Indian Institute of Technology (ISM)-Dhanbad
Department of Mathematics & Computing
Mathematics-II (Common) (Tutorial Sheet-1)

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(1) Which of the following sets V are vector space over field $\mathbb{R}$?
(i) The set of orders pairs $V = \{(x_1, x_2) : x_1, x_2 \in \mathbb{R}\}$
(ii) The Euclidean space $V = \{(x_1, x_2, ..., x_n) \in \mathbb{R}^n : x_1, x_2, ..., x_n \in \mathbb{R}\}$
(iii) Let $V = P_2 = \{polynomial\ of\ degree\ atmost\ 2\ i.e\ f(x) = a_2x^2 + a_1x + a_0,\ where\ a_0, a_1, a_2 \in \mathbb{R}\}$, and define addition and scalar multiplication as follows:
(a)Addition. Let $f(x) = a_2x^2+a_1x+a_0$ and $g(x) = b_2x^2+b_1x+b_0$. Then $(f + g)(x) = (a_2 + b_2)x^2 + (a_1 + b_1)x + (a_0 + b_0)$,
(b) Scalar multiplication. Let $r \in \mathbb{R}$, $f(x) = a_2x^2 + a_1x + a_0$. Define $(rf)(x) = (ra_2)x^2 + (ra_1)x + (ra_0)$.
(iv) Let $V = \{x : x \in \mathbb{R}, x > 0\}$, under usual addition and scalar multiplication of real numbers.
(v) Let $V = M_{m\times n} = \{m \times n\ matrices\ with\ real\ entries\}$, under the operations addition be the addition of Matrices, and scalar multiplication be the multiplication of Matrices by scalars.

(2) Examine whether the following sets of vectors are linearly independent or not? Justify your answers.
(i) $(1,\ 3,\ 2), (2,\ 8,\ -1). (-1,\ 9,\ 2)$
(ii) $(1,\ 1,\ 3), (2,\ -1,\ 3), (0,\ 1,\ 1), (4,\ 4,\ 3)$
(iii)$(1,\ 2,\ -1,\ 6), (3,\ 8,\ 9,\ 10), (2,\ -1,\ 2,\ -2)$
(iv) $(1,\ 0,\ 0,\ 0), (0,\ 1,\ 0,\ 0), (0,\ 0,\ 1,\ 0), (0,\ 0,\ 0,\ 1)$
(v) $(2,\ 3,\ -1,\ -1), (1,\ -1,\ -2,\ -4), (3,\ 1,\ 3,\ -2), (6,\ 3,\ 0,\ -7)$.

(3) Determine whether the vector $(6,\ 10,\ 2)$ is a linear combination of the vectors $(1,\ 3,\ 2), (2,\ 8,\ -1)$ and $(-1,\ 9,\ 2)$.

(4) Let $S = \{(1,\ 2,\ 1), (1,\ 1,\ -1), (4,\ 5,\ -2)\}$. Determine which of the following vectors are in $[S]$:
(i)$(0,\ 0,\ 0)$ (ii)$(1,\ 1,\ 0)$ (iii)$(2,\ -1, -8)$ (iv)$(1,\ 0,\ 1)$ (v)$(1,\ -3,\ 5)$.

(5) Which of the following subset S form a basis for the given vector space V?
(i) $S = \{(1,\ 2,\ 3),\ (3,\ 1,\ 0),\ (-2,\ 1,\ 3)\}$, $V = \mathbb{R}^3$
(ii) $S = \{(1,\ ,1,\ 1),\ (1,\ 2,\ 3),\ (-1,\ 0,\ 1)\}$, $V = \mathbb{R}^3$
(iii) $S = \{((1,\ -1,\ 0,\ 1),\ (0,\ 0,\ 0,\ 1),\ (2,\ -1,\ 0,\ 1),\ (3,\ 2,\ 1,\ 0)\}$, $V = \mathbb{R}^4$
(iv) $S = \{(0,\ 1,\ 2,\ 1),\ (1,\ 2,\ -1,\ 1),\ (2,\ -3,\ 1,\ 0),\ (4,\ -2,\ -7,\ -5)\}$, $V = \mathbb{R}^4$
(v) $S = \{x - 1,\ x^2 + x - 1,\ x^2 - x + 1\}$, $V = P_2$, be the space of all polynomials of degree atmost 2
(vi) $S = \{1,\ x,\ (x - 1)x,\ x(x - 1)(x - 2)\}, V = P_3$, be the space of all polynomial of degree atmost 3
(vii) $S = \{1,\ x,\ (3x^2-1)/2,\ (5x^3-2x)/2,\ (35x^4-30x^2+3)/8\}, V =$

P_4, be the space of all polynomial of degree atmost 4
(viii) $S = \{(1,\ i,\ 1+i),\ (1,\ i,\ 1-i),\ (i,\ -i,\ 1)\}, V = \mathbb{C}^3$.

(6) Let $\{(1,\ 1,\ 1,\ 1),\ (1,\ 2,\ 1,\ 2)\}$ be a linearly independent subset of the vector space $V = \mathbb{R}^4$. Extend it to a basis for $V = \mathbb{R}^4$.

(7) Transform the following Matrices into row-echelon form and hence find their rank

(i) $\begin{bmatrix} 2 & 1 & 0 & 5 \\ 3 & 6 & 1 & 1 \\ 5 & 7 & 1 & 8 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 2 & 1 & -4 & 1 \\ 2 & 3 & 0 & -1 & -1 \\ 1 & -6 & 3 & -8 & 7 \end{bmatrix}$

(8) By using the Gauss-jordan Elimination method, find the inverse of the following matrices

(i) $\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 & 1 \\ 5 & 1 & -1 \\ 2 & -3 & -3 \end{bmatrix}$

(9) Find the values of a and b for which the equations $x + ay + z = 3, x + 2y + 2z = b, x + 5y + 3z = 9$ are consistent. When will these have a unique solution?

(10) Find the values of λ for which the following equations may have non-trivial solutions: $3x + y - \lambda z = 0, 4x - 2y - 3z = 0, 2\lambda x + 4y + \lambda z = 0$. For each value of λ find the solution.

(11) Investigate for what values of λ and μ the simultaneous equations $x+y+z = 6, x+2y+3z = 10, x+2y+\lambda z = \mu$, have (i) unique (ii) no solution (iii) infinite number of solution.

(12) Consider a transformation $T : \mathbb{R}^3(\mathbb{R}) \to \mathbb{R}^3(\mathbb{R})$ given by the followings. Which of them are linear transformations:
(1) $T(x, y, z) = (x+y, z, 0)$ (2) $T(x, y, z) = (1, 0, 0)$ (3) $T(x, y, z) = (xy, z, 0)$ (4) $T(x, y, z) = (x^2, y, z)$ (5) $T(x, y, z) = (e^x, 0, 0)$.

(13) Consider a transformation $T : \mathbb{C}(\mathbb{C}) \to \mathbb{C}(\mathbb{C})$ defined by $T(z) = \overline{z},\ z \in \mathbb{C}$. Check whether T is Linear transformation or not. Justify your answer.

(14) Give all the linear transformations from $\mathbb{F}(\mathbb{F})$ to $\mathbb{F}(\mathbb{F})$, where $\mathbb{F}$ is a standard notation for field.

(15) If $T : \mathbb{R}^3(\mathbb{R}) \to \mathbb{R}^2(\mathbb{R})$ is linear transformation such that $T(1, 1, 1) = (1, 2),\ T(1, 2, 1) = (3, 4),\ T(0, 0, 1) = (1, 5)$. Find $T(x, y, z)$.

(16) Consider $T : \mathbb{R}^4(\mathbb{R}) \to \mathbb{R}^5(\mathbb{R})$ is linear transformation such that $T(x, y, z, w) = (x, x+y, y, z, 0)$. Find rank and nullity of the transformation as well as a basis of null space and a basis of range space.

(17) Consider vector space V of all twice differentiable function over $\mathbb{R}$ satisfying $f'' - 2f' + f = 0$ and further let us take the transformation $T : V(\mathbb{R}) \to \mathbb{R}^2(\mathbb{R})$ such that $T(f(x)) = (f'(0), f(0))$. Find rank and nullity of transformation. Further verify rank-nullity theorem for T.

(18) Let $T : \mathbb{R}^4(\mathbb{R}) \to \mathbb{R}^4(\mathbb{R})$ be the linear map satisfying $T(e_1) = e_2, T(e_2) = e_3, T(e_3) = 0, T(e_4) = e_3$, where $\{e_1, e_2, e_3, e_4\}$ is the standard basis of $\mathbb{R}^4$. Then show that T is nilpotent and further find the index of nilpotency.