Low-Complexity Blind Phase Noise Compensation in OFDM Systems

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***Abstract*—In Orthogonal Frequency Division Multiplexing (OFDM) system there is a random phase fluctuation of local oscillators both at the transmitter and receiver that is known as phase noise (PHN). This PHN causes the performance degradation in OFDM system. To compensate for the critical effect of PHN, several pilot based PHN estimation methods are developed in the literature with the cost of system bandwidth. In this paper, a blind PHN compensation method is proposed to abate the effect of common phase error (CPE) in OFDM system caused by PHN to retain the system bandwidth with reduced calculation complexity. The performance of the proposed method is demonstrated by several computer simulations.**

Keywords: OFDM, Phase Noise, Common Phase Error, Inter-carrier Interference, Phase Locked Loop.

I. Introduction

OFDM system has been widely adopted in various wireless and wireline communication standards, such as IEEE 802.11a/g, IEEE 802.16, HIPERLAN etc. It is robust against inter-carrier interference (ICI) and inter-symbol interference (ISI) caused by multipath frequency selective channel. But, the most important feature, the orthogonality among the sub-carriers, of OFDM system is threatened by the presence of PHN. To compensate for the effect of PHN, several pilot based PHN estimation methods are developed, such as in [1]-[3]. In most of the existing pilot based algorithms, some pilots are sent with each OFDM block to estimate the time-varying channel and these pilot are also used to estimate the phase noise. The usage of pilot symbols decreases the system bandwidth efficiency. Besides, due to the time-varying nature of PHN, the pilot sequence needs to be transmitted periodically, results further loss of system throughput. In order to improve the bandwidth efficiency, blind methods for PHN compensation have attracted much attention. In [4]-[6] the authors have proposed a blind algorithm that suffers from higher computational complexity. In most of the existing blind algorithms, there is a performance degradation arising from decision errors, since it exploits the decision-directed approach [7]. Recent advancement in wireless communication, such as WiMAX has been implemented with one out of three consecutive OFDM symbols are allocated for pilot while in LTE one full OFDM is devoted to channel estimation once every seven time slots. Therefore, it is necessary to detect data symbols with the help of pilot symbols, this motivation leads this research to develop an algorithm to estimate the data symbol as well as phase noise without pilot in each OFDM symbol. In this paper, a blind PHN compensation method is proposed to abate the effect of CPE in OFDM system caused by PHN. In the proposed algorithm, the angles of all signals corresponding to a particular constellation point are calculated w.r.t reference axis and averaged, then this process is repeated for all available constellation points. These average angles are compared with the corresponding ideal angles and averaged again to estimate the angle of rotation caused by CPE. The performance of the proposed method is demonstrated by the computer simulations.

II. System Model

A block diagram of OFDM transceiver system with PHN is shown in Fig. (1). At the transmitter, Inverse Fast Fourier Transform (IFFT) is applied to the N frequency-domain QAM symbols $X[k]$ to obtain time-domain signal $x[n]$ given by,

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi nk/N} \quad (1)$$

where, $k, n = 0, 1, \cdots N-1$. To prevent the effect of ISI, a cyclic prefix (CP) of length $N_{cp} \geq L$ samples are appended at the beginning of each OFDM signal, where L is the length of the multipath channel. At the receiver, it is assumed that there is a perfect frame synchronization. The carrier frequency offset and channel conditions are estimated in the training phase and compensated for in the data detection stage, hence the effect of frequency offset is minimized and the channel state information is available at the receiver. After removing the CP, the received complex baseband signal of m^{th} OFDM block in the presence of PHN can be written as

$$y_m[n] = e^{j\theta[n]} \sum_{i=0}^{L-1} h_m[i] x_m[n-i]_{modN} + \nu_m[n], \quad (2)$$

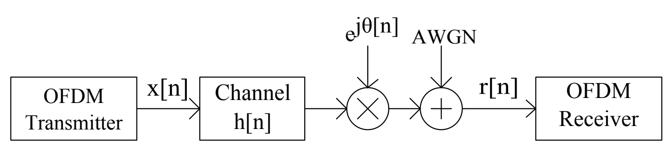

Fig. 1. OFDM transceiver system with phase noise

where, h is L taps baseband channel and ν is AWGN noise. After performing N-point FFT on the received time-domain signal, the frequency-domain signal $Y_m[n]$ is given by

$$\begin{aligned} Y_m[n] &= \sum_{l=0}^{N-1} H_m[l]X_m[l]C[l-n]_N + Z_m[n] \\ &= C_m[0]X_m[n]H_m[n] \\ &\quad + \sum_{l=0,l\neq n}^{N-1} H_m[l]X_m[l]C[l-n]_N + Z_m[n] \quad (3) \end{aligned}$$

It will discussed in the following sub-section that the PHN angle is small and can be expressed as $e^{j\theta[n]} \approx 1 + j\theta[n]$. Therefore,

$$C_m[0] \approx \frac{1}{N}\sum_{n=0}^{N-1}(1+j\theta[n]) = 1 + \frac{j}{N}\sum_{n=0}^{N-1}\theta[n] = 1 + j\bar{\theta} \quad (4)$$

where,

$$\bar{\theta} = \frac{1}{N}\sum_{n=0}^{N-1}\theta[n]$$

is the angle of rotation caused by CPE. In the first term of equation (3), $C_m[0]$ is the effect of CPE and its function is to rotate every received symbol by an average angle $\bar{\theta}$ given by equation (4), and the second term is ICI. The variation of generated PHN process is low and the effect of ICI is small compared to CPE. Besides, the ICI term can be approximated as zero-mean complex Gaussian noise, and can be merged with AWGN noise. So, the equation (3) can be rewritten as

$$Y_k = C_0 X_k H_k + \xi_k \quad (5)$$

where, ξ_k consists of ICI and AWGN noise. The main objective of this research is to mitigate the effect of CPE, i,e to estimate and compensate for the average angle that causes CPE to demodulate the received data bit efficiently.

III. Phase Noise Model

PHN is a tame-varying random process caused by the phase fluctuation of the local oscillators (LOs) both in transmitter and receiver. The most valuable feature, orthogonality among the sub-carriers of OFDM systems, is threatened by the presence of PHN that results degrade in system performance.

A. Source of PHN

The output voltage of an ideal LO can be described mathematically as a pure sine wave of constant frequency and amplitude. In practice, this type of oscillator is impossible. A real oscillator output voltage is expressed in [8] given by

$$v(t) = [A + \alpha(t)]cos\{2\pi f_c t + \theta(t)\} \quad (6)$$

where, $\alpha(t)$ and $\theta(t)$ are amplitude and phase noise modulation respectively. If the oscillator is well-designed, amplitude noise is less significant than phase noise. Practically, it is very difficult to implement this type of oscillators. The rapidly increasing demand of bandwidth leads to higher order and more compact carrier frequency overlapping modulation techniques. It gets pretty challenging to recover the transmitted data in the presence of phase noise. Since, the oscillator amplitude noise can be fixed-up by using an automatic gain controller (AGC), the main issue in this paper is to deal with the time-varying random PHN process.

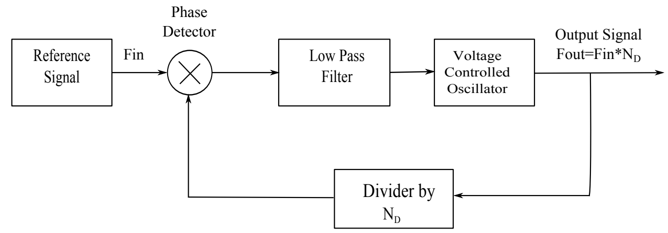

Fig. 2. Block diagram of frequency synthesizer.

PHN characteristics depend on the type of frequency generator. Carrier frequency can be generated from a free running oscillator or frequency synthesizer. Most of the wireless communication standards use phase lock loop (PLL) based frequency synthesizer [9], because it gives high level of stability, easy control, wide range of frequency generation and higher accuracy as well. The non-ideal characteristics of PLL results PHN that is random process caused by the phase fluctuation of the oscillators. Better hardware design may reduce the effect of PHN but improved estimation scheme is an important issue to estimate the PHN. Most of the existing research works assume Wiener model (Random walk) for phase noise [10]. Such a model is only suggested when the carrier signal is generated from a stand alone local oscillator. This type of design is almost never adopted as a free running oscillator that slowly drift away from the required phase and frequency. Whereas, the gain of a PLL is almost constant within the loop bandwidth and there is a 30dB/dec roll off outside the loop bandwidth. The block diagram of a PLL is shown in Fig. (2) that consists of a reference signal source, a phase detector (PD), a low pass filter (LPF), a voltage controlled oscillator (VCO) and a frequency divider.

The function of PD is to detect the phases of two input signals, then compare the phases and gives an error signal depending on the phase difference. If the reference input and the signal from divider are given by

$$s_r = A_r sin\{\omega_c t + \theta_r(t)\} \quad (7)$$
$$s_d = A_d cos\{\omega_c t + \theta_d(t)\} \quad (8)$$

then, the PD output will be

$$\begin{aligned} s_p = s_r \times s_d = \frac{A_r A_d}{2}&[sin\{\theta_r(t) - \theta_d(t)\} \\ &+ sin\{2\omega_c t + \theta_r(t) + \theta_d(t)\}] \quad (9) \end{aligned}$$

The high frequency component is filtered out by the LPF and the input voltage of the VCO is given by

$$s_{LP} = \frac{A_r A_d}{2} sin\{\theta_r(t) - \theta_d(t)\} \quad (10)$$

VCO produces periodic signal, the frequency of which

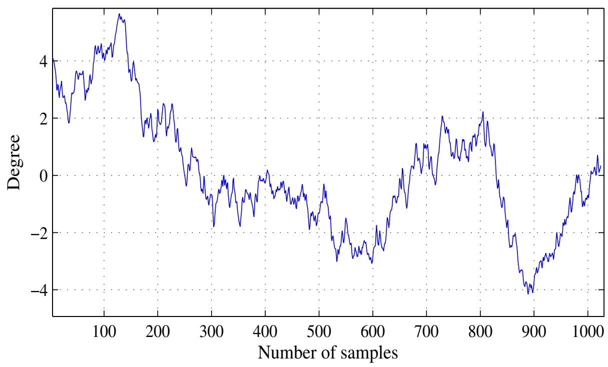

Fig. 3. The randomness of phase noise.

changes based on the control signal applied externally. If the control signal is zero, VCO runs freely and generates its center frequency. As long as the input signal of VCO present, the phase difference exists, phase of VCO output changes linearly, results decrease in phase difference. This changes in phase difference continue until the difference is zero, i.e. the input of VCO is zero. This zero input voltage of VCO is the desired state of PLL, called lock state. In the lock state, VCO produces its constant center frequency which is N_D times the reference frequency. Phase noise characteristics, its statistical properties are well defined in [8]. Using PLL and VCO, random phase noise is generated as shown in Fig. (3) that illustrates the PHN is a zero mean random variable.

B. Effect of PHN on OFDM

To verify the effect of PHN on OFDM systems, computer simulations were performed using the following parameters shown in Table I. It is clearly seen from the scatter plot shown in Fig. (4) that the received signals, blue marked (dot), are rotated counter-clockwise from its original location, the place

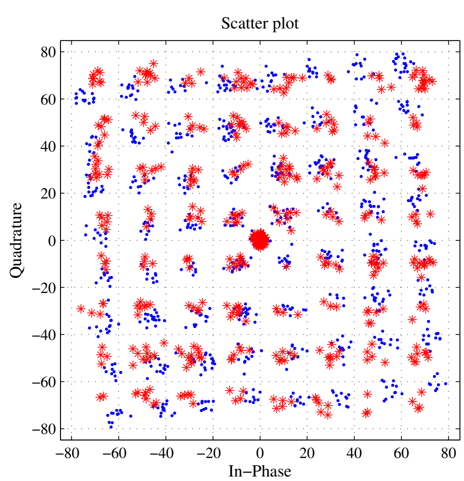

Fig. 4. Figure shows that the received signal is rotated from its ideal position due to CPE effect on 64-QAM OFDM

of signal without phase noise marked in red (star), by an average angle $\bar{\theta}$. The main objective of this research is estimate this average angle of rotation and to mitigate the effect of CPE.

IV. CPE Estimation

The CPE angles rotate the signal constellation points and if the angle of rotation is beyond a threshold angle, then it is quite difficult to detect the transmitted data symbols. Different threshold angles depending on different M-QAM constellations are discussed in [8]. In that case, different algorithms or special error correcting codes are necessary to efficiently detect the symbol. In this section, a CPE compensation method is proposed to abate the effect of CPE in OFDM systems caused by PHN. In the proposed algorithm, the angles of all signals corresponding to a particular constellation point are calculated w.r.t reference axis and averaged, then this process is repeated for all available constellation points. These average angles are compared with the corresponding ideal angles and averaged again to estimate the angle of rotation caused by CPE. PLL frequency synthesizer is regarded as the source of PHN [11], and for the simplicity, only the receiver phase noise is considered. The performance of the proposed algorithm is demonstrated by the computer simulations. To do this, 64-QAM modulation technique is used to modulate the frequency domain interleaved signal. Firstly, the conventional pilot based method of CPE estimation will be discussed, and then the simple but robust blind CPE estimation algorithm will be proposed.

A. Conventional Pilot Based Estimation

Suppose X_p presents the set of indices corresponding to pilots in an OFDM signal. To estimate the CPE angle $\bar{\theta}$, the least-square (LS) method [3] is applied by minimizing the cost function,

$$\underset{C_0}{min} \sum_{k \in X_p} \mid Y_k - C_0 X_k H_k \mid^2$$

To minimize the above function, it is differentiated w.r.t C_0 and set to zero. This results,

$$\frac{d}{dC_0} \sum_{k \in X_p} \mid Y_k - C_0 X_k H_k \mid^2 = 0$$

$$\Rightarrow \quad \sum_{k \in X_p} (-2Y_k(X_k^* H_k^*) + 2C_0 \mid H_k X_k \mid^2) = 0$$

$$\Rightarrow \quad C_0 = \frac{\sum_{k \in X_p} Y_k (H_k X_k)^*}{\sum_{k \in X_p} \mid H_k X_k \mid^2} \tag{11}$$

With the help of equation (4) for small angle assumption equation (11) can be written as

$$1 + j\bar{\theta} = \Re \left(\frac{\sum_{k \in X_p} Y_k (H_k X_k)^*}{\sum_{k \in X_p} \mid H_k X_k \mid^2} \right) + j\Im \left(\frac{\sum_{k \in X_p} Y_k (H_k X_k)^*}{\sum_{k \in X_p} \mid H_k X_k \mid^2} \right) \tag{12}$$

Equating imaginary part of equation (12), the pilot based estimation of CPE angle can be obtained as follows-

$$\hat{\hat{\theta}}_{pilot} = \Im\left(\frac{\sum_{k\in X_p} Y_k(H_k X_k)^*}{\sum_{k\in X_p} \mid H_k X_k \mid^2}\right) \tag{13}$$

B. Proposed Blind Estimation

To find out the angle of rotation caused by CPE as shown in Fig. (4), the received frequency-domain signal points were divided into four quadrants according to its coordinates that can be expressed as

$$R_g \subset Y_k \tag{14}$$

where, $g = 1, 2, 3, 4$ indicates 4 quadrants. In each R_g, there are maximum 16 constellation points that consist of several complex baseband signal points. To estimate the CPE, the signal of maximum amplitude in one of the four quadrants is detected with the corresponding angle w.r.t reference axis given by,

$$\theta_{max}(g) = \underset{\theta}{max}\|R_g\| \tag{15}$$

Then, the neighboring signal amplitudes are located by using a threshold angle. From this group of signals, average angle w.r.t reference axis is estimated. The average angle for one constellation point is expressed by

$$\theta_{av}(1) = \frac{1}{N_s}\sum_{j=1}^{N_s} arg\{R_j\} \tag{16}$$

This process is repeated for all constellation points in the same quadrants, and then for all quadrants. For all points there are individual ideal angles that can be denoted by $\theta_{ideal}(p)$ where, $p = 1, 2, ...M$. The averaged angles were compared with the corresponding ideal angles that gives M individual CPE angles,

$$\theta_{cpe}(p) = \theta_{ideal}(p) \sim \theta_{av}(p) \tag{17}$$

These CPEs were again averaged to figure out the desired angle of rotation, given by

$$\hat{\theta}_{proposed} = \frac{1}{M}\sum_{j=1}^{M} \theta cpe(j) \tag{18}$$

V. Simulation Results

To investigate the performance of the proposed method, several computer simulations were performed by using the system parameters shown in Table I.

TABLE I
SIMULATION PARAMETERS

Parameters	Value
Modulation Type	64-QAM
Number of sub-carriers	1024
Number of unused sub-carriers	200
Length of Cyclic Prefix	64
Channel length	10
Channel Type	Rayleigh fading
Sampling frequency	20 MHz
3dB bandwidth of PLL	10 kHz
rms value of PHN	3^0

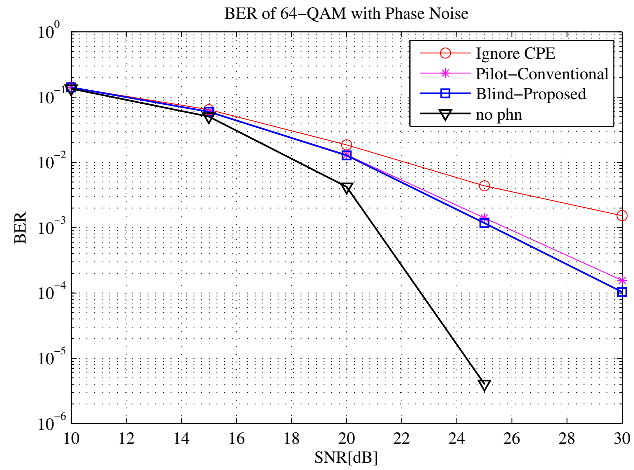

Fig. 5. Comparison of CPE compensation methods.

As shown in Fig. (5), the PHN causes considerable bit error rate (BER) performance loss, and the proposed method can compensate for the CPE much better than the conventional pilot based scheme.

VI. Conclusion

An improved and bandwidth efficient new algorithm with low calculation complexity has been developed for PHN compensation in OFDM systems. Simulations show that the proposed blind algorithm outperforms the conventional pilot based CPE compensation method.

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