CSCI 5446 - Chaotic Dynamics

Problem Set 5 - Solutions

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2. Lorenz System

a. Chaotic Attractor

Given *a = 16, r = 45, b = 4*

initial condition (x_0) *= [-13, -12, 52]*

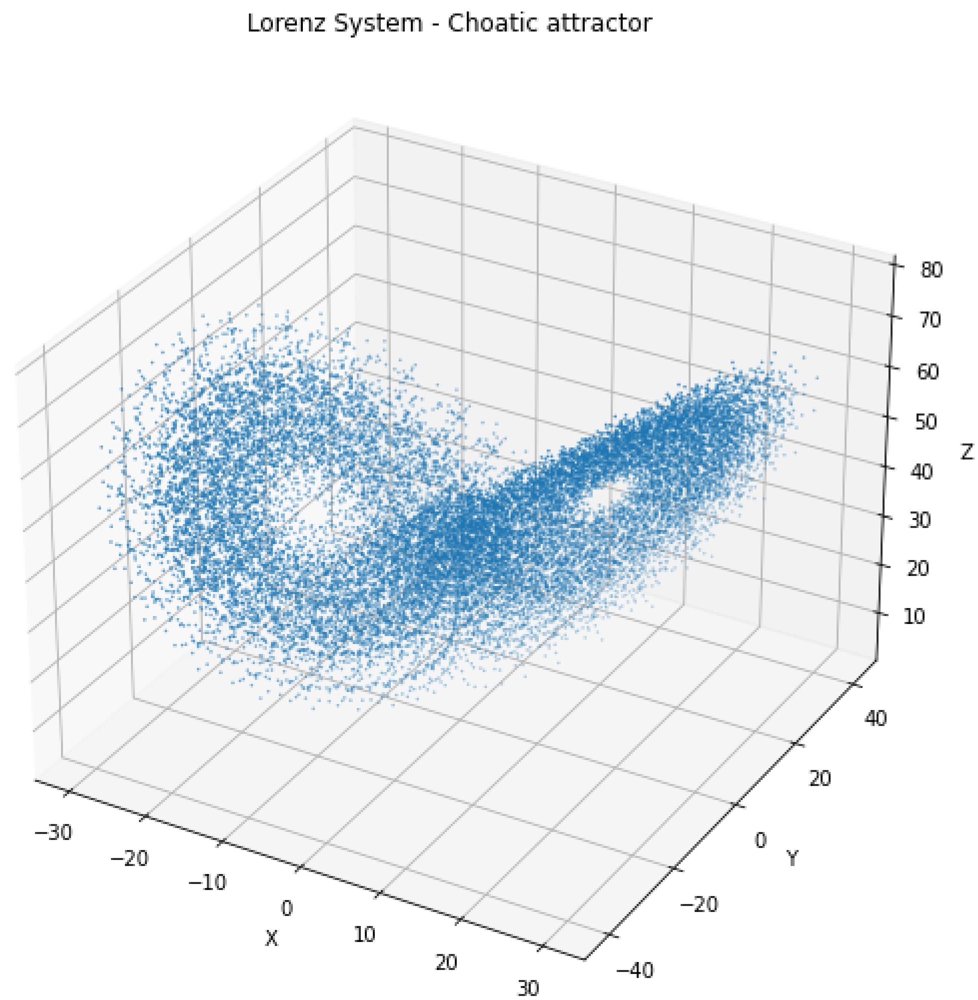


Fig1: Chaotic Attractor for Lorenz System for *a = 16, r = 45, b = 4, initial condition* (x_0) *= [-13, -12, 52] using Adaptive Runge-Kutta*

b. Adaptive and Non-Adaptive Runge-Kutta

Given *a = 16, r = 45, b = 4*

initial condition (x_0) *= [-13, -12, 52]*

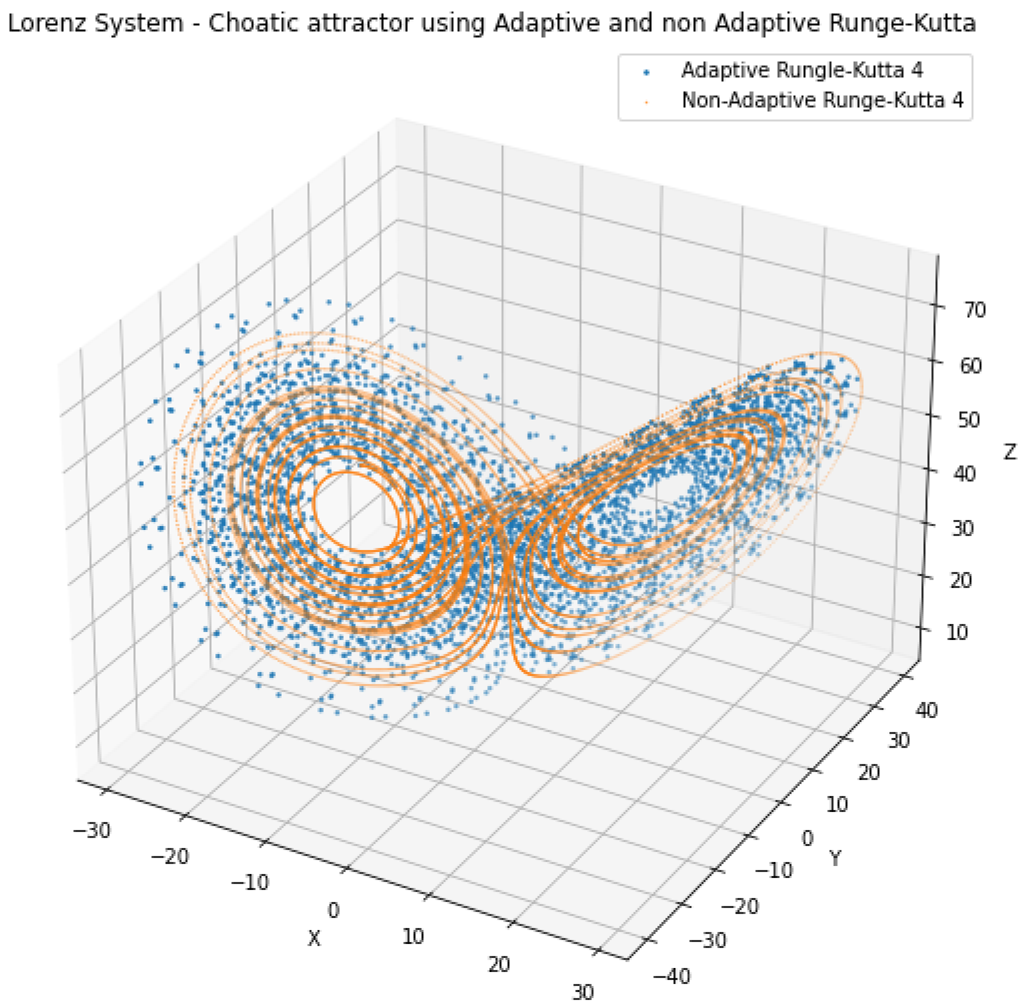


Fig2: Chaotic Attractor for Lorenz System for *a = 16, r = 45, b = 4, initial condition* (x_0) *= [-13, -12, 52]* *using Adaptive and Non-Adaptive Runge-Kutta*

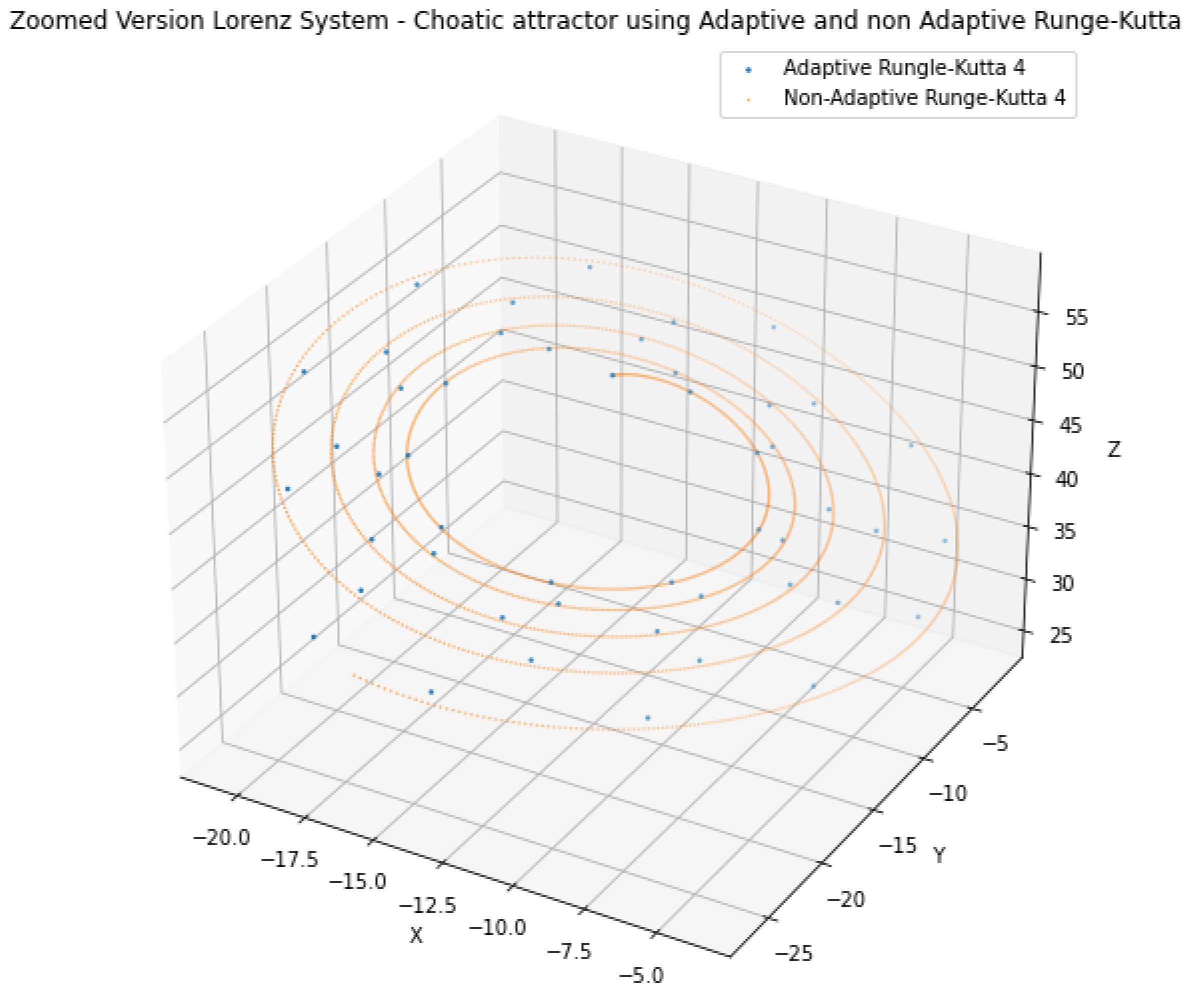


Fig3: Zoomed Version: Chaotic Attractor for Lorenz System for *a = 16, r = 45, b = 4, initial condition* (x_0) = *[-13, -12, 52]*

The two solutions are consistent when the adaptive Runge-Kutta 4th order approach is displayed (in blue) over the non-adaptive Runge-Kutta 4 (in orange) with a fixed time step. The blue dots representing the adaptive Runge-Kutta 4 solution follow a more jagged course than the non-adaptive orange trajectory, which is predicted, because the non-adaptive RK4 approach employs a tiny and set time step, resulting in a tightly curved path with closely spaced points.

From Fig3, On the other hand, the adaptive technique permits its time step to develop along pathways that do not change as quickly, resulting in more dispersed spots. The adaptive solution produces points that are closer together in time, where the curve is tighter and further apart when the curve expands. The Adaptive Runge-Kutta curve diverges somewhat from the Runge-Kutta 4 curve at the far end of the curve. The fundamental rationale for adopting an adaptive solution is the unequal spacing of points over time, which results in less cluttering of the plot.

c. Changing the r-value in Lorenz System

Changing the value of the parameter r in the Lorenz system has a significant impact on the behavior of the chaotic attractor. For, $0 < r < 1$, the system follows a single standout path that converges to a single fixed point. At r = 1, there is a bifurcation to two stable fixed points, and at r = 13.615, the system crosses a boundary between the two basins of attraction. The system spirals into one fixed point for r < 13.615, and into the other for r > 13.615. As r grows between 23 and 29.5 the spiral trajectory along the basin of attraction becomes much denser and the chaotic attractor becomes obvious once r exceeds 29.5.

3. Rossler System

Given *a = 0.398, b = 2, c = 4*
initial condition (x_0) *= [0.0001, 0.0001, 0.0001]*

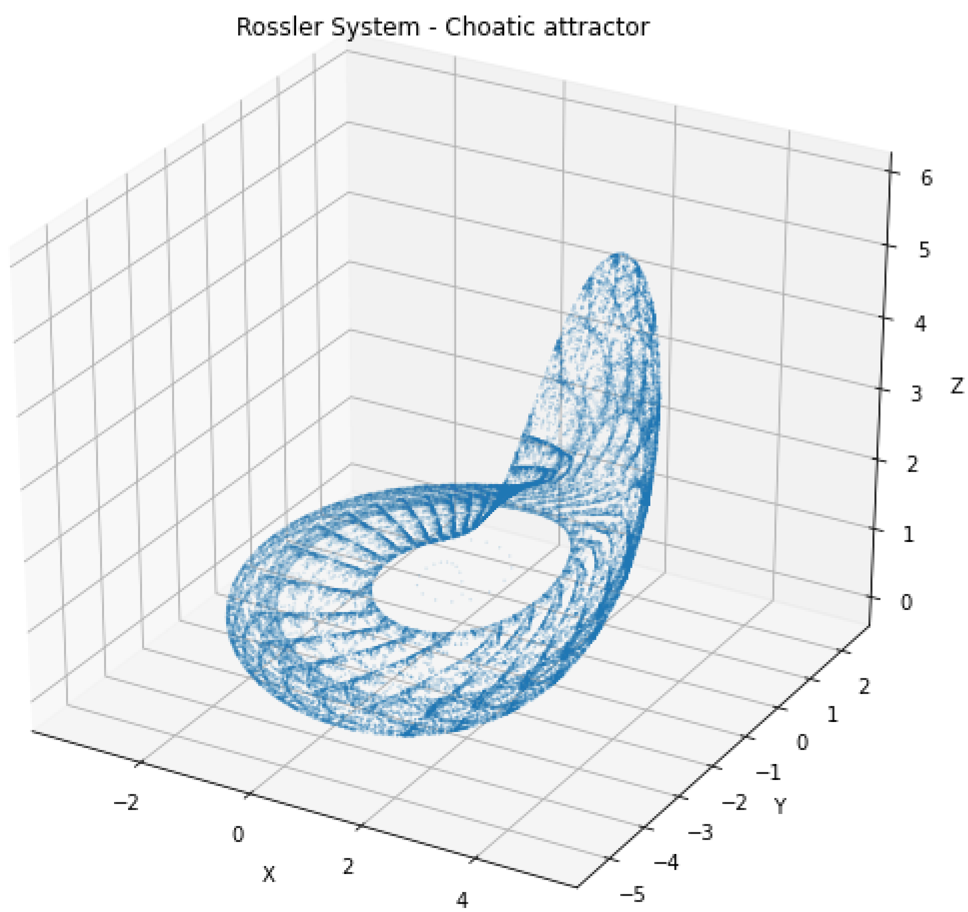


Fig4: Chaotic Attractor for Rossler System for *a = 0.398, b = 2, c = 4, initial condition* (x_0) *= [0.0001, 0.0001, 0.0001] using Adaptive Runge-Kutta*

4. Effects of Error Bounds

Given Lorenz System with *a = 16, r = 45, b = 4*
initial condition (x_0) *= [-13, -12, 52]*

The system dynamics start to degrade gradually as the tolerance is increased from 0.01. The chaotic attractor may still be seen but is rough and fragmented. The dynamics tend to sparse out as we increase the error bound from 0.01 (*Fig5*) to 0.15 (*Fig7).*

Fig5: Error Bound = 0.01

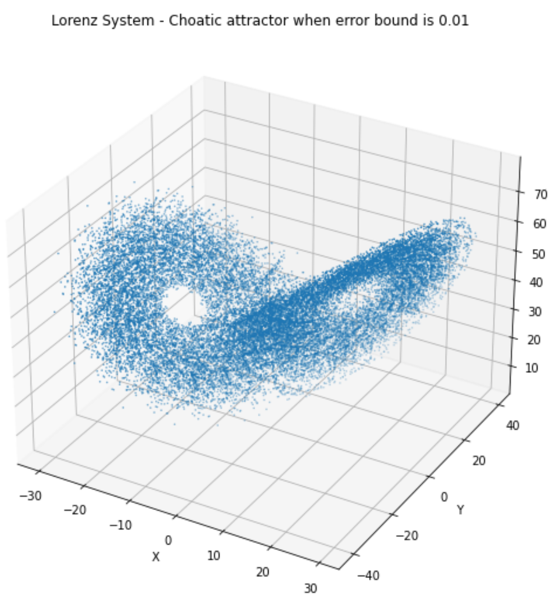
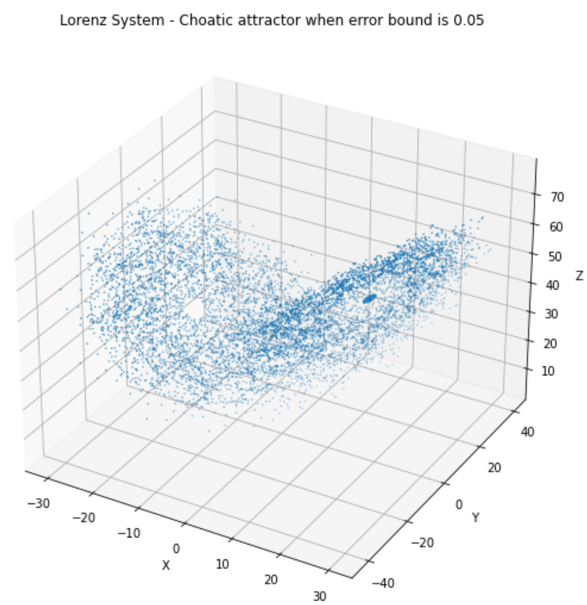


Fig6: Error Bound = 0.05



As the tolerance increases to 0.3 (*Fig8 on next page*) and larger, the dynamics break down completely. The form of the attractor is totally lost at tolerance 0.3. Instead, we witness a jagged spiral leading to what appears to be a fixed point. This is somewhat similar to the previous problem set's timestep experiment in that it shows compounding faults in an ODE integrator by relaxing the settings. We raised the timestep using RK4 until the system's dynamics broke down. This led to the approximation mistake to develop and multiply. Similarly, raising tolerance beyond a reasonable upper bound causes mistakes to proliferate and compound to undesirable levels.

– Fig 7 and Fig 8 on the next page

Fig7: Error Bound = 0.15

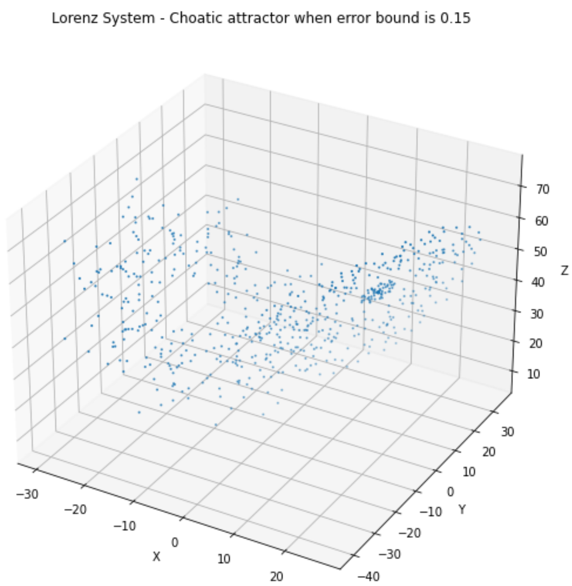


Fig8: Error Bound = 0.3