03 June 2022 13:59

35.12316

35.123168

35.12317

Solution of linear ~~equations~~ system of eq^n

Direct
- Gauss Elim.
- " Jordan
- LU.

Iterative
- Gauss Jacobi
- Gauss Seidal

Gauss Jacobi

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$(x_1^0, x_2^0, x_3^0)$$

$$x_1^{n+1} = \frac{1}{a_{11}}\left(b_1 - a_{12}x_2^n - a_{13}x_3^n\right)$$
$$x_2^{n+1} = \frac{1}{a_{22}}\left(b_2 - a_{21}x_1^n - a_{23}x_3^n\right)$$
$$x_3^{n+1} = \frac{1}{a_{33}}\left(b_3 - a_{31}x_1^n - a_{32}x_2^n\right)$$

$n = 0, 1, 2, \ldots$

$n = 0$

$$x_1^1 = \frac{1}{a_{11}}\left(b_1 - a_{12}x_2^0 - a_{13}x_3^0\right)$$
$$x_2^1 = \frac{1}{a_{22}}\left(b_2 - a_{21}x_1^0 - a_{23}x_3^0\right)$$
$$x_3^1 = \frac{1}{a_{33}}\left(b_3 - a_{31}x_1^0 - a_{32}x_2^0\right)$$

$$(x_1^1, x_2^1, x_3^1)$$

$n=1 \quad x_1^2 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^1 - a_{13}x_3^1)$

x_1, x_2, x_3

$|x_1^{(k+1)} - x_1^{(k)}| \leq \varepsilon$

$|x_2^{(k+1)} - x_2^{(k)}| \leq \varepsilon$

$|x_3^{(k+1)} - x_3^{(k)}| \leq \varepsilon$, $\varepsilon = 5\times 10^{-(r+1)}$

upto r decimal places

eg Q 7

upto 3 decimal

$\varepsilon = 5\times 10^{-4}$

$(x^0, y^0, z^0) = (0,0,0)$

$83x + 11y - 4z = 95$

$7x + 52y + 13z = 104$

$3x + 8y + 29z = 71$

$$x^{(n+1)} = \frac{95 - 11y^{(n)} + 4z^{(n)}}{83}$$

$$y^{(n+1)} = \frac{104 - 7x^{(n)} - 13z^{(n)}}{52}$$

5×10^{-4}

$$z^{(n+1)} = \frac{71 - 3x^{(n)} - 8y^{(n)}}{29}$$

$n=0$

$x^1 = \frac{95}{83} = 1.1445$

$|x^1 - x^0| = |1.445 - 0| = 1.445$

$\rightarrow y^1 = \frac{104}{52} = 2$

$|y^1 - y^0| = |2-0| = 2$

$z^1 = \frac{71}{29} = 2.448$

$n=1 \quad x^2 = \frac{95 - 11(2) + 4(2.448)}{}$

$n=1 \quad x^2 = \frac{25-11(2)+4(2.448)}{83} = 0.997.$

$|x^2 - x^1|$

$= |0.997 - 1.445|$

$= 0.147$

$y^2 = \frac{104-7(1.445)-13(2.448)}{52}$

$= 1.023385$

$z^2 = \frac{71-3(1.445)-8(2)}{29} = 1.7784$

$|x^{k+1} - x^k| \leq \varepsilon$

$|y^{k+1} - y^k| \leq \varepsilon$

$|z^{k+1} - z^k| \leq \varepsilon.$

Gauss Seidal

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

(x_1^0, x_2^0, x_3^0)

$x_1^{n+1} = \frac{1}{a_{11}}(b_1 - a_{12}x_2^n - a_{13}x_3^n)$

$a_{21}x_1^{n+1} + a_{22}x_2^{n+1} = b_2 - a_{23}x_3^n.$

$a_{31}x_1^{n+1} + a_{32}x_2^{n+1} + a_{33}x_3^{n+1} = b_3$

$\underline{Q}$

$$83x + 11y - 4z = 95$$
$$7x + 52y + 13z = 104$$
$$3x + 8y + 29z = 71$$

$$x^{n+1} = \frac{95 - 11y^n + 4z^n}{83} \qquad (x^0, y^0, z^0)$$
$$7x^{n+1} + 52y^{n+1} = 104 - 13z^n \qquad = (0, 0, 0)$$
$$3x^{n+1} + 8y^{n+1} + 29z^{n+1} = 71$$

$n = 0$

$$x^1 = \frac{95}{83} = 1.1445$$
$$7x^1 + 52y^1 = 104 - 13z^0 = 104$$

$$y^1 = \frac{104 - 7x^1}{52} = \frac{104 - 7(1.1445)}{52}$$
$$= 1.84591$$

$$3x^1 + 8y^1 + 29z^1 = 71$$

$$z^1 = \frac{71 - 3x^1 - 8y^1}{29}$$

$$|x^1 - x^0| = |1.1445 - 0|$$

$$= \frac{71 - 3(1.1445) - 8(1.84591)}{29}$$

$$|y^1 - y^0| = |1.84591 - 0|$$

$$= 1.82065$$

Convergence of Gauss Jacobi & Gauss Seidel

$A_{A} \to n \times n$

$\downarrow$

diagonally dominant

$|a_{ii}| \geq \sum_{j \neq i} |a_{ij}| \quad \forall i$

$83x + 11y - 4z = 95$

$7x + 52y + 13z = 104$

$3x + 8y + 29z = 71$

$$\begin{pmatrix} 83 & 11 & -4 \\ 7 & 52 & 13 \\ 3 & 8 & 29 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & \cancel{7} & 6 \\ 1 & 2 & 3 \end{pmatrix}$$

$A \to n \times n$

$Ax = b$

Sufficient condⁿ for convergence of Gauss Jacobi & Seidel is that A is diagonally dominant & there is atleast one inequality involved

$|a_{ii}| \geq \sum_{j \neq i} |a_{ij}| \quad \forall i = 1, \ldots, n$

If this condition is satisfied $\Rightarrow$ Gauss Jacobi & Seidel converges.

Q 4

$$x_1 - 2x_2 + 5x_3 = 12$$
$$5x_1 + 2x_2 - x_3 = 6$$
$$2x_1 + 6x_2 - 3x_3 = 5$$

$$\left[\begin{array}{l} 5x_1 + 2x_2 - x_3 = 6 \\ 2x_1 + 6x_2 - 3x_3 = 5 \\ x_1 - 2x_2 + 5x_3 = 12 \end{array}\right.$$

$$\left[\begin{array}{l} 5x + 4y + \cancel{z} = 11 \\ x + 2y + 4z = 4 \\ x + y + 4z = 5 \end{array}\right.$$

$$\left(\begin{array}{ccc|c} 5 & 4 & 0 & 11 \\ 1 & 2 & 4 & 4 \\ 1 & 1 & 4 & 5 \end{array}\right) \quad R_2 \to R_2 - R_3$$

$$\left(\begin{array}{ccc|c} 5 & 4 & 0 & 11 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & 4 & 5 \end{array}\right)$$

$$\begin{pmatrix} a_{11} & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$