Phase Equilibrium

Introduction:

Two or more different phases present in equilibrium with one another, constitute a heterogenous system. Such heterogenous system can be conveniently studied with the help of a generalization called **Gibbs Phase** rule. It is applicable to all heterogeneous systems. This rule was deduced on the basis of thermodynamic principles by J. Willard Gibbs . This rule predicts qualitatively the effect temperature, pressure and concentration on a heterogeneous equilibrium.

Gibbs phase rule may be stated as follows:

When the equilibrium between any numbers of phases is influenced only by temperature, pressure and concentration but not influenced by gravity, electrical and magnetic forces, then the number of degree of freedom (F) of the system is related to the number of components (C) and number of phases (P) by the Phase Rule equation:

$$F = C - P + 2$$

where

F = number of degree of freedom

C = number of components

P = number of phases

2 = variables depending on temperature and pressure besides the concentration variables.

Phase Rule:

Gibbs phase rule may be also mathematically stated as follows:

"In a heterogeneous system in equilibrium, the number of degrees of freedom plus the number of phases is equal to the number of components plus two".

Mathematically, F + P = C + 2or F = C - P + 2

where

F = number of degrees of freedom

C = number of components

P = number of phases

2 = variables depending on temperature and pressure besides the concentration variables.

Phase: The chemically homogeneous, physically distinct and mechanically separable part of a system which is separated by a definite boundary from other such parts of the system is called phase". It is denoted by P.

Examples

- (i) A gaseous mixture constitutes a single phase since gases are completely miscible. Air is a mixture of N₂, O₂, CO₂, water vapour etc. Which constitute a single phase.
- (ii) Two or more liquids which are miscible with one another constitute a single phase as there is no bounding surfaces separating the different liquids. e.g., water and alcohol, benzene & chloroform constitute one phase system.
- (iii) A system consisting of a liquid in equilibrium with its vapour constitute a two phase system

system

- (iv) Solid \rightleftharpoons Liquid \rightleftharpoons Vapour P = 3
- (v) $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g) P = 3$
- (vi) Water phenol P = 2

Component:

The number of component of a system at equilibrium is defined as the minimum number of independently variable constituents which are required to express the composition of each phase in the system. It is denoted by C.

In a chemically reactive system, the number of components is given by

$$C = N - E$$

where C = components.

N = Number of chemical species

E = Number of independent equations relating the concentrations of the N species.

- If you have equilibrium balance between reactants and products, the number of components will be reduced by one
- If you have equal amounts (concentrations) of products formed, the number of components will also be reduced by one

Examples

(i) Sulphur system: Consists of four phases namely monoclinic sulphur, rhombic sulphur, liquid sulphur and sulphur vapour. The composition of each phase of the system can be expressed in terms of sulphur only, so, it is a one component system. C = 1

Rhombic sulphur - Monoclinic sulphur - Sulphur liquid - Sulphur vapour

- (ii). Na₂SO₄ + water system: Certain salts are capable of existing as hydrates with different number of water molecules of crystallization. These hydrates correspond to different solids and hence to different phases. The system is a two component, because the composition of each phase of the hydrates is completely described in terms of the anhydrous salt and water alone. e.g., Na₂SO₄ + water C = 2
 - (iii). Water \rightleftharpoons Ice \rightleftharpoons Water vapour C = 1
- (iv) KCI NaCI H_2O system C = N E = 3-1 = 2
- (v) KCI NaBr H_2O system N = 5, E = 1 C = 5 – 1 = 4
- (vi) $CaCO_3(s) \implies CaO(s) + CO_2(g)$ C = N - E = 3 - 1 = 2

Component

Number of Components

- NaCl(s) dissolved in water
 - Available chemical constituents are four. Na⁺, Cl⁺, NaCl and H₂O
 - Because Na⁺ and Cl⁻ have the same amount "equal neutrality" as NaCl, then c = 2 and not
- · Decomposition of calcium carbonate

$$CaCO_3(s) = CaO(s) + CO_2(g)$$

- Available chemical constituents are three. Is it correct to say c = 3?
- Because of the equilibrium condition the number of independent components is reduced by one. Thus, c = 2 instead of 3; C = 2, P = 3 → F = 2 - 3 + 2 = 1
- · Decomposition of ammonium chloride

$$NH_4Cl(s) \rightleftharpoons NH_3(g) + HCl(g)$$

- Available chemical constituents are three. Is it correct to say c = 3?
- Because of the equilibrium condition the number of independent components is reduced by one. And also because the products formed form a single phase and are formed in equal amounts, the no. of independent components are further reduced by one.

$$PCl_5(s) \rightleftharpoons PCl_3(s) + Cl_2(g)$$

- · Decomposition of PCIs
 - Available chemical constituents are three. Is it correct to say c = 3?
 - Because of the equilibrium condition the number of independent components is reduced by one. C = 2, P = 3 → F = 2 - 3 + 2 = 1

Degree of Freedom:

The degree of freedom or variance of a system is defined as the minimum number of variable factors such as temperature, pressure and concentration which should be required in order to define the system completely. It is denoted by F.

Examples

(i) For a given sample of any gas PV = nRT. Any two of the three variables P, V, T define the system completely. Hence the system is bivariant or it has two degrees of freedom.

Conclusion-

- (i) The greater the number of components in a system, the greater is the degree of freedom for a given number of phases.
- (ii) The greater the number of phases, the smaller is the number of degrees of freedom.
- (iii) The number of phases is maximum when the number of degrees of freedom = Zero, for a given number of components. Thus, for

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one component system, P_{max.} = 3.
two component system, P_{max.} = 4
three component system, P_{max.} = 5
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Advantages of Phase' Rule:

(i) The phase rule confirms that the different systems having the same number of degrees of freedom behave in same manner.

Limitations:

- (i) The phase rule is applicable to heterogeneous systems in equilibrium, so, it is of no use for such systems which are slow in attaining the equilibrium state.
- (ii) In Gibbs phase rule, various variables are temperature, pressure and composition. It does not take in account the electric and magnetic influences. For consideration of such variables, the factor 2 of the Phase rule has to be adjusted accordingly.
- (iii) All the phases in the system must be present under the same temperature, pressure and gravitational force.

Phase diagrams:

The number of phases that exist in equilibrium depends upon the conditions of temperature and pressure, composition being constant or temperature and composition, pressure being constant. These conditions are determined experimentally and interdependence of values of the variables can be shown graphically using appropriates coordinates. These diagrams are termed phase diagram. A phase diagram is the description of the behaviour of the phases under equilibrium. It is very easy to describe the phase behaviour of a system by such diagrams and to investigate the conditions in which various phases will constitute the system.

Application of Gibbs Phase Rule to One Component System:

From the mathematical expression,

$$F = C - P + 2$$

For maximum value of degree of freedom $(F_{max.})$

$$C = 1, P = 1$$

$$F_{\text{max.}} = 1-1+2$$

$$= 2$$

Hence, all one component systems can be completely described graphically by stating only two variables, pressure and temperature on appropriate axis.

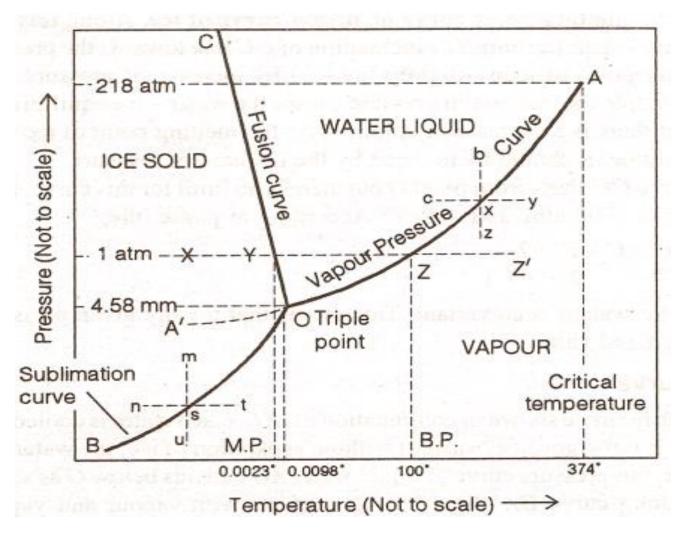
The Water System:

It is a one component system. Water exists in three possible phases viz. ice (solid), water (liquid), and vapour (gas). These three single phases may form four possible equilibria.

- (iii) Solid ⇌ Vapour
- (iv) Solid ⇒ Liquid ⇒ Vapour

The Phase diagram of water system is given as

Phase Diagram of water system



The phase diagram consists of :

(i) Stable curves: three OB, OA and OC

(ii) Metastable curve: one OA'

(iii) Areas: three AOB, COB and AOC

(iv) Triple point: One at O

(v) Critical point A: Critical temperature (374°C) and critical pressure (218 atm.). Above critical temp. only vapour phase exists whatever may be the value of pressure. The vapour pressure of water increases with increase in temperature. Above the critical temperature and critical pressure, liquid and vapour phase can not be distinguished and a **super critical fluid** is obtained.

(i) Stable curves

OA: It is known as **vapour pressure curve of water**. The curve OA starts from point O i.e., 0.0098°C under 4.58 mm of Hg pressure and ends at A, the critical point (374°C at 218 atm.).

The rate of increase of its vapour pressure with temperature is relatively higher at higher temperatures and therefore the curve OA slants upwards and slopes away from the temperature axis.

Liquid
$$\Rightarrow$$
 Vapour $P = 2$

From phase rule,

$$F = C - P + 2$$

= 1-2+2
= 1

The water vapour system is univariant

- **OB**: It is the **sublimation curve of ice**. Along this curve, solid ice is in equilibrium with its vapour.
- This curve is not the prolngation of curve A but falls of more steeply. At all points of the curve OB, ice is in equilibrium with vapour. Hence, there are two phases.

Solid
$$\Rightarrow$$
 Vapour P= 2
$$F = C - P + 2$$

$$= 1 - 2 + 2$$

$$= 1$$

Thus, the system is univariant. This means that for each temperature; there may be one pressure and for each pressure there may be one temperature.

OC: This curve is the **melting point curve or fusion curve of ice.** Along this curve two phases, ice and water are in equilibrium. The inclination of *OC* line towards the pressure axis indicates that the melting point of ice is slightly lowered by increase of pressure. (According to Le Chatelier's principle the increase in pressure causes the water - ice equilibrium to shift in such a direction that there is a decrease in volume). As the melting point of ice is accompanied by decrease in volume, it should be lowered by the increase of pressure.

Solid
$$\rightleftharpoons$$
 Liquid $P = 2$

According to phase rule,

$$F = C - P + 2$$

=1-2+2 = 1

Thus, the system is univariant. This means that for any given pressure, melting point must have one fixed value.

(ii) Metastable Curve

OA': It is a metastable curve shown in continuation of AO. When water is cooled below its freezing point (when it is not vigorously stirred) without separation of ice, the water is said to be **super cooled**. The vapour pressure curve of liquid water AO extends below O as shown by the dotted curve OA'. Along curve OA' liquid water coexists with vapour and vapour pressures are different than over the solid. This equilibrium is called metastable equilibrium as slight disturbance brings it to the stable region OB of the phase diagram.

Liquid
$$\Rightarrow$$
 Vapour $P = 2$
 $F = C - P + 2$
=1-2+2
= 1 Monovariant

(iii) Areas

The areas give the conditions of temperature and pressure under which single phase - ice (solid), water (liquid) and vapour(gas) can exist. It is necessary to specify both temperature and pressure to define a system within this area. In the area BOC, AOC and AOB exists, ice (solid), water (liquid) and vapour (gas) respectively. In these areas, the degrees of freedom for the system is two or they are bivariant. P = 1 F = 2

(iv) The Triple point O

The point O at which the curves AO, BO and CO meet is called the triple point. At this point all the three phases viz, ice, water and vapour co-exist. Thus, P=3.

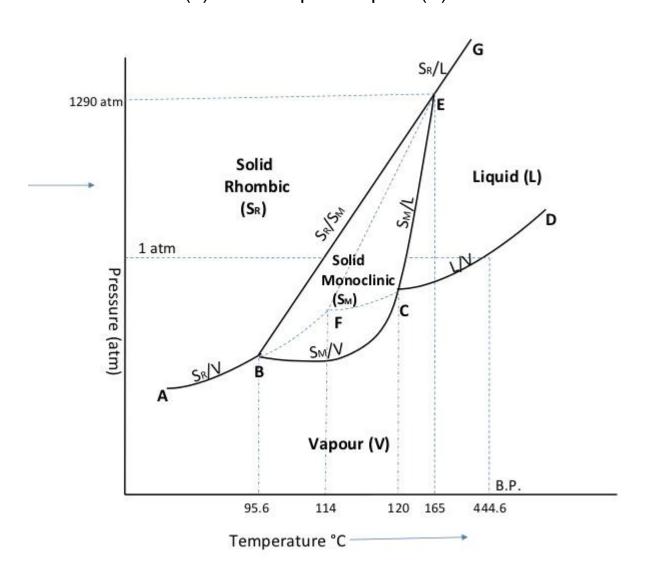
Solid
$$\rightleftharpoons$$
 Liquid \rightleftharpoons Vapour $F = C - P + 2$, $F = 1 - 3 + 2$ $F = 0$ Nonvariant

Thus, the degree of freedom at triple point is zero, which indicates that there is only one set of variables P, T at which all the three phases coexist.

The triple point O is a self defined point corresponding to 0.0098°C temperature and 4.58 mm of Hg pressure.

(v) Critical point A: Critical temperature (374°C) and critical pressure (218 atm.). Above critical temp. only vapour phase exists whatever may be the value of pressure. The vapour pressure of water increases with increase in temperature. Above the critical temperature and critical pressure, liquid and vapour phase can not be distinguished and a super critical fluid is obtained.

Phase Diagrame of Sulphur Sulphur exist in 4 phases Rhombic sulphur (S_R) , Monoclinic sulphur (S_M) , Sulpur liquid (L) and Sulphur vapour (V)



Sulphur

From phase rule, if P = 4 F = C - P + 2 = 1 - 4 + 2 = -1

Since degree of freedom may not be negative therefore all the four form of sulphur can not co-exist at any point of the phase diagram of sulphur.

- Curve AB is sublimation curve $S_R \rightleftharpoons V$ F = 1-2+2 = 1 monovariant
- Curve BC is sublimation curve $S_M \rightleftharpoons V$ F = 1-2+2 = 1 monovariant
- Curve CD is vapour pressure curve L

 V F = 1-2+2 = 1 monovariant
- Curve CE is fusion curve $S_M \rightleftharpoons L$ F = 1-2+2 = 1 monovariant
- Curve EG is fusion curve $S_R \rightleftharpoons L$ F = 1-2+2 = 1 monovariant
- Curve BE is transition curve $S_R \rightleftharpoons S_M$ F = 1-2+2 = 1 monovariant
- Curve BF is metastable curve $S_R \rightleftharpoons S_v$ F = 1-2+2 =1 monovariant
- Curve CF is metastable curve $L \rightleftharpoons V$ F = 1-2+2 =1 monovariant
- Curve EF is metastable curve $S_R \rightleftharpoons L$ F = 1-2+2 = 1 monovariant
- B is triple point $S_R \rightleftharpoons V \rightleftharpoons S_M$ F = 1-3+2 =0 nonvariant
- C is triple point $S_M \rightleftharpoons V \rightleftharpoons L$ F = 1-3+2 = 0 nonvariant
- E is triple point $S_R \rightarrow L \rightarrow S_M$ F = 1-3+2 =0 nonvariant
- F is metastable triple point $S_R \rightleftharpoons V \rightleftharpoons L$ F = 1-3+2 =0 nonvariant
- Under the areas, ABG, DCG, DGBA and BCEB, P = 1 and F = 2

POLYMORPHISM

POLYMORPHISM: The occurrence of the same substance in more than one crystalline forms is known as Polymorphism. This phenomenon is shown by both elements and compounds. In the case of elements the term allotropy is often used. The individual crystalline forms of an element are referred to as polymorphs or allotropes. Rhombic and monoclinic sulphur are two polymorphs or allotropes of sulphur. The polymorphic or allotropic forms of an element have distinct physical properties and constitute separate phases.

Allotropy can be divided into three types: Enantiotropy, Monotropy and Dynamic allotropy.

• Enantiotropy: In some cases one polymorphic form (or allotrope) can change into another at a definite temperature when the two forms have a common vapour pressure. This temperature is known as the transition temperature. One form is stable above this temperature and the other form below it. When the change of one form to the other at the transition temperature is reversible, the phenomenon is called enantiotropy and the polymorphic forms enantiotropes. For example, rhombic sulphur on heating changes to monoclinic sulphur at 95.6°C (transition temperature). Also, monoclinic sulphur, on cooling, again changes to rhombic sulphur at 95.6°C. That is, 95.6°C

Rhombic sulphur \Rightarrow Monoclinic sulphur

Thus Rhombic sulphur and Monoclinic sulphur are enantiotropic.

POLYMORPHISM:

- Monotropy: It occurs when one form is stable and the other metastable. The metastable changes to the stable form at all temperatures and the change is not reversible. Thus there is no transition temperature as the vapour pressures are never equal. This type of polymorphism is exhibited by phosphorus, White phosphorus —→ Red phosphorus. Another example is graphite and diamond, graphite being stable and diamond metastable, although the change is infinitely slow.
- **Dynamic allotropy:** Some substances have several forms which can coexist in equilibrium over a range of temperature. The amount of each is determined by the temperature. The separate forms usually have different molecular formulae but the same empirical formula. This form of allotropy, known as dynamic allotropy, resembles enantiotropy in that it is reversible but there is no fixed transition point. An example of dynamic allotropy is provided by liquid sulphur which consists of three allotropes S_{μ} , S_{π} and S_{λ} . $S_{\mu} \rightleftharpoons S_{\pi} \rightleftharpoons S_{\lambda}$ These three forms of sulphur differ in molecular structure. S_{λ} is S_{8} , S_{π} is S_{4} while formula of S_{μ} is not known. The composition of the equilibrium mixture at 120°C and 444.6°C (b.p. of sulphur) is :
- 120°C S_{μ} 0% S_{π} 3.7% S_{λ} 96.3%
- 444.6°C S_{μ} 37% S_{π} 4% S_{λ} 59%

Clapeyron – Clausius equation

The variation of vapour pressure with temperature is quantitatively given by Clapeyron – Clausius equation as follows

$$\frac{dP}{dT} = \frac{\Delta H_v}{T (V_g - V_l)}$$

$$\ln \frac{P_2}{P_1} = \frac{\Delta H_v}{R} \left\{ \frac{T_2 - T_1}{T_1 T_2} \right\}$$

Where V_I = Molar volume of liquid water, V_g = Molar volume of liquid vapour or steam, ΔH_v = Heat of vaporization

The variation of sublimation of ice with temperature is quantitatively given by Clapeyron – Clausius equation as follows

$$\frac{dP}{dT} = \frac{\Delta H_S}{T(V_g - V_S)} \qquad \ln \frac{P_2}{P_1} = \frac{\Delta H_S}{R} \left\{ \frac{T_2 - T_1}{T_1 T_2} \right\}$$

Where V_s = Molar volume of ice, V_g = Molar volume of liquid vapour or steam, ΔH_v = Molar heat of sublimation of ice

The variation of fusion of ice with temperature is quantitatively given by Clapeyron – Clausius equation as follows

$$\frac{dP}{dT} = \frac{\Delta H_f}{T (V_l - V_s)}$$

Where $V_s = Molar$ volume of ice, $V_l = Molar$ volume of water, $\Delta H_f = Molar$ heat of fusion of ice

Two component system

For two component system

$$C = 2$$

For maximum number of degree of freedom P = 1

Therefore
$$F_{\text{max.}} = 2-1+2=3$$

Thus, the variables (T, P, and composition) are required to define the system completely. But three variables can not be plotted in 2D figures (phase diagram). Therefore one of the degree of freedom is reduced by taking it as constant. Now we have taken P constant. Then

$$F = C - P + 2$$
 becomes

$$F = C - P + 2 - 1$$

F = C - P + 1 which is known as reduced phase rule equation. The reduced phase rule equation is used for calculation of degree of freedom for two component systems.

Phase diagram of two component Liquid – Liquid system Water – phenol system

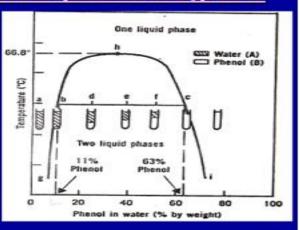
Water and phenol are partially miscible to each other. Therefore when we take water and phenol together, we get two phases, one is saturated solution phenol in water and other is saturated solution of water in phenol. The solution containing these two phases having different composition in equilibrium, are called conjugate solution. The mutual solubility of these two phases increases by increasing the temperature or decreasing the temperature and reach at a temperature where both the phases becomes completely miscible and two phase system is converted to one phase system. This temperature is known as critical solution temperature (CST). If two phase system is converted to one phase system by increasing the temperature, it is known as upper critical solution temperature (**UCST**). If two phase system is converted to one phase system by decreasing the temperature, it is known as lower critical solution temperature (LCST).

Liquid – Liquid System

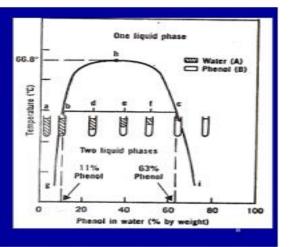
Phenol and water system phase diagram.

Temperature fixed at 50 °C

- Point a, system containing 100% pure water.
 - Addition of phenol to water will result in the formation of a single liquid phase until the point b is reached.
- *At point b, appears a second phase.
 - Phase A: water rich phase containing 11% phenol
 - Phase B: phenol rich phase containing 63% phenol



- increasing quantities of phenol, i.e., as we proceed across the diagram from point b to point c, we form systems in which the amount of the phenol-rich phase (B) continually increases
 - * At the same time the amount of the water-rich phase (A) decreases.
 Once the total conc. of phenol exceeds 63 % at 50 °C a single phenol-rich liquid phase is formed.



Liquid - Liquid System



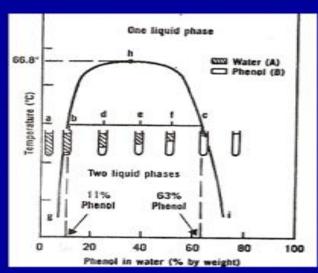
At 50°C

Aqueous phase saturated with phenol: contains 11% phenol (point b)

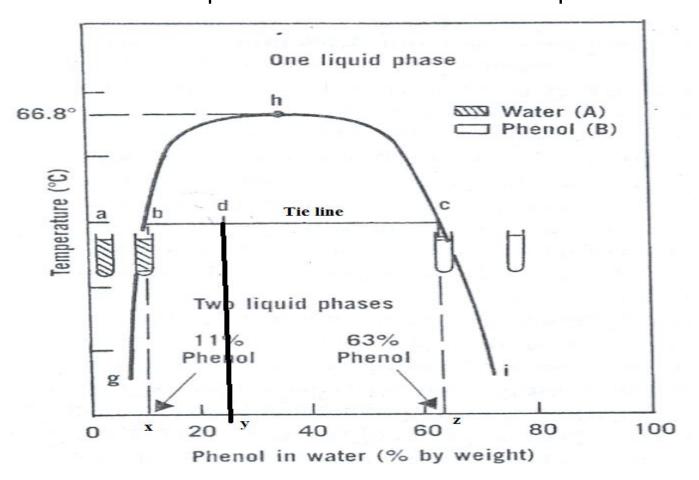
Phenolic phase saturated with water: contains 63% phenol (point c)

(1)

- The line bc drawn across the region containing two phases is termed a tie line; it is always parallel to the base line in two component systems.
- * all systems prepared on a tie line at 50° C will separate into phases of constant composition whose composition is b and c. These phases are termed conjugate phases.

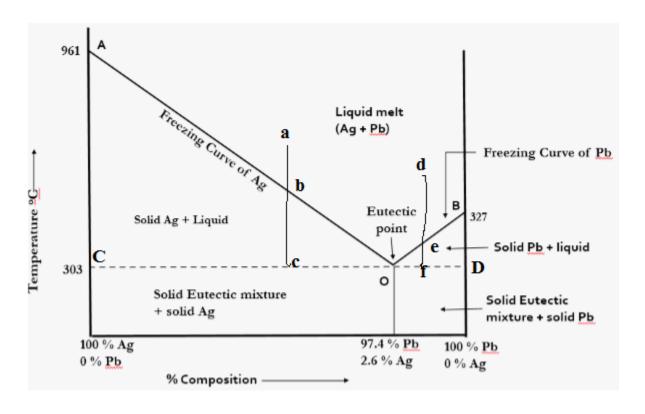


Lever Rule : It is used to determine the ratio of the amount of the two phases at a perticular point inside the phase diagram m1/m2 = dc/db = (z-y)/(y-x) m1 is amount of phase b and m2 is amount of phase c



Two component Solid – Liquid system

- Eutectic Mixture: If both the components are completely miscible in liquid (molten) state, immiscible in solid state and do not react with each other then combination of both form eutectic mixture. Example: Pb- Ag system.
- Phase diagram of Pb Ag system



Phase diagram of Pb – Ag system

• Curve AO. It is a freezing point curve of Ag. Solid Ag and liquid co-exists. P = 2

$$F = C - P + 1 = 2 - 2 + 1 = 1$$

Therefore, system is univariant

• **Curve BO.** It is a freezing point curve of Pb. Solid Pb and liquid co-exists. P = 2 F= C-P+1 = 2-2+1=1

Therefore, system is univariant

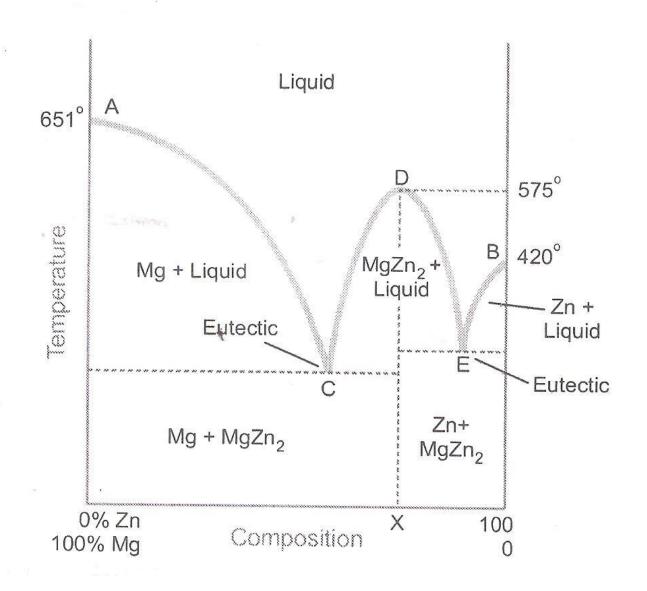
Point O. It is eutectic point. Here 3 phases (solid Ag, solid Pb and liquid) co-exists
 Eutectic temperature is the lowest temperature at which the liquid melt can exist. P=3
 F= C-P+1 = 2-3+1=0

Therefore, system is nonvariant and temperature =303 °C, Ag = 2.6 %

Area AOB. It represents solution of Pb Ag. P=1 F=2-1+1=2 bivariant

- Area ACO: In area ACO Solid lead and melt are in equilibrium. P=2 F = 1
- Area BDO: In area ACO Solid Ag and melt are in equilibrium. P=2 F = 1
- Area below line COD: No liquid phase in present below line COD. Solid silver and solid lead are present in this area as two different phases.
- F= C-P+1 = 2-2+1=1, monovariant
- Application: Used in Pattison's process of desilverisation of Pb.
- To increase % Ag in Argentiferous lead (0.1 % Ag) up to 2.6% Ag

Phase Diagram of Zn – Mg System



Phase Diagram of Zn – Mg System

Systems in which two components form a solid compound.

$$A + B \rightarrow C$$

The system will involve three solid phases, A, B and C. The fourth phase will be a liquid (or solution) containing varying concentration of A and B. There will be three freezing point or melting point curves.

- The curve AC, CDE and BE: AC is the freezing point curve of magnesium; BE is the freezing point curve of zinc; and CDE is that of the compound MgZn₂.
- The curve AC shows that the melting point of magnesium (651°C) is lowered on the addition of zinc. This continues until the point C is reached. Here a new phase, solid MgZn₂ appears. The curve CD shows the increase of concentration of zinc in the melt with the rise of temperature. At the maximum point D, the composition of the melt and the solid compound becomes the same i.e., MgZn₂. The point D, therefore, represents the congruent melting point of MgZn₂ (575°C). The curve DE now shows the lowering of the melting point with the addition of zinc until the lowest point is attained. Here solid zinc appears. The curve BE exhibits that the melting point of zinc (420°C) falls with the addition of magnesium until the point E is reached. Along the freezing point curves AC, CDE and BE, there are two phases in equilibrium viz., one solid phase (Mg, Zn, or MgZn₂) and the other liquid phase. Applying the reduced phase rule equation, we have

F = C - P + 1 = 2 - 2 + 1 = 1 This shows that the system Mg/liquid, Zn/liquid and MgZn₂/liquid are all monovariant.

Phase Diagram of Zn – Mg System

- **Eutectic points C and E.** There are two eutectic points in the phase diagram. The systems at the points C and E have two components and three phases in equilibrium.
- Phases Present
- C Solid Mg, solid MgZn₂, Liquid
- E Solid Zn, solid MgZn2, Liquid
- These systems are, therefore, nonvariant. F = C P + 1 = 2 3 + 1 = 0
- Congruent Melting Point. As already stated, the composition of the compound MgZn₂ and the melt at D is identical. The corresponding temperature is the congruent melting point of the compound. Here the system has two phases viz., the solid compound and the melt. Both these can be represented by one component (MgZn₂). Therefore the system at D is nonvariant,
- F = C P + 1 = 1 2 + 1 = 0
- The Areas. The area above the curves AC, CDE and BE represents the solution of magnesium and zinc (the melt). The single phase system at any point in this area is bivariant.
- F = C P + 1 = 2 1 + 1 = 2