CSCI 5446 - Chaotic Dynamics

Problem Set 10 - Solutions

Name : Gowri Shankar Raju Kurapati SID: 110568555

1. Box Counting Algorithm:

Used the traditional simple box-counting method with optimization of hashing. The Hashed Box Counting algorithm uses a set-based approach rather than a matrix of Boolean values to represent the filled spaces occupied by the attractor. This enables it to count the number of boxes required to cover the attractor more efficiently and with lower memory requirements

The Hashed Box Counting algorithm is designed to estimate the number of boxes N(ε) required to cover an attractor A in n dimensions using a specified ball size ε. The algorithm operates as follows:

1. Initialize an empty set X.
2. For each point p in the attractor A:
   - Discretize the point p in space by selecting a vector x = (x1, x2, ..., xn), where each xi is obtained by rounding pi/ε to the nearest integer value.
   - Store the discretized point x in the set X by hashing it, i.e., add it to X. This simple operation takes constant time when using a hash set data structure.
3. Compute the cardinality of the set X, corresponding to the number of unique boxes required to cover the attractor.

2. Box Dimension of Lorenz Attractor

a. Short Lorenz Choatic Trajectory

Lorenz system - a = 16, r = 45, b =4, IC = [-13 , -12, 52]

The trajectory consists of 40,000 steps of RK4 with h = 0.001 and the first 10,000 steps are removed to ignore the transient.

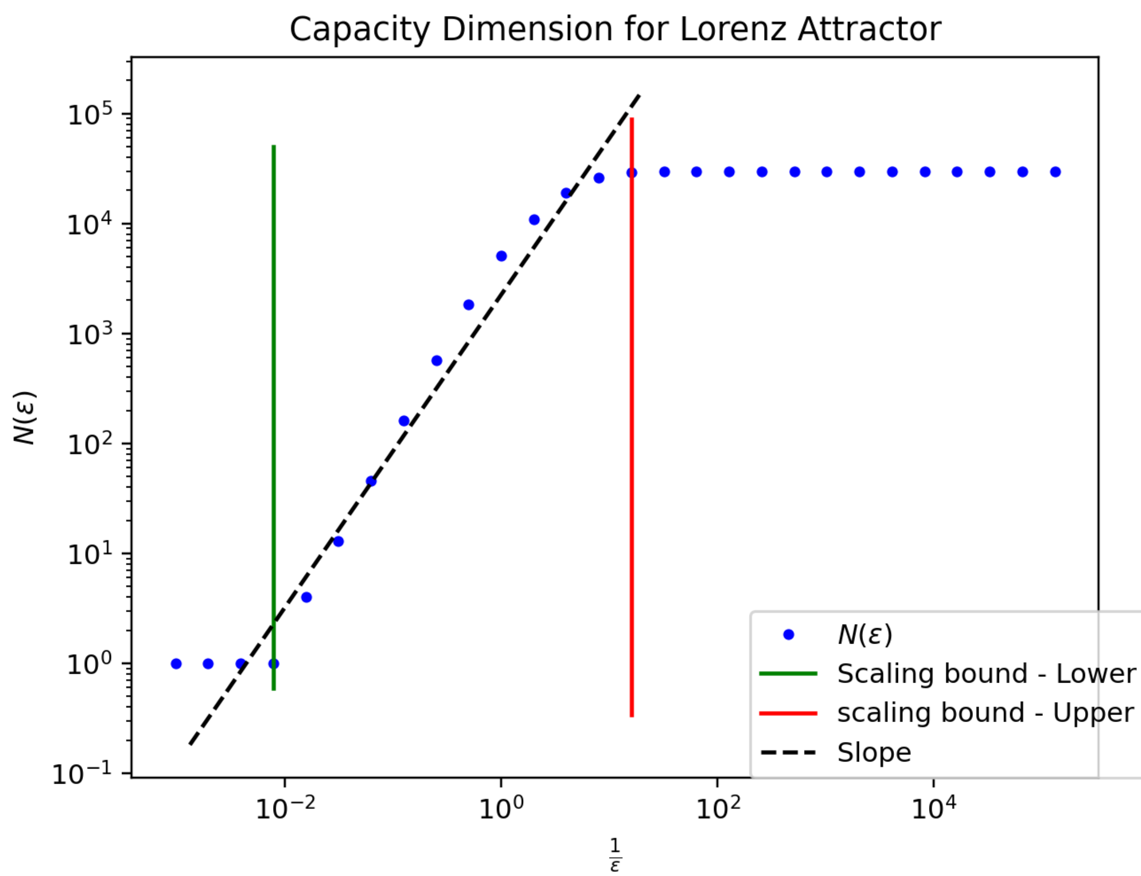

Fig1: Log(N(ϵ)) vs log(1/(ϵ)) to find the box dimension of Lorenz attractor with 30,000 points on trajectory

From Fig1, the shape of the log-log plot of N(ϵ) vs 1/ϵ is as expected as it has two numerical artifact regions and the scaling region in between. The region to the left of the green line is due to the entire set being covered by a single ϵ-ball and the shape of the curve to the right of the red line is due to each point being covered by a single ϵ-ball.

The region in between the green and red lines is the scaling region which has the power law relationship and the slope of the scaling region is the capacity dimension and it turns out to be **1.789 (= d_{cap}).**

b. Long Lorenz Choatic Trajectory

Lorenz system - a = 16, r = 45, b =4, IC = [-13, -12, 52]

The trajectory consists of 100,000 steps of RK4 with h = 0.001, and the first 10,000 steps are removed to ignore the transient.

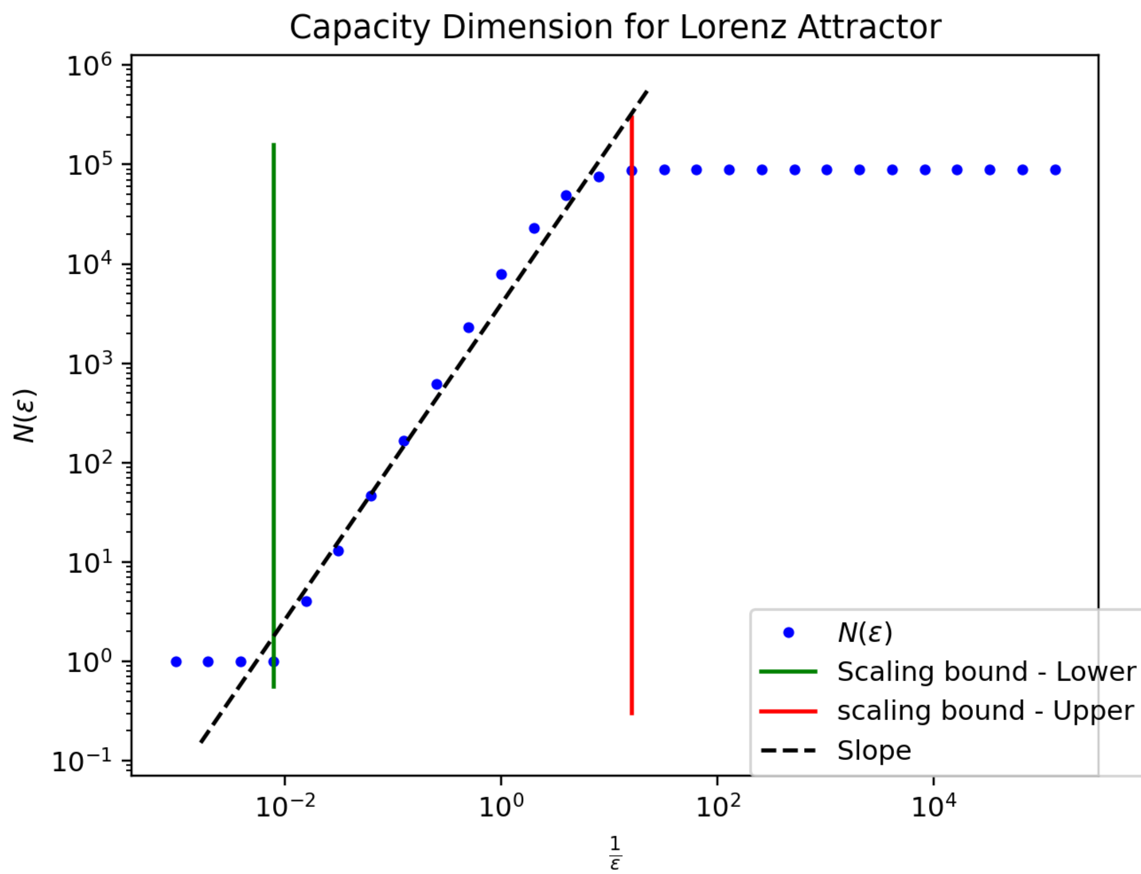

Fig2: Log(N(ϵ)) vs log(1/(ϵ)) to find the box dimension of Lorenz attractor with 90,000 points on trajectory

The slope of the scaling region is the capacity dimension and it turns out to be **1.851 (= d_{cap}).**

The capacity dimension obtained from the long run appear to be higher than those of the shorter run. This is because the longer run provides a more precise representation of the actual attractor, making it a superior approximation. If we were to examine a section of the longer run, it would more closely resemble the Cantor-like structure that we anticipate observing in the real attractor. On the other hand, a section of the shorter run would be less complete.

c. Reconstructed Lorenz Attractor from x-coordinate

The Capacity dimension for the Lorenz attractor reconstructed from just the delayed x-coordinate (using the zeroth and fifth columns of the embedding) turned out to be **1.576 (= d_{cap}).**

The outcome of the current result differs considerably from the outcome of part (a) & (b) , indicating a possible loss of dimensionality during the embedding process. The Lorenz attractor is a three-dimensional system, and reconstructing it using just one coordinate means that the resulting reconstruction is a lower-dimensional approximation of the original system.

The capacity dimension is a measure of how space-filling a set of points is, and it is related to the number of boxes of a certain size needed to cover the set. When reconstructing a system using only one coordinate, the resulting set of points is embedded in a lower-dimensional space, and this can affect its space-filling properties. In other words, the lower-dimensional reconstruction may not fill space as efficiently as the original system, leading to a lower capacity dimension.

3. Thought Experiment

Though the hashing method is a little optimal to prevent us from running out of memory constraints sooner than later compared to the simple box counting algorithm, one more possible way to overcome the box size/memory limitation in the previous problem is to use a parallel implementation of the box-counting algorithm, which would distribute the computation across multiple processors or nodes. Another approach would be to use a hierarchical grid-based method, to recursively partition the space and count the number of boxes at different levels of resolution, allowing for more efficient use of memory and computation.

4. Analysis of Correlation Sum

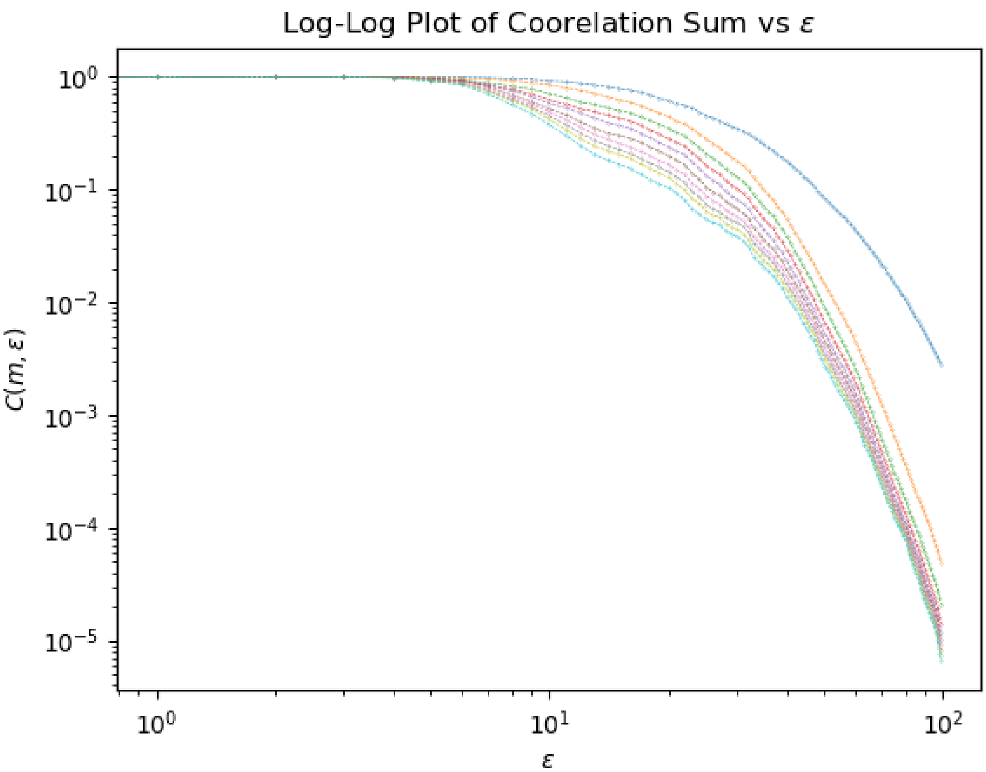

Fig3: Log-log plot of correlational integral C(m,ϵ) vs ϵ for x-dimension of Lorenz attractor for 40,000 point trajectory. The different curves represent m = 1,2,3,., 10, now counted from above.

Ideally, we have to choose a delay time for the embedding (τ = 0.108), a range of interesting embedding dimensions, and a correlation time t_{min} inorder to discard temporal neighbours which could affect the result adversely. We computed C(m,ϵ) and corresponding D(m,ϵ) in 2-10 dimensional embeddings.

The power law behavior of C(m,ϵ) as the signature of self-similarity can be found plotting the slope D(m,ϵ) of the double logarithmic plot of C(m,ϵ) versus ϵ.

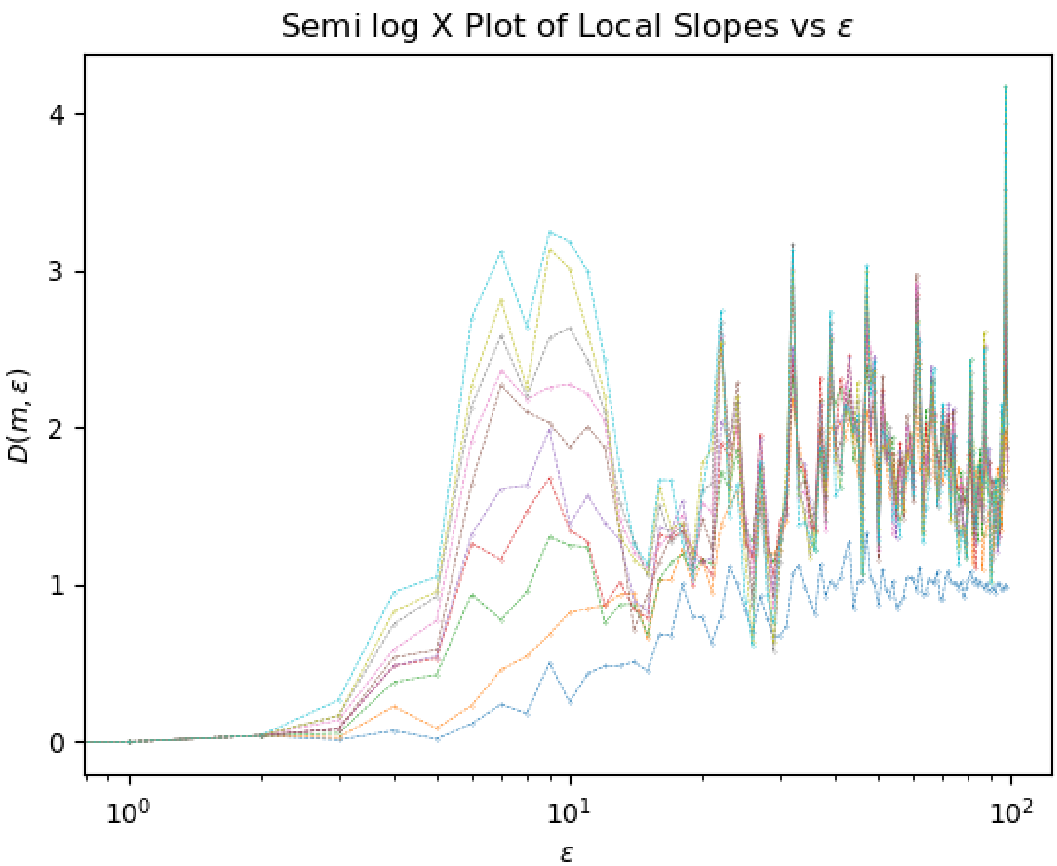

Fig4: Semi log X plot of Local slopes of Correlation integral D(m,ϵ) vs ϵ for x-dimension of Lorenz attractor for 40,000 point trajectory. The different curves represent m = 1,2,3,., 10, now counted from below.

For the self similar object, the local scaling exponent is constant and the same is true for all embedding dimensions larger than m_{min} > D. If there exists a plateau convincing enough for some range of smaller length scales of ϵ on D(m,ϵ) vs ϵ plot, that scaling exponent can be used as an estimate for the correlation dimension of that fractal set.

But from our graph in Fig4, we are unable to find any plateau region good enough for us to estimate the scaling exponent and thus the correlation dimension.So, one can not conclude any dimension for the system.