Analytical Error Rate Performance Evaluation of a Multi-Keyhole Wireless MIMO Channel based on Largest Eigenvalue Distribution

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***Abstract*— An analytical approach is presented to evaluate the symbol error rate (SER) of a multiple-input multiple-output wireless channel over keyhole Nakagami-m channel using the probability density function of the largest eigenvalue of the channel matrix. The results are evaluated numerically in terms of SER for 8-PSK coherent multiple-input multiple-output (MIMO) system. The results show that compared to independent and identically distributed channel, system with keyhole channel suffers power penalty in receiver sensitivity to achieve a given SER. Further, there is improvement in receiver sensitivity at a given SER as the number of keyhole is increased. The improvement is about 13.5 dB for n_T=n_R=4 when number of keyhole is 4 compared to mono-keyhole channel, at SER=10^{-6}.**

***Index terms*- Diversity, eigenvalue, keyhole, multiple-input multiple-output (MIMO) channels, space–time block code (STBC), symbol error rate (SER).**

I. Introduction

Multiple-input multiple-output (MIMO) systems have emerged as one of the most promising technology to combat the challenges of limited availability of radio frequency spectrum, complex space-time varying radio environment, increasing demand for higher data rate, better quality of service, and higher network capacity [1]. MIMO systems employ multi-element antenna arrays at both the transmit and the receive ends of a radio link to drastically improve the capacity over more traditional single-input multiple-output (SIMO) systems. With multiple antennas typically being used at the base stations only, SIMO systems provide diversity gain, array gain, and interference gain. In addition to these advantages, MIMO systems can provide the spatial multiplexing gain as well, by opening parallel spatial data pipes or channels within the same frequency band and no additional power expenditure [2].

MIMO systems exploit the rich scattering environment by employing multiple transmit and receive antennas at both ends, which makes the signal from every individual transmitter appear highly uncorrelated at each of the receive antennas resulting very high spectral efficiency [2]. The high spectral efficiency is reduced if signals arriving at the receiver are correlated [3]. The existence of rank-deficient *keyhole* or *pinhole* can significantly reduce the diversity gain and spatial multiplexing gain of the channel. It has been demonstrated through physical experiments that MIMO systems even with uncorrelated transmit and receive signals can only have a single degree of freedom [2], [3].

In the presence of keyhole, the channel matrix is a product of a complex Gaussian column vector and a complex Gaussian row vector. Thus each entry of the channel matrix is a product of two complex Gaussian random variables. Therefore, the channel matrix is of rank one although the spatial fading gains are uncorrelated. As a result, this rank deficiency reduces achievable spectral efficiency and link quality in MIMO systems [4].

Mono-keyhole (or pinhole) MIMO propagation channels lead to the extreme scenario of rank-deficiency channel with a rank-1 channel matrix and a degenerate capacity performance [2]–[6]. The phenomenon of mono-keyhole has been validated both theoretically [3] and experimentally [6] and a great deal of research initiatives has been conducted to study the performance of mono-keyhole channels in various settings [4], [7]-[12].

Performance of MIMO wireless system in the presence of keyhole over Nakagami-*m* fading channel is reported based on the moment generating function (MGF) of instantaneous SNR after space-time block coding [8]. The results are reported for mono-keyhole effects. In multi-keyhole propagation channels, the distribution of the eigenvalues of the multi-keyhole MIMO channels is recently reported [13] based on Wishart distribution. The analysis also extended to cascaded multi-keyhole MIMO channels. The results are presented in terms of distribution of largest eigenvalues for multi-keyhole and cascaded multi-keyhole channels and also in the form of capacity distribution of the channels.

In this paper we present an analytical approach to find the symbol error probability (SEP) of wireless MIMO system with multi-keyhole channels based on the largest eigenvalue distribution of the channels as presented in the ref [13]. We compute the average symbol error rate (SER) with the probability density function (PDF) of the largest eigenvalue of the keyhole channel.

The remainder of this paper is organized as follows. Section II presents the system model. In Section III, analysis of instantaneous signal-to-noise ratio (SNR) and average SER for the proposed multi-keyhole model are presented. In Section IV, we present the analytical results. Finally, conclusive remarks are made in Section V.

II. System Model

Assuming non-line of sight local rich scattering model, we consider a MIMO system model with n_T transmit and n_R receive antennas at the transmit end (TE) and the receive end (RE), respectively as depicted in Fig. 1. The channel (T-R) is divided into two links (T-K and K-R) through the keyhole. The keyhole is the only way for radio waves to propagate

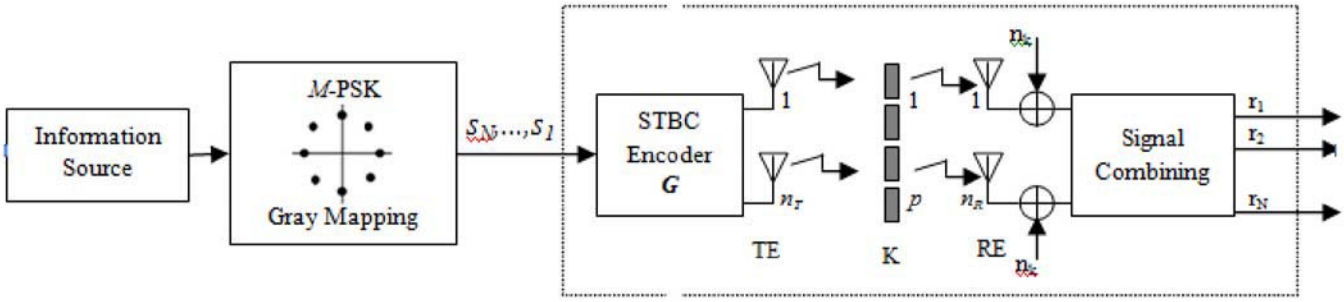

Fig.1. Space–time block-coded MIMO system with multi-keyholes in between.

from the TE to the RE. Thus a rank-deficiency fading channel comes into being. With p keyholes between the TE and RE, both the T-K and K-R links are divided into p sub-links, i.e. T-K_k and K-R_k with k = 1, 2, . . . , p. Such multi-keyhole channel may arise in some typical scenarios, e.g. the multiple roof-top edges diffraction propagation, the outdoor-to-indoor propagation etc. From the information source, *Nb* information bits are mapped as symbols s_1, s_2, ...,s_N, which are selected from *M*-PSK signal constellations with average energy E_0 by Gray mapping, where $b = log_2 M$. Then, $\{s_n\}_{n=1}^{N}$ are encoded by a space-time block code (STBC) defined by a column orthogonal transmission matrix G. At the receive end, maximum likelihood (ML) receiver computes the decision matric.

III. System Analysis

The transmitted STBC signal can be represented as

$$\boldsymbol{x} = \left[x_1, \ldots \ldots x_{n_T}\right]^T \tag{1}$$

The received signal vector can be expressed as,

$$\boldsymbol{y} = \boldsymbol{Hx} + \boldsymbol{n} \tag{2}$$

where, $\boldsymbol{n} \sim CN(0, \sigma^2 \boldsymbol{I})$ denotes the zero mean Gaussian noise vector, and $\boldsymbol{H}$ is the channel transfer matrix.

On the assumption of a quasi-static, frequency-flat and uncorrelated MIMO channel, according to the multi-keyhole structure [9]-[11], we can express the channel transfer matrix of the multi-keyhole case as follows [13],

$$\boldsymbol{H} = \sum_{k=1}^{N_k} g_k\, \boldsymbol{h}_{R_k} \boldsymbol{h}_{T_k}^H = \boldsymbol{H_R G H_T^H} \tag{3}$$

where, $\boldsymbol{H_T} = [\boldsymbol{h_{T_1}}, \boldsymbol{h_{T_2}}, \ldots, \boldsymbol{h_{T_p}}]^H$, $\boldsymbol{H_R} = [\boldsymbol{h_{R_1}}, \boldsymbol{h_{R_2}}, \ldots, \boldsymbol{h_{R_p}}]$ and g_k denotes the complex gain for the k^{th} keyhole, $\boldsymbol{G} = diag([g_1, g_2, \ldots, g_p])$. Moreover, $\boldsymbol{H_R}$ and $\boldsymbol{H_T}$ are mutually independent matrices $\sim\mathcal{CN}(0, \boldsymbol{I})$. For each k, $\boldsymbol{h_{T_k}}$ and $\boldsymbol{h_{R_k}}$ with independent distributions are induced by the T-K_k and K_k-R sub-links. The capacity performance of MIMO systems are determined by rank($\mathbf{H}$) $= \min(n_T, n_R, p)$. $\boldsymbol{H}$ can be normalized with $\mathbb{E}[\|\boldsymbol{H}\|^2] = n_T \times n_R$ and $\sum_{k=1}^{p} \mathbb{E}[|g_k|^2] = 1$, thus $\mathbf{G}$ is normalized by the keyhole number p, i.e. $\boldsymbol{G} = \sqrt{1/p} \cdot \boldsymbol{I_{p\times p}}$, where, $\boldsymbol{I_{p\times p}}$ denotes a $p\times p$ identity matrix. Therefore, we can get the normalized multi-keyhole channel transfer matrix as follows,

$$\boldsymbol{H} = \sqrt{1/p} \cdot \boldsymbol{H_T H_R} \tag{4}$$

For mono-keyhole scenario, the channel matrix $\boldsymbol{H}$ is given by (5), shown at the bottom of the page, where, $h_j = \{\alpha_j\}_{j=1}^{n_T}$ and $h_i = \{\beta_i\}_{i=1}^{n_R}$ describe the rich scattering at transmit and receive arrays, respectively. The squared Frobenious norm of the channel matrix is [8],

$$\|\boldsymbol{H}\|_F^2 = \sum_{i=1}^{n_R} \sum_{j=1}^{n_T} \left|h_j\right|^2 \cdot |h_i|^2 \tag{6}$$

IV. Analysis of Instantaneous SNR and Expression of Average SER

For a mono-keyhole channel, the largest eigenvalue λ_1 can be expressed as [24]

$$\lambda_1 = tr\left[\left(h_j h_j^H\right)\left(h_i h_i^H\right)\right] = \sum_{i=1}^{n_R} \sum_{j=1}^{n_T} \left|h_j\right|^2 \cdot |h_i|^2 \tag{7}$$

where, $tr(\cdot)$ denotes trace operator, and superscript H denotes the transpose conjugate of a matrix.

$$H = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1n_T} \\ h_{21} & h_{22} & \cdots & h_{2n_T} \\ \vdots & \vdots & \cdots & \vdots \\ h_{n_R 1} & h_{n_R 2} & \cdots & h_{n_R n_T} \end{pmatrix} = \begin{pmatrix} \beta_1 e^{j\theta_1} \\ \beta_2 e^{j\theta_2} \\ \vdots \\ \beta_i e^{j\theta_i} \\ \vdots \\ \beta_{n_R} e^{j\theta_{n_R}} \end{pmatrix} \left(\alpha_1 e^{j\varphi_1} \quad \alpha_2 e^{j\varphi_2} \quad \ldots\ldots \alpha_j e^{j\varphi_j} \ldots\ldots \quad \alpha_{n_T} e^{j\varphi_{n_T}}\right)$$

$$= \begin{pmatrix} \alpha_1\beta_1 e^{j(\varphi_1+\theta_1)} & \alpha_2\beta_1 e^{j(\varphi_2+\theta_1)} & \cdots & \alpha_j\beta_1 e^{j(\varphi_j+\theta_1)} & \cdots & \alpha_{n_T}\beta_1 e^{j(\varphi_{n_T}+\theta_1)} \\ \alpha_1\beta_2 e^{j(\varphi_1+\theta_2)} & \alpha_2\beta_2 e^{j(\varphi_2+\theta_2)} & \cdots & \alpha_j\beta_2 e^{j(\varphi_j+\theta_2)} & \cdots & \alpha_{n_T}\beta_2 e^{j(\varphi_{n_T}+\theta_2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_1\beta_i e^{j(\varphi_1+\theta_i)} & \alpha_2\beta_i e^{j(\varphi_2+\theta_i)} & \cdots & \alpha_j\beta_i e^{j(\varphi_j+\theta_i)} & \cdots & \alpha_{n_T}\beta_i e^{j(\varphi_{n_T}+\theta_i)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\ \alpha_1\beta_{n_R} e^{j(\varphi_1+\theta_{n_R})} & \alpha_2\beta_{n_R} e^{j(\varphi_2+\theta_{n_R})} & \cdots & \alpha_j\beta_{n_R} e^{j(\varphi_j+\theta_{n_R})} & \cdots & \alpha_{n_T}\beta_{n_R} e^{j(\varphi_{n_T}+\theta_{n_R})} \end{pmatrix} \tag{5}$$

The instantaneous SNR per symbol of MIMO keyhole Nakagami-m channel at the receiver output after space-time block decoding can be expressed as [8]

$$\gamma_{STBC} = \frac{\kappa^2 \|\boldsymbol{H}\|_F^4 E_s}{\kappa \|\boldsymbol{H}\|_F^2 N_0} = \frac{\|H\|_F^2 E_s}{n_T R N_0} \tag{8}$$

Comparison of (6) and (7) reveals that

$$\gamma_{STBC} = \lambda_1 \frac{E_s}{n_T R N_0} \tag{9}$$

The PDF of λ_1 for mono-keyhole channel is given by [12],

$$p_{mono}(\lambda_1) = \frac{2\lambda_1^{\frac{n_T+n_R}{2}-1}}{\Gamma(n_T)\Gamma(n_R)} \cdot K_{n_T-n_R}\left(2\sqrt{\lambda_1}\right) \tag{10}$$

and the PDF of λ_1 in the multi-keyhole channel, is given as [13],

$$p(\lambda_1) = \frac{2\lambda_1^{\frac{n_T+n_R}{2}+l-1}\eta^{-\frac{n_T+n_R}{2}-l}}{\Gamma(n_T+l)\Gamma(n_R+l)} \cdot K_{n_T-n_R}\left(2\sqrt{\lambda_1/\eta}\right) \tag{11}$$

where, $\eta = \frac{n_T+n_R}{(n_T+l)(n_R+l)}$ denotes diversity gain adjusting factor and $l(n_T, p, n_R) = \mu[n_T(p-1)n_R]^{\theta}$ with the equivalent adjusting factors μ, θ. $K_v(\cdot)$ represents the modified Bessel function of the second kind.

The conditional SER for a coherent M-PSK receiver conditioned on a given value of λ_1 is given by [14], [15]

$$P_S^{MPSK}(E|\lambda_1) = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} exp\left(-\frac{\lambda_1 \frac{E_s}{n_T N_0} g_{MPSK}}{sin^2\theta}\right) d\theta \tag{12}$$

where, $g_{MPSK} = sin^2\left(\frac{\pi}{M}\right)$. Averaging the conditional SER over the PDF $p(\lambda_1)$, the expression for average SER can be written as,

$$\text{SER} = \frac{1}{\pi}\int_0^{\infty}\int_0^{\pi-\frac{\pi}{M}} exp\left(-\frac{\lambda_1 \frac{E_s}{n_T N_0} g_{MPSK}}{sin^2\theta}\right) p(\lambda_1) d\theta d\lambda_1 \tag{13}$$

where, $p(\lambda_1)$ is the largest eigenvalue PDF of λ_1 in a mono or multi-keyhole channel, [12], [13] as shown in eqn. (10) and (11).

V. Results and Discussion

Following the analytical approach presented in Section IV, we evaluate the average SER of a wireless communication system over Nakagami-m fading channels operating in a mono or multi-keyhole environment. We provide the results for mono and multi-keyhole MIMO fading channel for 8-PSK signal constellation, STBC $\mathcal{H}_3$ (rate, R= 3/4). The plots of SER are presented as a function of average SNR per antenna for mono-keyhole, multi-keyholes as well as for independent and identically distributed (i.i.d.) channel without keyhole. The plots of SER vs. SNR are depicted in Fig. 2 and Fig. 3 for (n_T=4, n_R=4) and (n_T=3, n_R=3) respectively. It is noticed that, compared to i.i.d. channel, the SER of mono-keyhole channel suffers significantly due to the presence of a keyhole. Whereas in the presence of multi-keyholes, the degradation in SER is less compared to mono-keyhole channel and there is a significant amount of receiver sensitivity improvement at a given SER (say, 10^{-6} and 10^{-9}) with increase in the number of keyholes. It is also observed that the SER performance is better for (4×4) system compared to (3×3) system due to diversity gain.

The plots of receiver sensitivity (dB) at a given SER are shown in Fig. 4 and Fig. 5 for various system configurations. It is noticed that as the number of keyhole increases, there is improvement in receiver sensitivity. For example, the improvement in receiver sensitivity is about 13.5 dB when the number of keyholes is 4 compared to a mono-keyhole system.

However, compared to i.i.d. channel, the keyhole channels suffer power penalty at a given SER (say, 10^{-6}). The plots of power penalty are depicted in Fig. 6 as a function of number of keyholes. It is noticed that penalty reduces significantly with increase in number of keyholes.

Fig. 7 shows the plots of receiver sensitivity improvement due to increase in the number of keyholes at a given SER. It is noticed that improvement is higher for higher number of keyholes. For example, improvement is 8.5 dB, 11.5 dB, and 13.5 dB over mono-keyhole system with (n_T = 4, n_R = 4) antenna combination at SER=10^{-6} corresponding to number of keyholes 2, 3, and 4 respectively.

VI. Conclusions

An analytical approach is presented to evaluate the SER performance of a wireless MIMO-STBC channel with mono- and multi-keyhole channels using PDF of the largest eigenvalue of the channel transfer matrix. It is found that the system suffers penalty due to keyhole compared to i.i.d. channel at a given SER. Further, the results show that there is significant improvement in receiver performance as the number of keyholes is increased. The improvement is of the order of 15 dB when the number of keyholes is 4 compared to mono-keyhole channel.

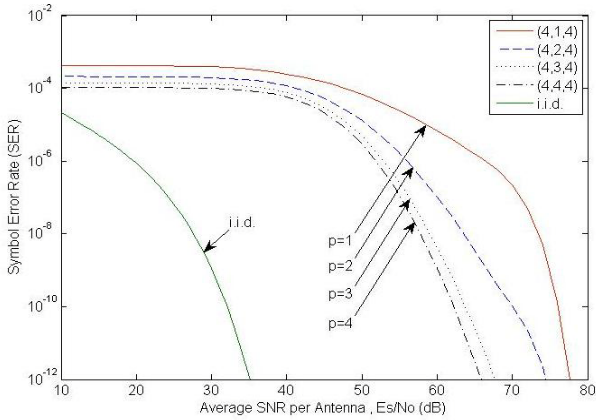

Fig. 2. SER vs. Average SNR per receive antenna for 8-PSK constellation over multi-keyhole fading channel. The parameters in (n_T, p, n_R) denote the number of the transmit antenna, the receive antenna and the keyholes in between.

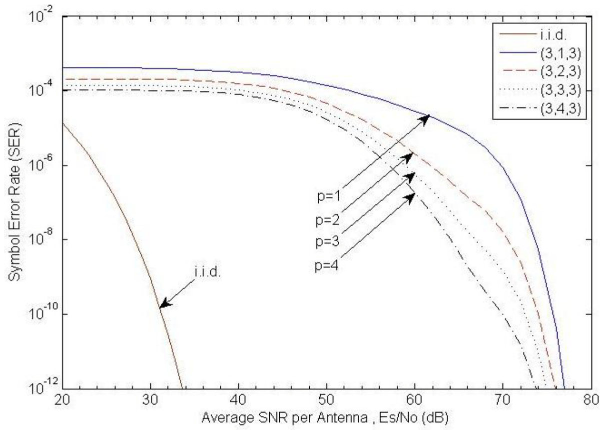

Fig.3. SER vs. Average SNR per receive antenna for 8-PSK constellation over multi-keyhole fading channel. The number of the transmit antenna, n_T =3, the receive antenna, n_R =3, and the keyholes in between, p=1, 2, 3, 4.

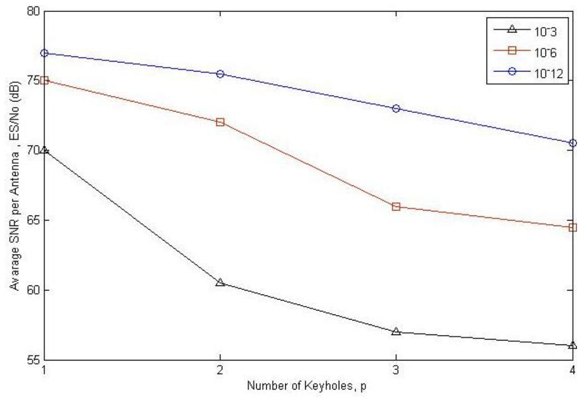

Fig.4. Average SNR per receive antenna for SER= 10^{-3}, 10^{-6} and 10^{-12} vs. the number of keyholes for 8-PSK constellation over multi-keyhole fading channel with $n_T = n_R = 4$.

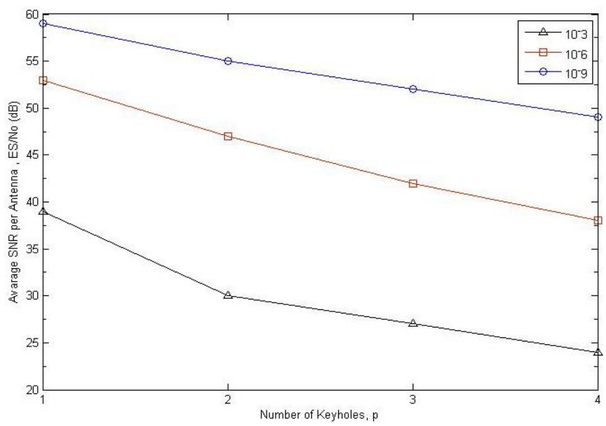

Fig.5. Average SNR per receive antenna for SER= 10^{-3}, 10^{-6} and 10^{-9} vs. the number of keyholes for 8-PSK constellation over multi-keyhole fading channel with $n_T = n_R = 3$.

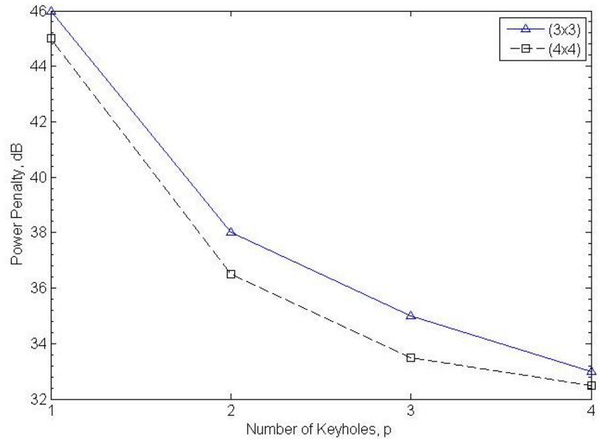

Fig.6. Power penalty vs. number of keyholes at SER=10^{-6} for the antenna combinations (n_T =4, n_R =4) and (n_T =3, n_R =3).

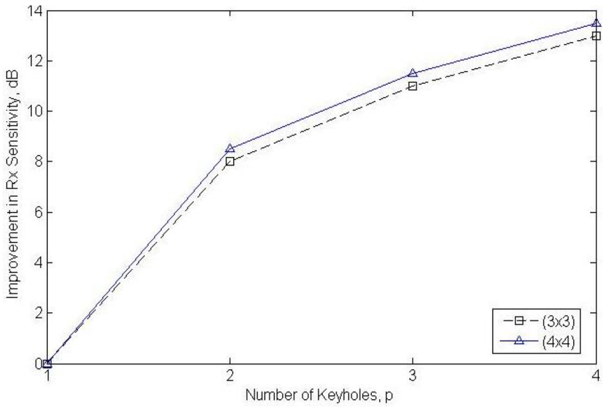

Fig. 7. Improvement in receiver sensitivity vs. number of keyholes for the antenna combinations (n_T =4, n_R =4) and (n_T =3, n_R =3) investigated at SER=10^{-6}.

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