

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



Q1 (C) $2x - 3y + z = -1$
 $x + 4y - 3z = 0$
 $3x - y + 4z = 13$

$$\begin{bmatrix} 2 & -3 & 1 & -1 \\ 1 & 4 & -3 & 0 \\ 3 & -1 & 4 & 13 \end{bmatrix} \quad \begin{array}{l} R_2 \rightarrow R_2 - \frac{1}{2}R_1 \\ R_3 \rightarrow R_3 - \frac{3}{2}R_1 \end{array}$$

$\times \frac{3}{2}$

$$\begin{bmatrix} 2 & -3 & 1 & -1 \\ 0 & \frac{5}{2} & -\frac{7}{2} & \frac{1}{2} \\ 0 & \frac{7}{2} & \frac{5}{2} & \frac{29}{2} \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 1 & -1 \\ 0 & 11 & -7 & 1 \\ 0 & 7 & 5 & 29 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - \frac{7}{11}R_2$$

$$\begin{bmatrix} 2 & -3 & 1 & -1 \\ 0 & 11 & -7 & 1 \\ 0 & 0 & 104 & 312 \end{bmatrix}$$

$$11y - 7z = 1$$

$$y = 2$$

$$104z = 312$$

$$z = 3$$

$$2x - 3y + z = -1$$

$$x = 1$$

$$R_1 \rightarrow R_1 + \frac{3}{11}R_2$$

$$\begin{bmatrix} 2 & 0 & \frac{10}{11} & -\frac{8}{11} \\ 0 & 11 & -7 & 1 \\ 0 & 0 & 104 & 312 \end{bmatrix}$$

$$-1 + \frac{3}{11} =$$

$$= 1 - \frac{21}{11}$$

$$1 - \frac{3 \times 7}{11}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & \frac{-10}{11} & -\frac{8}{11} \\ 0 & 11 & -7 & 1 \\ 0 & 0 & \cancel{12} & \cancel{3} \end{array} \right] \quad -\frac{8}{11} + \frac{10}{11} \times 3$$

$$R_1 \rightarrow R_1 + \frac{10}{11} R_3 \quad 1+1$$

$$R_2 \rightarrow R_2 + 7 R_3$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & 11 & 0 & 22 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Q5

$$x - 4y + z = 5$$

$$4x + 2y - z = 10$$

$$x + y - 4z = 0$$

(Iteration)

→
(Gauss ~~Jacobi~~,
Seidel)

$$4x + 2y - z = 10$$

$$x - 4y + z = 5$$

$$x + y - 4z = 0$$

$$x^{n+1} = \frac{10 - 2y^n + z^n}{4}$$

$$(x^0, y^0, z^0) = (0, 0, 0)$$

$$x^1 = 2.5$$

$$y^{n+1} = -\frac{1}{4}(5 - x^n - z^n)$$

$$y^1 = -\frac{5}{4} = -1.25$$

$$z^{n+1} = \frac{1}{4}(x^n + y^n)$$

$$z^1 = 0 \quad x^2 = 10 - \frac{2(-1.25) + 0}{4} = 3.125$$

$$y^2 = \frac{-1}{4}(5 - 2 \cdot 5 - 0) = -0.625$$

$$|x^1 - x^0| = 2.5$$

$$|x^2 - x^1| = |3.125 - 2.5| = 0.625 \quad z^2 = \frac{1}{4}(2.5 - 10.25) = 0.3125$$

$$|x^3 - x^2| \quad x^3 = \frac{1}{4}(10 - 2(-0.625) + 0.3125)$$

$$|2.89062 - 3.125| = \frac{185}{64} = 2.89062$$

$$y^3 = \frac{-1}{4}(5 - 3.125 - 0.3125) = -0.39062$$

$$z^3 = \frac{1}{4}(3.125 - 0.625) = 0.625$$

$$x - 4y + z = 5$$

$$4x + 2y - z = 10$$

$$x + y - 4z = 0$$

Gauss Seidel

~~$$x = 5 + 4y - z$$~~

$$4x + 2y - z = 10$$

$$x - 4y + z = 5$$

$$x + y - 4z = 0$$

$$17x_1 + 65x_2 - 13x_3 + 50x_4 = 84$$

$$x^{n+1} = \frac{10 - 2y^n + z^n}{4}$$

$$x'' = \frac{10 - 2y'' + z''}{4}$$

$$x - 4y'' = 5 - z''$$

$$x + y'' - 4z'' = 0$$

$$= 5 - z''$$

$$= 0$$

$$n=0$$

$$x' = 2.5$$

$$x' - 4y' = 5 - 0 \Rightarrow x' = 5$$

$$2.5 - 4y' = 5$$

$$y' = -0.625$$

$$x' - y' - 4z' = 0$$

$$2.5 - (-0.625) - 4z' = 0$$

$$z' = 0.469$$

LU decomposition Crout's

Q9

$$2x - y + z = 3$$

$$x + y + z = 6$$

$$3x + y - z = 2$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$l_{11} = 2$$

$$l_{11} u_{12} = -1 \Rightarrow u_{12} = -\frac{1}{2}$$

$$l_{11} u_{13} = 1 \Rightarrow u_{13} = \frac{1}{2}, \quad l_{21} = 1$$

$$l_{21} u_{12} + l_{22} = 1 \Rightarrow -\frac{1}{2} + l_{22} = 1 \Rightarrow l_{22} = \frac{3}{2}$$

$$l_{21} u_{13} + l_{22} u_{23} = 1 \Rightarrow 1\left(\frac{1}{2}\right) + \frac{3}{2} u_{23} = 1$$

$$u_{23} = \frac{1}{3}$$

$$l_{31} = 3$$

$$l_{31} u_{12} + l_{32} = 1 \Rightarrow 3 \times -\frac{1}{2} + l_{32} = 1$$

$$l_{32} = \frac{5}{2}$$

$$l_{31} u_{13} + l_{32} u_{23} + l_{33} = -1$$

$$3\left(\frac{1}{2}\right) + \frac{5}{2}\left(\frac{1}{3}\right) + l_{33} = -1$$

$$l_{33} = -\frac{19}{6}$$