CSCI 5446 - Chaotic Dynamics

Problem Set 4 - Solutions

Name : Gowri Shankar Raju Kurapati SID: 110568555

2. Undamped ($\beta = 0$) and Undriven ($\alpha = A = 0$) Pendulum:

Given m = 0.1 Kg, l = 0.1m, $\beta = 0$, and $\alpha = A = 0$, (pendulum without a drive and no damping) the state space trajectory is as follows:

a. Initial Condition : **[θ, ω] = [3, 0.1]**

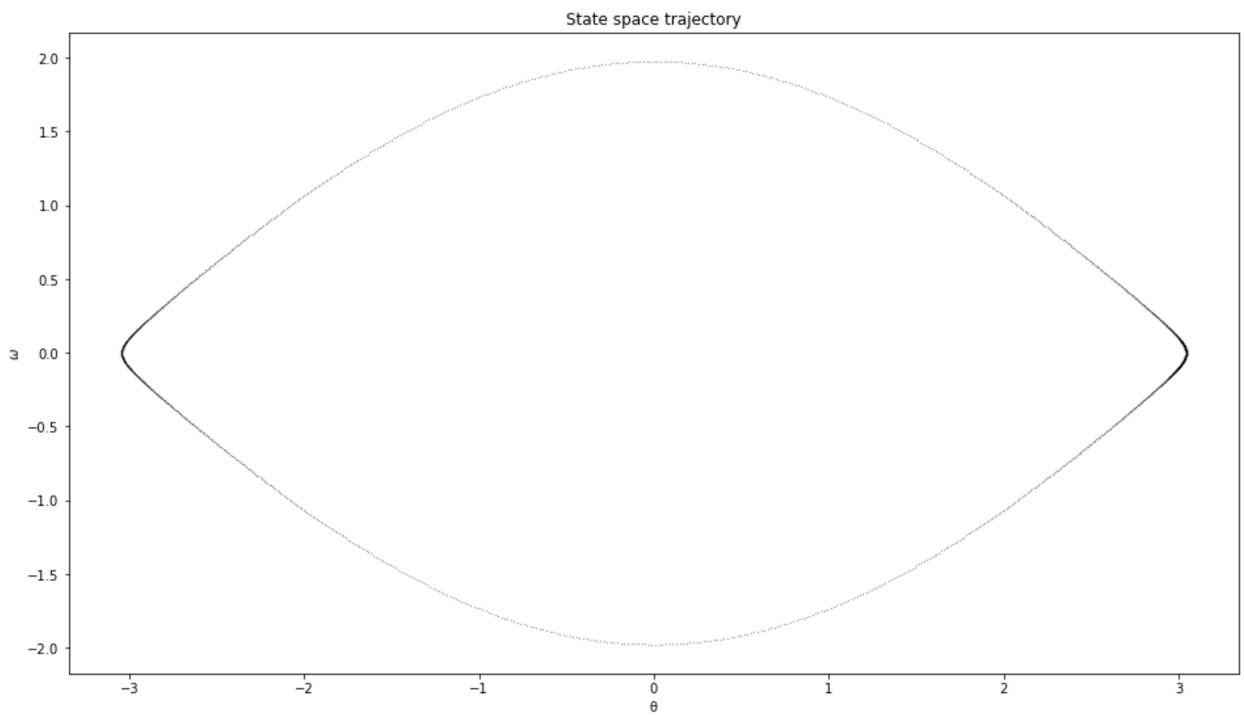


Fig1: State Space Trajectory for IC [θ, ω] = [3, 0.1] m = 0.1 Kg, l = 0.1m, $\beta = 0$, $\alpha = A = 0$, $\Delta t = 0.005$

The initial condition [θ, ω] = [3, 0.1] is near the unstable fixed point [θ, ω] = [π, 0], This path is comparable to taking up a pendulum and lowering it while standing vertically but not quite straight up. The pendulum returns to the same height it was dropped at and comes back, oscillating endlessly in an undamped and undriven system. The system won't move away from that point on its own, but if it is disturbed, it will travel away from that point and not return.

b. Initial Condition : **[θ, ω] = [0.01, 0]**

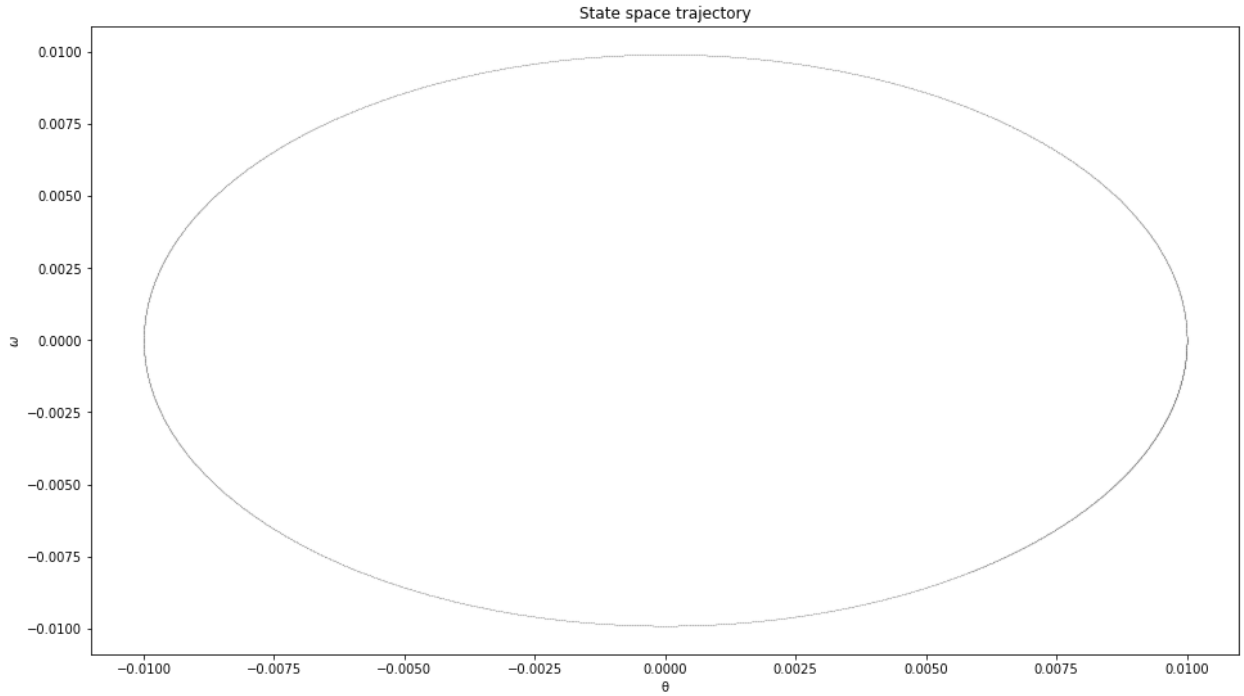


Fig2: State Space Trajectory for IC [θ, ω] = [0.01, 0] m = 0.1Kg, l = 0.1m, β = 0 , α = A = 0, Δt = 0.005

Compared to the trajectory depicted in Fig 1, which is huge and shaped like a diamond with smooth edges, this one is far smaller and more completely elliptical.

The difference between the shapes mostly is because of the runge-kutta algorithm that we use, which takes in more steps around the corners, and also because of the length of the steps taken, as the ranges of parameter space that we are dealing with in Fig1 are magnitudes times larger than the steps taken in Fig2.

3. State Space Portrait of Undamped Pendulum:

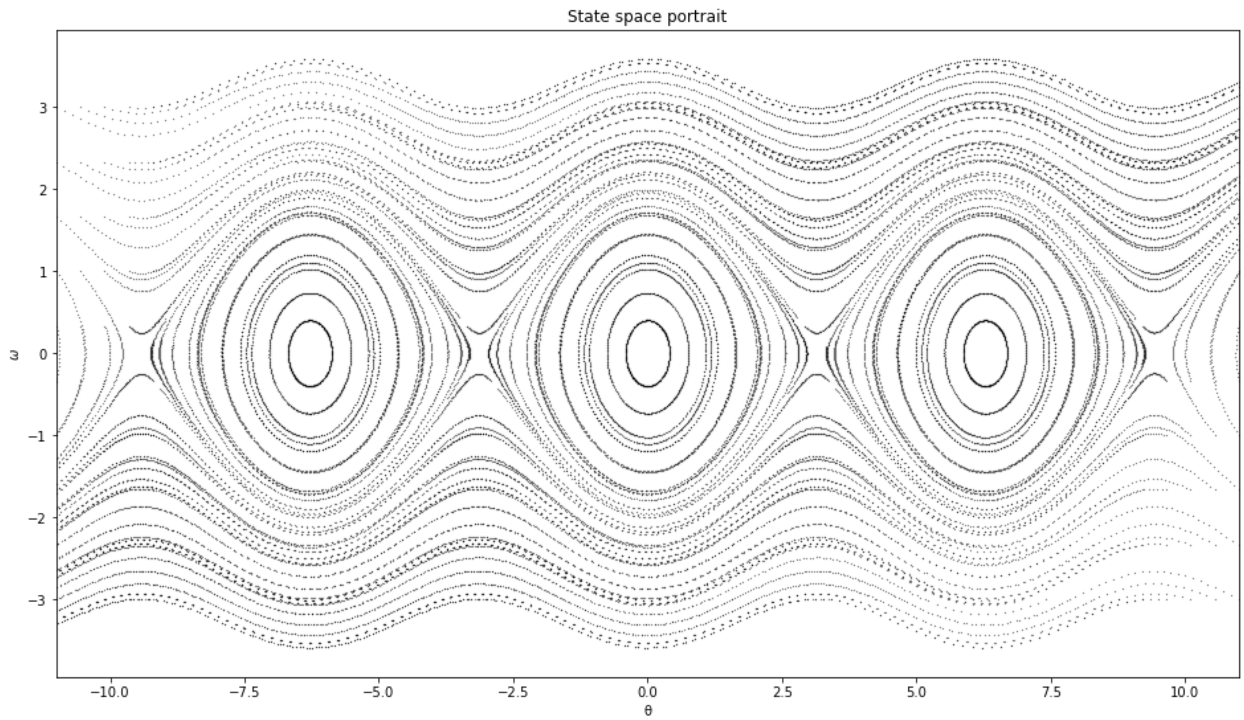


Fig3: State Space Portrait m = 0.1 Kg, l = 0.1m, β = 0 , α = A = 0

4. State Space Portrait of damped Pendulum:

m = 0.1 Kg, l = 0.1m, **β = 0.25**, and α = A = 0

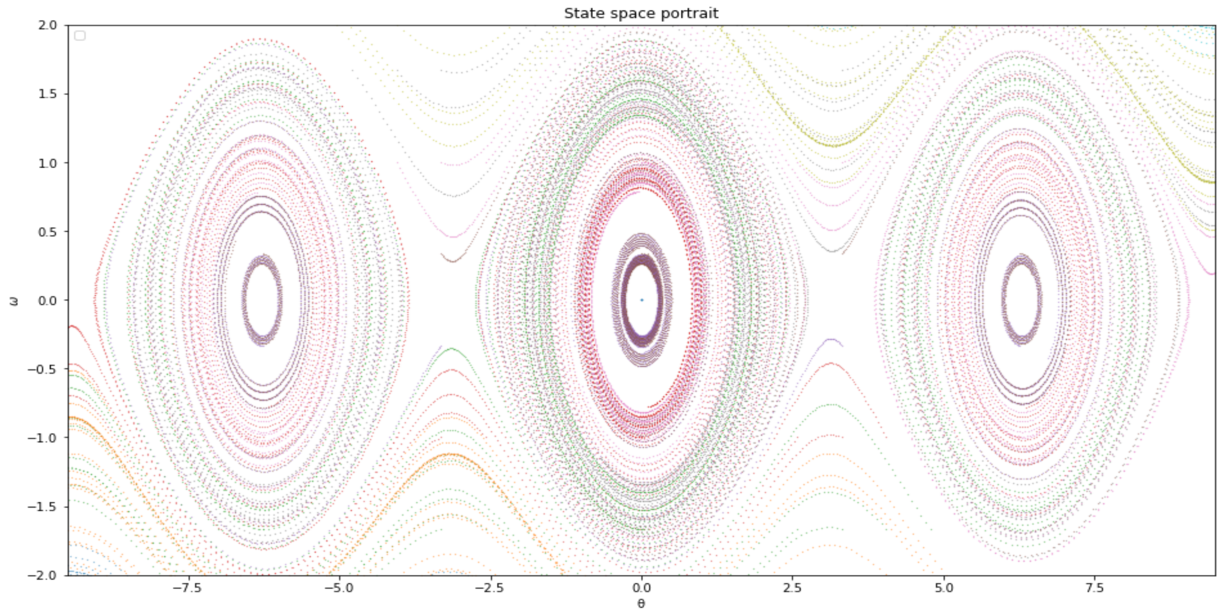


Fig4: State Space Portrait m = 0.1 Kg, l = 0.1m, **β = 0.25** , α = A = 0

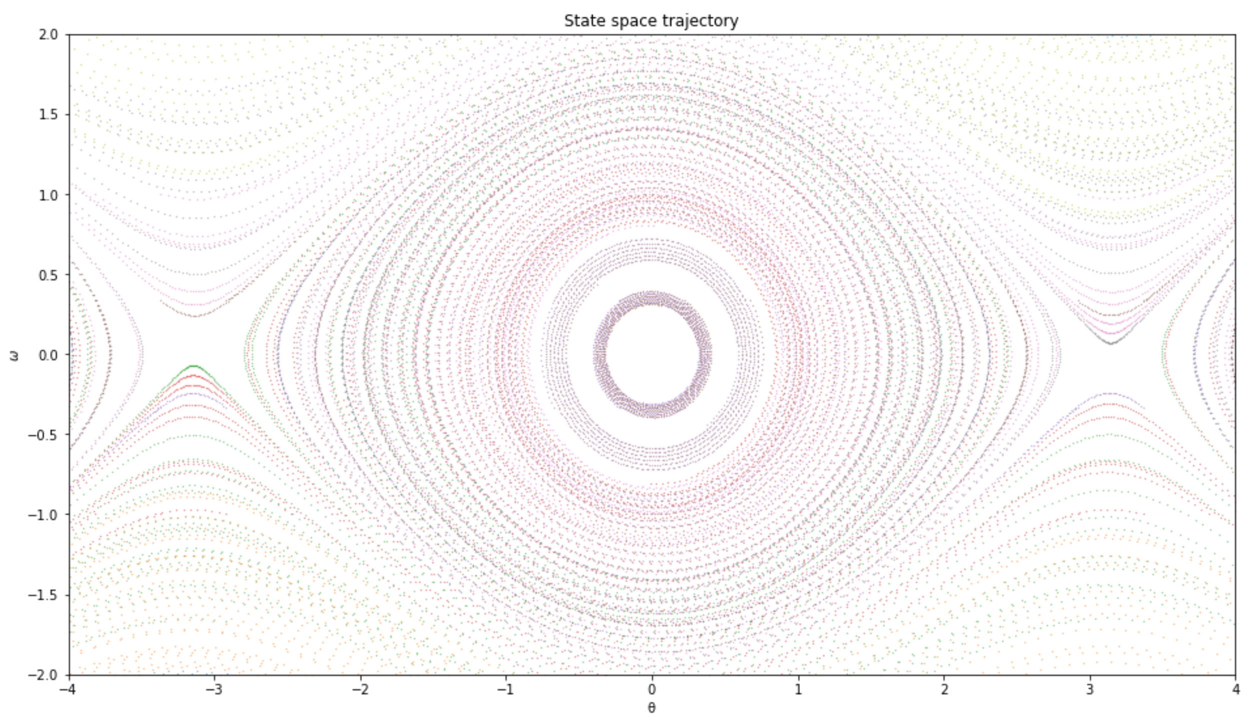


Fig5: Zoomed version of Fig4, State Space Portrait m = 0.1 Kg, l = 0.1m, **β = 0.25** , α = A = 0

m = 0.1 Kg, l = 0.1m, **β/m = 0.25**, and α = A = 0

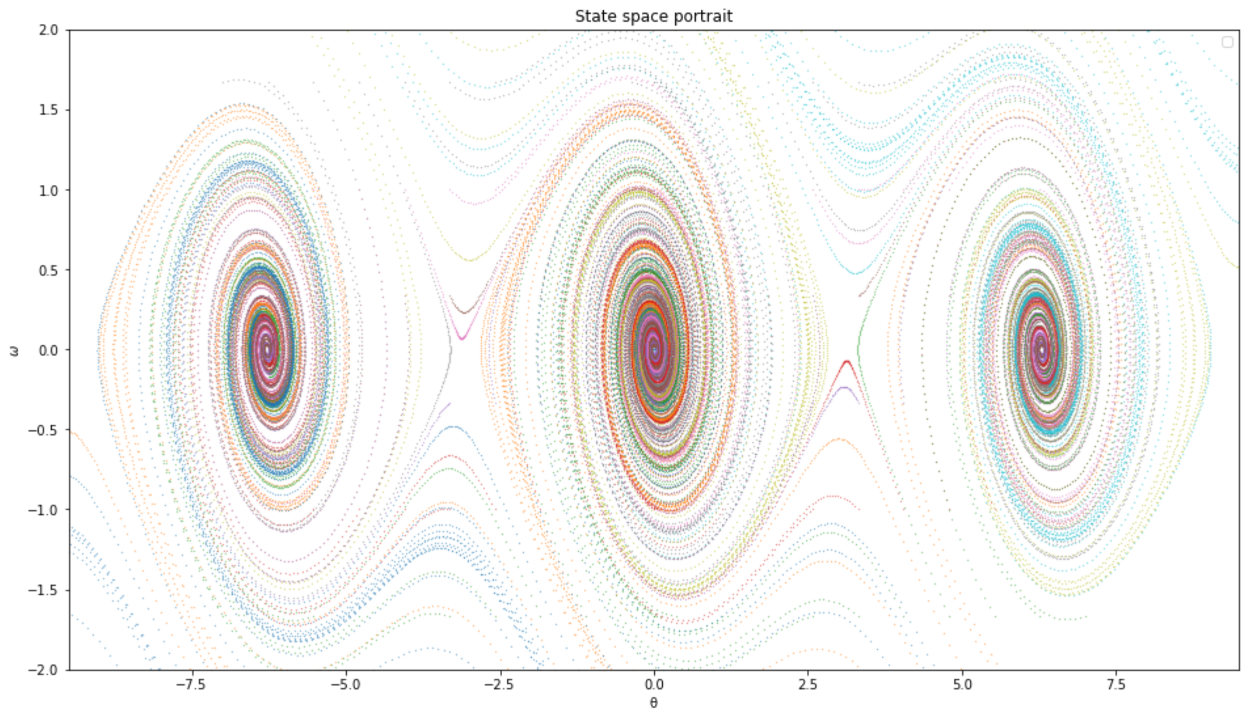


Fig4: State Space Portrait m = 0.1 Kg, l = 0.1m, **β/m = 0.25** , α = A = 0

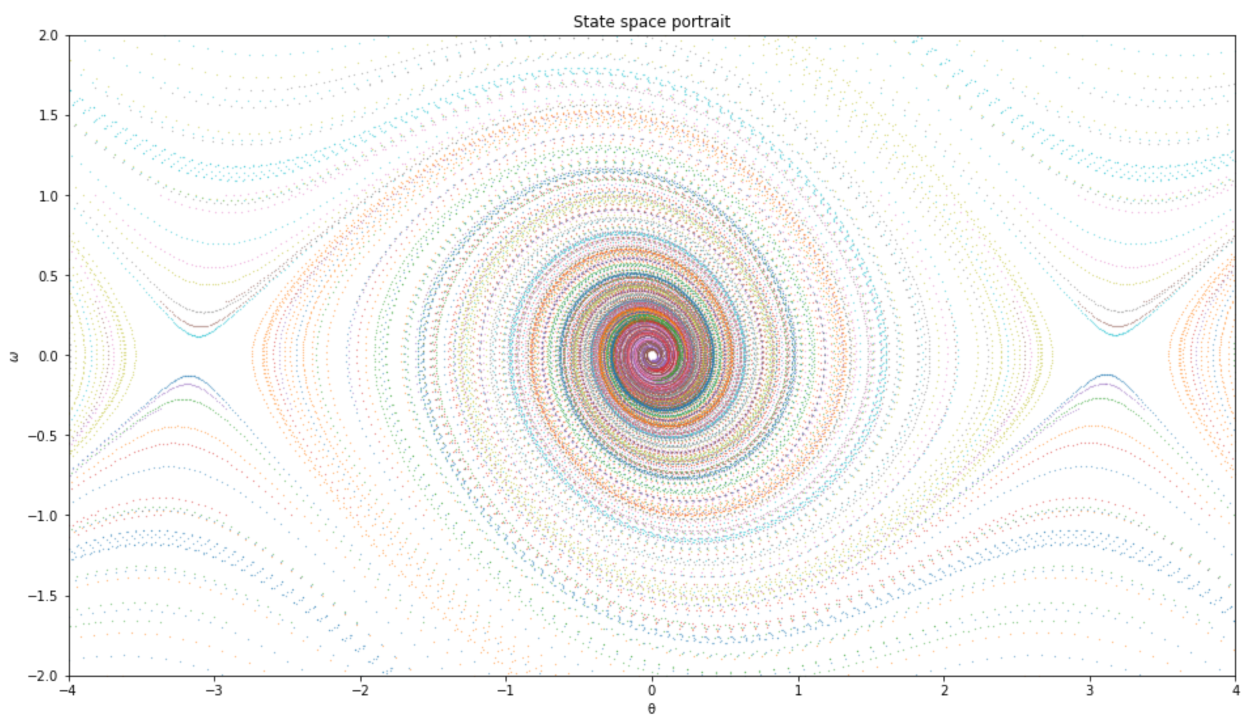


Fig5: Zoomed version of Fig4, State Space Portrait m = 0.1 Kg, l = 0.1m, **β/m = 0.25** , α = A = 0

Fig5 shows how the damping mechanism affects the system. Trajectories spiral into the center, when in the undamped system they would have looped endlessly or swirled over the top. Of course, the position when the pendulum is halted and pointing straight down is represented by the center.
This fixed point is stable at [nπ/2, 0] = [0, 0].

As beta grows, the plot's elliptical elements are expanding vertically, skewing to the right, and generally becoming less oval., the system is also visibly getting less dense towards the outside while remaining dense at the central points. This system's physical dynamics must be deteriorating since it is increasingly likely to deviate from its orbits. As there is more damping, we would expect the trajectory to collapse to the stable state faster, which means there would be fewer inward spirals and thus, we see sparse spirals at higher theta.
The trajectories of the system would have to become more elliptical and less skewed if the beta were to be lower. As there is less damping, it would take more spirals(less time) before collapsing to the center. With smaller beta values, there is a more even spread in the system's density from the inside to the periphery.

m = 0.1 Kg, l = 0.1m, **β/m = 0.5**, and α = A = 0

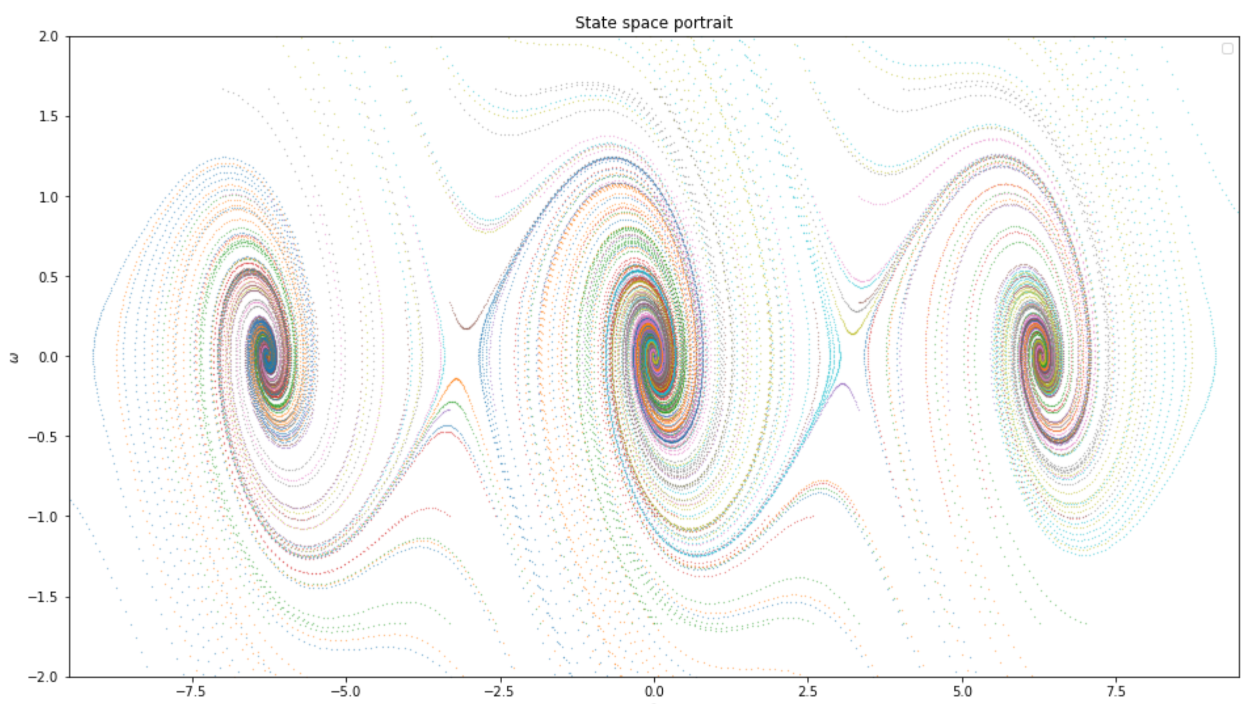


Fig6: State Space Portrait m = 0.1 Kg, l = 0.1m, **β/m = 0.5** , α = A = 0

5. Θ modulo 2Π

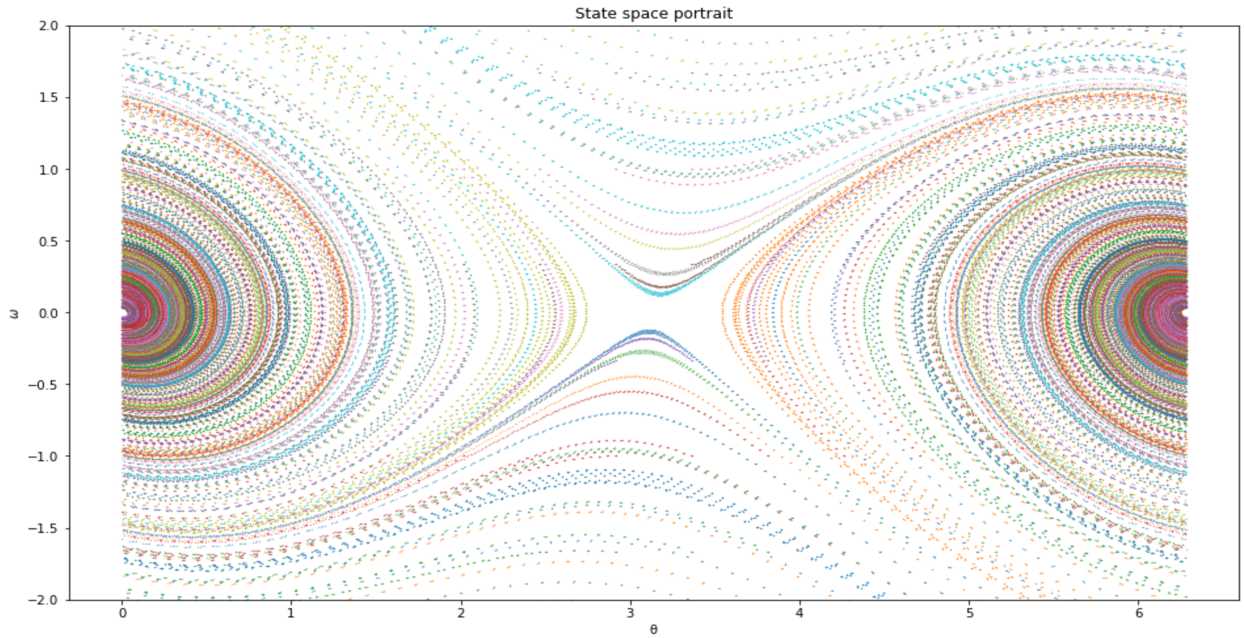


Fig7: Θ mod 2π - State Space Portrait m = 0.1 Kg, l = 0.1m, **β/m = 0.25** , α = A = 0

Plot appears zoomed in / truncated along the $(0,\ 2\pi)$ horizontal (Θ) axis. An unstable fixed point has now appeared in the middle. This position is equivalent to a pendulum pointing straight up, not falling in either direction, but remaining stationary.

6. Damped and Driven Pendulum

When you turn on the drive, the system might radically alter. Things remain about the same when the amplitude A is still extremely tiny, i.e .01, and alpha varies from around 0 to the natural frequency. However, as A matures, things begin to shift, and alpha begins to have a greater influence on the centers and saddles. By A =.6, the left and right ellipses have changed form and shape, with the left becoming more compact and dense and the right becoming larger and more spread out, with lower alpha values making the centers thicker than higher alpha values. However, when alpha exceeds 80% of the natural frequency, the trajectory becomes jumbled (overlaps) at the ellipses' centers.

Changes in alpha have less of an effect on how the system operates than changes in amplitude A. When A exceeds.9, the dynamics become rather weird, with overlapping and ambiguous trajectories that begin to take on a different overall phase picture shape.

One of the chaotic state space portraits is seen for **α = 7.5, A = 1**

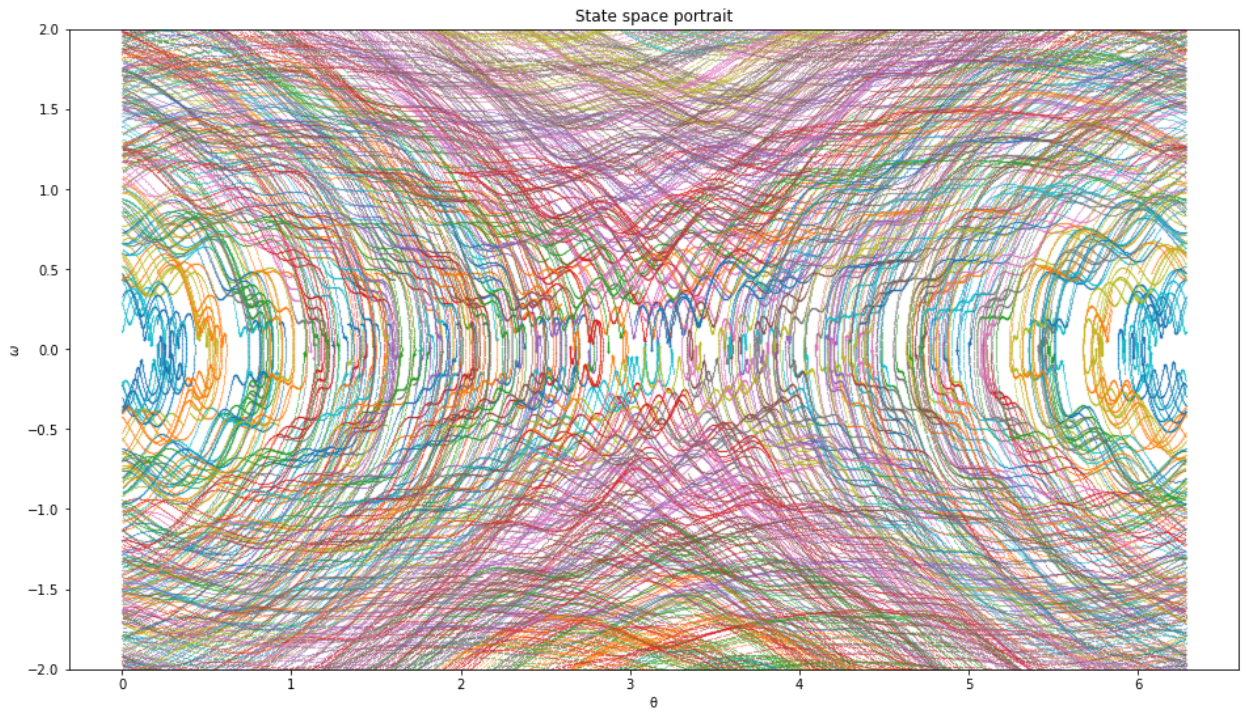
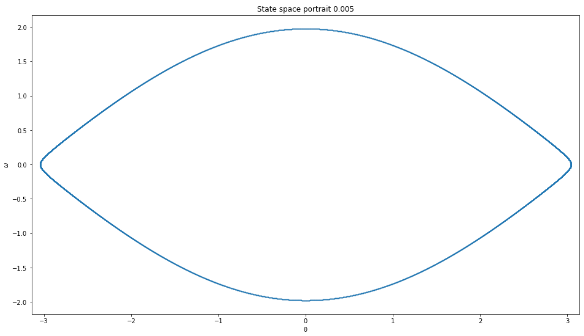


Fig7: θ mod 2π - State Space Portrait m = 0.1 Kg, l = 0.1m, β = 0.25 , **α = 7.5, A = 1**

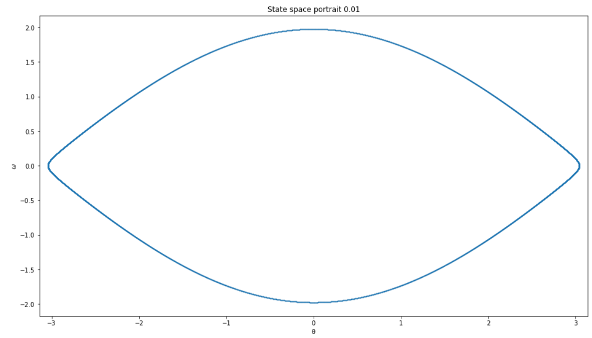
7. Effects of Δt on an undriven and undamped pendulum

For smaller values of 0.005 < Δt < 0.1, we see the complete trajectory describing the actual state space of the never-ending pendulum as there is no damping involved. The plot gets denser from Δt = 0.2, where the trajectory follows an inward spiral path, which states that the Runge-Katta algorithm is deviating from the actual trajectory. This effect of spiraling gets more pronounced as the time step increases (0.2 < Δt < 0.4). At around 0.4, we observe that the points get denser around the center while it gets sparse when moved radially outward. We also observe that there are concentric elliptical-like structures with dense points at regular intervals along those elliptical paths. They seem more like Fractal structures. As the time step is further increased(0.4 < Δt < 0.7), we see the effect of more pronounced, dense points formation around the center with the concentric elliptical paths being pushed down near the center and sparse points on the outside of the dense cloud. At around Δt = 0.7, we see the structure of a spiral resembling the structure when a 2d surface is whirled around the center, and this effect is more pronounced as Δt increased to 0.9.

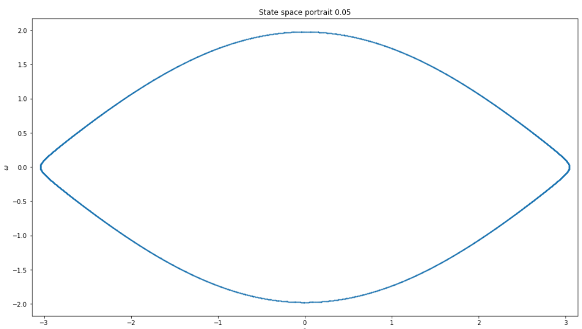
The plots for m = 0.1 Kg, l = 0.1m, β = 0, and α = A = 0 and varying Δt are shown below:



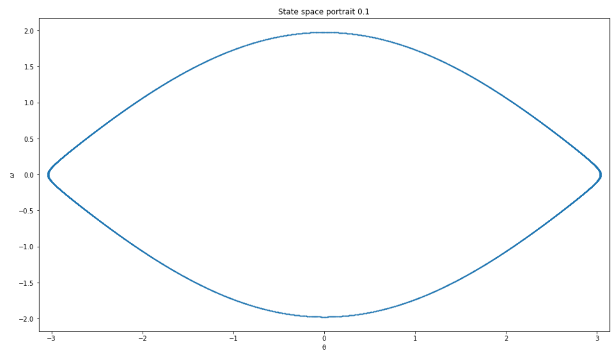
Δt = 0.005



Δt = 0.01

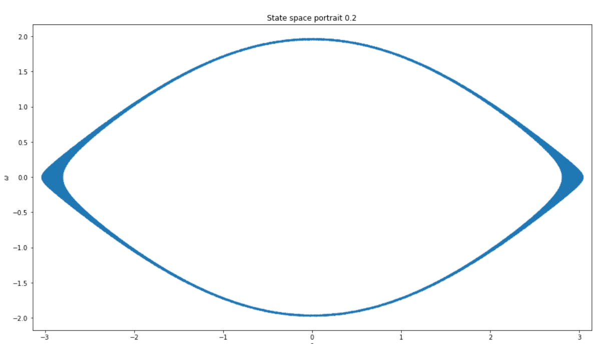


Δt = 0.05

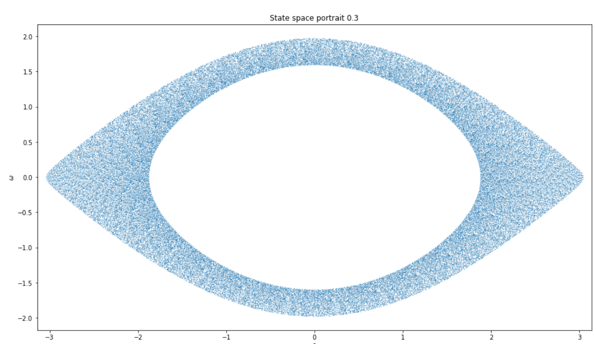


Δt = 0.1

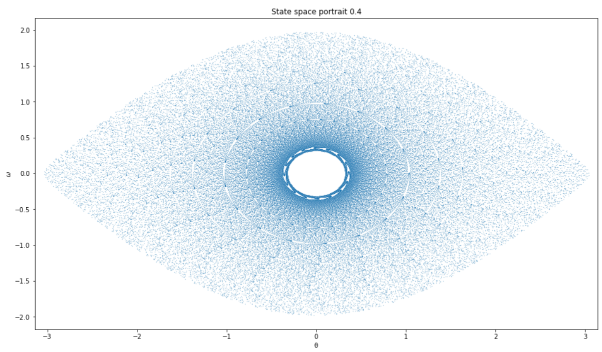
Δt = 0.2



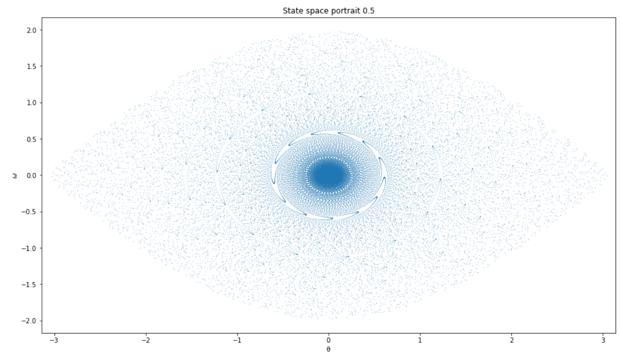
Δt = 0.3



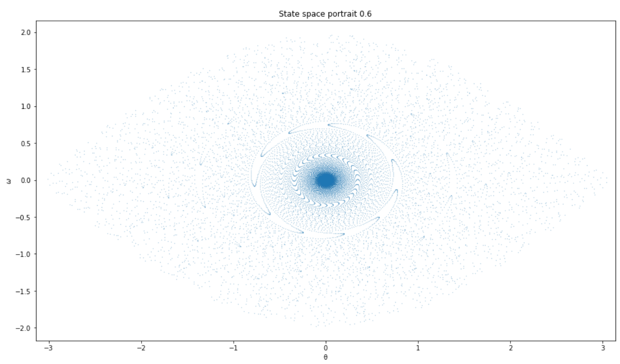
Δt = 0.4



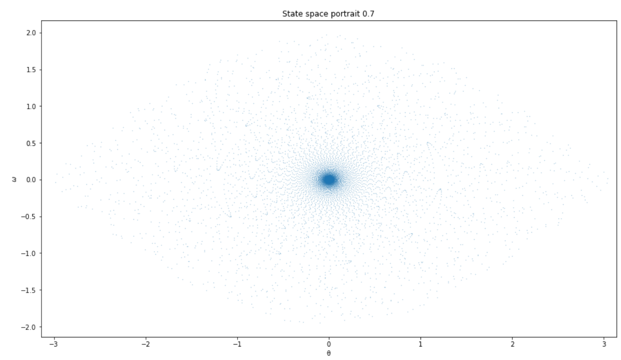
Δt = 0.5



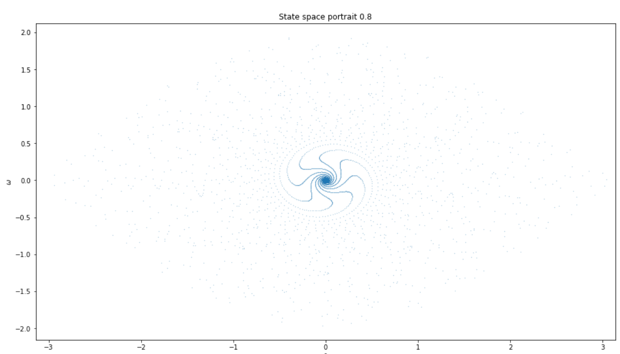
Δt = 0.6



Δt = 0.7



Δt = 0.8



Δt = 0.9