$$\frac{dy}{dx} = f(x,y)$$

$$a \le x \le b$$

$$y(a) = y_0$$

$$y_{(L+1)} = y_L + h y_L' + \frac{h^2}{2} y_L'' + \cdots + \frac{h^n}{n!} y_L^n.$$

$n = 1$ (Euler Method)

$n = 2$ (Second order Taylor)

$n = 3$ (Third ''   '')

$n = 4$ (4th   ''   '')

$$\frac{1}{h} \left(\frac{h^{n+1}}{(n+1)!} y_L^n \right) = h^n x_-$$

order of method $O(n)$

Modified Euler Method.

↓ order of this method 2.

$$\int_{x_L}^{x_{L+1}} \frac{dy}{dx} dx = \int_{x_L}^{x_{L+1}} f(x,y) dx$$

$$y_{L+1} - y_L = \frac{h}{2} \left[f(x_L, y_L) + f(x_{L+1}, y_{L+1}) \right]$$

$$y_{L+1} = y_L + \frac{h}{2}\left(f(x_i, y_i) + f(x_{L+1}, y_{(L+1)})\right)$$

$$y_{L+1}^{K+1} = y_L + \frac{h}{2}\left(f(x_L, y_L) + f(x_{L+1}, y_{(L+1)})\right)$$

$$k = 0, 1, \cdots$$

using $\rightarrow$ 0th iteration
of y_{L+1}

Euler $\leftarrow$ $y_{L+1}^0 = y_L + h f(x_i, y_i)$
method

RK (Runge kutta) Method

RK Method of order 4.

$$\int_{x_v}^{x_{L+1}} \frac{dy}{dx} dx = \int_{x_L}^{x_{L+1}} f(x, y) \, dx$$

$$\frac{h}{2}\left(f(x_L, y_L) + f(x_{L+1}, y_{L+1})\right)$$

$$= w_1 k_1 + w_2 k_1$$

$$w_1 = \frac{h}{2}, \quad w_2 = \frac{h}{2}$$

$$k_1 = f(x_v, y_L)$$

$$k_2 = f(x_{L+1}, y_{L+1})$$

$$y_{L+1} - y_L = w_1 k_1 + w_2 k_2 + w_3 k_3 + w_4 k_4$$

$$w_1, w_2, w_3, w_4$$

$$y_{L+1} = y_L + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_i, y_i) \checkmark$$

$$k_2 = f(x + h \cdots \cdots$$

$$k_2 = f\left(x_c + \frac{h}{2}, y_c + \frac{k_1}{2}\right) \checkmark$$

$$k_3 = f\left(x_c + \frac{h}{2}, y_c + \frac{k_2}{2}\right)$$

$$k_4 = f(x_c + h, y_c + k_3)$$

Q 9

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

$$y(0) = 1$$

$$h = 0.2$$

$$y(0.2) \& y(0.4)$$

$$y_{i+1} = y_c + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$k_1 = f(x_c, y_c), \quad k_2 = f\left(x_c + \frac{h}{2}, y_c + \frac{k_1}{2}\right)$$

$$h = 0.2 \qquad k_3 = f\left(x_c + \frac{h}{2}, y_c + \frac{k_2}{2}\right), \quad k_4 = f(x_c + h, y_c + k_3)$$

$$y_1 = y_0 + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \qquad f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$$

$$k_1 = f(x_0, y_0) = f(0, 1) = 1$$

$$k_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = f\left(0.1, 1 + \frac{1}{2}\right) = f(0.1, 1.5)$$

$$= \frac{(1.5)^2 - 0.1^2}{1.5^2 + 0.1^2} = 0.9911$$

$$k_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = f\left(0.1, 1 + 0.99111 \over 2\right))$$

$$= 0.95104$$

$$k_4 = f(x_0 + h, y_0 + k_3) = f(0.2, 1 + 0.95104)$$

$$= 0.96537$$

$$y_1 = 1 + \frac{0.2}{6} \left(1 + 2 \times 0.99111 + 2 \times 0.95104 + 0.96537\right)$$

$$\overset{\shortparallel}{y}(0.2)$$