CSCI 5446 - Chaotic Dynamics

Problem Set 9 - Solutions

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2. Wolf Algorithm on Lorenz System:

The Lyapunov exponent or Lyapunov characteristic exponent of a dynamical system is a quantity that characterizes the rate of separation of infinitesimally close trajectories. Ideally, there are n different Liapunov exponents for an n-dimensional system and the most positive one usually dominates the others as the time tends to infinity.

Wolf's algorithm gives the most prominent positive Lyapunov exponent.
The initial condition taken to generate the dataset is [-13, -12, 52]

For the Choatic trajectory, generated with the Lorenz system (a = 16, r = 45, and b = 4), using Wolf's algorithm the largest Lyapunov experiment is around **2.145.** From the paper, "Determining Lyapunov exponents from a time series", we see that the largest Lyapunov exponent for r = 45.92 is 2.16, so for r =45 and all the parameters same, I would expect a number close to 2.16.

For the non-chaotic trajectory generated with the Lorenz system (a = 16, r = 18, and b = 4), using Wolf's algorithm the largest Lyapunov experiment is 0. This makes sense because the Lorenz attractor with r = 18 follows a long curve from the starting point to a single attractor where it makes its only spiral inward, where the original trajectory is primarily that spiral and the z trajectory where neighbors are sought to be that long curve that eventually makes its way into the spiral.

For Chaotic Systems, we expect to have at least one positive Lyapunov exponent and all the Lyapunov exponents are non-positive for non-chaotic systems. Thus, the above-calculated Lyapunov exponents are expected.

3. Wolf Algorithm on Lorenz System:

For Chaotic Trajectory, doing a mutual, the tau picked is at 108 (108 * 0.001 = 0.108 seconds) and the minimum m chosen was six after since the box dimension is three for Lorenz System, according to Takens theorem. The m range is taken from 4 to 8. *lyap_k foo -m4 -M8 -d108 -o pr9_3a.out* and from the output, the Lyapunov exponent is `0.0027071.`

It is very surprising that the Lyapunov exponent from Wolf's algorithm is around 2 and the value from this experiment is nearly 1000 times less, but still, the exponent is positive which would describe the chaotic nature.

For Non-Chaotic Trajectory, doing a mutual, the tau picked is at 159 (159* 0.001 = 0.159 seconds) and the minimum m chosen was six after since the box dimension is three for Lorenz System, according to Takens theorem. The m range is taken from 4 to 8. *lyap_k foo -m4 -M8 -d159 -o pr9_3b.out* and from the output, the Lyapunov exponent is `-0.00041394.` The Lyapunov exponent is nearly zero agreeing with Wolf's algorithm which is strengthening the fact it is a non-chaotic trajectory.

4. Lyapunov Experiments using Variation

a.

Integrating the variational system and using the final eigenvalues to find λ_i, we find $\lambda_1 = 0.013334$, $\lambda_2 = 0.012097$, $\lambda_3 = 0.011929$. This is a numerical limit at t = 10 seconds. This approach does not produce a credible value since we are approximating with the 10000th step over a 0.001s timestep, which is only 10 seconds into the trajectory while the actual value needs the limit as time goes to infinity, and 10 seconds is not even close to infinite time.

b.

The λ_i 's approach to the actual values when the time gets to infinity. So theoretically, the values change as the integrations time change.
I repeated the calculation using longer and shorter integration times for the variational equation. For shorter integration times, the Lyapunov exponents varied widely and were not very reliable. For longer integration times, the Lyapunov exponents did not change significantly, which is expected since they represent the long-term behavior of the system.

It is important to choose a suitable integration time that balances the need for accuracy with the avoidance of numerical issues, such as ill-conditioned matrices. In this case, a 10000-point integration run provided a good compromise between accuracy and numerical stability.I also varied the initial conditions and found that the Lyapunov exponents were different but still within the expected range for the Lorenz system. This confirms that the Lyapunov exponents are a property of the system itself and not just a consequence of the specific initial conditions.