20 June 2022 15:04

$$I = \oint \frac{h}{2}\left(f(a) + f(b)\right) \qquad h = b-a$$

$$\int_a^b f(x)\,dx \to \text{Exact} \qquad \simeq \frac{b-a}{2}\left(f(a) + f(b)\right)$$

$$f(x) = c$$

$$I = \int_a^b f(x)\,dx = c(b-a)$$

$$I = \frac{h}{2}(c + c) = c(b-a)$$

$$f(x) = cx$$

$$\int_a^b f(x)\,dx = \frac{c(b^2-a^2)}{2}$$

$$I = \frac{h}{2}(ca + cb) = c\frac{(b-a)}{2}(b+a)$$

$$= \frac{c(b^2-a^2)}{2}$$

$$f(x) = cx^2$$

$$\int_a^b f(x)\,dx = \frac{c(b^3-a^3)}{3}$$

$$I = \frac{(b-a)}{2}(ca^2 + cb^2) = \frac{c(b-a)(a^2+b^2)}{2}$$

Degree of Precision $\to$

Degree of Precision →
max degree of polynomial for which the exact & approximate value of integral are same.

Degree of Precision of Trapezoidal → 1.

→ Simpson's $\frac{1}{3}$ Rule.

$\int_{x_0}^{x_n} f(x)\,dx \simeq$ $n = 2$

$\parallel$

$I = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{1}{24}n(n-2)^2\Delta^3 y_0 + \cdots\right)$

(x_0, y_0), (x_1, y_1) (x_2, y_1)

x_0 y_0

x_1 y_1 Δy_0 $\Delta^2 y_0$

x_2 y_2 Δy_1

$$\int_{x_0}^{x_2} f(x)\,dx = 2h\left[y_0 + \Delta y_0 + \frac{2}{12}\Delta^2 y_0\right]$$

$$= 2h\left(y_0 + y_1 - y_0 + \frac{1}{6}(y_2 - 2y_1 + y_0)\right)$$

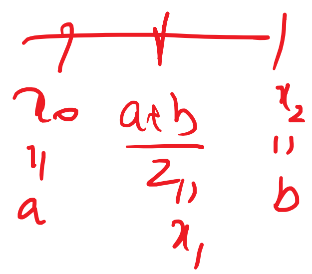
$$= 2h\left(y_1 + \frac{y_2}{6} - \frac{y_1}{3} + \frac{y_0}{6}\right)$$

$$= 2h \quad (\ldots$$

$h = \frac{b-a}{2}$

$$= \frac{2h}{6}\left(6y_1 + y_2 - 2y_1 + y_0\right)$$

$h = \frac{b-a}{2}$

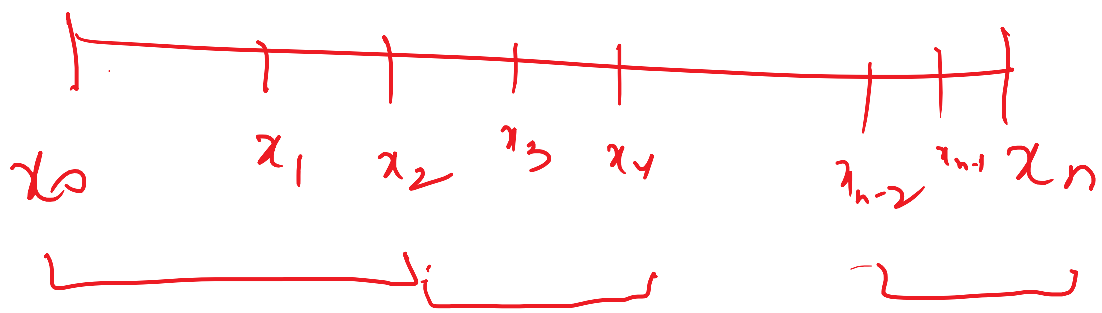


$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left(y_0 + 4y_1 + y_2\right)$$

$$= \frac{h}{3}($$

$$= \frac{(b-a)}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right) \to \text{Simple Simpson's } \tfrac{1}{3} \text{ Rule}$$

$$\int_{x_0}^{x_n} f(x)\,dx =$$



Composite 1/3 Rule

$n \to$ even,

$n+1 \to$ odd.

$\underline{n \to \text{even}}$

$$\int_{x_0}^{x_n} f(x)\,dx = \int_{x_0}^{x_2} f(x)\,dx + \int_{x_2}^{x_4} f(x)\,dx + \dots + \int_{x_{n-2}}^{x_n} f(x)\,dx$$

$$= \frac{h}{3}\left(y_0 + 4y_1 + y_2\right) + \frac{h}{3}\left(y_2 + 4y_3 + y_4\right)$$

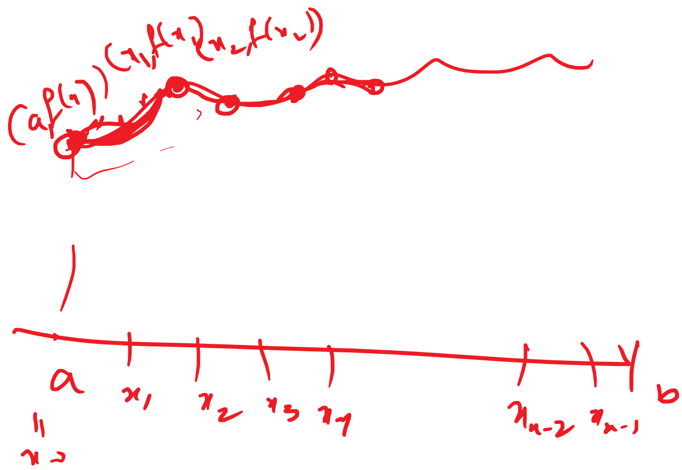
$$+ \left(y_4 + 4y_5 + y_6\right) + \frac{h}{3}\left(y_{n-2} + 4y_{n-1} + y_n\right)$$

$$= \frac{h}{3}\left(y_0 + y_n + 4y_1 + 4y_3 + \dots + 4y_{n-1}\right.$$

$$+ 2y_2 + 2y_4 + \dots + 2y_{n-2})$$

$$= \frac{h}{3}\left(y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})\right)$$

Composite Trapezoidal Rule



Q 6

$$\int_1^2 \frac{x^2}{1+x^3}\,dx \qquad \text{Simpson's } \tfrac{1}{3} \text{ Rule}$$

with (1) 2 subinterval (n=2)

(2) 4 subinterval (n=4)

$$\frac{h}{3}\left(y_0 + y_n + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})\right)$$

$n = 2 \qquad h = \frac{b-a}{2} = \frac{2-1}{2} = 0.5 \qquad y_0 = \frac{1}{2}$

$$= \frac{2-1}{2} = 0.5$$

$$\frac{h}{3}(y_0 + y_2 + 4y_1) =$$

$$\frac{0.5}{3}\left(0.5 + 0.44444 + 4(0.51428)\right)$$

$$= \underline{0.500}$$

$y_1 = \frac{(1.5)^2}{1+(1.5)^3}$

$= 0.51428$

$y_2 = \frac{2^2}{1+2^3}$

$= 0.44444$

$n = 4$. $\quad h = \frac{b-a}{4} = \frac{1}{4} = 0.25$

$1 \quad 1.25 \quad 1.5 \quad 1.75 \quad 2$

$$\frac{h}{3}\left(y_0 + y_4 + 4(y_1 + y_3) + 2y_2\right)$$

y_0

y_1

y_2

y_3

y_4

$= 0.50130$.

$$f(x) = c, \; c\,x, \; c\,x^2, \; c\,x^3$$