CSCI 5446 - Chaotic Dynamics

Problem Set 8 - Solutions

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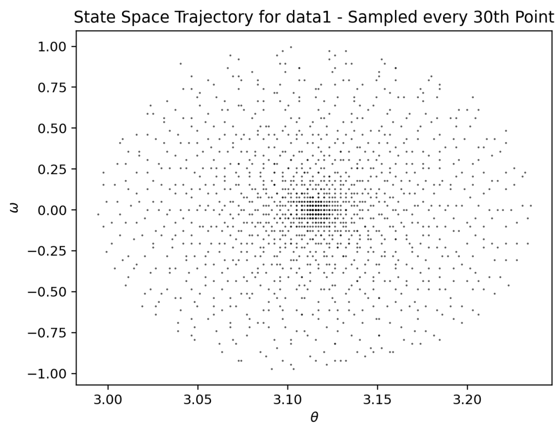
1. State Space Trajectory on *data1* :



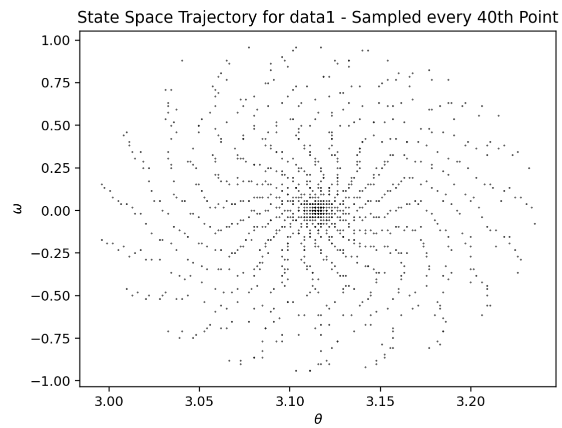


Fig1: State Space Trajectory plotted for *data1* by plotting every 30th point and 40th point

The plot's form is not a neat spiral, despite being circular and spiraling into the center. It's a bit jagged, crossing over itself and losing symmetry in the middle. Yet, both of these plots are highly organized, with the first being more compact and rounded, and the second, with a considerably higher down-sampling rate, displaying more of a spiral towards the center. The goal was to utilize a down-sample rate that was not so low that the points were completely reliant on each other, but also not so high that the points sampled were completely independent of each other.

There are two clear elements that can be influencing this behavior. Firstly, because this data originates from an actual experiment, the accuracy of the observations might be off. Each value in the data set has a precision of 16 digits. It seems questionable that the device used to measure is indeed that exact. Second, the mathematical operations that computers carry out pose a challenge to numerical differentiation. When using a smaller step or a more exact formula, the machine is forced to divide by a lower amount, raising the possibility of round-off error.

2. Delay Coordinate Embedding :

a. τ = 0.15, m = 7, j = 0, k = 2 on data2

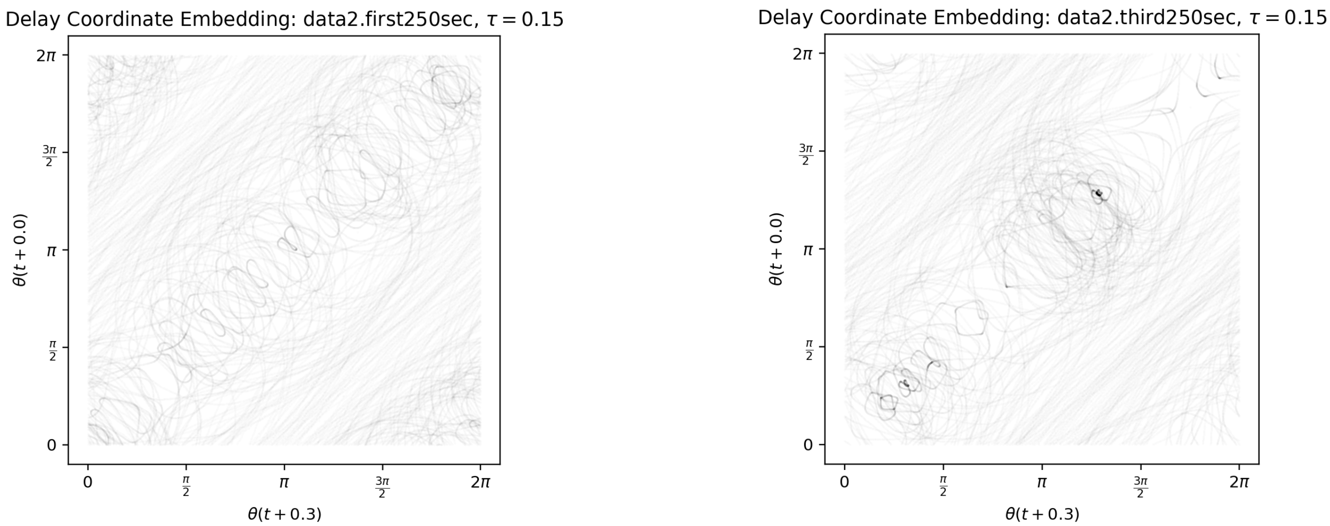


Fig2: Delay Coordinate embedding for τ *= 0.15, m = 7, j = 0, k = 2 on data2.first 250 seconds and third 250 seconds*

The delay coordinate embedding is plotted for the four chunks of the data2 which correspond to the first 250 seconds, second 250 seconds, third 250 seconds, and fourth 250 seconds. From all the plots, we observe that the trajectory is Chaotic Trajectory as the trajectory is tangled like a nest. As time passes, it's unclear where the trajectories will go.

b. *m = 7 on data3*

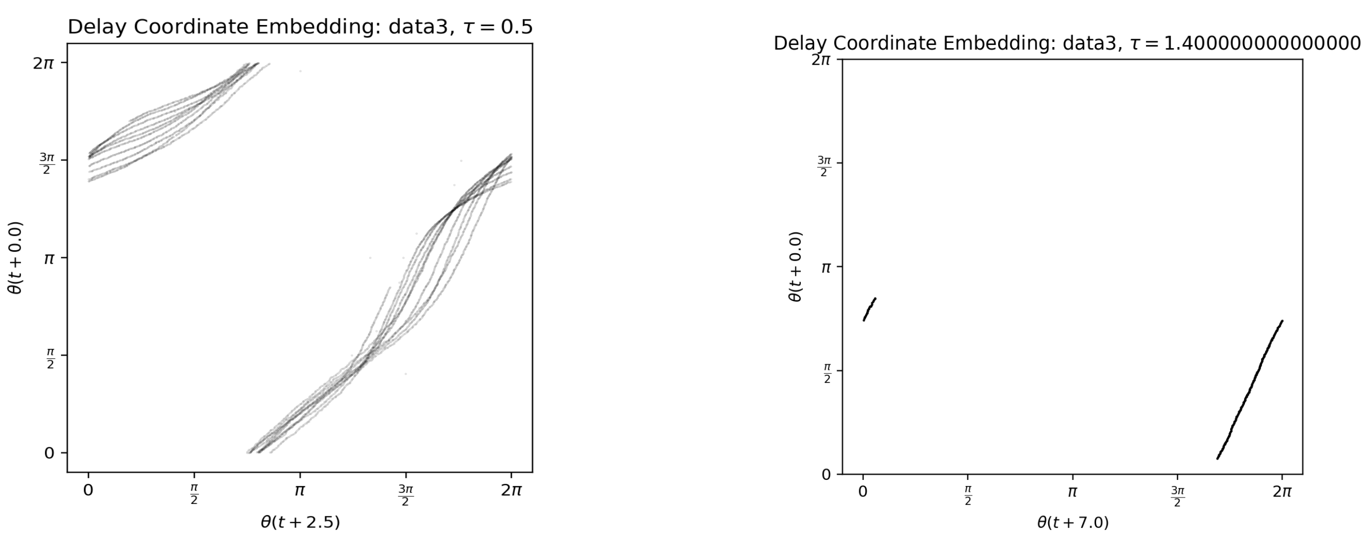


Fig3: Delay Coordinate embedding $\theta(t)\ vs\ \theta(t\ +\ 5\,\tau)$ for τ = 0.5 and τ = 1.4

From the plots, it seems as if the system is oscillating back and forth.
At τ = 0.01, the reconstruction is more compact but as the τ increases, it starts to unfold till τ = 0.5, but as it is further increased, it starts folding again as the trajectory is twisted across the trajectory axis. This can be seen as the trajectory unfolds more and more, like it is twisted across more as τ increases. At around τ = 1.4, it is so much twisted as if it is a thin line and as τ is further increased, the plot is then left blank as the system returns to the same condition without recording anything in between.

We also see that, as the τ is increased, trajectories twist and folded more and more and shift towards the right with a modular operation top of the displacement towards the right.

3. Thought Experiments:

a. Box Dimension requirements on Taken's Theorem

In (θ, ω) , the driven pendulum is a nonautonomous system. As a result, the dimension of a driven pendulum should be 3 (θ, ω, t). According to Takens' theorem, the system can be embedded in m dimensions if m is more than twice the attractor's box-counting dimension. Because the box-counting dimension can never be more than three, m = 7 is always enough to incorporate the driven pendulum system. In (θ, ω) , the undriven pendulum is an independent system. Because its box-counting dimension cannot be larger than 2, m = 5 would suffice for the undriven pendulum system.

b. Effect of dimensions used for reconstruction from Taken's Theorem

Given that the box dimension of a driven pendulum is 3, from Taken's Theorem, the trajectory would not accurately represent the dynamics of the system if m = 2 is used.
Although Taken's Theorem's sufficient condition is met if m = 25, the reconstructed trajectory would not produce an exact or interpretable plot because the noise might be amplified.

c. Effect of τ used for reconstruction from Taken's Theorem

If $\tau = 10^{-6}$ is used for reconstruction, there would be insufficient change and would only be able to see a diagonal line along $\theta(t + k\tau) = \theta(t)$.

On the contrary, if $\tau = 10^{6}$ is chosen, we might see points scattered randomly unless the natural period of the pendulum is divided 10^{6} evenly, in which case you'd see a diagonal line because it's sampling at the natural frequency.

4. TISEAN'S *mutual*:

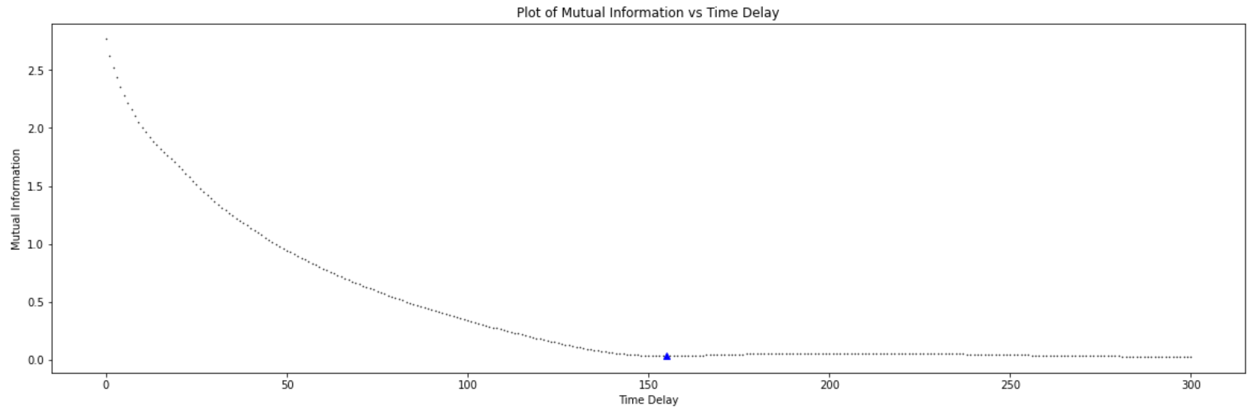


Fig4: Plot of Mutual Information vs Time Delay ($\Delta t \ = \ 0.002s$)

The blue triangle point is where Mutual Information at time delay is greater than the one before it and the delay at which the minimum is 155., *data2* sampled at 0.002s. Thus, 155*.002s = **0.31s** is the time delay

108 * 0.001 = 0.108 seconds

in seconds at which the first minimum occurs.

The command used: ***mutual data2.first250sec -o pr8_4a.txt -D 300***

5. TISEAN'S *false-nearest* :

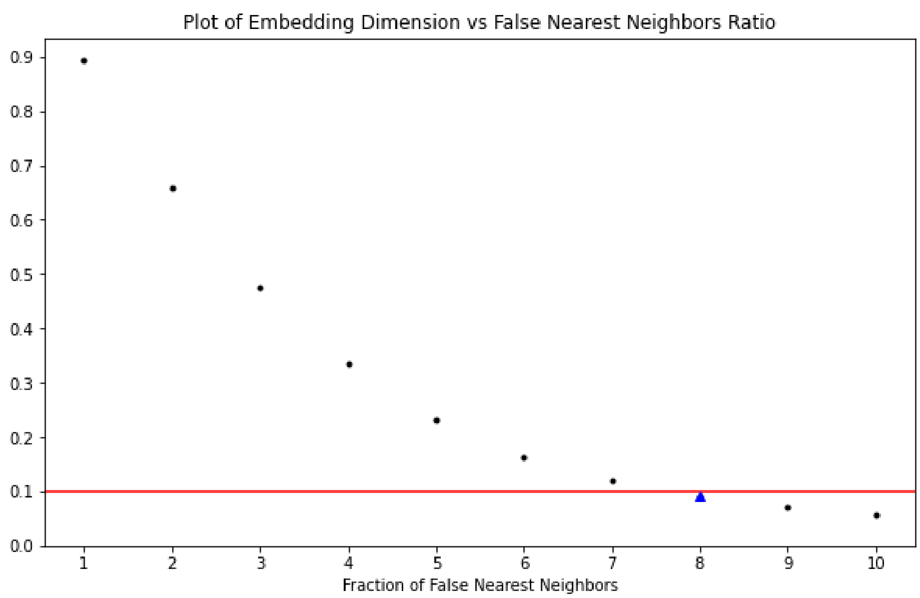


Fig5: Plot of Embedding Dimension vs False Nearest Neighbors ratio

The lowest value of embedding dimension just below 10% false neighbors is **8** and that point is marked as a blue triangle in Fig5 and the red line is the 10% false neighbors.
The command used: ***false_nearest data2.first250sec -M 0,10 -d 155 -o pr8_5.txt***