22 June 2022 17:09

$f(x) = x^4$

$x = 1 \quad 2 \quad 3$

$2x + y + 4z = 12$

$8x - 3y + 2z = 20$

$4x + 11y - z = 33$

$$A = \begin{pmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

→ Simpson's $\frac{3}{8}$ Method

$$\rightarrow \int_{x_0}^{x_n} f(x)\,dx = \frac{3h}{8}\Big(y_0 + y_n + 2(y_3 + y_6 + \dots + y_{n-3}) + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{n-2} + y_{n-1})\Big)$$

Q 8 $\int_0^1 \frac{1}{1+x^2}\,dx$ Simpson's $\frac{3}{8}$.

(i) with 4 nodal points $h = \frac{b-a}{n} = \frac{1-0}{3}$

(1) with 4 nodal points $h = \frac{b-a}{n} = \frac{1-0}{3}$

(2) 7 nodal points $\rightarrow h = \frac{1-0}{6} = \frac{1}{6}$

$h = \frac{1}{3}$

$x_0 = 0, \quad x_1 = \frac{1}{3}, \quad x_2 = \frac{2}{3}, \quad x_3 = 1$

$y_0 = 1$

$y_1 = y(\frac{1}{3}) = \frac{1}{1+\frac{1}{9}} = 0.9$

$y_2 = y(\frac{2}{3}) = 0.6923$

$y_3 = 0.5$

$$\frac{3h}{8}\left(y_0 + y_3 + 3(y_1 + y_2)\right)$$

$$= \frac{3 \times 1}{3 \times 8}\left(1 + 0.5 + 3(0.9 + 0.6923)\right)$$

$$= 0.78461$$

$h = \frac{1}{6}$

$x_0 = 0, \quad x_1 = \frac{1}{6}, \quad x_2 = \frac{2}{6}, \quad x_3 = \frac{3}{6}, \quad x_4 = \frac{4}{6}, \quad x_5 = \frac{5}{6}, \quad x_6 = 1$

$$\frac{3h}{8}\left(y_0 + y_6 + 2(y_3) + 3(y_1 + y_2 + y_4 + y_5)\right)$$

$$= 0.78539$$

$$\int_0^1 \frac{1}{1+x^2}\,dx = \tan^{-1} 1 = \frac{\pi}{4} = 0.78539$$

$\rightarrow$ Degree of precision of Simpson's $\frac{3}{8}$.

$$I = \frac{3h}{8}\left(y_0 + 3y_1 + 3y_2 + y_3\right) \qquad h = \frac{b-a}{3}$$

$$= \frac{3}{8} \times \frac{(b-a)}{3}\left(f(a) + 3f\left(a + \frac{b-a}{3}\right) + 3f\left(a + 2\left(\frac{b-a}{3}\right)\right) + f(b)\right)$$

$$= \frac{b-a}{8}\left(f(a) + 3f\left(\frac{2a+b}{3}\right) + 3f\left(\frac{a+2b}{3}\right) + f(b)\right)$$

$$f(x) = C \qquad \int_a^b f(x) = C(b-a)$$

$$I = \frac{b-a}{8}\left(C + 3C + 3C + C\right) = C(b-a)$$

...

$$f(x) = Cx \qquad \int = I.$$

$$Cx^2 \qquad \int = I$$

$$Cx^3 \qquad \int = I,$$

$$Cx^4 \qquad \int \neq I,$$

Degree of Precision $= 3$

Error bounds → max error that the method will give.

→ Trapezoidal

$$\frac{b-a}{12} h^2 M_2, \quad M_2 = \max_{x \in [a,b]} |f''(x)|$$

$b - a = 6h$

$x \in [a,b]$

$b-a = 6h$

$\Rightarrow$ Simpson's $\frac{1}{3}$ Rule $\rightarrow \dfrac{b-a}{180} h^4 M_4$, $M_4 = \max |f''''(x)|$
$x \in [a,b]$

Simpson's $\frac{3}{8}$ Rule $\rightarrow \dfrac{3}{80} h^5 M_5$ $M_4 = \max |f''''(x)|$
$x \in [a,b]$

$n = 6$, $h = \dfrac{b-a}{6}$

$$\frac{b-a}{180} \times \left(\frac{b-a}{6}\right)^4 M_4 .$$

$$= \frac{(b-a)^5 M_4}{}$$

$$\frac{b-a}{180} \times h^4 M_4 = \frac{6h}{180} \times h^4 M_4$$

$$= \frac{h^5 M_4}{30} .$$

Simpson's $\frac{3}{8}$ $\dfrac{3h^5 M_4}{80}$