$$l_0(x) = \frac{(x-x_1)}{x_0-x_1} \rightarrow l_0(x_0)=1,\ l_0(x_1)=0$$

$$l_1(x) = \frac{(x-x_0)}{x_1-x_0} \rightarrow l_1(x_0)=0,\ l_1(x_1)=1$$

$$P_1(x) = \frac{x-x_1}{x_0-x_1} f(x_0) + \frac{x-x_0}{x_1-x_0} f(x_1)$$

$\forall x \in [x_0, x_1]$

$P_1(x_0) = f(x_0)$, $P_1(x_1) = f(x_1)$

$f(0) = 3$

$f(1) = -1$

x	0	1
y	3	-1

$P_1(x)$

$$P_1(x) = l_0(x) f(x_0) + l_1(x) f(x_1)$$

$$= l_0(x) \times 3 + l_1(x) \times -1$$

$$l_0(x) = \frac{x-x_1}{x_0-x_1} = \frac{x-1}{0-1} = -(x-1)$$

$$l_1(x) = \frac{x-x_0}{x_1-x_0} = \frac{x-0}{1-0} = x$$

$$P_1(x) = -(x-1) \times 3 + x \times -1$$

$$= -3x + 3 \oplus -x = -4x + 3$$

$P_1(x) = -4x + 3$ $\quad P_1(0) = 3,\ P_1(1) = -1$

Quadratic Interpolation

$= f(x_0)$ $\quad l(x_1)$ $\quad l($

Quadratic Interpolation

$(x_0, y_0) = (x_0, f(x_0))$, $(x_1, y_1) = (x_1, f(x_1))$, $(x_2, y_2) = (x_2, f(x_2))$ $n = 2$

x	x_0	x_1	x_2
y	y_0	y_1	y_2

$$P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$$

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

Method-3 (Lagrange)

eg - Q2

approximate $f(1.5)$

x	x_0 -1	x_1 0	x_2 2	x_3 3
y	-8 $f(x_0)$	3 $f(x_1)$	1 $f(x_2)$	12 $f(x_3)$

$(n+1)$

$\| \quad 4 \quad n = 3$

$P_3(x) \rightarrow$ polynomial of degree ≤ 3

$$P_3(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2) + l_3(x) f(x_3)$$

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}$$

$$= -\frac{x(x-2)(x-3)}{12}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x-(-1))(x-2)(x-3)}{(0-(-1))(0-2)(0-3)}$$

$$= \frac{(x+1)(x-2)(x-3)}{6}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x-(-1))(x-0)(x-3)}{(2-(-1))(2-0)(2-3)} = \frac{(x+1)(x-2)(x-3)}{6}$

$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x-(-1))(x-0)(x-3)}{(3-(-1))(3-0)(3-2)} = \frac{x(x+1)(x-3)}{-6}$

$l_0(x) = \frac{(x+1)(x-0)(x-2)}{(-1+1)(-1-0)(-1-2)} = \frac{x(x+1)(x-2)}{12}$

$P_3(x) = l_0(x)f(x_0) + l_1(x)f(x_1) + l_2(x)f(x_2) + l_3(x)f(x_3)$

$= \frac{x(x-2)(x-3)}{12} \times (-8) + \frac{(x+1)(x-2)(x-3)}{6} \times (+3) + \frac{x(x+1)(x-3)}{-6} \times 1 + \frac{x(x+1)(x-2)}{12} \times 12$

$P_3(1.5) = \frac{(1.5)(-0.5)(-1.5)}{12} \times -8 + \frac{(2.5)(-0.5)(-1.5)}{2} + \frac{(1.5)(2.5)(-1.5)}{6} + (1.5)(2.5)(-0.5)$

$= 0.75$

$f(1.5) \cong P_3(1.5) = 0.75$

Q4

wt.	0-40	40-60	60-80	80-100	100-120
No. of students	250	120	100	70	50