Cognitive Algorithms - Assignment 4 (30 points)

Cognitive Algorithms
Summer term 2018
Technische Universität Berlin
Fachgebiet Maschinelles Lernen

Due on June 13, 2018 10am via ISIS

After completing all tasks, run the whole notebook so that the content of each cell is properly displayed. Make sure that the code was ran and the entire output (e.g. figures) is printed. Print the notebook as a PDF file and again make sure that all lines are readable - use line breaks in the Python Code '\' if necessary. Points will be deducted, if code or content is not readable!

Upload the PDF file that contains a copy of your notebook on ISIS.

Group: 21

Members: Raj, Sourabh, Cejudo Grano de Oro, José Eduardo Peng, Yizhou Pipo, Aiko Lars Xiao, Shijian

Part 1: Theory (3 points)

A) (2 points) Explain briefly the goal of classification and regression. What is the difference between both tasks?

A classification problem consists in, given an input vector x, determining the corresponding class y that x belongs to. Therefore, the output is a discrete variable. This is equivalent to divide the high dimensional space of input vectors into regions associated to each class.

In a regression problem the goal is to, given an input vector x, predict the value for y=f(x). Hence, the output of a regression problem is a continuous variable. It can be also known as interpolation, i.e., given a data-point cloud, finding a function which approximate the behaviour of the data.

B) Which statement is true?

- [ ] Classification is a supervised learning task, regression is an unsupervised learning task.
- [ ] Classification is an unsupervised learning task, regression is a supervised learning task.
- [X] Classification and regression are both supervised learning tasks.
- [ ] Classification and regression are both unsupervised learning tasks.

Part 2: Programming (27 points)

Note that part 2 of this assignment consists of two tasks.

Task 1: Ordinary Least Squares (9 points)

In this assignment you will implement a linear regression and predict two dimensional hand positions from electromyographic (EMG) recordings obtained with high-density electrode arrays on the lower arm. Download the data set `myo_data.mat` from the ISIS web site, if not done yet.

In [11]:

```
import pylab as pl
import scipy as sp
from numpy.linalg import inv
from scipy.io import loadmat
%matplotlib inline
```

In [16]:

```
def load_myo_data(fname):
    ''' Loads EMG data from <fname>
    '''
    # load the data
    data = loadmat(fname)
    # extract data and hand positions
    X = data['training_data']
    X = sp.log(X)
    Y = data['training_labels']
    #Split data into training and test data
    X_train = X[:, :5000]
    X_test = X[:, 5000:]
    Y_train = Y[:, :5000]
    Y_test = Y[:, 5000:]
    return X_train,Y_train,X_test, Y_test

def train_ols(X_train, Y_train, llambda = 0):
    ''' Trains ordinary least squares (ols) regression
    Input:       X_train  -  DxN array of N data points with D features
                 Y        -  D2xN array of length N with D2 multiple labels
                 llabmda  -  Regularization parameter
    Output:      W        -  DxD2 array, linear mapping used to estimate labels
                             with sp.dot(W.T, X)
    '''
    #your code here
    D,N = sp.shape(X_train)
    A = X_train.dot(sp.transpose(X_train)) + llambda*sp.eye(D)
    W = inv(A).dot(X_train.dot(sp.transpose(Y_train)))
    return W

def apply_ols(W, X_test):
    ''' Applys ordinary least squares (ols) regression
    Input:       X_test    -  DxN array of N data points with D features
                 W        -  DxD2 array, linear mapping used to estimate labels
                             trained with train_ols
    Output:     Y_test    -  D2xN array
    '''
    #your code here
    D,N = sp.shape(X_test)
    Y_test = sp.transpose(W).dot(X_test)
    return Y_test

def predict_handposition():
    X_train,Y_train,X_test, Y_test = load_myo_data('myo_data.mat')
    # compute weight vector with linear regression
    W = train_ols(X_train, Y_train)
    # predict hand positions
    Y_hat_train = apply_ols(W, X_train)
    Y_hat_test = apply_ols(W, X_test)

    pl.figure(figsize=(8,6))
    pl.subplot(2,2,1)
    pl.plot(Y_train[0,:1000],Y_train[1,:1000],'.k',label = 'true')
    pl.plot(Y_hat_train[0,:1000],Y_hat_train[1,:1000],'.r', label = 'predicted')
    pl.title('Training Data')
    pl.xlabel('x position')
    pl.ylabel('y position')
    pl.legend(loc = 0)
```

```
    pl.subplot(2,2,2)
    pl.plot(Y_test[0,:1000],Y_test[1,:1000],'.k')
    pl.plot(Y_hat_test[0,:1000],Y_hat_test[1,:1000],'.r')
    pl.title('Test Data')
    pl.xlabel('x position')
    pl.ylabel('y position')

    pl.subplot(2,2,3)
    pl.plot(Y_train[1,:600], 'k', label = 'true')
    pl.plot(Y_hat_train[1,:600], 'r--', label = 'predicted')
    pl.xlabel('Time')
    pl.ylabel('y position')
    pl.legend(loc = 0)

    pl.subplot(2,2,4)
    pl.plot(Y_test[1,:600],'k')
    pl.plot(Y_hat_test[1,:600], 'r--')
    pl.xlabel('Time')
    pl.ylabel('y position')

def test_assignment4():
    ##Example without noise
    x_train = sp.array([[ 0,  0,  1 , 1],[ 0,  1,  0, 1]])
    y_train = sp.array([[0, 1, 1, 2]])
    w_est = train_ols(x_train, y_train)
    w_est_ridge = train_ols(x_train, y_train, llambda = 1)
    assert(sp.all(w_est.T == [[1, 1]]))
    assert(sp.all(w_est_ridge.T == [[.75, .75]]))
    y_est = apply_ols(w_est,x_train)
    assert(sp.all(y_train == y_est))
    print('No-noise-case tests passed')

    ##Example with noise
    #Data generation
    w_true = 4
    X_train = sp.arange(10)
    X_train = X_train[None,:]
    Y_train = w_true * X_train + sp.random.normal(0,2,X_train.shape)
    #Regression
    w_est = train_ols(X_train, Y_train)
    Y_est = apply_ols(w_est,X_train)
    #Plot result
    pl.figure()
    pl.plot(X_train.T, Y_train.T, '+', label = 'Train Data')
    pl.plot(X_train.T, Y_est.T, label = 'Estimated regression')
    pl.xlabel('x')
    pl.ylabel('y')
    pl.legend(loc = 'lower right')
```

A) (5 points) Implement ordinary least squares regression (OLS) with an optional ridge parameter by completing the function stubs `ols_train` and `ols_apply`. In `ols_train`, you estimate a linear mapping W,

$$W = (X_{\text{train}} X_{\text{train}}^\top + \lambda I)^{-1} X_{\text{train}} Y_{\text{train}}^\top$$

that optimally predicts the training labels from the training data, $X_{\text{train}} \in \mathbb{R}^{D_X \times N_{tr}}$, $Y_{\text{train}} \in \mathbb{R}^{D_Y \times N_{tr}}$. Here, $\lambda \in \mathbb{R}$ is the (optional) Ridge regularization parameter.
The function `ols_apply` than uses the weight vector to predict the (unknown) hand positions of new test data $X_{\text{test}} \in \mathbb{R}^{D_X \times N_{te}}$

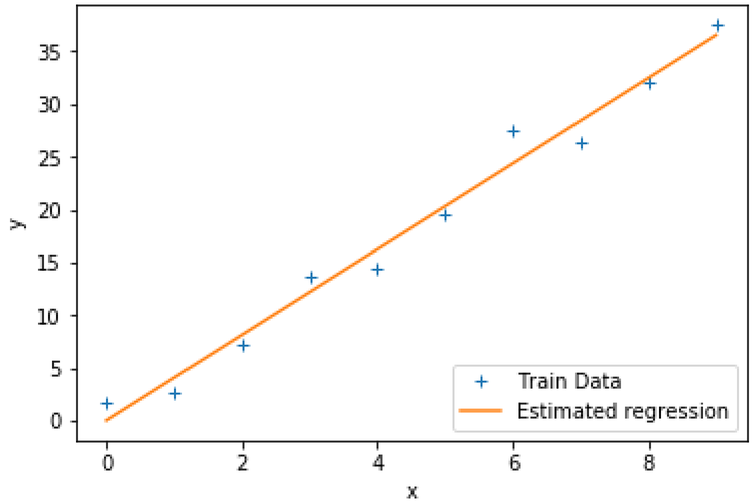
$$Y_{\text{test}} = W^\top X_{\text{test}}$$

The function `test_assignment4` helps you to debug your code.

In [13]:

```
test_assignment4()
```

```
No-noise-case tests passed
```



B) (1 point) The data set `myo_data.mat` consists of preprocessed EMG data X and 2-dimensional stimulus labels Y. Labels are x/y positions of the hand during different hand movements. The function `load_myo_data` loads the data and splits it into train and test data. Familiarize yourself with the data by answering the following questions:
How many time points N_{tr} does the train set contain? How many time points N_{te} does the test set contain? At each time point, at how many electrodes D_X was the EMG collected?

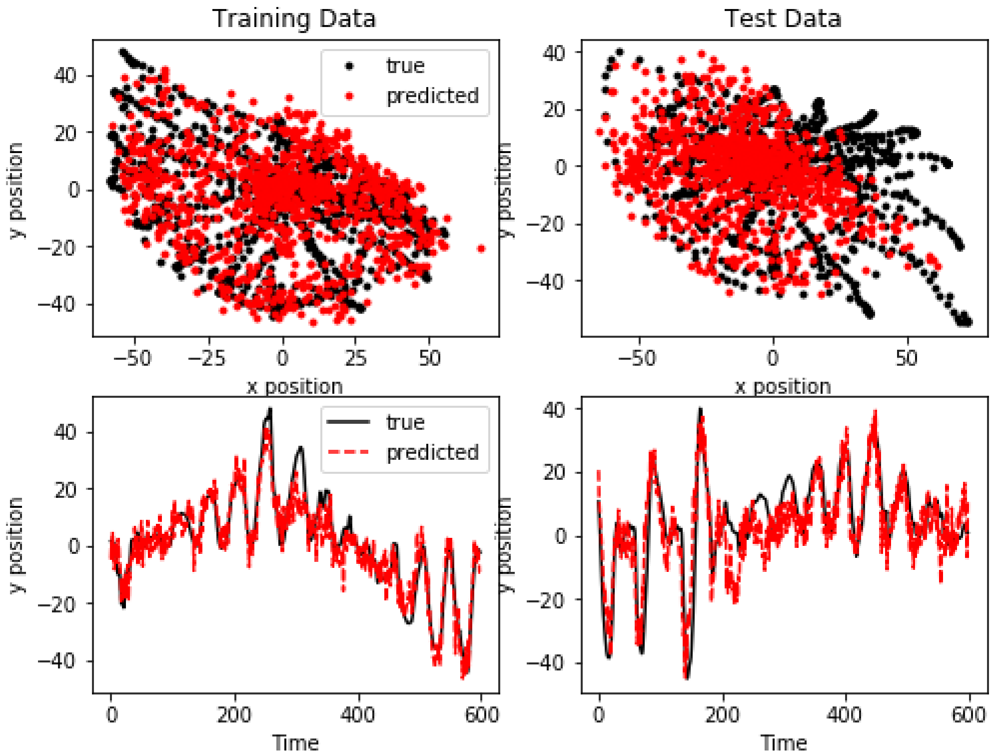
N_{tr} = **5000**
N_{te} = **5255**
D_X = **192**

C) (1 points) Predict two dimensional hand positions by calling the function `predict_handpositions`. It plots, for the train and the test data, the true hand position versus the estimated hand position. Do you notice a performance difference between train and test data set? Is this a surprising result?

Answer : **Test Data performs better than Train data, it can be said as an inbuilt property of OLS algorithm**

In [14]:

```
predict_handposition()
```

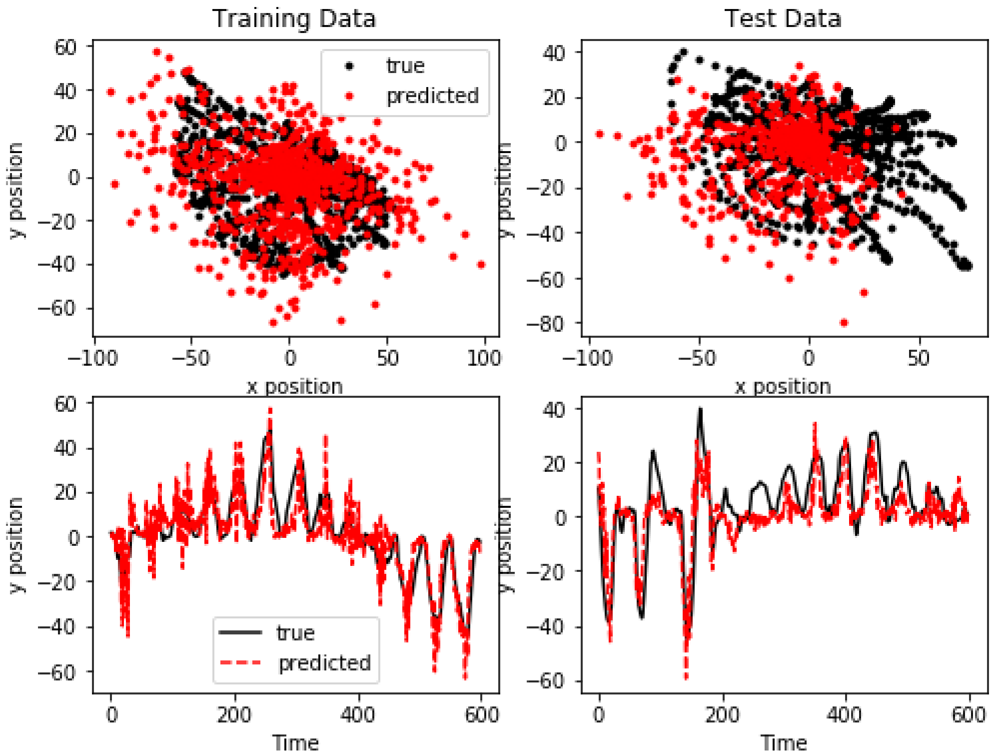


D) (1 points) In the previous tasks, we have used the logarithmized muscle activiations to predict the hand positions. Comment the line where we logarithmize the EMG features in the function `load_myo_data` and call `predict_handpositions` again. Do you notice a performance difference compared to the logarithmized version? Why?

Answer: **The performance of Training data has reduced compared to the logarithmic version. As, logarithmic function linearize the data relation and so good in performance**

In [17]:

```
predict_handposition()
```



E) (1 points) If we cannot predict the labels Y perfectly by a linear regression on X, does this imply that the relationship between X and Y is non-linear? Explain your decision.

Answer : **Yes, the relatioship between X and Y is Non-linear and that is why we take the logarithmic function to make this relation linear and so efficient.**

Task 2: Polynomial Regression (18 points)

In task 1 you implemented linear regression. However, you will see in this task, that aboves code can be generalized to polynomial regression.

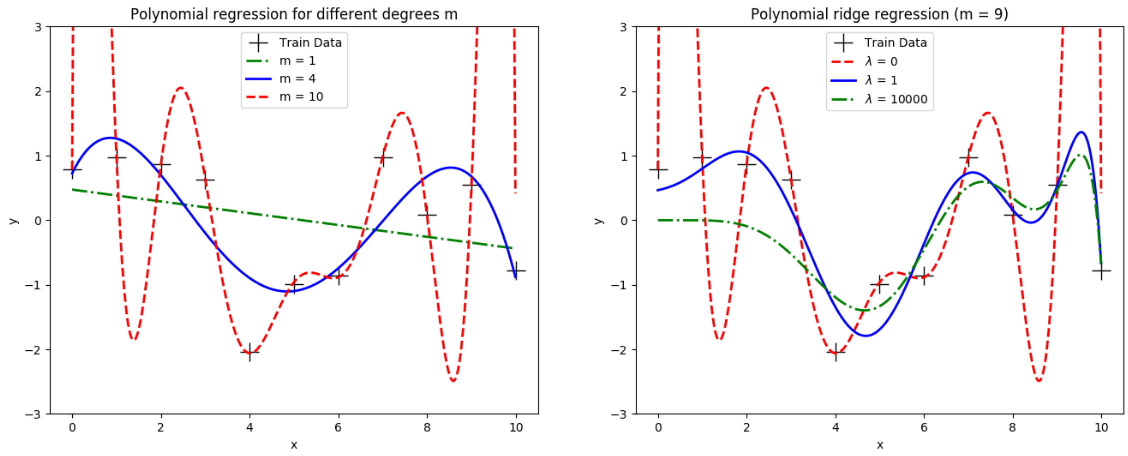
A) (9 points) Write a function `test_polynomial_regression` which generates toy data and visualizes the results from a polynomial regression. The goal is to create two plots as in the Figure below (Note that your figure will look slightly different, because the data is generated randomly.)
To do so, first create toy data from a sine function as follows:

$$x_i \in \{0, 1, 2, \ldots, 10\}, y_i = \sin(x_i) + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}(0, 0.5)$$

where $\mathcal{N}$(mean, standard deviation) denotes the Gaussian distribution and $i \in \{1, 2, \ldots, 11\}$is an index. Then implement polynomial regression, which models the relationship between y and x as an mth order polynomial, i.e. $\hat{y} = w_0 + w_1 x + w_2 x^2 + \ldots + w_m x^m$. The parameters $w_0, w_1, \ldots, w_m \in \mathbb{R}$are estimated by Ridge Regression.

Hint: You can use your functions `ols_train` and `ols_apply`, if you build an appropriate data matrix (for loops are allowed to do so).

Apply and visualize polynomial ridge regression for different parameters.


Polynomial regression for different degrees m
Train Data
m = 1
m = 4
m = 10
x
y
Polynomial ridge regression (m = 9)
Train Data
λ = 0
λ = 1
λ = 10000
x
y

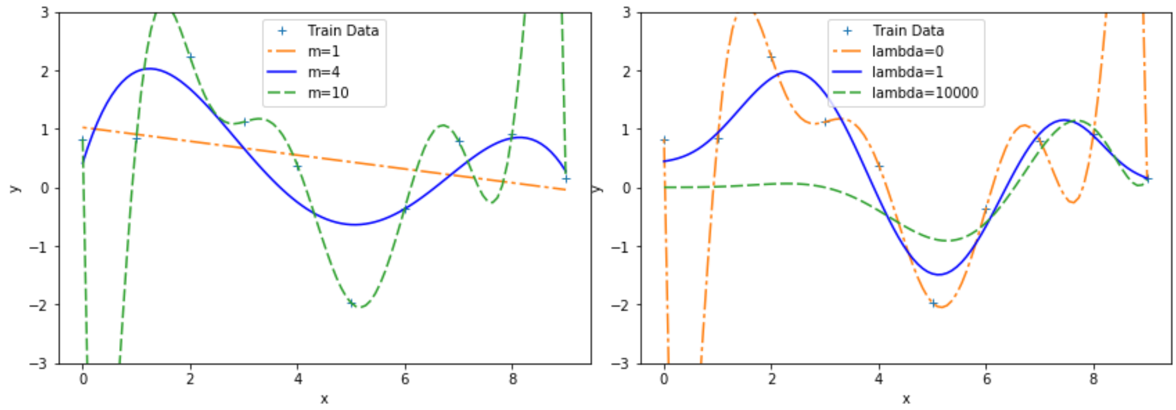
In [213]:

```
def plot_style():
    plt.tight_layout()
    pl.ylim(-3, 3)
    pl.xlabel('x')
    pl.ylabel('y')
    pl.legend(loc = 'upper center')

def datapoint():
    X = sp.arange(10)
    Y_train = sp.sin(X) + sp.random.normal(0,0.5,X.shape)
    return X,Y_train;

def getdata(X,Y_train,m,lam):
     #the data matrix contains powers of the x coordinates of the data generated
        X_train = sp.ones((1,10))
        for k in range(1,m+1):
            X_train = sp.vstack((X_train,X**k))
        W = train_ols(X_train, Y_train,llambda=lam)

        #testing
        N=100 #number of lattice nodes
        Xt = sp.linspace(0,9,N)
        X_test = sp.ones((1,N))
        for k in range(1,m+1):
            X_test = sp.vstack((X_test,Xt**k))
        Y_test = apply_ols(W, X_test)
        return Xt,Y_test;

def test_polynomial_regression():

    #generate toy data
    X,Y_train = datapoint();

    pl.figure(figsize=(12,8))
    // M subplot
    pl.subplot(2,2,1)
    pl.plot(X,Y_train,'+', label = 'Train Data')

    for m in [1,4,9] :#order of the polynomial
        #training
        #the data matrix contains powers of the x coordinates of the data generated
        Xt,Y_test = getdata(X,Y_train, m,0)

        if m == 1:
             pl.plot(Xt,Y_test, dashes=[2, 2, 10, 2], label = 'm=1')
        elif m == 4:
             pl.plot(Xt,Y_test, 'b', label = 'm=4')
        elif m == 9:
             pl.plot(Xt,Y_test, dashes=[6, 2], label = 'm=10')

    plot_style();

    #Lambda subplot
    pl.subplot(2,2,2)
    pl.plot(X,Y_train,'+', label = 'Train Data')

    for lam in [0,1,10000] :#value of lambda
        #training
        m = 9
```

```
        Xt,Y_test = getdata(X,Y_train, m,lam)
        if lam == 0:
            pl.plot(Xt,Y_test, dashes=[2, 2, 10, 2], label = 'lambda=0')
        elif lam == 1:
            pl.plot(Xt,Y_test, 'b', label = 'lambda=1')
        elif lam == 10000:
            pl.plot(Xt,Y_test, dashes=[6, 2], label = 'lambda=10000')

    plot_style();
```

In [214]:

```
test_polynomial_regression()
```



B) (9 points) Run your code of task A) multiple times.

- (4 points) What do you observe for the different values of the parameters m and λ? Explain this behaviour.
- (2 points) Decide for each of the two figures which values of the parameters yield the best fit.
- (3 points) Do you expect those parameters to perform good on all possible data sets? Explain your decision.

Answer :

1. m=1, hardly fits any data point m=4, fit couple of data points m=9, fit every data point nicelly. m basically determines the degree of polynomial to fit all the training data point but the higher degree of m may lead to overfitting of test data and may increase the test error. Also less dgree of m could lead to the problem of Underfitting. λ = 0, fits all the data nicely λ = 1, fir couple of data λ = 100000, hardly fit any data point Having low value of λ gives more flexibility but again that can lead to the overfitting problem and higher value of λ could lead to the problem of Underfitting.
2. The best fit is m=4 and λ=1.
3. No, parameters ought to change to fit the data points accuratley by avoiding both overfitting and Underfitting. These parameters will depend upon the type of data we have, suppose the data point is linear in that case lower value of m and higher value λ would be appropriate.