30 May 2022 15:03

→ Numerical Method.

→ 4 Units (26 L)

1. $x^2-1=0$

$\rightarrow x e^{x} = \cos x$.

$$\int_{2}^{5} e^{x^2}(\cos x)^{10} \ln x \, dx$$

$f(1.5)$

$\left.\frac{df}{dx}\right|_{x=1.5} = f(x)$

$y = \underbrace{f(x)}_{\text{unknown}}$

time

$x \rightarrow$	1	2	3
y = dist	3	5	8

$$\frac{dy}{dx} = x^2 + x^9 y^p \sin x$$

1) System of eq^n (linear)

2) Interpolation

3) num Diff & Inter

4) DE numerically.

Books

1. Jain & Iyengar - Numerical methods for scientific & engineering.

math. | Numerical Methods

2. S. S Sastry → Introductory Methods ~~to~~ of Numerical Analysis

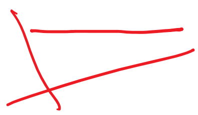
3. Gerald & Wheatley → Applied Numerical Analysis

System of linear equations.

$n \times n$

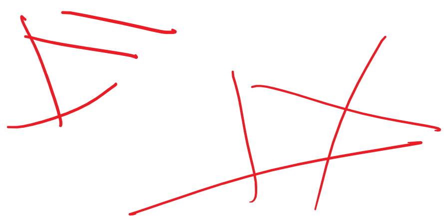
(x_1, y_1)
(x_2, y_2)

$ax + by = c$
$dx + ey = f$



Consistent — Inconsist

Consistent: unique — infinitely



$ax + by = c$
$dx + ey = f$

(x_1, y_1)
(x_2, y_2)

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\begin{cases} a x_1 + b y_1 = c \\ d x_1 + e y_1 = f \end{cases}$

$\begin{cases} a x_2 + b y_2 = c \\ d x_2 + e y_2 = f \end{cases}$

$a\frac{(x_1 + x_2)}{2} + b\frac{(y_1 + y_2)}{2} = a x_1 + b y_1 /2 + a x_2 + b y_2 /2$

$= \frac{2c}{2}$

c

$$(2x_1 - x_2 \, , 2y_1 - y_2)$$

$$a(2x_1 - x_2) + b(2y_1 - y_2)$$
$$= 2(ax_1 + by_1) - (ax_2 + by_2)$$

$$= 2c - c = c$$

$$\cancel{\left(\frac{k_1 x_1 + k x_2}{k_1}\right.} \qquad \frac{k_1 x_1 + k_2 x_2}{k_1 + k_2}$$

$$(\lambda x_1 + (1-\lambda)x_2 \, , \lambda y_1 + (1-\lambda) y_2) \quad \lambda \in \mathbb{R}$$

$$ax + by = c$$
$$dx + ey = f$$

$$a(\lambda x_1 + (1-\lambda)x_2) + b(\lambda y_1 + (1-\lambda)y_2)$$
$$= \lambda(ax_1 + by_1) + (1-\lambda)(ax_2 + by_2)$$
$$= \lambda c + (1-\lambda) = c$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

$(u_1, u_2, \ldots, u_n)$, $(v_1, v_2, \ldots, v_n)$

$(\lambda u_1 + (1-\lambda)v_1, \lambda u_2 + (1-\lambda)v_2, \ldots, \lambda u_n + (1-\lambda)v_n)$ $\lambda \in \mathbb{R}$

$$\left.\begin{aligned} x+y &= 1 \\ 2x+2y &= 3 \end{aligned}\right\} \text{No soln}$$

$$\left.\begin{aligned} x+y &= 1 \\ x-y &= 2 \end{aligned}\right\} \text{unique sol}$$

$$\left.\begin{aligned} x+y &= 1 \\ 2x+2y &= 2 \end{aligned}\right\} \to \text{inf.}$$

$Ax = b$

$b = 0$

$\downarrow$

Homogeneous.

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$A \to n \times n$

$Ax = b$

if A^{-1} exist ($\det A \neq 0$)

$x = A^{-1}b$

Numerical Methods

Direct Methods

- Gauss Elimi
- Gauss Jordan
- —

Iterative method

$Ax = b$

$A \to n \times n$

$kx + ky = 5k$ $k \neq 0$

$x + y = 5$

$2x + y = 8$

$0 \cdot (x+y) = 5 \cdot 0$

Elementary Row op.

1) $R_i \longleftrightarrow R_j$

2) $R_i \to kR_i$, $k \neq 0$

3) $R_i \to R_i + kR_j$

$x + y = 5$

$2x + y = 8$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 2 & 1 & 8 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 1 & 2 & 8 \end{array}\right]$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} y \\ x \end{bmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$x + y = 5$

$x + 2y = 8$

$Ax = b$

$$\begin{bmatrix} a_{11} & a_{12} & a_{1n} \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

$a_{11} \neq 0$

$R_2 \to R_2 - \frac{a_{21}}{a_{11}} R_1$

$R_3 \to R_3 - \frac{a_{31}}{a_{11}} R_1$,

$a_{21} - \frac{a_{21}}{a_{11}} \times a_{11}$

$a_{22}^{(1)} = a_{22} - \frac{a_{21}}{a_{11}} \times a_{12}$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22}^{(1)} & a_{23}^{(1)} & b_2^{(1)} \\ 0 & a_{32}^{(1)} & a_{33}^{(1)} & b_3^{(1)} \end{array}\right]$$

$R_3 \to R_3 - \frac{a_{32}^{(1)}}{a_{22}^{(1)}} R_2$, $a_{22}^{(1)} \neq 0$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22}^{(1)} & a_{23}^{(1)} & b_2^{(1)} \\ 0 & 0 & a_{33}^{(2)} & b_3^{(2)} \end{array}\right]$$

Backward Substitution

$a_{33}^{(2)} x_3 = b_3^{(2)}$

$x_3 = \frac{b_3^{(2)}}{a_{33}^{(2)}}$

$a_{22}^{(1)} x_2 + a_{23}^{(1)} x_3 = b_2^{(1)}$

$Ax = b$

$Ux = b'$

(Backward substitution)

$Ax = b \to Lx = b'$

(Forward)

$Ax = b \Rightarrow Ux = b'$

$U \to$ upper $\triangle$ matrix

Gaussian Elimination

$Ax = b \Rightarrow Dx = b'$

(Gauss Jordan)

$$\begin{bmatrix} d_{11} & & 0 \\ & d_{22} & \\ 0 & & d_{nn} \end{bmatrix} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$d_{11} \neq 0$

$d_{11} x_1 = b_1 \Rightarrow x_1 = \frac{b_1}{d_{11}}$

$d_{nn} x_n = b_n = x_n = \frac{b_n}{d_{nn}}$

eg

$x + 3y + 3z = 16$

$x + 4y + 3z = 18$

$$x + 3y + 4z = 19$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 4 & 19 \end{array}\right]$$

$R_2 \to R_2 - R_1$, $\quad R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$