Department of Mathematics and Computing

B. Tech., Semester-2, Subject: Numerical Methods

Tutorial Sheet-IV

Note: Answers are given at the end of each question.

1. Given $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$, obtain the Taylor's series for $y(x)$ and compute $y(0.1)$ correct to four decimal places. [Answer: 1.1053]

2. Compute the values of $y(1.1)$ and $y(1.2)$ using Taylor's series method for the solution of the problem $y'' + y^2y' = x^3,\ y(1) = 1, y'(1) = 1$. [Answer: 1.1002, 1.2015]

3. Apply Taylor's series method of second order to integrate $\frac{dy}{dx} = 2x + 3y$, $y(0) = 1$, $x \in [0, 0.4]$ with $h = 0.1$. [Answer: 1.355, 1.855475, 2.551614, 3.510921]

4. Using the fourth order Taylor's series method, solve the equation $\frac{dy}{dx} = 3x + y^2$ for $x = 0.1$ given that $y(0) = 1$. [Answer: 1.127]

5. Solve the initial value problem $\frac{dy}{dt} = \frac{t}{y}$, $y(0) = 1$ using Euler's method with $h = 0.2$ to get $y(0.2)$.

6. Consider the initial value problem $\frac{dy}{dx} = x(y + x) - 2$, $y(0) = 2$. Use Euler's method with step sizes $h = 0.3$, $h = 0.2$, $h = 0.15$ to compute $y(0.6)$ correct to five decimal places.
   [Answer: 0.95300, 1.00576, 1.03273]

7. Given the differential equation by $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$, compute $y(0.02)$ using Euler's modified method. [Answer: 1.0202]

8. Solve by Euler's modified method, the problem $\frac{dy}{dx} = x + y$, $y(0) = 0$. Choose $h = 0.2$ and compute $y(0.2)$, $y(0.4)$. [Answer: 0.0222, 0.0938]

9. Use Runge-Kutta fourth order method to find $y(0.2)$ and $y(0.4)$ given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$. [Answer: 1.19598, 1.3751]

10. Use Runge-Kutta fourth order formula to find $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx} = 1 + \frac{2xy}{1 + x^2}$, $y(0) = 0$. [Answer: 0.1006, 0.2052]

[P.T.O.]

11. Find the solution of the Laplace equation $u_{xx} + u_{yy} = 0$ in the given region R , subject to the boundary conditions, using the standard five point formula.

(a) R is a square of side 3 units. Boundary conditions are $u(0, y) = 0, u(3, y) = 3+y,\ u(x, 0) = x, u(x, 3) = 2x$. Assume step length as $h = k = 1$. [Answer: $u_1 = 5/3, u_2 = 10/3, u_3 = 4/3, u_4 = 8/3$.]

(b) R is a square of side 1 unit. $u(x, y) = x - y$ on the boundary. Assume $h = k = 1/3$. [Answer: $u_1 = -(1/3), u_2 = 0, u_3 = 0, u_4 = 1/3$. (by symmetry $u_2 = u_3$)]

12. Find the solution of the Poisson's equation $u_{xx} + u_{yy} = G(x, y)$ in the given region R , subject to the boundary conditions.

(a) $R : 0 \leq x \leq 1, 0 \leq y \leq 1$. $G(x, y) = 4$. $u(x, y) = x^2 + y^2$ on the boundary and $h = k = 1/3$. [Answer: $u_1 = (5/9), u_2 = 8/9, u_3 = 2/9, u_4 = 5/9$.]

(b) $R : 0 \leq x \leq 1, 0 \leq y \leq 1$. $G(x, y) = 3x + 2y$. $u(x, y) = x - y$ on the boundary and $h = k = 1/3$. [Answer: $u_1 = -(101/216), u_2 = -35/216, u_3 = -25/216, u_4 = 41/216$.]

(c) $R : 0 \leq x \leq 3, 0 \leq y \leq 3$. $G(x, y) = x^2 + y^2$. $u(x, y) = 0$ on the boundary and $h = k = 1$. [Answer: $u_1 = -(5/2), u_2 = -13/4, u_3 = -7/4, u_4 = -5/2$. (by symmetry we can start by setting $u_1 = u_4$]