Performance Analysis of a Space-Frequency Block Coded OFDM Wireless Communication System with MSK and GMSK Modulation

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***Abstract*—Space frequency block codes (SFBC) are very much efficient in overcoming the effect of frequency selective fading channel in a wireless communication system. In this paper, bit error rate performance analysis is carried out for a SFBC-OFDM system with MSK and GMSK modulation schemes. Results are evaluated numerically for SISO and MIMO communication links. It is shown that, for a fixed bit error rate the improvement in SNR for both SFBC coded MSK and GMSK modulation is noticeable. Also the receiver sensitivity is evaluated for system BER. It is shown that sensitivity improves for the change in code rate but remains nearly same for the same combination of transmit and receive antennas both for SFBC coded MSK and GMSK modulation scheme.**

***Keywords*—Space-frequency block coding (SFBC), Orthogonal frequency division multiplexing (OFDM), Rayleigh fading, Bit error rate (BER), Minimum shift keying (MSK) modulation, Gaussian minimum shift keying (GMSK) modulation.**

I. INTRODUCTION

Now a days multiple input multiple output (MIMO) technology is one of the most used and necessary technique of modern wireless communication which is the result of combining high data rate orthogonal frequency division multiplexing (OFDM) with antenna arrays. Multiple antennas are possible to be used both at transmitting and receiving end in this technology. Without increasing the transmitting power or adding extra bandwidth, MIMO increases the rate of data transmission. MIMO reduces fading and enhances signal diversity[1]. By using space time block coding [2]-[6] a single signal is being transmitted with orthogonal nature from each of the transmitting antenna of MIMO. In STBC coding technique data streams are coded in blocks.

Most of the channels are frequency selective. As a result space frequency block code (SFBC) [7] originated from the space time block code is introduced in wireless communication system. To transmit data through faded environment space frequency block code is now being utilized.

In previous works, performance analysis of space time coding with imperfect channel information is studied with MIMO. Because of frequency selective channels number of works have done by using space frequency (SFC) [8] or space time frequency (STFC) coding on OFDM system for broadband wireless communication [9]. Space frequency block code is proposed and evaluated by performance to utilize the diversities of frequency and space of channels. Closed form expression of BER for SFBC coded OFDM system with MPSK and MQAM modulation scheme for Rayleigh faded channel [10] have already been evaluated. In this paper we have derived the closed form expressions of BER for OFDM with MSK and GMSK modulation schemes. The expressions are derived for Rayleigh faded channels. In the performance analysis several forms of SFBC-OFDM are considered with various numbers of transmitting and receiving antennas. Results and comparisons of average BER are provided for MSK and GMSK modulation with SISO and MIMO techniques.

In the remainder of this paper, the system model of SFBC-OFDM-MIMO is presented in Section II. In Section III, the analysis of BER is evaluated with MSK and GMSK modulation. Results and comparisons are provided in Section IV. The concluding remark is presented in Section V.

II. SYSTEM MODEL

The system model for SFBC-OFDM is shown in Fig. 1. The system has M_T transmit and M_R receive antennas. Information signal is first converted from serial to parallel. Assuming an OFDM system with N_s subbands each having q adjacent subchannels and the system also has N subcarriers, where N_s=N/q, q is the symbol period of the SFBC system. All subbands are modulated using MSK or GMSK modulation scheme. A signal vector $\boldsymbol{S} = \{s[0], s[1], \ldots, s[N_t - 1]\}$ is provided to the SFBC encoder. Here the SFBC code rate is R_C. SFBC-OFDM is based on space time block code. The input blocks for each transmit antenna of OFDM should be length of N. SFBC provides M_T blocks, each of length N, S_1, S_2, S_3,….S_{MT}. After the IFFT and addition of Cyclic prefix blocks X_1, X_2, X_3,…X_{MT} are generated from 1st , 2nd, 3rd and M_T th antenna respectively. It is assumed that the fading process remains static during each time slot and changes from one OFDM block to another.

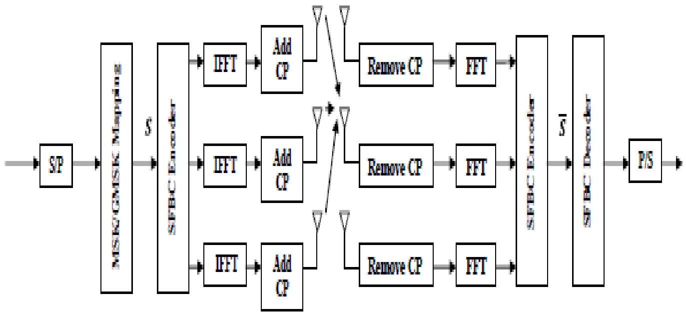

Fig. 1. SFBC-OFDM Block diagram

Uncorrelated fading process with perfect synchronization for different transmit-receive antenna pairs are considered. The fading process impulse response of the link between the i-th transmit antenna and j-th receive antenna, $(i=1,\ldots,M_T; j=1,\ldots,M_R)$ can be expressed as $h_{j,i}(t)=\sum_{m=0}^{L-1}\alpha_{m,j,i}(t)\delta(t-\tau_m(t))$ [11] where $\alpha_{m,j,i}(t)$ is the tap weight, $\tau_m(t)$ is the time delay of the m-th path and L is the total number of resolvable paths. The $\alpha_{m,j,i}(t)$'s are complex Gaussian random processes with zero mean and variance 1/L (equal power). Then after the removal of cyclic prefix and performing FFT on the received signal, the signal can be expressed as, $r_j=\sum_{i=1}^{M_T}H_{j,i}S_i+W_j, \qquad 1\le j\le M_R$ (1)

where $r_j=\left(r_j[0],\ldots,r_j[N-1]\right)^T$, $S_i=\left(s_i[0],\ldots,s_i[\frac{N}{q}-1]\right)^T$ is the transmitted signal at the i-th antenna, $W_j=(W_j[0],\ldots,W_j[N-1])^T$ denotes the AWGN and $H_{j,i}=diag\{H_{j,i}[k]\}_{k=0}^{N-1}$ is an N×N diagonal matrix with elements corresponding to the DFT of the channel response between the i-th transmit and j-th receive antennas.

III. Analysis of Bit Error Rate

The bit error rate performance of OFDM system is reviewed in this section. Let S be the MSK/GMSK input signal and r be the signal after removing the cyclic prefix and performing FFT of the remaining signal. The received signal is given by,

$$r[k]=H[k]s[k]+W[k], \qquad k=0,\ldots,N-1 \tag{2}$$

which can be expressed as,

$$r=HS+W \tag{3}$$

where $r=\left(r[0],\ldots,r[N-1]\right)^T$, $S=\left(s[0],\ldots,s[N-1]\right)^T$ is the transmit signal, $W=(W[0],\ldots,W[N-1])^T$ denotes the AWGN, and H is a diagonal matrix of size N×N given by $H_{j,i}=diag\{H_{j,i}[k]\}_{k=0}^{N-1}$, with H[k]'s elements corresponding to the DFT of the multipath channel response $h=[\alpha(0),\ldots,\alpha(L-1)]^T$ with L the total number of resolvable paths $(H[k]=\sum_{l=0}^{L-1}\alpha(l)e^{-j\frac{2\pi}{N}kl})$. The α(l)'s are complex Gaussian random variables with zero mean and variance 1/L.

The BER expression of the OFDM system can be written as:

$$BER=\frac{1}{N}\sum_{k=0}^{N-1}BER[k] \tag{4}$$

where BER[k] is the instantaneous BER of the k-th subchannel in the OFDM block.

A. BER Performance of MSK-OFDM system

Assuming MSK modulation scheme and negligible degradation due to the cyclic prefix in the OFDM, the expression for the instantaneous BER of the k-th subchannel can be written as:

$$BER_{MSK}[k]=\frac{1}{2}erfc\left(\sqrt{\frac{E_S|H[k]|^2}{N_0}}\right)=\frac{1}{2}erfc\left(\sqrt{\gamma_S|H[k]|^2}\right) \tag{5}$$

where, $\gamma_S=\frac{E_S}{N_0}$, with E_S the symbol energy at the transmitter and $\frac{N_0}{2}$ the variance of the real/imaginary part of the AWGN and $erfc(x)$ is the complementary error function: $erfc(x)=\int_0^\infty \exp\left(-t^2\right)dt$ [10].

By using approximate exponential approximation of complementary error function an approximate expression for (5) is obtained as:

$$BER_{MSK}=\frac{1}{2}\exp(-\gamma_S|H[k]|^2) \tag{6}$$

The BER expression for the OFDM system of (4) can be written as

$$BER_{MSK}=\frac{1}{2N}\sum_{k=0}^{N-1}erfc\left(\sqrt{\gamma_S|H[k]|^2}\right) \tag{7}$$

The above expression has the following closed form expression for BER:

$$BER_{MSK}=\frac{1}{2N}\sum_{k=0}^{N-1}\exp(-\gamma_S|H[k]|^2) \tag{8}$$

The average bit error rate can be obtained using,

$$\overline{BER_{MSK}}=\int_0^\infty BER_{MSK}\,\rho(\gamma)d\gamma \tag{9}$$

where $\rho(\gamma)$ is the probability density function (PDF) of $\gamma=\gamma_S|H[k]|^2$. $|H[k]|$ is Rayleigh distributed with variance one, also $|H[k]|^2$ has a chi square PDF with two degrees of freedom. γ is also chi-square distributed according to $\rho(\gamma)=\frac{1}{\bar{\gamma}}\exp(-\frac{\gamma}{\bar{\gamma}}), \gamma\ge 0$ where $\bar{\gamma}$ is defined as a function of the average of $|H[k]|^2$, i.e. $\bar{\gamma}=\gamma_S E\{|H[k]|^2\}=\gamma_S$. Then by substituting (8) into (9), the average BER of the MSK-OFDM system can be expressed as:

$$\overline{BER_{MSK}} = \int_0^{\infty} \frac{1}{2N} \sum_{k=0}^{N-1} \exp(-\gamma_S |H[k]|^2) \rho(\gamma) d\gamma \tag{10}$$

Finally, the average bit error rate can be written as,

$$\overline{BER_{MSK}} = [2(1+\gamma_S)]^{-1} \tag{11}$$

B. BER Performance of GMSK-OFDM system

Assuming GMSK modulation scheme and negligible degradation due to the cyclic prefix in the OFDM, the BER expression for the k-th subchannel is given as [11]:

$$BER_{GMSK} = \frac{1}{2} erfc \sqrt{\frac{\alpha E_S |H[k]|^2}{2N_0}} = \frac{1}{2} erfc \sqrt{\frac{\alpha \gamma_S |H[k]|^2}{2}} \tag{12}$$

The above expression can be approximated as:

$$BER_{GMSK}[k] = \frac{1}{2} \exp\left(\frac{-\alpha \gamma_S |H[k]|^2}{2} \right) \tag{13}$$

Now, the BER expression for GMSK-OFDM can be written as:

$$BER_{GMSK} = \frac{1}{2N} \sum_{k=0}^{N-1} erfc \sqrt{\frac{\alpha \gamma_S |H[k]|^2}{2}} \tag{14}$$

The BER expression is approximated as:

$$BER_{GMSK} = \frac{1}{2N} \sum_{k=0}^{N-1} \exp\left(\frac{-\alpha \gamma_S |H[k]|^2}{2} \right) \tag{15}$$

Now, the average bit error rate (BER) of GMSK-OFDM can be given as,

$$\overline{BER_{GMSK}} = \int_0^{\infty} BER_{GMSK} \rho(\gamma) d\gamma \tag{16}$$

$$\text{or,} \quad \overline{BER_{GMSK}} = \int_0^{\infty} \frac{1}{2N} \sum_{k=0}^{N-1} \exp\left(\frac{-\alpha \gamma_S |H[k]|^2}{2} \right) \rho(\gamma) d\gamma \tag{17}$$

Now, finally the average BER is approximated as,

$$\overline{BER_{GMSK}} = [2 + \alpha \gamma_S]^{-1} \tag{18}$$

C. BER Performance of MSK-SFBC-OFDM system

Considering an MSK-SFBC-OFDM system with M_T transmit and M_R receive antennas, decision metric is minimized by the decoder as:

$$|\tilde{s}[k] - s[k]|^2, \qquad k = 0, \ldots, N-1 \tag{19}$$

Now it can be shown here that,

$$\tilde{s}[k] = \frac{1}{R_C} \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2 s[k] + \eta[k] \tag{20}$$

where, $|H_{j,i}[k]|$ is the k-th subchannel with the i-th transmit and j-th receive antennas, R_C is the code rate of the SFBC system and $\eta(k)$ is the noise component. The normalized SNR can be written as:

$$\gamma = \frac{1}{M_T R_C} \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2 \gamma_S \tag{21}$$

The BER of MSK-SFBC-OFDM over frequency selective fading channel can be written as:

$$BER_{MSK} = \frac{1}{2N} \sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\gamma_S \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{R_C M_T}} \right) \tag{22}$$

The above expression can be approximated as:

$$BER_{MSK} = \frac{1}{2N} \sum_{k=0}^{N-1} \exp\left(\frac{-\gamma_S \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{R_C M_T} \right) \tag{23}$$

Average BER is obtained as:

$$\overline{BER_{MSK}} = \int_0^{\infty} \ldots \int_0^{\infty} BER_{MSK} \rho(\gamma_{1,1}) \ldots \rho(\gamma_{M_R,M_T}) d\gamma_{1,1} \ldots d\gamma_{M_R,M_T} \tag{24}$$

$$\text{or,} \quad \overline{BER_{MSK}} = \int_0^{\infty} \ldots \int_0^{\infty} \frac{1}{2N} \sum_{k=0}^{N-1} \exp\left(\frac{-\gamma_S \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{R_C M_T} \right) \rho(\gamma_{1,1}) \ldots \rho(\gamma_{M_R,M_T}) d\gamma_{1,1} \ldots d\gamma_{M_R,M_T} \tag{25}$$

On the above expression of average BER, $|H_{j,i}[k]|$ are independent and identically distributed Rayleigh random variables with variance one, $\gamma_{j,i} = \gamma_S |H_{j,i}[k]|^2$. The probability distribution function is expressed as,

$$\rho(\gamma_{j,i}) = \frac{1}{\overline{\gamma}_{j,i}} \exp\left(-\frac{\gamma_{j,i}}{\overline{\gamma}_{j,i}} \right), \qquad \gamma_{j,i} \geq 0, j = 1, \ldots, M_R, i = 1, \ldots, M_T \tag{26}$$

By substituting (23) and (26) in (25) the average BER expression is found. The average BER for MSK-SFBC-OFDM is given by,

$$\overline{BER_{MSK}} = \frac{1}{2N} \left[1 + \frac{\gamma_S}{M_T R_C} \right]^{-M_R M_T} \tag{27}$$

D. BER Performance of GMSK-SFBC-OFDM system

Applying the same approach like MSK, for GMSK-SFBC-OFDM system the BER expression can be written as,

$$BER_{GMSK} = \frac{1}{2N} \sum_{k=0}^{N-1} erfc\left(\sqrt{\frac{\alpha \gamma_S \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{2 R_C M_T}} \right) \tag{28}$$

Again it can be approximated as,

$$BER_{GMSK} = \frac{1}{2N}\sum_{k=0}^{N-1}\exp\left(\frac{-\alpha\gamma_S\sum_{j=1}^{M_R}\sum_{i=1}^{M_T}\left|H_{j,i}[k]\right|^2}{2R_C M_T}\right) \quad (29)$$

The average BER expression of GMSK-SFBC-OFDM is obtained as,

$$\overline{BER_{GMSK}} = \int_0^{\infty}\cdots\int_0^{\infty} BER_{GMSK}\,\rho(\gamma_{1,1})\cdots\rho(\gamma_{M_R,M_T})\,d\gamma_{1,1}\cdots d\gamma_{M_R,M_T} \quad (30)$$

The closed form expression of the average BER is,

$$\overline{BER_{GMSK}} = \frac{1}{2}\left[1+\frac{\alpha\gamma_S}{2R_C M_T}\right]^{-M_R M_T} \quad (31)$$

IV. RESULTS AND DISCUSSIONS

The BER performance with MSK and GMSK modulation scheme for SFBC-OFDM is evaluated in this section. Frequency selective multipath fading channels are considered here with Rayleigh fading environment. Here number of resolvable paths are L=4, and number of subcarriers included in the OFDM system is N=128. Slow varying and stationary fading process is considered for BER evaluation.

The average BER performance for MSK and GMSK SFBC-OFDM system are shown in Fig. 2 and Fig. 3 respectively. The code rate used here is 1bits/s/Hz. The results are shown for uncoded and coded signals. The results of coded SFBC-OFDM is shown with 1×1, 2×2, 2×3, 3×3 and 4×4 combinations of transmit and receive antennas. It is observed from the figures that, in both case of MSK and GMSK- SFBC-OFDM the average BER performance gain with respect to their SNR have improved than the uncoded OFDM. Also for the coded OFDM the performance have improved with the increament of number of antennas in transmitting and receiving

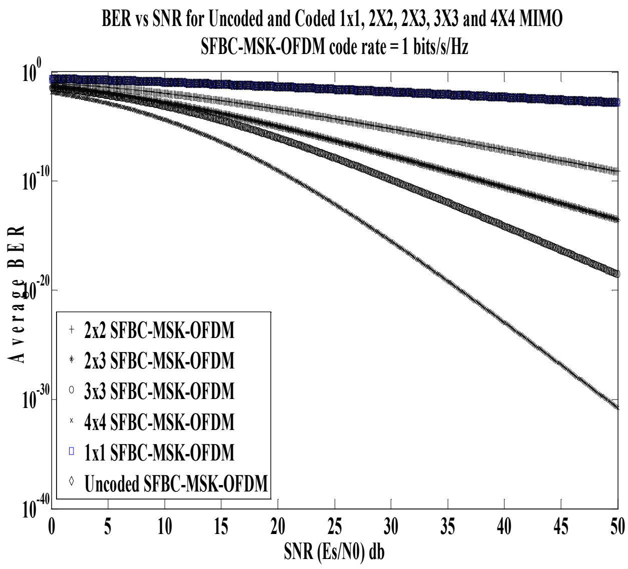

Fig. 2. Average BER performance of coded and uncoded MSK-OFDM with code rate 1bits/s/Hz

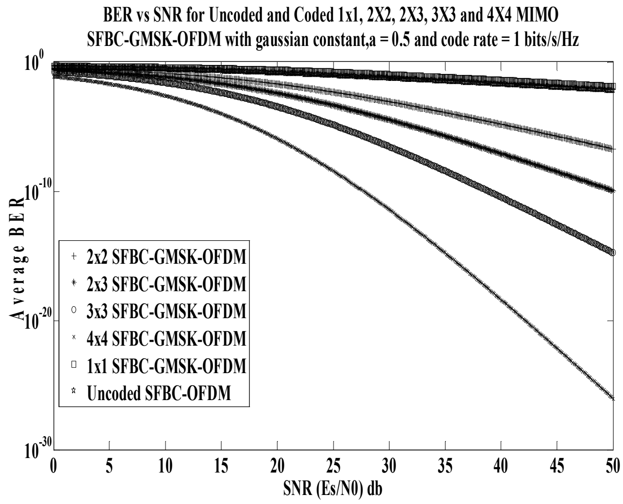

Fig. 3. Average BER performance of coded and uncoded GMSK-OFDM with code rate 1 bits/s/Hz

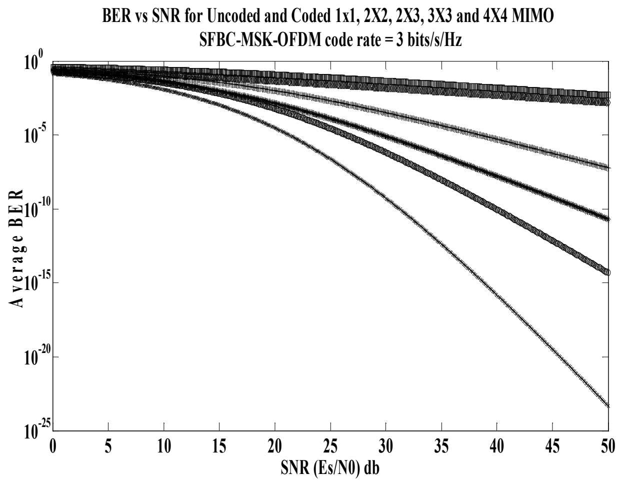

Fig. 4. Average BER performance of coded and uncoded MSK-OFDM with code rate 3 bits/s/Hz

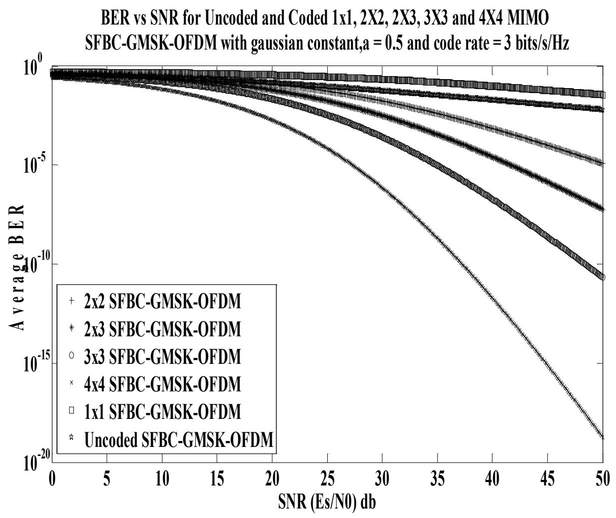

Fig. 5 Average BER performance of coded and uncoded GMSK-OFDM with code rate 3 bits/s/Hz

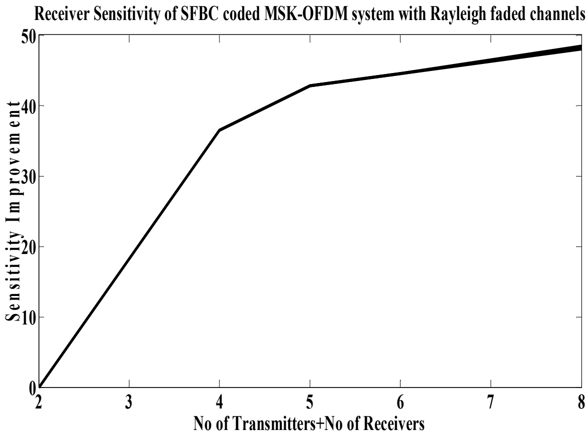

Fig.6 Receiver Sensitivity of SFBC coded MSK-OFDM system with Rayleigh faded channels for code rate ½, 1 and 3.

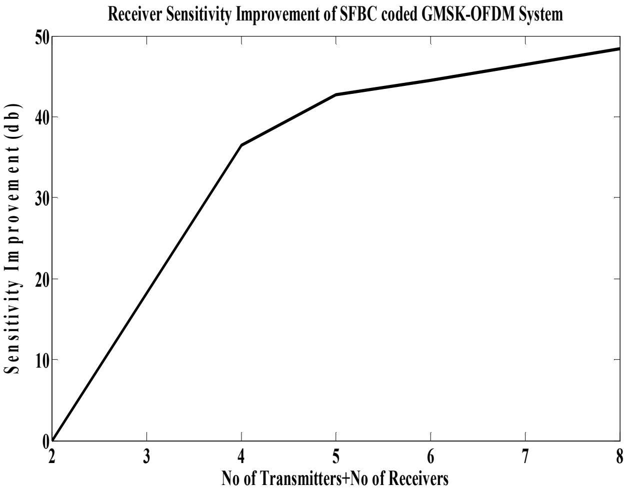

Fig. 7. Receiver Sensitivity of SFBC coded GMSK-OFDM system with Rayleigh faded channels for code rate ½, 1 and 3.

end. Fig. 4 and Fig. 5 show the average BER performance for SFBC coded and uncoded MSK and GMSK-OFDM with

Fig. 6, shows the receiver sensitivity improvement of the coded MSK-OFDM system with code rate ½, 1 and 3 respectively. For every case the receiver sensitivity improves nearly at a same rate depending on the number of transmitters and receivers.

Fig. 7 shows the sensitivity improvement of the SFBC coded GMSK-OFDM system with the Gaussian constant α=0.5 and code rate ½, 1 and 3. It is also seen here that for every case of code rate the sensitivity improves nearly at a same way and depends on the number of transmit and receive antennas.

It is seen from the above two figures that both SFBC coded MSK and GMSK modulation scheme perform similarly to improve the receiver sensitivity.

V. Conclusion

The closed form approximate expression of BER for the MSK and GMSK modulated space frequency block coded OFDM system is presented in this paper. Here frequency selective fading channels under Rayleigh fading environment are considered. The effect of coding on the system BER performance is shown in the paper. The receiver sensitivity improvement for both of the modulation system is also shown. It is seen that despite of different code rates the change in sensitivity remain nearly same for the described combinations of transmit and receive antennas. The proposed expressions can be used for finding out the effects of coding under other fading environment with their sensitivity improvement performance.

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