Performance Analysis of a Free Space Optical Link in the Presence of Pointing Errors with Space Diversity

A. K. M. Nazrul Islam
PhD student, Dept of EEE, BUET
e-mail: nazrul_4620@yahoo.com
* Faculty member, MIST

S P Majumder
Dept of EEE, BUET, Dhaka-1000, Bangladesh
e-mail: spmajumder@eee.buet.ac.bd

Abstract—**An analytical approach is presented to evaluate the bit error rate (BER) performance of an optical free space link considering the effect of pointing error. The analysis is presented to evaluate the BER with multiple photo detectors in the presence of pointing error. The results are evaluated at a bit rate of 10 Gbps for various system parameters. The receiver sensitivity with pointing error is evaluated for SISO and SIMO FSO links for various system parameters. It is noticed that SIMO FSO links can tolerate more pointing error compared to SISO.**

Key Words – Bit Error Rate (BER); Free-Space Optical (FSO), Pairwise Probabilities (PP), pointing error on-off keying (OOK), Gamma-Gamma fading, Diversity Technique; etc.

I. Introduction

FREE space optical (FSO) communication has become very popular recently for effectively transferring data at high rates over short distances. In order to provide line of sight (LOS), the transmitter and receiver are placed on high-rise buildings separated by several hundred meters [1]. FSO has a numbers of merits over its rivals: it is lightweight, easily deployable, provides high data rates without any requirement for licensing. However, FSO systems are impaired by adverse weather conditions, such as fog, cloud and turbulences and by pointing errors due to loss of alignment between transmitter and receiver in the presence of building sway [1].

Continuous alignment between transmitter and receiver are required in FSO communication. Dynamic and load thermal expansion and earthquake cause buildings to sway resulting in misalignment between the transmitter and receiver [1]. The effect of pointing error (jitter) and atmospheric turbulence on terrestrial FSO links are investigated in [2] and [3]. In these works, the detector aperture is considered negligible with respect to the beam width at the receiver. In [4] Farid and Hranilovic have derived a statistical model for FSO link which model the fading due to atmospheric turbulence and pointing errors considering beam width, pointing error variance, and detector size. In [5], Sandalidies et al, present a closed form probability density function (pdf) of the statistical model considering the joint effects of K-distributed strong turbulence fading and BER in terms of the Meijer's-G functions. In [6], Deva K. Borat et al, investigated the effect of beam pointing errors on the capacity of optical links affected by atmospheric turbulence. They have used a wave-optics based approach to evaluate the capacity expression of a FSO links affected by atmospheric turbulence and beam pointing error.

Spatial diversity i.e. the use of multiple transmitters and receivers, provide an attractive technique to mitigate the fades in the received signal. Spatial Diversity to mitigate turbulence induce fading is investigated in [7-11]. In [9], Navidpour et al. have evaluated the BER performances of FSO links with spatial diversity over log-normal atmospheric turbulence fading channels, assuming both independent and correlated channels among transmitter/receiver apertures. In [10], Tsiftsis et al. have studied the BER performances of MIMO FSO links in Gamma-Gamma fading by expressing pairwise probabilities (PP) as a power series with respect to the signal-to-noise ratio (SNR). The use of spatial diversity to combat misalignment induced fading in a FSO system is suggested in [1]. Analytical results are also reported on FSO system considering pointing error for MISO and MIMO optical diversity scheme [12]

In this paper, we consider a SIMO FSO system affected by pointing errors only. We assume intensity modulation/direct detection (IM/DD) with on-off keying (OOK). We first consider a single-input-single-output (SISO) FSO system and obtain via numerical integration the BER performance of the system considering the beam width, detector size and pointing error variance. From these we obtain the receiver power penalty for different pointing error variances considering beams of different beam widths. Further the BER expressions for SIMO FSO link impaired by pointing errors are developed and the BER performance of the system is evaluated numerically.

II. SYSTEM MODEL

We consider a FSO link with M transmitters and N receivers, using IM/DD with OOK. The received signal at the nth receive aperture is given by:

$$r_n = R_d x \sum_{m=1}^{M} h_{mn} + \upsilon_n \text{ , n=1,....,N} \tag{1}$$

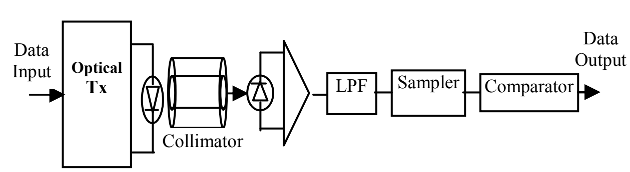

Fig. 1(a). Block diagram of FSO link with one transmitter and one receiver (SISO).

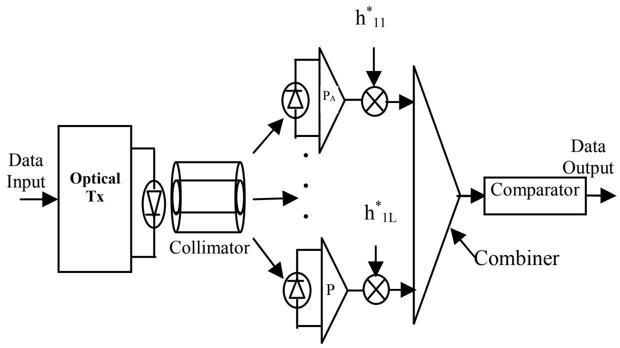

Fig. 1(b). Block diagram of FSO link with one transmitter and multiple receiver (SIMO).

where, x is the transmitted signal intensity, R_d is the detector responsivity and υ_n is the additive white Gaussian noise with variance σ_n, and h_{mn} is the normalized fading channel coefficient which models the channel from the m^{th} transmit aperture to the nth receive aperture. The transmitted signal x is either 0 or $2P_t$, where, P_t is the average transmitted optical power. The channel state h_{mn} is the attenuation due to geometric spread and pointing errors. By considering a Gaussian beam and a circular detection aperture of radius r, the pdf of h for a SISO channel is derived in [4] as:

$$f_h(\mathrm{h}) = \frac{\gamma^2}{A_0^{\gamma^2}} h^{\gamma^2-1}, \qquad 0 \le h \le A_0 \tag{2}$$

where, $\gamma = \omega_{zeq} / 2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation at the receiver, ω_{zeq} is the equivalent beam width at the receiver calculated using the relation:

$$\upsilon = \frac{\sqrt{\pi r}}{\sqrt{2\omega_z}}, \; \omega_{zeq}^2 = \omega_z^2 \frac{\sqrt{\pi} erf(\upsilon)}{2\upsilon \exp(-\upsilon^2)}, \; \mathrm{A}_0 = [\mathrm{erf}(\upsilon)]^2$$

here, ω_z is the beam waist i. e. radius calculated at e^{-2} at a distance z from the transmitter. The pointing error is defined as radial displacement between the detector and the received light beam on the detector plane.

III. AVERAGE BER

The transmitted optical power OOK signal is given by:

$$S(\mathrm{t}) = 2\,\mathrm{P}_t\, e^{j\omega_c t}.a_k \tag{3}$$

A_k is the k^{th} bit either 1 or 0. Where, P_t is the average transmitted optical power and ω_c is the angular carrier frequency. The received optical intensity at the input of n^{th} photodetector is given by:

$$r_n(\mathrm{t}) = \sum_{m}^{M} h_{mn}.S(\mathrm{t}) + n(\mathrm{t}); \quad \mathrm{n}=1, 2 \ldots \mathrm{N} \tag{4}$$

where, M is the number of transmitter and N is the number of receiver.

The output current on n^{th} receiver is given by:

$$i_{\theta_n}(\mathrm{t}) = R_d.r_n(\mathrm{t}) = 2R_d P_t \sum_{m=1}^{M} h_{mn}.a_k + i_n(\mathrm{t}) \tag{5}$$

For a MIMO FSO link the output current is given by:

$$i_0(\mathrm{t}) = \sum_{n=1}^{N} i_{on}(\mathrm{t}) = \sum_{m=1}^{M}\sum_{n=1}^{N} 2R_d P_t h_{mn}.a_k + i_n(\mathrm{t}) \tag{6}$$

and for a SIMO FSO link, the output current is given by:

$$i_0(\mathrm{t}) = \sum_{n=1}^{N} i_{on}(\mathrm{t}) = \sum_{n=1}^{N} 2R_d P_t h_{1n}.a_k + i_n(\mathrm{t}) \tag{7}$$

When $i_n(t)$ represent the output noise current with zero mean and variance σ^2_n is given by:

$$\sigma_n^2 = 4eR_d P_t B + 4kTB / \mathrm{R}_L \tag{8}$$

B is the bandwidth of receive LPF, K is the Boltzmann's constant and R_L is the receiver load resistance.

A. SISO FSO Link

For a SISO Link, M=1, N=1 and the signal to noise ratio at the output of receiver at a given value of h_{11}, can be represented as:

$$snr(\mathrm{h}) = \frac{\left(2R_d P_t h_{11}\right)^2}{\sigma_n^2} \tag{9}$$

The conditional bit error rate (BER) conditioned on a given value of h_{11} is given by:

$$P_b(\mathrm{h}) = 0.5\,\mathrm{erfc}\left[\sqrt{snr(\mathrm{h})} / 2\sqrt{2}\right] \tag{10}$$

The average BER can be obtained as:

$$BER = \int_0^{A_0} P_b(\mathrm{h}) f_h(\mathrm{h})\,\mathrm{dh} \tag{11}$$

B. SIMO FSO Link

For a SIMO Link with N output receiver we consider equal gain combining (EGC). The output of the combiner is given by equation (5) and signal to noise ratio at the output is given by:

$$\left[snr(\mathrm{h})\right]_{SIMO} = \frac{\sum_{n=1}^{N} (2R_d P_t h_{1n})^2}{\sigma_n^2} = \frac{(2R_d P_t)^2}{\sigma_n^2} \sum_{n=1}^{N} h_{1n}^2 \tag{12}$$

The conditional BER of a SIMO FSO link in presence of pointing error is given by:

$$P_b(\mathrm{h}) = 0.5\,\mathrm{erfc}\sqrt{snr(\mathrm{h})_{SIMO}} / 2\sqrt{2} \tag{13}$$

The average BER can be evaluated as:

$$BER = \int_0^{A_0} P_b(\mathrm{h})\, \mathrm{f}_h(\mathrm{h}) dh \tag{14}$$

The equation (14) can be re-written as:

$$BER_{,SIMO} = \int_h f_h(h)\, Q\left[\frac{\sqrt{2}R_d P_t}{\sigma_n \sqrt{N}} \sum_{n=1}^{N} h_n^2\right] dh_n \qquad (15)$$

The above expression is almost impossible to iterate, we use the approximation for the Q –function:

$$Q(x) \approx \frac{1}{12} e^{-x^2/2} + \frac{1}{4} e^{-2x^2/3} \qquad (16)$$

Thus the BER can be expressed as:

$$BER_{,SIMO} = \int_h f_h(h) \left[\frac{1}{12} e^{-\frac{R_d P_t}{N\sigma_N^2}\sum_{n=1}^{N} h_n^2} + \frac{1}{4} e^{-\frac{2R_d P_t}{3N\sigma_N^2}\sum_{n=1}^{N} h_n^2} \right] dh_n \qquad (17)$$

IV. RESULTS AND DISCUSSIONS

Following the analytical formulation presented in sec III, we evaluate the bit error rate performance of a free space optical link with pointing error considering SISO and SIMO link configurations. The noise standard deviation σ_n is taken to be 10^{-7}. The receiver diameter is taken to be 20 cm and the detector responsivity 0.8 A/w. The results are evaluated in terms of average BER for different values of normalized jitter standard deviation σ_s/r and normalized beamwidth ω_z/r. The plots of BER versus transmitter power P_t (dBm) are depicted in fig. 2 for $\omega_z/r=5$ considering SISO FSO links with σ_s/r as a parameter. It is noticed that system BER is sensitive to pointing jitter and the BER drastically increases with increase in normalized jitter standard deviation σ_s/r. However, BER decreases with increase in transmitter power. Similar results for $\omega_z/r=10$ are depicted in fig. 3 with σ_s/r as a parameter. Comparison of fig.2 and fig. 3 revels that the BER performance is further degraded with increase in beamwidth and a higher transmitter is required to achieve the same BER as ω_z/r is increases from 5 to 10.

The BER performance results for a SIMO free space optical link are depicted in fig. 4 and fig. 5 for two number of photodetector receivers with σ_s/r as a parameter with $\omega_z/r=5$ and 10 respectively. It is noticed that there are significant improvements in BER performance with two receivers compared to SISO FSO link performance. As noticed that at a given value of P_t(dBm), the BER is reduced from 10^{-3} to 10^{-5} at P_t(dBm)= -10 at $\sigma_s/r=0.6$ and $\omega_z/r=10$. Thus there are significant reductions in required transmitter power for SIMO FSO link compared for SISO FSO link for a given BER. The required transmitter power to achieve a given BER of 10^{-10} are plotted in fig. 6 and fig. 7 for SISO and SIMO FSO link as the function of σ_s/r with ω_z/r as a parameter. It is noticed that the system requires higher power as the jitter standard deviation σ_s/r is increased to cope up with the pointing error. Using multiple receiver (SIMO), the required transmitter power can further be reduced at given values of σ_s/r and ω_z/r. The SIMO system shows almost 6 dB improvement in receiver sensitivity at $\sigma_s/r=0.8$, $\omega_z/r=10$ and it is found to be higher at higher values of pointing jitter variance.

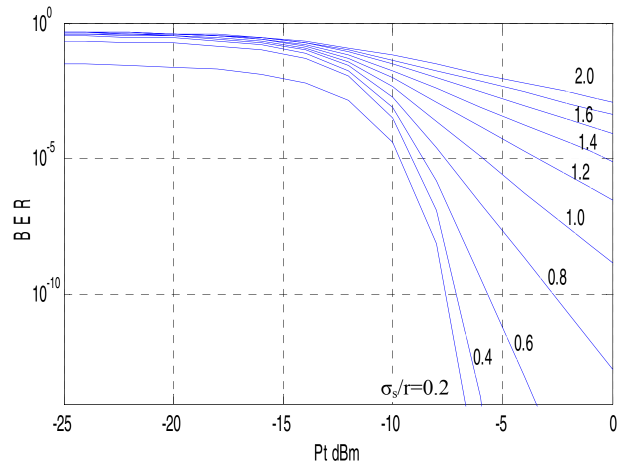

Fig. 2. BER vs. transmitted power at $\omega_z/r=5$ for various σ_s/r and L=1 (SISO)

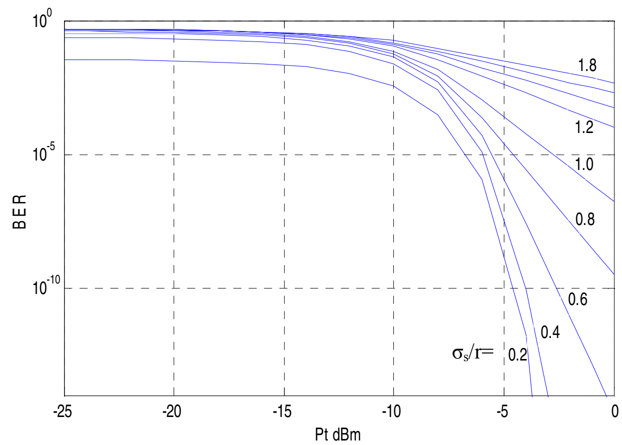

Fig. 3. BER vs. transmitted power at $\omega_z/r=10$ for various σ_s/r and L=1 (SISO).

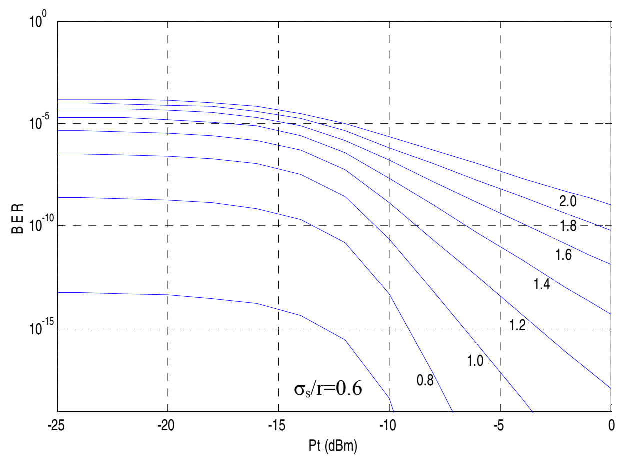

Fig. 4. BER vs. transmitted power at $\omega_z/r=5$ for various σ_s/r and L=2 (SIMO)

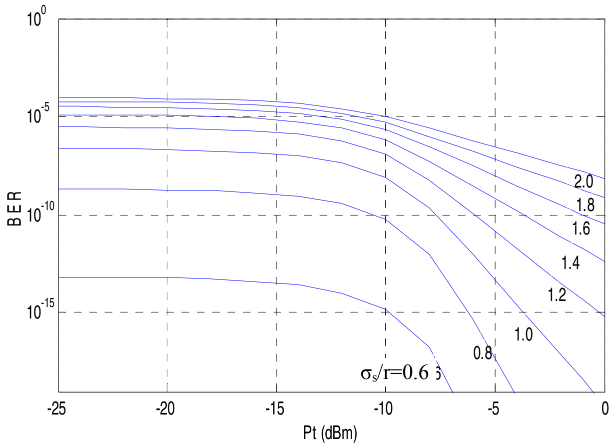

Fig. 5. BER vs. transmitted power at $\omega_z/r=10$ for various σ_s/r and L=2 (SIMO)

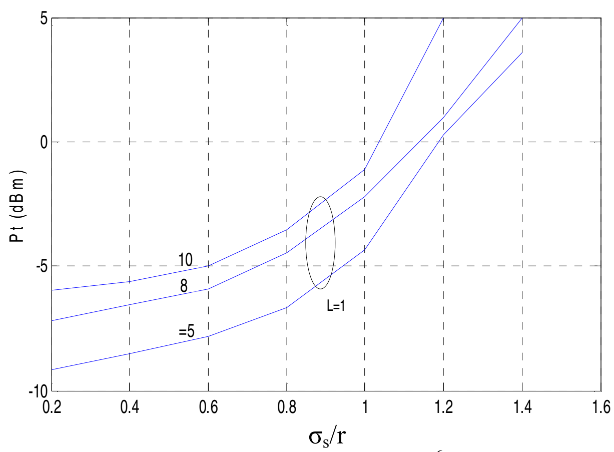

Fig. 6. P_t (dBm)vs. σ_s/r at BER=10^{-6} for SISO System

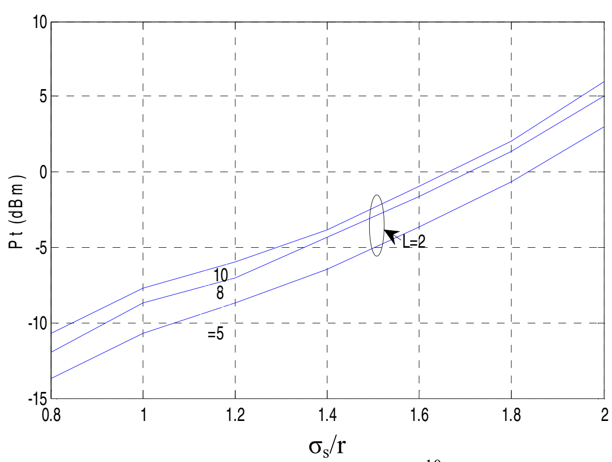

Fig. 7. P_t (dBm) vs. σ_s/r at BER=10^{-10} for SIMO system.

V. CONCLUSIONS

An analytical approach is presented to evaluate the effect of pointing error in a SISO and SIMO free space optical links in the presence of pointing error. It is noticed that, system suffers significantly due to pointing error and there is penalty due to pointing error which is higher at higher values of jitter variance and beamwidth of source. The penalty can be reduced by 6 to 10 dB by using multiple receivers at a given system performance.

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