Indian Institute of Technology (ISM)-Dhanbad
Department of Mathematics & Computing
Mathematics-II (Common) (Tutorial Sheet-II)

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(1) Find the eigen values and eigen vectors of the following matrices:

(i) $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$.

(2) Verify the Cayley Hamilton theorem for the following matrix

$\begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

Also (*i*) obtain A^{-1} and A^3, (*ii*) find eigen values of A, A^2 and verify that eigen values of A^2 are squares of those of A, (*iii*) find the spectral radius of A.

(3) By using the Cayley-Hamilton theorem, find A^8, if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

and A^4, if $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

(4) Find the characteristic equation of matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$

and hence also find A^{-1}, by using Cayley Hamilton theorem. Also, find the values of $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$.

(5) Examine whether A is similar to B, where

(i) $A = \begin{bmatrix} 5 & 5 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$.

(ii) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

(6) Show that the matrix

$\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

is diagonalizable. Hence, find P such that $P^{-1}AP$ is a diagonal matrix. Then, obtain the matrix $B = A^2 + 5A + 3I$.

(7) Examine whether the matrix A, where A is given by

(*i*) $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ (*ii*) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

is diagonalizable. If so, find P such that $P^{-1}AP$ is a diagonal matrix.

(8) The eigen vectors of a 3×3 matrix A corresponding to the eigen values $1, 1, 3$ are $[1, 0, -1]^T, [0, 1, -1]^T, [1, 1, 0]^T$ respectively. Find the matrix A.

(9) Obtain the symmetric matrix for the quadratic form:
(i) $2x_1^2 + 3x_1x_2 + x_2^2$, (ii) $x_1^2 + 2x_1x_2 - 4x_1x_3 + 6x_2x_3 - 5x_2^2 + 4x_3^2$.

(10) Reduce the following quadratic form to the canonical form by an orthogonal transformations. Also, specify the matrix of transformation in each case:

(i) $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$
(ii) $x^2 + 2y^2 + 2z^2 + -2yz + zx - 2xy$.

(11) Determine the nature, index and signature of the following:
(i) $x^2 + 2y^2 + 2z^2 - 2yz + zx - 2xy$ (ii) $2x_1x_2 + 2x_1x_3 + 2x_2x_3$.

(12) Solve the following differential equations (initial value problems) by matrix method:
(i) $x'' - 2x' - 3x = 0,\ x(0) = 0, x'(0) = 1$ (ii) $y'' + \mu^2 y = 0,\ y(0) = 1, y'(0) = \mu$
(iii) $\dfrac{dx_1}{dt} = x_1 + x_2,\ \dfrac{dx_2}{dt} = lx_1 + x_2,\ x_1(0) = 10, x_2(0) = 70.$

(13) Show that:
(i) The eigen values of an Hermitian matrix are real.
(ii) The eigen values of a Skew-Hermitian matrix are zero or pure imagenary.
(iii) The eigen values of an unitary matrix are of magnitude 1.
(iv) $\begin{bmatrix} 2 & -2 & -4 \\ -3 & 3 & 4 \\ 1 & -2 & -2 \end{bmatrix}$ is idempotent. Further, Show that $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is involutory.
(v) Every skew-Hermitian matrix A can be expressed as $B + iC$, where B is real and skew-symmetric matrix, and C is real and symmetric matrix.
(vi) $\lambda = 0$ is an eigen value of the matrix A iff A is singular matrix.

(14) Determine the value of a, b, c when the matrix $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal.