

$$f(x) = x^4$$

$$x = 1 \quad 2 \quad 3$$

$$2x + y + 4z = 12$$

$$8x - 3y + 2z = 20$$

$$4x + 11y - z = 33$$

$$A = \begin{pmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} u_{12} & u_{13} \\ 0 & u_{23} \\ 0 & 0 \end{pmatrix}$$

→ Simpson's $\frac{3}{8}$ Method

$$\rightarrow \int_{x_0}^{x_n} f(x) dx = \frac{3h}{8} (y_0 + y_n + 2(y_3 + y_6 + \dots + y_{n-3}) + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{n-2} + y_{n-1}))$$

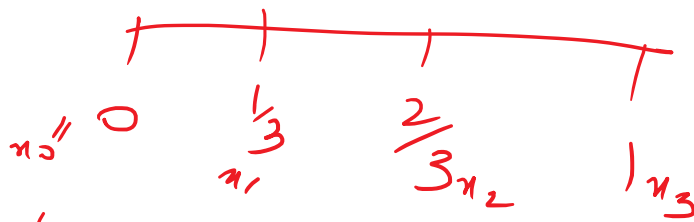
Q 8 $\int_0^1 \frac{1}{1+x^2} dx$ Simpson's $\frac{3}{8}$

(1) with 4 nodes points $h = \frac{b-a}{n} = \frac{1-0}{2}$

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(2) 7 nodes points $\rightarrow h = \frac{1-0}{6} = \frac{1}{6}$

$$h = \frac{1}{3}$$



$$y_0 = 1$$

$$y_1 = y\left(\frac{1}{3}\right) = \frac{1}{1+\frac{1}{9}} = 0.9$$

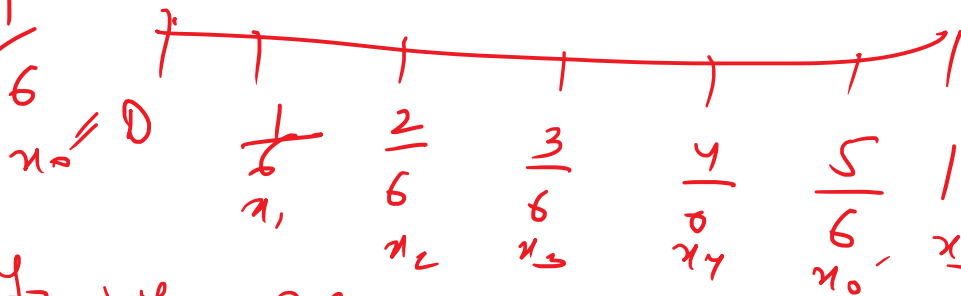
$$y_2 = y\left(\frac{2}{3}\right) = 0.6923$$

$$\frac{3h}{8} (y_0 + y_3 + 3(y_1 + y_2))$$

$$= \frac{3 \times 1}{3 \times 8} (1 + 0.5 + 3(0.9 + 0.6923)) \quad y_3 = 0.5$$

$$= 0.78461$$

$$h = \frac{1}{6}$$



$$\frac{3h}{8} (y_0 + y_6 + 2(y_3) + 3(y_1 + y_2 + y_4 + y_5))$$

$$= 0.78539$$

$$\int_0^1 \frac{1}{1+x^2} dx = \tan^{-1} 1 = \frac{\pi}{4} = 0.78539$$

\rightarrow Degree of precision of Simpson's $\frac{3}{8}$.

$$I = \frac{3h}{8} (y_0 + 3y_1 + 3y_2 + y_3)$$

$$h = \frac{b-a}{3}$$

$$= \frac{3}{8} \times \frac{(b-a)}{3} \left(f(a) + 3f\left(\frac{a+b-a}{3}\right) + 3f\left(a + 2\left(\frac{b-a}{3}\right)\right) + f(b) \right)$$

$$= \frac{b-a}{8} \left(f(a) + 3f\left(\frac{2a+b}{3}\right) + 3f\left(\frac{a+2b}{3}\right) + f(b) \right)$$

$$f(x) = c \quad \int_a^b f(x) = c(b-a)$$

$$I = \frac{b-a}{8} (c + 3c + 3c + c) = c(b-a)$$

$$\dots f(x) = cx \quad \int = I$$

$$cx^2 \quad \int = I$$

$$cx^3 \quad \int = I$$

$$cx^4 \quad \int \neq I$$

Degree of Precision

= 3

Error bounds

→ max error that the method will give

→ Trapezoidal

$$\frac{b-a}{12} h^2 M_2, \quad M_2 = \max_{x \in [a,b]} |f''(x)|$$

$$b-a = 6h$$

$$b-a=6h$$

$$x \in [a, b]$$

$$\rightarrow \text{Simpson's } \frac{1}{3} \text{ Rule} \rightarrow \frac{b-a}{180} h^4 M_4, M_4 = \max_{x \in [a, b]} |f^{(4)}(x)|$$

$$\text{Simpson's } \frac{3}{8} \text{ Rule} \rightarrow \frac{3}{80} h^5 M_5, M_5 = \max_{x \in [a, b]} |f^{(5)}(x)|$$

$$n=6, h=\frac{b-a}{6}$$

$$\frac{b-a}{180} \times \left(\frac{b-a}{6}\right)^4 M_4$$

$$= \frac{(b-a)^5 M_4}{180 \times 6^4}$$

$$\frac{b-a}{180} \times h^4 M_4 = \frac{6h}{180} \times h^4 M_4$$

$$= \frac{h^5 M_4}{30}$$

$$\text{Simpson's } \frac{3}{8} \text{ Rule} \rightarrow \frac{3h^5 M_5}{80}$$