CSCI 5446 - Chaotic Dynamics

Problem Set 3 - Solutions

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1. Middle-sixth-removed Cantor set:

Consider a line segment of unit length and let us remove ⅙ of the line (1/12 on both sides from the midpoint) and this leaves us two line segments each of length 5/12. Iterate each line segment at each level similarly by removing the middle-sixth. The table below demonstrates the number of $\epsilon - radius$ balls required for removing the middle-sixth at each level,

n	N(ϵ)	ϵ
1	1	1
2	2	5/12
3	4	$(5/12)^2$
...	...	...
n	2^{n-1}	$(5/12)^{n-1}$

The capacity dimension is given by the formula,

$$d_c = \lim_{\epsilon -> 0} \frac{log(N(\epsilon))}{log(1/\epsilon)}$$

As $\epsilon -> 0,\ n -> \infty,$

$$d_c = \lim_{\epsilon -> 0} \frac{log(2^n)}{log((12/5)^n)}$$

$$d_c = \frac{log(2)}{log((12/5)}$$

$$d_c = 0.7917$$

2. ODE

a. Third Order ODE to first-order ODE

The equation given can be written as the following,

$$2\dddot{x} - 3\tan\left(\frac{\ddot{x}}{2}\right) + 16\log(\dot{x}) - x = 0$$

$$\Rightarrow \dddot{x} = \frac{3}{2}\tan\left(\frac{\ddot{x}}{2}\right) - 8\log(\dot{x}) + \frac{x}{2}$$

Let,

$$x_1 = x \Rightarrow \dot{x}_1 = \dot{x} = x_2$$

$$x_2 = \dot{x} \Rightarrow \dot{x}_2 = \ddot{x} = x_3$$

$$x_3 = \ddot{x} \Rightarrow \dot{x}_3 = \dddot{x} = \frac{3}{2}\tan\left(\frac{x_3}{2}\right) - 8\log(x_2) + \frac{x_1}{2}$$

$$\dot{x}_1 = x_2 \longrightarrow (1)$$

$$\dot{x}_2 = x_3 \longrightarrow (2)$$

$$\dot{x}_3 = \frac{3}{2}\tan\left(\frac{x_3}{2}\right) - 8\log(x_2) + \frac{x_1}{2} \longrightarrow (3)$$

Equations (1), (2), and (3) are the first-order differential equations. We can also observe that the three equations CANNOT be written as, where the matrix A is just numbers,

$$\dot{\vec{X}} = AX$$

b. First Order ODE to high-order ODE

Given

$$\dot{x} = y \quad (1)$$
$$\dot{y} = z \quad (2)$$
$$\dot{z} = yz + \log(y) \quad (3)$$

$$\dot{y} = z \Rightarrow \ddot{x} = z$$
$$\dot{z} = \dddot{x}$$
$$\dddot{x} = \dot{x}\ddot{x} + \log(\dot{x})$$
$$\Rightarrow \boxed{\dddot{x} - \dot{x}\ddot{x} - \log(\dot{x}) = 0}$$

The first order differential equations (1), (2), and (3)are transformed into a single third-order differential equation.

Also, Equations (1), (2), and (3) are first-order differential equations. We can also observe that the three equations CANNOT be written as, where the matrix A is just numbers,

c.

We can observe that for the systems described in parts (a) and (b) the first-order differential equations for the system CAN NOT be written as

$$\dot{\vec{x}} = Ax$$

where the matrix A is just numbers. Thus **both systems are non-linear**.

3. Fractal Trees

Let *r* define the ratio of the current segment to the previous segment for the iteration.

a.

After iterating about 13 times, observation has been made that there is no longer a difference between successive segments. The plot after 13 iterations is as follows,

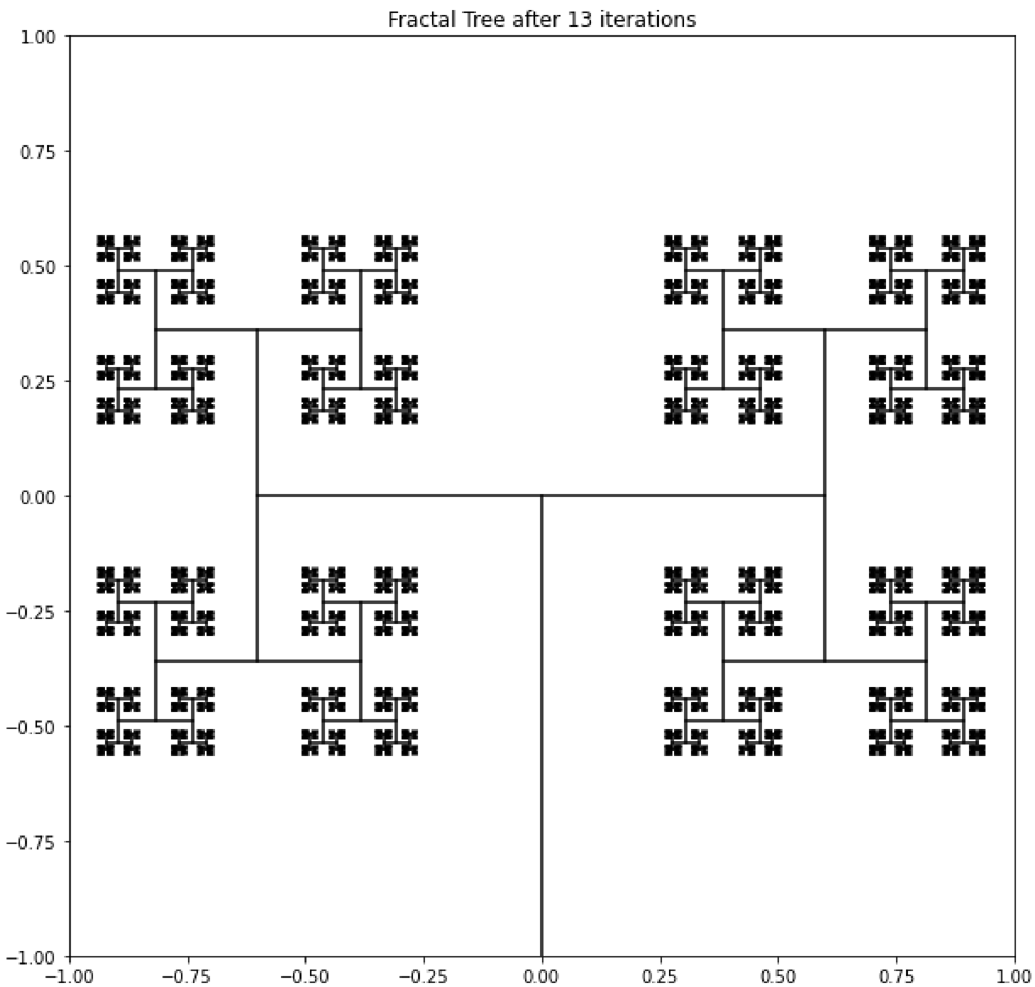

Fig 1: Fractal Tree for r = 0.6, left angle = 90 degrees, right angle = 90 degrees

b.

The tree looks to shrink out when the segment length ratio drops to 0.5 and below. More room becomes available between the branches.

If the length ratio is 1/√2, more balanced appearance, with branches that are neither too close together nor too spread out. We see a very uniform plot with spaces between them forming repeating patterns on their own, as shown below

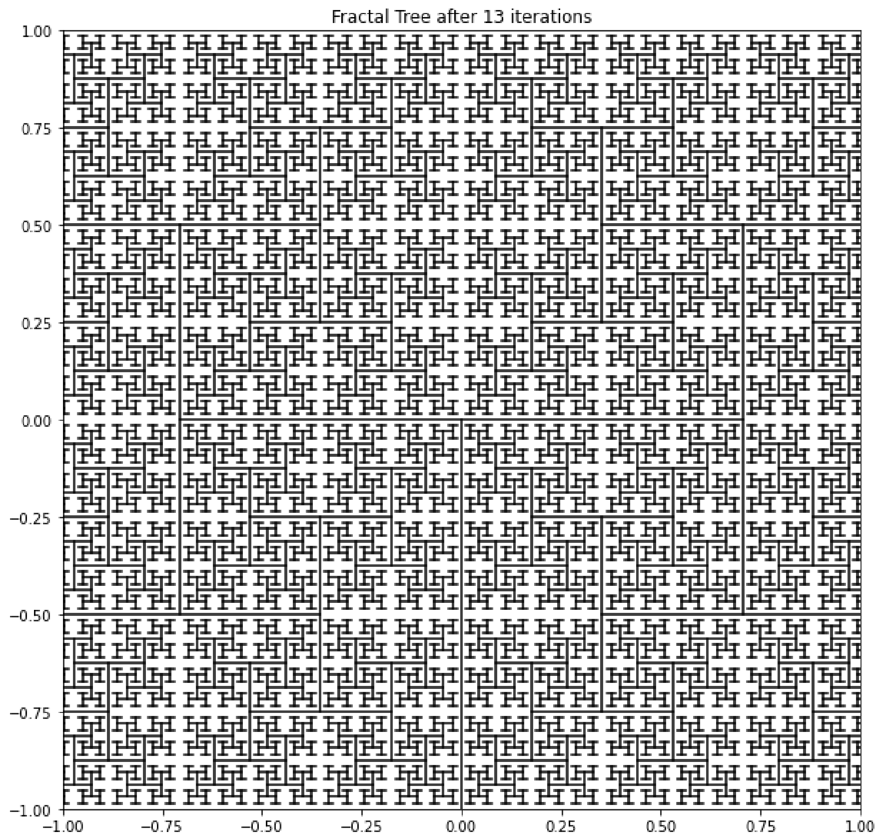

Fig 2: Fractal Tree for r = 1/√2, left angle = 90 degrees, right angle = 90 degrees

If the length ratio is greater than 1/√2, more balanced appearance is lost and we observe noisy grids.

If the length ratio is smaller than 0.5, the tree will grow at a much slower rate, and the branches will become very close together. The tree will be much shorter than the other two cases. In this scenario, the branches will also become much shorter and narrower over time, due to the rapid reduction in length at each iteration.

c.

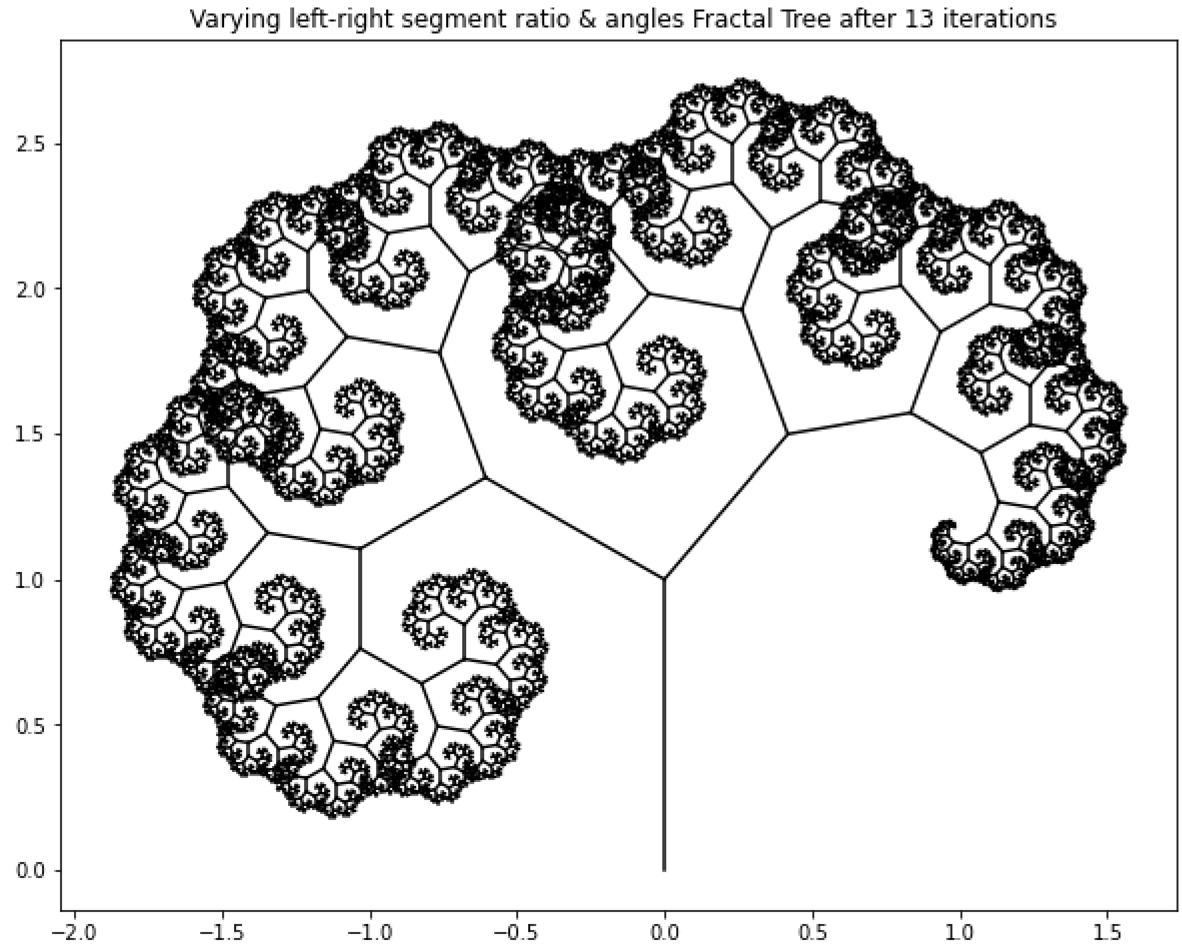

Fig 3: Fractal Tree for left-ratio = 0.7, right-ratio = 0.65, left-angle = 60 degrees, right-angle = 40 degrees

Figure 3 shows unequivocally how altering branching angles and segment length ratios alter the structure of the tree while maintaining self-similarity. We can also observe that the tree is slightly skewed towards the left as the left -ratio, and angle are greater than that of the right.

d.

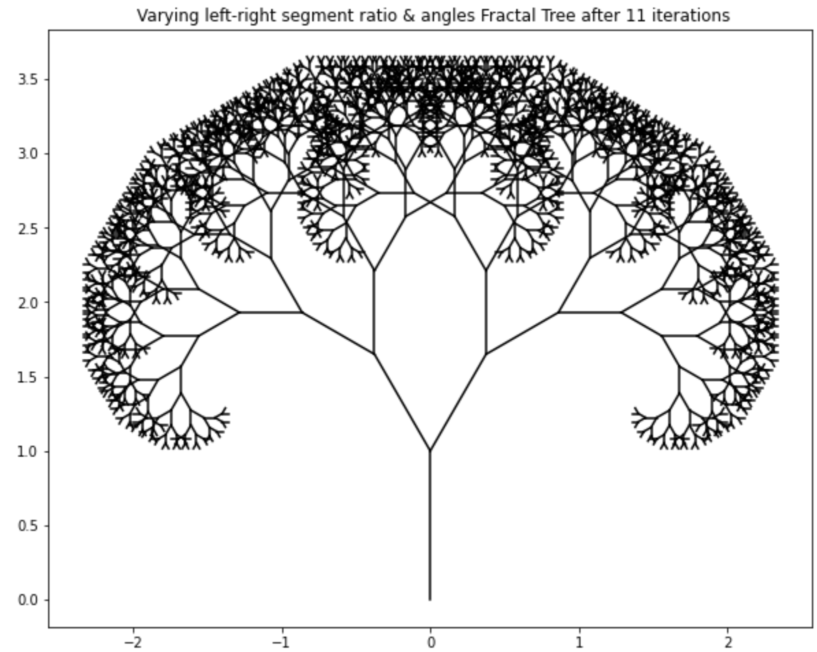

Fig 4: Fractal Tree for left-ratio = 0.7, right-ratio = 0.65, left-angle = 60 degrees, right-angle = 40 degrees

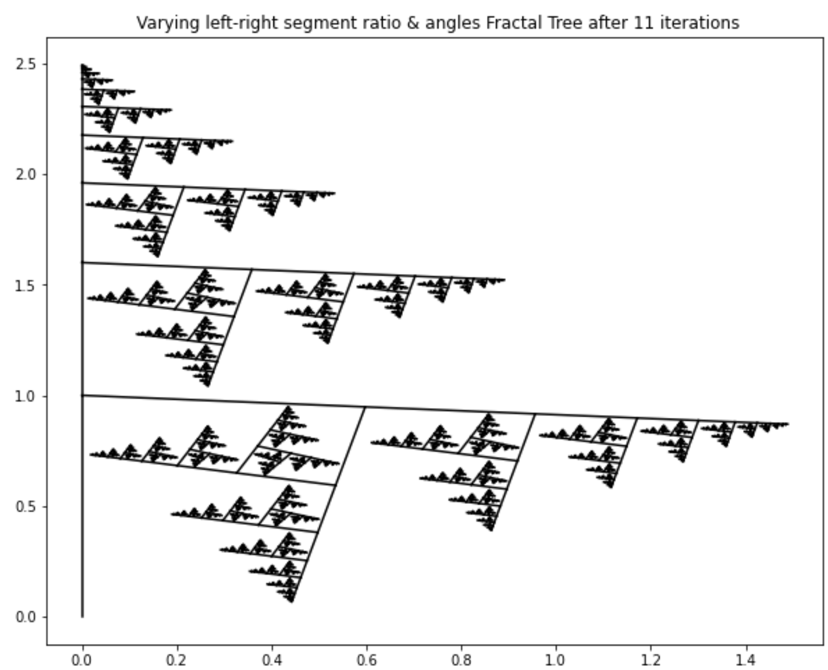

Fig 5: Fractal Tree for left-ratio = 0.6 , right-ratio = 0.6, left-angle = 0 degrees, right-angle = 100 degrees

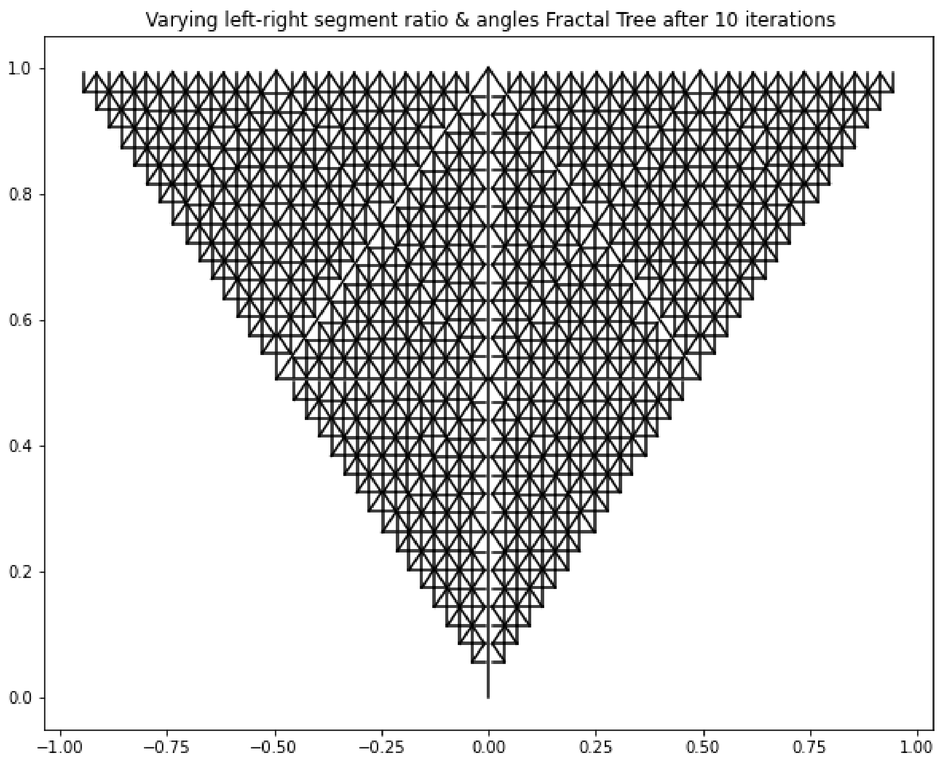

Fig 6: Fractal Tree for left-ratio = 0. 7, right-ratio = 0.7, left-angle = 135 degrees, right-angle = 135 degrees