CSCI 5446 - Chaotic Dynamics

Problem Set 1 - Solutions

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Logistic Map:

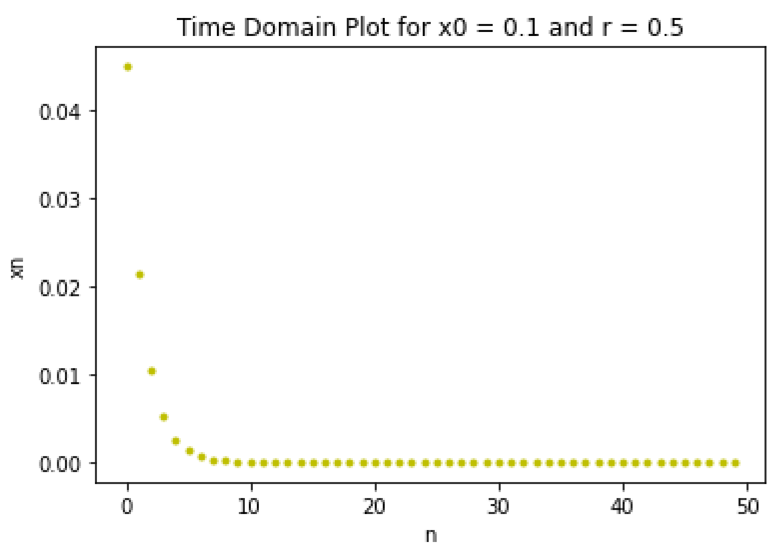
$x_{n+1} = rx_n(1 - x_n)$ where r is the parameter

x_0 is the initial condition

x_0 is varied between 0 and 1 for a few of the experiments below.

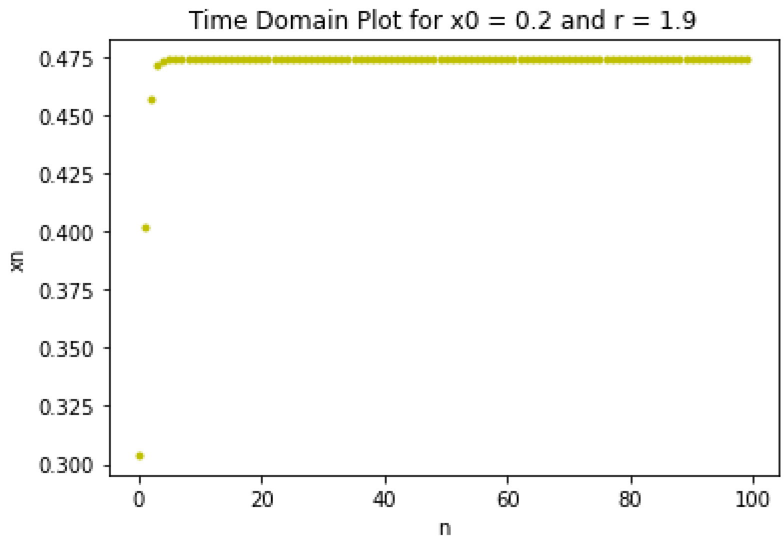
Interesting Observations when playing with r :

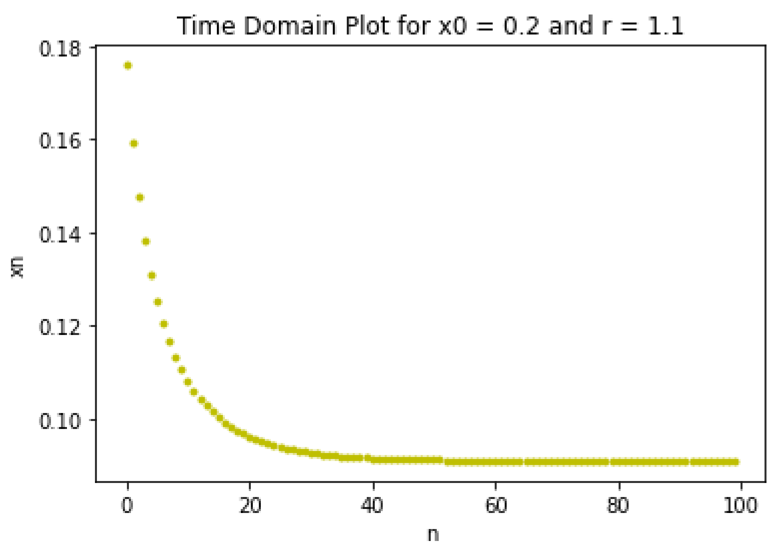
1. For the values of r between 0 and 1, irrespective of the initial condition, we observe that x_n dies out to zero as n increases.



   a.

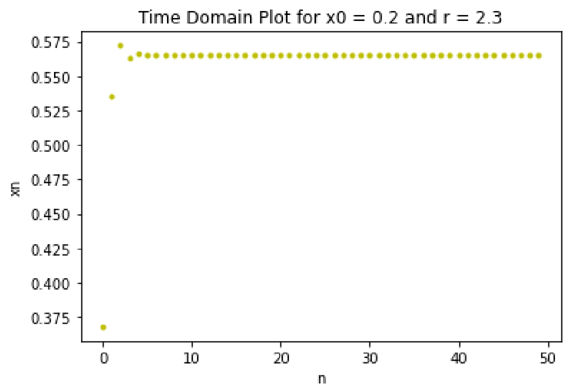
2. For the values between 1 and 2, we observe that it converges to a fixed point gradually. We also observe that the fixed point value increases as the r increases.

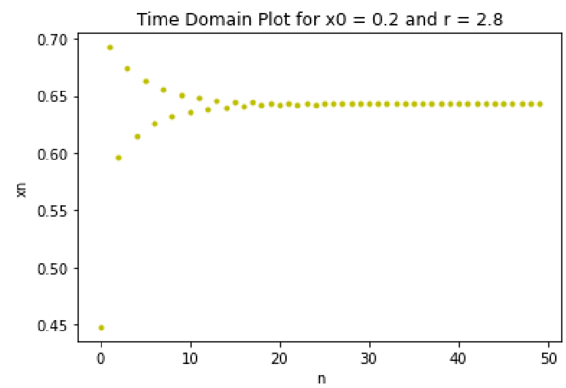




3. When we increase the **r from 2 to 2.9** at steps of 0.1,
   a. For each r, the trajectory converges to a fixed point , which is an "Attracting Fixed Point".
   b. Different values of r converge to different fixed points, the value of attracting fixed points increases as the r increases.
   c. For a particular r, the fixed point remains the same, Irrespective of the initial condition (x_0 = [0,1]) .
   d. The time it takes to reach the fixed point , aka Transient Time, also increases as we increase the r.
   e. We also observe that in the transient state, the trajectory has oscillations and these oscillations are more pronounced as r increases
   .

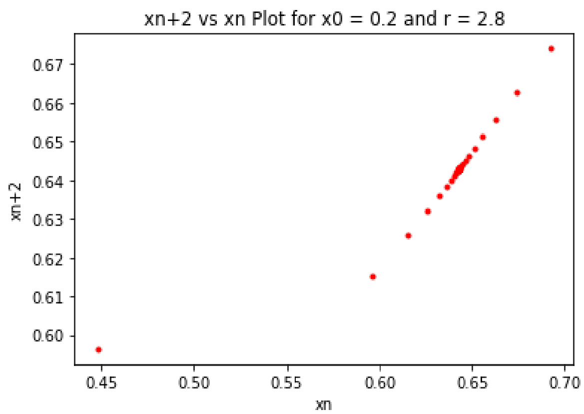
From the plots below, we can see that the above statements hold true.

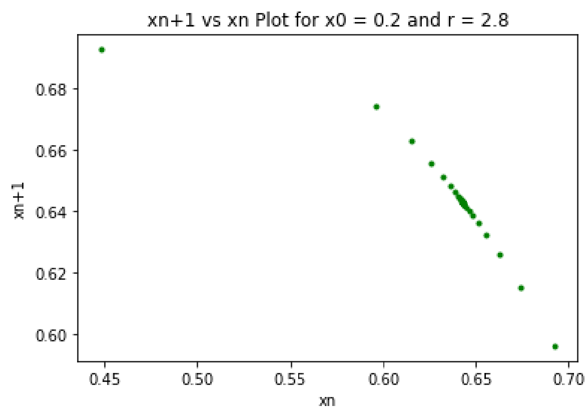




The Time Domain plots for x0 = 0.2 and for r values of 2.3 and 2.8

The oscillations can also be visualized from the 'x_{n+1} vs x_n ' and 'x_{n+2} vs x_n ' plots..

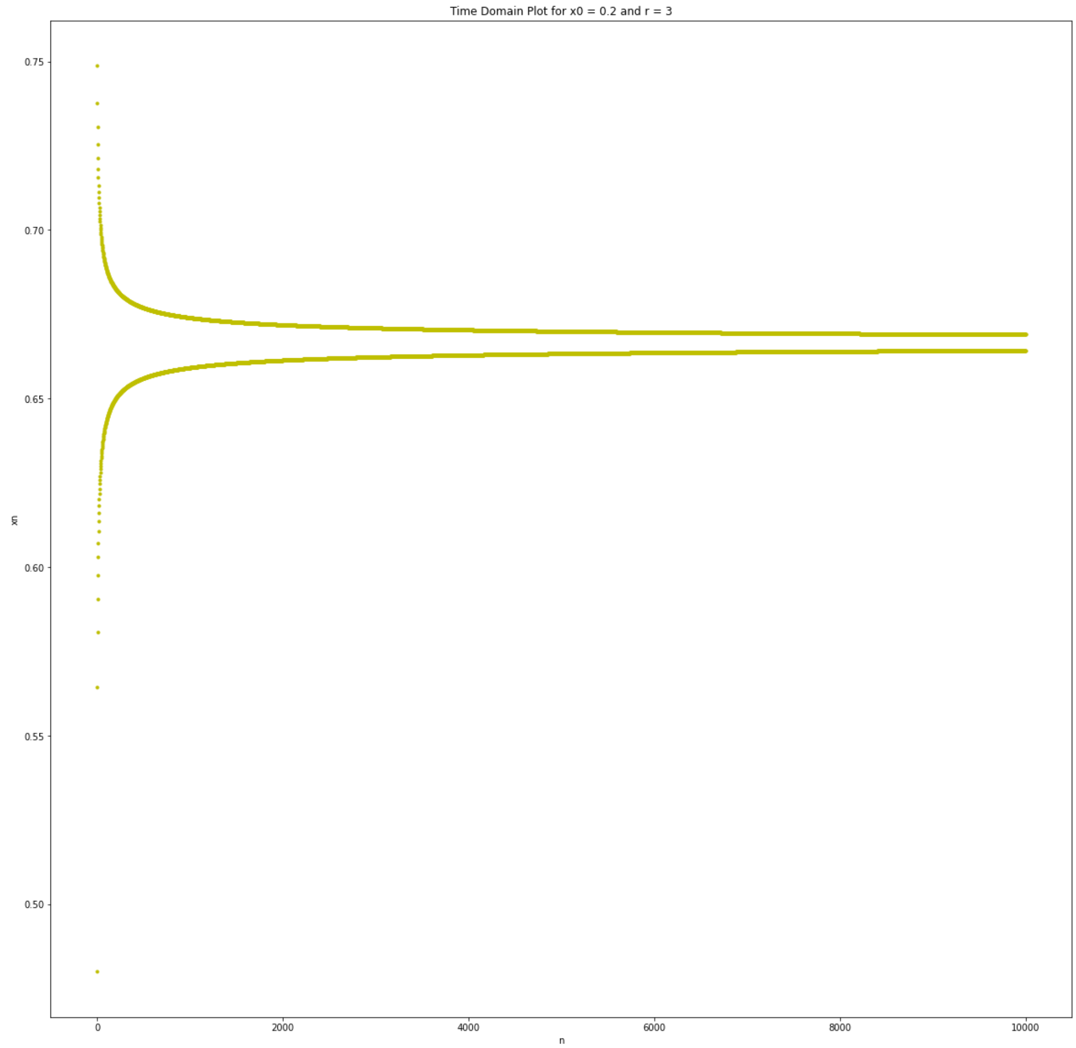




As we can see, for the slopes of the lines of above plots, the oscillations decrease as the trajectory moves from transient to fixed state value.

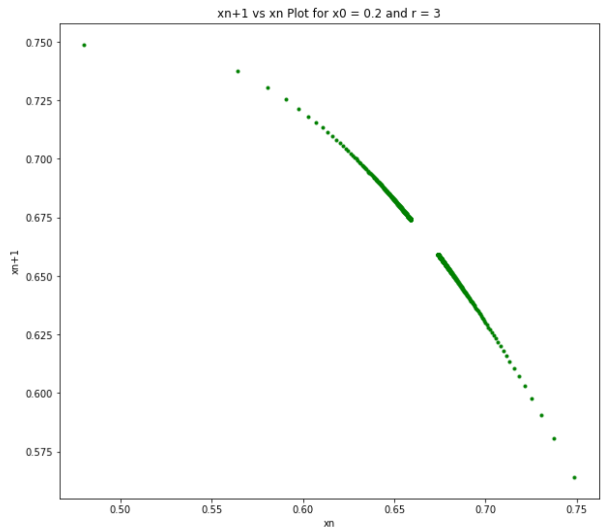
4. We observe that as r moves as near to 3 as possible, the oscillations of the trajectory are so much pronounced that it oscillates between two points and they never converge to one single point.

a.

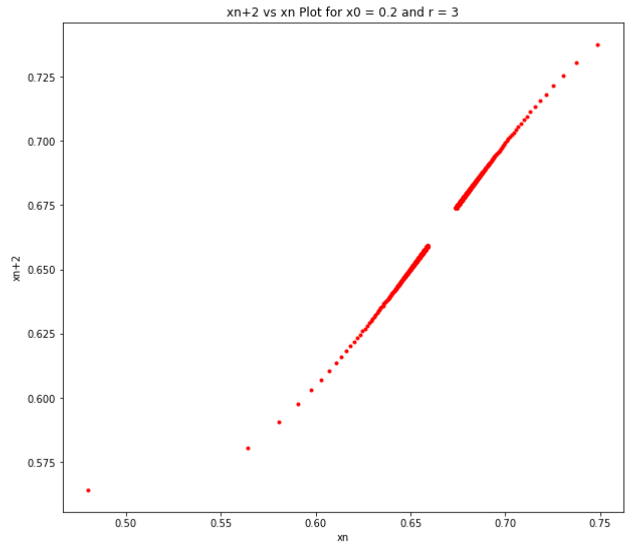


i. Time Domain plot for Even after 10000 iterations, it doesn't seem to converge.

We can also observe the same thing from the 'x_{n+1} vs x_n' and 'x_{n+2} vs x_n' plots..

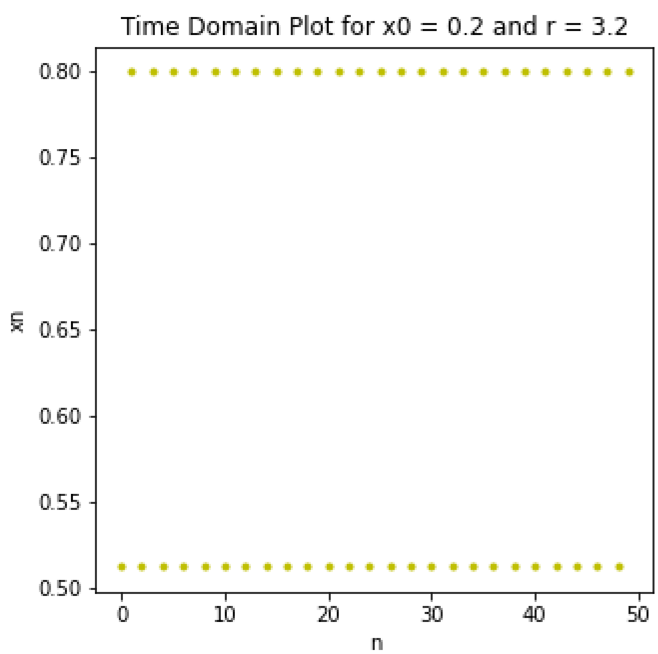


x_{n+1} vs x_n

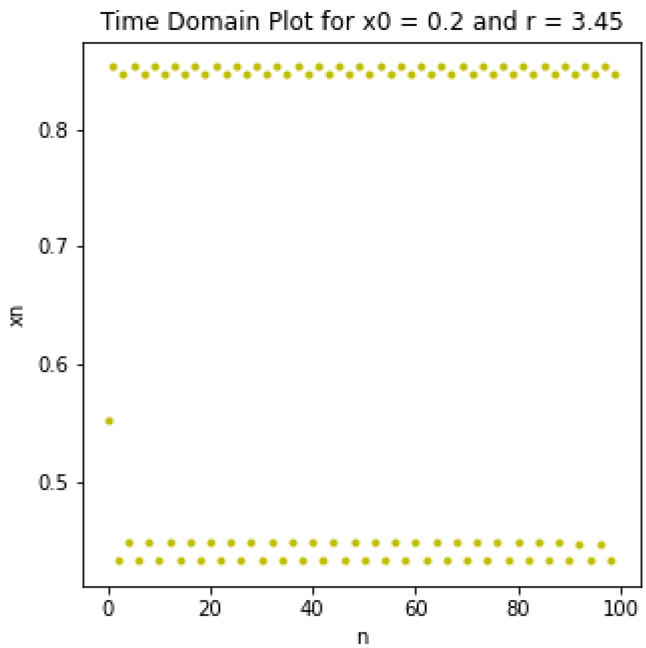


x_{n+2} vs x_n

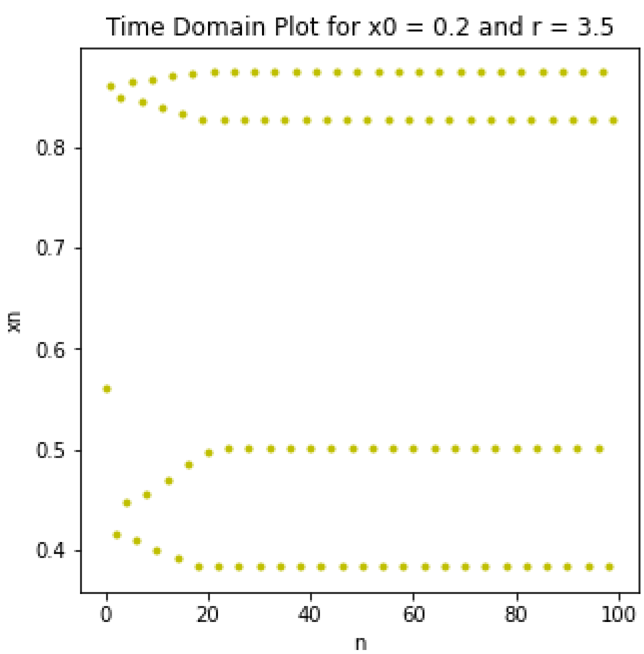
5. We observe the same phenomenon of bifurcation from r = [3, 3.45) , which we call as the **2 Cycle Attracting Periodic Orbit.**



At r = 3.45, each bifurcation seen to be oscillating between two fixed values, making the whole system reach a **4 Cycle Attracting Periodic Orbit.**

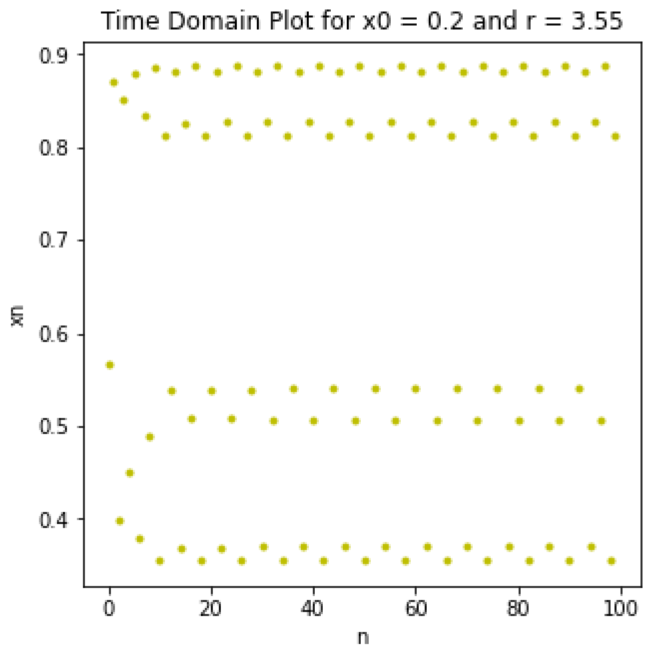


r = 3.45, x0 = 0.2

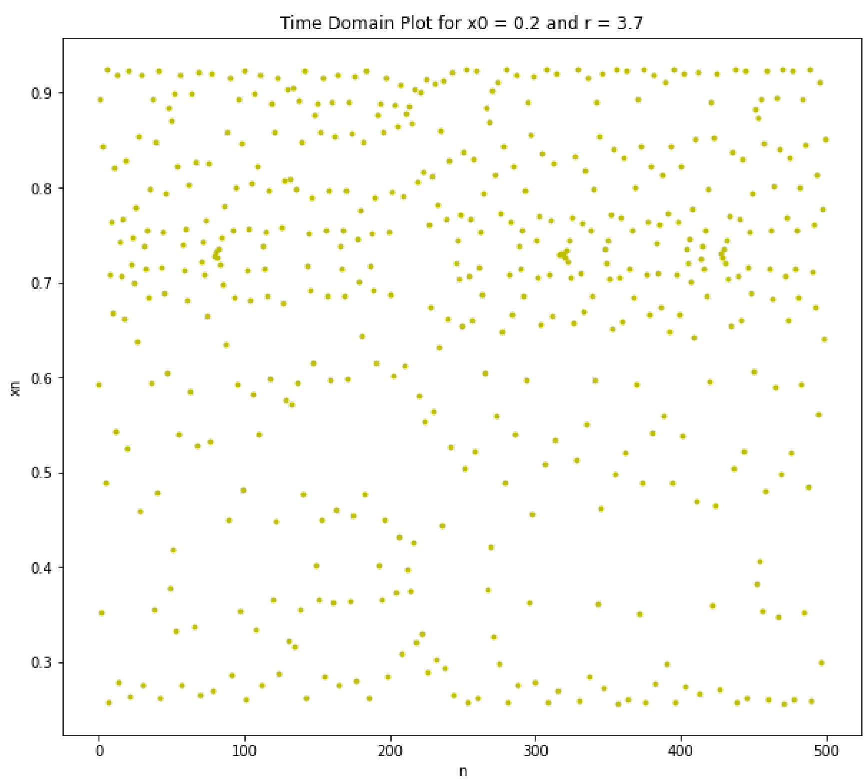


r = 3.5, x0 = 0.2

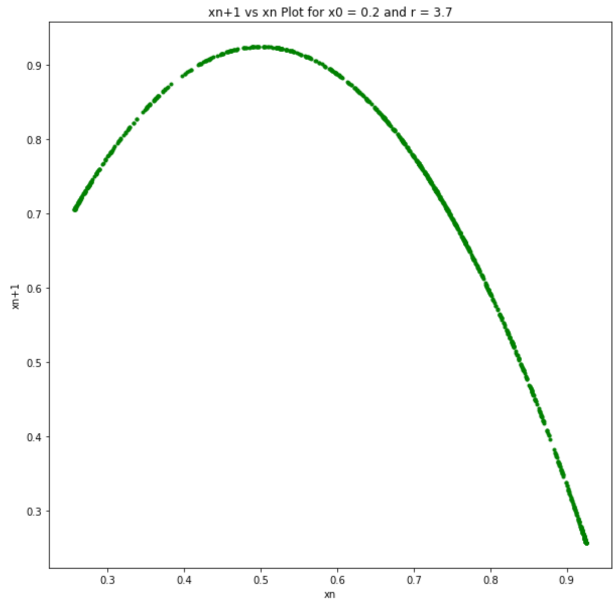
6. At r = 3.55, we see a bifurcation again , making it a **8 Cycle Attracting Periodic Orbit.**



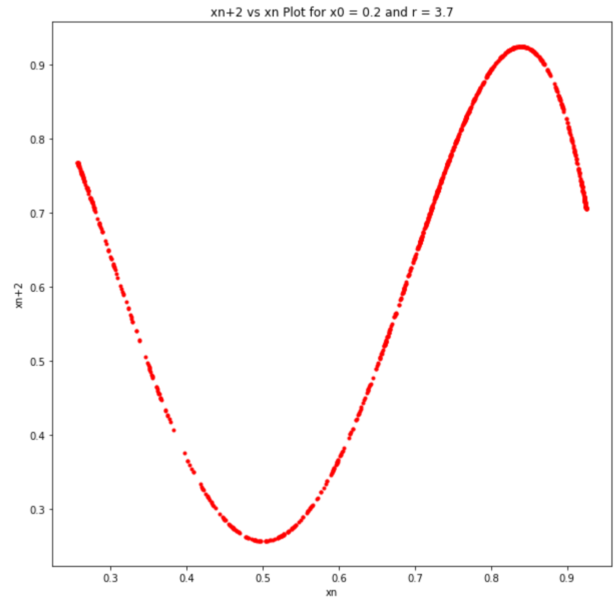
Gradually , the time domain plot turns chaotic as we increase r from 3.55. For example at r = 3.7, the time domain plot looks **Chaotic**



We can also confirm the chaotic behavior from the 'x_{n+1} vs x_n' and 'x_{n+2} vs x_n' plots..

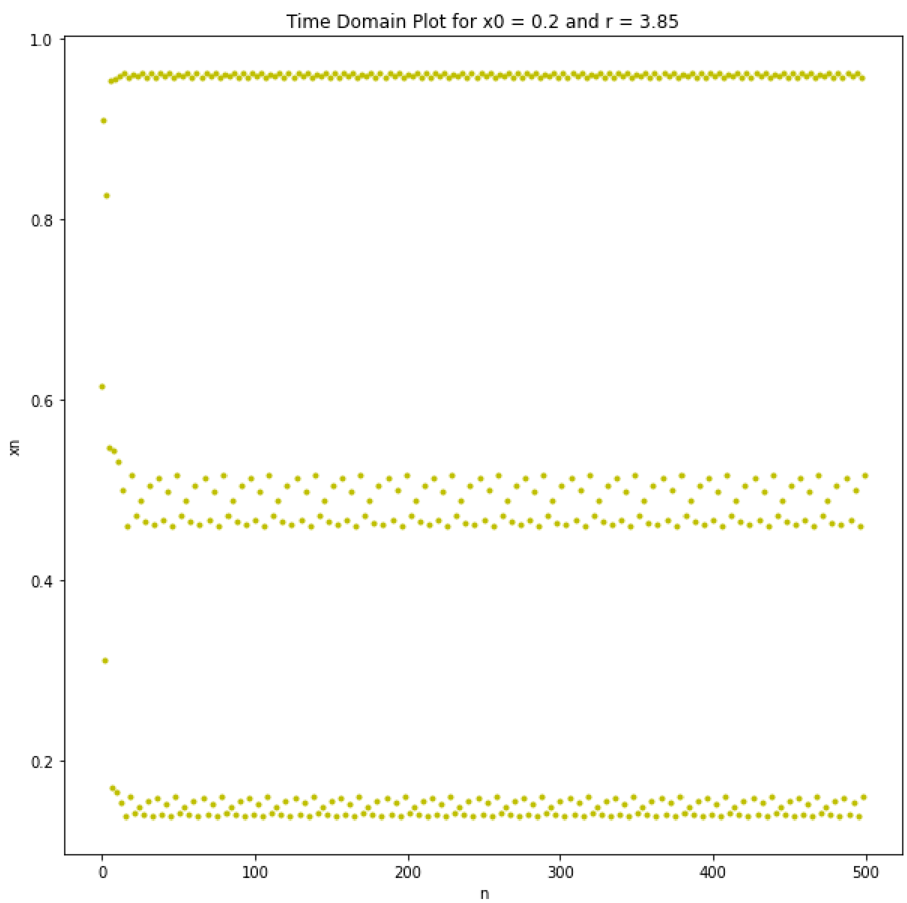


x_{n+1} vs x_n



x_{n+2} vs x_n

Surprisingly , at r = 3.85 , we observe patterns emerge again, The plot for r = 3.85 for 500 iterations is plotted below



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For **r > 4**, we observe that the values diverge away from 1 and very fast, and the graph doesn't make much sense. Even for the values between 0 and 1, we see no pattern and remain chaotic.

For r =2.5, irrespective of the initial conditions (x0 ranges from o to 1), the value of the attracting fixed point remains constant. Based on the initial condition, the transient iterations may vary but the attracting fixed point is always 0.6. The set of such initial conditions is known as **" Basin of Attraction".**