→ Permanence Property.

x	0	2	3	5	8	9
y.						

Loss of permanence.

→ Newton Forward Interpolation

(x_i, y_i) $x_{i+1} - x_i = h$

$x_0, x_1, x_2, \ldots, x_n$

$x_1 - x_0 = h$

$x_2 - x_1 = h, \ldots x_n - x_{n-1} = h$

x	0	2	4	6	8

0	1	3	5	6	9

$y_i = f(x_i) = f_i$

x_0	x_1		x_n
y_0	y_1		y_n
$\|\|$	$\|\|$		$\|\|$
$f(x_0)$	$f(x_1)$		$f(x_n)$

E → Increment operator

$E(f(x_i)) = f(x_{i+1})$

$E^2(f(x_i)) = E \circ E(f(x_i)) = E(f(x_{i+1})) = f(x_{i+2}) = f_{i+2}$

$E^2(f_i) = f_{i+2}$

$E^k(f_i) = f_{i+k}$

$E^k(f_i) = f_{i+k} \quad \forall k$

$E^{1/2}(f_i) = f_{i+1/2}$

$E^2(f_0) = f_{1/2} = f(x_{1/2})$

$E^{-1}(f_3) = f_2$

$E^{-2}(f_3) = f_1$

(2) Forward Difference Operator

$\Delta f(x_i) = f(x_{i+1}) - f(x_i)$

$= f_{i+1} - f_i$

x_0	x_1	x_2	x_3	x_4
f_0	f_1	f_2	f_3	f_4
$f(x_0)$	$f(x_1)$			

$\Delta f_3 = f_4 - f_3$

$\Delta^2 f_i = \Delta(\Delta f_i) = \Delta(f_{i+1} - f_i)$

$= \Delta f_{i+1} - \Delta f_i$

$= f_{i+2} - f_{i+1} - (f_{i+1} - f_i)$

$= f_{i+2} - 2f_{i+1} + f_i$

$$\Delta^3 f_i = \Delta(\Delta^2 f_i)$$

$$= \Delta(f_{i+2} - 2f_{i+1} + f_i)$$

$$= \Delta f_{i+2} - 2\Delta f_{i+1} + \Delta f_i$$

$$= f_{i+3} - f_{i+2} - 2(f_{i+2} - f_{i+1}) + f_{i+1} - f_i$$

$$= f_{i+3} - 3f_{i+2} + 3f_{i+1} - f_i$$

$f_{i+3} - 3f_{i+2} + 3f_{i+1} - f_i$

→ Relation b/w Δ & E

$\Delta f_i = f_{i+1} - f_i$

$Ef_i = f_{i+1}$

$\Delta f_i = Ef_i - f_i = (E-1)f_i$

$\Delta = E - 1$

Forward Difference Table

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
x_0	f_0	$\Delta f_0 = (f_1 - f_0)$	$\Delta^2 f_0 = (\Delta f_1 - \Delta f_0)$	$\Delta^3 f_0 = \Delta^2 f_1 - \Delta^2 f_0$	$\Delta^4 f_0 = \Delta^3 f_1 - \Delta^3 f_0$
x_1	f_1	$\Delta f_1 = (f_2 - f_1)$	$\Delta^2 f_1 = \Delta f_2 - \Delta f_1$	$\Delta^3 f_1 = \Delta^2 f_2 - \Delta^2 f_1$	
x_2	f_2	$\Delta f_2 = f_3 - f_2$	$\Delta^2 f_2 = \Delta f_3 - \Delta f_2$		
x_3	f_3	$\Delta f_3 = f_4 - f_3$			
x_4	f_4				

$\Delta^4 f_0 = \Delta^3(\Delta f_0)$