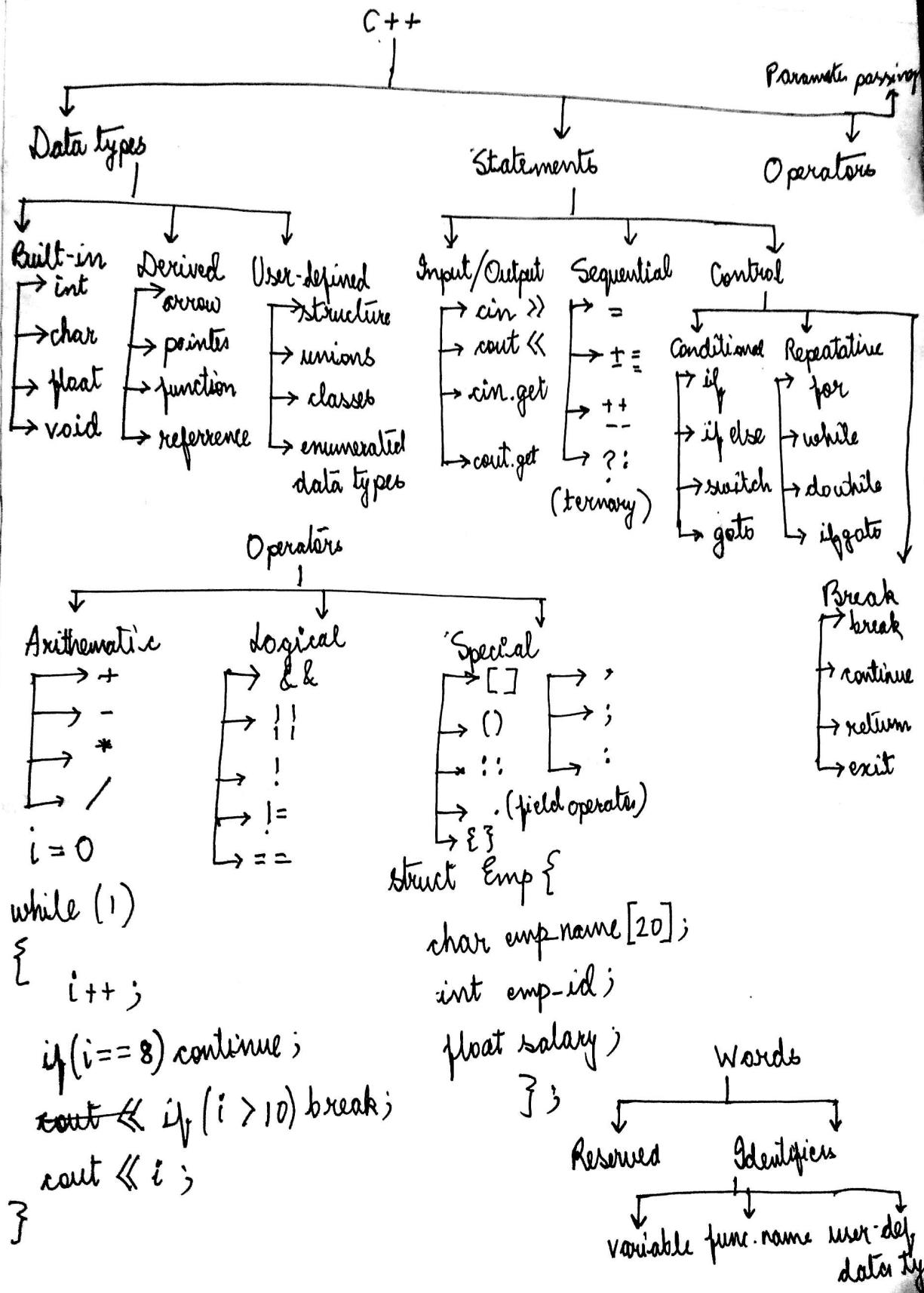
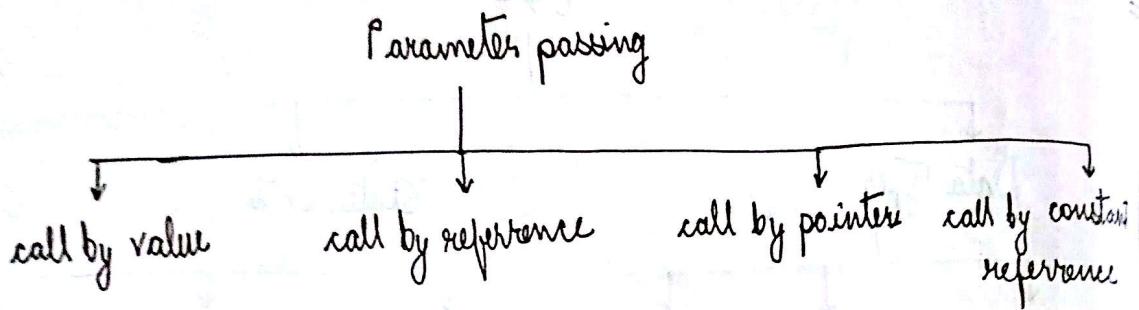


A decorative banner featuring five white squares arranged horizontally. Each square contains a large, bold, red letter: 'I' on the first square, 'N' on the second, 'D' on the third, 'E' on the fourth, and 'X' on the fifth. The letters are slightly tilted, giving the banner a dynamic appearance.

NAME: Shibam Basu STD: B.Tech SEC: A ROLL NO: 157153 SUB: DSA





```

# main()
{
    int a, b;
    a = 3; b = 5;
    cout << a << b;
    swap(a, b);
    cout << a << b;
}
  
```

```

void swap(int *p, int *q) //swap(int& p, int& q)
{
    int temp;
    temp = *p;
    *p = *q;
    *q = temp;
}
  
```

→ Calendar program (use 10 statements in a function) Write for 2016.

→ Parameter passing (all 3)

Mon Tue Wed Thu

Mon  
Tue  
Wed  
Thu

## Horizontal Calender

```
#include <iostream>
using namespace std;
{
    int month[13] = {0, 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
    int d, m, y, val;
    string s[] = {"", "January", "February", "March", ...};
    cout << "Enter year " << endl;
    cin >> y;
    if(((y % 4 == 0) && (y % 100 != 0)) || (y % 400 == 0)) month[2] = 29;
    y -= 1;
    val = (y + y / 4 - y / 100 + y / 400 + 1) % 7;
    for (int i = 1; i <= 12; i++)
    {
        cout << s[i] << endl;
        cout << "Sun Mon Tue Wed Thu Fri Sat" << endl;
        for (int space = 0; space < val; space++) cout << " ";
        for (int days = 1; days <= month[i]; days++, val++)
        {
            if (val == 7) { val = 0;
                cout << endl;
            }
            if (days < 10).cout << days << " ";
            else cout << days << " ";
        }
        cout << endl;
    }
    return 0;
}
```

## Vertical Calendar

```
#include <iostream>
using namespace std;

string month[] = {"", "JAN", "FEB", "MAR", "APR", "MAY", "JUN", "JUL", "AUG", "SEP", "OCT", "NOV", "DEC"};
string day[] = {"", "MON", "TUE", "WED", "THU", "FRI", "SAT", "SUN"};
int daysinmonth[13] = {0, 31, 29, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};

int i, j, temp, val, firstday = 5;
for(i = 1; i <= 12; i++)
{
    val = 9 - firstday;
    cout << month[i] << endl;
    for(j = 1; j <= 7; j++, val++)
    {
        cout << day[j] << " ";
        temp = val - 7;
        if(temp > 0)
        {
            while(temp <= 31)
            {
                cout << temp << " ";
                if(temp == daysinmonth[i]) firstday = j + 1;
                temp += 7;
            }
        }
        else
        {
            temp += 7;
            cout << " ";
            while(temp <= 31)
            {
                cout << temp << " ";
                if(temp == daysinmonth[i]) firstday = j + 1;
                temp += 7;
            }
        }
    }
    cout << endl;
}
```

## # Demonstration of Call by Reference -

```
#include <iostream>
using namespace std;
int main()
{
    int a = 3, b = 5;
    cout << a << b << endl;
    swap(a, b);
    cout << a << b << endl;
    return 0;
}
void swap(int &x, int &y)
{
    int t;
    t = x; x = y; y = t;
}
```

## # Demonstration of Call by Pointer

```
#include <iostream>
using namespace std;
void swap(int *x, int *y);
int main()
{
    int a = 3, b = 5;
    cout << a << b << endl;
    swap(&a, &b);
    cout << a << b;
    return 0;
}
void swap(int *x, int *y)
{
    int t;
    t = *x; *x = *y; *y = t;
}
```

## # Call By Value :-

```
#include <iostream>
using namespace std;
void swap(int, int);
int main()
{
    int a = 3, b = 5;
    cout << a << b << endl;
    swap(a, b);
    cout << a << b << endl;
}
void swap(int x, int y)
{
    int t;
    t = x; x = y; y = t;
}
```

## # Call by ~~reference~~ and Pointer

```
#include <iostream>
using namespace std;
void swap(int &x, int &y);
int main()
{
    int a = 3, b = 5;
    cout << a << b << endl;
    swap(a, b);
    cout << a << b << endl;
}
void swap(int &x, int &y)
{
    int t;
    t = x; x = *y; *y = t;
}
```

## Recursion

Any recursive program should have 3 statements

- ① Termination statement
- ② Self-call
- ③ Change

```
#include <iostream>  
using namespace std;  
  
int sum(int i)  
{  
    if (i >= 10) return 0;  
    else return i + sum(i+1);  
}  
  
int main()  
{  
    int s = 0;  
    cout << sum(1);  
    return 0;  
}
```

```
#include <iostream>  
using namespace std;  
  
int sum(int i)  
{  
    if (i >= 10) return 0;  
    else return i + sum(i+1);  
}
```

```
#include <iostream>  
using namespace std;  
  
void sum(int i)  
{  
    static int s = 0;  
    if (i > 10) cout << s;  
    else { s = s + i; sum(i+1); }  
}
```

```
#include <iostream>  
using namespace std;  
  
int sum(int i)  
{  
    if (i == 10) return i;  
    else  
    {  
        return i + sum(i+1);  
    }  
}
```

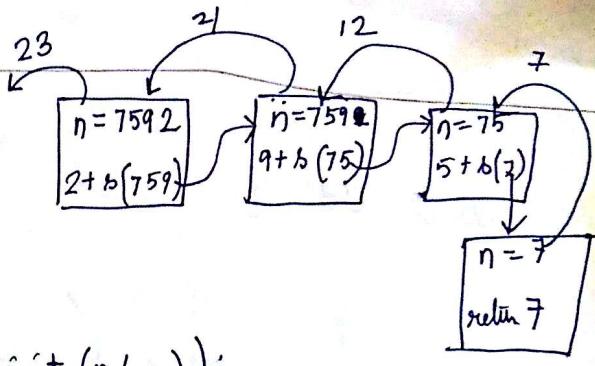
```
#include <iostream>  
using namespace std;  
  
void sum(int i, int &s)  
{  
    if (i > 10) return;  
    else  
    {  
        s = s + i;  
        sum(i+1, s);  
    }  
}
```

# Printing sum of digits  
int sumofdigit (int n)

{  
    if ( $n < 10$ ) return n;

    else return ( $n \% 10 + \text{sumofdigit}(n / 10)$ );

}



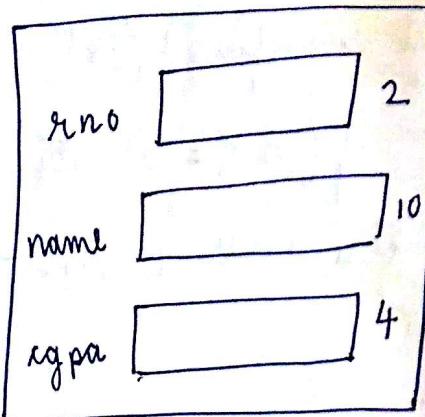
# void print (int n)  
{  
    if ( $n < 10$ ) cout  $\ll n$ ;  
    else {  
        print( $n / 10$ );  
        cout  $\ll n \% 10$ ;  
    }  
}

# Recursive function to find min, max, sum of element of array  
single (int a[],

```
struct std {
```

```
    int rno;  
    char name[20];  
    float cgpa;
```

```
};
```



```
struct std s;
```

No bytes are allocated while defining structure

```
# int i=5, j, k, *p, *q = &j, *r;
```

```
*p = 7;
```

```
p = &j;
```

```
*p = i;
```

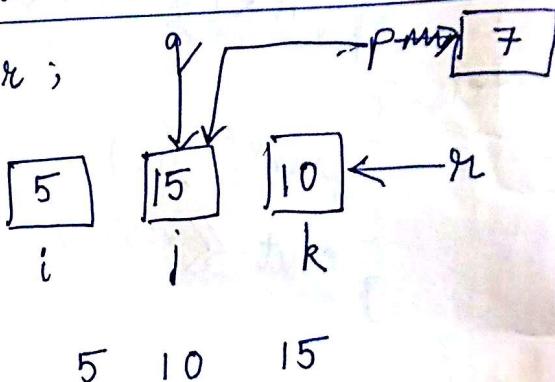
```
p = q;
```

```
r = &k;
```

```
*r = *p + *q;
```

```
*q = k + i;
```

```
cout << i << *r << *q;
```



```
struct std
```

```
{ int rno;
```

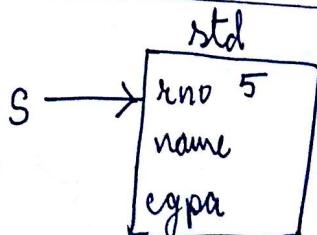
```
char name[10];
```

```
float cgpa;
```

```
};
```

```
struct std *s;
```

```
s = new std;
```



```
s -> rno = 5
```

struct add {

```
int street ;  
int cityno ;
```

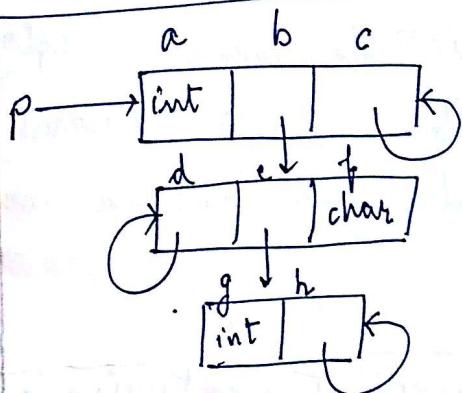
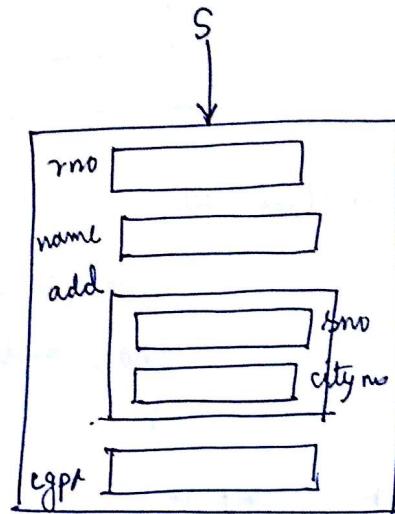
};

three semi-colons three statements.

struct std {

```
int rno;  
char name [10];  
struct add address;  
float cgpa;  
};
```

```
struct std *s = new std;  
s->address.street = 7;
```



*p* = new(*struct b3*);

*p* → *b* = new(*struct b2*);

*p* → *b* → *e* = new(*struct b1*);

*p* → *b* → *e* → *g* = 5;

struct s1 {

4 declarations;  
int g;  
struct s1 \*h;  
};

struct s2 {

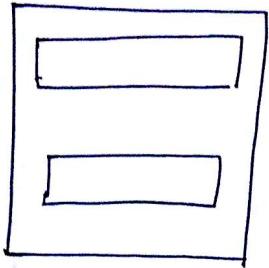
char f;  
struct s2 \*d;  
struct s1 \*e;  
};

struct s3 {

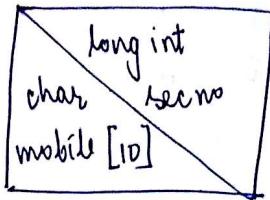
int a;  
struct s2 \*b;  
struct s3 \*c;  
};

struct b3 \*p;

adr



Union address



} either one  
(we use union)

union adr {

```
    long int seeno;  
    char mobile [10];  
};
```

struct std {

```
    int sno;  
    char name [10];  
    int flag;  
    union adr address;  
};
```

struct std s[10]; 320 bytes

```
for (i = 0; i < 10; i++)
```

{ cout << s[i].sno;

cout << s[i].name;

~~cout << if (s[i].flag == 1) cout << s[i].address.seeno;~~

else cout << s[i].address.mobile;

```
int n;
```

```
for (i = 0; i < 10; i++)
```

{

cout << "Enter roll no " << endl;

cin >> s[i].sno;

cout << "Enter name " << endl;

cin >> s[i].name;

cout << "Security or mob.no? " << endl;

cin >> ~~s[i].flag~~ n; s[i].flag = n;

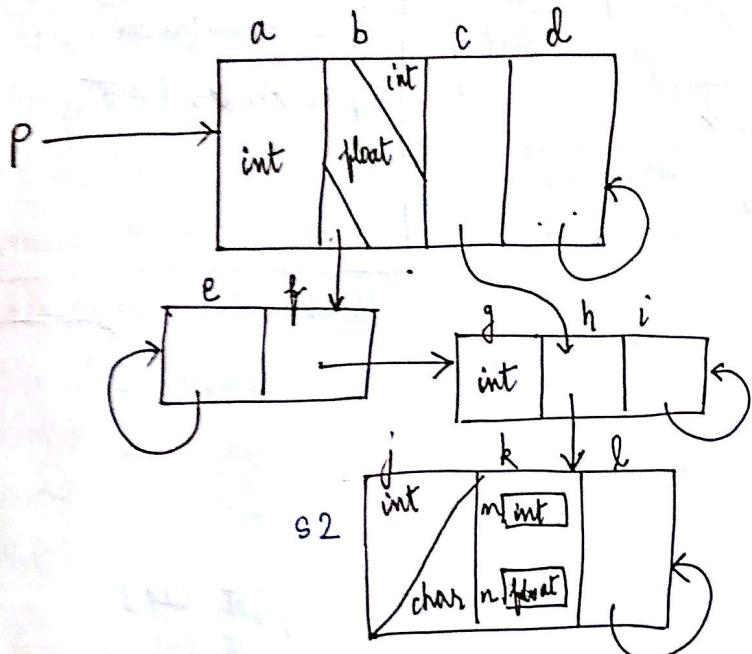
if (n == 1) cin >> s[i].address.seeno;

else cin >> s[i].address.mobile;

}

every structure containing a union must contain a flag field for choosing union field

# In union take larger field for calculating memory field .



```
struct s1 {
    int m;
    float n;
};
```

```
struct s2 {
    struct s1 k;
    union u1 j;
    struct s2 *l;
    int tag;
};
```

```
struct s5 {
    int a;
    union u2 b;
    struct s5 *d;
    struct s3 *c;
    int tag
};
```

```
union u1 {
    int o;
    char p;
};
```

```
struct s3 {
    int g;
    struct s3 *i;
    struct s2 *h;
};
```

~~struct s5 \*p = new (struct s5);~~

~~p->c = new (struct s3);~~

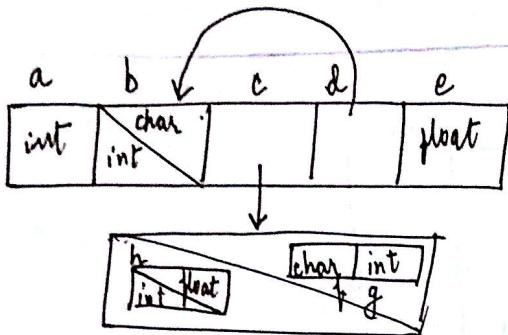
~~p->c->h = new (struct s2);~~

~~p->c->h->k.m = 5;~~

```
union u2 {
    int q;
    float r;
    struct s4 *s;
};
```

```
struct s4 {
    struct s4 *e;
    struct s3 *f;
};
```

#  
③



struct s1 \*obj = new(struct s1);  
obj->c = new(union u2);  
obj->c->h = 5; i = 5

→

union u1 {  
 int i;  
 float j;  
};

struct s1 {  
 char p;  
 int q;  
};

union u2 {  
 union u1 h;  
 struct s1 k;  
};

union u3 {  
 char l;  
 int m;  
};

struct s2 {  
 int a; → int tag1;  
 int tag; → int tag3;  
 union u3 b;  
};  
union u2 \*c;  
union u3 \*d;  
float e;  
};

① Array  
② Print  
③ Given  
④ Find

# in  
uni,

stru

## ②. Printing digits using recursion -

```
#include <iostream>
using namespace std;
void printdigit(int n);
int main()
{
    printdigit(7942);
}
```

```
void
^ printdigit(int n)
{
    if(n < 10) cout << n << endl;
    else
    {
        printdigit(n/10);
        cout << n % 10;
    }
}
```

### Assignment:-

- ① Array of student structures using union → program.
- ② Printing of 9742 using recursion
- ③ Given diagram
- ④ Finding max, min, sum, avg using single recursion.

```
#include<iostream>
using namespace std;
union contact {
    long mobile;
    int tele;
};

struct std {
    char name[20];
    int roll;
    contact c;
    float cgpa;
};

int main()
{
    struct std s[10]; int ch;
    for(int i=0; i<10; i++)
    {
        cout << " Details of student " << i+1;
        cout << " Enter name " << endl;
        cin >> s[i].name;
        cout << " Enter roll number " << endl;
        cin >> s[i].roll;
        cout << " 1. Mobile 2. Telephone ? " << endl;
        cin >> s[i].t;
        cout << " Enter " << endl;
        if (s[i].t == 1)
        {
            cin >> s[i].c.mobile;
        }
        else cin >> s[i].c.tele;
        cout << " Enter CGPA " << endl;
        cin >> s[i].cgpa;
    }
    for (i=0; i<10; i++)
    {
        cout << s[i].name << endl;
        cout << s[i].roll;
        if (s[i].t == 1) cout << s[i].mobile;
        else cout << s[i].c.tele;
        cout << s[i].cgpa;
    }
    return 0;
}
```

④

```
#include<iostream>
```

```
using namespace std;
```

```
void recursion(int [], int, int, int, int);
```

```
int main()
```

```
{ int a[10], n;
```

```
cout << "Enter the number of integers";
```

```
cin >> n;
```

```
for (int i=0; i<n; i++) cin >> a[i];
```

```
recursion(a, a[0], a[0], a[0], n);
```

```
}
```

```
void recursion(int a[], int min, int max, int sum, int n)
```

```
{ static int i = 0;
```

```
if (i == n)
```

```
{ cout << "Minimum = " << min << " Max = " << max << " Sum = " << sum
```

```
<< "Average = " << (sum/n); (float)
```

```
. return;
```

```
else
```

```
{ if (a[i] < min) min = a[i];
```

```
if (a[i] > max) max = a[i];
```

```
sum += a[i];
```

```
i++;
```

```
recursion(a, min, max, sum, n);
```

```
}
```

## Data Structures

linear  
 → Arrays (static)  
 → linked list (dynamic)  
 → Stack  
 → Queue

Tree

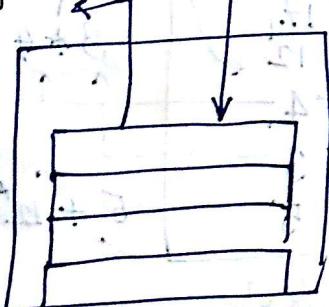
graph

Diagram → Definition Declaration Operation Application Variation

Abstract Data Type - type which has data structure as well as operations associated with it.

### Stacks (LIFO)

Diagram - pop



push (stk, n)      Definition

struct stack {

int size;  
int elements [50];

int top;

};

struct stack stk;

stk.top = -1;

stk.size = 50;

### Declaration

### Operation -

```
void push(struct stack & s, int x)
{
  if (s.size - 1 == s.top)
  {
    cout << "stack is full" << endl;
  }
  else
  {
    s.top++;
    s.elements [s.top] = x;
  }
}
```

```

int pop(struct stack s)
{
    if(s.top == -1)
        cout << " Stack is empty & null ";
    else
        return s.elements[s.top--];
}

```

### Application -

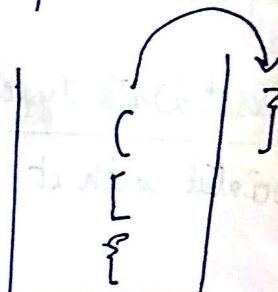
①

Balancing symbols eg - {a + [b \* c - (d \* e + f)]}

Take an array,

opensym[] = {'{', '[', '('}

closesy[] = {'}', ']', ')'}



②

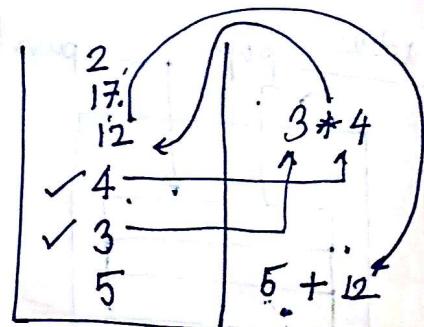
Evaluating post-fix expression -

Infix  $\rightarrow 5 + 3 * 4 - 2$

Postfix  $\rightarrow 5 3 4 * + 2 -$

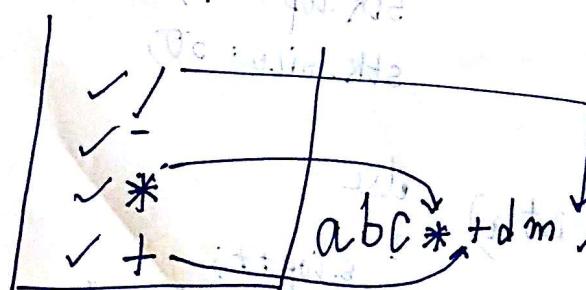
Infix  $\rightarrow a / b * c / d / m$

Postfix  $\rightarrow abc * + dm / -$



stack of  
operands

$$17 - 2 = 15$$



stack of operators

Infix  $\rightarrow$  postfix

(~~stack~~ stronger operator sits on weaker one)

An operator cannot sit on operator of same rank.

Every operator should enter stack ~~one~~ one atleast.

Queues (FIFO) → First In First Out.

Definition

struct queue {

```
int size;  
int elements[50];  
int front;  
int rear;  
};
```



Queue is full is  $(r+1) \% \text{size} = f$

since queue is round

Declaration - struct queue Q;

→ Initialisation - Q.size = 50;

Q.front = -1;

Q.rear = -1;

void enqueue(queue &Q, int x);

{

if ((Q.rear + 1) % Q.size == Q.front) cout << "Queue is full" << endl;

else if (Q.front == -1) Q.front = 0;

Q.rear = (Q.rear + 1) % Q.size;

Q.elements[Q.rear] = x;

}

}

int dequeue(queue &Q)

{ if (Q.front == -1) { cout << "Queue is empty" << endl;

else

①  
②  
③  
x ④

```
{ length rear = max no. elements front
```

if (~~B. front~~ == B.size - 1)  
    ~~rear~~  
    too ~~B. front~~ = 0 ;  
    return B.elements[B.rear++];  
else {  
    temp = B.front ;  
    if (~~B. front~~ == B.size - 1) B.front = 0;  
    else B.front ++;  
    return temp ;  
}

```
int dequeue(queue &B)
```

```
{ int t ;
```

```
    if (B.front == -1) cout << "Q is empty" << endl ;
```

```
    else {  
        t = B.elements[B.front] ;  
        if (B.front == B.rear) {  
            B.front = -1 ;  
            B.rear = -1 ;  
        }  
        else {  
            B.front = (B.front + 1) % size ;  
        }  
    }  
}
```

```
else {  
    t = B.elements[B.front] ;  
    B.front = (B.front + 1) % size ;  
}
```

```
} } return t ;
```

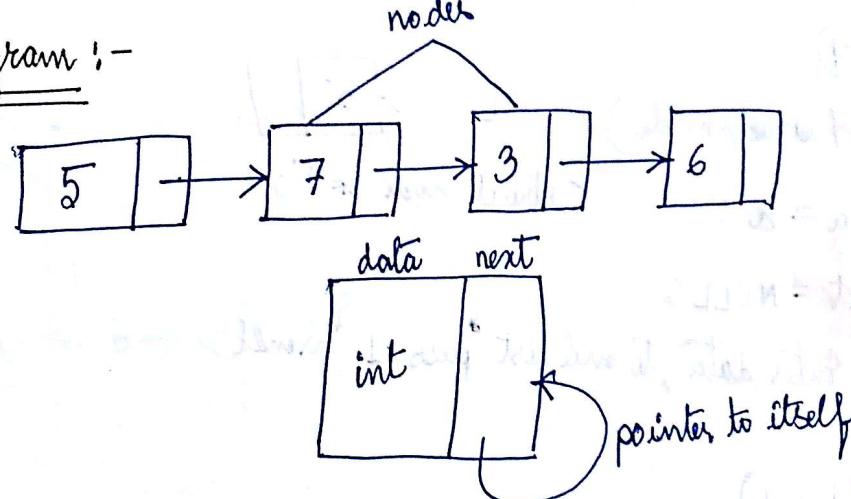
Note:

## Assignments :-

- ① Reverse a queue using stack
- ② Implementation of queue using two stacks
- ③ Implementation of stack using queue.
- X ④ enqueue, dequeue, eval.

## linked list

### Diagram :-



### Definition :-

```
struct node {
```

```
    int data;
```

```
    struct node *next;
```

```
};
```

### Declaration

```
struct node *L;
```

(Declaration is initialisation itself)

### Note :-

Use complex user-defined data types through pointers so that you can deallocate the memory later.

## Operations -

- |   |           |                                  |
|---|-----------|----------------------------------|
| ① | addbegin  | ④ <del>removes</del> deletebegin |
| ② | addend    | ⑤ deleteend                      |
| ③ | addafter  | ⑥ createlist                     |
| ⑦ | addbefore | ⑦ printlist                      |

## Creating linked list -

```

cout << "Enter data" << endl;
cin >> d;
if (d != -1)
    L = new (struct node);
    L->data = d;
    L->next = NULL;
cout << "Enter data, to end list press -1" << endl;
cin >> d;
while (d != -1)
{
    T = L;
    while (T->next != NULL) T = T->next;
    struct node *p = new (struct node);
    T->next = p;
    p->data = d;
    p->next = NULL;
}

```

★ typedef int length; → renaming int as length.

struct node → typedef

```

    struct node {
        int data;
        struct node* next;
    }*Lptr;
  
```

} 0 bytes allocation.

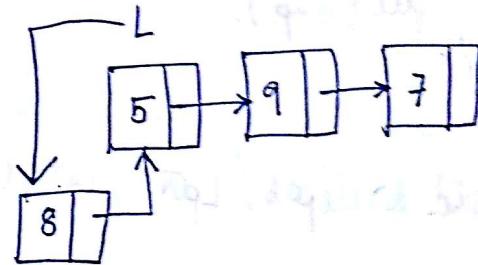
Lptr is a pointer to node structure type.

Lptr L; (2 bytes)

# void addbegin(Lptr L, int d) (Call by value):

```

{
    Lptr temp = new(struct node);
    temp->data = d;
    temp->next = L;
    L = temp;
}
  
```



# void addend(Lptr L, int d)

```

{
    Lptr temp = new(struct node), v = L;
    temp->data = d; temp->next = NULL;
    while (v->next != NULL) v = v->next;
    v->next = temp;
}
  
```

}

# void deletion (Lptr L)

{

Lptr temp = L, v = L;

while (temp → next != NULL) temp = temp → next;

while (v → next != temp) v = v → next;

v → next = NULL;

free (temp);

}

# void deletibegin (Lptr L)

{

Lptr temp = L;

~~if (temp == NULL)~~ L = L → next;

free (temp);

}

# void deletepos (Lptr L, int d)

{

Lptr t = L, temp;

while (t → next → next != d) t = t → next;

temp = t → next;

t → next = t → next → next;

~~free (temp);~~ delete temp;

}

1018  
L      1018  
2

```
# void reprint(Lptr t)
{
    if (t != NULL)
    {
        cout << t->data << " ";
        reprint(t->next);
    }
}
```

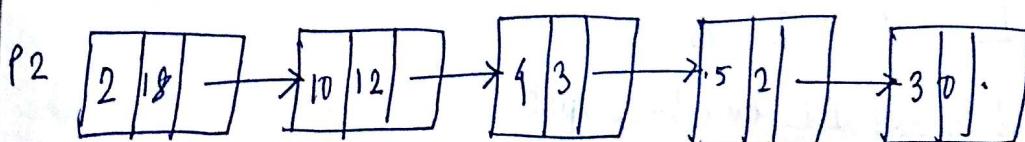
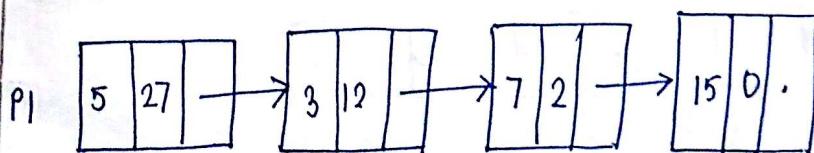
```
# void reverseprint(Lptr t)
{
    if (t == NULL) return;
    else
    {
        reverseprint(t->next);
        cout << t->data << " ";
    }
}
```

Polynomial addition -

$$P_1: 5x^{27} + 3x^{12} + 7x^2 + 15$$

$$P_2: 2x^{18} + 10x^{12} + 4x^3 + 5x^2 + 3$$

A [i] → 3 information  
 ↓  
 exponent  
 value of array element  
 = co-efficient .



void MergeL(Lptr L1, Lptr L2, Lptr L)

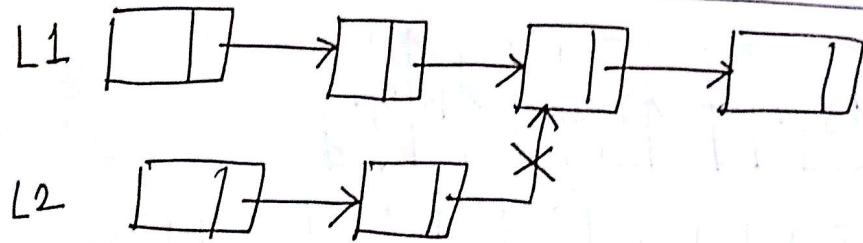
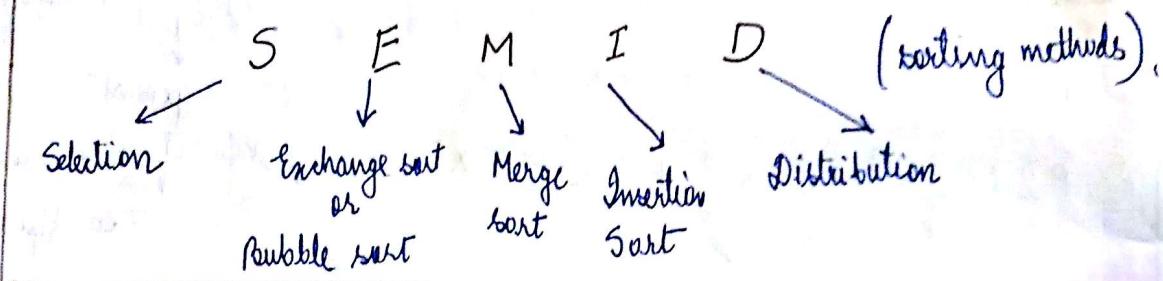
```
{
    Lptr t1, t2, l;
    while (t1 != NULL & t2 != NULL)
    {
        if (t1->data > t2->data) { addend(t, t2->data); t2=t2->next; }
        else { addend(t, t1->data); t1=t1->next; }
    }
}
```

```

while (t1 != NULL)
{
    addend(t, t1->data);
    t1 = t1->next;
}
while (t2 != NULL)
{
    addend(t, t2->data);
    t2 = t2->next;
}

```

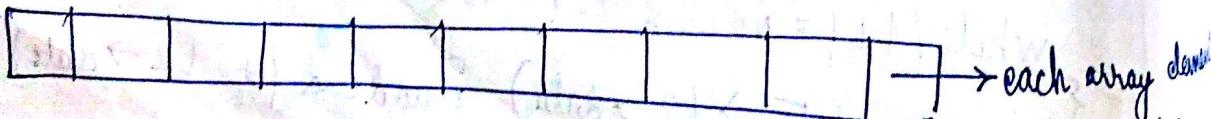
- ① Polynomial Addition
- ② Union Intersection of 2 lists.
- ③ Insertion Sort using lists
- ④ Insertion Sort Using linked lists Recursively
- ⑤ Remove duplicates



Create 2 linked lists and then split.

Radix Sort :- (Bucket Sort)

98    517    33    26    801    745    4    19

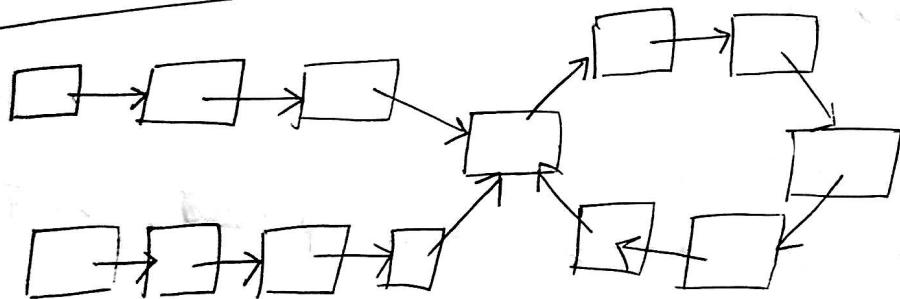
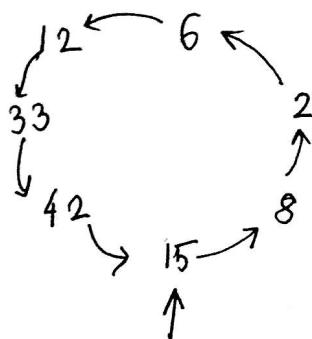


$\text{int } n = (\text{rand}() \bmod 100)$

⑧ Stack as linked list

⑨ Queue as linked list

⑩ Joseph's Problem



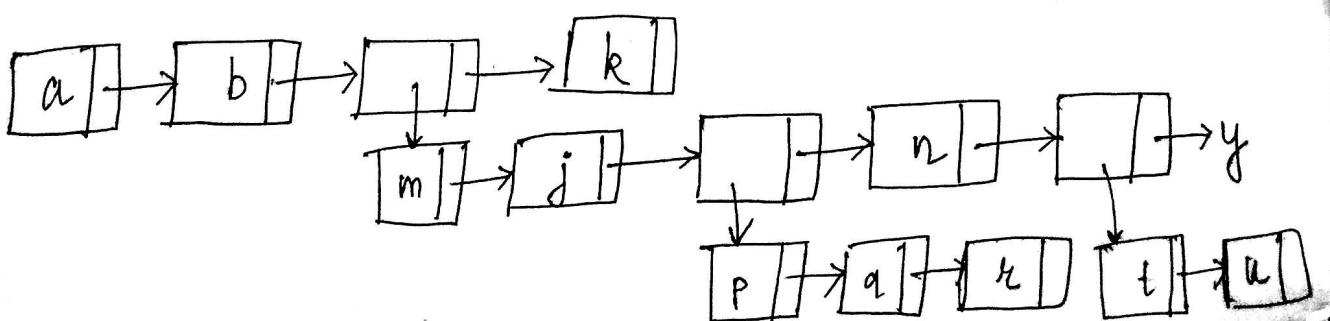
$$\# S = (a, b, (m, j, (p, q, r), n, (t, u), y), k) \quad \begin{matrix} 4 \text{ elements} \\ 4 \text{ sets} \end{matrix}$$

$$\text{let } A = (p, q, r)$$

$$B = (t, u)$$

$$C = (m, j, An, B, y)$$

$$\therefore S = (a, b, C, k)$$



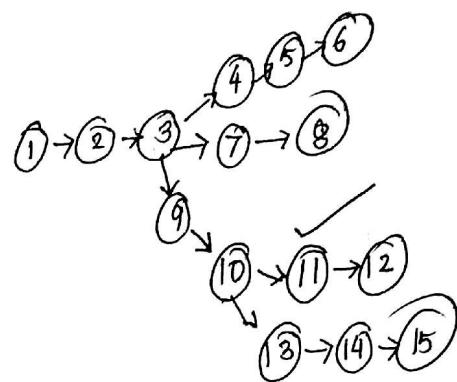
```
struct list {  
    int tag;  
    union u {  
        char data;  
        struct list * head;  
        } obj;  
    struct list * next;  
};
```

```
void print(struct list * L)  
{  
    if (L == NULL) return;  
    else  
    {  
        if (L->tag == 1) cout << L->obj.data;  
        else  
        {  
            cout << "(";  
            print(L->obj.head);  
            cout << ")";  
        }  
        print(L->next);  
    }  
}
```

```

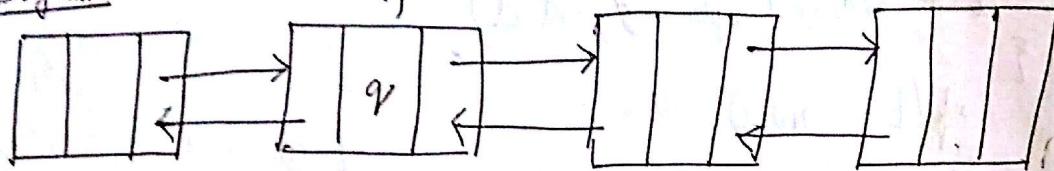
bool
search(Lptr L, int d)
{
    if (L == NULL) return ;
    else if (L->data == d)
    {
        cout << " Item found ";
        return true ;
    }
    search (L->next 1) ;
    search (L->next 2) ;
    search (L->next 3) ;
    cout << " Item not found " << endl ;
    return false ;
}

```



## Doubly linked list

Diagram



Definition -

struct dNode {

int data;

struct dNode\* left;

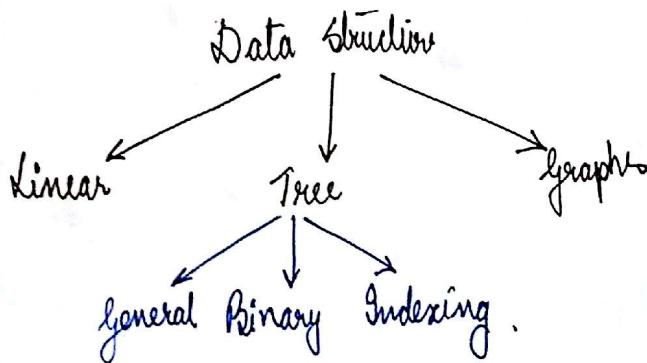
struct dNode\* right;

};

Declaration:- struct dNode\* DPTR;

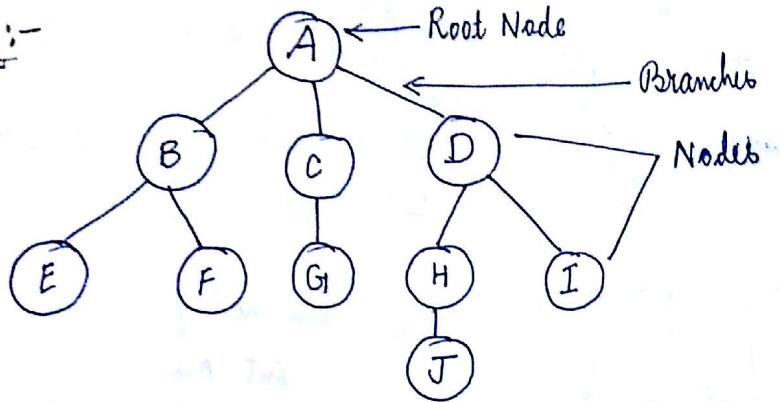
Deleting node T :-  $T \rightarrow \text{left} \rightarrow \text{right} = T \rightarrow \text{right}$

$T \rightarrow \text{right} \rightarrow \text{left} = T \rightarrow \text{left}$

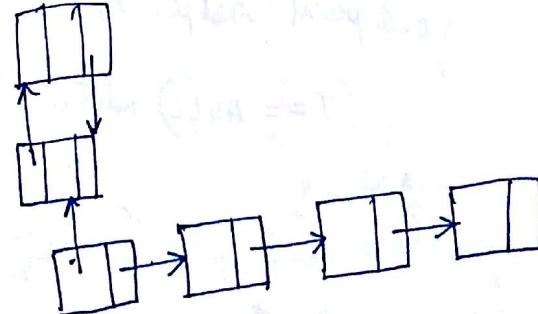
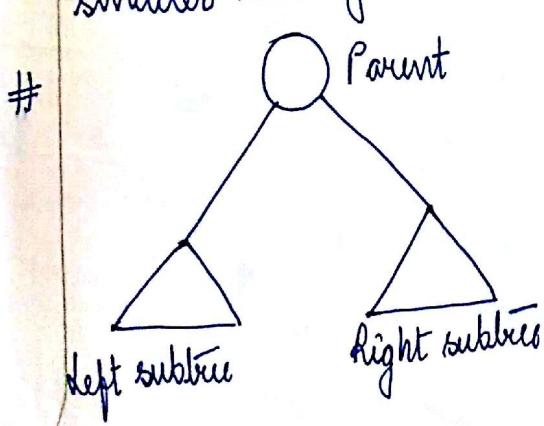


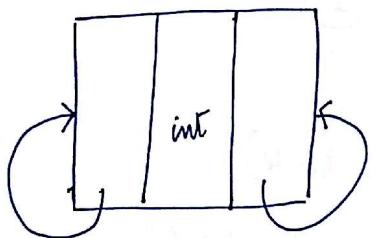
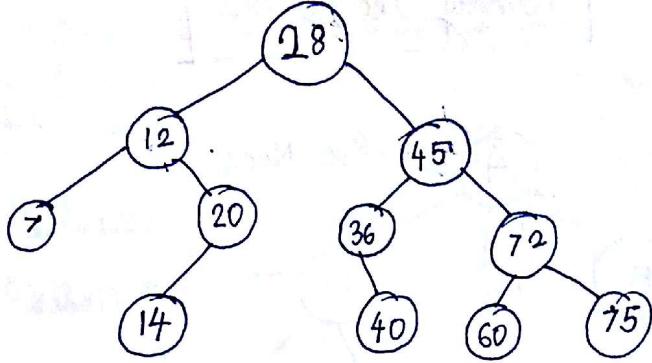
## Binary Search Tree

Tree:-



- # B C D → siblings (child of same parent node).
- # The number of branches from root to a node is its pathlength.
- # A node without children are called external nodes / terminal nodes / leaf nodes.
- # A node having atleast one child is internal nodes / non-terminal / non-leaf nodes.
- # A binary tree is a tree, that each node has atmost 2 children.
- # A binary search tree is a tree with such property that at every node the key value must be greater than all the values in its left subtree and smaller than right subtree.





```
struct bst_node {
    int key;
    bst_node *leftchild;
    bst_node *rightchild;
};
```

```
void insert(bst_ptr T, int k)
{
    if (T == NULL) { T = new createnode(k); return; }
    else { if (k > T->key) insert(T->rightchild, k);
           else if (k < T->key) insert(T->leftchild, k);
    }
}
```

---

```
void print(bst_ptr T)
{
    if (T == NULL) return;
    else
    {
        cout << print(T->left);
        cout << *T->key;
        cout << print(T->rightchild);
    }
}
```

Height of tree :- height of a tree is pathlength from deepest node to root.

Depth of a node - branches from root to that node

Binary tree - Tree in which a node has atmost 2 children.

struct bstnode {

    int data ;

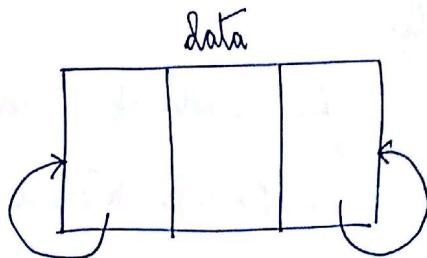
    bstnode \*lchild ;

    bstnode \*rchild ;

}

typedef bstnode\* BSTPTR

# Binary search tree is a BT such that at every node the data value is greater than data values of all nodes in left subtree and ~~greater~~ smaller than those of all nodes in right subtree.



Q- Find node at which search property is not satisfied.

bool ~~find~~ find (BSTPTR root, int k)

{ if (root == NULL) return false;

else

{ if (root->data == k) return true;

else if (find (root->l

{ ~~see~~ find (root->lchild, k);

else if (root->data > k)  
return find (root->lchild, k);  
else if (root->data < k)  
return find (root->rchild, k);  
else return false;

}

```

void insert(BSTPTR T, int k)
{
    if (T == NULL)
    {
        T = new (bst_node);
        T->data = k;
        T->leftchild = null;
        T->rightchild = null;
    }
    else
    {
        if (T->data > k) insert(T->leftchild, k);
        if (T->data < k) insert(T->rightchild, k);
    }
}

```

<u>Inorder printing</u>	<u>Preorder printing</u>	<u>Postorder printing</u>	<u>Level Order</u>
LDR	DLR	LRD	ABCDEFG DBEAFCG (inorder) ABODECFG (preorder) DEBGFCA (postorder)

```

void preorder(BSTPTR t)
{
    if (t == NULL) return;
    else
    {
        cout << t->data;
        preorder(t->leftchild);
        postorder(t->rightchild);
    }
}

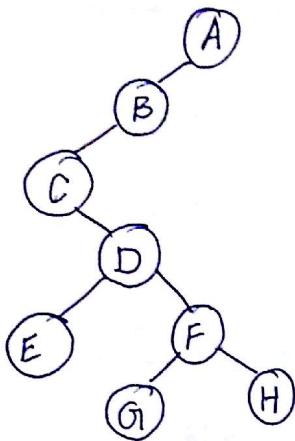
```

```
void postorder(BSTPTR t)
```

```

{
    postorder(t->rightchild);
    postorder(t->leftchild);
    cout << t->data;
}

```



LRD: EGH FDCBA

LDR: C E D G F H B A

```

int findmin(BSTPTR T)
{
    if (T == NULL) return ;
    else
        if ((T->rightchild)) while (T->lchild) T = T->lchild
        return T->data;
}
  
```

```

int findmax(BSTPTR T)
{
    if (T == NULL) return ;
    while (T->rightchild) T = T->rightchild;
    return T->data;
}
  
```

### Insertion (Iterative) -

```

void insert(BSTPTR t, int key)
{
    if (t == NULL) return;
    else
    {
        while (key < t->data) t = t->lchild;
        while (key > t->data) t = t->rchild;
    }
}

```

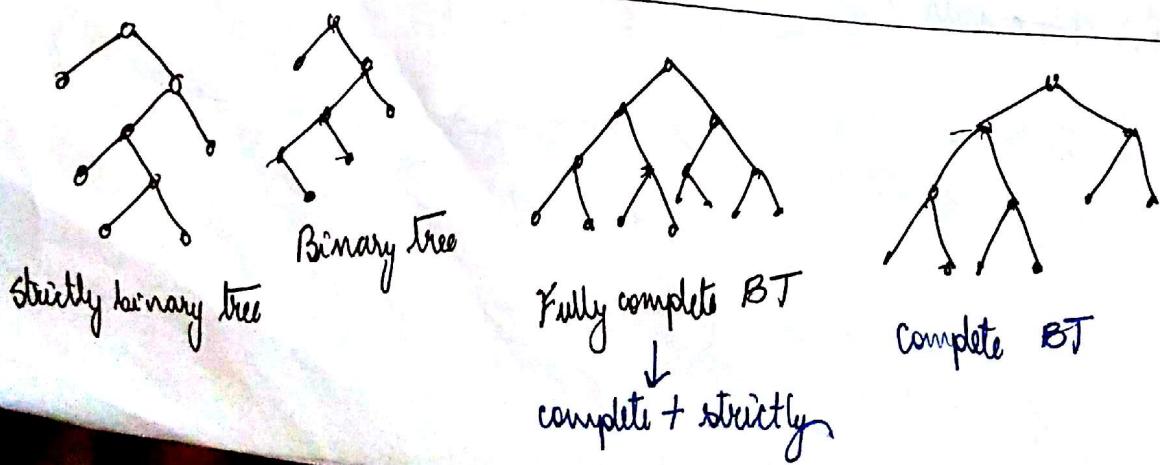
```
void insert(BSTPTR t, int val)
```

```

{
    while (T)
    {
        if (T->data > val) T = T->lchild;
        else if (T->data < val) T = T->rchild;
    }
    T = new bstnode;
    T->data = val;
    T->lchild = T->rchild = NULL;
}

```

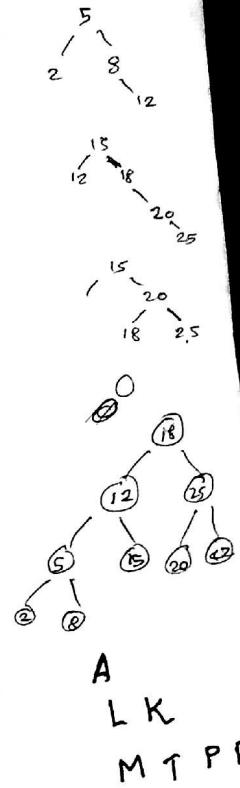
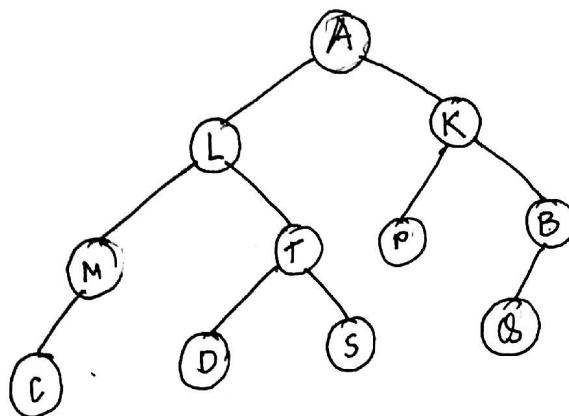
### Tree Traversal (LDR, DLR or LRD or level order (breadth first)).



Q. Construct BST if <sup>i/p</sup> is such that binary search tree is complete .  
 2 5 8 12 15 18 20 25 42  
 0 1 2 3 4 5 6 7 8

Q. Construct BST in iterative manner.

Binary Tree



while ( $\theta$  is not empty)

{  
 $T = \text{dequeue}(\theta)$

cout  $\ll T \rightarrow \text{data}$  ;

if ( $T \rightarrow \text{lchild} \neq \text{NULL}$ ) enqueue ( $\theta, T \rightarrow \text{lchild}$ )

if ( $T \rightarrow \text{rchild} \neq \text{NULL}$ ) enqueue ( $\theta, T \rightarrow \text{rchild}$ )

}

Print a binary tree line by line (level order printing)

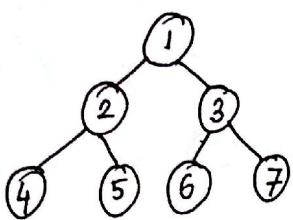
Q. Print a binary tree line by line (level order printing)

Q. Construct binary tree (user input  $\rightarrow A \ L \ M \ C \dots$ )

If a node does not have a left or right child he will put '.' when asked.

Q- Find maximum element in binary tree

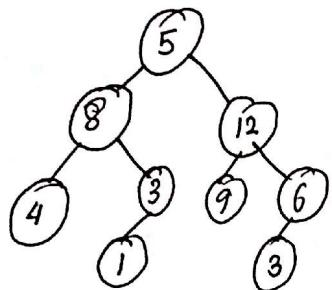
Q-



Print 4 5 6 7 2 3 1.

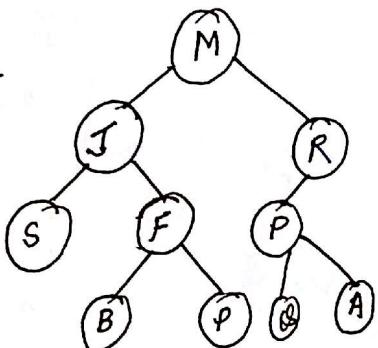
int  
{  
}

Q-



Print all possible paths.

Q-



Print a binary tree as it is.

Y co-ordinate  $\rightarrow$  level order

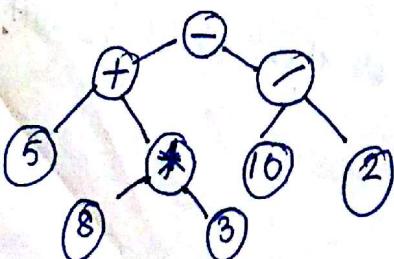
X co-ordinate  $\rightarrow$  inorder

3

### Expression Trees

$\rightarrow$  it is a binary tree where internal or non-leaf nodes are operators and external/leaf nodes are operands.

Infix :-  $5 + 8 * 3 - 10 / 2$ .



```

int eval(BPTR T)
{
    if (T->data == '+' || T->data == '-')
        char x, y;
        if (T->tag == 1)
            return T->leftchild; T->data;
    }
    else
    {
        eval(T->leftchild) ; int x = eval(T->leftchild)
        eval(T->rightchild) ; int y = eval(T->rightchild)
        return function(f)
        switch (T->data):
            case '+': return x + y;
            :
    }
}

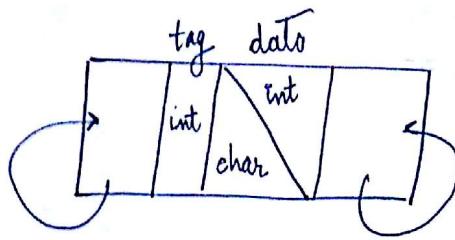
```

```

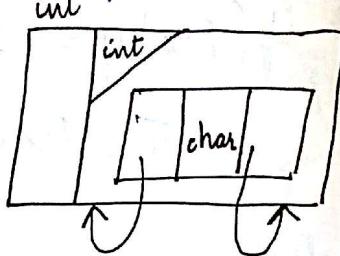
int eval(BPTR T)
{
    if (T->tag == 1) return T->data;
    else
    {
        int a = eval(T->leftchild);
        int b = eval(T->rightchild);
        switch(T->data):
            case '+': return a+b;
    }
}

```

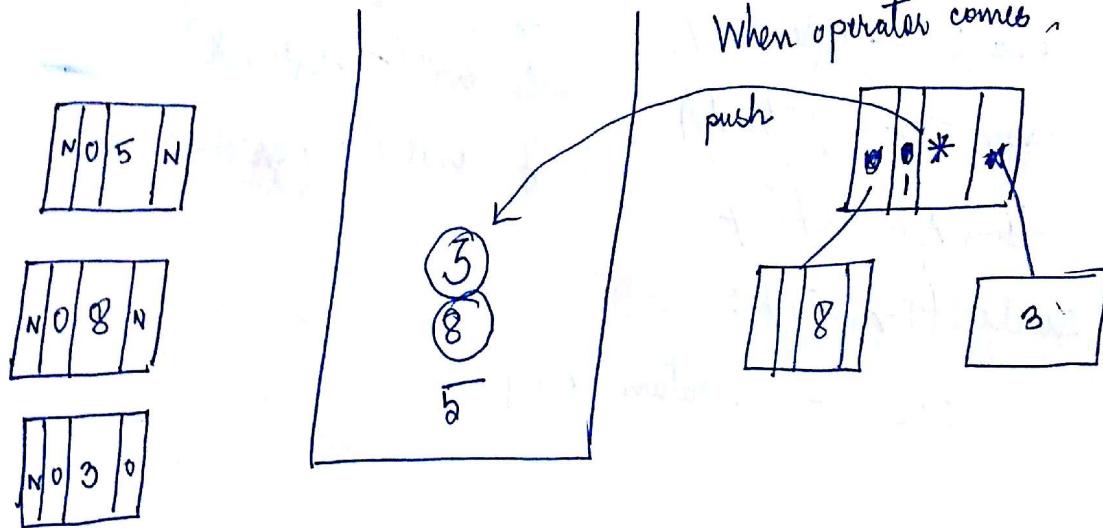
Postfix :-  $5 \ 8 \ 3 * \ ^{+} \ 10 \ 2 / -$



Node structure of expression tree.



Postfix evaluation of expression tree :-



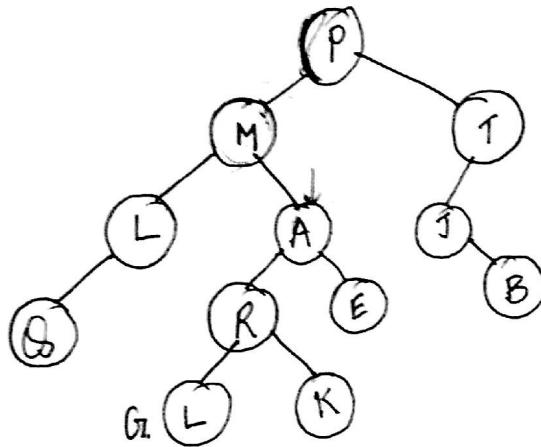
When operator comes,

- ① Multidigit evaluation
- ② ET construction and evaluation
- ③ ET construction using 2<sup>nd</sup> structure
- ④ BST deletion
- ⑤ LDR, DLR
- ⑥ LDR, LRD
- ⑦ Top View of Binary Tree

Remember :- To delete a node, substitute leftmost child or of right subtree or  
rightmost child of left subtree.

LDR: Q L M L R K A E P J B T

DLR: P M L Q A R L K E T J B .



root → left = dNode(R → left);

temp = Node + root;

root = root → right;

delete temp;

return root;

LRD: Q L G K R E A M B J T P ← root of binary tree

LDR: Q L M L R K A E P J B T

## Height of a Binary Tree :-

```
int calcheight (Bpt & R)
```

```
{ if (R==NULL) return -1;
```

```
else
```

~~int calcheight~~

```
    if (calcheight (R->left) > calcheight (R->right)) return
```

```
        1 + calcheight (R->left)
```

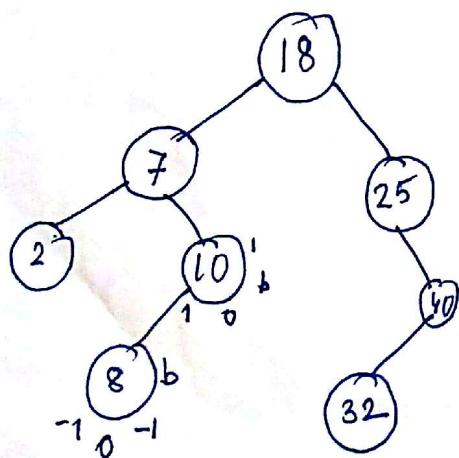
```
    else return 1 + calcheight (R->right);
```

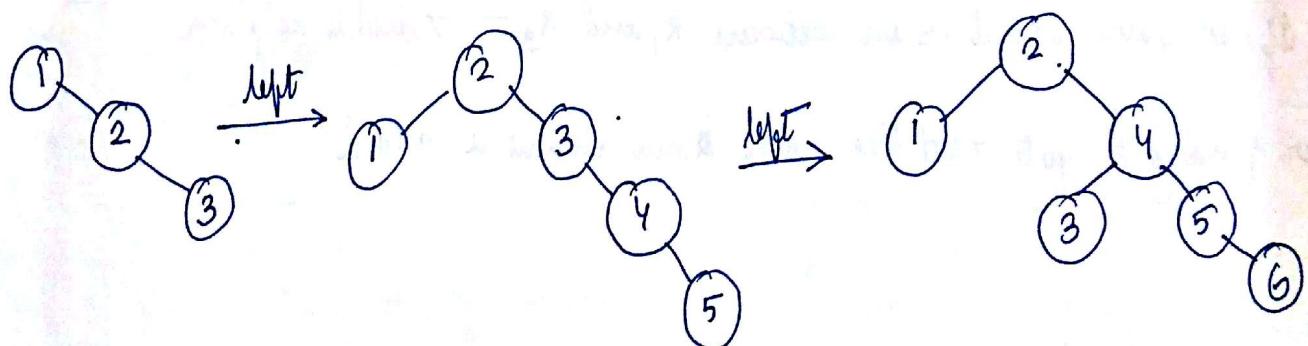
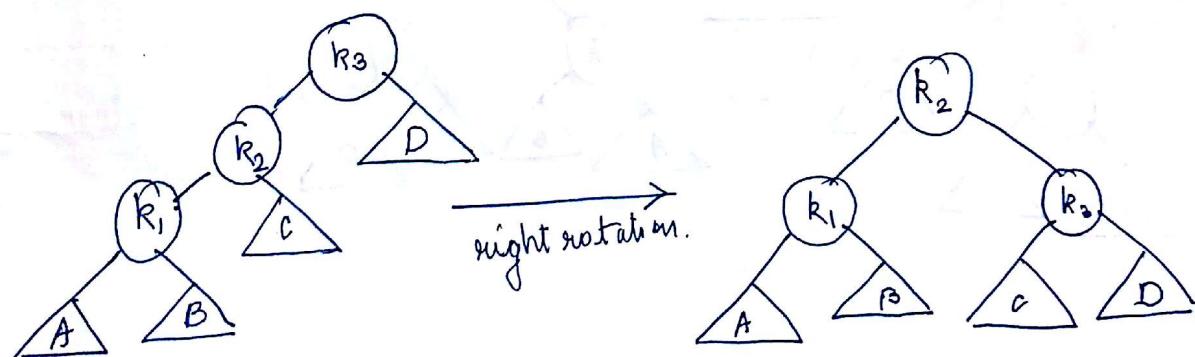
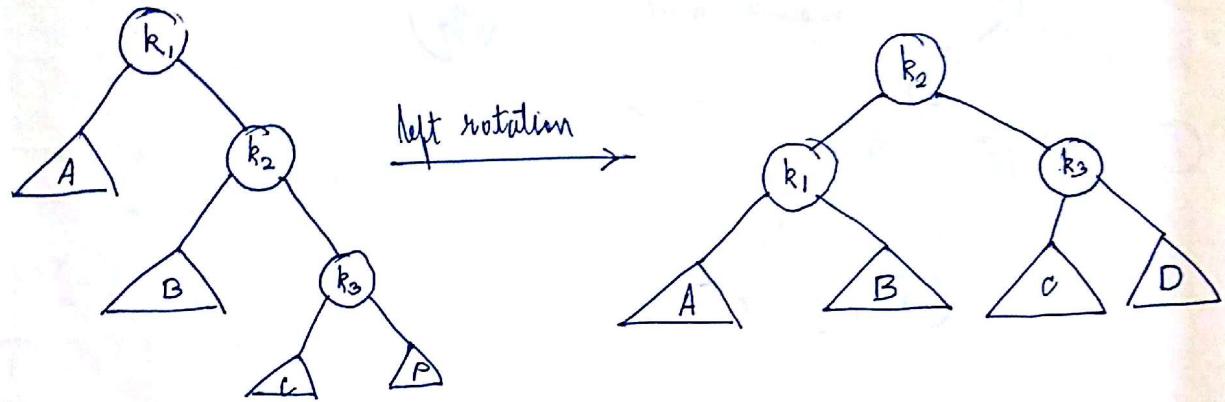
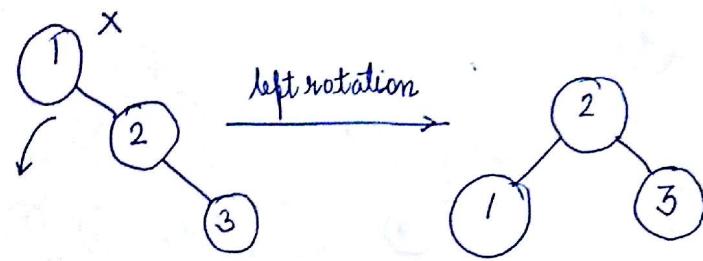
3 7

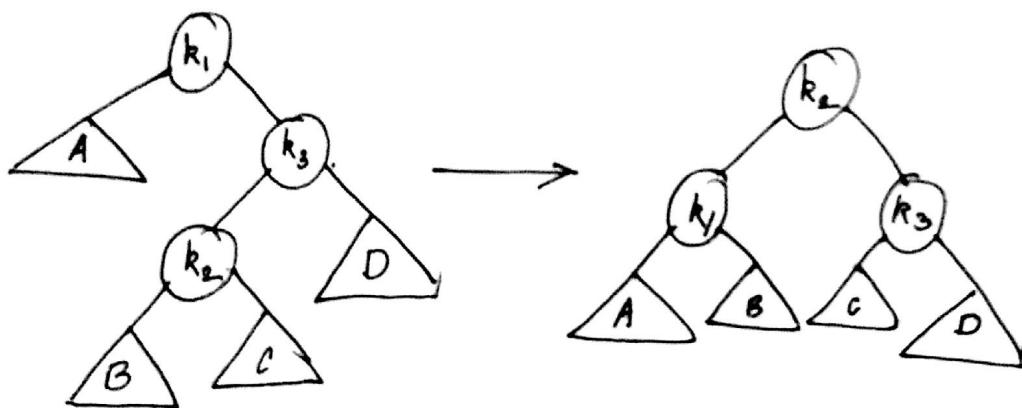
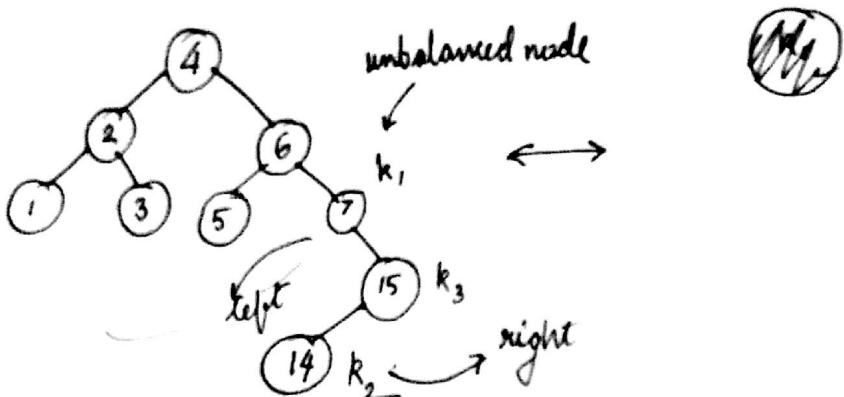
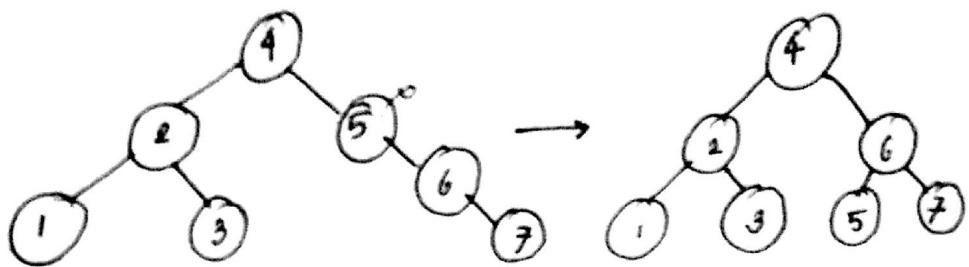
## AVL Tree

AVL Tree is a Binary Search Tree with height balanced condition.

At every node height of left subtree and height of right subtree must differ at most by 1.

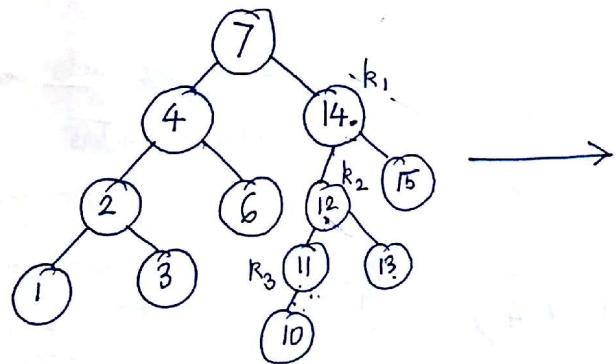
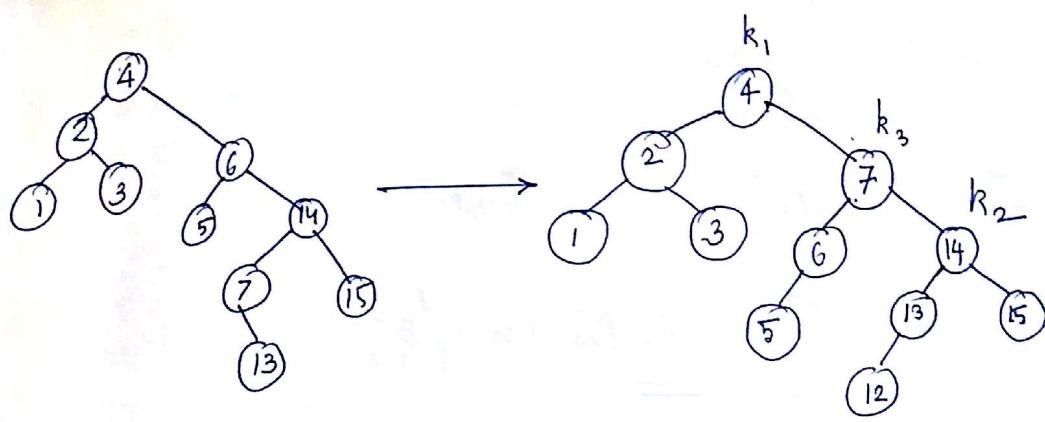




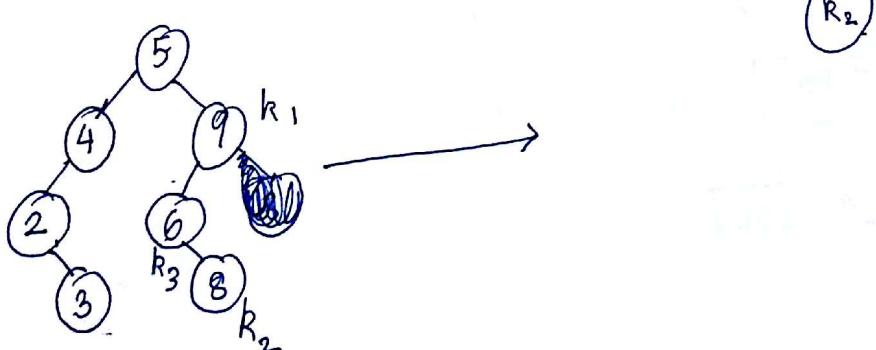


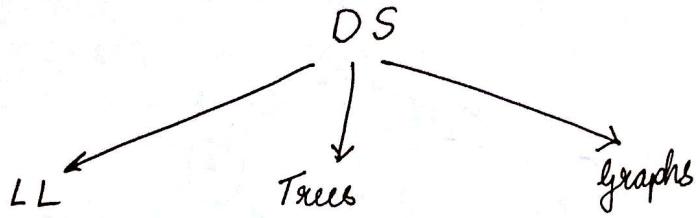
# If new value entered is in between  $k_1$  and  $k_3 \rightarrow$  double rotation

For finding  $k_2$ , go to that tree where  $k$ .new element is added.



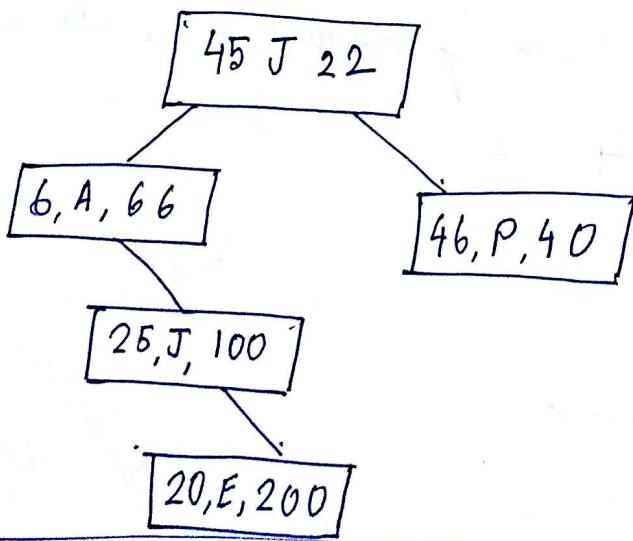
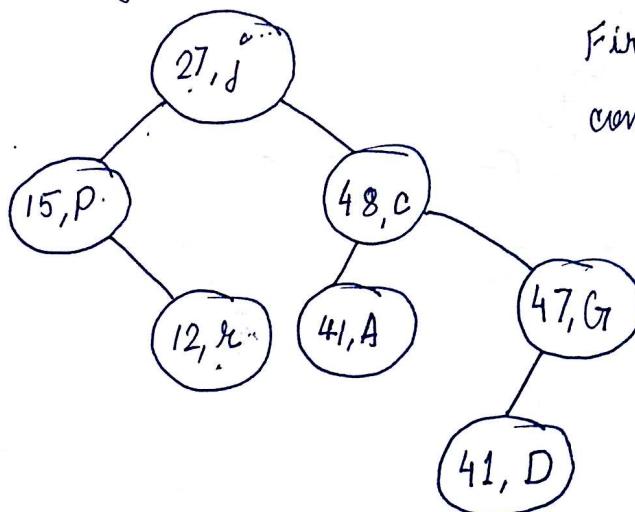
Q Create AVL Tree and tell the no. of single and double rotation -  
 5 9 4 2 6 3 8





K-d tree (variation of BST).

each node has two keys - a number and a character



struct kdNode {

kdNode \* leftchild ;  
kdNode \* rightchild ;  
int data [d] ;  
} → keys .

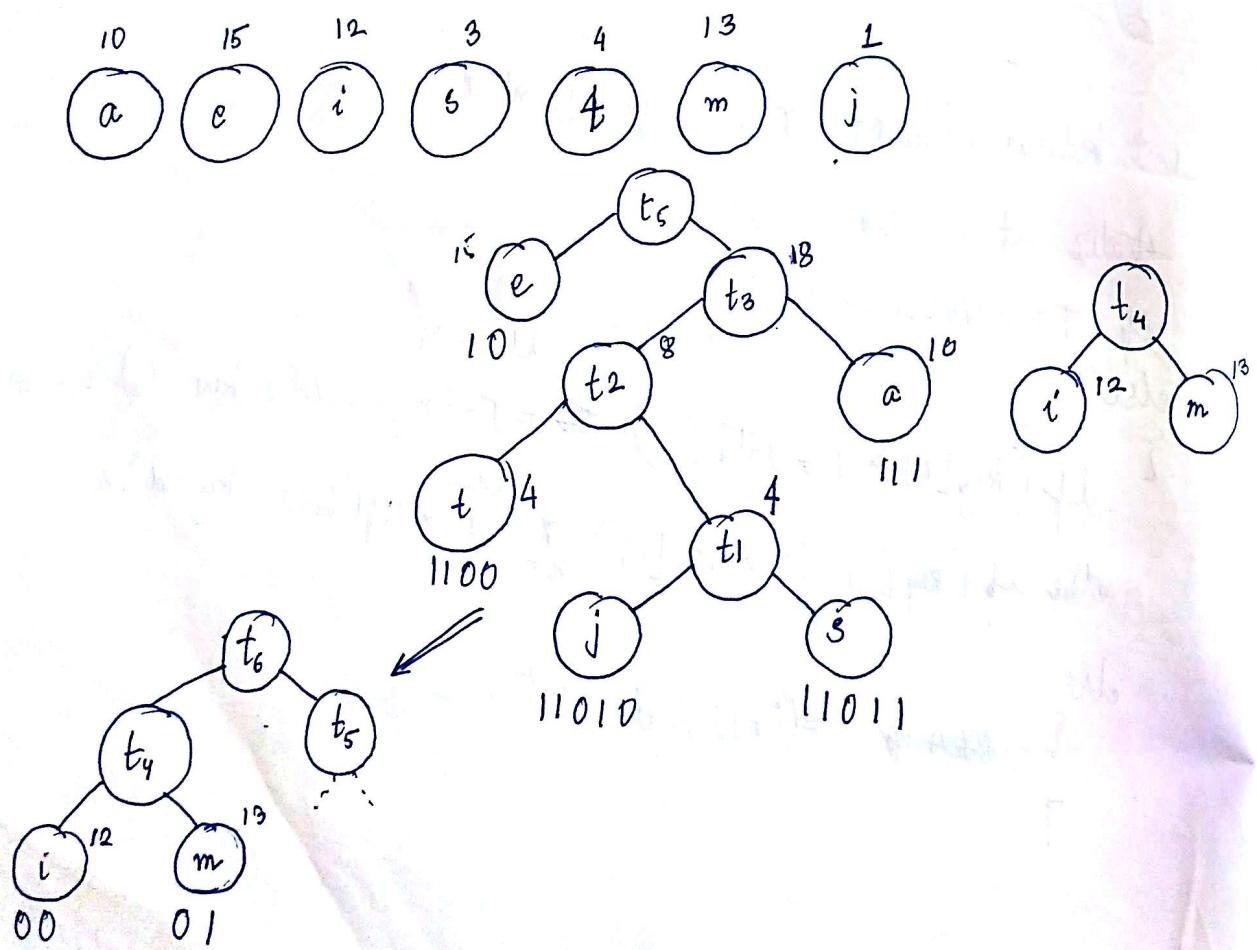
int Kdsearch (kdPTR T, int key [], int d)  
{ static int i = 0 ;  
if (T == NULL) return -1 ;  
else  
{ if (key [i] > T → data [i]) T = T → left ;  
else T = T → right ; i++ ;  
else for (i = 0 ; i < d ; i++)  
{ if (key [i] > T → data [i]) kdsearch (T → right , key , d) ;  
else if (key [i] < T → data [i]) kdsearch (T → left , key , d) ;  
else  
{

int kdsearch (kdPTR T, int key [], int d)

{ static int i = 0 , x = 0 ;  
if (T == NULL) return -1 ;  
else  
{ if (key [i] > T → data [i]) x = kdsearch (T → rightchild , key , d) ;  
else if (key [i] < T → data [i]) x = kdsearch (T → leftchild , key , d) ;  
else  
{ if (key [(i + 1) % d] > T → data [(i + 1) % d]) x = kdsearch (T → rightchild , key , d) ;  
else if (key [(i + 1) % d] < T → data [(i + 1) % d]) x = kdsearch (T → leftchild , key , d) ;  
else  
{ if (key [(i + 1) % d] == T → data [(i + 1) % d]) return 1 ;  
else  
{

### Huffman Code Tree

	<u>frequency</u>		<u>Codes got from HCT</u>	
a	10	000	111	30
e	15	001	10	30
i	12	010	00	24
s	3	011	11011	15
t	4	100	1100	16
m	13	101	01	26
j	1	110	11010	5
			174 bits	146 bits ← bits (each character uses $\frac{5}{3}$ bits)
				58 bytes → 464 bits



CT

30

30

24

15

16

26

5

$\frac{4+6}{4 \text{ bits}} \leftarrow \text{bits}$

Now we follow, for going left  $\rightarrow 0$ , for going right  $\rightarrow 1$ .

Suppose data is sent  $\rightarrow 100001110111010$  (eims)

Sender and receiver share same HCT.

Assignment :-

- ① AVL Tree
- ② K<sub>d</sub> Tree
- ③ Huffman Code Tree  $\rightarrow$  do everything taught
- ④ Digital Search Tree
- ⑤ DLR
- ⑥ LDR } Iteratively.
- ⑦ LRD

Print jump push

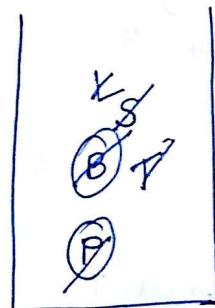
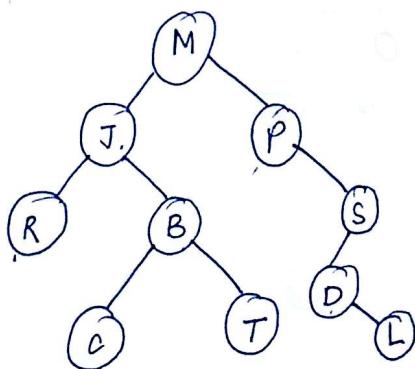
D L R

middle

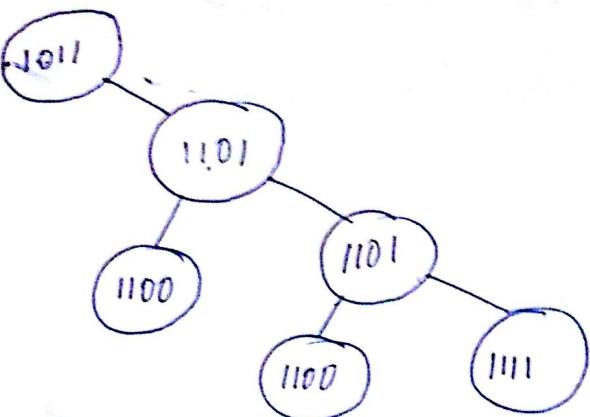
To eliminate recursion use stack compulsorily.

Pre-order M J R B C T P S D L

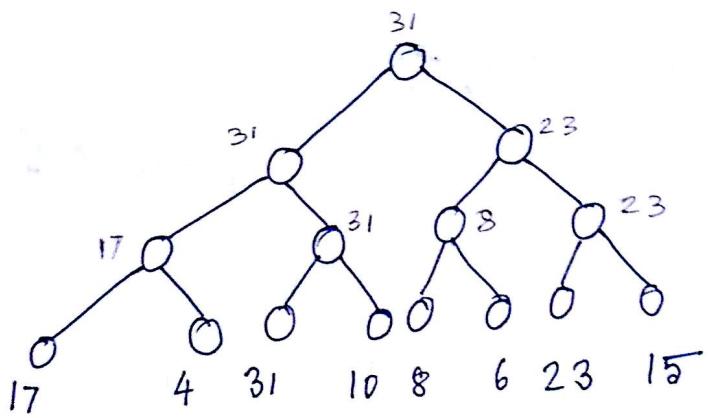
M J R B C T P D S D L



## Digital Search Tree



## Tournament Trees



31 23 17 15 10

void visitleafnodes(T)

{ if (T == NULL) return ;

if (T->left == NULL && T->right == NULL)

{ insertend(s, T) ;

else { visitleafnodes(T->left) ;

visitleafnodes(T->right) ;

void insertatend (stacks &, BTptr &)

{ if ( $s.top == -1$ ) push (s, T);

else

{ temp = pop if ( $T \rightarrow data < s.peek() \rightarrow data$ )

{ temp = pop(s);

insertatend (s, T);

push (s, T);

}

}

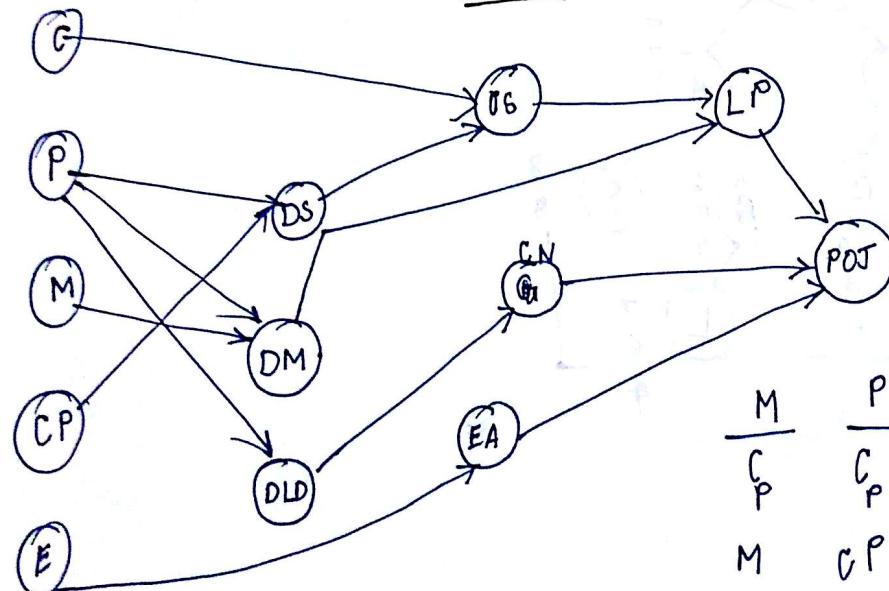
}

void printstack (stacks &)

{ while ( $s.top > -1$ ) cout << pop(s) << endl;

}

### Topological Sort



DLD	DS	DM	OS
DS	DS	OS	
DM	PM	DM	
DLD			

CN	LP
LP	CN

M	P	C	E	EA	CP
CP	CP	CP	CP	DM	DM
M	CP	E	E	DLD	DLD
CP	E	DM	DM	DL	CP
E	DLD	CP	CP	EA	

struct courses {

```
    char subject[10];  
    int; int links;  
    courses** b;  
};
```

```
typedef courses* Cptr;
```

```
Cptr T = NULL;
```

```
T = new courses;
```

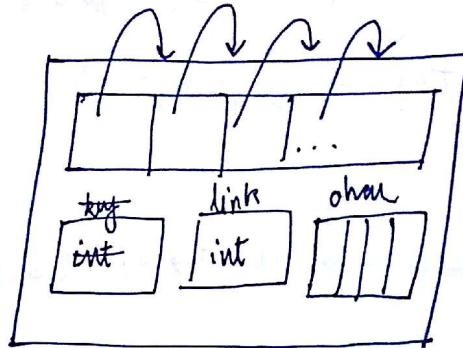
```
cout << "Enter name of course";
```

```
cin >> name;
```

```
strcpy(T->subject, name);
```

```
cout << "Enter no. of links"; cin >> T->links;
```

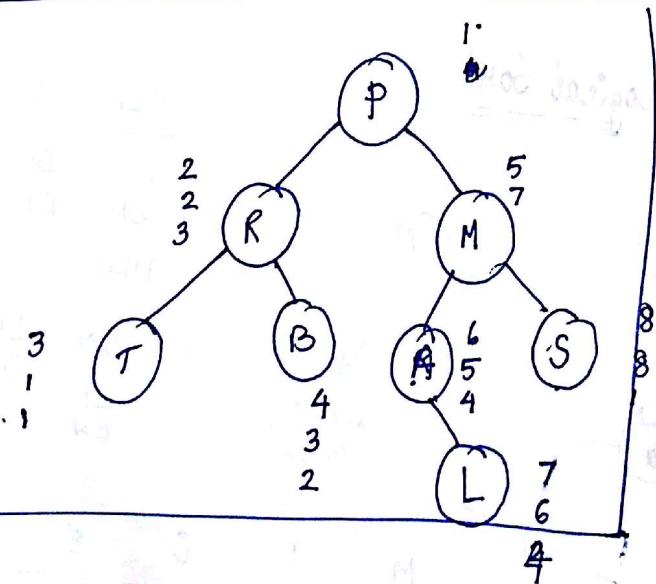
```
0 T->b = new Cptr[links];
```

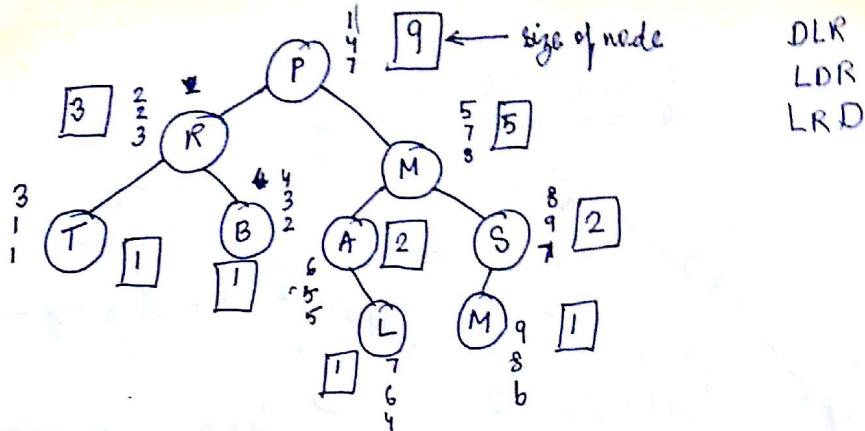


2/09/16

com  
↓  
number  
keys fil

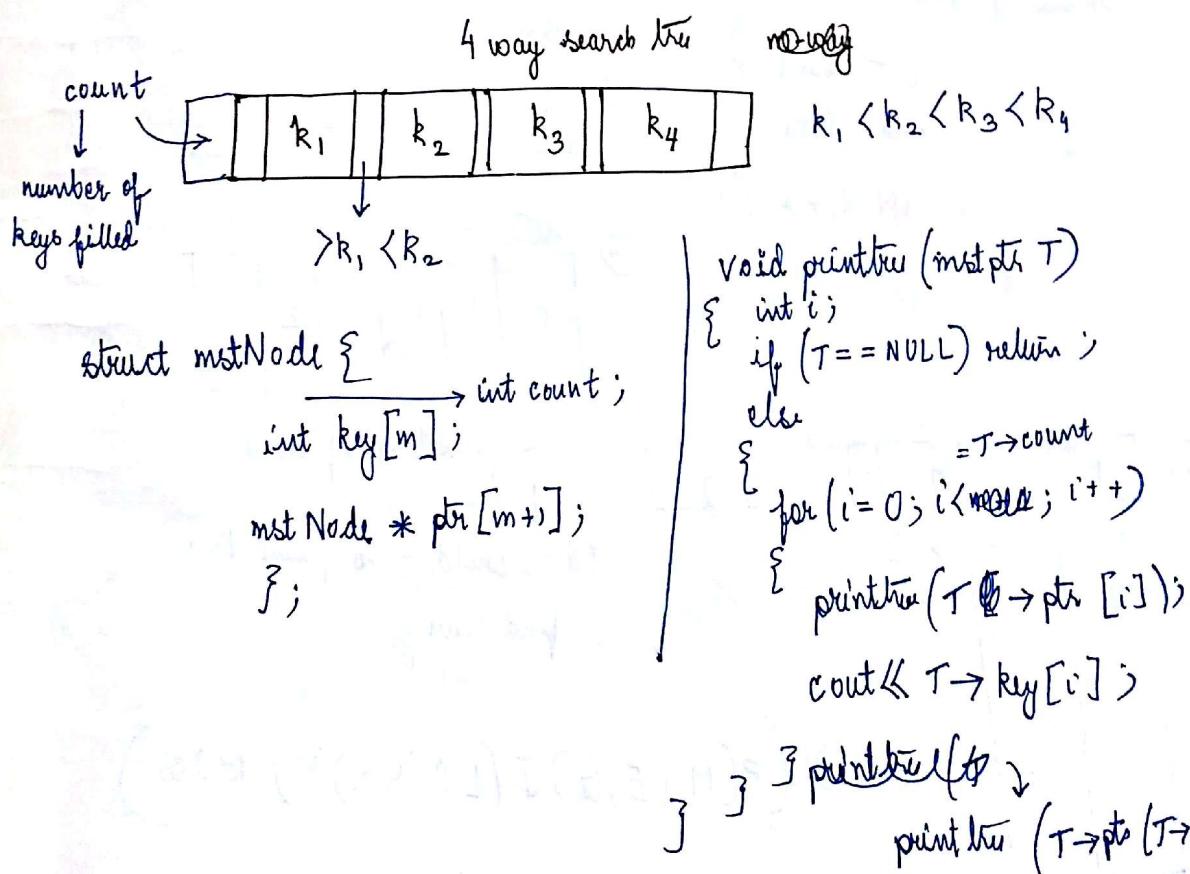
st



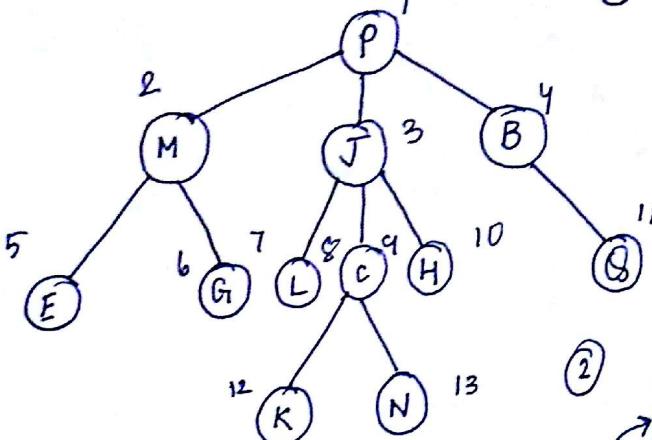


2/09/16

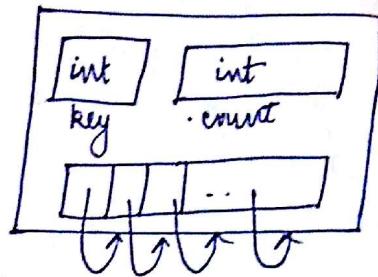
### M-Way Search Tree (multi-way)



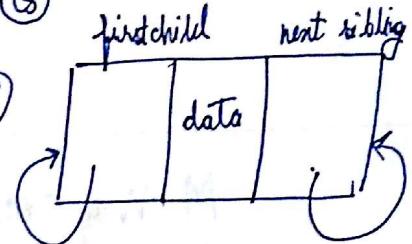
## General Trees



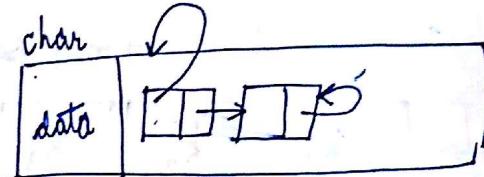
①



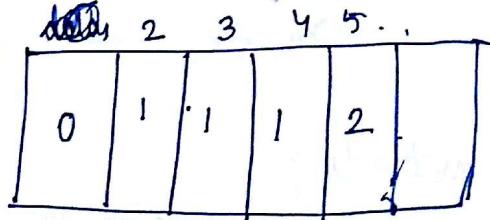
②



③

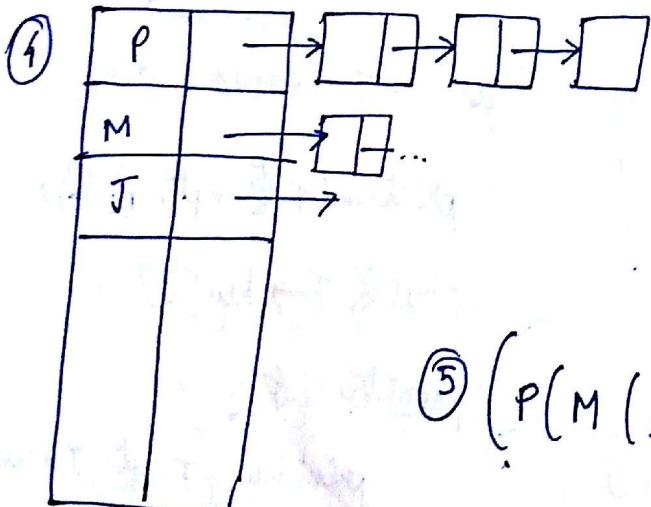


④

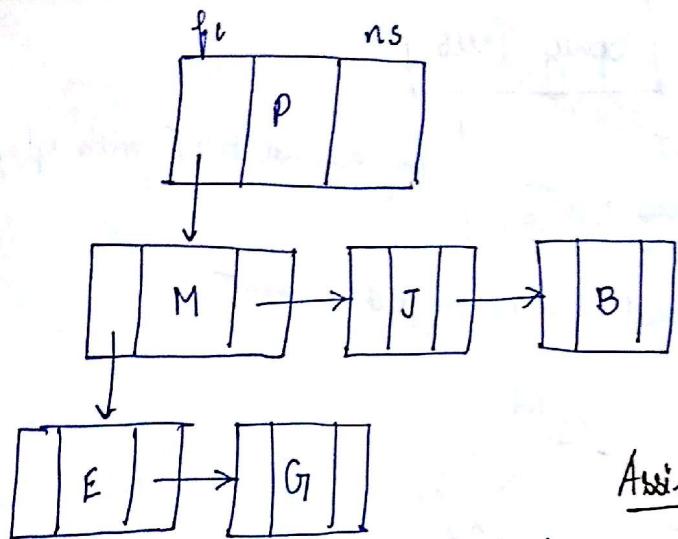


information of parent

From child goes to parent then  
find sibling.

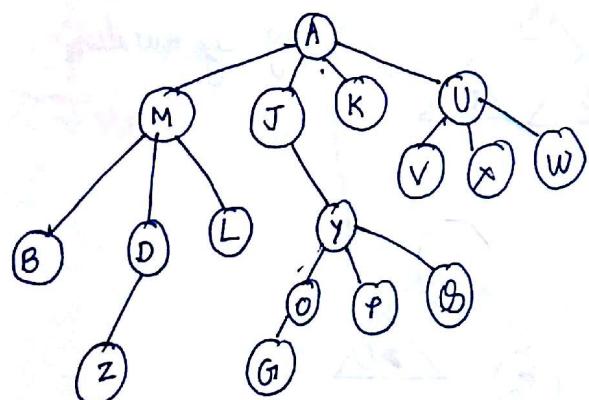


⑤  $(P(M(E,G))J(L(C(K,N))H)B)S$



### Assignments

- ① Topological Sort
- ② Tournament Sort
- ③ general Tree - Recursive & Iterative
  - ↓
  - ④ Print BFT, DFT
  - ⑤ general Tree as it is
  - ⑥ m-way search tree, creation, insertion, finding printing.
  - ⑦ m-way search delete
  - ⑧ Split m tree into binary trees



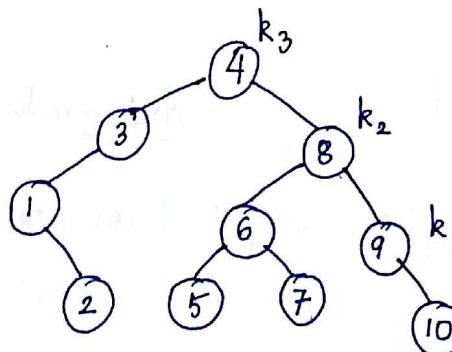
## Splay Trees

→ it is a BST

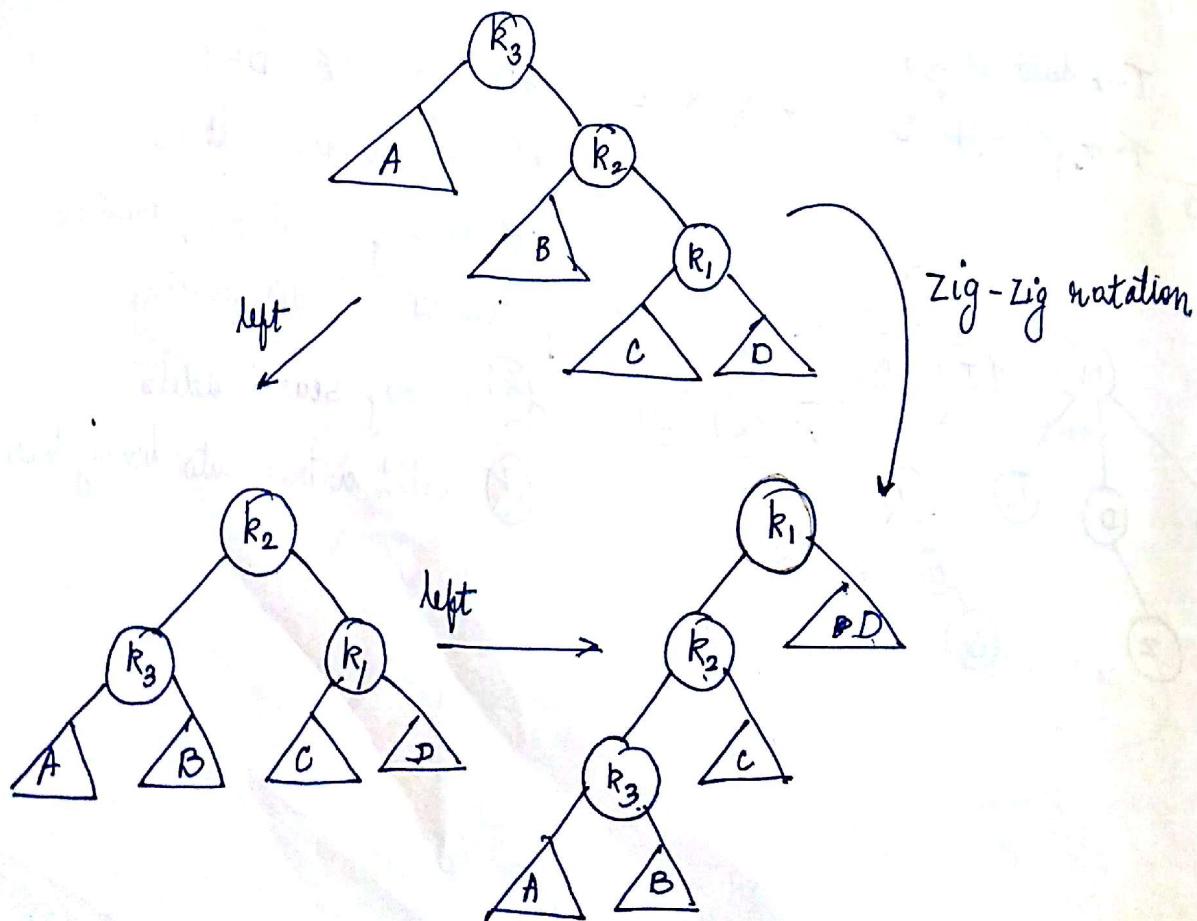
→ self-adjusting property

Splay trees are BST with splaying property.

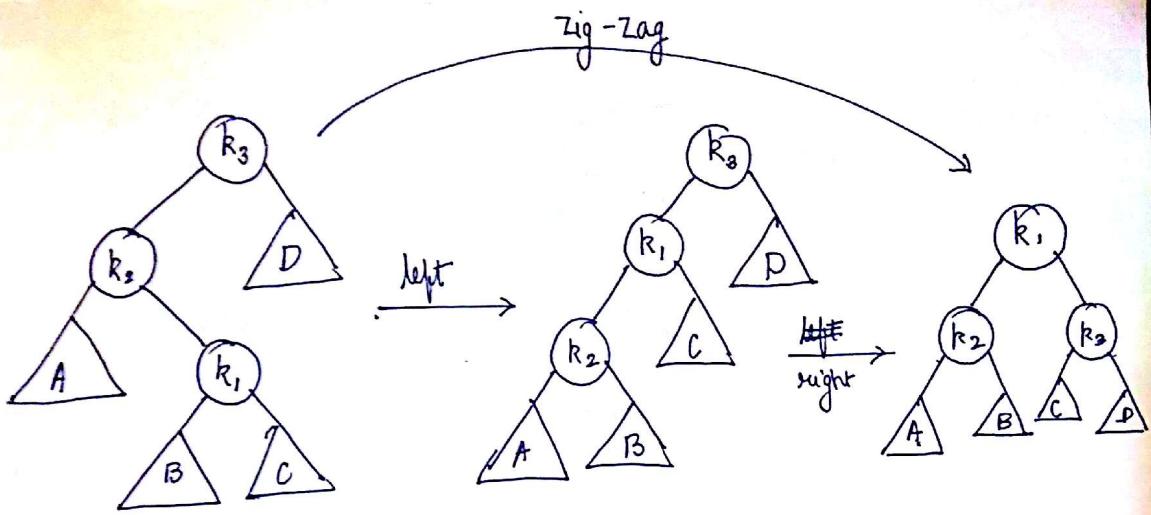
Most frequently searched node is shifted to root.



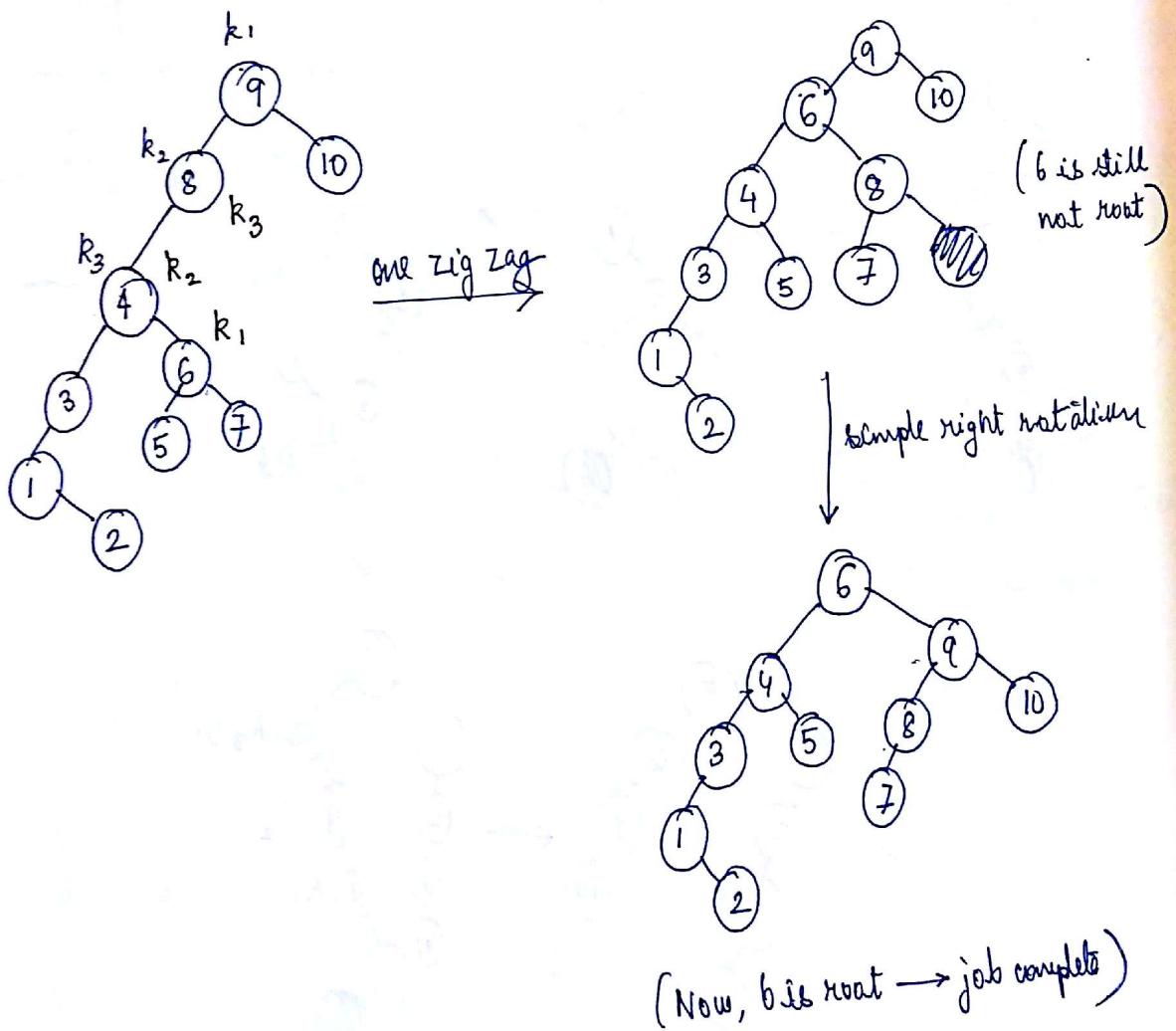
$k_1 \rightarrow$  most frequently searched.

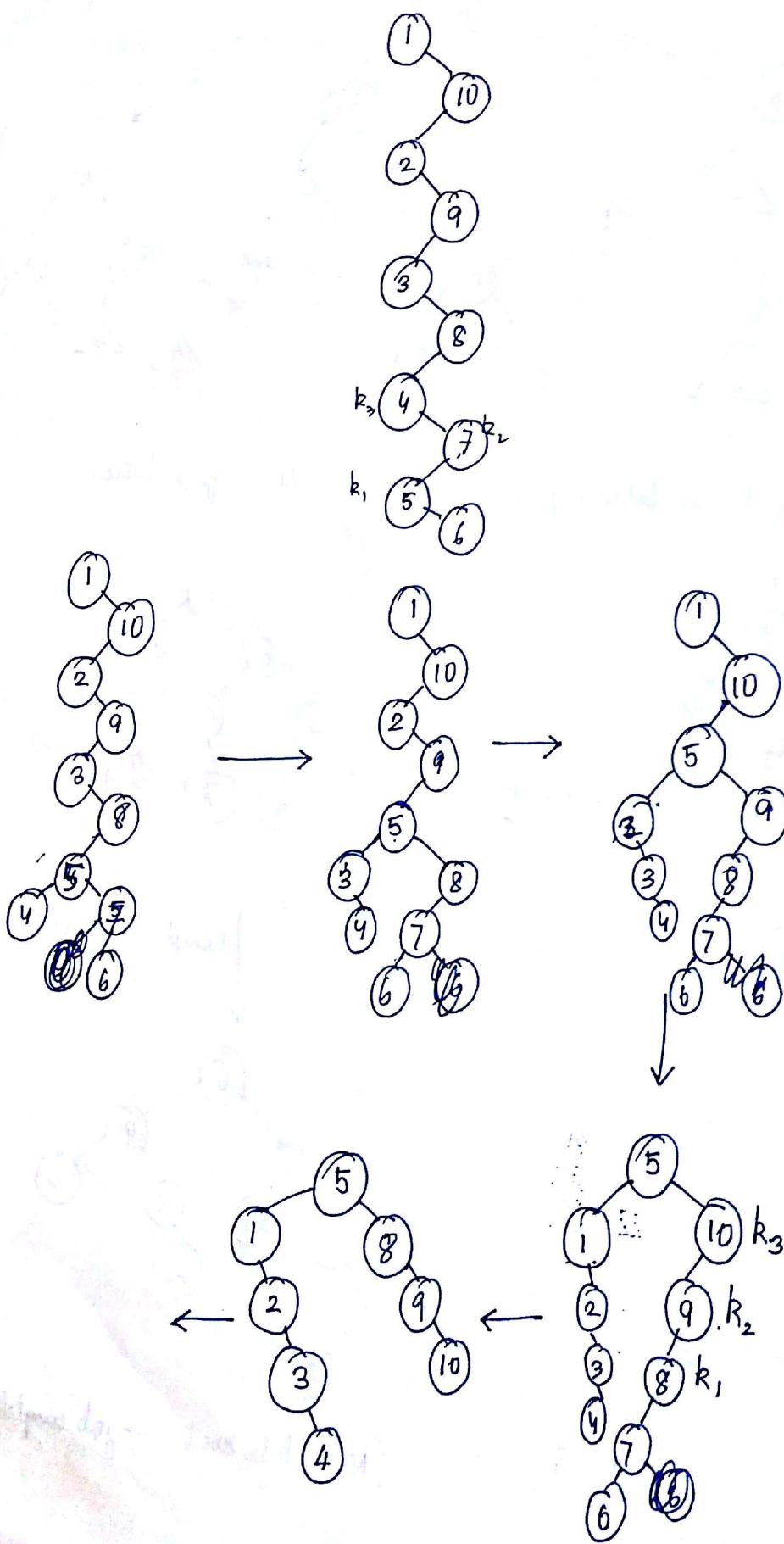


laying properties

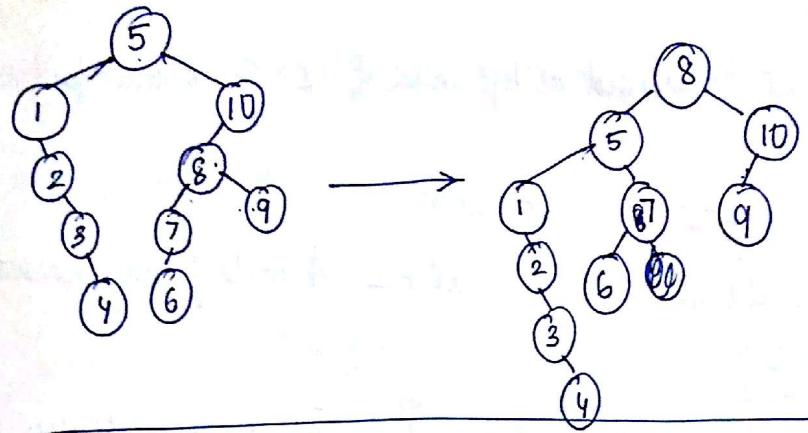


Don't bother height balancing in zig-zig and zig-zag rotations.



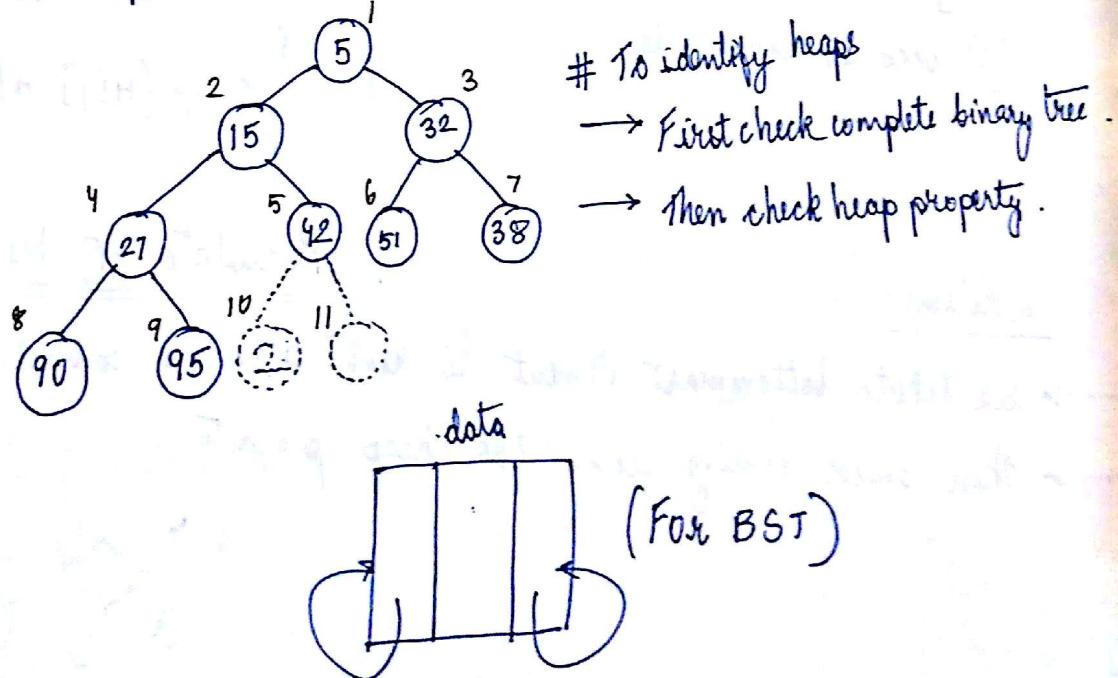


$\rightarrow$  Heap  
 $\rightarrow$  Heap  
 $\rightarrow$  Heap  
 value  
 $\rightarrow$  Heap



## Heaps (implement heap using tree in Nids)

- Heap is a complete, binary tree with heap property.
- Heap generally means min heap.
- Heap property - At every node the node's value is smaller than left child's value and right child value.
- Heap is one of the priority queue (It is a special form of priority queue)



The next child must be inserted as left child of 42. So no need for BST data structure.

We have to use array to implement heaps.

A node  $i$  has children in  $2^i$  and  $2^i + 1$ . A node  $j$  has parent  $\lfloor j/2 \rfloor$ .

Heap array  $H[n]$ .

Suppose ② is added as left child of 42 to ensure completeness.

Now, ensure heap property.

array\_p( $H, n, s, k$ )

{  
     $j = s+1; s++; H[j] = k;$

    while ( $j \neq 1$ )

        if ( $H[j] < H[\lfloor j/2 \rfloor]$ ) swap ( $H[j], H[\lfloor j/2 \rfloor]$ );

$j = \lfloor j/2 \rfloor;$

}

② becomes root node.

Better code

while ( $H[j] > H[\lfloor j/2 \rfloor] \& j \neq 1$ )

{  
    swap ( $H[j], H[\lfloor j/2 \rfloor]$ )

}

Percolate Up Method

Deletion -

- substitute bottommost element to root to ensure completeness
- Then check coming down for heap property.

er BST

```
void PercolateDown(int H, int n, int s, int k)
{
    cout << H[0] << endl;
    int j = 1; int t[H[s-1]];
    while(j <= s/2)
    {
        cout << H[j];
        if(H[j] > H[2*j] || H[j] > H[2*j+1])
        {
            if(H[2*j] > H[2*j+1])
            {
                swap(H[j], H[2*j+1]);
                j = 2*j+1;
            }
            else
            {
                swap(H[j], H[2*j]);
                j = 2*j;
            }
        }
    }
}
```

---

```
void func(Hptr T)
{
    if(T == NULL) return;
    if(T->lc == T->rc == NULL) return;
    else
    {
        if(T->data < T->lc->data && T->data > T->rc->data)
        {
            func(T->lc);
            func(T->rc);
        }
        else
        {
            if(T->data > T->lc->data > T->rc->data) cout << "Not heap";
            return;
        }
    }
}
```

```

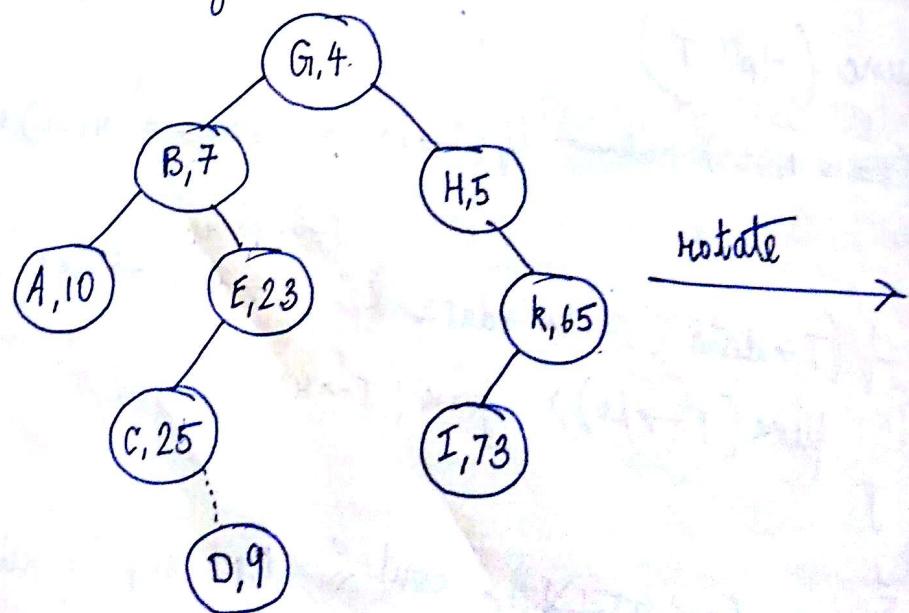
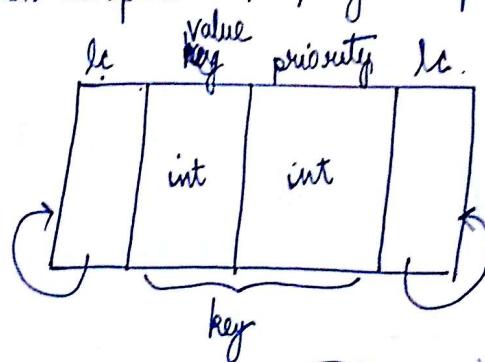
bool
void check( BstNode T1, BstNode T2)
{
    if (T1->data == T2 == NULL) return true;
    else if (T1->data == T2->data)
        return check(T1->lc, T2->lc) && check(T1->rc, T2->rc);
    else return false;
}

```

Treap. [in Mids]

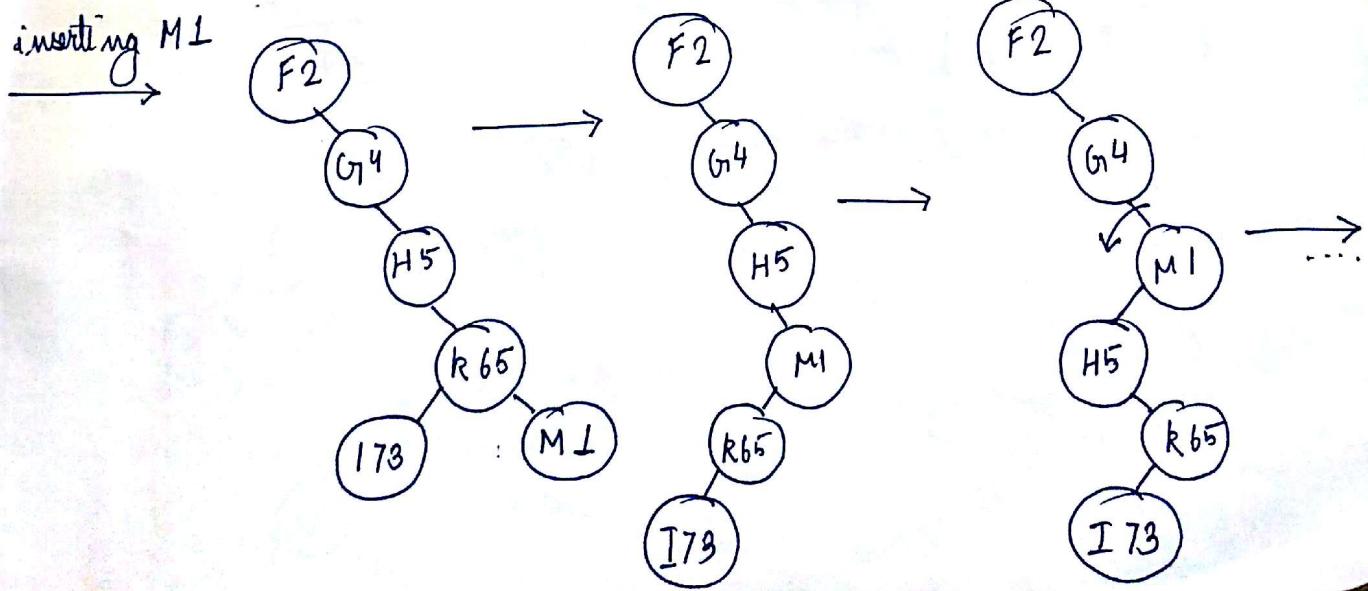
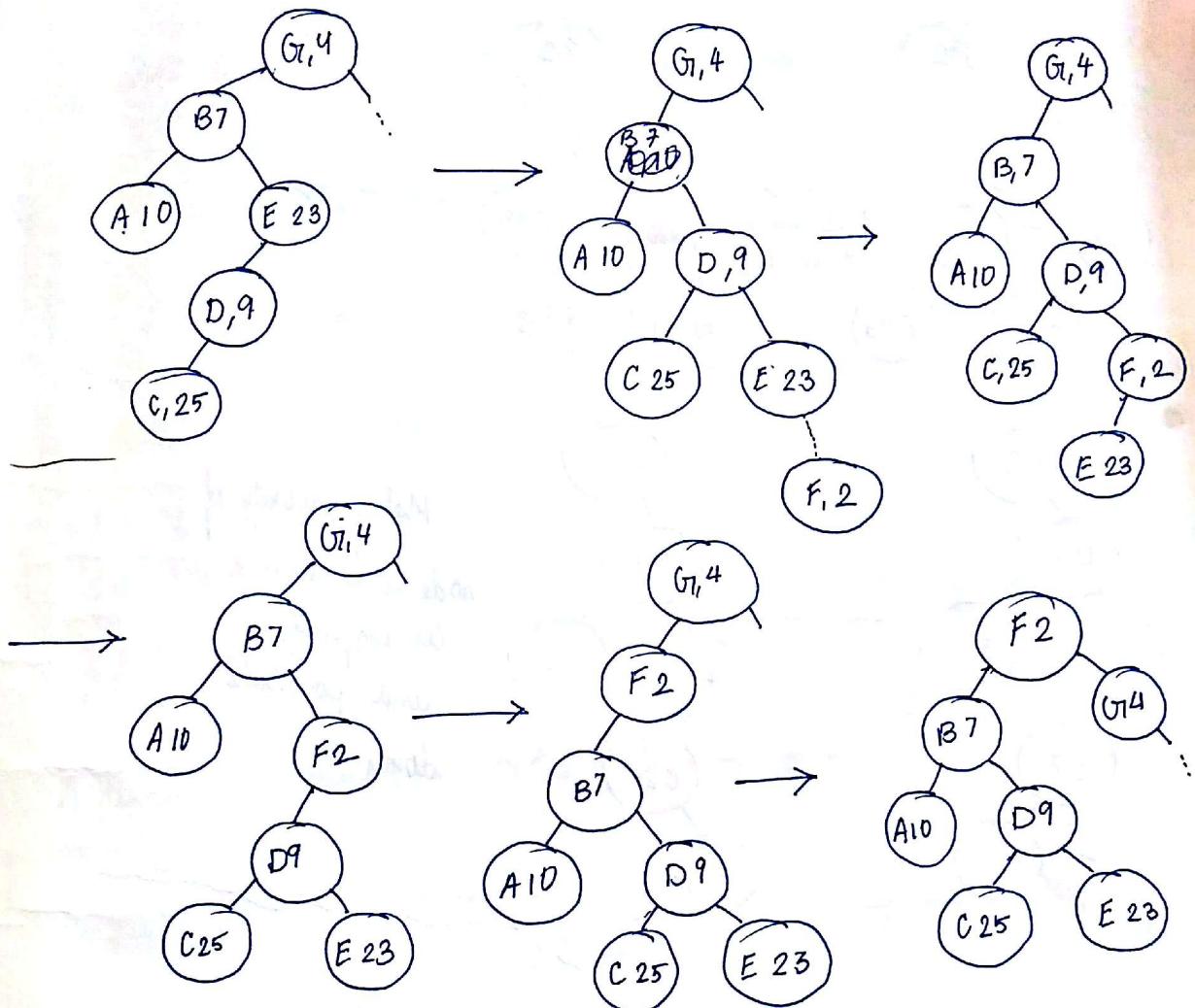
$$T_{\text{heap}} = BST + \text{heap}.$$

A node in heap has lo, rc, key and priority value.

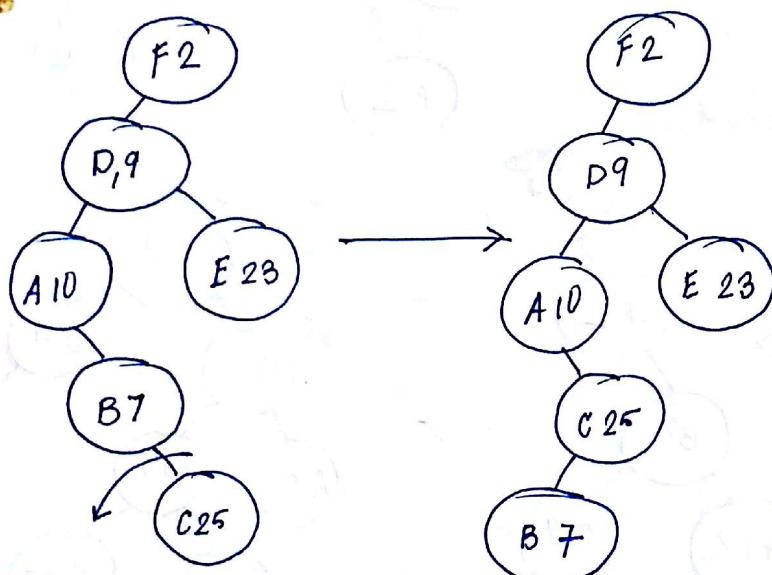
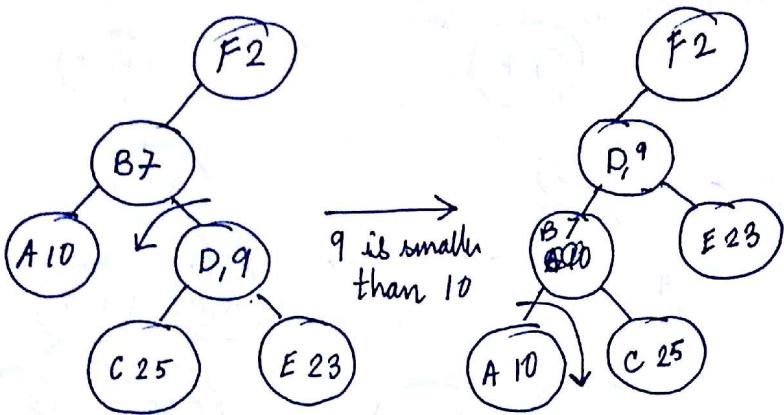


BST

While inserting in tree, insert as in ~~top~~. Next you maintain heap property.



## Deletion - (B, 7)



Make priority of  
node to be deleted  
as infinity  
and percolate  
down.

d-heap

[any node will contain at max d children].

If parent is at index  $i$  of array, then children are at  $di$  and  $di + 1 \dots di + d - 1$ .

### Disjoint Sets (used in graph algorithms)

1 2 5 7 8 4 6 9 3

(2,9) (3,6) (3,9) (7,5) (1,5)

Set-1 = {2, 3, 6, 9}  $\rightarrow$  2

Set-2 = {1, 5, 7}  $\rightarrow$  1

Set-3 = {4} Set-4 = {8}

A disjoint set can be implemented using array S.

1	2	3	4	5	6	7	8	9	
0	0	0	0	0	0	0	0	0	

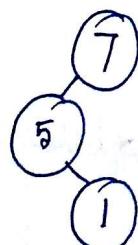
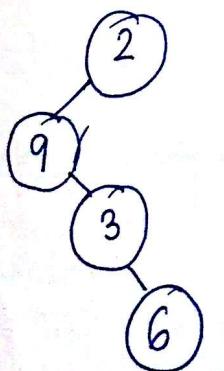
Initially all 0.

For a pair  $(i, j)$   $S[j] = S[i]$ .

1	2	3	4	5	6	7	8	9
5	0	9	0	7	3	0	0	2

(2,9), (3,9)

$\rightarrow \because S[9]$  is filled we put  $S[3] = 9$ .



$$f = \sum m(3, 4, 6, 7)$$

```

int find(int s[], i)
{
    if (s[i] == 0) return i;
    else return find(s, s[i]);
}

```

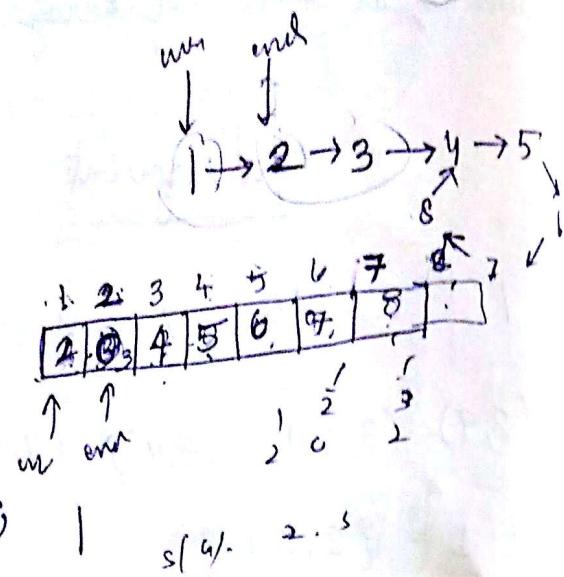
} For inserting.



```
void union (int s[], int i, int j)
```

```
{
    if (s[j] == 0) s[j] = i;
    else if (s[i] == 0) s[i] = j;
}
```

```
else s[find(s, i)] = find(s, j); }
```



(B-tree)

(Bayer's Tree)

**B-Trees**

(Bushy Tree) or (Balanced Tree) or

B-Tree is a multiway search tree

Just like in m way search tree we mention count of a node, in B-tree we use notation  $B_d$ -tree for telling count.

[ $d$  is minimum number of keys in a node, maximum is  $2d$ ].

Definition - No node except root can have less than  $d$  keys. All other nodes must have  $d$  keys ( $d+1$  pointers) and can have maximum  $2d$  keys ( $2d+1$  pointers).

③ All leaf nodes must be at same level  $\rightarrow$  balanced.

```
struct btNode {
```

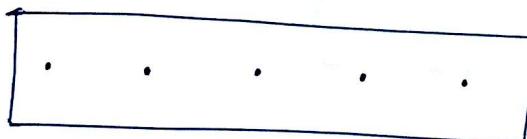
```
    int key *key;           int key[2*d];  
    btNode* pointers;     btNode* ptr[2*d+1];  
    int count;             int count;  
};
```

```
typedef btNode* Bptr;
```

} 4d+1 fields in  
B<sub>2</sub> tree.

count field is just  
for our use.

#



B<sub>2</sub> tree node [has 5 pointers]

B<sub>2</sub> tree construction grows from bottom to top.

Construction - (62, 51, 7, 19, 49, 24, 57, 11, 1, 21, 39) Insert & Sort

Overflow function -

while adding 49 overflow occurs,

1. handle

2. Node splitting ()

1. create a new node

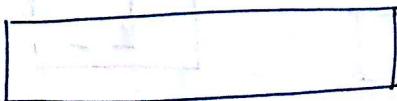
2. find middle element of overflow node

3. add left elements to original node and right elements to new node. Add mid element

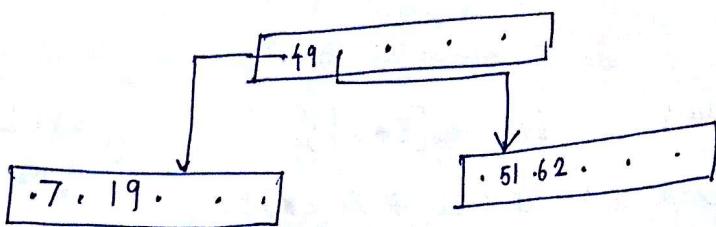
to parent node. If parent is not there create parent and adjust pointer.

Parents new elements left pointer → old node, right pointer → new node

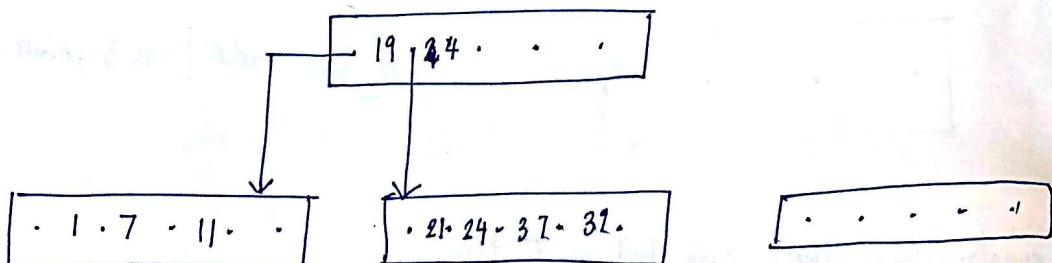
• 7 • 19 • 51 • 62 •



7 19 (49) 52 62



To add a node find leaf node where it should be added. To check if a node is leaf check its leftmost pointer.



Whenever overflow occurs do node splitting and insert-root mid element to parent. While inserting shift keys and pointers accordingly.

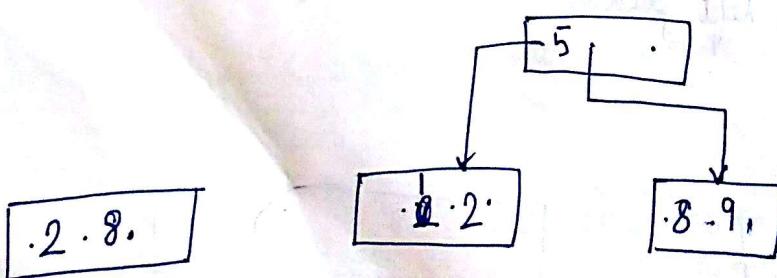
# B-tree  $\rightarrow (3, 9, 1, 8, 2, 6, 4, 5, 7)$ .

(2 8 5 9 1 6 4 7 3)

1, 2, 4

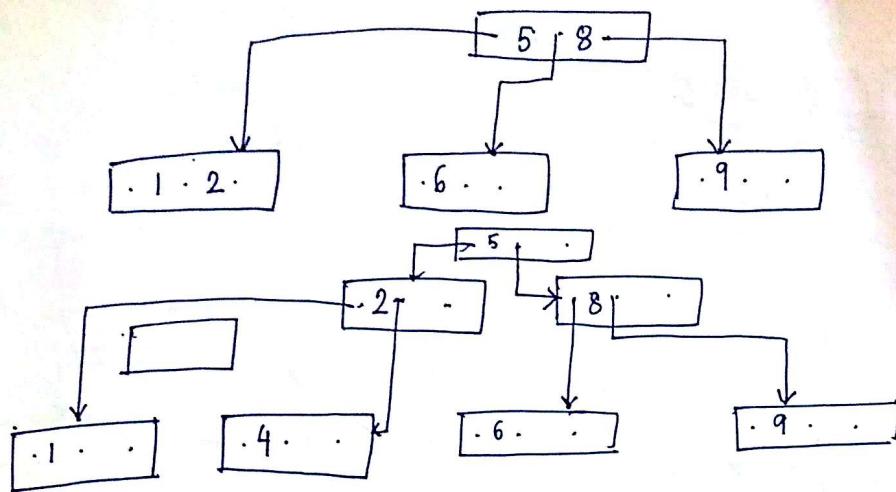
Overflow

① Key 8

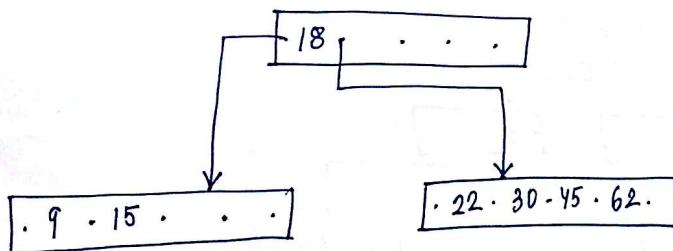


2 5 8

6 8 9

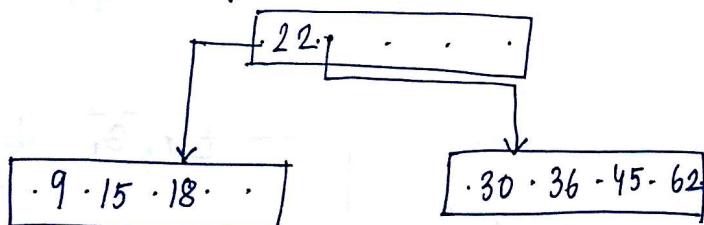


## B<sub>2</sub>-tree Creation And Node splitting

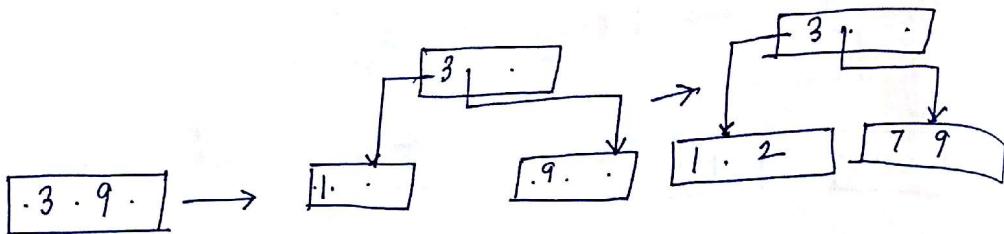


Overflow :-

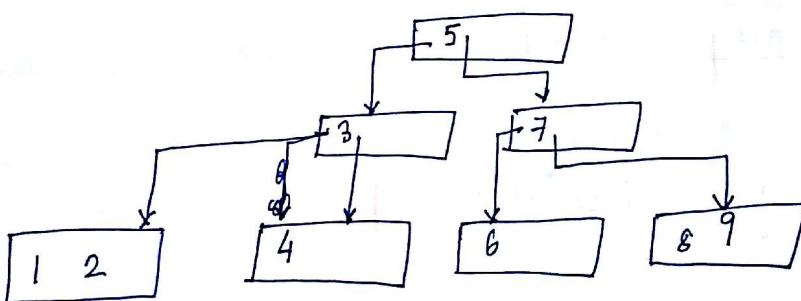
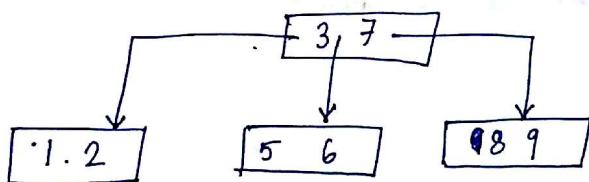
- ① Key redistribution → check if immediate left or right sibling has empty space. If yes then key redist., else node splitting.



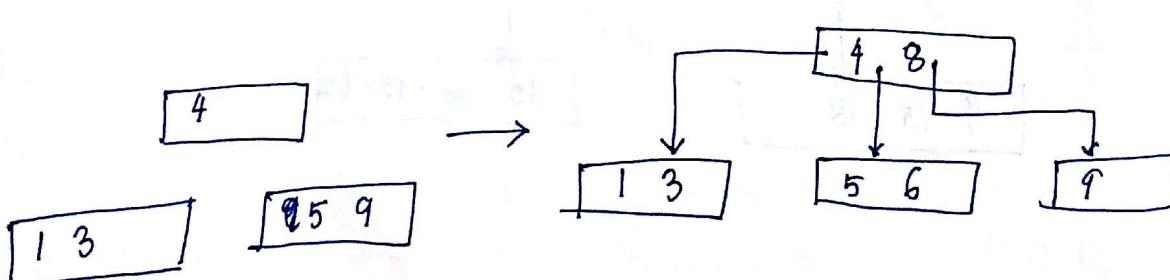
# 3 9 1 7 2 5 8 6 4



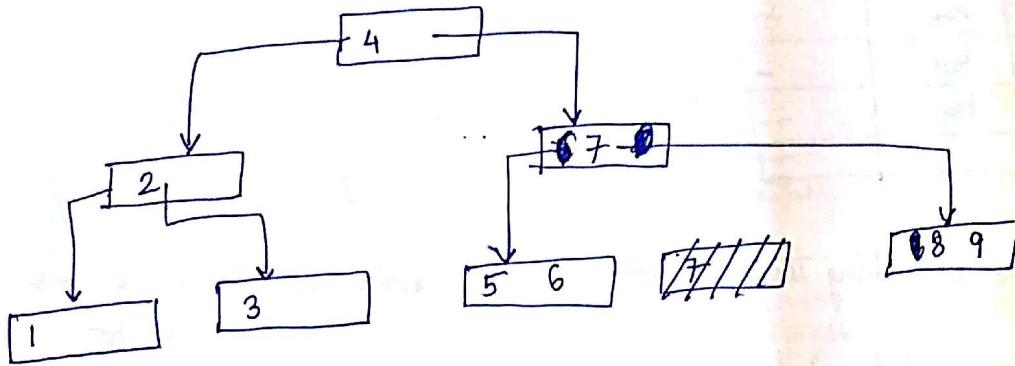
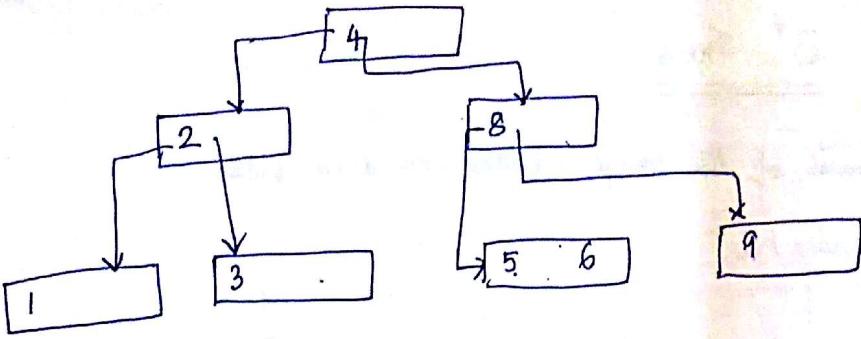
1, 3, 9



# 4 9 1 3 5 8 6 2 7



insert ()  
 {  
 i = T →  
 while (d  
 {  
 T → k  
 T →  
 i ←  
 }  
 T → key [ ]  
 }



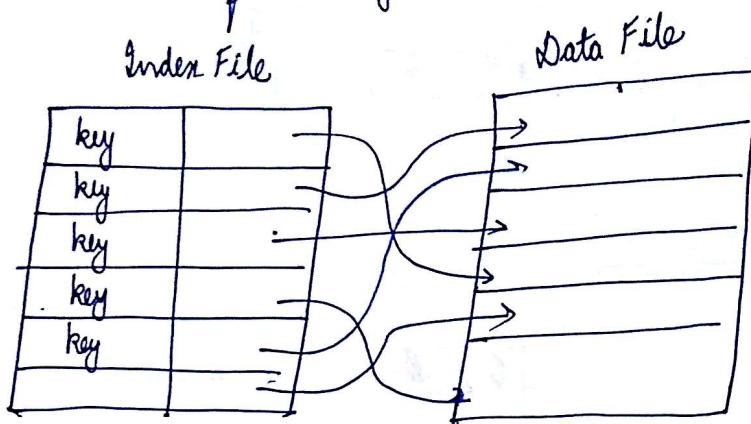
insert () → Bst & T, int d

```

{
    i = T->count;
    while (d < T->key[i-1]key[i-1] & & i > 0)
    {
        T->key[i] = T->key[i-1];
        T->ptr[i] = T->ptr[i-1];
        i--;
    }
    T->key[i] = d;
}
  
```

## $B^+$ - Trees

Index File consists of two things - index and data file.



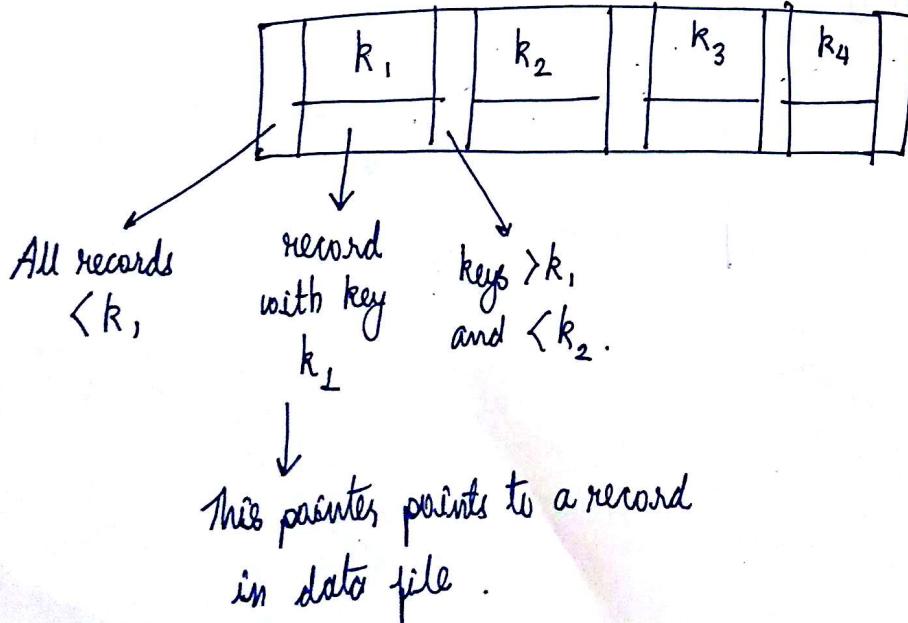
In  $B_d$  trees searching time is  $\log_{2d+1} n$ .

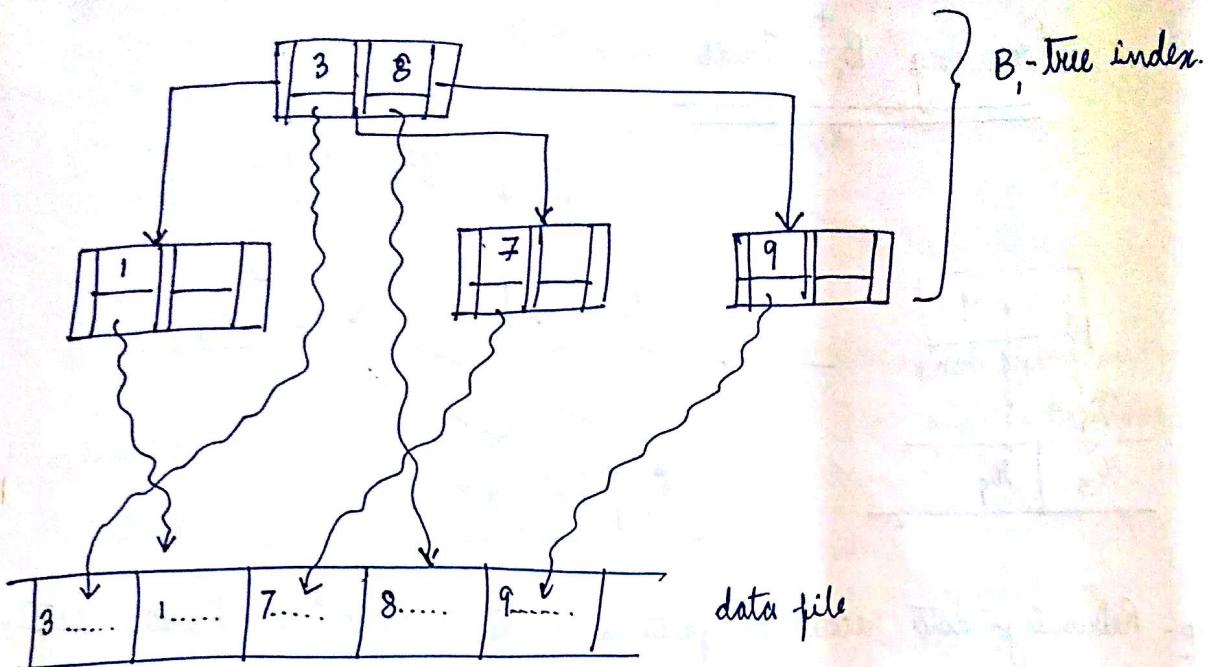
$B^+$ -Trees are used for indexing purpose.

Suppose we want to create  $B^+$ -indexed tree.

Note:  $B_d$  tree is not used for searching.

$B_2$  tree indexed node structure -



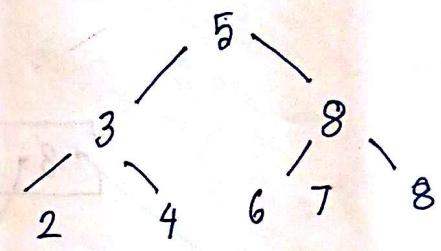


The moment we write B+ tree index in harddisk it becomes B+ tree index file.  
Each record in B+ tree index file has 7 fields! 6.1+1).

tpt1	key1	sptr1	tpt2	key2	sptr2	tpt2
1	5	7	2	-1	-1	-1

level order numbering of child.

relative record number  
in data file.



### Index Node

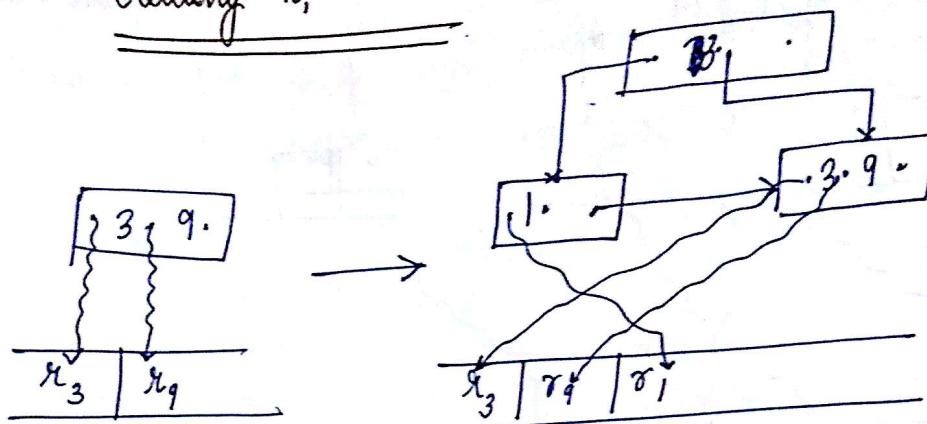
Non leaf nodes → has only index  
leaf nodes → has only data points

(Data Node)

Total number of fields in B+ tree =  $4d + 1$ .

B+ tree has heterogeneous structure

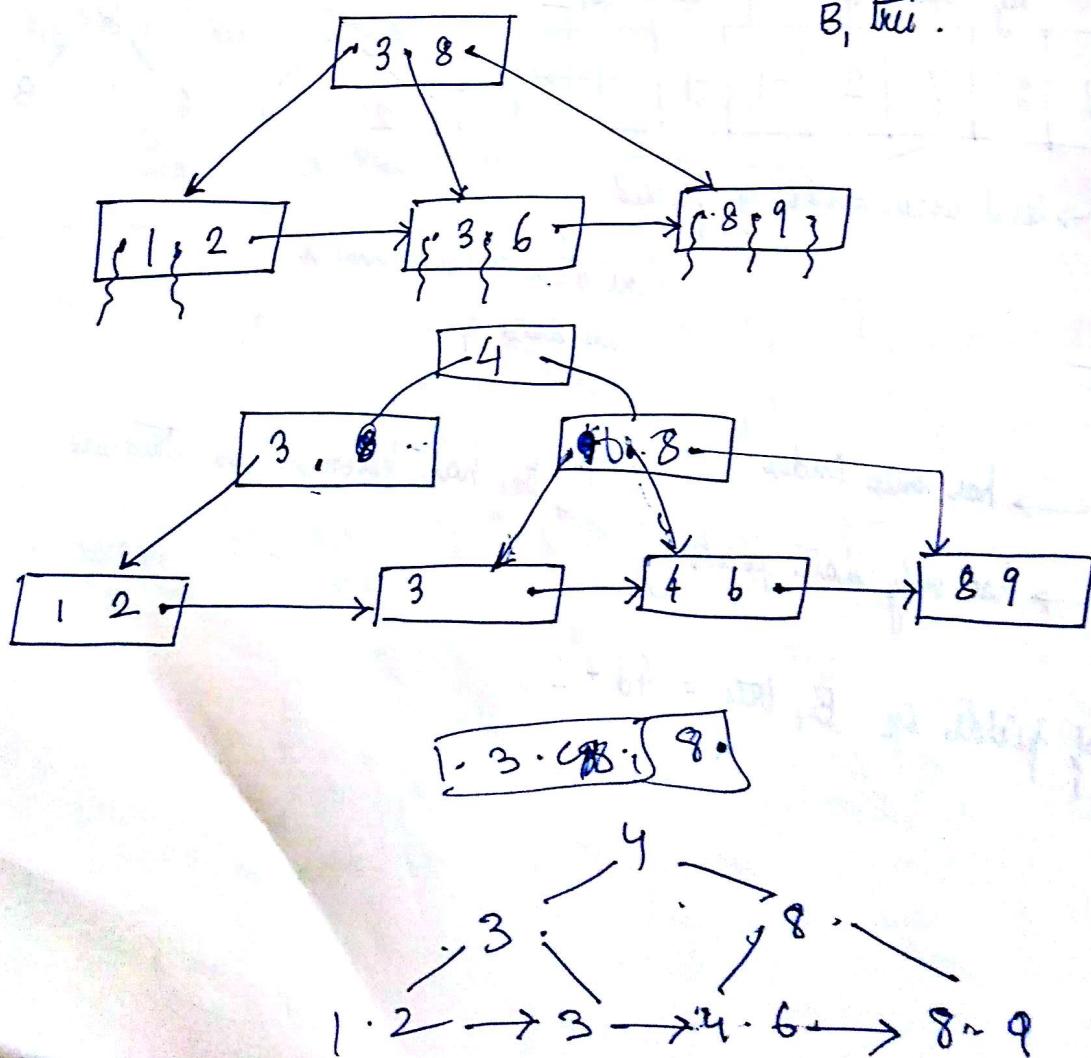
## Creating $B_+^+$ -Trees



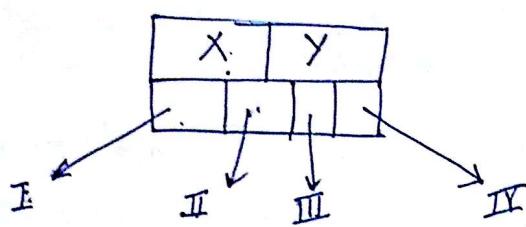
V. Imp - Retrieving data records is faster in  $B_+$ -tree than  $B^+$  tree because data pointer is there itself.

But  $B_+$  tree is used for static data structures

Non-leaf node splitting is same as  $B^+$  tree.



## Quad Trees



Always insert at leaf.

Look upon internet  
for applications.

```
struct qNode {  
    int x, y;  
    qNode *pointer[4];  
};
```

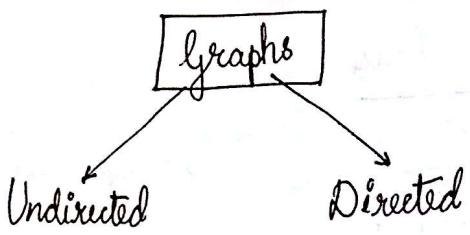
```
typedef qNode* Qptr;
```

```
void insert (Qptr &T, int x, int y)
```

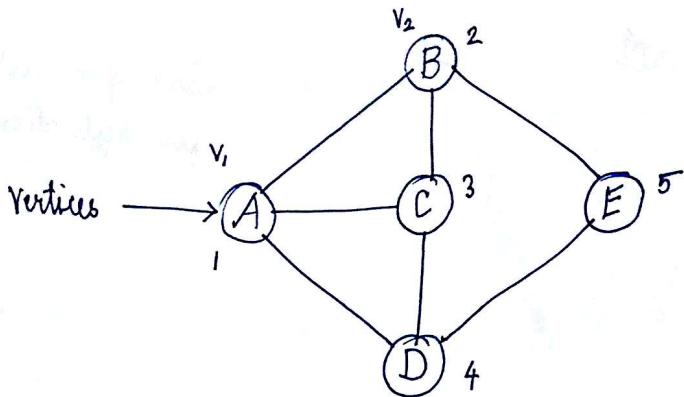
```
{  
    if (T == NULL)  
    {  
        T = new qNode;  
        T->x = x; T->y = y;  
        for (int i = 0; i < 4; i++) T->pointer[i] = NULL;  
        return;  
    }
```

```
    if (x < T->x && y < T->y) insert (T->pointer[0], x, y);  
    else if (x < T->x && y > T->y) insert (T->pointer[1], x, y);  
    else if (x > T->x && y > T->y) insert (T->pointer[2], x, y);  
    else insert (T->pointer[3], x, y);
```

```
}
```



$A, B, C \dots \rightarrow$  value of vertex

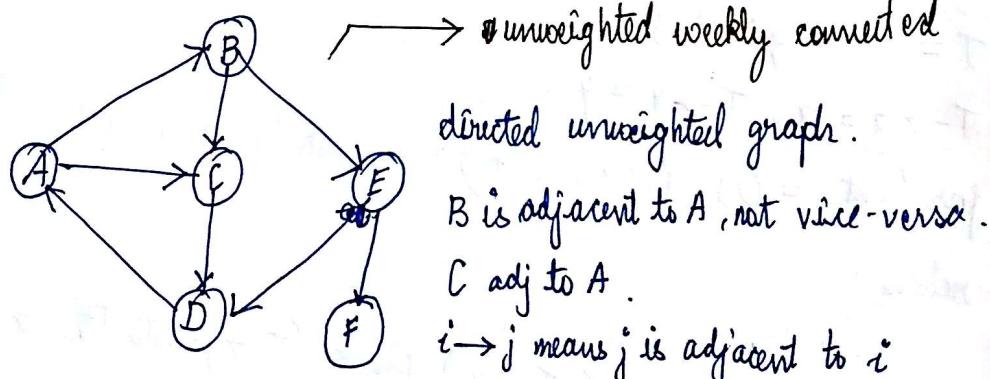


$$\text{Vertices } V = \{v_1, v_2, \dots, v_n\}$$

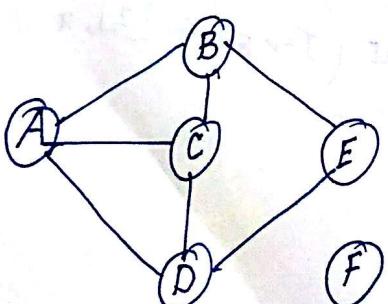
$$\text{Edges } E = \{(u, v) / u, v \in V \}$$

adjacent

If edges are labelled then graph is weighted.



Connected graph is used for ~~un~~ undirected.



excluding F → unweighted disconnected

including F → unweighted connected

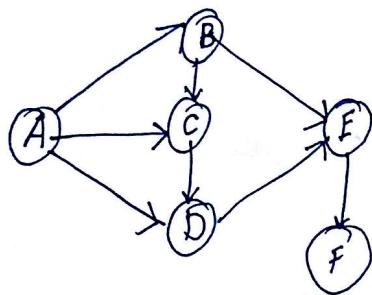
In a directed graph if we can reach from any vertex to other than  $\rightarrow$  strongly connected graph. otherwise weakly connected graph.

Path:- In undirected graph, each edge contributes 1 to path length.

In directed graph, weight of each edge contributes to path length.

If a directed graph has cycles it is cyclic. Mostly strongly connected graph is cyclic.

A cycle is a path from  $i$  to  $i$  whose pathlength is more than 1.



excluding  $F \rightarrow$  directed acyclic graph (DAG)  
↓  
neither weakly connected  
neither strongly "

including  $F \rightarrow$  weakly connected DAG.

The number of edges incident on a vertex is its degree.

In directed graph- Indegree (Fan-in) Outdegree (Fan-out).

Topological sort is only for DAG.

Representation of matrix:-  $\text{size} = |V|^2$

	1	2	3	4	5
1	1	1	1	1	0
2	1	1	1	0	1
3	1	1	1	1	0
4	1	0	1	1	1
5	0	1	0	1	1

adjacency matrix of undirected graph

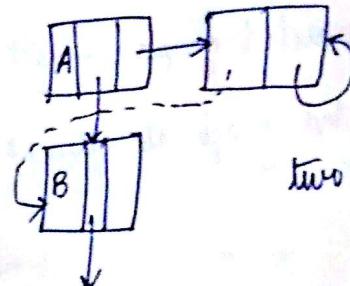
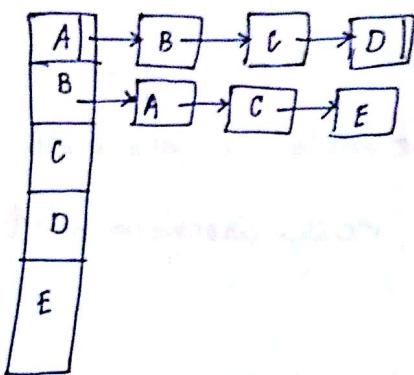
is symmetric.

represent using A in program

$|V| \times |V|$

Adjacency list :- (requires only 1 structure) represented by  $G_1$ .

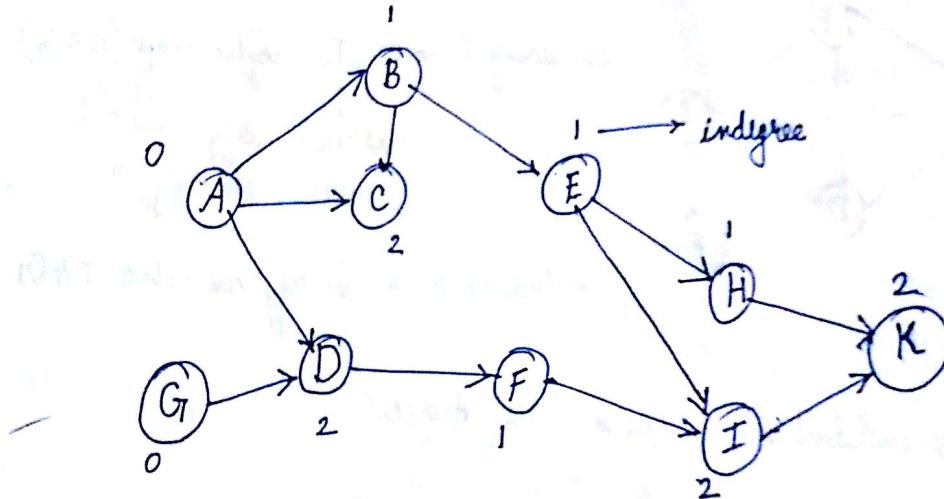
For saving memory we can use two structures.



two structures

$$\text{size} = |V| + 2|E|.$$

P [ ] B [ ]



Topological Sort -

Calculate indegree of each vertex and store in indegree array  $[ ]$ .

A	B	C	D	E	F	G	H	I	J
0	1	2	2	1	1	0	1	2	2

 $a[ ] = 0 \ 1 \ 2 \ 2 \ 1 \ 1 \ 0 \ 1 \ 2 \ 2$ 

Check for 0.  $\rightarrow$  print "A".

Delete vertex A by decreasing degrees of every vertex adjacent to A by:

- ① a [ ]
- ② Queue
- ③ Print
- ④ Find
- Time

①  $a[]$

② Queues.

③ Print all paths.

④ Find cyclic or acyclic.

Third representation

1	2	3	4	5
3	6	9	12	1

P[]

B	C	D	A	C	E	A	B	D	A	C	F	B	D
---	---	---	---	---	---	---	---	---	---	---	---	---	---

1 2 3 4 5

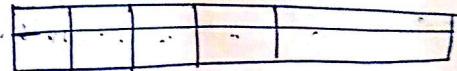
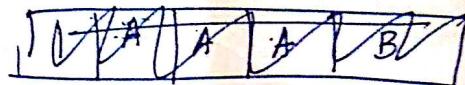
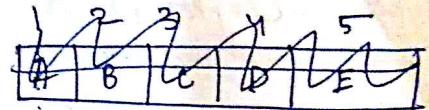
0	3	6	1.9	12
---	---	---	-----	----

Packed Array Representation

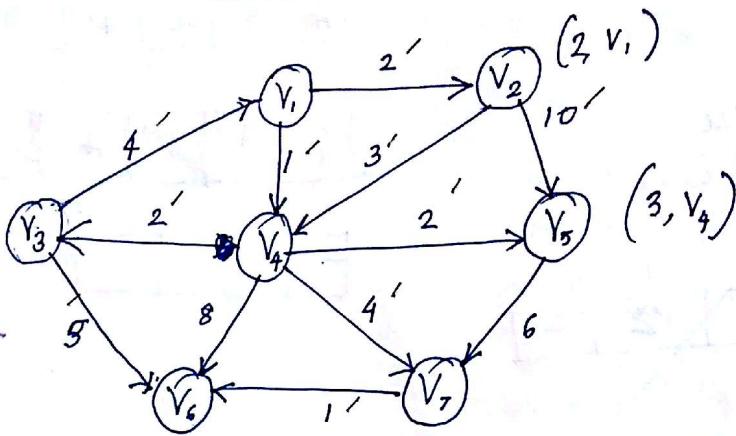
12

B	C	D	A	C	E
---	---	---	---	---	---

A by 1.



## Dijkstra's Algorithm for Shortest Path



source vertex, destination vertex, current.

Update adjacent, unvisited vertices of current node.

Among all unvisited vertices, take the vertex with min distance from source.

distance	previous	visited

general Tree -

For level order  $\rightarrow D \ Ns \ Fc$

D Fc Ns for A

B  
D  
J  
C

## Traversal in graphs :-

Level order  $\rightarrow$  First print node and then visit all of its adjacent vertices.

BFS  $\Rightarrow$

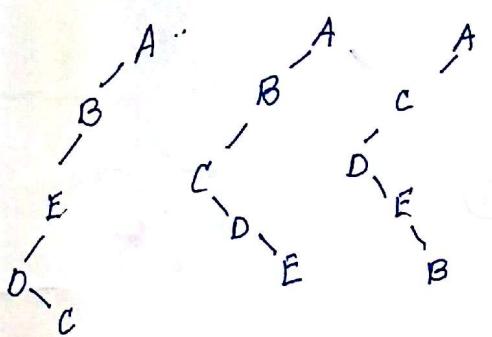
① BFT (Breadth First Search)  $\rightarrow$  Traversal

② DFT (Depth First Traversal)  $\rightarrow$  easier in recursion.

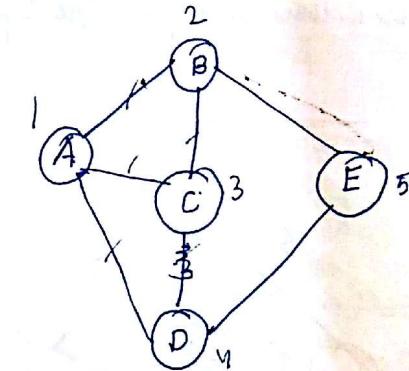
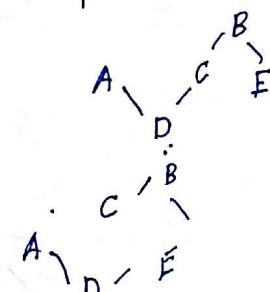
For graph, compulsorily give starting vertex.

void dfs()

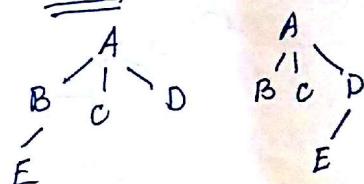
DFT : - (V. imp)  $\rightarrow$  no. of DFT possible



6 possibilities



BFT (2 possibilities)



Starting from F

E  $\rightarrow$  B  $\rightarrow$  A  $\rightarrow$  C  $\rightarrow$  D

E  $\rightarrow$  D  $\rightarrow$  C  $\rightarrow$  B  $\rightarrow$  A

E  $\rightarrow$  B  $\rightarrow$  C  $\rightarrow$  D  $\rightarrow$  A

E  $\rightarrow$  B  $\rightarrow$  A  $\rightarrow$  D  $\rightarrow$  C

E  $\rightarrow$  D  $\rightarrow$  A  $\rightarrow$  B  $\rightarrow$  C

E  $\rightarrow$  D  $\rightarrow$  A  $\rightarrow$  C  $\rightarrow$  B

E  $\rightarrow$  B  $\rightarrow$  C  $\rightarrow$  A  $\rightarrow$  D

void dfs(v)

{

visit(v) = 1;

Print(v);

for each adjacent vertices w of v,

if (!visited(w))

{

dfs(w);

}

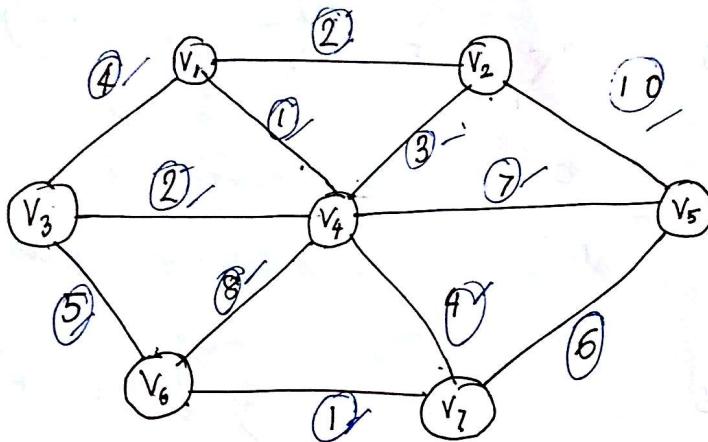
$$2 \leftarrow \frac{1}{3} \rightarrow 4$$

BFT:-

~~A B C D~~  $\rightarrow$

1 As soon it enters queue, mark it visited.

For unconnected graph enqueue initial vertex



### Applications of graphs -

Traversal graph	BFT	DFT
Undirected	Prim	Euler Path
Directed	Dijkstra's Alg. Topological Sort	Strongly Connected Component

Min

- ① Prim
- ② Kruskal

① Pr

9

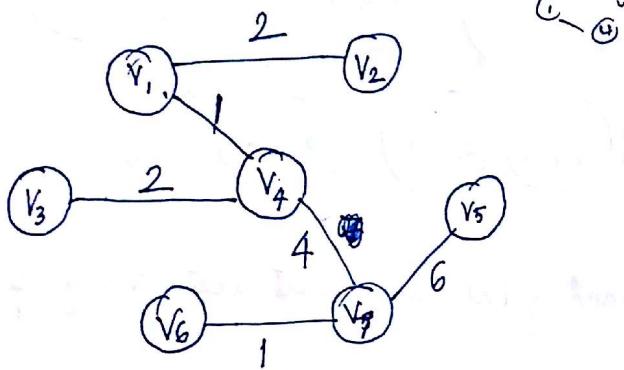
Minimum Spanning Trees - (tree with minimum sum of edges and no loop).

- ① Prim's Algorithm
- ② Kruskal's Algorithm.

① Prim's Algorithm -

Input as starting (vertex).

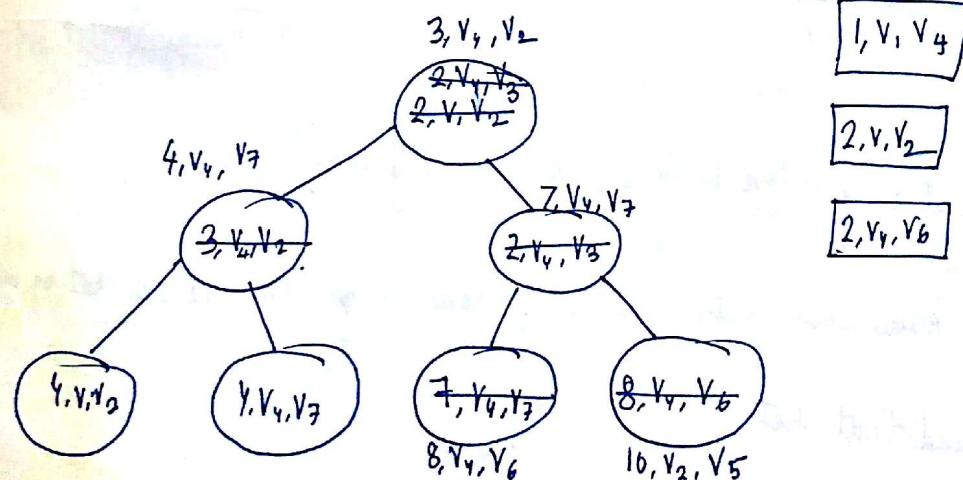
(~~V<sub>1</sub>~~)



V<sub>1</sub>, V<sub>4</sub>  
V<sub>1</sub>, V<sub>2</sub>  
V<sub>4</sub>, V<sub>3</sub>  
V<sub>4</sub>, V<sub>7</sub>  
V<sub>1</sub>, V<sub>6</sub>  
V<sub>7</sub>, V<sub>5</sub>

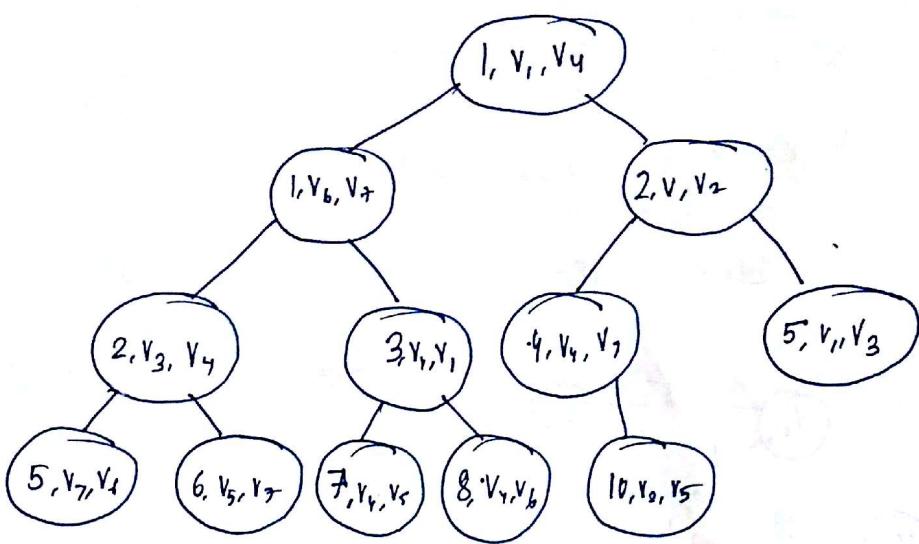
# sequence of edges selected in finding minimum spanning tree.

Using heap - (always keep track of last visited vertex).



Add all edge weights and adjacent unvisited vertices of present visited vertex (V<sub>4</sub>). Delete the minimum.

① Insert all edges in heap while creating graph.



Delete from heap and if the edge does not contain any of visited vertices store it somewhere.

If the deleted edge is adjacent to only one of the visited vertices, add that edge to MST.

If deleted edge contains ~~only~~ two vertices as visited then discard as cycle is formed.

② Kruskal - (unlike starting vertex in Prim's, use starting edge).

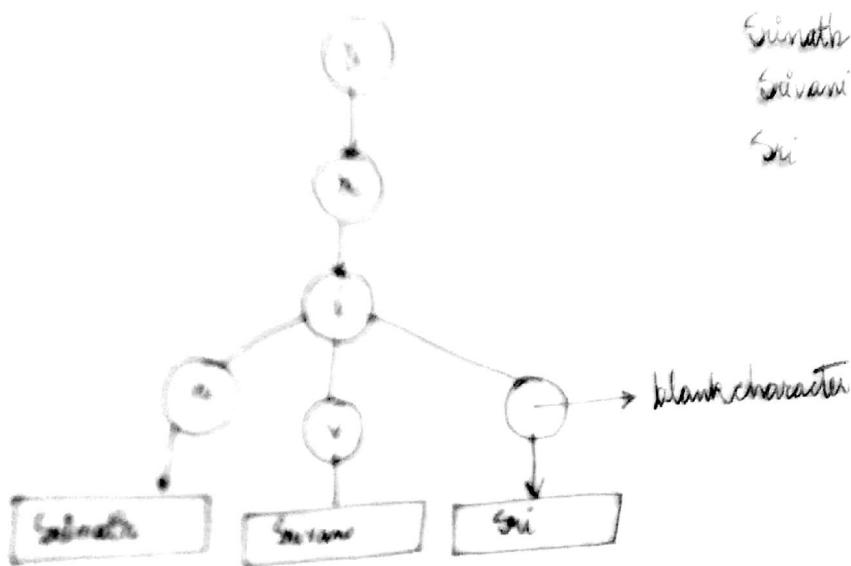
Use disjoint sets to know that both vertices of edge ~~is~~ are in same set or not.

If same set don't add that set.

Aug

- ① Point with DJ change
- ② Point every loop
- ③ Recall

[Task] by (not me)

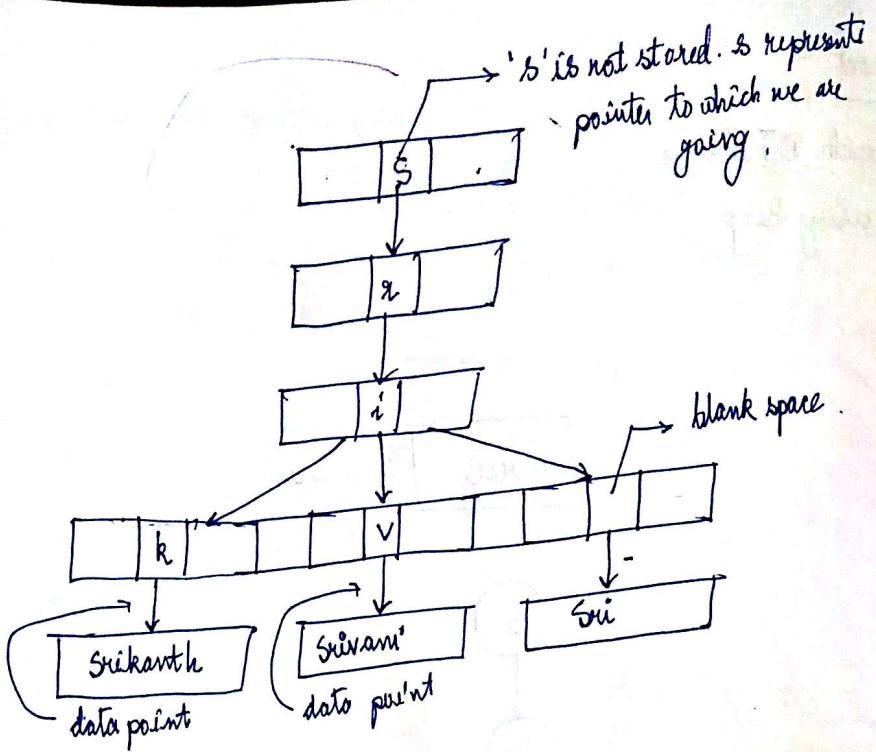


The letters present in key are called Alphabet (denoted by  $A[.]$ ).

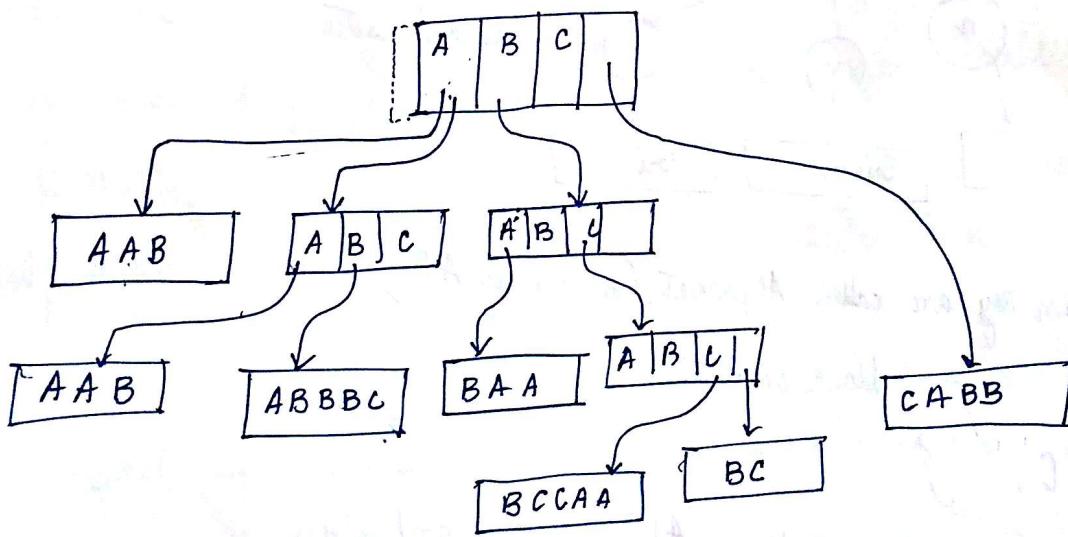
$$A = \{A, B, C, \dots\}$$

e.g. the keys have a lot of different types.  $A[.]$  will have no / entries.

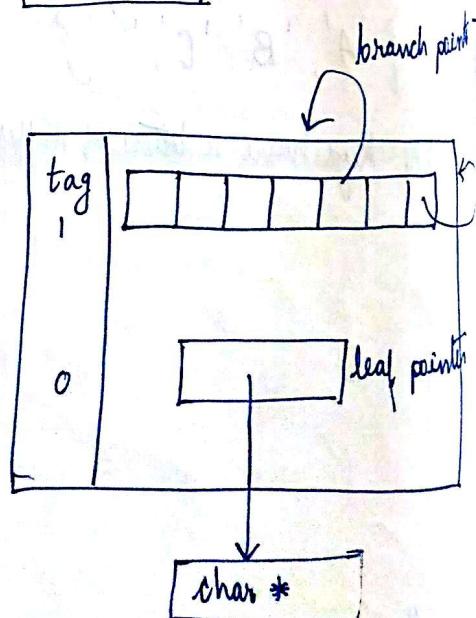
AAB  
 ABBC C  
 BAA  
 BCCAA  
 BC  
 CABBB



### Branch Node Structure -



```
struct trienode {
  int tag;
};
```

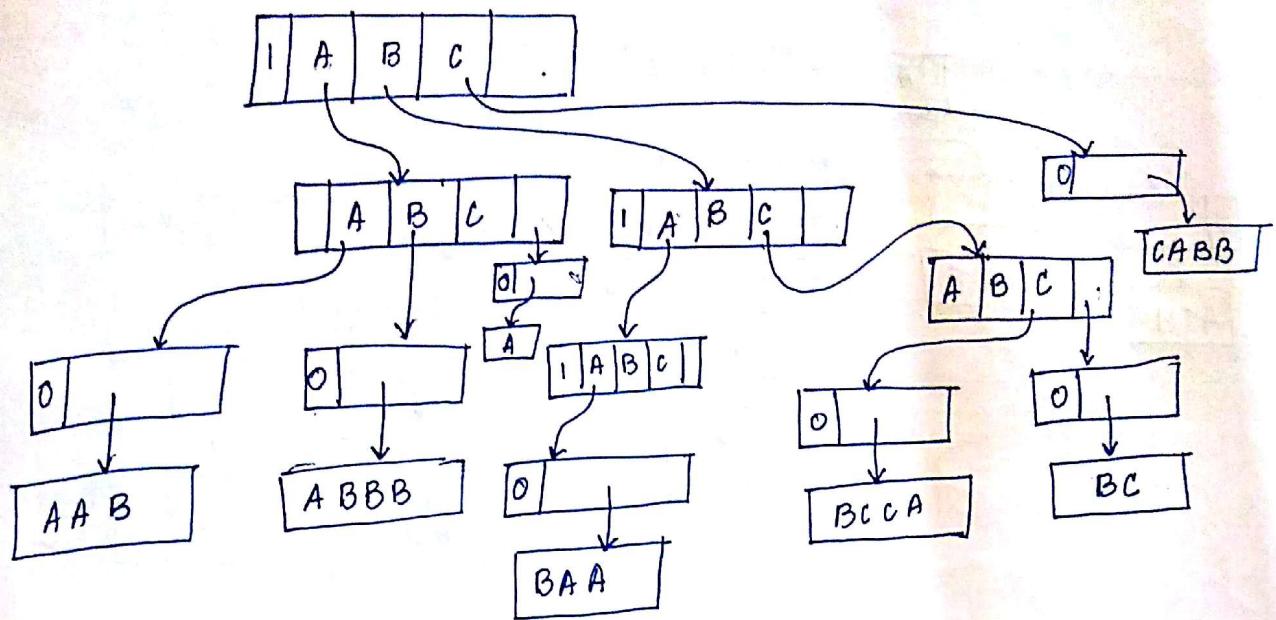


Insertion

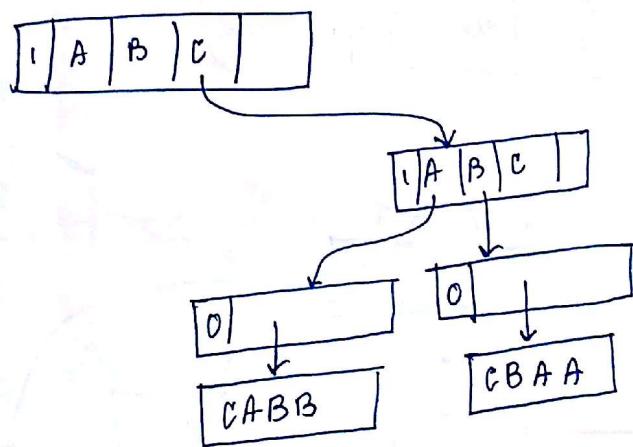
O

A

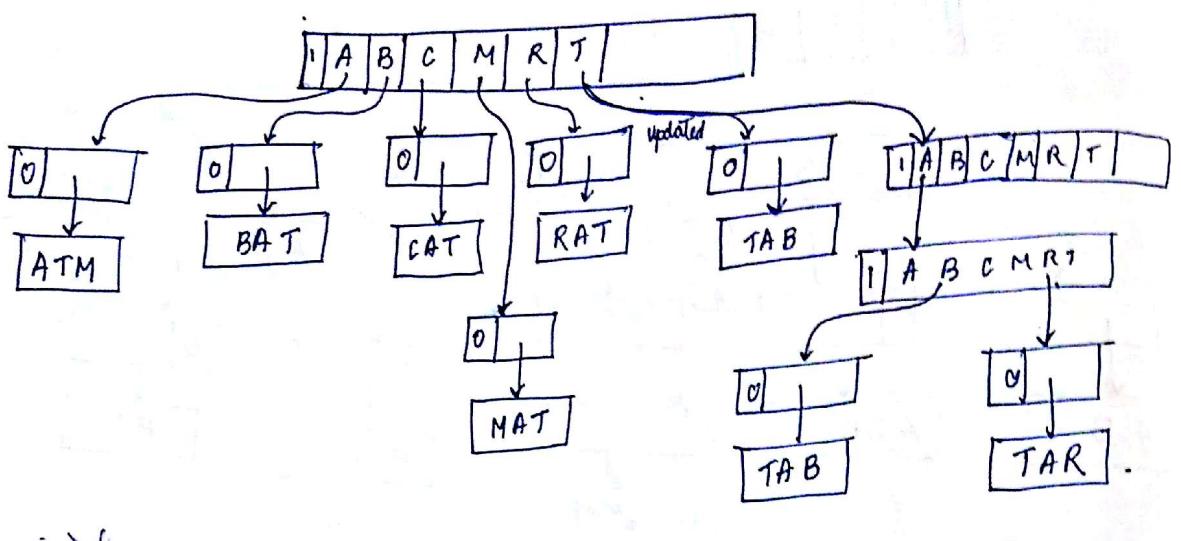
CA



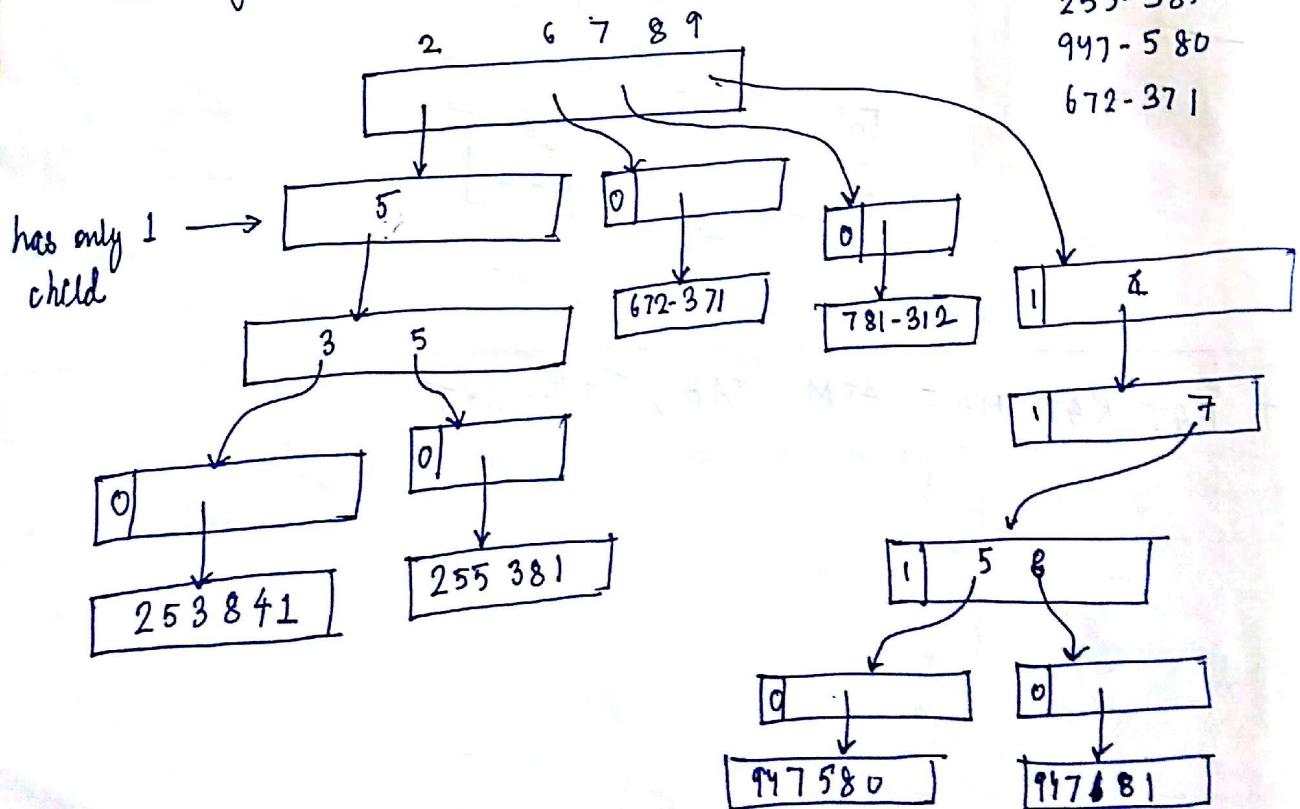
Inserting CBAA



CAT BAT RAT MAT ATM TAB , TAR , ART,



Constructing Trie:



253-841  
781-312  
947-81  
255-381  
947-580  
672-371

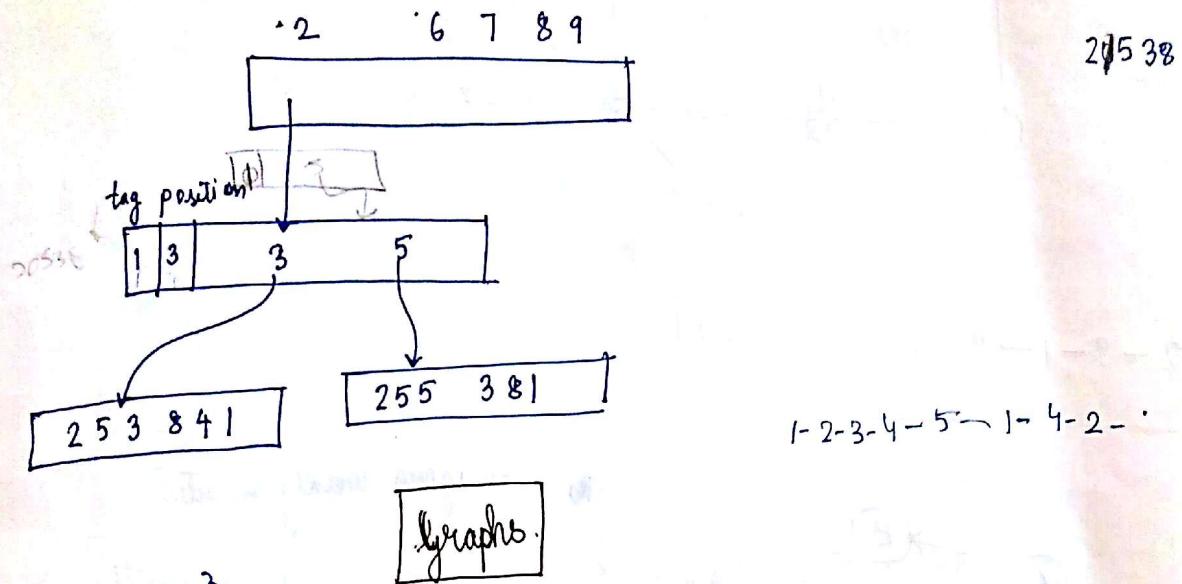
→ All

→ Ex

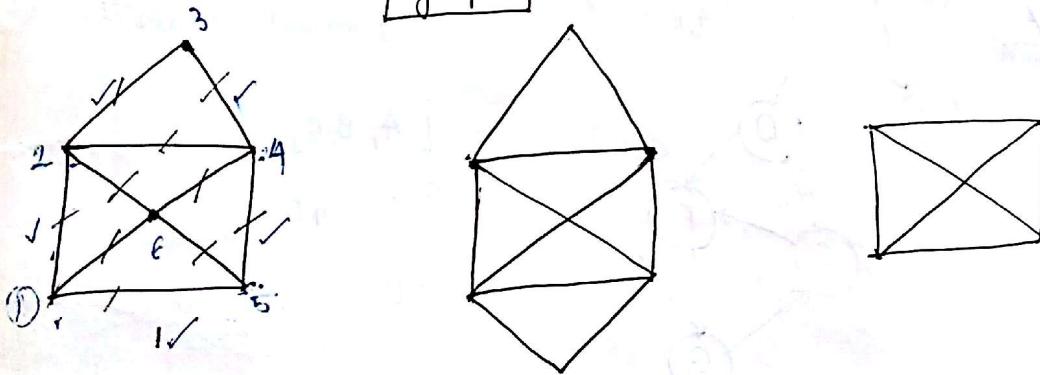
In a G-tree every branch node should have atleast 2 children.

This is done by 3 methods :-

① Character partition method - store position or index of string in branch node.



1-2-3-4-5 → 1-4-2-



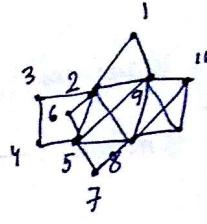
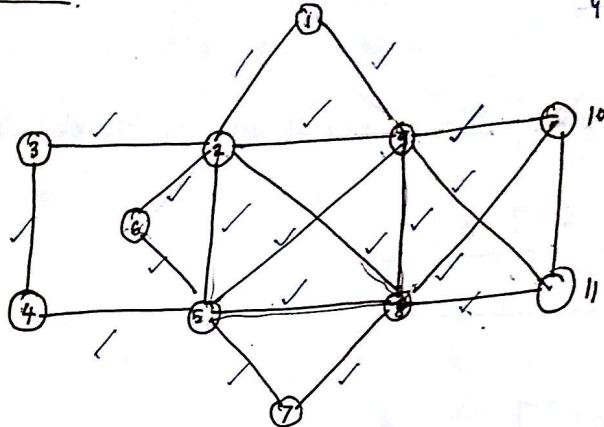
unweighted undirected connected graphs

→ All vertices <sup>must</sup> have even degree → start and end at the same vertex

OR

→ Exactly two vertices must have odd degree → start and end at two different odd degree vertices.

## Euler's Path :-



10 - 9 - 1 - 2 - 9 - 11 - 8 - 10 - 11.

Algorithm

① do DF

② reverse

③ do DF

{

{ H

{ A

{ D

{ E

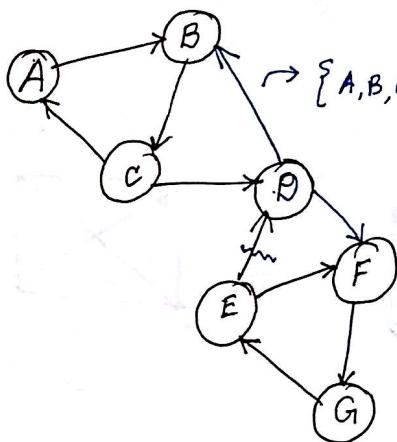
Unweighted, weakly connected

strongly connected components,

{ A, B, C }

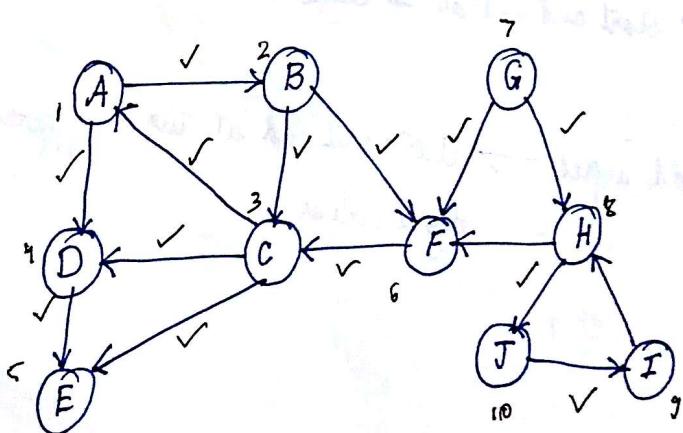
{ E, F, G }

{ D }



Biconnected

## Strongly Connected Components -



Directed, weakly connected

{ A, B, C, F }

{ H, I, J }

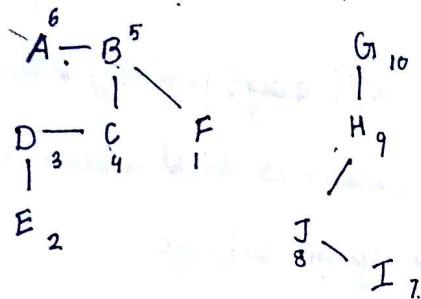
{ G } { D } { E }

Algorithm

D queue

### Algorithm - (3 steps)

① do DFT and number vertices in post-order.



② reverse the graph

③ do DFT on graph. (start on highest numbered vertex).

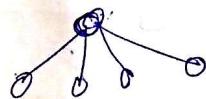
{G}

{H, I, J}.

{A, ~~B~~, C, B, F}.

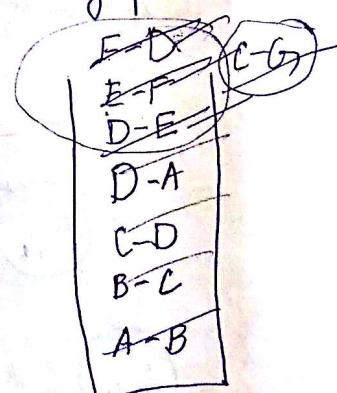
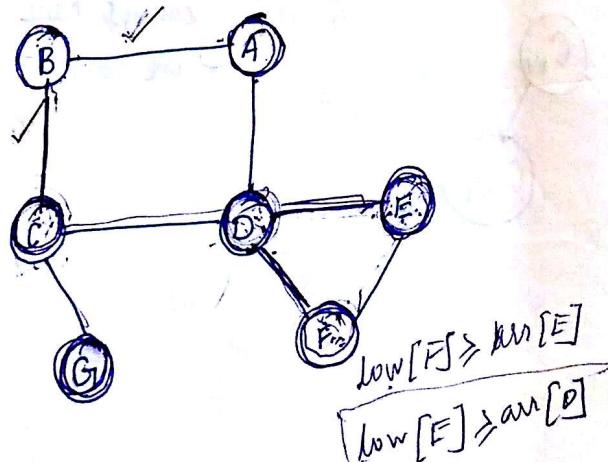
{D, F}

{E}.



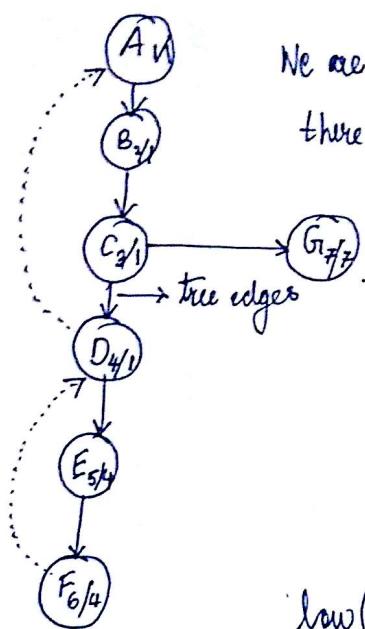
### Articulation Point - Cut-vertices

Biconnected component - Removal of any vertex still keeps the ~~rest~~ graph connected.



### Algorithm -

① value preorder numbering after dft.



We are creating a dft tree. From D we can go again to A so there is a backedge

Write a function to assign numbering  $\sigma$  to vertex.

Calculate  $\text{low}(v)$  which is lowest numbered vertex you can reach using only one back edge.

$$\text{low}(E) = \text{low}(4).$$

$$\text{low}(G) = 7.$$

$\text{low}(v)$  :- minimum of

(1)  $\text{num}(v)$

(2)  $\min \text{low}(w)$  if  $v \rightarrow w$  is an edge.

(3)  $w$  is backedge of  $v$  take  $\text{num}(w)$

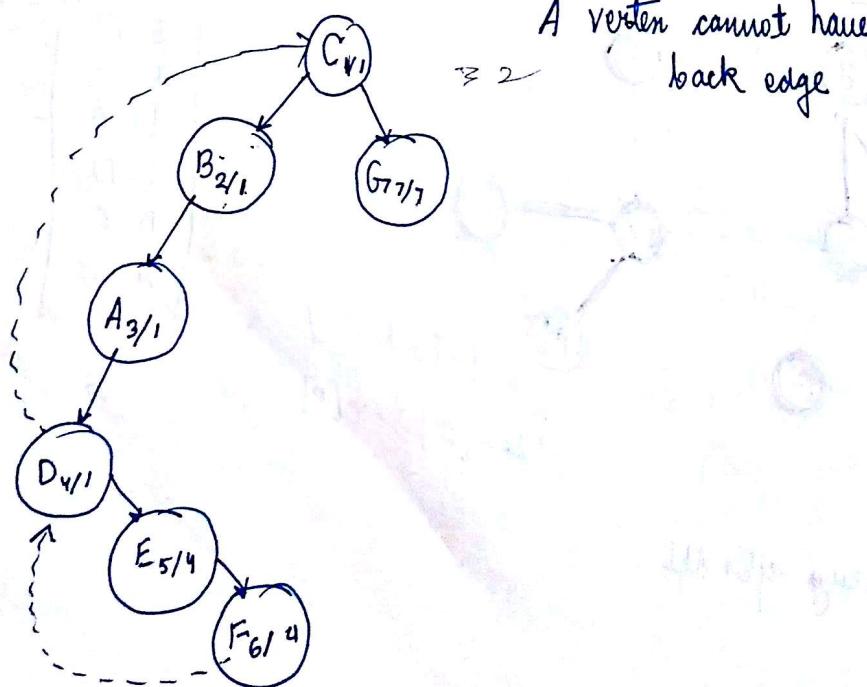
$v$  is an articulation point if

$$\text{low}(w) \geq \text{num}(v) \text{ in the tree.}$$

$v$  and  $w$  are adjacent vertices.

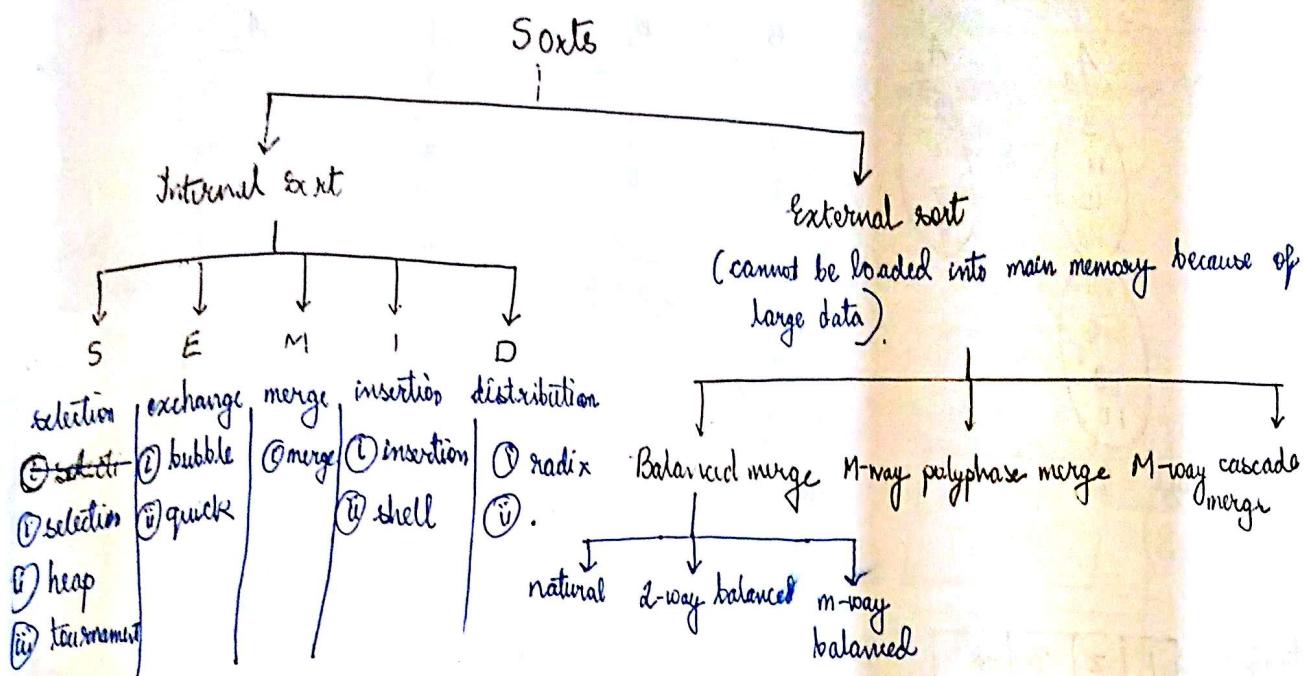
If  $v$  is a root and has one child then the rule is relaxed.

A vertex cannot have more than one back edge



first part  
of two parts

## Sorting: (questions on either FS or DS)

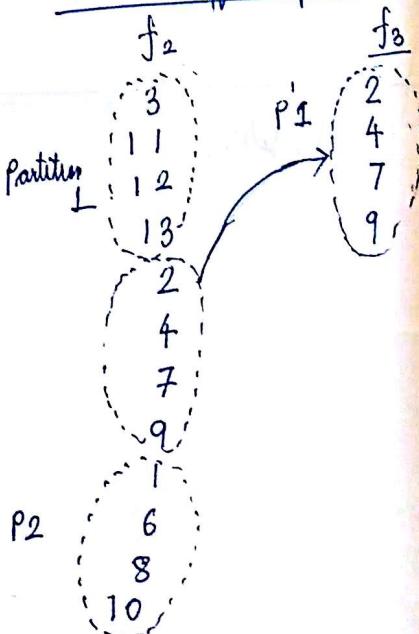


## External Sort

File contains,

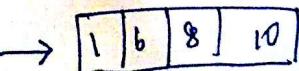
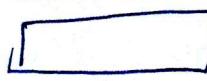
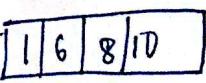
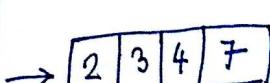
$f_1$   
 { 3  
 13  
 12 } → at a time sort  
 first 4 numbers!  
 11  
 7  
 9  
 4  
 2  
 6  
 1  
 10  
 8

Write in different file

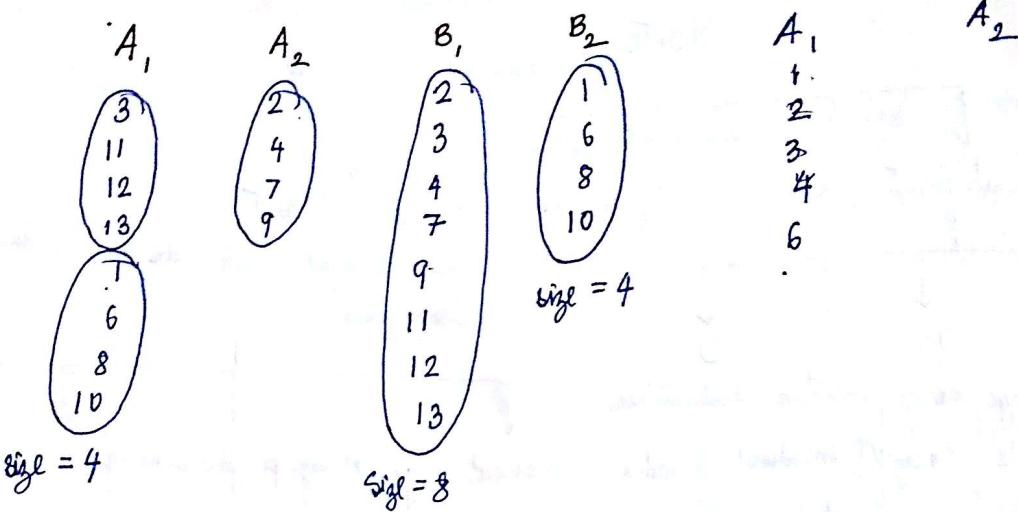


$f_1$   
 2  
 3  
 4  
 7  
 9  
 11  
 12  
 13  
 1  
 6  
 8  
 10

first partition  
of two files.



Second logic . (2-way balanced merge sort) (2 files used)



24  
16

Sorted partition

buffer 

1	2	3	4
---	---	---	---

12	3	4	7
----	---	---	---

1	6	8	10
---	---	---	----

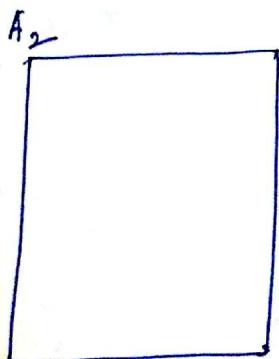
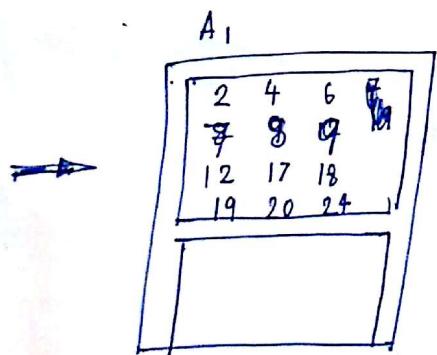
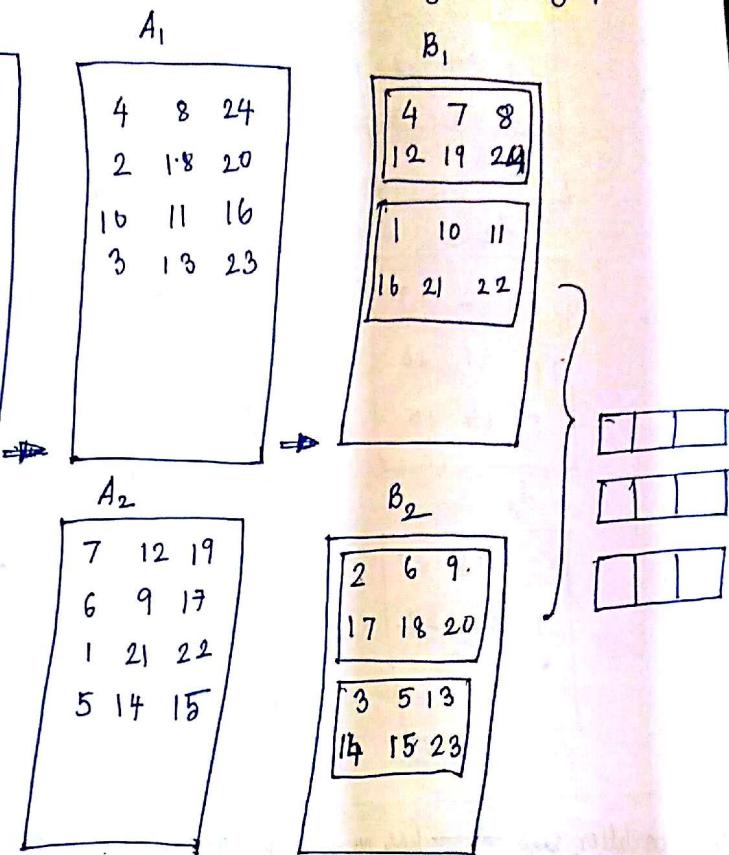
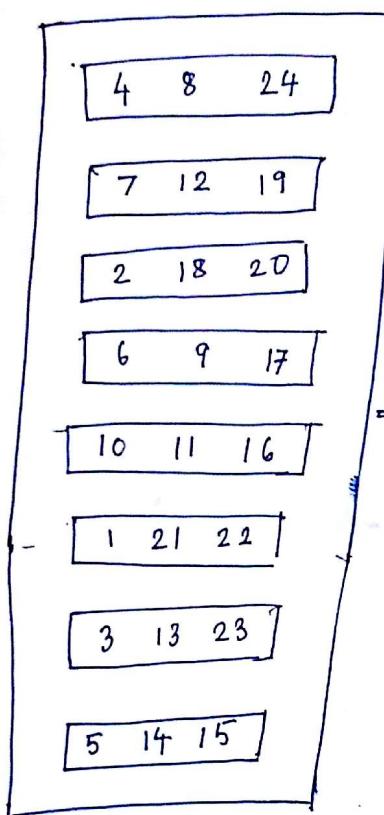
---

m-way balanced merge sort -  $(2m+1)$  files.

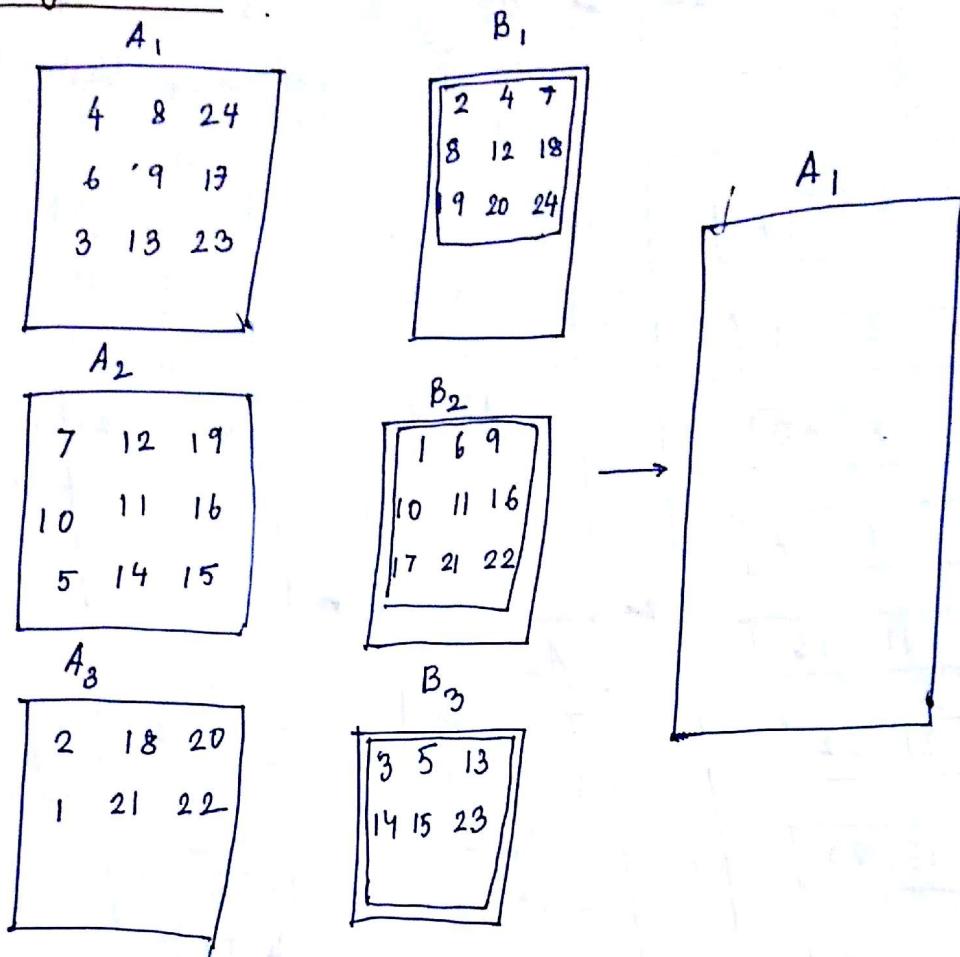
24      4      8      19      12      7      2      20      18      6      17      9  
 16      10     11     21     22      1      23      3      13     15     14      5

Partition size = blocking factor

Sorted partition

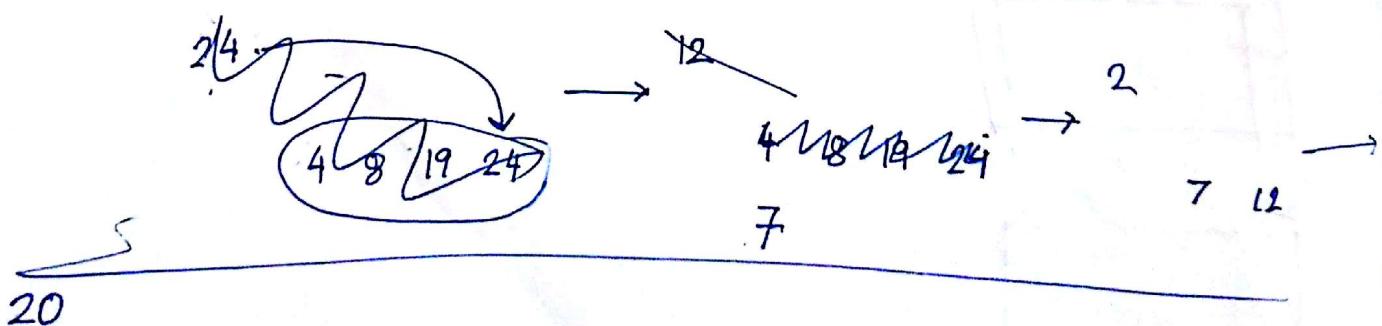


### 3-way balanced



More partition size → less number of merges.

24	4	8	19	12	7	2	20	18	6	17	9	16
10	11	21	1	23	3	13	15	14	5			



thus variables use no.

I 24  
J 19  
K 12  
~~I 24  
J 19  
K 12~~

Sorted partition generate -

I 24  
J 19  
K 12

4 8 12 19 24

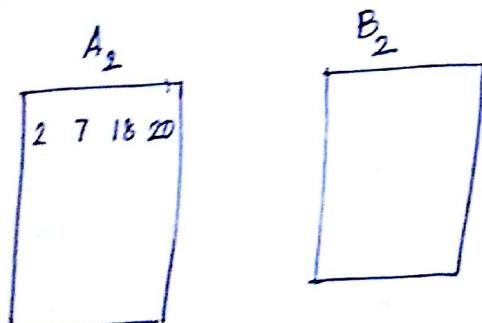
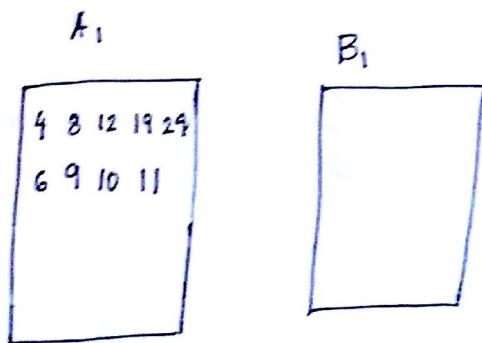
I 26  
J 7  
K 2

2 7 18 20

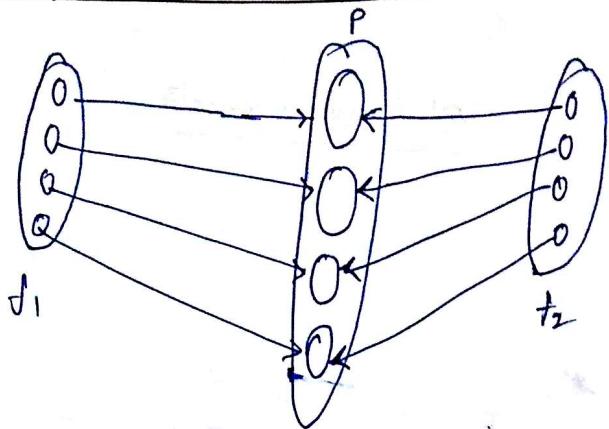
I 7  
J 16  
K 10 X

6 9 10 11

New merge,



m-way natural balanced merge sort - (compulsory 3 files).



Again distribute p in  $f_1$  and  $f_2$  and repeat process until  $P$  contains one partition.

m-way polyphase merge -  $(m+1)$  files required compulsorily.

3-way (17 partitions initially)

	$f_1$	$f_2$	$f_3$	$f_4$
1	7	6	4	0
2	3	2	0	4
3	1	0	2	2
4	0	1	1	1
5	1	0	0	0

Since this case is not for balanced so distribute the partitions such that partitions in  $f_1$ ,  $f_2$  and  $f_3$  are not equal in number.

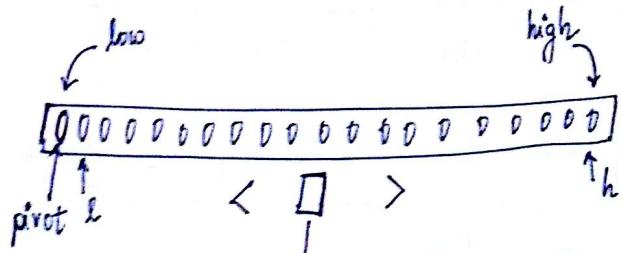
## 6-way polyphase merge

125 partitions of size 1

	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$\pi$	$\tau$	partitions now	No. of merges
	125	123	119	111	95	63	0	636	0		
1.	62	69	56	48	32	0	63	321	$63 \times 6 = 378$		
2.	30	28	24	16	10	32	31	161	$32 \times 11$		
3.	14	12	8	0	16	16	15	81	336		
4.	6	4	0	8	8	8	7	41	328		
5.	2	0	4	4	4	4	3	21	324		
6.	0	2	2	2	2	2	1	11	322		
7.	1	1	1	1	1	1	0	6	321		
8.	0	0	0	0	0	0	1	636			

## Quick Sort

Time Complexity =  $\Theta(n \log n)$ .



Putting pivot into position is partition

$$\text{pivot} = a[\text{low}]$$

void quicksort (int A[], int low, int high)

{

    if (low < high)

{

        int j = partition(A, low, high);

        quicksort(A, low, j-1);

        quicksort(A, j+1, high);

}

int partition (int A[], int low, int high)

{

    int j, l, h; p;

    l = low + 1; h = high;

    p = A[low];

    while (l < h)

    { while (A[l] < p) l++;

        while (A[h] > p) h--;}

    if (h > l) swap (A[l], A[h]);

    if  $a[l] < \text{pivot} \rightarrow l = l + 1$ ;

    if  $a[h] > \text{pivot} \rightarrow h = h - 1$ ;

$k=5$

21

$k=2$  14

$k=1$  9

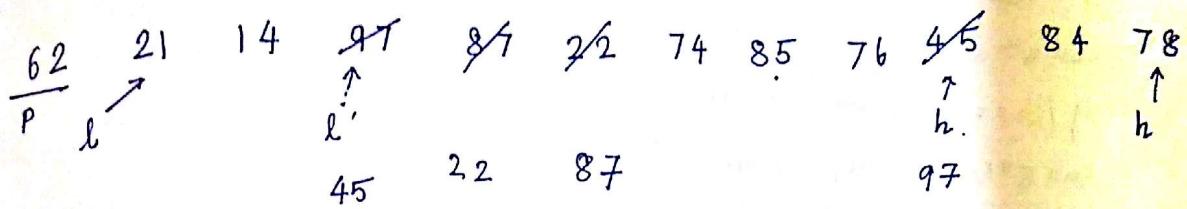
Still sort i

Take ini

(ii)

(iii)

if ( $low < h$ ) swap ( $A[low]$ ,  $A[h]$ ) ;  
 return  $h$  ;  
}



### Shell Sort

$k=1$	1	2	3	4	5	6	7	8	9	10
	27	62	14	9	30	21	80	25	70	55
	21	62	14	9	30	27	80	25	70	55
$k=2$	14	9	21	62	30	28	70	21	80	77
$k=1$	9	14	21	30	62	25	55	70	77	80

Shell sort is called diminishing increment sort.

Take initial  $k = n/2$ .

(i) or take prime numbers

(ii) or take decreasing Fibonacci number

```
void createtree(Tptr &T, istream &in)
{
    cout << "Enter value" << endl; in >> d;
    T = createnode(d);
    cout << "Lc ?" << endl; in >> ch;
    if (ch == 1) createtree(T->lc);
    cout << "Rc ?" << endl; in >> ch;
    if (ch == 1) createtree(T->rc);
}
```