(1) $\frac{d^4x}{dt^4} = m^4x$

$D^4 - m^4 = 0$

$(D^2 - m^2)(D^2 + m^2) = 0$

$D = \pm m, \quad D = \pm im$

$x = C_1 \cos mt + C_2 \sin mt + C_3 e^{mt} + C_4 e^{-mt}$

$\cosh mt = \frac{e^{mt} + e^{-mt}}{2}$

$\sinh mt = \frac{e^{mt} - e^{-mt}}{2}$

$\cosh mt + \sinh mt = e^{mt}$

$\cosh mt - \sinh mt = e^{-mt}$

$\therefore x = C_1 \cos mt + C_2 \sin mt$

$+ C_3 (\cosh mt + \sinh mt)$

$+ C_4 (\cosh mt + \sinh mt)$

$= C_1 \cos mt + C_2 \sin mt + C_3 \cosh mt$

$+ C_4 \sinh mt.$

① $(D^3 - D)y = 2x+1 + 4\cos x + 2e^x$

$D^3 - D = 0 \Rightarrow D(D^2-1) = 0, \quad D = 0, D = \pm 1 \quad \therefore y_c = c_1 + c_2 e$

$$P_1 = \frac{2x+1}{D^3-D} = \frac{2x+1}{D(D^2-1)} = \frac{2x+1}{D(-1+D^2)}$$

$-1+D^2) \; 2x+1 \; (-2x$

$-2x$

1

$+1$

0

$$P_1 = \left(\frac{-2x-1}{D}\right) = -x^2 - x$$

$$P_2 = \frac{4\cos x}{D^3 - D}\Bigg|_{\substack{a=1 \\ D^2=-1}} = \frac{4\cos x}{-2D} = -2\sin x$$

$$P_3 = \frac{2e^x}{D^3-D}\Bigg|_{\substack{a=1 \\ D=1}} = \frac{2xe^x}{3D^2-1}\Bigg|_{D=1} = \frac{2xe^x}{2} = xe^x$$

② $(D^2+1)^2 y = x^4 + 2\sin x \cos 3x$

(1) $y'' - 3y' + 2y = xe^{3x} + \sin 3x$

Soln

$$D^2 - 3D + 2 = 0$$

$$(D-2)(D-1) = 0$$

$$D = 1, 2$$

$$\therefore C.F = c_1 e^{x} + c_2 e^{2x}.$$

$$P.I = \frac{xe^{3x}}{D^2 - 3D + 2} + \frac{\sin 3x}{D^2 - 3D + 2}$$

Replace $D = D+3$ Put $D^2 = -9$

$$= e^{3x} \cdot \frac{x}{(D+3)^2 - 3(D+3) + 2} + \frac{\sin 3x}{-3D - 7}$$

$$= e^{3x} \frac{x}{D^2 + 3D + 2} - \frac{(3D+7)\sin 3x}{9D^2 - 49}$$

$$= e^{3x}\left(\frac{x}{2} - \frac{3}{4}\right) - \left[\frac{9\cos 3x + 7\sin 3x}{-140}\right]$$

$$= e^{3x}\left(\frac{x}{2} - \frac{3}{4}\right) + \frac{1}{140}\left[9\cos 3x + 7\sin 3x\right]$$

$$\therefore y = C.F + P.I = c_1 e^{x} + c_2 e^{2x} + e^{3x}\left(\frac{x}{2} - \frac{3}{4}\right) + \frac{1}{140}\left[9\cos 3x + 7\sin 3x\right]$$

Side working:

$D^2 + 6D + 9 - 3D - 9 + 2$

$2 + 3D + D^2 \,)\, x \,(\, x/2 - 3/4$

$\underline{x + 3/2}$

$-3/2$

$\underline{-3/2}$

$0.$

(2) $y'' - 4y = x \sinh x$

Soln: $D^2 - 4 = 0, \quad D = \pm 2,$ $\therefore C.F = c_1 e^{2x} + c_2 e^{-2x}$

$$P.I = \frac{1}{D^2-4} x\left(\frac{e^x - e^{-x}}{2}\right)$$

$$= \frac{1}{2}\left[\frac{xe^x}{D^2-4} - \frac{xe^{-x}}{D^2-4}\right]$$

$$= \frac{1}{2}\left[e^x \frac{x}{(D+1)^2-4} - e^{-x}\frac{x}{(D-1)^2-4}\right]$$

$$= \frac{1}{2}\left[e^x \cdot \frac{x}{D^2+2D-3} - e^{-x}\cdot\frac{x}{D^2-2D-3}\right]$$

$$-3 + 2D + D^2 \overline{)\, x \;(\; -x/3 - 2/9}$$
$$x + 2/3$$
$$-2/3$$
$$-2/3$$
$$0$$

$$-3 - 2D + D^2 \overline{)\, x \;(\; -x/3 + 2/9}$$
$$x + 2/3$$
$$-2/3$$
$$-2/3$$
$$0$$