LU Decomposition

$A \to$ square matrix

if principal minors are non singular

$\Rightarrow$ LU decomposition of A exists

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{32} \end{bmatrix}$$

$|a_{11}| \neq 0$

$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$

$|A| \neq 0$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{32} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$\Downarrow$

Decomposition is unique

$||$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \to \text{infinitely}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

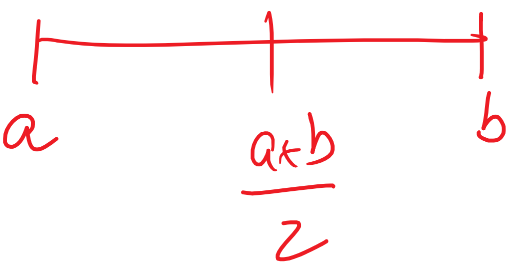
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}\begin{bmatrix} b & c \\ 0 & d \end{bmatrix}$$

$$\begin{bmatrix} a & 0 \\ b & c \end{bmatrix}\begin{bmatrix} d & e \\ 0 & f \end{bmatrix} \qquad = \begin{bmatrix} b & c \\ ab & ac+d \end{bmatrix} \qquad \begin{array}{l} b=0 \\ ab=0 \neq 1 \end{array}$$

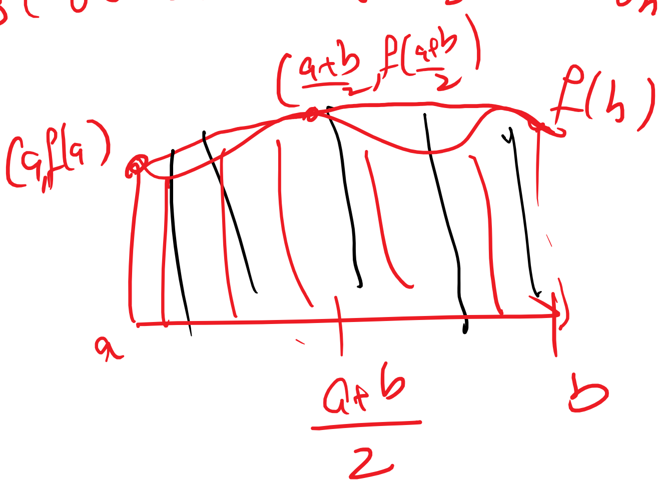
→ Simpson's $\frac{1}{3}$ Rule.

$$\int_a^b f(x)\,dx = \frac{h}{3}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right)$$



n: even.

$$= \frac{h}{3}\left(y_0+y_n+4(y_1+y_3+\dots+y_{n-1})+2(y_2+y_4+\dots+y_{n-2})\right)$$



$$- \quad I = \frac{h}{3}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right)$$

$$= \frac{b-a}{6}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right)$$

$\int_a^b f(x)\,dx$.

1. $f(x)=c$

2. $f(x)=cx$

$$f(x)=c \Rightarrow \int_a^b f(x)\,dx = c(b-a)$$

$$I = \frac{b-a}{6}(c+4c+c) = c(b-a)$$

$$f(x)=cx \Rightarrow \int_a^b f(x)\,dx = \frac{c(b^2-a^2)}{2}$$

$$I = \frac{b-a}{6}\left(ca+4c\left(\frac{a+b}{2}\right)+cb\right)$$

$$= \frac{b^2-a^2}{2}$$

$$f(x)=cx^2 \qquad \int_a^b f(x)\,dx = \frac{c(b^3-a^3)}{3}$$

$$I = \frac{b-a}{6}\left(ca^2+4c\left(\frac{a+b}{2}\right)^2+cb^2\right)$$

$$= \frac{c(b-a)}{6}\left(a^2+(a+b)^2+b^2\right)$$

$$= \frac{c(b-a)}{6}\left(a^2+ab+b^2\right) = \frac{c(b^3-a^3)}{3}$$

$$f(x)=cx^3 \qquad \int_a^b f(x)\,dx = \frac{c(b^4-a^4)}{4}$$

$$I = \frac{b-a}{6}\left(ca^3+4c\left(\frac{a+b}{2}\right)^3 \quad +cb^3\right)$$

$$I = \frac{b-a}{6}\left(ca^3 + 4c\left[\frac{a+b}{2}\right]^3 + cb^3\right)$$

$$= \frac{c(b-a)}{6}\left(\_\_\_\_\_\_ \right) = \frac{c(b^4-a^4)}{4}$$

$f(x) = cx^4$ $\quad \int_a^b f(x)dx \neq I$

Degree of precision in Simpson's $\frac{1}{3}$ Rule

$\rightarrow 3$

Simpson's $\frac{3}{8}$ Rule

$n = 3$

$$\int_{x_0}^{x_n} f(x)dx \doteq nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 - \right)$$

(x_0, x_1, x_2, x_3)

x_0	y_0			
		Δy_0		
x_1	y_1		$\Delta^2 y_0$	
		Δy_1		$\Delta^3 y_0$
x_2	y_2		$\Delta^2 y_1$	
		Δy_2		
x_3	y_3			

$$\int^{x_3} f(x)\,dx = 3h\left[y_0 + \frac{3}{2}\Delta y_0 + \frac{3(3)}{\ }\Delta^2 y_0 + 3 \ \ 3.1\right.$$

$$\int_{x_0}^{x_3} f(x)\,dx = 3h\left[y_0 + \frac{3}{2}\Delta y_0 + \frac{3(3)}{12}\Delta^2 y_0 + \frac{3}{24}\Delta^3 y_0 \right)$$

$$= 3h\left[y_0 + \frac{3}{2}(y_1 - y_0) + \frac{9}{12}(y_2 - 2y_1 + y_0) \right)$$

$$+ \frac{3}{24}(y_3 - 3y_2 + 3y_1 - y_0)$$

$= - - - -$

$$= \frac{3h}{8}(y_0 + 3y_1 + 3y_2 + y_3)$$

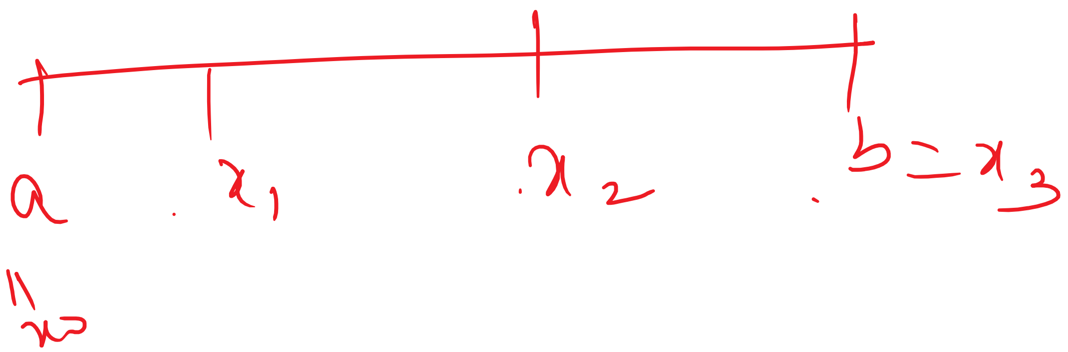
$$h = \frac{b-a}{3}$$

$$\downarrow$$

$$\frac{3}{8}\frac{(b-a)}{3}(y_0 + 3y_1 + 3y_2 + y_3)$$

$$= \frac{b-a}{8}(y_0 + 3y_1 + 3y_2 + y_3)$$

Simple Simpson's $\frac{3}{8}$ Rule.



$a = x_0$ ... x_{n-3}

$n \to \underline{\text{multiple of } 3}$

Composite Simpson's $\frac{3}{8}$ Rule $\to n$

$$\int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_3} f(x)dx + \int_{x_3}^{x_6} f(x)dx + \cdots + \int_{x_{n-3}}^{x_n} f(x)dx$$

$$= \frac{3h}{8}(y_0+3y_1+3y_2+y_3) + \frac{3h}{8}(y_3+3y_4+3y_5+y_6)$$

$$+ \cdots \quad + \frac{3h}{8}(y_{n-3}+3y_{n-2}+3y_{n-1}+y_n)$$

$$= \frac{3h}{8}\left[y_0+y_n+3y_1+3y_2+3y_4+3y_5+\cdots+3y_{n-2}+3y_{n-1} + 2y_3+2y_6+\cdots + 2y_{n-3}\right)$$

$$= \frac{3h}{8}\left(y_0+y_n+3(y_1+y_2+y_4+y_5+\cdots+y_{n-2}+y_{n-1}) + 2(y_3+y_6+\cdots+y_{n-3})\right)$$

$$h = \frac{b-a}{n}$$