

# Performance Analysis of OFDM based Decode-and-Forward relay over Nakagami fading channel

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**Abstract**—Cooperative communication is a promising technology for significant efficient spectrum uses in future wireless communication. This paper derives the signal to noise interference ratio (SNIR) and average bit error rate (BER) for orthogonal frequency division multiplexing (OFDM) based decode and forward (DF) relay scheme over Nakagami fading channels. The resulting expressions are single relay with the presence of frequency offset and phase noise. Numerical results are provided to show that of OFDM based DF relay scheme with diversity combining technique is better performance in cooperative wireless communication.

**Keywords**— Orthogonal Frequency Division multiplexing (OFDM); Decode and Forward (DF); Nakagami-m fading; Equal Ratio Combining (ERC) and Signal to Noise Interference Ratio (SNIR).

## I. INTRODUCTION

OFDM based cooperative communication is promising the high data-rate coverage make sure a wide range of multimedia services such as speech, image, and data transmission in upcoming wireless communications systems. There are two major benefits of this technology; mitigate the fading in wireless communication transmission and achieved cooperative relay protocols. In this paper OFDM based DF relay scheme have been proposed for wireless networks [1]. The basic ideas of OFDM based DF relay nodes receive the information symbols and decode the original message ensure significant performance gains in terms of link reliability, spectral efficiency, system capacity and transmission range [1].

Performance analysis of OFDM based DF relay scheme cooperative communication yielded many interesting results including signal-to-noise interference ratio (SNIR), with diversity, without diversity, and average bit error rate (BER) expressions over Nakagami-fading channels. The Nakagami-m fading channel is regularly employed to model the multipath fading wireless communication broadcast in municipal regions, wherever the arbitrary fluctuations of the instantaneous received signal control are extremely common and fast [2].

The present study is to develop a mathematical model and simulation model of OFDM based DF relay cooperative wireless communication system with single number of relays that uses equal ratio combining (ERC) technique at the destination for both cases such as with diversity and without diversity. The performance analysis is based on the average bit error rate (BER) and signal to noise interference ratio (SNIR). In addition, we have also evaluated the result of frequency offset and phase noise in OFDM based DF relay scheme.

## II. COMMUNICATION SYSTEM MODEL

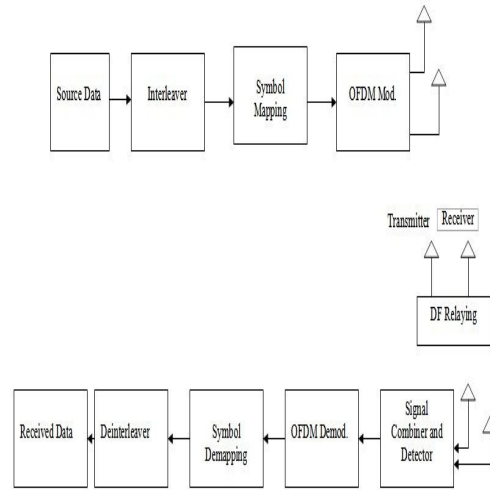


Fig. 1. Block diagram of an OFDM based cooperative DF Relay wireless communication scheme

The block diagram of the simulated OFDM based cooperative DF Relay wireless communication scheme is shown in Figure 1. In such communication systems the source data are simultaneously transmitting. The source data are converted into binary bits and these information bits are multiplied with Interleaver. The interleaved information bits are modulated digitally by M-Quadrature Amplitude Modulation (QAM).

According to the symbol are then sent up into OFDM modulation. Finally the signal is fed to digital to analog converter (DAC) followed by high power amplifier (HPA) and up converter before transmission. In OFDM based DF relay scheme, relay receives the information symbols and translates the original information [1]. The symbols are afterward forward into two ways in phase I (source to destination) and phase II (relay to destination) [1]. Then it forwards the information to the signal combiner and detector. In receiving section, the transmitted signals are detected with linear signal detection schemes and the detected signals are subsequently passed low noise amplifier (LPA) and converted from analog to digital using the analog to digital converter (ADC) and transferred by the parallel to serial both antenna using OFDM demodulator. The retrieved signals processed through the demapped, deinterleaved and recover the transmitted original information.

### III. THEORETICAL ANALYSIS OF DECODE AND FORWARD (DF) RELAY SCHEME

In OFDM based decode and-forward (DF) relaying scheme, the relay at initial translate the received information then ahead the received information to the destination. In Phase 1, the source-relay and the source-destination received signals is expressed as

$$\begin{aligned}
 y_{k,s-r} &= \sum_{l=0}^{N-1} x_{l,s} H_{l,s-r} Q_{l-k} + w_r \\
 &= x_{k,s} H_{k,s-r} Q_0 + \sum_{\substack{l=0 \\ l \neq k}}^{N-1} x_{l,s} H_{l,s-r} Q_{l-k} + w_r \\
 &= x_{k,s} H_{k,s-r} Q_0 + I_{ICI}(s-r) + w_r \\
 y_{k,s-d} &= \sum_{l=0}^{N-1} x_{l,s} H_{l,s-d} Q_{l-k} + w_d^{(1)} \\
 &= x_{k,s} H_{k,s-d} Q_0 + \sum_{\substack{l=0 \\ l \neq k}}^{N-1} x_{l,s} H_{l,s-d} Q_{l-k} + w_d^{(1)}
 \end{aligned} \tag{1}$$

*Inter-Carrier interference (ICI):* ICI is the crosstalk involving adjacent sub channels or frequency bands of the equal FFT [3].

$$\begin{aligned}
 \text{Where } I_{ICI}(s-d) &= \sum_{\substack{l=0 \\ l \neq k}}^{N-1} x_{l,s} H_{l,s-d} Q_{l-k} \\
 &= x_{k,s} H_{k,s-d} Q_0 + I_{ICI}(s-d) + w_d^{(1)}
 \end{aligned} \tag{2}$$

Here  $x_l$ ,  $H_{k,s-r}$  and  $H_{k,s-d}$  are the transmitted signal, channel coefficients of the source to relay (s-r) links and the source to destination(s-d) links correspondingly. The terms  $w_r$  and  $w_d^{(1)}$  is the additive white Gaussian noise (AWGN) at the relay and the destination respectively.  $\varepsilon$  is the normalized frequency offset and is given by  $\Delta f T$ .  $T$  is the subcarrier symbol period.

$Q_L$  is defined as follows [4],

$$\begin{aligned}
 Q_L &= \frac{1}{N} \sum_{n=0}^{N-1} e^{j l \left[ \frac{2\pi}{N} (L+\varepsilon) n + \varphi(n) \right]} \\
 &= \exp[j \{ 2\pi (L+\varepsilon) + \varphi \} (1/2 - 1/2N)] \frac{\sin[ \{ 2\pi (L+\varepsilon) + \varphi \} / 2]}{N \sin[ \{ 2\pi (L+\varepsilon) + \varphi \} / 2N]}
 \end{aligned} \tag{3}$$

OFDM based DF relay scheme, the received signal at the destination in Phase 2 can be expressed as

$$\begin{aligned}
 y_{k,r-d} &= \sum_{l=0}^{N-1} x_{l,r} H_{l,r-d} Q_{l-k} + w_d^{(2)} \\
 &= x_{k,r} H_{k,r-d} Q_0 + \sum_{\substack{l=0 \\ l \neq k}}^{N-1} x_{l,r} H_{l,r-d} Q_{l-k} + w_d^{(2)}
 \end{aligned}$$

Where  $I_{ICI}(r-d) = \sum_{\substack{l=0 \\ l \neq k}}^{N-1} x_{l,r} H_{l,r-d} Q_{l-k}$

$$= x_{k,r} H_{k,r-d} Q_0 + I_{ICI}(r-d) + w_d^{(2)} \tag{4}$$

Where  $|x_{k,r}|^2$  is the relay broadcast power,  $H_{k,r-d}$  is the channel coefficient of the relay-to-destination links and  $w_d^{(2)}$  is the AWGN of relay to destination links. The information's incoming at the destination can be exploited for recognition of OFDM based DF relay scheme of with or without diversity combining.

#### A Case 1: Without Diversity

In OFDM based DF relay scheme without diversity technique, the destination receives only the signal from relay. Using equation (4), SNIR can be written as

$$\gamma_{SNIR\_WOD} = \frac{|x_{k,r}|^2 |H_{k,r-d}|^2 |Q_0|^2}{P_{ICI}(r-d) + \sigma_d^2} \tag{5}$$

#### B Case 2: With Diversity

In OFDM based DF relay scheme with diversity combining technique, the signals received in source to destination links and relay to destination links, be able to be optimally combined at the destination with equal ratio combining (ERC) to acquire the output signal is written as [5],

$$\begin{aligned}
 \tilde{y}_{k,d\_WD\_DF} &= y_{k,s-d} + y_{k,r-d} \\
 &= x_{k,s} H_{k,s-d} Q_0 + I_{ICI}(s-d) + w_d^{(1)} \\
 &\quad + x_{k,r} H_{k,r-d} Q_0 + I_{ICI}(r-d) + w_d^{(2)}
 \end{aligned} \tag{6}$$

The effectual average SNIR at the output of the OFDM based DF relay scheme in ERC is written as

$$\gamma_{SNIR\_WD} = \frac{|x_{k,s}|^2 |H_{k,s-d}|^2 |Q_0|^2}{P_{ICI}(s-d) + \sigma_d^2} + \frac{|x_{k,r}|^2 |H_{k,r-d}|^2 |Q_0|^2}{P_{ICI}(r-d) + \sigma_r^2} \tag{8}$$

For an OFDM based DF relay scheme in Nakagami M-QAM fading Channel, the probability density

function (PDF) of the instantaneous channel of SNIR  $\gamma$  is given by [6]

$$f_{\gamma}(\gamma_w)_{DF} = \left(\frac{m}{\gamma_w}\right)^m \frac{\gamma_w^{m-1}}{\Gamma(m)} e^{-\frac{m\gamma_w}{\gamma_w}}, \gamma_w \geq 0 \quad (9)$$

Where  $\gamma_w$  = average SNIR of OFDM based DF relay scheme. This OFDM based DF relay scheme of average BER is able to be calculated accurately as the ratio of the average number of bits in error above the total number of transmitted bits [6]:

$$BER_{avg_{DF}} = \frac{\sum_{k=1}^K b_k X_{k_{DF-QAM}}}{\sum_{k=1}^K b_k \delta_k} \quad (10)$$

Where  $\delta_k$  likelihood to the effectual is obtain SNIR of OFDM based DF relay scheme in the  $k^{th}$  separator province can be written as

$$\delta_{k_{DF}} = \int_{\gamma_n}^{\gamma_{n+1}} f_{\gamma}(\gamma_w)_{DF} d\gamma_w \quad (11)$$

And  $X_k$  is the BER of the normal SNIR of OFDM based DF relay scheme province which is determined as

$$X_{k_{DF-QAM}} = \int_{\gamma_n}^{\gamma_{n+1}} X_{M_k, QAM}(\gamma_w) f(\gamma_w)_{DF} d\gamma_w \quad (12)$$

Where  $X_{M_k, QAM}(\gamma_w)$  is the BER of an OFDM based DF relay scheme of M-QAM in an AWGN channel, with coherent detection and Gray coding is written as [7]

$$X_{M_k, QAM}(\gamma_w) = \sum_i A_i Q(\sqrt{a_i \gamma_w}) \quad (13)$$

$$\text{Where } Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-\frac{x^2}{2}} dx \quad (14)$$

This equation is the average Gaussian Q-function defined is the conventional OFDM based DF relay scheme of SNIR,  $A_i$  and  $a_i$  is invariables particular to the QAM constellation [7]. Employing (9), (12) and (13) term for  $X_k$  can be consequent as

$$\begin{aligned} X_{k_{DF-QAM}} &= \frac{m}{\gamma_w} \sum_i A_i \int_{\gamma_n}^{\gamma_{n+1}} Q(\sqrt{a_i \gamma_w}) \gamma_w^{m-1} e^{-\frac{m\gamma_w}{\gamma_w}} d\gamma_w \\ &= \sum_i A_i \left\{ -e^{-\frac{m\gamma_w}{\gamma_w}} Q(\sqrt{a_i \gamma_w}) \sum_{i=0}^{r-1} \left( \frac{m\gamma_w}{\gamma_w} \right)^i \frac{\gamma_w^{m-1}}{\Gamma(i+1)} \right\}_{\gamma_n}^{\gamma_{n+1}} + \sum_{i=0}^{r-1} X_i(a_i, \gamma_w) \Big|_{\gamma_n}^{\gamma_{n+1}} \end{aligned} \quad (15)$$

Substituting  $X_{k_{DF-QAM}}$  and  $\delta_{k_{DF}}$  in equation (10) and (15)

the OFDM based DF relay scheme of average BER can be achieved.

The average BER performance of an OFDM based DF cooperative relay scheme under M-QAM over Nakagami fading channels where the assumption of frequency offset and phase noise of OFDM based DF cooperation is derived. While the OFDM based DF relay scheme has the benefit above the OFDM based AF relay scheme in dropping the effects of additive noise at the relay, the DF relay scheme difficulty will be improved to promise accurate signal detection.

#### IV. RESULTS AND DISCUSSION

We have conducted computer simulations to evaluate the average BER performance analysis of OFDM based DF relay scheme of cooperative wireless communication. The graphical illustrations presented in Figure 2 through Figure 4, show system performance comparison with implementation of OFDM based DF relay scheme of Nakagami fading parameter under various frequency offset and phase noise.

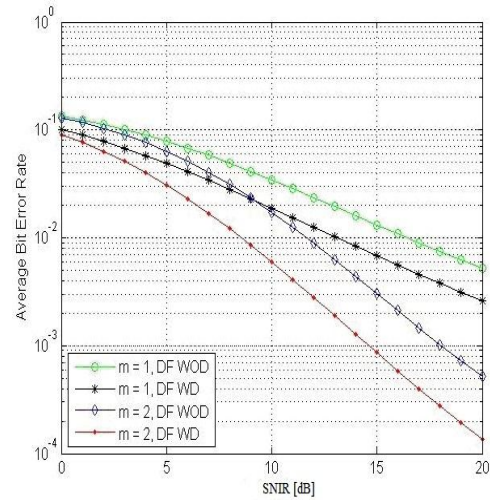


Fig. 2. Effect of Nakagami fading parameter on average BER for DF scheme

Figure 2 indicates the effect of average probability of error for DF relay scheme of Nakagami fading parameter. For a given fading parameter  $m$ , average bit error rate decreases as SNIR increases. For example, at SNIR = 15dB,  $m = 1$  and  $m = 2$  for DF scheme with diversity value of average bit error rate approximately are  $0.67 \times 10^{-2}$  and  $0.86 \times 10^{-3}$ .

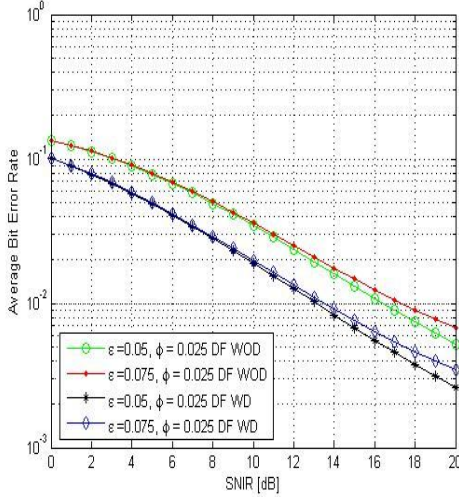


Fig. 3. Effects of frequency offset on average BER in DF scheme

Figure 3 shows the performance analysis of average BER vs. SNIR in DF relay scheme. DF with lower frequency offset is better than higher frequency offset. For a typically assumed SNIR value of 15 dB, the average BER values are approximately 0.006798, 0.01315, 0.007625 and 0.01469 in the case of  $\varepsilon = 0.05$  DF WD, DF WOD and  $\varepsilon = 0.075$  DF WD, DF WOD schemes. It is observable that at very high SNIR, the various relay schemes performance is comparatively better under deployment of the OFDM based cooperative DF relay scheme. However, the best performance is that of the  $\varepsilon = 0.05, \phi = 0.025$  DF WD relay scheme.

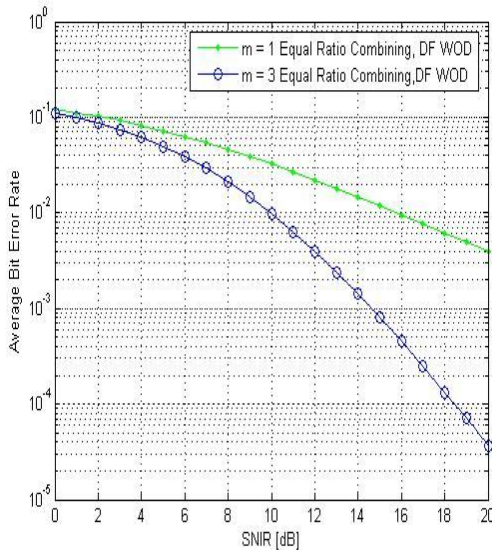


Fig. 4. Average bit error rate of the Cooperative OFDM system under implementation of DF without diversity relaying protocols with equal ratio combining technique of Nakagami fading parameter.

Figure 4 shows of Average bit error rate of the cooperative OFDM system under implementation of DF without diversity relaying protocols with equal ratio combining technique of Nakagami fading parameter. It is observed that the system with higher order Nakagami fading parameter ERC has been better performance. For example, when SNIR value of 16 dB,  $m=1$  Equal Ratio Combining, DF WOD and  $m=3$  Equal Ratio Combining, DF WOD in the system have the value of BER are approximately 0.00045 and 0.009572 respectively. OFDM based DF relaying protocol with ERC has the least approximately 20.40 dB improvement.

## V. CONCLUSION

A detailed theoretical analysis is carried out to evaluate the effects of frequency offset and phase noise in terms of SNIR and average BER in OFDM based DF relay cooperative communication scheme over Nakagami- $m$  fading channel. In the content of system performance, it can be concluded that the implementation of M-QAM Nakagami fading channel in OFDM based DF relay scheme over cooperative wireless communication system with low order frequency offset and phase noise provides satisfactory performance.

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