CSCI 5446 - Chaotic Dynamics

Problem Set 6 - Solutions

Name : Gowri Shankar Raju Kurapati SID: 110568555

1. Simplistic Temporal Poincare Sections of Pendulum:

a. **[θ, ω] = [0.01, 0]**

Given m = 0.1 Kg, l = 0.1m, β = 0, and α = A = 0

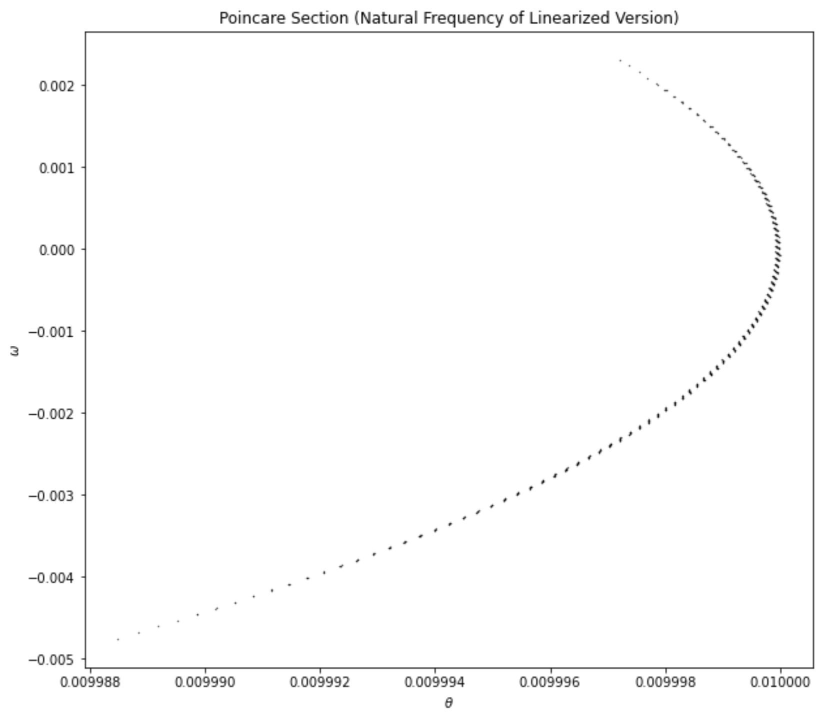


Fig1: Poincare Section for State Space Trajectory for IC [θ, ω] = [0.01, 0] m = 0.1 Kg, l = 0.1m, β = 0, α = A = 0, Δt = 0.005

Ideally, sampling at natural frequency should result in a single point **[0.01, 0]** on the state space portrait. But we see dense points around **[0.01, 0]** and some points in sparse, around the initial condition, as the natural frequency at which we sampled is not exactly accurate numerically. There is an approximation involved at smaller theta and this approximation mixed with runge-kutta four approximaions causes to sample a few points around the trajectory as seen in the Fig1.

b. Irrational Multiple of Natural Frequency

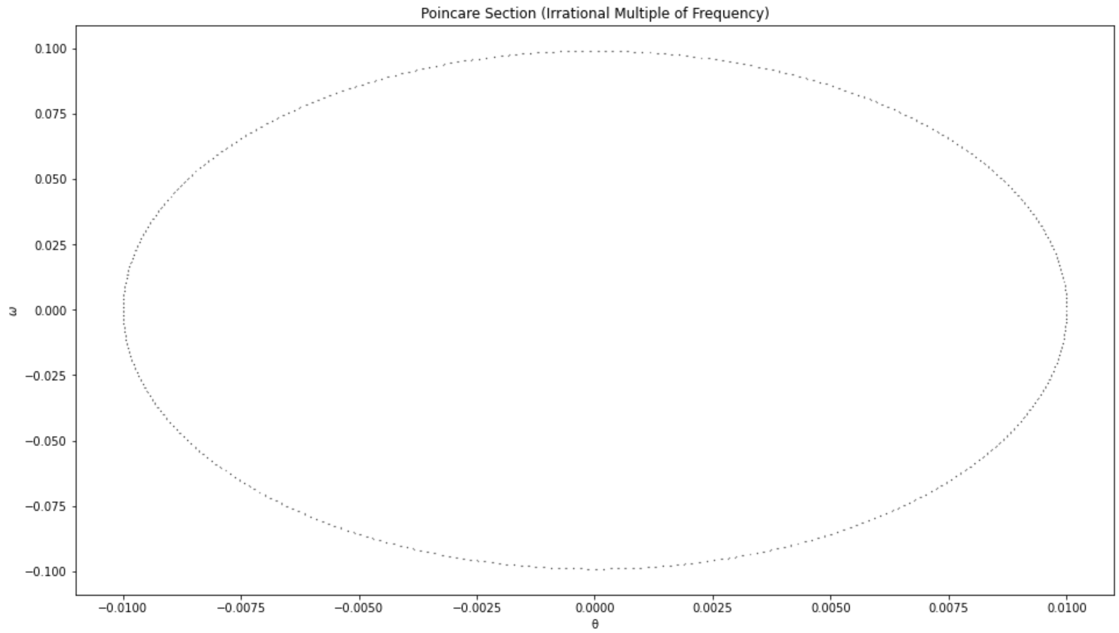
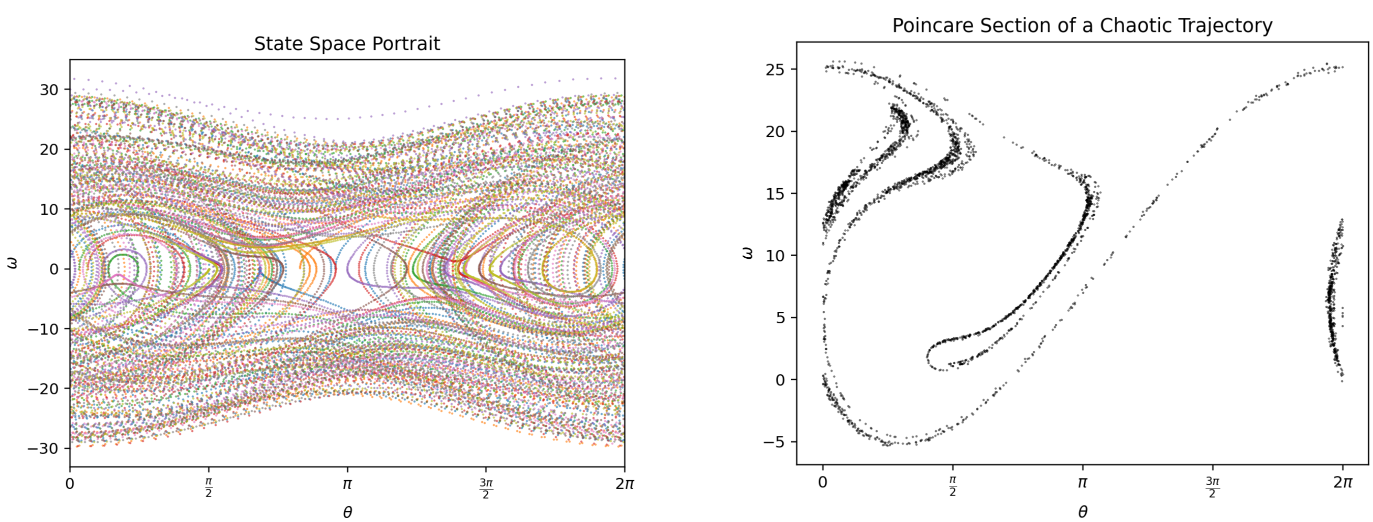


Fig2: Poincare Section $\Sigma : t = 1/\sqrt{2}\,T_0$ for IC [θ, ω] = [0.01, 0] m = 0.1 Kg, l = 0.1m, β = 0, α = A = 0, Δt = 0.005

When the section is sampled at an irrational multiple of the natural frequency, it finally intersects the trajectory at every point on the trajectory and this is what we observe in Fig2.

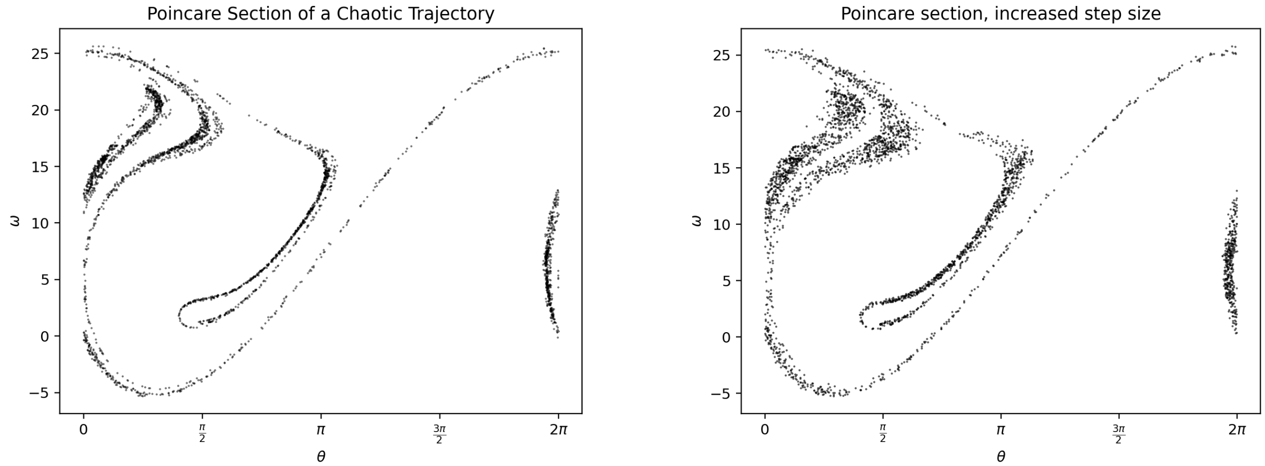
c. Choatic Trajectory



(a) (b)

Fig3: m = 0.1 Kg, l = 0.1m, β = 0, α =4.02, A = 0.9, Δt = 0.005
(a)State Space Portrait (b) Poincare Section : IC [θ, ω] = [0.01, 0]

d. Effects of Δt

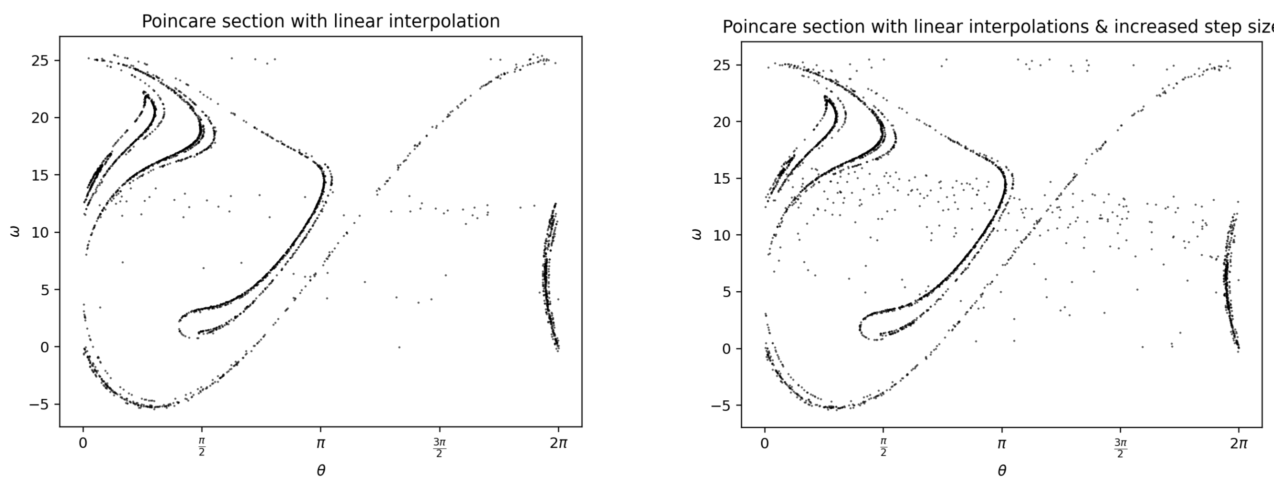


(a) Δt = 0.005 (b) Δt = 0.02

Fig4: Poincare sections of a choatic trajectory at different timestep sizes

By keeping the timespan covered by the trajectory constant, we observe that the form the section remains same but it becomes more spread out and becomes more sloppy as the time step increases. We know that for Runge-Kutta 4, the error is $O(h^5)$, $i.e$ the error increases as the time step increases. As the error is more on higher timestep size, there is more deviation of the trajectory from original one and this effect compounds over time, making the points more spread out from the original trajectory when crossing the section.

2. Temporal Poincare Sections using Linear Interpolation:



(a) Δt = 0.005 (b) Δt = 0.02

Fig5: Poincare sections of a choatic trajectory with linear interpolation at different timestep sizes

Comparing Fig4 (a) and Fig5 (a), it is evident that this approach, which employs linear interpolation, catches the points on the section more correctly than the original method. The flow tightens up along the left side of this route, generating more defined curves rather than blobs. Fig5 (a) clearly shows the improved accurate section with the better algorithm.

As the timestep size is increased by keeping the timespan covered by the trajectory constant, we still observe the same crisp nature of the section without any sloppyness or spreading out as seen when interpolation isn't used. But we see some points inbetween which are like noise and these noisy points increase as the timestep size is increased.

3. Spatial Section of Lorenz attractor:

Lorenz System a = 16, r = 50, b = 4
initial condition (x_0) *= [-13, -12, 52]*

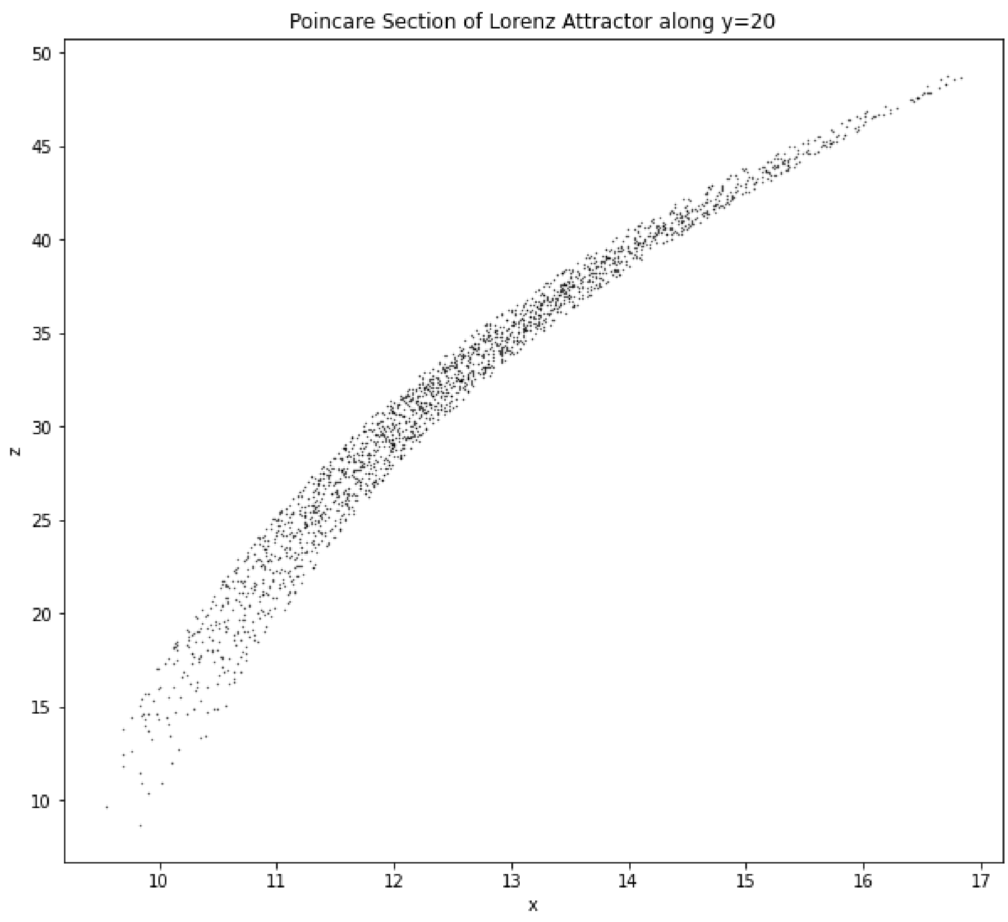


Fig6: Poincare sections of a Lorenz Attractor at y = 20 with *a = 16, r = 50, b = 4, IC = [-13, -12, 52]*

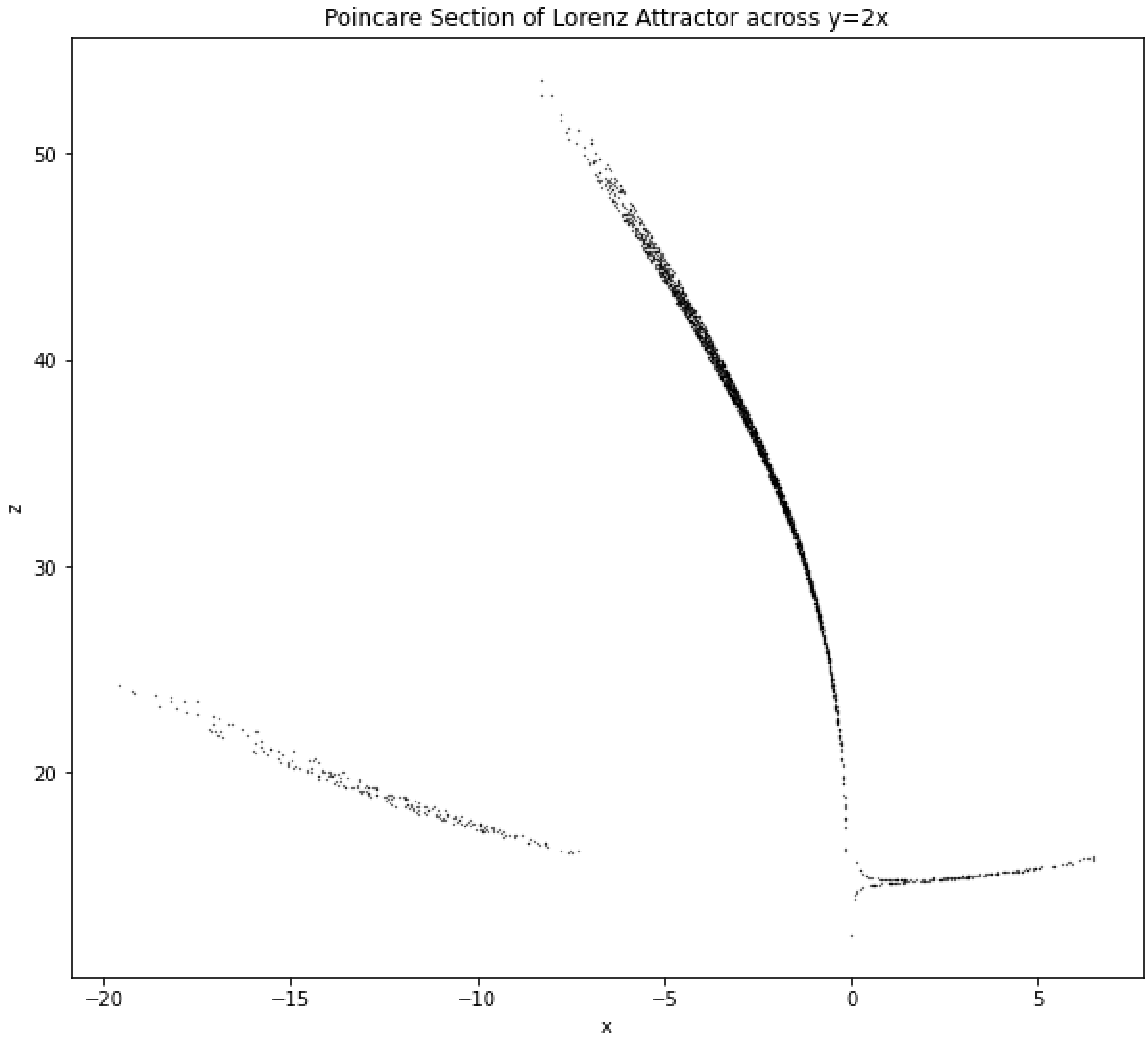


Fig7: Poincare sections of a Lorenz Attractor at y = 2x with *a = 16, r = 50, b = 4, IC = [-13, -12, 52], projected the section onto XZ plane*