An Eigenvalue and Superposition Approach Based Cooperative Spectrum Sensing in Cognitive Radio Networks

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Abstract— **In this paper, we propose an Eigenvalue and superposition approach based cooperative spectrum sensing scheme: firstly, we calculated the Eigenvalue from the received autocorrelation matrix and secondly, the reporting time is utilized to increase the number of smoothing factors for autocorrelation calculation. In addition, we define optimization problem in smoothing factors with reserved higher detection probability. Simulation results show that the proposed scheme has better detection probability while requiring less sensing time as compared with both the conventional Energy detection (ED) and Eigenvalue-based detection (EVD) scheme.**

Keywords— cognitive radio network, spectrum sensing, energy detection, eigenvalue-based detection, superposition approach

I. INTRODUCTION

Cognitive Radio (CR) [1][2] as a revolutionary intelligent technology can maximize the utilization of the frequency resources by allowing secondary users to access the spectrum bands allocated to Primary Users (PUs) when they are idle temporally. Energy detection scheme can be used by a single Cognitive Radio User (CRU) for idle spectrum. However, it cannot deal with the hidden terminal problem [3] which arises due to multi-path fading and shadow effect, which results in severe performance degradation. Recently, cooperative spectrum sensing scheme was proposed to overcome the hidden terminal problem in single node sensing [4][5][6][7]. To implement conventional cooperative spectrum sensing, each CRU makes a local decision and those decisions are reported to a fusion center to make a final decision according to some fusion rules (e.g. OR, AND, Half voting, Majority rule etc). However, more reporting data are required for each CRU to report its local decision to a fusion center as the number of cooperative user's increases. Subsequently, more reporting time will be required.

The energy detection (ED) has lower complexity and is widely used as a signal detection method in real time systems. However, ED relies on knowledge of noise power, and inaccurate estimation of the noise power will lead to high probability of false alarm as well as miss detection [8]. Thus ED is vulnerable to the noise uncertainty. Moreover, ED is not optimal for real environment due to the received signal from the PU for different times are correlated. Recently, many papers have investigated the application of Eigenvalue based spectrum sensing which outperforms energy detection methods [9][10]. In Eigenvalue-based spectrum sensing, covariance matrix of the received signal was used for spectrum detection. In [11], T. Ratnarajah et. al. performed asymptotic analysis of exact decision thresholds for maximum Eigenvalue detector (MED) and maximum–minimum Eigenvalue (MME) detector. They also pointed out that MED has better spectrum detection probability. In [12], M. M. Sipon et. al. performed cooperative spectrum sensing method based on Eigenvalue and superposition for cognitive radio network. However, they cannot show the data fusion logic at cognitive radio for local and global. Also, they cannot show the value of threshold and cannot define the optimization problem in smoothing factors.

In this paper, we propose a minimum-to-maximum Eigenvalue and superposition approach based cooperative detection scheme. The sensing performance of Eigenvalue based cooperative detection depends on smoothing factors. The larger smoothing factor will result in the better sensing performance. However, it will cause larger sensing time. Thus, to achieve better detection performance within given sensing time, in this paper a superposition approach [13] is applied to the conventional Eigenvalue-based cooperative detection scheme. In the proposed scheme, next CRU will utilize immediately previous CRUs' reporting time for sensing more samples. Therefore, the sensing performance of the proposed scheme can be improved without adding extra spectrum sensing time.

Remaining of the paper is structured as follows. In Section II, we introduce the conventional cooperative spectrum sensing model based on Energy Detection and Eigenvalue. In Section III, we propose the Eigenvalue and superposition approach based cooperative spectrum sensing scheme. In Section IV, simulation results are shown, and conclusion of the paper is drawn in Section V.

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II. Conventional Cooperative Spectrum Sensing Model Based Energy Detection and Eigenvalue

For CRU $i(i=1,2,......,Q)$, the spectrum sensing is a binary hypothesis test that can be formulated as follows [14][15][16]:

$$\begin{cases} H_0 \ : y_i(t)=\eta_i(t) \quad t=1,2,...,N_x \\ H_1 \ : \ y_i(t)=x_i(t)+\eta_i(t) \quad t=1,2,...,N_x \end{cases} \tag{1}$$

where t is the sample index, N_x is the total number of samples, $y_i(t)$ is the received signal at CRU, $x_i(t)$ is the transmitted signal samples through a wireless channel consisting of path loss, multipath fading and time dispersion effects, and $\eta_i(t)$ is an independent and identically distributed (i.i.d) with zero mean Additive White Gaussian Noise (AWGN), i.e., $N(0,\sigma_\eta^2)$. The mean and variance of $x_i(t)$ are zero and σ_x^2, respectively as well.

A. Conventional Energy Detection Scheme

The decision statistic for energy detector [17] is

$$T_{ED}=\frac{1}{N_x}\sum_{n=0}^{N_x-1}\left[y_i(t)\right]^2 \tag{2}$$

The performance of detection is measured by the couple of its detection and false alarm probabilities (P_d,P_f). Each couple is associated with a threshold λ of the decision statistic. If T_{ED} is greater than λ, the primary signal is present. Otherwise, the primary signal is absent.

B. Conventional Eigenvalue Based Detection Scheme

We can define the received signal at CRU in the following matrix form under H_1 hypothesis:

$$\begin{bmatrix} y_i(t) \\ y_i(t-1) \\ ... \\ ... \\ y_i(t-F+1) \end{bmatrix} = \begin{bmatrix} x_i(t) \\ x_i(t-1) \\ ... \\ ... \\ x_i(t-F+1) \end{bmatrix} + \begin{bmatrix} \eta_i(t) \\ \eta_i(t-1) \\ ... \\ ... \\ \eta_i(t-F+1) \end{bmatrix} \tag{3}$$

Here, F is smoothing factor. Also, we can define the received signal at CRU in the following matrix form under H_0 hypothesis:

$$\begin{bmatrix} y_i(t) \\ y_i(t-1) \\ ... \\ ... \\ y_i(t-F+1) \end{bmatrix} = \begin{bmatrix} \eta_i(t) \\ \eta_i(t-1) \\ ... \\ ... \\ \eta_i(t-F+1) \end{bmatrix} \tag{4}$$

The statistical covariance matrices of the received signal, the transmitted signal, and noise can be defined respectively, as

$$C_y = E\left[Y_i(t)Y_i^T(t)\right] \tag{5}$$

$$C_x = E\left[X_i(t)X_i^T(t)\right] \tag{6}$$

$$C_\eta = E\left[\eta_i(t)\eta_i^T(t)\right] \tag{7}$$

where

$$Y_i(t)=\begin{bmatrix} y_i(t) \\ y_i(t-1) \\ ... \\ ... \\ y_i(t-F+1) \end{bmatrix} \tag{8}$$

$$Y_i(t)=\left[y_i(t) \quad y_i(t-1) \quad ... \quad ... \quad y_i(t-F+1)\right]^T$$

$$X_i(t)=\begin{bmatrix} x_i(t) \\ x_i(t-1) \\ ... \\ ... \\ x_i(t-F+1) \end{bmatrix} \tag{9}$$

$$X_i(t)=\left[x_i(t) \quad x_i(t-1) \quad ... \quad ... \quad x_i(t-F+1)\right]^T$$

$$\eta_i(t)=\begin{bmatrix}\eta_i(t)\\ \eta_i(t-1)\\ \ldots\\ \ldots\\ \eta_i(t-F+1)\end{bmatrix} \quad (10)$$

$$\eta_i(t)=[\eta_i(t) \quad \eta_i(t-1) \quad \ldots \quad \ldots \quad \eta_i(t-F+1)]^T$$

Here, the superscript $[.]^T$ stands for transpose and $E[.]$ stands for expectation operation. We can verify the statistical covariance matrix of the received signal defined as C_y under the two hypotheses that is

$$C_y=\begin{cases} C_x+\sigma_\eta^2 I_F, & H_1 \\ \sigma_\eta^2 I_F, & H_0 \end{cases} \quad (11)$$

where σ_η^2 is the variance of the noise and I_F is the identity matrix of order F.

If the signal $x_i(t)$ is absent, then the statistical covariance matrix of transmitted signal, $C_x = 0$. If the signal $x_i(t)$ is present, then the statistical covariance matrix of transmitted signal, $C_x \neq 0$.

The Eigenvalue have the following characteristics: Firstly, the statistical covariance matrix of the received signal C_y is a symmetrical matrix. All Eigenvalue are the real numbers, and the sum of all Eigenvalue is related to the number of CRUs Q and smoothing factor F.

Secondly, if the samples of cognitive users are completely uncorrelated, the statistical covariance matrix of the received signal C_y is a unitary matrix and all Eigenvalue are equal to 1. Therefore, this situation corresponds to the assumption H_0.

Finally, if the samples of CRUs are completely correlated, all the elements of matrix are greater than 1s, and the maximum Eigenvalue is equal to the number of CRUs Q. Therefore, this situation corresponds to the assumption H_1.

In an ideal environment, the correlations of the received signal samples at different times would be larger than one if the time separations are smaller than the data symbol duration whereas the correlations of the received noise samples at different times should be one due to the AWGN channel. Let's define the sample auto-correlations of the received signal as following [18]:

$$A(k)=\frac{1}{N_x}\sum_{n=0}^{N_x-1} y_i(n)y_i(n-k), \quad k=0, \ldots, F-1 \quad (12)$$

where N_x is a total number of available samples and F is a positive integer called the smoothing factor.

In real environment, the sample covariance matrix of received signal $(\dot{C}_y)$ can only be calculated by using a limited number of signal samples. Hence, we can only obtain the sample covariance matrix other than the statistical covariance matrix i.e., $\left(C_y \approx \dot{C}_y\right)$. The sample covariance matrix of the received signal $(\dot{C}_y)$ is formed using Toeplitz matrix as following [18]:

$$\dot{C}_y(N_x)=\begin{bmatrix} A(0) & A(1) & A(F-1) \\ A(1) & A(0) & A(F-2) \\ \ldots & \ldots & \ldots \\ A(F-1) & A(F-2) & A(0) \end{bmatrix} \quad (13)$$

C. Conventional Cooperative Spectrum Sensing Scheme

To find sample autocorrelations of the received signal expressed by Eq.13, in the Eigenvalue based spectrum sensing, the essential number of computational time for a single user can be calculated as [10]:

$$T_{ssc} = F \times N_x \quad (14)$$

From above Eq.14, it is obvious that for a fixed number of samples, the sensing time is only dependent on different number of smoothing factors.

In conventional Eigenvalue based detection (EVD) scheme, if we increase the smoothing factor, the necessary sensing time will be increased while each CRU has a fixed sensing time and reporting time for their corresponding smoothing factor. The main diagram of the conventional EVD scheme is shown in Fig. 1. In this Fig. 1, each CRU reporting time is not utilized to calculate autocorrelation of the samples. The total time is denoted by T_t which consists of sensing time T_s and reporting time T_r. When the number of CRU is Q, the total time in the conventional EVD scheme can be calculated as following:

$$T_t = T_s + Q \times T_r \times N_x \sum_{i=1}^{4} F_i \quad (15)$$

where $F_1 = 4, F_2 = 8, F_3 = 16$ and $F_4 = 32$.

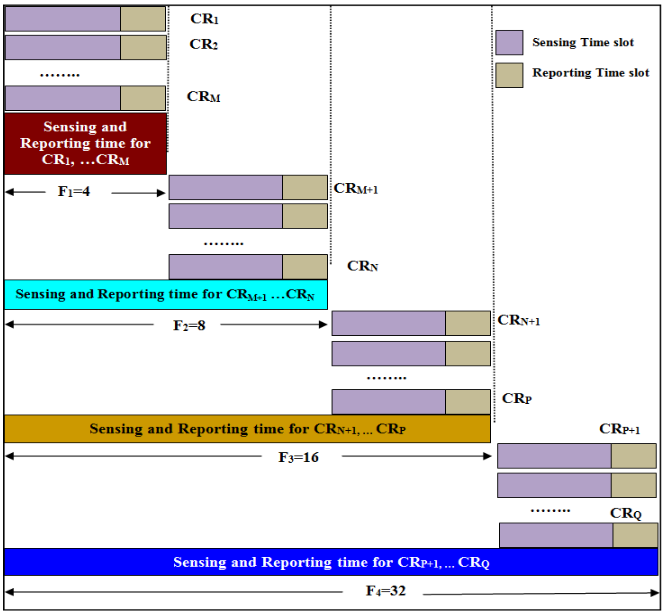

Fig. 1. Conventioal EVD scheme with different number of smoothing factors

III. Proposed Eigenvalue and Superposition Approach Based Cooperaitve Spectrum Sensing

A. *Proposed Eigenvalue and Superposition Approach Based Spectrum Sensing*

We propose the following signal detection procedure according to smoothing factor and sample covariance matrix:

- Select appropriate smoothing factors F and calculate the sample covariance matrix of the received signal $\left(\dot{C}_y\right)$ from the Eq. 13 for the fixed samples.
- Determine the minimum $(\zeta_{\min})$ and maximum $(\zeta_{\max})$ Eigenvalue from the sample covariance matrix of the received signal $\left(\dot{C}_y\right)$ and also calculate their ratio, $R_{mtm} = \frac{\zeta_{\min}}{\zeta_{\max}}$. Here, R_{mtm} stands for minimum-to-maximum Eigenvalue ratio.
- Compare R_{mtm} with the threshold value ℓ. If $R_{mtm} > \ell$ then the signal is present; otherwise the signal is absent.
- Consider local decision (u_i) is the i th CRU and it can be expressed as

$$u_i = \begin{cases} 1, & if\ R_{mtm} > \ell \\ 0, & Otherwise \end{cases} \tag{17}$$

here, threshold ℓ [9] is

$$\ell = \frac{\left(\sqrt{N_x} - \Psi F\right)^2}{\left(\sqrt{N_x} + \Psi F\right)^2} \times \left(1 + \frac{\left(\sqrt{N_x} - \Psi F\right)^{\frac{-2}{3}}}{\left(N_x \Psi F\right)^{\frac{1}{6}}} T^{-1}\left(1 - P_f\right)\right)$$

where parameter Ψ indicates the number of receivers used, in this paper the number is always one (Ψ =1) and T represents the Tracy-Widom distribution.

- Making a final global decision G of Q CRUs for the two data fusion rules are made upon a following simple forms:

$$G = \begin{cases} 1, & \begin{cases} if \sum_{i=1}^{Q} u_i \geq 1, & OR\ rule \\ if \sum_{i=1}^{Q} u_i \geq \frac{1}{2} Q, & M\ rule \end{cases} \\ 0, & Otherwise \end{cases} \tag{18}$$

The main objective is to justify minimum number of smoothing factors to obtain target performance. Therefore, large number of smoothing factors to obtain target performance is defined as the optimization problem and defined below:

$$\begin{aligned} &F_{\min} = \min(F) \\ &s.t. G_d(F) \geq 0.9 \end{aligned} \tag{19}$$

B. *Cooperative Spectrum Sensing Based on Superposition Approach*

In the proposed Eigenvalue and superposition approach based spectrum sensing scheme, the reporting time shown in Eq.15 is exploited for computational purpose for different number of smoothing factors to calculate the autocorrelation of the sample. The entire CRUs in cooperative network have the same number of samples.

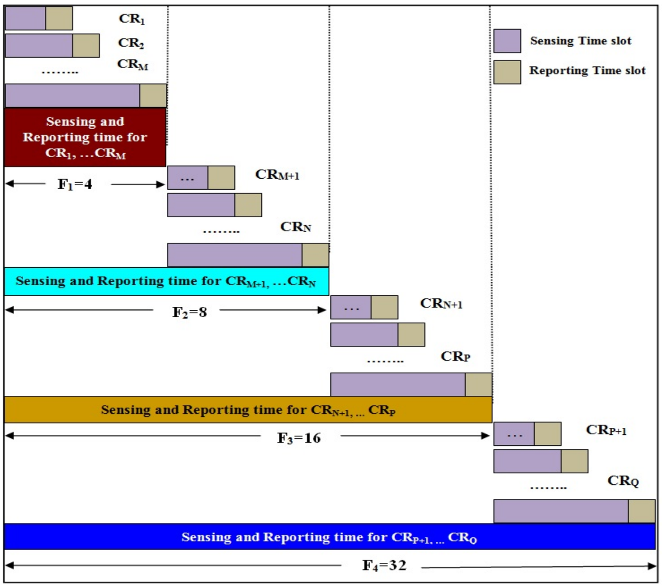

Fig. 2. Proposed Eigenvalue based detection schme with different number of smoothing factors

Since the sensing duration is fixed, except for first CRU, all the CRUs have to adopt a different waiting time for its reporting to the fusion center. As an example, the 2nd CRU can utilize reporting time of 1st CRU for sensing, and 3rd CRU can utilize reporting time of the1st CRU and the 2nd CRU for sensing and so on. In the proposed Eigenvalue and superposition-based detection scheme, this reporting time is utilized to calculate autocorrelation of the signal. The main diagram of the proposed scheme is shown in Fig. 2. From the Fig. 2, it is observed that, for the lowest smoothing factor $\left(F_1 = 4\right)$, total M CRUs sense the signal for a fixed duration. In addition, for $F_2 = 8$, idle reporting time of previous CRUs utilized by $(M+1)^{th}$ to N^{th} CRU for computational and reporting purpose can be calculated as:

$$T_{F_2} = T_s + M \times T_r \tag{20}$$

Likewise, for $F_3 = 16$, idle reporting time of previous CRUs utilized by $(N+1)^{th}$ to P^{th} CRU for computational and reporting purpose can be written as:

$$T_{F_3} = T_s + N \times T_r \tag{21}$$

Similarly, for $F_4 = 32$, idle reporting time of previous CRUs utilized by $(P+1)^{th}$ to Q^{th} CRU for computational and reporting purpose is as:

$$T_{F_4} = T_s + P \times T_r \tag{22}$$

Therefore, from above equations (20), (21) and (22), it is obvious that the proposed EVD scheme utilizes reporting time of previous CRU's for increasing number of smoothing factors. Thus, the required time of the proposed EVD scheme can be calculated:

$$T_{OPT} = T_s + 4 \times (Q \times T_r \times N_x) \tag{23}$$

IV. Simulation Results

Firstly, we evaluate the required normalized time for different number of smoothing factors. From Fig. 3, it is evident that the time complexity is increased when the smoothing factors F_1, F_2, F_3, and F_4 are also increased.

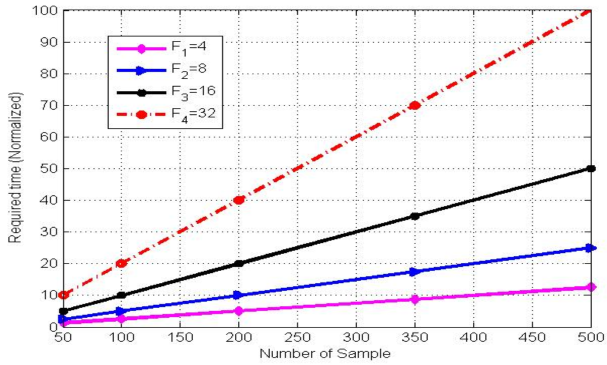

Fig. 3. Normalized time for different number of smoothing factors according to the number of samples

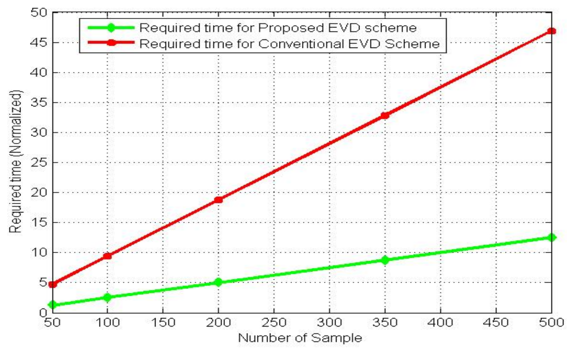

Fig. 4. The normalized time of conventional EVD scheme and proposed EVD schemes according to the number of samples

Fig. 4 shows the required normalized times for the proposed EVD scheme and the conventional EVD scheme. The conventional EVD scheme requires additional time for different number of smoothing factors since it does not utilizes idle reporting time.

Secondly, the proposed EVD scheme utilizes reporting time for different number of smoothing factor, it does not need to take any extra time due to it utilizes idle reporting time. Consequently, minimum time is sufficient for its data processing for additional smoothing factors.

In order to evaluate the proposed scheme, the local sensing performance is shown in the Fig. 5 for different number of smoothing factors. For the simulation, the following parameters are considered; the sampling frequency f_c is 300 kHz, the local sensing time T_s is 1 milli second (ms), the local reporting time T_r is 0.125 milli second (ms), and the number of samples N_x is 1000. Moreover, it is assumed that the signal is independent and identically distributed (i.i.d), and the channel is under AWGN with signal-to-noise ratio (SNR) is -20 dB. Under such condition, the Receiver Operating Characteristics (ROC) curves illustrated in Fig. 5 shows that the proposed EVD scheme has the highest probability of detection compared with both the conventional EVD scheme and ED scheme. Moreover, larger smoothing factor (F) means better the probability of detection. However, it is very difficult to choose the best F due to it is related to signal property. Therefore, the probability of detection and probability of false for the ED do not change with F as shown in Fig. 5.

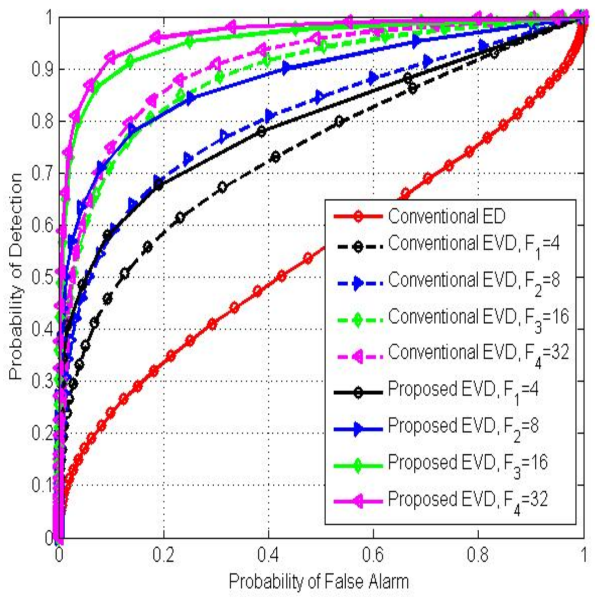

Fig. 5. The local sensing performance for the conventional ED scheme, conventional EVD scheme and proposed EVD scheme for different smoothing factors

Finally, Fig. 6 shows the global sensing performance for the conventional cooperative ED scheme, the conventional cooperative EVD scheme, and the proposed cooperative EVD scheme. For our simulation, it is assumed that four CRUs are spread in the network to perform local sensing where distributed CRUs have different SNR values. We consider the condition that the received signals of all four CRUs are -28 dB, -20 dB, -15 dB, and -10 dB, respectively. Also, we have considered both OR rule and M rule as a data fusion rule at the fusion center. Under such condition, probability of detection of OR rule is always largest than M rule. The conventional EVD scheme where all CRUs utilize smoothing factor has better detection performance than the conventional ED scheme for both OR rule and M rule. However, by utilizing OR rule it is clarified that the proposed EVD scheme has outperformance compared with both the conventional EVD scheme and ED scheme and so forth for the M-rule as well. The simulation results proved that the propose EVD scheme has the ability to significantly improve the sensing detection performance in CRU networks.

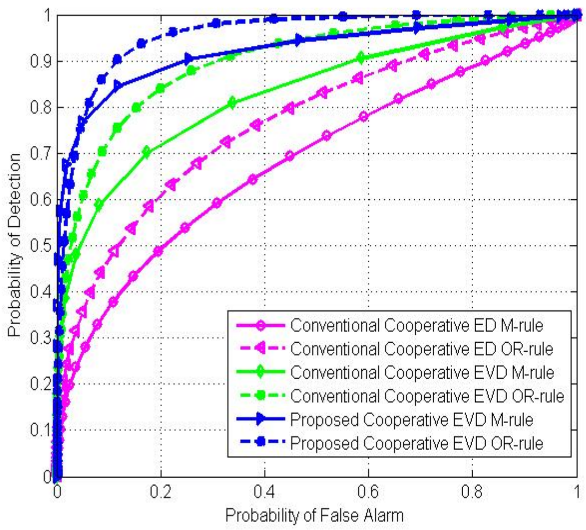

Fig. 6. Global probability of detection and false alarm of conventional cooperative ED, conventional cooperative EVD scheme and proposed cooperative EVD scheme when four CRUs are -28 dB, -20 dB, -15 dB and -10 dB, respectively

V. CONCLUSION

In this paper, we have proposed a minimum-to-maximum Eigenvalue and superposition approach based cooperative detection in cognitive radio networks where next CRU is utilized reporting time in previous CRU as a sensing time. Simulation results showed that the proposed scheme outperforms compared with both the conventional EVD scheme and ED scheme.

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