Department of Mathematics & Computing, IIT (ISM) Dhanbad

B. Tech., Semester-II, Subject: Numerical Methods

Tutorial Sheet-I

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1. Solve the following system by Gauss elimination

   (a) $2x_1 + x_2 - x_3 = -1,\ x_1 - 2x_2 + 3x_3 = 9,\ 3x_1 - x_2 + 5x_3 = 14.$ (Ans: $x_1 = 1, x_2 = -1, x_3 = 2$)

   (b) $2x_1 + 2x_2 + x_3 + 2x_4 = 7,\ x_1 - 2x_2 - x_4 = 2,\ 3x_1 - x_2 - 2x_3 - x_4 = 3,\ x_1 - 2x_4 = 0.$

   (c) $2x - 3y + z = -1,\ x + 4y - 3z = 0,\ 3x - y + 4z = 13.$ (Ans: (1,2,3))

   (d) $2y - 3z = -5,\ x + 4y - 7z + t = -8,\ 2x - y - t = -4,\ x + y + z = 6.$ (Ans: (1,2,3,4))

2. Solve the following system by Gauss-Jordan method.

   (a) $x - y + 2z = 5,\ 3x + y - z = 2,\ 2x - 3y + z = -1.$ ‘ (Ans: (1,2,3))

   (b) $2x_1 + 3x_2 + x_3 = 13,\ x_1 - x_2 - 2x_3 = -1,\ 3x_1 + x_2 + 4x_3 = 15.$ (Ans: (3,2,1))

3. Solve the system by (a) Gauss elimination (b) Gauss-Jordan method. In both the cases, check your answers by substituting them into the original equations

   $5x - 2y + z = 4,\ 7x + y - 5z = 8,\ 3x + 7y + 4z = 10$

4. Determine the approximate solution by performing three iterations of Jacobi method.

   $x_1 - 2x_2 + 5x_3 = 12,\ 5x_1 + 2x_2 - x_3 = 6,\ 2x_1 + 6x_2 - 3x_3 = 5.$ (Ans: (1.04, 0.71, 1.71)

5. Determine the approximate solution by performing three iterations of Gauss-Seidel method.

   $x - 4y + z = 5,\ 4x + 2y - z = 10,\ x + y - 4z = 0$ (Ans: (1.35,1.50,3.0))

6. Solve the following systems, viz.,

   (a) $10x + 2y + z = 9,\ 2x + 20y - 2z = -44,\ -2x + 3y + 10z = 22.$

   (b) $17x_1 + 65x_2 - 13x_3 + 50x_4 = 84,\ 12x_1 + 16x_2 + 37x_3 + 18x_4 = 25,\ 56x_1 + 23x_2 + 11x_3 - 19x_4 = 36,\ 3x_1 - 5x_2 + 47x_3 + 10x_4 = 18.$

   by (i) Jacobi's method, and (ii) Gauss-Seidel method. In each case, carry your computations to two decimal places, and proceed up to 10 iterations.

   (Ans: (a) $x = 1, y = -2, z = 3$, (b)$x_1 = 4.84, x_2 = -4.70, x_3 = -1.64, x_4 = 5.72$)

7. Find the solution to three decimals, of the system using Jacobi and Gauss-Seidel methods,

   $83x + 11y - 4z = 95,\ 7x + 52y + 13z = 104,\ 3x + 8y + 29z = 71.$ (Ans: $x = 1.06, y = 1.37, z = 1.96$)

8. Decompose the matrix

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$$

into the form LU and hence solve the system $Ax = b$ where $b = [4\ \ 8\ \ 10]^T$. Determine also L^{-1} and U^{-1} and hence find A^{-1}.

(Ans:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 7/5 & 1 & 0 \\ 3/5 & 41/19 & 1 \end{bmatrix}, U = \begin{bmatrix} 5 & -2 & 1 \\ 0 & 19/5 & -32/5 \\ 0 & 0 & 327/19 \end{bmatrix}, (x = 122/109, y = 284/327, z = 46/327)).$$

9. Solve the equations by LU factorization method.

   (a) $2x - y + z = 3,\ x + y + z = 6,\ 3x + y - z = 2.$ (Ans: (1,2,3))

   (b) $x_1 + x_2 + 2x_3 = 4,\ 3x_1 + x_2 - 3x_3 = -4,\ 2x_1 - 3x_2 - 5x_3 = -5$ (Ans: (1,-1,2))