

$$x + y + z = 3$$

$$2x + 2y + z = 1$$

$$4x + 3y + 3z = 2$$

$$\begin{bmatrix} 1 & 1 & 1 & | & 3 \\ 2 & 2 & 1 & | & 1 \\ 4 & 3 & 3 & | & 2 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 4R_1$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & | & 3 \\ 0 & 0 & -1 & | & -5 \\ 0 & -1 & -1 & | & -10 \\ 0 & -2 & -3 & | & 4 \end{bmatrix} \quad \begin{array}{l} \text{partial} \\ \text{pivoting} \end{array}$$

eg

Solve with
Gauss elimination with
partial pivoting

$$\begin{array}{rcl} x_1 + 10x_2 - x_3 & = & 3 \\ 2x_1 + 3x_2 + 20x_3 & = & 7 \\ 10x_1 - x_2 + 2x_3 & = & 4 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 10 & -1 & 3 \\ 2 & 3 & 20 & 7 \\ 10 & -1 & 2 & 4 \end{array} \right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 2 & 3 & 20 & 7 \\ 1 & 10 & -1 & 3 \end{array} \right]$$

$$R_2 \rightarrow R_2 - \frac{2}{10} R_1$$

$$R_3 \rightarrow R_3 - \frac{10}{10} R_1$$

$$\left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & 3.2 & 19.6 & 6.2 \\ 0 & 10.1 & -10.2 & 2.6 \end{array} \right]$$

$$R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & 10.1 & -10.2 & 2.6 \\ 0 & 3.2 & 19.6 & 6.2 \end{array} \right]$$

$$R_3 \rightarrow R_3 - \frac{3.2}{10.1} R_2$$



Gauss Jordan. $Ax=b \rightarrow Ux=b$

Gauss Jordan. $Ax=b \rightarrow Ux=b$
 $(Lx=b_1)$

$$Ax=b \rightarrow Ix=b_1$$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array} \right]$$

$$R_2 \rightarrow R_2 - \frac{a_{21}}{a_{11}} R_1$$

$$R_3 \rightarrow R_3 - \frac{a_{31}}{a_{11}} R_1$$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22}^{(1)} & a_{23}^{(1)} & b_2^{(1)} \\ 0 & a_{32}^{(1)} & a_{33}^{(1)} & b_3^{(1)} \end{array} \right]$$

$$R_1 \rightarrow R_1 - \frac{a_{12}}{a_{22}^{(1)}} R_2$$

$$R_3 \rightarrow R_3 - \frac{a_{32}^{(1)}}{a_{22}^{(1)}} R_2$$

$$\left[\begin{array}{ccc|c} a_{11} & 0 & a_{13}^{(2)} & b_1^{(2)} \\ 0 & a_{22}^{(1)} & a_{23}^{(1)} & b_2^{(1)} \\ 0 & 0 & a_{33}^{(2)} & b_3^{(2)} \end{array} \right]$$

$$R_1 \rightarrow R_1 - \frac{a_{13}^{(2)}}{a_{33}^{(2)}} R_3$$

$$R_2 \rightarrow R_2 - \frac{a_{23}^{(1)}}{a_{33}^{(2)}} R_3$$

$$\left[\begin{array}{ccc|ccc} a_{11} & 0 & 0 & b_1^{(3)} & & \\ 0 & a_{22}^{(3)} & 0 & b_2^{(3)} & & \\ 0 & 0 & a_{33}^{(2)} & b_3^{(3)} & & \end{array} \right] \begin{array}{l} R_1 / a_{11} \\ R_2 / a_{22}^{(3)} \\ R_3 / a_{33}^{(2)} \end{array}$$

eg

$$\left[\begin{array}{ccc|ccc} 1 & & & 3 & & \\ & 0 & & & 4 & \\ & & 5 & & & \end{array} \right]$$

$$x + 3y + 3z = 16$$

$$x + 4y + 3z = 18$$

$$x + 3y + 4z = 19$$

Gauss Jordan \rightarrow more no. of operations
(1-5 times more mult & division)

$$A \rightarrow A^{-1} \text{ exist (det } A \neq 0)$$

Find A^{-1} through Gauss Jordan

$$\begin{array}{c}
 \begin{array}{l}
 \text{eg} \\
 Ax = b
 \end{array}
 \end{array}
 \begin{array}{c}
 \leftarrow \\
 [A|I] \rightarrow [I|B]
 \end{array}
 \begin{array}{c}
 \rightarrow \\
 Bn
 \end{array}$$

$$[A|b] \rightarrow [I|x] \quad \begin{array}{l} \swarrow \\ \searrow \end{array} \quad \begin{array}{l} A^{-1} = B \\ A(B) = I \end{array}$$

$$[A|I] \rightarrow [I|B]$$

$$\text{eg} \quad \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 5R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 0 & 1 & 0 \\ 0 & -13 & -7 & -5 & 0 & 1 \end{array} \right]$$

$$R_1 \rightarrow R_1 - \frac{3}{4} R_2$$

$$R_3 \rightarrow R_3 + \frac{13}{4} R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{11}{4} & \frac{1}{4} & -\frac{3}{4} & 0 \\ 0 & 4 & 1 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} & -\frac{5}{4} & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 4 & \frac{11}{4} & -\frac{3}{4} & 0 & 0 \\ 0 & 0 & -3.75 & -5 & \frac{13}{4} & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array} \right]$$

$$C_2 \rightarrow C_2 - \frac{a_{12}}{a_{11}} C_1$$

$$C_3 \rightarrow C_3 - \frac{a_{13}}{a_{11}} C_1$$

$$\left[\begin{array}{ccc|ccc} a_{11} & 0 & 0 & 1 & - & - \\ a_{21} & a_{22}^{(1)} & a_{23}^{(1)} & - & - & - \\ a_{31} & a_{32}^{(1)} & a_{33}^{(1)} & - & - & - \end{array} \right]$$

$$\cancel{C_1} \rightarrow \cancel{C_1} - \frac{a_{21}^{(1)}}{a_{22}^{(1)}} \cancel{C_2}$$

$$C_1 \rightarrow C_1 - \frac{a_{21}}{a_{22}^{(1)}} C_2 ; C_3 \rightarrow C_3 - \frac{a_{23}^{(1)}}{a_{22}^{(1)}} C_2$$

$$\rightarrow A = [A|I]$$

$$\rightarrow [I|B]$$

$$\left[\begin{array}{ccc} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 3 & 3 \end{array} \right]$$

$$\rightarrow R_3 \rightarrow R_3 \times 4$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 3 & 3 \end{bmatrix}$$

$$I_3 \rightarrow I_{3 \times 4}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 12 & 12 & 12 \end{bmatrix}$$

Row Operation $I \cdot A = A$

$$A = I \cdot A$$

Column Operation

$$A = A \cdot I$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 3 & 3 \end{bmatrix}$$

$$I_3 \rightarrow I_{3 \times 4}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 3 & 3 \end{bmatrix}$$

$$C_3 \rightarrow C_3 \times 4$$

$$\begin{bmatrix} 1 & 2 & 12 \\ 2 & 3 & 16 \\ 3 & 3 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$