Code A

Ques No.	Option	Ans
1		True
2	D	3
3	D	a may be any number, b = 2
4	B	infinite solutions
5	C	$x \in (-\infty, 1)$
6		0.5
7	D	None of these
8	C	0
9	D	$y(x) = 0.5(3x - x^{-1})$
10	A	$y = 4 \tan 4x$ and $\|x\| < 1/8$
11	A	$(D^3 + D^2 + 4D + 4)\, y = 0$
12	C	$f(x, y) = x^2 \cos^2 y + y \sin x$ on R : $\|x\| \le 1$, $\|y\| < \infty$.
13	A	$2, 2 + 7x, 2 + 7x + 10.5x^2, 2 + 7x + 10.5x^2 + 10.5x^3$.

Code B

Ques No.	Option	Ans
1	D	None of these
2	C	0
3	D	$y(x) = 0.5(3x - x^{-1})$
4	A	$y = 4 \tan 4x$ and $\|x\| < 1/8$
5	A	$(D^3 + D^2 + 4D + 4)\, y = 0$
6	C	$f(x, y) = x^2 \cos^2 y + y \sin x$ on R : $\|x\| \le 1$, $\|y\| < \infty$.
7	A	$2, 2 + 7x, 2 + 7x + 10.5x^2, 2 + 7x + 10.5x^2 + 10.5x^3$.
8		True
9	D	3
10	D	a may be any number, b = 2
11	B	infinite solutions
12	C	$x \in (-\infty, 1)$
13		0.5

Code C

Ques No.	Option	Ans
1	A	$y = 4 \tan 4x$ and $\|x\| < 1/8$
2	A	$(D^3 + D^2 + 4D + 4)\, y = 0$
3	C	$f(x, y) = x^2 \cos^2 y + y \sin x$ on R : $\|x\| \le 1$, $\|y\| < \infty$.
4	A	$2, 2 + 7x, 2 + 7x + 10.5x^2, 2 + 7x + 10.5x^2 + 10.5x^3$.
5		True
6	D	3
7	D	None of these
8	C	0
9	D	$y(x) = 0.5(3x - x^{-1})$
10	D	a may be any number, b = 2
11	B	infinite solutions
12	C	$x \in (-\infty, 1)$
13		0.5

Code D

Ques No.	Option	Ans
1	B	infinite solutions
2	C	$x \in (-\infty, 1)$
3		0.5
4	D	None of these
5	C	0
6	D	$y(x) = 0.5(3x - x^{-1})$
7		True
8	D	3
9	D	a may be any number, b = 2
10	A	$y = 4 \tan 4x$ and $\|x\| < 1/8$
11	A	$(D^3 + D^2 + 4D + 4)\, y = 0$
12	C	$f(x, y) = x^2 \cos^2 y + y \sin x$ on R : $\|x\| \le 1$, $\|y\| < \infty$.
13	A	$2, 2 + 7x, 2 + 7x + 10.5x^2, 2 + 7x + 10.5x^2 + 10.5x^3$.

Admission No. _____ **Section** _____

Semester: Winter Code: A **Session: 2021-2022.**
Examination & Semester: Quiz-2, II Sem. B.Tech.
Subject: Mathematics-II (MCI102) , IIT (ISM) Dhanbad **Max. Marks: 13.**
Time: 30 Minutes.

Instructions: 1. All questions are compulsory.
2. Each question carries one marks.
3. Each MCQ has only a single correct option

Q.1. Consider $\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+q(x)y=0$, $x \in I$, where $I \subset \mathbb{R}$ is an interval and $p(x)$, $q(x)$ are continuous functions on I. Decide if the following statement is true or false. If $y_1(x)$ and $y_2(x)$ are linearly independent solution of the above ODE on an interval I, they are linearly independent on any interval contained in I. If it is false, give a counterexample.
_____ [1]

Q.2. If $x^2+xy^2=c$, where c is an arbitrary constant, is the general solution of the exact differential equation $M(x,y)dx+2xydy=0$, then $M(1,-1)$ is
(A) 0 (B) 1 (C) 2 (D) 3. [1]

Q.3. $(3a^2x^2+by\cos x)+(2\sin x-4ay^3)\frac{dy}{dx}=0$ is exact for
(A) a may be any number, $b=3$
(B) $a=3,\ b=4$
(C) $a=2,\ b=5$
(D) a may be any number, $b=2$ [1]

Q.4. Given initial value problem $2x\frac{dy}{dx}=3(2y-1)$ defined on the real line $\mathbb{R}$, such that $y(0)=\frac{1}{2}$, then it has
(A) unique solution (B) infinite solutions (C) no solution (D) finite solutions [1]

Q.5. The solution of the differential equation $\frac{dy}{dx}=y^2,\ \ y(0)=1$ exists for all
(A) $x \in (-\infty,\infty)$
(B) $x \in (0,a]$, where $a>1$
(C) $x \in (-\infty,1)$
(D) $x \in [1,a]$, where $a>1$ [1]

Q.6. Find largest b such that in the interval $[0,b]$, the existence of the solution is assured by Picard's Theorem for the following IVP:

$$\frac{dy}{dx}=y^2+\cos^2 x;\ \ x>0;\ y(0)=0$$

$b=$ _____ [1]

Q.7. Which of the following functions is **not** an integrating factor for the differential equation $ydx - xdy = 0$?
(A) $-1/x^2$ (B) $1/y^2$ (C) $1/xy$ (D) None of these. [1]

Q.8. Consider two solutions $y_1(t)$ and $y_2(t)$ of differential equation y''–$3y' + 2y = 0$ such that $y_1(0) = 2$, $y_1'(0) = 2$, $y_2(0) = 1$ and $y_2'(0) = 1$. The Wronskian $W(y_1, y_2)$ at the point $t = -1$ is
(A) $2e^3$ (B) $2/e^3$ (C) 0 (D) e^2 [1]

Q.9. If a set of functions $\{x, 1/x\}$ forms a basis of solutions of the equation $x^2y''+xy'-y = 0$. A solution when $y(1) = 1, y'(1) = 2$ is
(A) $y(x) = 0.5(3x - x)$
(B) $y(x) = 0.5(2x - x^{-1})$
(C) $y(x) = 0.5(2x - x)$
(D) $y(x) = 0.5(3x - x^{-1})$ [1]

Q.10. Find the solution and the largest possible interval for x, for which the existence theorem guarantees at least one solution of the IVP $y' = 16 + y^2, y(0) = 0$.
(A) $y = 4\tan 4x$ and $|x| < 1/8$
(B) $y = 4\tan 4x$ and $|x| < 1/4$
(C) $y = 2\tan 4x$ and $|x| < 1/2$
(D) None of the above. [1]

Q.11. The differential equation whose linearly independent solutions are $\cos 2x, \sin 2x$, and e^{-x} is $\left[Assume \;\; D \equiv \frac{d}{dx}\right]$.

(A) $(D^3 + D^2 + 4D + 4)\, y = 0$
(B) $(D^3 - D^2 + 4D - 4)\, y = 0$
(C) $(D^3 + D^2 - 4D - 4)\, y = 0$
(D) $(D^3 - D^2 - 4D + 4)\, y = 0$ [1]

Q.12. Which of the following function satisfies the Lipschitz condition (with respect to y) on R?
(A) $f(x, y) = y^{2/3}$ on $R : |x| \le 1, |y| \le 1$.
(B) $f(x, y) = y^{1/2}$ on $R : |x| \le 1, 0 \le y < \infty$.
(C) $f(x, y) = x^2\cos^2 y + y\sin x$ on $R : |x| \le 1, |y| < \infty$.
(D) None of these [1]

Q.13. The first four successive approximations of solutions of the IVP $y' = 3y+1,\ y(0) = 2$ are
(A) 2, $2 + 7x$, $2 + 7x + 10.5x^2$, $2 + 7x + 10.5x^2 + 10.5x^3$.
(B) 2, $2 + 7x$, $2 + 7x - 10.5x^2$, $2 + 7x - 10.5x^2 + 10.5x^3$.
(C) 2, $2 - 7x$, $2 - 7x - 10.5x^2$, $2 - 7x - 10.5x^2 - 10.5x^3$.
(D) None [1]