

# A Case Study of Satellite Navigation Reliability: Bangladesh Perspective

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**Abstract**—Space-based navigation was not only the first method of long-distance navigation but is the most advanced today. It proved superior to any other guidance system as it is all weather, immune to precipitation static and propagates predictably through the atmosphere. GPS (Global Positioning System), GLONASS (Global Navigation Satellite System), Galileo are the systems that are now harbinger of navigation. In the field of navigation achieving reliability is one of the most concerns of any system. Reliability is the consistency of the solutions. A study of the satellite navigation system reliability is carried out in this paper in the context of Bangladesh. A frequency analysis has been done and shown that how multiple number of frequency reduce biases hence improve reliability.

**Keywords**—GPS, Galileo, Reliability, Bias, Frequency and Interoperable.

## I. INTRODUCTION

For the last 70 years the navigator relied on the radio transmission located at land. A new navigation system based on satellites began development in 1970s. Space-based navigation was not only the first method of long-distance navigation but is the most advanced today. It proved superior to any other guidance system as it is all weather, immune to precipitation static and propagates predictably through the atmosphere. The first attempt at satellite navigation was initiated by US with introducing NAVSTAR GPS for military purpose primarily. GPS provides continuous positioning and timing information, anywhere in the world under any weather conditions. Because it serves an unlimited number of users as well as being used for security reasons, GPS is a one-way-ranging (passive) system.

GLONASS, the second satellite navigation system was developed by USSR from mid 1970s, in parallel to GPS. Following the dissolution of Soviet Union, GLONASS development was continued by Russia. Galileo is in its development phase which is initiated by European Union and its initial operational capability is planned for 2010–2012 with full operational capability by 2014.

A number of studies have been done on the navigation system analysis in respect of availability of satellite, biases, and outliers. In this paper three cases (GPS, Galileo and interoperable GPS-Galileo system) has been taken. For the analyzing purposes the co-ordinates of Bangladesh has been

chosen. So, all the simulation result is specific for Bangladesh. For each case the reliability of the navigation system solution has analyzed for different frequency. Design parameters, which don't need any actual observations, are considered in this purpose. Parameters considered are internal reliability, represented as minimal detectable bias (MDB) and external reliability represented as bias-to-noise ratio (BNR).

## II. CASE STUDY

### A. Case 1: Standalone GPS

To ensure continuous worldwide coverage, GPS satellites are arranged so that four satellites are placed in each of six orbital planes. With this constellation geometry, four to ten GPS satellites will be visible anywhere in the world, if an elevation angle of  $10^\circ$  is considered. Our first case comprises navigation reliability analysis of GPS with its different frequency bands. GPS works in three frequency bands: where the carrier frequencies are: 1575.42 MHz for L1, 1227.80 MHz for L5 and 1176.45 MHz for L5.

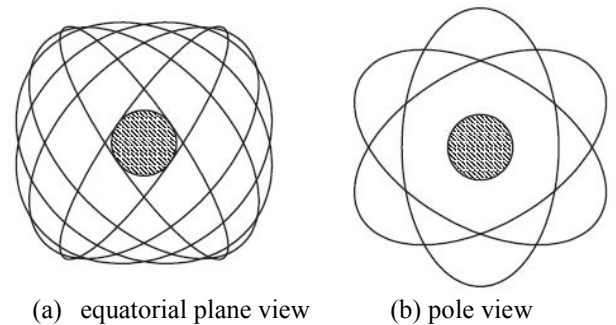


Figure 1: GPS satellite orbit

### B. Case 2: Standalone Galileo

The fully deployed Galileo system consists of 27 operational satellite and 3 satellites at active sphere. So far four operational satellites launched, which is minimum requirement for navigation solution. Galileo broadcasts 10 different navigation signals across three frequency bands: E5, E6, and E1-L1-E2. The E5 band is centered at 1191.795 MHz. It is partitioned into E5a and E5b sub bands, with carrier frequencies of 1176.45 and 1207.14 MHz, respectively. The E6

and E1-L1-E2 bands are both 40.92 MHz wide and centered at 1278.75 and 1575.42 MHz, respectively.

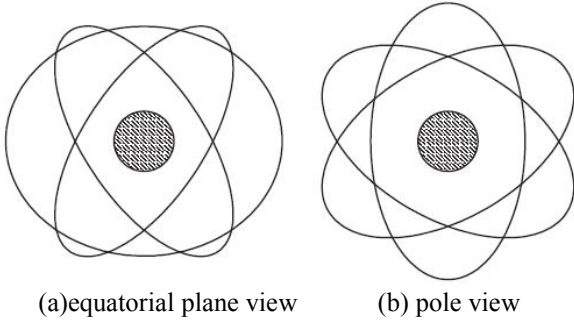


Figure 2: Galileo satellite orbit

### C. Case 3: Interoperability: GPS and Galileo

Interoperability comprises combining information such as navigation data, carrier measurements taken from two or more satellite system simultaneously. While combining information three parameters must be taken into account. These are: signal in space; geodetic coordinate reference frame and time reference frame. Galileo is made with the provision that it is interoperable with GPS. GPS and Galileo share two frequency bands, L5/E5a and L1. Both service broadcast navigation message using Code Division Multiple Access (CDMA) with high chipping rate and moderate code lengths, so they can coexist without a problem. The GALILEO Terrestrial Reference Frame (GTRF) shall be an independent realization of the International Terrestrial Reference System (ITRS). WGS-84 is the coordinate reference frame for GPS. WGS-84 is also a realization of the ITRS. The differences between WGS-84 and the GTRF are expected to be on the order of a few centimeters. This implies that the WGS-84 and GTRF will be identical within the accuracy of both realizations (i.e., the coordinate reference frames are compatible). In our case-3 the simulation or navigation reliability is done for the sharing frequencies of these two systems. Here the constellations of these two systems are used separately.

### III. POSITIONING SOLUTION OF GNSS

Pseudo range code  $P$  and carrier range code  $\Phi$  are the observations of satellite navigation. Observation can be of single difference (observation of single receiver) and double difference (observations collected by two receivers). Considering Double Difference (DD) observation equations for satellite  $r$  and  $s$  can be given by:

$$p_i^{rs}(t) = \rho^{rs}(t) + \frac{f_{L1}^2}{f_i^2} I^{rs}(t) + e_i^{rs}(t) \quad (1)$$

$$\Phi_i^{rs}(t) = \rho^{rs}(t) - \frac{f_{L1}^2}{f_i^2} I^{rs}(t) + \lambda_i N_i^{rs} + \epsilon_i^{rs}(t) \quad (2)$$

These equations are applicable in satellite navigation. Unknown parameters are the DD satellite-receiver range,  $\rho$ , the DD ionospheric effect,  $I$ , the DD integer carrier ambiguities,  $N$ , the measurement noise,  $e$  and  $\epsilon$ , of code and phase respectively. The carrier wavelengths are denoted by,  $\lambda$ . And  $i$  is the frequency any L-band/E-band frequency. Geometry free (GF) baseline model is considered, where observation equations

remain parameterized in terms of the unknown DD receiver-satellite ranges.

The three baseline models for  $k$  epochs of data can all be written in a form as:

$$y = [I_k \otimes M \quad e_k \otimes N] + n$$

The measurement models observed so far can be applied for both GPS and Galileo observations. For the interoperability and compatibility property of both the GPS and Galileo it would be interesting to observe at the model for the receivers that collect observations of both systems. The baseline unknowns in the GB models are common for both systems, but of course the ambiguities are not. The combined measurement model for one epoch is written as:

$$\begin{pmatrix} y_{GPS} \\ y_{GAL} \end{pmatrix} = \begin{pmatrix} M_{GPS} & N_{GPS} \\ M_{GAL} & N_{GAL} \end{pmatrix} \begin{pmatrix} b \\ a_{GPS} \\ a_{GAL} \end{pmatrix} + n \quad (3)$$

In probability theory, a purely stochastic system is one whose state is non-deterministic so that the subsequent state of the system is determined probabilistically. Any system or process that must be analyzed using probability theory is stochastic at least in part. The variance-covariance (vc) matrix of the single differenced (SD) observations of one satellite without eliminations of the ionospheric parameters is given by:

$$C_{p\phi} = \begin{pmatrix} C_p & \\ & C_\phi \end{pmatrix}$$

where,  $C_p$  and  $C_\phi$  are the vc- matrices of the code and phase observations respectively. The variance-covariance (vc) matrix of DD satellite observations is:

$$Q_y = I_k \otimes C \otimes E$$

The vc-matrix corresponding to the combined measurement model of GPS and Galileo is given by,

$$Q_y = I_k \otimes \begin{pmatrix} C_{GPS} \otimes E_{GPS} & \\ & C_{GAL} \otimes E_{GAL} \end{pmatrix} \quad (4)$$

### IV. NAVIGATION RELIABILITY MODEL

Reliability is the description of property performing a certain function without failure under given conditions for a specified period of time. Actually the definition of the reliability will in any case have to pertain to variants, either observation variants or estimators' variants. To avoid possible confusion between the definitions of reliability of pertaining two groups, we shall use the terms "Internal reliability" and "External reliability" respectively.

Internal reliability refers to the extent to which a measure is consistent within itself. It is the ability to detect outliers. Internal reliability is dictated by the lower bound for detectable outliers and is expressed by the Minimal Detectable Biases (MDBs).

Corresponding size of the bias can be obtained as:

$$|\nabla| = \sqrt{\frac{\lambda_0}{c^T Q_y^{-1} P_A^{-1} c}} \quad (5)$$

$$P_A^{-1} = I - P_A = I - A(A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1}$$

$I$  is the identity matrix,  $P_A$  is the orthogonal projector on the range space of  $A$ ,  $\lambda_0$  is non-centrality parameter which depends on the chosen values of the level of confidence (the probability of rejecting  $H_0$  when in fact it is true), and the detection power (the probability of accepting  $H_a$  correctly).

External reliability refers to the extent to which a measure varies from one user to another. It is applied to determine the effect of undetected gross and/or systematic error of observation on adjusted coordinate. BNR is a dimensionless parameter, binding the effect of a single observation on all coordinates.

$$\nabla \hat{x} = (A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} c \nabla = Q_{\hat{x}} A^T Q_y^{-1} c \nabla$$

BNR is the square root of  $\nabla \hat{x}$ :

$$\lambda_{\hat{x}} = \|\nabla \hat{x}\|_{Q_{\hat{x}}}^2 = (\nabla \hat{x})^T Q_{\hat{x}}^{-1} (\nabla \hat{x}) \quad (6)$$

## V. SIMULATION REQUIREMENTS

A Matlab® software tool, VISUAL, is used to analyze the result. The input parameters chosen for the simulations are:

- System: GPS, GALILEO and combined GPS-GALILEO;
- Almanac: a Yuma almanac file for current GPS and GALILEO ;
- Time and date: 29-11-2013 0:00-0:24h ;
- Number of epochs: 1-300s;
- Frequencies : 1, 2 or 3;
- Cutoff elevation: 15° ;
- Ionospheric model: weighted,  $s=0.02m$  ;
- Tropospheric model: fixed;
- Receiver type: Stationary;
- Baseline model: Geometry Free;
- Location: Bangladesh 24° N, 90° E

## VI. SATELLITE AVAILABILITY

The number of satellite available is one of the very basic parameter while evaluating the performance of GNSS. The precision of any position solution is always dependent on the visible satellite available in the user range. More number of visible satellites increases the probability of a better solution. Satellite availability depends on the geometry of the satellites, location and time. When the satellite geometry is strong good positioning accuracy can be obtained. Depending upon satellite

constellations number of visible satellite available varies with time at specific place. The constellations which contain more number of satellites will consequently provide more number of satellites. So, obviously combined GPS-Galileo constellation will provide more availability than any of the combined system.

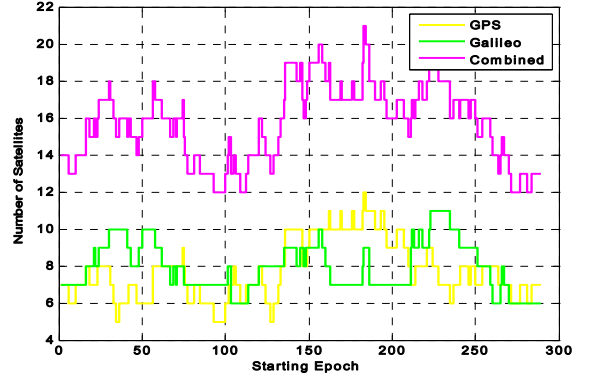


Figure 3: Satellite availability at the day of simulation in Bangladesh

In case of GPS, minimum four satellites are required for positioning accuracy. For Bangladesh, at that day from figure it is obtained that for GPS the lowest number of satellite is 5 (five), for Galileo it is 6 (six) and for combined system it is 12(twelve), twice the number of any of them.

## VII. RESULT ANALYSIS

To study internal reliability MDB for code outlier is evaluated for different number of frequency in three combinations and the difference is calculated. Mainly MDB depends on the precision of code data, the number of satellite tracked  $m$  and through  $\delta$  on the presence of a second/third frequency. The size of model error that can be detected with a certain power and level of significance (quantified by the MDB) decreases with increasing number of used epochs and available frequencies. External reliability is concerned with the effect of undetected outliers upon the final solution. So the probability of detecting outliers can be quantified if they do occur and the effect they will have if they are not detected. BNR is evaluated to analyze the effect of single observation on all coordinates.

### A. MDB Analysis

Through simulation the impact on reliability due to the increased number of available frequencies and the improved code accuracy is investigated. As MDB is the bias so decreasing MDB indicates improved reliability.

It is obtained that in case-1, for single frequency the error is so large and is on the order of 150 meter. Sometimes gaps are found in the graph which indicates worst reliability. Adding a second frequency provides more reliability by decreasing MDB in an efficient manner where it reduces to a range of 2 ~ 3. When a combination of triple frequency is considered it is found that MDB decreases to minimum which is less than 2.5 providing better reliability. This happens

because with increasing the number of frequency increases the code accuracy which effectively reduces the error.

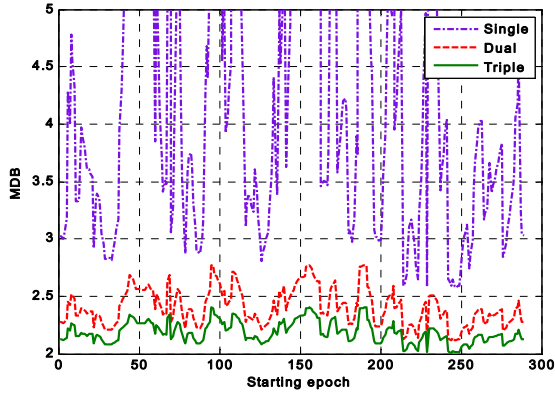


Figure 4: MDB for case-1

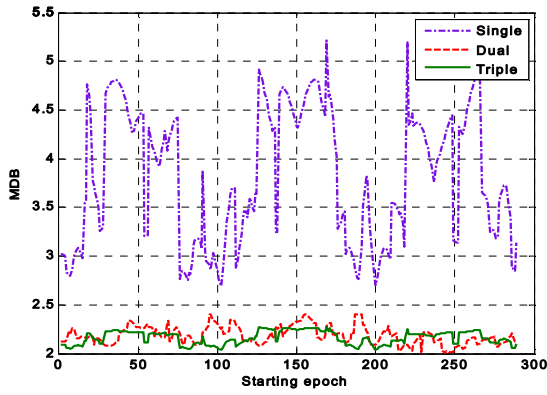


Figure 5: MDB for case-2

In case-2 the result is different than case-1. The MDB for single frequency combination is very much less than that single frequency combination and lies within 5.5. No gap is found in the graph which indicates a better reliable system than case-1.

The reason behind the better reliability is the even distribution of Galileo (case-2) satellites, where GPS satellites are divided unevenly over six planes. Also due to the lower precision assigned to the GPS observations, the detect ability of code outlier is far worse than Galileo.

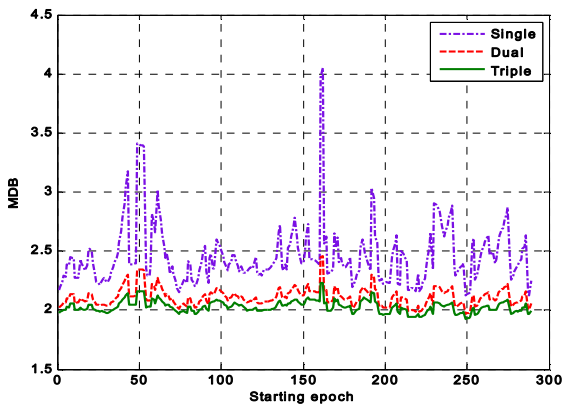


Figure 6: MDB for case-3

Now case-3 which shows combined system provides more satellite availability as the combined constellation contains more satellites. Here the improvement in reliability compared to Galileo is marginal. But considering the overall comparison, from case-3 triple frequency combined combination can provide possible best solution. The overall reliability data for Bangladesh is shown in Table I.

TABLE I. COMPARISON OF MDB

Case	Frequency	Error bounds	Remarks
Case-1 (GPS)	Single	2.6~15	Highest MDB worst reliability
	Dual	2.1 ~ 2.8	Low reliability; MDB decreases than single system
	Triple	2 ~ 2.45	MDB decreases than dual combination
Case-2 (Galileo)	Single	3~5.2	Better reliability than GPS single combination.
	Dual	2 ~ 2.6	MDB lower than GPS dual combination
	Triple	2 ~ 2.3	MDB decreases than dual combination
Case-3 (Combined)	Single	2.3~4.1	Best reliability than any single frequency system
	Dual	1.95~2.35	Improves up to 19% than present GPS dual combination
	Triple	1.95 ~ 2.2	Improves up to 27% than present GPS dual combination

### B. BNR Analysis

BNR shows the MDB effect. BNR simulation is carried out for one point calculation. For temporal variation BNR represents the MDB effect of the systems.

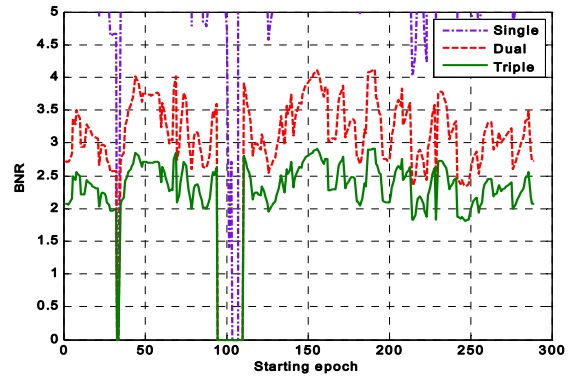


Figure 7: BNR for case-1

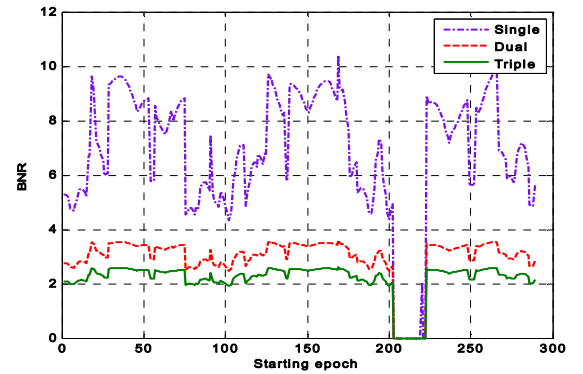


Figure 8: BNR for case-2



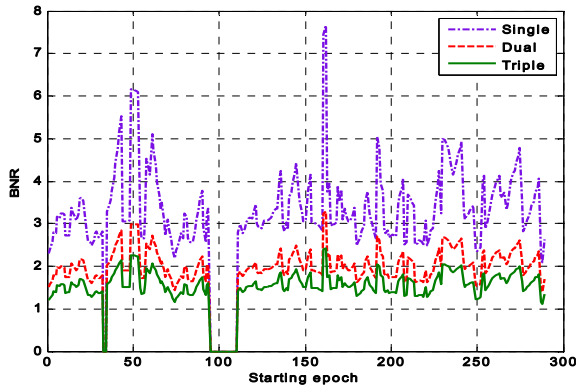


Figure 9: BNR for case-3

The BNR in case-1 for single frequency is mainly in the order of 1000 meter, which reveals the worst effect can be caused by MDB. As the MDB of dual frequency is higher than triple frequency, the BNR of dual frequency is also higher than triple frequency.

From the simulation of case-2 (Galileo), it has been found that the BNR for single frequency is in the order of 10 meter, far less than GPS. In every case BNR reduced that is effect of MDB is much less than case-1 (GPS). In case-3 (Combined system) triple frequency combination reduced the BNR to less than 2. Analysis reveals BNR for a triple frequency combined GPS-Galileo system from case-3 is the best of all of them. Table II shows the comparison of BNR in different case.

TABLE II. COMPARISON OF BNR

Case	Frequency	Error bounds	Remarks
Case-1 (GPS)	Single	1~100	Higher MDB causes BNR to be high
	Dual	1~8	MDB decreases so as BNR
	Triple	0.5~5	BNR much lower than dual combination
Case-2 (Galileo)	Single	5~10	Better solution than GPS single combination.
	Dual	0.2~1.8	BNR lower than GPS dual combination
	Triple	0.2~1.2	BNR decreases than dual combination
Case-3 (Combined)	Single	1.5~7.5	Best reliability than any single frequency system
	Dual	0.2~1.2	Improves up to 66% than present GPS dual combination
	Triple	0.1~0.8	Improves up to 150% than present GPS dual combination

## VIII. CONCLUSION

An analysis of reliability using various frequency combinations over Bangladesh has been done. In this region lowest error bounds is 1.95 in case of internal reliability and 0.1 in case of external reliability. In any time the combined system gives highest number of satellite than any other system. It is shown that multiple frequency reduces biases thus improve reliability. It also reveals how increasing frequency reduces error. Also combinations of system enhance reliability. It is shown that when using triple frequency gives the best solution.

In the context of Bangladesh, analyzing the data and simulation it is found that reliability improves 150% than present GPS dual solution.

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## REFERENCES

- [1] Principles of GNSS, Inertial and Multisensor Integrated system. Paul D. Groves, Artech House Inc, 2008.
- [2] Understanding GPS, Principles and Applications. Elliott D. Kaplan, Christopher J. Hagerty, Artech House, Inc, 2006.
- [3] Hawkins D.M. Identifications of outliers, Chapman & Hall, London/ New York, 1980.
- [4] Gao Y. Reliability assurance for GPS integrity test, ION GPS 1992, Salt Lake City, Utah, pp 567-574, September 22-24. 1992.
- [5] Teunissen, P.J.G. Minimal detectable biases of GPS data. Journal of Geodesy, vol. 72, 236-244, 1998.
- [6] Salgado, G., S. Abbondanza, R. Blondel, and S. Lannelongue, Constellation availability concepts for Galileo. Proc. Of ION NTM, Long Beach CA, pp 778-786, January 22-24. 2001.
- [7] Verhagen, S. Performance Analysis of GPS, Galileo and Integrated GPS-Galileo, ION GPS 2002, Portland, Oregon, September 24-27. 2002, 2208-2215.
- [8] Verhagen, S. A New Software tool: Studying the Performance of Global Navigation Satellite Systems. GPS World June 2002.
- [9] Dinwiddie, S. E., E. Breeuwer, and J. H. Hahn, "The Galileo System," Proc. ENC-GNSS 2004, Rotterdam, Netherlands, 2004.
- [10] The U.S. Coast Guard Navigation Centre website. Current almanac file. [Online]. Available: <http://navcen.uscg.gov/?pageName=gpsAlmanacs>.