CSCI 5446 - Chaotic Dynamics

Problem Set 2 - Solutions

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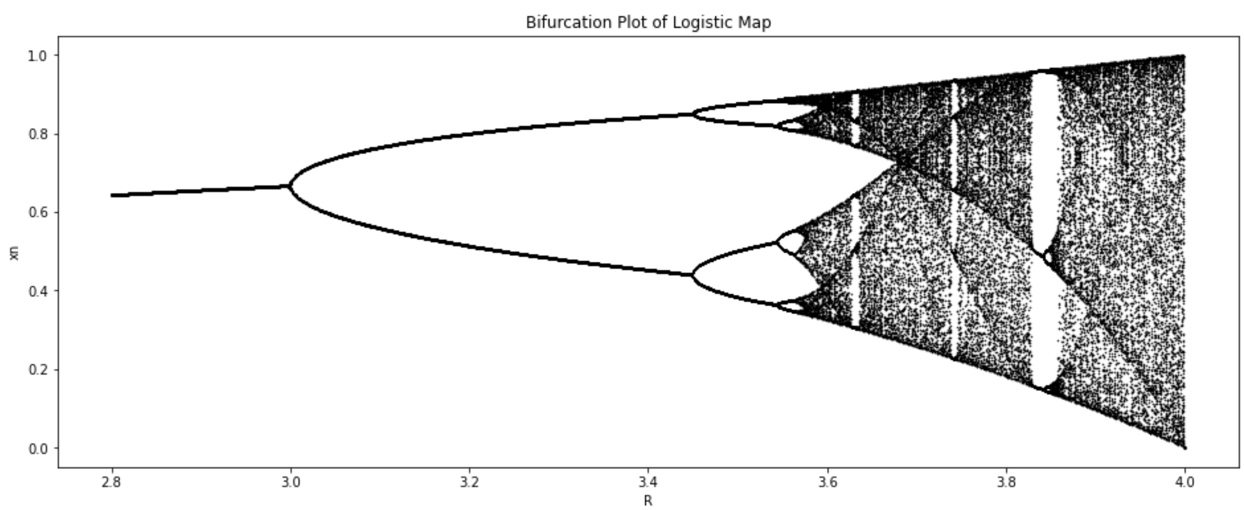
1. Bifurcation Plot of the Logistic Map:

$x_{n+1} = rx_n(1 - x_n)$ where r is the parameter

x_0 is the initial condition

For every r, ranging from 2,8 to 4 , 1000 iterations of the map are computed. The first 900 points are discarded for each run of the map. The interval between the successive values of r is 0.001.



2. Calculating Feigenbaum number (using Logistic Map) :

After adding the routine to the programme to identify the bifurcation points, the corresponding first four r values are 3.000, 3.446, 3.542, 3.564 and 3.566.
If $a_1, a_2, \ldots a_n$ (n > 0) is the sequence of bifurcation points on the same cascade, the

Feigenbaum number is calculated as

$$F = \frac{a_{k+1} - a_k}{a_{k+2} - a_{k+1}}, \quad 1 <= k <= n$$

Calculating Feigenbaum number from the period doubling r values,

Period	Bifurcation 'r'	Approx F
2	3.000	N/A
4	3.446	N/A
8	3.542	4.645
16	3.562	4.800
32	3.566	5.000

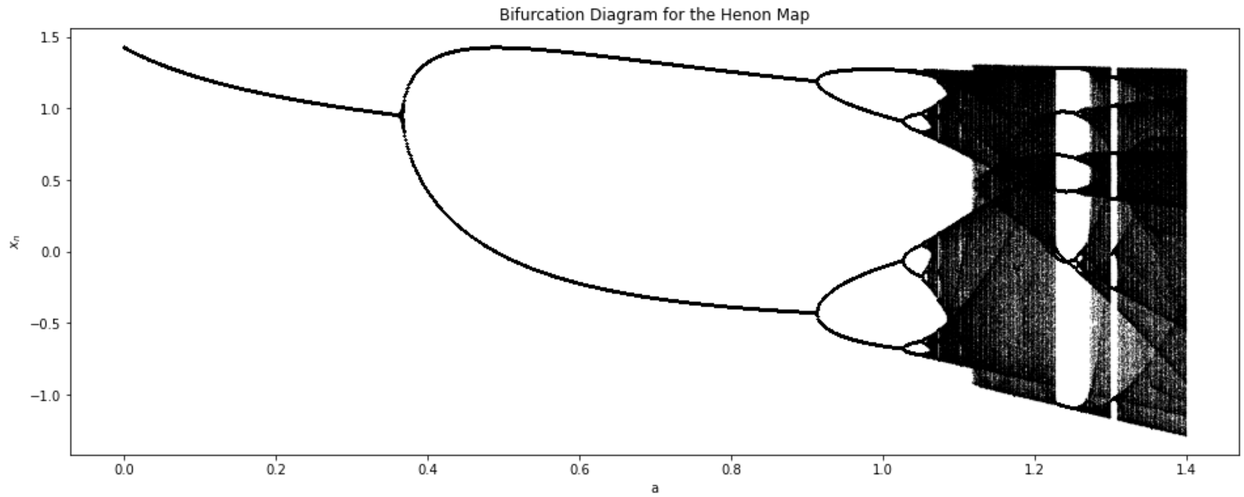
We know that the actual value of F is 4.667. but from the above table we notice that it diverges from the original value. Also, observe how the approximated value diverges from the actual we go down the table. This can be reasoned from the fact that higher resolution is needed for r as the gaps between bifurcations get smaller. More resolution in the interval between successive values of r would improve the approximate value of the Feigenbaum number calculated.

3. Calculating Feigenbaum number (using Henon Map) :

Bifurcations are found at 'a' values of 0.367, 0.907, 1.021, 1.053 and 1.057

Period	Bifurcation 'a'	Approx F
2	0.367	N/A
4	0.907	N/A
8	1.023	4.655
16	1.047	4.834
32	1.051	4.800

The bifurcation diagram of Henon map is



4. Comparing Feigenbaum number calculated using Henon Map and Logistic Map :

The outcomes of the Henon map and the logistic map are pretty similar. This is not unexpected given that the logistic and Henon maps are all 1D maps with quadratic maximums and the Feigenbaum number holds for all of them. We should observe comparable outcomes. The higher period doubling cascade, which approximates the Feigenbaum number to three decimal places, is of special importance.