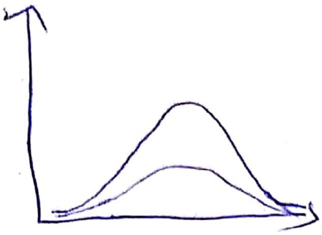
2.

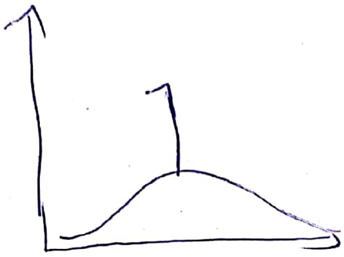
(a) $p(z(\varphi))$

$\varphi = 0$



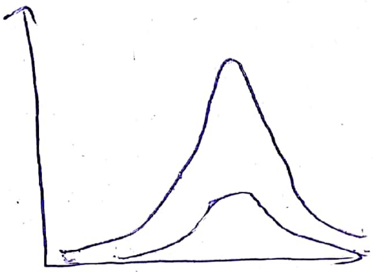
$\varphi = 0$

$\varphi = \pi/4$



$\theta = \pi/4$

$\varphi = \pi/8$



$\theta = \pi/8$

2 (b)

The projection.

$$P(z(\varphi)) = \underbrace{\tfrac{1}{2} N_1(\varphi)}_{\text{Projection of 1st comp }(\pi/4)} + \underbrace{\tfrac{1}{2} N_2(\varphi)}_{\text{Projection of 2nd com }(-\pi/4)} \quad \text{—(1)}$$

mean $= 0$ after projection.

$$\hat{\sigma}_1^2 = |\cos(\varphi - \pi/4)| \quad \text{—(2)}$$

$$\sigma_2^2 = |\sin(\varphi - \pi/4)|$$

$$p(z|\varphi)) = \frac{1}{2} N_1(0, \hat{\sigma}_1^2) + \frac{1}{2} N_2(0, \hat{\sigma}_2^2)$$

The variance

$$V[z] = \int z^2 p(z|\varphi)) \, dz$$

$$V[z] = \frac{1}{2}\hat{\sigma}_1^2 + \frac{1}{2}\hat{\sigma}_2^2 \quad \text{— (3)}$$

— by gaussian integral

To maximize $V[z]$, replacing by (2) in (3)

$$V[z] = |\cos(\varphi - \pi/4)| + |\sin(\varphi - \pi/4)|$$

in projection φ and $\varphi - \pi$ will have same variance so $\underline{\cos(\varphi - \pi/4) \geq 0}$

$$\text{so,} \quad V = \cos(\varphi - \pi/4) + |\sin(\varphi - \pi/4)|$$

a. for $\underline{\varphi \geq \pi/4}$

$$\frac{dV}{d\varphi} = -\sin(\varphi - \pi/4) + \cos(\varphi - \pi/4) = 0$$

$$= \cos(\varphi - \pi/4) = \sin(\varphi - \pi/4)$$

$$\Rightarrow \underline{\varphi = \pi/2} \quad \text{— (4)}$$

for $\varphi \leq \pi/4$

$$\frac{dV}{d\varphi} = -\sin(\varphi - \pi/4) - \cos(\varphi - \pi/4) = 0$$

$$\underline{\underline{\varphi = 0}} \quad \text{— (5)}$$

Solution :

The principle components are

$(1, 0)$ and $(0, 1)$

angles 0 and $\pi/2$

2. (c) $$\mathrm{Kurtz}[z(\varphi)] = \frac{E[(z(\varphi) - E[z(\varphi)])^4]}{(\mathrm{Var}[z(\varphi)])^2}$$

as mean $= 0$

$$E[z(\varphi)] = 0$$

$$\mathrm{Kurtz}[z(\varphi)] = \frac{E[z(\varphi)^4]}{V[z(\varphi)]^2}$$

$$= \frac{3/2\,(\hat{\sigma}_1^4 + \hat{\sigma}_2^4)}{(1/2\,\sigma_1^2 + 1/2\,\sigma_2^2)^2} \rightarrow \text{gaussian integral.}$$

Gaussian itegral

$$\alpha \int x^{2n} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \left(\frac{(2n-1)!!}{(2\alpha)^n}\right)$$

$$= \frac{3/2\,(\cos^2(\varphi - \pi/4) + \sin^2(\varphi - \pi/4))}{(1/2\,|\cos(\varphi - \pi/4) + 1/2\,(\sin(\varphi - \pi/4)))^2}$$

$$\Rightarrow \frac{3/2}{1/4 + 1/2\,|\cos(\phi - \pi/4)|\,|\sin(\phi - \pi/4)|} = \text{Kurtz}[\tau_2(\phi)]$$

To maximize the kurtz function denomenator should be minimized

so, $\underline{\underline{\phi = \pm \pi/2}}$ $\Rightarrow$ <u>solution.</u>

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