

Q.7

1) If Daniel goes to party then Bob and Cody come too.

$$\rightarrow D \Rightarrow CB \wedge C)$$

$$2) D \Leftrightarrow (C \wedge \neg A)$$

$$3) (D \Rightarrow C) \wedge (D \Rightarrow \neg B)$$

$$4) A \Rightarrow (C \wedge B \wedge \neg C \Rightarrow D)$$

$$5) (A \wedge B \wedge C \Rightarrow \neg D) \wedge (C \wedge \neg A \wedge \neg B \Rightarrow (D \Rightarrow C))$$

$A \Rightarrow$ Andy comes to party

$B \Rightarrow$ Bob comes to party

$C \Rightarrow$ Cody comes to party

$D \Rightarrow$ Daniel comes to party

(2) CONVERT TO CNF: \rightarrow

$$(1) (A \Rightarrow B) \Rightarrow C$$

Step 1: $\rightarrow (A \vee B) \Rightarrow C$ Eliminate \Rightarrow by transforming $d \Rightarrow e$ to $(\neg d \vee e)$

Step 2: $\rightarrow \neg(A \vee B) \vee C$ Eliminate \Rightarrow by transforming $d \Rightarrow e$ to $(\neg d \vee e)$

Step 3: $\rightarrow (A \wedge \neg B) \vee C$ De Morgan's law $\neg(A \vee e) \Rightarrow (\neg A \wedge \neg e)$

Step 4: $\rightarrow (A \vee C) \wedge (\neg B \vee C)$ De Morgan's law

Step 5: $\rightarrow (A \vee C) \wedge (\neg B \vee C)$

$$(2) (P \Rightarrow (Q \Rightarrow R)) \Rightarrow (P \Rightarrow (R \Rightarrow Q))$$

Step 1: $\rightarrow P \Rightarrow (\neg(Q \vee R)) \Rightarrow P \Rightarrow (\neg R \vee Q)$

Step 2: $\rightarrow (\neg P \vee (\neg(Q \vee R))) \Rightarrow \neg P \vee (\neg R \vee Q)$

Step 3: $\rightarrow \neg(\neg P \vee \neg(Q \vee R)) \vee (\neg P \vee \neg R \vee Q)$

Step 4: $\rightarrow (P \wedge Q \wedge \neg R) \vee (\neg P \vee \neg R \vee Q)$

Step 5: $\rightarrow (P \vee \neg P \vee \neg R \vee Q) \wedge (Q \vee \neg P \vee \neg R \vee Q) \wedge (\neg R \vee \neg P \vee \neg R \vee Q)$

Step 6: $\rightarrow \neg P \vee Q \vee \neg R$

(3) Language \Rightarrow r : "Tom is behind red door"
 b : "Tom is behind blue door"
 g : "Tom is behind green door"

(1) axiom: behind one door is Tom and other 2 doors is wrong.

Logic $\Rightarrow (r \wedge \neg b \wedge \neg g) \vee (\neg r \wedge b \wedge \neg g) \vee (\neg r \wedge \neg b \wedge g)$

(2) atleast one of three is true.

$$r \vee \neg b$$

(3) atleast one of three is false

$$\neg r \vee b$$

Solution \Rightarrow

r	b	g	$r \vee \neg b$	$\neg r \vee b$	$2 \wedge 3$
T	F	F	T	F	F
F	T	F	F	T	F
F	F	T	T	T	<u>T</u>

From truth table, Tom is behind the green door.

- Q.4 Let A - Alan goes for scuba diving.
B - Brianna goes for scuba diving.
C - Cody goes for scuba diving.
D - Daneil goes for scuba diving.

So here, only 2 can go, we have $(A \wedge B) \vee (C \wedge D) \vee (C \wedge B \wedge D)$
any 2 can go but there are constraints are.

1) Alan will go only if Brianna goes too.

$$A \Rightarrow B$$

2) Daneil will go only if Cody goes too $D \Rightarrow C$

3) Brianna had found that she did not complete 6150 home, so she could not go. $B = \text{false}$, So $\neg B$.

[B] we have $B = \text{false}$, $(A \wedge B) = \text{false}$, $(C \wedge B \wedge D) = \text{false}$,
 $(C \wedge D) = \text{true}$.

Any literal and false gives false. Now, we have possible pairs. $(A \wedge C) \vee (A \wedge D) \vee (C \wedge D)$

Also, $A \Rightarrow B \Rightarrow \neg A \vee B$ put $B = \text{false} \Rightarrow \neg A$

$\neg A$ means Alan does not go for scuba diving. Hence $(A \wedge C) \vee (A \wedge D)$ not possible.

So, only one pair is possible that is $(C \wedge D)$.

[Cody and Daneil go for scuba diving.]

1) No leader is higher than itself.

$\forall x \text{ leader}(x) \rightarrow \neg \text{higher}(x, x)$

2) Every person who buys a stock is smart.

$\text{person}(x) : x \text{ is person. } \text{stocks}(y) : y \text{ is policy.}$

$\text{Buys}(x, y) : \text{person } x \text{ buys policies } y.$

$\text{Smart}(x) : \text{person } x \text{ is smart.}$

3) $\forall x \exists y : (\text{person}(x) \wedge \text{stocks}(y)) \wedge (\text{Buys}(x, y)) \Rightarrow \text{Smart}(x)$
Every student who takes Algorithm passes it.

$\text{Student}(x) : x \text{ is student } | \text{Takes}(x, \text{algo}) : x \text{ takes Algorithm.}$

$\text{Passes}(x, \text{Algo}) : x \text{ passed algorithm.}$

$\forall x : (\text{Student}(x) \wedge \text{Takes}(x, \text{Algo})) \Rightarrow \text{Passes}(x, \text{Algo})$

4) Only one student took CCN in Spring 2021.

$\text{Student}(x) : x \text{ is a student.}$

$\text{TakesInSpring2021}(x, \text{CCN}) : \text{student } x \text{ takes CCN in Spring 2021}$

$\exists x \neq y : \text{Student}(x) \wedge \text{TakesInSpring2021}(x, \text{CCN}) \wedge$

$(\text{Student}(y) \wedge \text{TakesInSpring2021}(y, \text{CCN})) \Rightarrow$

$x = y$

(5) There is at least two temples in USA.

$\exists x \forall y$ Temples $(x) \wedge$ Temple $(y) \rightarrow$ InUSA $(x) \wedge$ InUSA $(y) \wedge x \neq y$

(6) No coat is waterproof, unless it has been specially treated

coat $(x) : x$ is a coat, waterproof $(x) : x$ is waterproof.

sot $(x) : x$ has been specially treated.

$\forall x$ (Coat $(x) \rightarrow \neg$ waterproof $(x) \vee$ sot (x))

(7) All students get good grades if they study.

$\forall x$ student $(x) \wedge$ study $(x) \rightarrow$ get good grade (x) .

6.a

→ Vehicle Routing: → Initially one Route may be assigned to the vehicle, by considering the current traffic scenario. It will now be iterated again and again to find the best route at every instance.

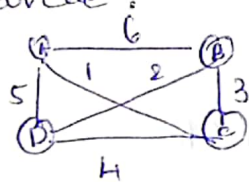
→ Traveling Salesman problem → First of all, a solution that visits each city can be obtained. In further iterations, the first solution can be revisited to obtain the shortest path for Traveling salesman problem.

6.b.

Problem statement: → The main idea of TSP is the problem faced by a salesman who has to travel to a number of cities or towns in an area by visiting every city exactly once.

GOAL: Find the optimal Route in terms of lowest cost, lowest distance.

Example: →



goal is to visit n-cities in above example 4, with minimum tour cost.

Solution using Hill climbing: → Solutions are generated for every state and it is checked to see if the goal is reached.

Lexicographic order: Part of tree is represented below.



Next state is generated based on the value returned by the heuristic function. The state which is better than current is selected.

State space: → The above figure shows state space as a set of tours.

Heuristic function: → Distance between the cities is the heuristic function. minimum distance heuristic is the value returned by the heuristic function.

ABCD
BACD ACBD ABDC DBCA
CABD ACDB

The hill climbing solution for TSP generates all possible routes. For each state -

Actions: → Travel to any city adjacent to current city.

Successor state are all possible combinations of the cities, the state with smallest value, smaller than current is selected. For expansion. The process continues until we get back to start. Algorithm created larger number of successor states for small cities. Hence the time complexity is sufficiently large for the travelling sales man problem.

- 7) For to CNF: \rightarrow 1) Eliminate \Leftrightarrow $B_{1,1} \Leftrightarrow (C_{1,2} \vee P_{2,1})$
- 1) Replace $x \Leftrightarrow B$ with $(x \Rightarrow B) \wedge (B \Rightarrow x) \Rightarrow (C_{1,1} \Rightarrow (C_{1,2} \vee P_{2,1})) \wedge (C_{1,2} \vee P_{2,1} \Rightarrow B_{1,1})$
- 2) Eliminate \Rightarrow , Replace $x \Rightarrow B$ with $\neg x \vee B$
- $(\neg B_{1,1} \vee C_{1,2} \vee P_{2,1}) \wedge ((\neg C_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$
- 3) move \neg inwards using de Morgan rule double negation.
- $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg C_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$
- 4) Apply distributivity law (over \wedge) and Flatten.
- $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg C_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$
- 5) Drop universal quantifiers and distribute \wedge or \vee

Example \Leftrightarrow eliminate \Leftrightarrow Eg. convert $p \Leftrightarrow q$ to $(p \Rightarrow q) \wedge (q \Rightarrow p)$

- 2) Eliminate \Rightarrow Eg convert $p \Rightarrow q$ to $\neg p \vee q$
- 3) move \neg inwards using de Morgan law. convert $\neg \neg p$ to p .
- convert $\neg(p \wedge q)$ to $\neg p \vee \neg q$, $\neg(p \vee q)$ to $\neg p \wedge \neg q$, $\neg \forall x p$ to $\exists x \neg p$
- 4) Standardize variable part: \rightarrow Each quantifier must have a unique variable name to avoid confusion while eliminating quantifiers

- 5) Eliminate Existential quantifiers.
- convert $\exists x p(x)$ to $p(c)$
- convert $\forall x y \exists z p(x, y, z)$ to $\forall x y p(x, y, f(x, y)) \rightarrow$ skolem function

Example Everyone has a surname.

$$\forall x \text{ Person}(x) \Rightarrow \exists y \text{ Surname}(y) \wedge \text{Has}(x, y)$$

- 6) Drop quantifiers: \rightarrow All variables are only universally quantified after eliminating existential quantifier.

Eg. convert $\forall x p(x) \vee \exists y q(y)$ to $p(x) \vee q(y)$

- 7) Distribute \wedge over \vee to get conjunctions of disjunctions.

Eg. convert $(p \wedge q) \vee r$ to $(p \vee r) \wedge (q \vee r)$

$$A \vee (B \vee (C \wedge D)) \Rightarrow (A \vee (B \vee C)) \wedge (A \vee D)$$