3.3 Types of Matrices

In this section, we shall discuss different types of matrices.

(i) **Column matrix**

A matrix is said to be a *column matrix* if it has only one column.

For example, $A = \begin{bmatrix} 0 \\ \sqrt{3} \\ -1 \\ 1/2 \end{bmatrix}$ is a column matrix of order 4 × 1.

In general, $A = [a_{ij}]_{m \times 1}$ is a column matrix of order $m \times 1$.

(ii) **Row matrix**

A matrix is said to be a *row matrix* if it has only one row.

For example, $B = \begin{bmatrix} -\frac{1}{2} & \sqrt{5} & 2 & 3 \end{bmatrix}_{1\times 4}$ is a row matrix.

In general, $B = [b_{ij}]_{1 \times n}$ is a row matrix of order $1 \times n$.

(iii) **Square matrix**

A matrix in which the number of rows are equal to the number of columns, is said to be a *square matrix*. Thus an $m \times n$ matrix is said to be a square matrix if $m = n$ and is known as a square matrix of order 'n'.

For example $A = \begin{bmatrix} 3 & -1 & 0 \\ \frac{3}{2} & 3\sqrt{2} & 1 \\ 4 & 3 & -1 \end{bmatrix}$ is a square matrix of order 3.

In general, $A = [a_{ij}]_{m \times m}$ is a square matrix of order m.

Note If $A = [a_{ij}]$ is a square matrix of order n, then elements (entries) $a_{11}, a_{22}, ..., a_{nn}$ are said to constitute the *diagonal*, of the matrix A. Thus, if $A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & 4 & -1 \\ 3 & 5 & 6 \end{bmatrix}$.

Then the elements of the diagonal of A are 1, 4, 6.

(iv) **Diagonal matrix**

A square matrix B = $[b_{ij}]_{m \times m}$ is said to be a *diagonal matrix* if all its non diagonal elements are zero, that is a matrix B = $[b_{ij}]_{m \times m}$ is said to be a diagonal matrix if $b_{ij} = 0$, when $i \neq j$.

For example, $A = [4]$, $B = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -1.1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, are diagonal matrices of order 1, 2, 3, respectively.

(v) **Scalar matrix**

A diagonal matrix is said to be a *scalar matrix* if its diagonal elements are equal, that is, a square matrix B = $[b_{ij}]_{n \times n}$ is said to be a scalar matrix if

$$b_{ij} = 0, \quad \text{when } i \neq j$$
$$b_{ij} = k, \quad \text{when } i = j, \text{ for some constant } k.$$

For example

$$A = [3], \quad B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$$

are scalar matrices of order 1, 2 and 3, respectively.

(vi) **Identity matrix**

A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an *identity matrix*. In other words, the square matrix A = $[a_{ij}]_{n \times n}$ is an identity matrix, if $a_{ij} = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}$.

We denote the identity matrix of order n by I_n. When order is clear from the context, we simply write it as I.

For example [1], $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 1, 2 and 3, respectively.

Observe that a scalar matrix is an identity matrix when $k = 1$. But every identity matrix is clearly a scalar matrix.

(vii) **Zero matrix**

A matrix is said to be *zero matrix* or *null matrix* if all its elements are zero.

For example, [0], $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, [0, 0] are all zero matrices. We denote zero matrix by O. Its order will be clear from the context.

3.3.1 *Equality of matrices*

Definition 2 Two matrices A = $[a_{ij}]$ and B = $[b_{ij}]$ are said to be equal if

(i) they are of the same order

(ii) each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

For example, $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ are equal matrices but $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ are not equal matrices. Symbolically, if two matrices A and B are equal, we write A = B.

If $\begin{bmatrix} x & y \\ z & a \\ b & c \end{bmatrix} = \begin{bmatrix} -1.5 & 0 \\ 2 & \sqrt{6} \\ 3 & 2 \end{bmatrix}$, then $x = -1.5$, $y = 0$, $z = 2$, $a = \sqrt{6}$, $b = 3$, $c = 2$

Example 4 If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$

Find the values of a, b, c, x, y and z.

Solution As the given matrices are equal, therefore, their corresponding elements must be equal. Comparing the corresponding elements, we get

$$x + 3 = 0, \qquad z + 4 = 6, \qquad 2y - 7 = 3y - 2$$
$$a - 1 = -3, \qquad 0 = 2c + 2 \qquad b - 3 = 2b + 4,$$

Simplifying, we get

$$a = -2,\ b = -7,\ c = -1,\ x = -3,\ y = -5,\ z = 2$$

Example 5 Find the values of a, b, c, and d from the following equation:

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Solution By equality of two matrices, equating the corresponding elements, we get

$$2a + b = 4 \qquad 5c - d = 11$$
$$a - 2b = -3 \qquad 4c + 3d = 24$$

Solving these equations, we get

$$a = 1,\ b = 2,\ c = 3 \text{ and } d = 4$$

EXERCISE 3.1

1. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

(i) The order of the matrix, (ii) The number of elements,

(iii) Write the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23}.

2. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

4. Construct a 2 × 2 matrix, $A = [a_{ij}]$, whose elements are given by:

(i) $a_{ij} = \dfrac{(i+j)^2}{2}$ (ii) $a_{ij} = \dfrac{i}{j}$ (iii) $a_{ij} = \dfrac{(i+2j)^2}{2}$

5. Construct a 3 × 4 matrix, whose elements are given by:

(i) $a_{ij} = \dfrac{1}{2}|-3i + j|$ (ii) $a_{ij} = 2i - j$

6. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ (iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

7. Find the value of a, b, c and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$