* Independent Component Analysis (ICA):

$$X = ED^{-1}E^{T}X \quad \text{(Mathematical eqn)}$$

where,

D = Diagonal matrix of eigenvalues

E = Orthogonal matrix of eigenvectors.

Example: whiten following matrix

$$X = \begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix}$$

Compute eigenvalues of matrix X.

$$\begin{vmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{vmatrix} = 0$$

or $(9-\lambda)(3-\lambda) - 16 = 0$

or $27 - 9\lambda - 3\lambda + \lambda^2 - 16 = 0$

$\therefore \lambda^2 - 12\lambda + 11 = 0$

After solving this equation we get

$\lambda = 1, 11$ {Eigenvalues}

$$\therefore D = \begin{bmatrix} 1 & 0 \\ 0 & 11 \end{bmatrix}$$

Now, Compute eigenvectors ;

$$(X - \lambda I)x = 0$$

$$\begin{bmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} (9-\lambda)x_1 + 4x_2 \\ 4x_1 + (3-\lambda)x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

Put $\lambda = 1$, we get

$$8x_1 + 4x_2 = 0 \text{ --- (i)}$$
$$4x_1 + 2x_2 = 0 \text{ --- (ii)}$$

$$8x_1 = -4x_2$$

$$2x_1 = -x_2$$

$$\frac{x_1}{-1} = \frac{x_2}{2} = (t = 1)$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Again put $\lambda = 11$, we get

$-2x_1 + 4x_2 = 0$ ——— (i)
$4x_1 - 8x_2 = 0$ ——— (ii)

$-2x_1 = -4x_2$

$x_1 = 2x_2$

$$\frac{x_1}{2} = \frac{x_2}{1} = (t = 1, \text{suppose})$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\therefore E = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$$

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we have;

$$X = ED^{-1}E^{T}X$$

$$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1/11 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2/11 \\ 2 & 1/11 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 22 & 11 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

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