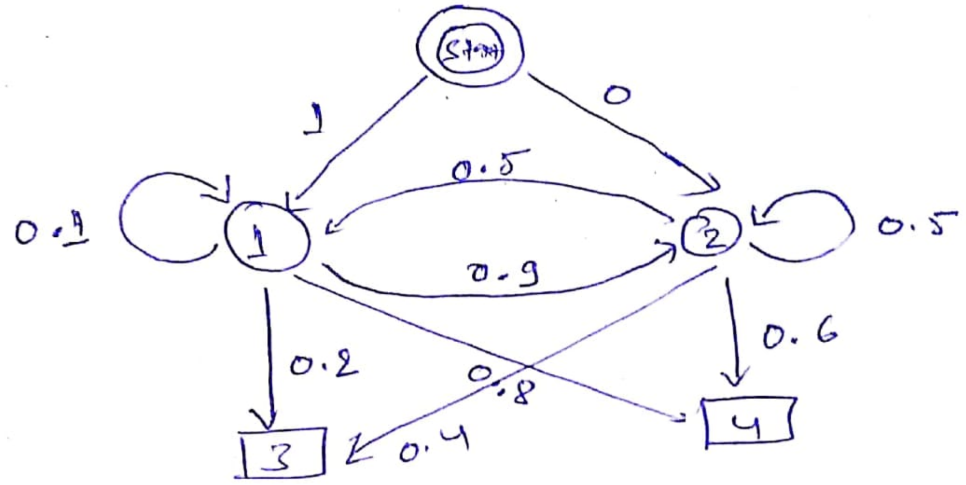
Ex - 6

1. (a) States are hidden but the output dependent on those states are visible.



(b) in HMM → states are hidden but the output is visible.
If we assume the matrix given in Q. 1 we can re-write in following form.

states = (coin 1, coin 2) → hidden state ~~hidden~~
observations = (coin 3, coin 4) → visible coins
Initial probability = { coin 1 : 1,
coin 2 : 0 }

Transition probability.

coin 1 : {coin 1 : 0.1 , coin 2 : 0.9}

coin 2 : { coin 1 : 0.5 , coin 2 : 0.5}

Emission probability

coin 1 : { coin 3 : 0.2 , coin 4 : 0.8}

coin 2 : { coin 3 : 0.4 , coin 4 : 0.6}

Start probability is that there is only chance to start with coin 1. which can be then picke again coin 1 or coin 2 with the probability of Transition probality matrix. However, whichever coin is picked from coin 1 or coin 2, there would be a emission output in form of coin 3 or coin 4 which can be decided based on emmision probability matrix

1. c

using Bayes formula.

$$P\big((q_1,q_2) \mid (O_1,O_2)=(T,T)\big)$$

$$= \frac{P\big((O_1,O_2)=(T,T) \mid (q_1,q_2)\big)\, P(q_1,q_2)}{P\big((O_1,O_2)=(T,T)\big)}$$

⇒ To find $P(q_1,q_2)$

$$= P(q_2 \mid q_1)\, P(q_1)$$

q_1	q_2	$P(q_1)$	$P(q_2 \mid q_1)$	$P(q_2 \mid q_1)\,P(q_1)$
1	1	1	0.1	0.1
1	2	1	0.9	0.9
2	1	0	0.5	0
2	2	0	0.5	0

⇒ To find

$$P\big((O_1,O_2)=(T,T) \mid (q_1,q_2)\big) =$$

~~just given a same~~

q_1	q_2	$P\big((O_1,O_2)=(T,T) \mid (q_1,q_2)\big)$
1	1	$0.8 \times 0.8 = 0.64$
1	2	$0.8 \times 0.6 = 0.48$
2	1	$0.8 \times 0.6 = 0.48$
2	2	$0.8 \times 0.8 = 0.64$

Now the numerator.

$p((o_1, o_2) = (T, T) \mid (q_1, q_2)) \; p(q_1, q_2) = N$

q_1	q_2	N	
1	1	0.64×0.1 =	0.064
1	2	0.48×0.9 =	0.432
2	1	0.48×0 =	0
2	2	0.64×0 =	0

~~hence~~ hence-forth we can remove the last two rows, as they are 0

Now denominator.

$p((o_1, o_2) = (T, T))$

This can be obtained by summing up the numerator as the probability of the $o_1 = T \mid o_2 = T$ will be the combined probility from q_1, q_2 to T

$p((o_1, o_2) = (T, T)) = \underline{0.496}$

solution

Probability of

$$P_s = P((q_1, q_2) \mid (o_1, o_2) = (\text{tails}, \text{tails}))$$

just a notation

q_1	q_2	P_s
1	1	$0.064/0.496 = 0.129$
1	2	$0.432/0.496 = 0.87$