PortfolioExample

Ryan

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MA610 + MA611 ish, sort of?

Portfolio Problem

We are managing a portfolio of three risky asset(Let's say Microsoft, Coke Cola and Starbucks), and trying to minimize the risk.

Load relevant packages

```
library(PerformanceAnalytics)
library(zoo)
library(tseries)
```

Get history adjusted closing price from yahoo

```
get_price <- function(stock){
        get.hist.quote(instrument=stock,
                    start="2010-01-01",end="2015-08-31",
                    quote="AdjClose",provider="yahoo",
                    origin="1970-01-01",
                    compression="m",
                    retclass="zoo",
                    quiet=TRUE)
}
SBUX_prices <- get_price('sbux')
MSFT_prices <- get_price('msft')
COKE_prices <- get_price('coke')

index(SBUX_prices) <- as.yearmon(index(SBUX_prices))
index(MSFT_prices) <- as.yearmon(index(MSFT_prices))
index(COKE_prices) <- as.yearmon(index(COKE_prices))
```

Calculating returns

```
all_prices = merge(MSFT_prices, COKE_prices, SBUX_prices)
head(all_prices)
```

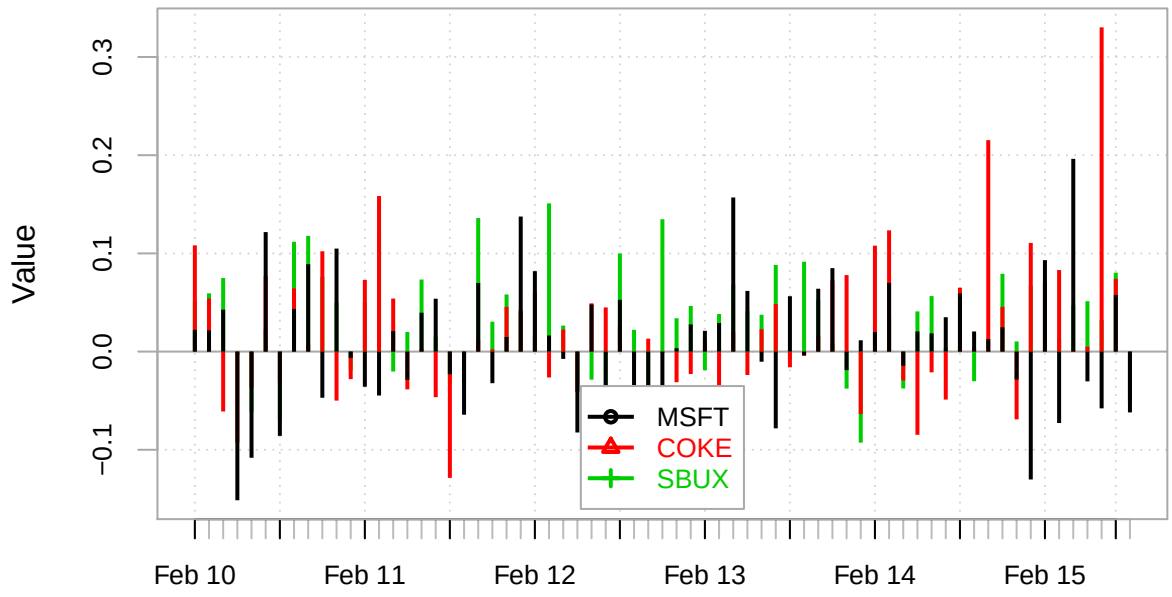
```
##          AdjClose.MSFT_prices AdjClose.COKE_prices AdjClose.SBUX_prices
## Jan 2010             24.21383             46.30931             10.04807
## Feb 2010             24.75006             51.31770             10.56453
## Mar 2010             25.28529             54.09337             11.19167
## Apr 2010             26.36438             50.79207             12.02984
## May 2010             22.37297             46.09480             11.98816
## Jun 2010             19.95356             44.39033             11.25192
```

```
colnames(all_prices) <- c("MSFT", "COKE", "SBUX")
simple_returns <- diff(all_prices)/lag(all_prices, k=-1);
head(simple_returns)
```

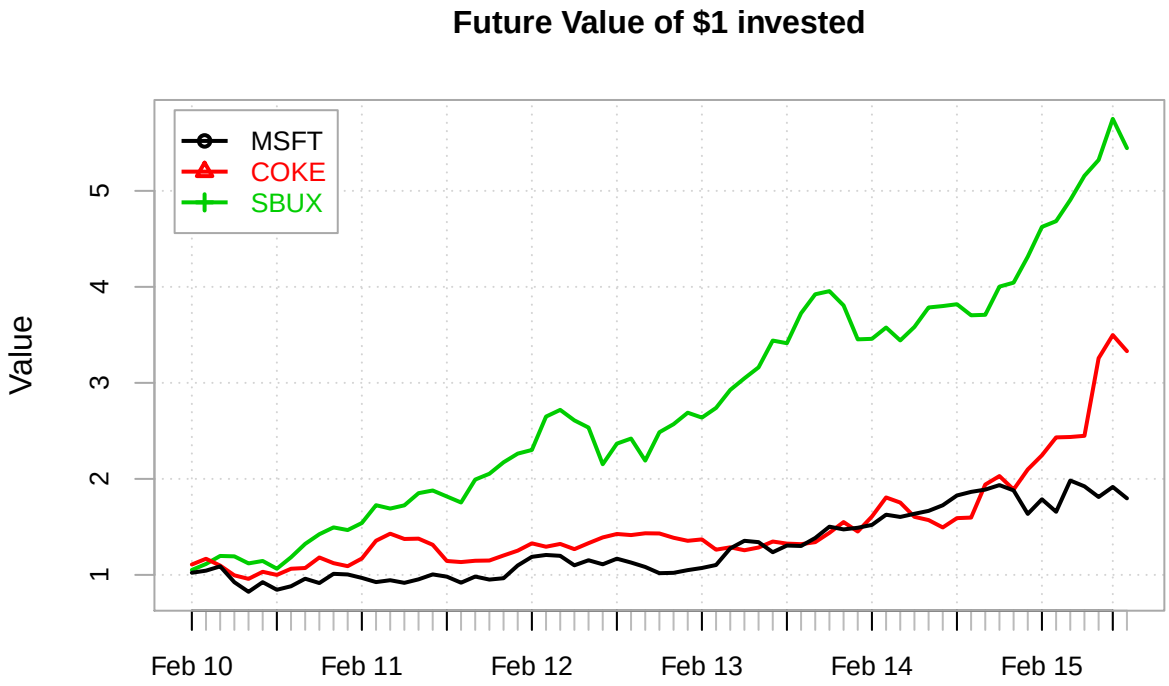
```
##                MSFT        COKE        SBUX
## Feb 2010  0.02214578  0.10815068  0.05139974
## Mar 2010  0.02162544  0.05408803  0.05936268
## Apr 2010  0.04267667 -0.06102966  0.07489175
## May 2010 -0.15139436 -0.09248044 -0.00346422
## Jun 2010 -0.10813949 -0.03697745 -0.06141375
## Jul 2010  0.12168623  0.07679465  0.02263382
```

Plot

```
chart.Bar(simple_returns, legend.loc="bottom", main=" ")
```



```r
chart.CumReturns(simple_returns, wealth.index = T, legend.loc="topleft", main = "Future Value of $1 inv
```



using mean and sd to make investment decision(CER model?)

$$R_i \sim iidN(\mu_i, \sigma_i^2)$$
$$cov(R_i, R_j) = \sigma_{ij}$$

```r
return_matrix <- coredata(simple_returns)
mu_hat_annual <- apply(return_matrix, 2, mean)*12
cov_mat_annual <- cov(return_matrix)*12
```

Portfolio return

- $R_{p,x} = x_A R_A + x_B R_B + x_C R_C$
- $\mu_{p,x} = \mathbb{E}[R_{p,x}] = x_A \mu_A + x_B \mu_B + x_C \mu_C$
- $\sigma_{p,x}^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + x_B^2 \sigma_B^2 + 2x_A x_B \sigma_{AB} + 2x_B x_C \sigma_{BC} + 2x_A x_C \sigma_{AC}$

With the condition that the proportions sum up to 1

$$x_A + x_B + x_C = 1$$

In matrix notation

$$\mu_{p,x} = \mathbf{x}' \mu$$

$$\sigma^2_{p,x} = var(\mathbf{x}'\mu) = \mathbf{x}'\mathbf{\Sigma}\mathbf{x}$$
$$\mathbf{x}'\mathbf{1} = 1$$

Optimization Problem

$$\min_{x_A,x_B,x_C} \sigma^2_{p,m} = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + x_B^2\sigma_B^2 + 2x_Ax_B\sigma_{AB} + 2x_Bx_C\sigma BC + 2x_Ax_C\sigma_{AC}$$
$$s.t. \qquad x_A + x_B + x_C = 1$$

Again in Matrix Notation

$$\min_{\mathbf{x}} \mathbf{x}'\mathbf{\Sigma}\mathbf{x}$$
$$s.t. \qquad \mathbf{x}'\mathbf{1} = 1$$

Solve using Quadratic Programming

Setting up the problem

```
top.mat = cbind(2*cov_mat_annual, rep(1,3))
bot.vec = c(rep(1,3),0)
Am.mat = rbind(top.mat, bot.vec)
b.vec = c(rep(0,3),1)
z.m.mat = solve(Am.mat) %*%  b.vec
x.vec = z.m.mat[1:3,1]
x.vec
```

```
##      MSFT      COKE      SBUX
## 0.3298210 0.2846464 0.3855326
```

```
mu.gmin = as.numeric(crossprod(x.vec, mu_hat_annual))
mu.gmin
```

```
## [1] 0.2397456
```