```
import numpy as np
import pandas as pd
from scipy.optimize import curve_fit
import matplotlib.pyplot as plt
# Define the model function for nonlinear fitting (Method 1)
def model(x, b):
  return x**b # note: a is fixed to 1
# True parameter and settings
b_true = 2.0
n_points = 100
# We use logarithmically spaced x to cover several orders of magnitude
x_data = np.logspace(0, 2, n_points) # from 1 to 100
# Set a noise level parameter (controls heteroscedastic noise)
noise_level = 0.15 # 15% noise factor
# Function to generate a dataset and compute b1 and b2
def generate_and_fit():
 # Generate the noiseless model
 y_true = model(x_data, b_true)
 # Add multiplicative noise (heteroscedastic: error scales with the magnitude of y)
  noise = np.random.normal(loc=0.0, scale=noise_level, size=n_points)
 y_noisy = y_true * (1 + noise)
 # Method 1: Nonlinear least squares on original data.
 # Provide an initial guess for b (say b=1.5)
  popt, _ = curve_fit(model, x_data, y_noisy, p0=[1.5])
```

```
b1 = popt[0]
  # Method 2: Linear regression on log-log data.
  # Use only positive y_noisy values for the log
  mask = y_noisy > 0
  logx = np.log(x_data[mask])
  logy = np.log(y_noisy[mask])
  # Fit a line: logy = b2 * logx + c (ignore intercept if noise is multiplicative)
  b2, _ = np.polyfit(logx, logy, 1)
  return x_data, y_noisy, b1, b2
# Loop until the relative difference condition is met
reldiff = 0
max_attempts = 1000
attempt = 0
while attempt < max_attempts:
  attempt += 1
  x_data, y_noisy, b1, b2 = generate_and_fit()
  reldiff = abs(b1 - b2) / (abs(b1) + abs(b2))
  if reldiff \geq 0.05:
    break
print(f"Dataset accepted after {attempt} attempt(s)")
print(f"Method 1 (nonlinear fit) estimated b1: {b1:.4f}")
print(f"Method 2 (log-log linear fit) estimated b2: {b2:.4f}")
print(f"Relative difference: {reldiff:.4f}")
# Save the dataset to a CSV file
df = pd.DataFrame({'x': x_data, 'y': y_noisy})
```

```
df.to_csv("fitted_dataset.csv", index=False)
print("Dataset saved to 'fitted_dataset.csv'.")
```

## Output:

X     y       1     0.765234       1.047616     1.170004       1.097499     1.194084       1.149757     1.449197       1.204504     1.879505       1.261857     1.668063       1.321941     1.652892       1.384886     2.056362
1.047616 1.170004 1.097499 1.194084 1.149757 1.449197 1.204504 1.879505 1.261857 1.668063 1.321941 1.652892
1.097499 1.194084 1.149757 1.449197 1.204504 1.879505 1.261857 1.668063 1.321941 1.652892
1.1497571.4491971.2045041.8795051.2618571.6680631.3219411.652892
1.2045041.8795051.2618571.6680631.3219411.652892
1.2618571.6680631.3219411.652892
1.321941 1.652892
1.384886 2.056362
2.000002
1.450829 2.197361
1.519911 2.777851
1.592283 2.179298
1.668101 1.69354
1.747528 3.778223
1.830738 2.397797
1.91791 2.989858
2.009233 3.698275
2.104904 4.102591
2.205131 3.587138
2.31013 5.200557
2.420128 4.979276
2.535364 6.602883
2.656088 6.205483
2.782559 7.256903
2.915053 6.146997
3.053856 10.85664
3.199267 7.780755
3.351603 12.9709
3.511192 12.85465
3.67838 13.90132
3.853529 18.10666
4.037017 16.87349
4.229243 19.09603
4.430621 20.34349
4.641589 20.83338
4.862602 23.54733
5.094138 23.81628
5.336699 31.20776
5.59081 35.16435
5.857021 37.55755
6.135907 36.2021

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6.428073 47.17524
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- 9.326033 75.51671
- 9.7701 102.703
- 102.3601 10.23531
- 10.72267 106.2021
- 11.23324 130.2852
- 11.76812 157.5292
- 12.32847 142.5205
- 12.9155 168.607 13.53048 203.0366
- 14.17474 180.2727
- 14.84968 232.4851
- 15.55676 255.6854
- 16.29751 275.5233
- 17.07353 358.4761
- 17.8865 216.5476
- 18.73817 219.466
- 19.63041 336.6514
- 20.56512 423.0802
- 21.54435 478.8513
- 22.5702 502.6289
- 23.64489 553.2851
- 24.77076 640.9958
- 25.95024 665.5803
- 27.18588 636.4808
- 28.48036 729.494
- 29.83647 787.0009 934.2408
- 31.25716
- 32.74549 839.6125
- 1217.283 34.30469
- 35.93814 1074.596
- 37.64936 1377.634
- 1195.349 39.44206
- 41.32012 1721.129
- 43.28761 2015.797
- 45.34879 1913.385
- 47.5081 2207.625
- 49.77024 2337.048
- 52.14008 2888.802
- 54.62277 2611.474
- 57.22368 3237.682

<sup>6.734151</sup> 45.86843

59.94843	2810.196
62.80291	4315.799
65.79332	4763.372
68.92612	4185.007
72.20809	5244.086
75.64633	5996.094
79.24829	6807.084
83.02176	6327.984
86.9749	7562.091
91.11628	5037.857
95.45485	8552.367
100	11489.58