24 June 2022 14:03

S	0	10	20	30	40	50	60
V	47	58	64	65	61	52	38
$\frac{1}{V}$	$\frac{1}{47}$	$\frac{1}{58}$	$\frac{1}{64}$	$\frac{1}{65}$	$\frac{1}{61}$	$\frac{1}{52}$	$\frac{1}{38}$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\frac{h}{3}\left(y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)\right)$$

Unit 4

ODE → Only one independent variable

$$\frac{dy}{dx}+\frac{dz}{dx}=x. \quad y=y(x)$$

$x \to$ ind. var.

$y \to$ dep var.

$z=z(x)$

PDE → more than one indep variable

$$z=z(x,y)$$

$x, y \to$ ind.

$z \to$ depend.

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=1$$

$$\frac{dy}{\cdot}=e^{x^3}x^4(\sin x)^{100}+\log x^{100}$$

$$\frac{dy}{dx} = e^{x^2} x y (\sin x)^{100} + \log x^{100}$$

Linear DE

degree $\rightarrow 1$

$$a_0(x)y(x) + a_1(x)y'(x) + a_2(x)y''(x) + \cdots + a_n(x)y^{n}(x) = b(x)$$

$$\frac{d^5y}{dx^5} + x^2\frac{d^3y}{dx^3} + \sin x^2 \frac{dy}{dx} = xy.$$

$\longrightarrow$

$$\frac{dy}{dx} = f(x,y), \quad a \le x \le b$$

initial value problem $\left[\; y(a) = y_0 \right.$

$x_{i+1} - x_i = h$

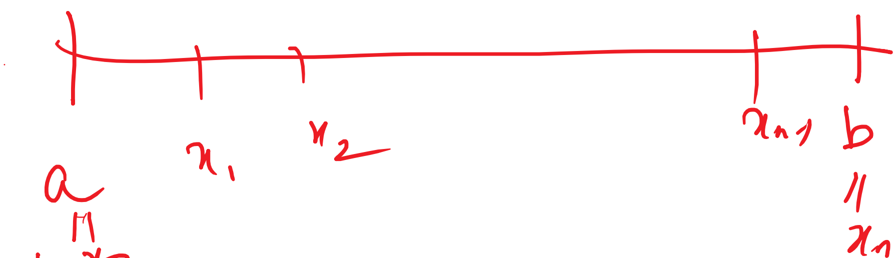
$$\frac{dy}{dx} = xy \qquad 1 \le x \le 2$$

$$\frac{dy}{y} = x\,dx$$

$$\ln y = \frac{x^2}{2} + C$$

~~$y(x) = e^{x^2/2} A$~~

$$y(x) = Ae^{\frac{x^2}{2}}$$



$y(a) = y_0$

$y(x_1) = y_1$

$y(x_2) = y_2 \ldots$ $\qquad y(x_n) = y(b) = y_n.$

$$
\begin{array}{ccc}
2 & 2.5 & 3 \\
\| & \| & \| \\
x_0 & x_1 & x_2 \\
y_0 & y_1 & y_2
\end{array}
$$

$y_0 \quad y_1 \quad y_2 \qquad y_n$

Taylors Theorem

$f : \mathbb{R} \longrightarrow \mathbb{R}$

f is $(n+1)$ diff. , f is n times cont

$x_0 \in \mathbb{R}$. $\qquad \exists\, c \in (x_0, x)$

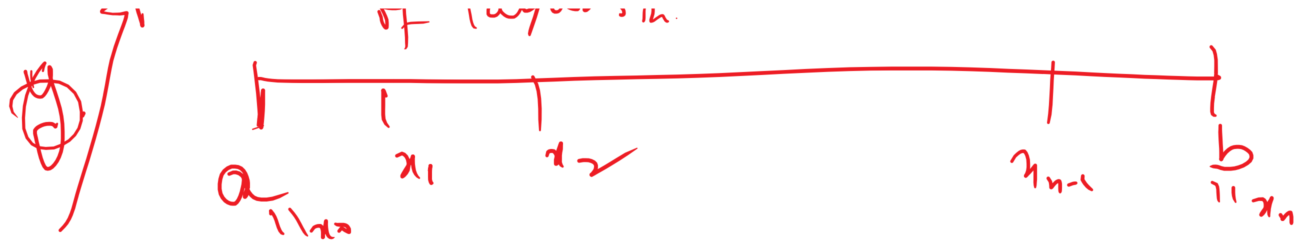
$$f(x) = f(x_0) + (x-x_0) f'(x_0) + \frac{(x-x_0)^2}{2!} f''(x_0) + \frac{(x-x_0)^3}{3!} f'''(x_0) + \cdots + \frac{(x-x_0)^n f^n(x_0)}{n!} + \frac{(x-x_0)^{n+1}}{(n+1)!} f^{n+1}(c)$$

~~$c = x_0 + \theta h$~~

$$\frac{dy}{dx} = f(x,y) \quad , \; a \le x \le b$$

$y(a) = y_0$

y satisfies all hypothesis of Taylor's Th.



$$y(x) = y(x_i) + (x-x_i)\,y'(x_i) + \frac{(x-x_i)^2}{2!}\,y''(x_i) + \cdots + \frac{(x-x_i)^n}{n!}\,y^{n}(x_i) + \frac{(x-x_i)^{n+1}}{(n+1)!}\,y^{n+1}(c_i)$$

$c_i \in (x_i, x) \subseteq (a,b)$

At $x = x_{i+1}$

$[\, y(x_i) = y_i$

$\cancel{y(x_{i+1}) = y(x}$

$$y_{i+1} = y_i + (x_{i+1}-x_i)\,y'_i + \frac{(x_{i+1}-x_i)^2}{2!}\,y''_i + \cdots + \frac{(x_{i+1}-x_i)^n}{n!}\,y^n_i + \underbrace{\frac{(x_{i+1}-x_i)^{n+1}}{(n+1)!}\,y^{n+1}(c_i)}$$

$i = 0, 1, \ldots, n-1$

$$y_{i+1} \cong y_i + h\,y'_i + \frac{h^2}{2!}\,y''_i + \cdots + \frac{h^n}{n!}\,y^n_i$$

For $n = 1$ $\Rightarrow$ Euler's Method.

↳ For $n=1$ → Euler's Method.

$\frac{dy}{dx} = f(x,y)$ ✓, $a \le x \le b$, $y(a) = y_0$.

$$y_{i+1} = y_i + h y_i', \quad i = 0, \ldots, n-1, \quad a \le x \le b$$

$$y(a) = y_0$$

$$y_{i+1} = y_i + h f(x_i, y_i), \quad i = 0, \ldots, n-1$$

$$y(a) = y_0.$$

$$y y' = x, \quad 0 \le x \le 1$$

$$y(0) = 1$$

$$y' = \frac{x}{y} \Rightarrow \frac{dy}{dx} = \frac{x}{y}, \quad 0 \le x \le 1, \quad y(0) = 1$$

$h = 0.5$

0	0.5	1
x_0	x_1	x_2

$$y_{i+1} = y_i + h f(x_i, y_i)$$

$y_0 = 1$

$$y(0.5) = y_1 = y_0 + h f(x_0, y_0)$$

$$= 1 + 0.5\left(\frac{x_0}{y_0}\right) = 1 + 0 = 1$$

$$y(1) = y_2 = y_1 + h f(x_1, y_1)$$

$$= 1 + 0.5\left(\frac{x_1}{y_1}\right) = 1 + 0.5\left(\frac{0.5}{1}\right)$$

$$= 1.25$$

$h = 0.2.$

$h = 0.2.$