

Department of Mathematics and Computing
Numerical Methods
Tutorial Sheet-III

1. Find the first and second derivatives of $f(x)$ at $x=1.5$ if

| | | | | | | |
|----------|-------|-------|--------|--------|--------|--------|
| $x :$ | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| $f(x) :$ | 3.375 | 7.000 | 13.625 | 24.000 | 38.875 | 59.000 |

Answer: 4.75, 9

2. Find the first and second derivatives of the function tabulated below at the point $x = 1.1$

| | | | | | | |
|-------|-----|-------|-------|-------|-------|-------|
| $x :$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| $y :$ | 0 | 0.128 | 0.544 | 1.296 | 2.432 | 4.000 |

Answer: 0.63, 6.6

3. Given the following table of values of x and y . Find the first and second derivatives at (a) $x = 1.05$,
(b) $x = 1.25$

| | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|
| $x :$ | 1.00 | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 |
| $y :$ | 1.000 | 1.025 | 1.049 | 1.072 | 1.095 | 1.118 | 1.140 |

Answer: (a) 0.493, -1.165, (b) 0.4473, -0.1583

4. Evaluate $\int_0^{\pi} x \sin x$ using trapezoidal rule with five nodal points.

Answer: π .

5. Evaluate $\int_0^{\pi/2} e^{-x} \cos x dx$, using the trapezoidal rule with $\pi/4$ and $\pi/8$.

6. Evaluate $\int_1^2 \frac{x^2}{1+x^3} dx$ using the Simpson's $\frac{1}{3}$ rule with two and four sub-intervals. Compare with the exact solution.

Answer: 0.8148, 0.7317.

7. Evaluate $\int_1^2 \sqrt{1+4x^2} \sin x dx$ using the Simpson's $\frac{1}{3}$ rule with $h = 1/2, 1/4$.

Answer: 3.0414, 3.0426.

8. Using the Simpson's $\frac{3}{8}$ rule, evaluate the following integrals with 4 and 7 nodal points. Compare with the exact solution.

(a) $\int_0^1 \frac{1}{1+x^2} dx$ **Answer:** 0.784616, 0.785396, Exact: 0.785398

(b) $\int_0^2 \frac{x+1}{x^2+2x+2} dx$ **Answer:** 0.802432, 0.804505, Exact: 0.804712.

9. Evaluate $\int_0^6 x \sec x dx$ using eight intervals by Trapezoidal rule.

Answer: -6.436.

10. Evaluate using Simpson's $\frac{1}{3}$ rule (i) $\int_0^6 \frac{e^x}{1+x} dx$ **Answer:** 70.16.

(ii) $\int_0^2 e^{-x^2} dx$ (Take $h = 0.25$) **Answer:** 0.635.

11. Evaluate using Simpson's $\frac{1}{3}$ rule (i) $\int_0^\pi \sin x dx$ taking 11 ordinates. **Answer:** 2.0009.

(ii) $\int_0^{\pi/2} \sqrt{\cos \theta} d\theta$ taking 9 ordinates. **Answer:** 1.1873.

12. Evaluate correct to 4 decimal places, by Simpson's $\frac{3}{8}$ rule (i) $\int_0^9 \frac{dx}{1+x^3}$ **Answer:** 1.1249.

(ii) $\int_0^{\pi/2} \sin x dx$ **Answer:** 0.911.

13. The table below shows the temperature $f(t)$ as a function of time. Using Simpson's $\frac{1}{3}$ rule, estimate

$$\int_1^7 f(t) dt.$$

| | | | | | | | |
|----------|----|----|----|----|----|----|----|
| $t :$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $f(t) :$ | 81 | 75 | 80 | 83 | 78 | 70 | 60 |

Answer: 403.67

14. A curve is drawn to pass through the points given by the following table. Estimate the area bounded by the curve, x-axis and the lines $x = 1, x = 4$.

| | | | | | | | |
|-------|---|-----|-----|-----|---|-----|-----|
| $x :$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| $y :$ | 2 | 2.4 | 2.7 | 2.8 | 3 | 2.6 | 2.1 |

Answer: 7.78

15. The velocity v of a particle at distance s from a point on its path is given by the table. Estimate the time taken to travel 60 ft by using Simpson's $\frac{1}{3}$ rule. Also, compare the result with Simpson's $\frac{3}{8}$ rule.

| | | | | | | | |
|---------------|----|----|----|----|----|----|----|
| $s(ft) :$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
| $v(ft/sec) :$ | 47 | 58 | 64 | 65 | 61 | 52 | 38 |

Answer: 1.063 sec, 1.064 sec