3 First & second derivate.

x

y.

$\rightarrow \frac{dy}{dx}, \frac{d^2y}{dx^2}$

Q3

x	1	1.05	1.07	1.1	1.15	1.2	1.25	1.3
y	0	1.025		1.049	1.072	1.095		1.118 1.14

at $x = 1.05$ & $x = 1.25$

$S = \frac{x - x_0}{h} = 1$ $\quad S = \frac{x - x_n}{h} = -1$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.0	1.025	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
1.1	1.049	Δy_1	$\Delta^2 y_1$	$\Delta^3 y_1$	$\Delta^4 y_1$	$\Delta^5 y_1$
1.15	1.072	Δy_2	$\Delta^2 y_2$	$\Delta^3 y_2$		
1.2	1.095	Δy_3	$\Delta^2 y_3$	$\Delta^3 y_3$		
1.25	1.118	Δy_4	$\Delta^2 y_4$			
1.3	1.14					

$x_0 = 1.05$ $\quad S = \frac{x - x_0}{h} = \frac{1.05 - 1.05}{h} = 0$

$\frac{1}{h}\left(\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \frac{\Delta^5 y_0}{5}\right)$

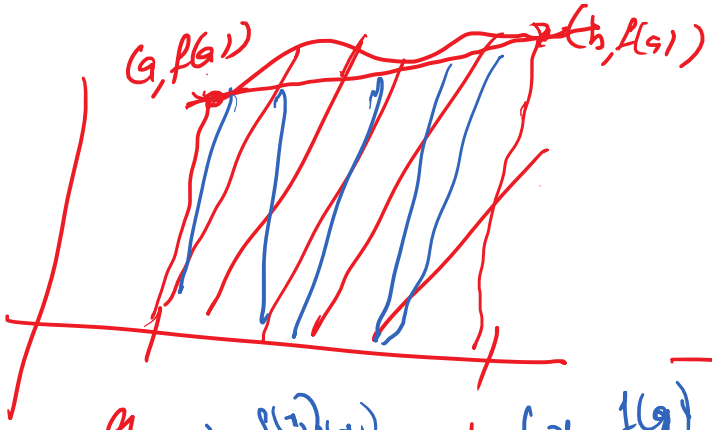
$$\bar{R}\left(\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \frac{\Delta^5 y_0}{5} - \right)$$

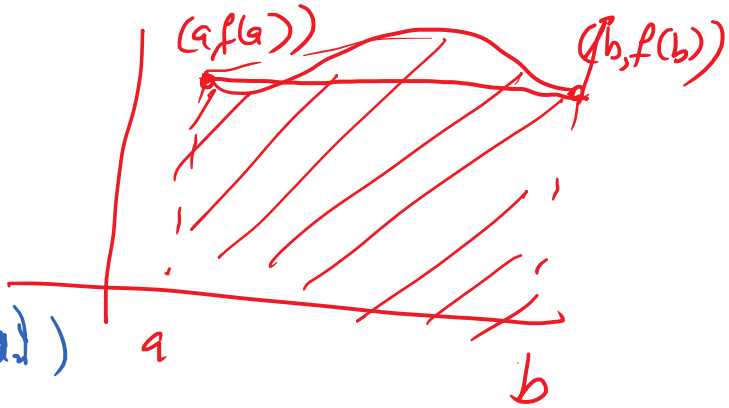
at $x = 1.25$ $\quad x_n = 1.25$ $\quad S = \frac{x - x_n}{h} = 0,$

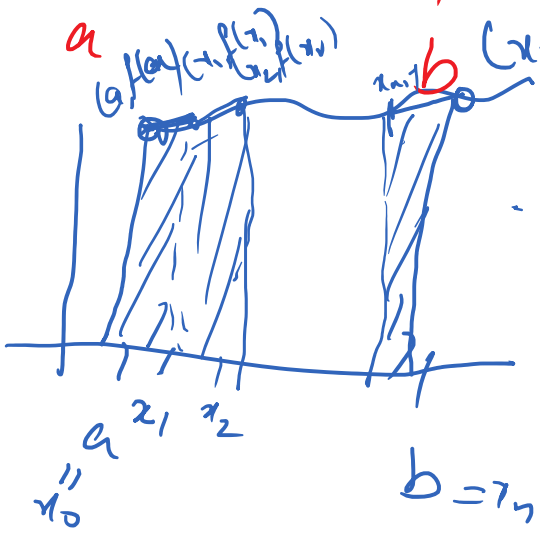
Numerical Integration.

$f \to$ cont on $[a, b]$ $\quad \int_a^b f(x)\,dx$ ✓

$$f(x) = x^{\sqrt{5}} (\sin x)^{100} e^{x^5} \qquad x \in [1, 2]$$







$x_{i+1} - x_i = h$

Newton forward ~~enter~~ polation

$$P_n(x) = f_0 + \Delta f_0 (x - x_0) + \Delta^2 f_0 (x - x_0)(x - x_1)$$

$$P_n(x) = f_0 + \Delta f_0 \frac{(x-x_0)}{h} + \frac{\Delta^2 f_0}{2!h^2}(x-x_0)(x-x_1) + \cdots + \frac{\Delta^n f_0}{n!h^n}(x-x_0)\cdots(x-x_{n-1})$$

$x_0 = a$, $x_n = b$.

$$\int_a^b f(x)\,dx \approx \int_{x_0}^{x_n} P_n(x)\,dx = \int_{x_0}^{x_n} \left(f_0 + \Delta f_0 \frac{(x-x_0)}{h} + \cdots + \frac{\Delta^n f_0}{n!h^n}(x-x_0)\cdots(x-x_{n-1})\right) dx$$

$s = \frac{x_n - x_0}{h}$, $s = \frac{x-x_0}{h} \Rightarrow ds = \frac{dx}{h} \Rightarrow dx = h\,ds$

$= \frac{x_0 + nh - x_0}{h} = n.$

$$h\int_0^n \left(f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \cdots + \frac{s(s-1)\cdots(s-(n-1))}{n!}\Delta^n f_0\right) ds$$

$s^2 - s$, $(s^2-s)(s-2) = s^3 - 3s^2 + 2s$

$$= h\left[f_0 n + \frac{s^2}{2}\Delta f_0\Big|_0^n + \frac{1}{2}\left(\frac{s^3}{3} - \frac{s^2}{2}\right)\Big|_0^n \Delta^2 f_0 + \frac{1}{6}\left(\frac{s^4}{4} - s^3 + s^2\right)\Big|_0^n \Delta^3 f_0 + \cdots\right)$$

$$= h\left[f_0 n + \frac{n^2}{2}\Delta f_0 + \frac{1}{2}\left(\frac{n^3}{3} - \frac{n^2}{2}\right)\Delta^2 f_0 + \frac{1}{6}\left(\frac{n^4}{4} - n^3 + n^2\right)\Delta^3 f_0 - \cdots\right)$$

$$= hn\left[f_0 + \frac{n}{2}\Delta f_0 + \frac{1}{12}(2n^2 - 3n)\Delta^2 f_0 + \frac{1}{24}(n^3 - 4n^2 + 4n)\Delta^3 f_0 + \cdots\right)$$

$$= hn\left[f_0 + \frac{n}{2}\Delta f_0 + \frac{n}{12}(2n-3)\Delta^2 f_0 + \frac{n}{24}(n-2)^2\Delta^3 f_0 + \cdots\right)$$

$(a, f(a))$ → $f(x)$ → $(b, f(b))$

Case 1

$\rightarrow n=1 \rightarrow$ Trapezoidal Rule.

x_0 y_0

x_1 y_1 Δy_0

$$\int_{x_0}^{x_1} f_1(x)\,dx = h\left[f_0 + \frac{1}{2}\Delta f_0\right]$$

$$= h\left[f_0 + \frac{1}{2}(f_1 - f_0)\right]$$

Simple Trapezoidal Rule

$$= \frac{h}{2}\left[2f_0 + f_1 - f_0\right]$$

$$= \frac{h}{2}\left[f_0 + f_1\right] =$$

$h = x_1 - x_0 = b - a$

$$\frac{b-a}{2}\left(f(a)+f(b)\right)$$

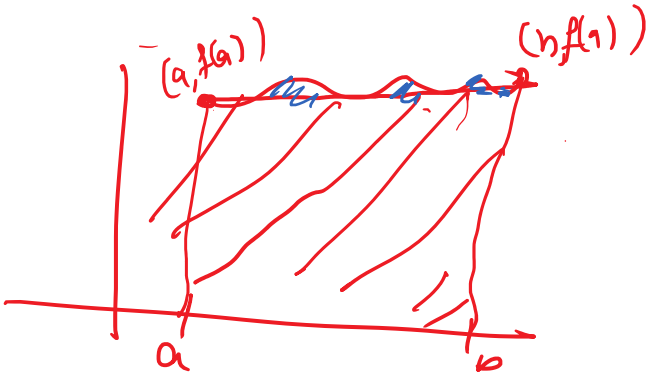
$(n=1$

$|(x_i, y_i)_{i=0}^{n}$

a (x_0, y_0),

(x_1, y_1)

b

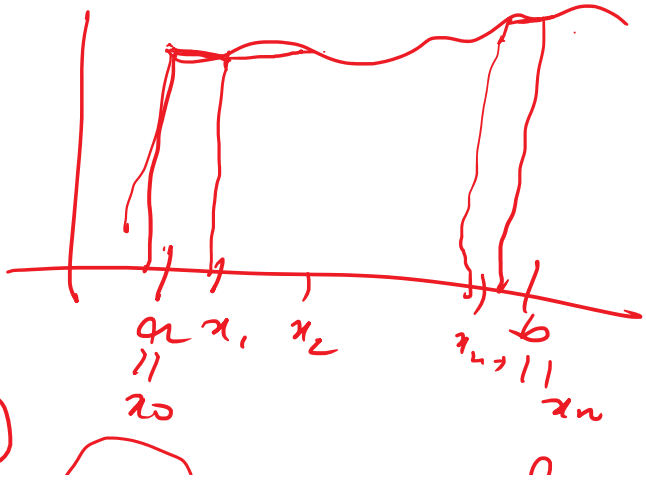


Composite Trapezoidal

$$\int_{x_0}^{x_n} f(x)\,dx = \int_{x_0}^{x_1} f(x)\,dx + \int_{x_1}^{x_2} f(x)\,dx$$

$$+ \cdots + \int_{x_{n-1}}^{x_n} f(x)\,dx.$$

$$\frac{h}{2}(f_0 + f_1) + \frac{h}{2}(f_1 + f_2)$$



$$y \frac{h}{2}(f_0 + f_1) + \frac{h}{2}(f_1 + f_2) \quad x_0 \quad x_n$$

$$+ \frac{h}{2}(f_2 + f_3) + \cdots + \frac{h}{2}(f_{n-1} + f_n) \qquad h = \frac{b-a}{n}$$

$$= \frac{h}{2}(f_0 + f_n + 2f_1 + 2f_2 + \cdots + 2f_{n-1})$$

$$= \frac{h}{2}(f_0 + f_n + 2(f_1 + f_2 + \cdots + f_{n-1}))$$

↓

Composite Trapezoidal Rule.

(n+1) points

means n subintervals

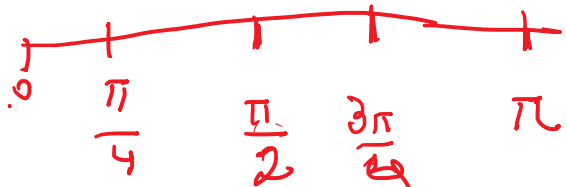
$h = \frac{b-a}{n}$

Q4 $\int_0^{\pi} x \sin x \, dx$

with 5 nodal points

(Trapezoidal Rule)

$$h = \frac{\pi - 0}{4} = \frac{\pi}{4}$$



$f(0) = 0$ $\quad f(\frac{3\pi}{4}) = \frac{3\pi}{4} \sin\frac{3\pi}{4} =$

$f(\frac{\pi}{4}) = \frac{\pi}{4}\sin\frac{\pi}{4}$ $\quad f(\pi) = 0$

$f(\frac{\pi}{2}) = \frac{\pi}{2}\sin\frac{\pi}{2}$

$h = \frac{\pi}{4}$.

$$\frac{h}{2}(f_0 + f_4 + 2(f_1 + f_2 + f_3))$$

$$= \frac{h}{2}(0 + 0 + 2(\downarrow \quad \downarrow \downarrow))$$

$= \frac{h}{2}(0+0+2($ $)$

(1) $\int_0^{\pi} x \sin x \, dx \rightarrow$ Exact.

(2) For $n = 6$ ✓

For $n = 4$ ✓

Absolute error $|$ Exact $-$ Approximate $|$