Time Dependent Force Outside a Complex Magneto-Dielectric Particle: Abraham-Minkowski Controversy

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***Abstract*—Abraham and Minkowski momenta in material medium have been a long standing dilemma for the last hundred years. It is well-known that the difference between the outside forces of a dielectric particle calculated from Abraham and Minkowski formula is generally difficult to observe due to the cancellation of their time dependent part in the averaging process. However, the missing information can be closely observed from the transient parts of the corresponding formulation of the force densities. Here, in this work, we have theoretically demonstrated the discrepancy between Abraham and Minkowski forces in time domain which clearly indicates their different nature for both absorbing and gain media. In addition, the study broadly supports the fact that the distinction between the two rival force densities is more obvious in a gain medium than in an absorbing one. The results can be applied in case of theoretical and practical optical manipulation of both the force and torque.**

Keywords*—Optical Force, Force Density, Maxwell Stress Tensor, Optical Momentum, Abraham- Minkowski Controversy, Mie Scattering Force.

I. Introduction

Until now, Electrodynamics is the only branch that has been completely understood. We have a classical, as well as a Quantum version of this theory. Although there is an alluring development in Electroweak theory and Chromodynamics in recent years, they were both inspired by Electrodynamics. Classical Electrodynamics is based on Maxwell's four equations. Another auxiliary equation, Lorentz force equation describes the interaction between particles and fields. There are no more measurable quantities in Electrodynamics. Despite all those success, scientists have been quietly aware of a dilemma for a century. The problem is the two proposed form of rival momentum densities inside a media. The well-known *Abraham-Minkowski controversy*. When the size of a particle is smaller than $1\mu m$, the dominant force acting on it is the radiation pressure and the Lorentz force [1]. Kepler suggested that the pressure of light explains why comet tails point away from the sun [2, 3]. The theory was later developed by many other scientists specially Maxwell [2,3]. But the notion that fields carry momentum leads to several intriguing problems, some of which are not entirely resolved after more than a century of debate [2]. As there is no ambiguity in understanding the radiation pressure or photon momentum [3], much attention turned to the corresponding properties in material media [3, 4]. Inside matter, which is subject to polarization and magnetization [4], the effective field momentum density is modified [2,3]. There are different definitions of momentum densities Each momentum density $(\mathbf{g})$ supports corresponding stress tensors $(\mathbf{T})$ and forces. Myriad of research papers have claimed to find results in favor of one or the other; experiments have been performed with unambiguous results. In recent years the dispute has become intense [5,6]. Minkowski Momentum Density has been reported in many experiments especially in [3], [7], [8]. It is convenient to define Abraham systems that support Abraham Momentum Density (AMD) of field and KM of the medium. However, the total momentum is considered as a conserved quantity [9]

$$\mathbf{p}_{Kinetic}^{medium} + \mathbf{p}_{Abraham} = \mathbf{p}_{canonical}^{medium} + \mathbf{p}_{Minkowski}$$

At present Abraham-Minkowski controversy has been studied because of the critical importance of optical forces in nanotechnology [5,6,7,8].As a result, significant resources are expended in attempts to realize technologies based on the assumption of one formulation or another. [9, 10] However, it is now established that the issue is not which momentum is correct, but which is measurable in a particular experiment. [11] As it is well known, in a medium with complex magneto-dielectric particle, the time average force calculated from the Minkowski and Abraham approach are the same due to the cancellation of the time dependent momentum densities. But, here, we will show that the time derivative of momentum density, which cancels out when we take time average, play a central role to distinguish these two rival forms of momentum outside a complex magneto-dielectric particle.

Here, we calculate the time dependent force outside a complex magneto-dielectric particle separately with both Abraham and Minkowski formulation considering a homogenous background medium. In Sec. II the corresponding theory is summarized Classical and Semi-classical approach respectively. Section III is devoted to the calculation of the force densities and cast a quick look at some possible manipulation of the dilemma.

II. THEORY

A. Classical Approach

A century has now passed since the origins of the Abraham-Minkowski controversy; both claim to be the correct form of optical momentum in media. [12-14] Whenever we describe optical momentum transfer to any particle, it is better to consider appropriate momentum of the medium at first (kinetic momenta (KM) or canonical momenta (CM) [15,16]). KM is associated with Einstein-Balaz's box experiment that supports Abraham momentum Density (AMD) of field inside the closed system [15,16]. But CM is associated with wave like behavior, translational effect on the medium and Minkowski momentum density (MMD) of field. In the presence of materials subject to polarization $(\mathbf{P})$ and magnetization $(\mathbf{M})$ it is convenient to express the laws of electrodynamics in terms of free charges and free currents, since these are the ones we directly control.

$$\rho = \rho_f + \rho_b \tag{1}$$

$$\mathbf{J} = \mathbf{J_f} + \mathbf{J_b} + \mathbf{J_p} \tag{2}$$

The 'bound charge', 'bound current', and 'polarization current'can be written as [20],

$$\rho_b = -\nabla \cdot \mathbf{P} \tag{3}$$

$$\mathbf{J_b} = \nabla \times \mathbf{M} \tag{4}$$

$$\mathbf{J_p} = \frac{\partial \mathbf{P}}{\partial t} \tag{5}$$

The corresponding auxiliary fields [20],

$$\mathbf{D} \equiv \epsilon_0 \mathbf{E} + \mathbf{P} \tag{6}$$

$$\mathbf{H} \equiv \frac{1}{\mu_o} \mathbf{B} - \mathbf{M} \tag{7}$$

Finally, Maxwell's equations become [20],

$$\nabla \cdot \mathbf{D} = \rho_f \tag{8}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{9}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{10}$$

$$\nabla \times \mathbf{H} = \mathbf{J_f} + \frac{\partial \mathbf{D}}{\partial t} \tag{11}$$

If the medium is linear, $\mathbf{D}$ and $\mathbf{H}$ are related to $\mathbf{E}$ an $\mathbf{B}$ by the constitutive relations,

$$\mathbf{D} = \epsilon \mathbf{E} \tag{12}$$

$$\mathbf{B} = \mu \mathbf{H} \tag{13}$$

The resulting electromagnetic energy density in matter is [20]

$$u_m = \frac{1}{2}\left(\varepsilon \mathbf{E}^2 + \frac{1}{\mu}\mathbf{B}^2\right) = \frac{1}{2}\left(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H}\right) \tag{14}$$

and the Poynting vector is [20],

$$\mathbf{S}_m = \frac{1}{\mu}\left(\mathbf{E}\times\mathbf{B}\right) = \left(\mathbf{E}\times\mathbf{H}\right) \tag{15}$$

Thus, we might calculate the force per unit volume on the free charges in the material:

$$\mathbf{f}_M = -\nabla \cdot \overset{\leftrightarrow}{\mathbf{T}} - \frac{\partial \mathbf{g}_M}{\partial t} \tag{16}$$

Where, $T_{ij} = D_i E_j + B_i H_j - \frac{1}{2}\delta_{ij}\left(\mathbf{B}.\mathbf{H} + \mathbf{D}.\mathbf{E}\right)$

It is the Minkowski stress tensor [17]. Here, δ_{ij} is the usual Kronecker delta defined by,

$$\delta_{ij} = \begin{cases} 1 & \text{when} \quad i = j \\ 0 & \text{when} \quad i \neq j \end{cases}$$

with Minkowski momentum density $\mathbf{g_M} \equiv (\mathbf{D}\times\mathbf{B})$ This was Minkowski's proposal [21].

However, Minkowski's tensor is not symmetric, and hence does not conserve angular momentum. To solve the problem, Abraham suggested that the electromagnetic momentum in matter is, [22,23,24]

$$\mathbf{f}_A = -\nabla \cdot \mathbf{T} - \frac{\partial \mathbf{g}_A}{\partial t} \tag{17}$$

Where, the Abraham stress tensor[17],

$$T_{ij} = \frac{1}{2}\left[D_i E_j + E_i D_j + B_i H_j + H_i B_j - \delta_{ij}\left(\mathbf{B}\cdot\mathbf{H} + \mathbf{D}\cdot\mathbf{E}\right)\right]$$

with AMD, $\mathbf{g}_A \equiv \frac{1}{c^2}\left(\mathbf{E}\times\mathbf{H}\right)$

If a force law is good enough, it should be consistent with its associated stress tensor. As Griffiths [2] pointed out,

> *"Only the total stress-energy tensor carries unambiguous physical significance, and how one apportions it between an "electromagnetic" part and a "matter" part depends on context and convenience."*[2, 25, 26, 27, 28]

B. Semi-Classical Approach

It is not necessary to quantize the electromagnetic field in order to realize the problem, but it is helpful to understand it in terms of the properties of a single photon of angular frequency ω. Our explanation will be parallel to the one found in [19]. The total electromagnetic energy within our volume is simply that of the photon $\hbar\omega$.

$$\int \frac{1}{2}\left(\mathbf{D}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{H}\right) dv = \hbar\omega \tag{18}$$

This energy is (on average) shared equally between the electric and magnetic parts so that

$$\int \frac{1}{2}\left(\mathbf{D}\cdot\mathbf{E}\right) dv = \frac{\hbar\omega}{2} \int \frac{1}{2}\left(\mathbf{B}\cdot\mathbf{H}\right) dv \tag{19}$$

If we consider, for simplicity, a linear isotropic and homogeneous medium with relative permittivity ϵ and permeability μ then we are led to

$$\int \mathbf{g}_{\min} dv = n\hbar\mathbf{k} \tag{20}$$

$$\int \mathbf{g}_{Abr} dv = \frac{\hbar\mathbf{k}}{n} \tag{21}$$

Where $\mathbf{k}$ is the wavevector in vacuum (with magnitude ω/c) and $n = \sqrt{\epsilon\mu}$ is the refractive index of the medium. Hence, Minkowski would assert that the momentum of a photon in a medium is its value in vacuum multiplied by the refractive index, while Abraham would have us believe that it is the vacuum [15].

III. FORCE CALCULATION FROM THE OUTSIDE OF A SPHERICAL MIE PARTICLE

To illustrate our proposals, we apply generalized Lorentz-Mie theory to spherical Mie particle. We have limited our discussion for linear, homogenous and isotropic media throughout the article. Let us consider the diffraction of a plane polarized monochromatic wave with wavelength $\lambda = 1024nm$ by a sphere of radius $\mathbf{a}$, immersed in a homogeneous, isotropic medium. Assume, also, as usual, the time dependence

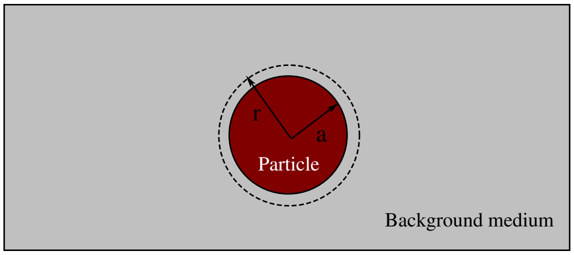

Fig. 1. Transfer of optical momentum: a particle is being shined by a light source. The force density is calculated through a surface of radius $r = a^+ = 1.001a$ with wavelength $\lambda = 1024nm$

$\exp(-j\omega t)$. The time dependent part of the electric and magnetic vectors both outside and inside the sphere satisfy Maxwell's equation in their time free form,

$$\nabla \times \mathbf{H} = -\mathbf{k_1 E} \quad (22)$$
$$\nabla \times \mathbf{E} = \mathbf{k_2 H} \quad (23)$$

where, $\mathbf{k_1}$ and $\mathbf{k_2}$ are the wavevector in the background medium and inside the particle respectively. Hence,

$$\mathbf{k^2} = -\mathbf{k_1 k_2}$$

Total transfer of pure Electromagnetic momentum from the outside of the particle due to incident, reflected and transmitted wave can be expressed as follows,

$$\mathbf{A_{incident}} + \mathbf{A_{scattered}} = \mathbf{A_{transmitted}} \quad (24)$$

Here, $\mathbf{A}$ can be the electric or magnetic field. As we are calculating outside force, we are only concern about the left hand side of (24).

We follow the Debye approach to calculate Mie scattering [29]. Following this, we shall represent the solution as a superposition of two linearly independent fields ($\mathbf{^eE}$,$\mathbf{^eH}$) and ($\mathbf{^mE}$,$\mathbf{^mH}$) each satisfying Maxwell's equations such that,

$$^eE = E_r, \qquad ^eH_r = 0$$
$$^mH = H_r, \qquad ^mE_r = 0$$

Hence, the components become,

$$\mathbf{E_r} = {}^e\mathbf{E_r} + {}^m\mathbf{E_r} = \frac{\partial^2 (r^e\Pi)}{\partial r^2} + \mathbf{k}^2 r^e\Pi \quad (25)$$

$$\mathbf{E}_\theta = {}^e\mathbf{E}_\theta + {}^m\mathbf{E}_\theta = \frac{1}{r}\frac{\partial^2 (r^e\Pi)}{\partial r \partial\theta} + \frac{\mathbf{k_2}}{rsin\theta}\frac{\partial (r^m\Pi)}{\partial\varphi} \quad (26)$$

$$\mathbf{E}_\varphi = {}^e\mathbf{E}_\varphi + {}^m\mathbf{E}_\varphi = \frac{1}{rsin\theta}\frac{\partial^2 (r^e\Pi)}{\partial r \partial\varphi} - \frac{\mathbf{k_2}}{r}\frac{\partial (r^m\Pi)}{\partial\theta} \quad (27)$$

$$\mathbf{H_r} = {}^e\mathbf{H_r} + {}^m\mathbf{H_r} = \mathbf{k}^2 r^m\Pi + \frac{\partial^2 (r^m\Pi)}{\partial r^2} \quad (28)$$

$$\mathbf{H}_\theta = {}^e\mathbf{H}_\theta + {}^m\mathbf{H}_\theta = -\frac{\mathbf{k_1}}{rsin\theta}\frac{\partial (r^e\Pi)}{\partial\varphi} + \frac{1}{r}\frac{\partial^2 (r^m\Pi)}{\partial r \partial\theta} \quad (29)$$

$$\mathbf{H}_\varphi = {}^e\mathbf{H}_\varphi + {}^m\mathbf{H}_\varphi = \frac{\mathbf{k_1}}{r}\frac{\partial (r^e\Pi)}{\partial\theta} + \frac{1}{rsin\theta}\frac{\partial^2 (r^m\Pi)}{\partial r \partial\varphi} \quad (30)$$

Where $^e\Pi$ and $^m\Pi$ are the Debye potentials [30] which satisfies the following wave equation.

$$\frac{1}{r}\frac{\partial^2 (r^e\Pi)}{\partial r^2} + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial^e\Pi}{\partial\theta}\right) + \frac{1}{r^2 sin\theta}\frac{\partial^2 (^e\Pi)}{\partial\varphi^2} + k^2 (^e\Pi) = 0$$

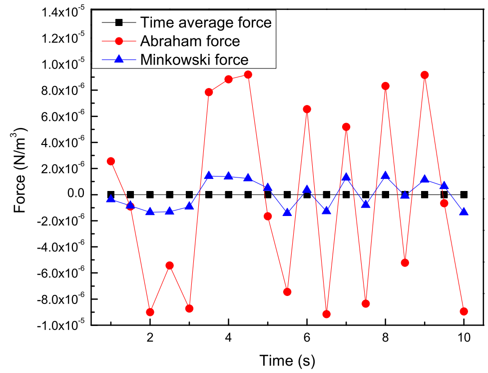

Fig. 2. Abraham and Minkowski force density for a particle having $\epsilon_s = 4 + j, \mu_s = 1$ in water medium $\epsilon_b = 1.33, \mu_b = 1$.

In a similar way, $^m\Pi$ also satisfy the above equation.

After calculating the electric and magnetic fields, the following equation is used to calculate the force densities (z *component*),

$$(\mathbf{f}_{A,M})_z = -(\nabla \cdot \mathbf{T})_z - \frac{\partial (\mathbf{g}_{A,M})_z}{\partial t}$$

The divergence of the stress tensor in Cartesian coordinate is given by [20],

$$(\nabla \cdot \mathbf{T})_z = \frac{\partial T_{xz}}{\partial x} + \frac{\partial T_{yz}}{\partial y} + \frac{\partial T_{zz}}{\partial z}$$

The force density, in all the cases, was calculated at a point $\theta = 45^0$ and $\phi = 60^0$. As the particle is spherically symmetric, this choice is arbitrary.

However, the time average force is

$$f_z = \frac{1}{2t}\int_{-t}^{t} \mathbf{f}_{A,M} dt = -\frac{1}{2}\langle\nabla \cdot \mathbf{T}\rangle_z \quad (31)$$

The above equation (31) shows the reason why time averaging prevents us to observe the difference between the two rival form of momentum transfer in calculation of the outside force as the time dependent part cancels out.

A detailed study has then been carried out to offer supporting evidence with both absorbing and gain medium. Note that, with our sign convention,

$$\varepsilon = a + jb = \begin{cases} b > 0 & gain \;\; medium \\ b < 0 & absorbing \;\; medium \end{cases}$$

In Fig. 2 and Fig. 3 both Abraham and Minkowski force densities are plotted (*red and blue line respectively*) for a gain and absorbing particle medium submerged in water respectively. The figures offer a clear illustration of the $\partial \mathbf{g_{A,M}}/\partial \mathbf{t}$ part in the Abraham and Minkowski formalism. Also in the figure time average force density is shown.

In Fig. 4 and Fig. 5 again Abraham and Minkowski force densities are plotted (*red and blue line respectively*), this time for a gain medium $15 + j$ and an absorbing particle medium $15 - j$, embedded in a magneto-dielectric background.

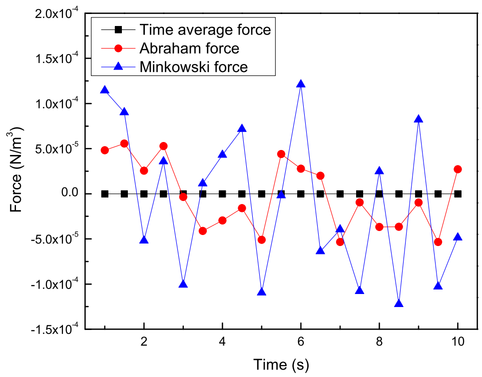

Fig. 3. Abraham and Minkowski force density for a particle having $\epsilon_s = 4 - j, \mu_s = 1$ in water medium $\epsilon_b = 1.33, \mu_b = 1$

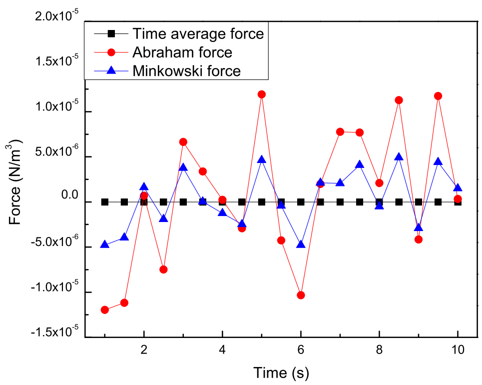

Fig. 4. Abraham and Minkowski force density for a particle having $\epsilon_s = 15 + j, \mu_s = 3$ in a background medium with $\epsilon_b = 4, \mu_b = 2$.

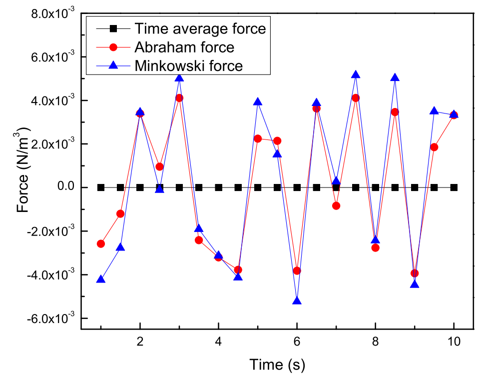

Fig. 5. Abraham and Minkowski force density for a particle having $\epsilon_s = 15 - j, \mu_s = 3$ in a background medium with $\epsilon_b = 4, \mu_b = 2$

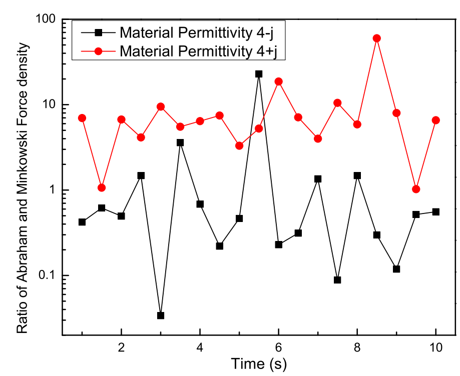

Fig. 6. Ratio of the Abraham and Minkowski force density for the system of Fig. 2 and Fig. 3

Perhaps it is facile to look in a different way. In Fig. 6, we attempt to establish a connection between the force densities calculated by Abraham and Minkowski formalism for a particle medium with $\epsilon_s = 4 + j$ and $\epsilon_s = 4 - j$ respectively. It can clearly be observed that the ratio of Abraham to Minkowski force densities in the gain medium is greater than 1 (*red line*), indicating that Abraham force density is greater than Minkowski's one. On the other hand, the ratio in the absorbing medium is in most cases less than or close to 1 (*black line*), indicating that Abraham force density is less than or close to Minkowski's one. Similar results were obtained for both $\epsilon_s = 15 + j$ and $\epsilon_s = 15 - j$ media. These are the clear illustrations of the importance of the imaginary part of permittivity of the particle medium or their gain or absorption property. Also notice that, in a gain medium, Abraham and Minkowski force densities show significant difference. While in absorbing medium the distinction is less obvious.

Intriguingly, the dilemma is extended from force to torque. Recently, Brevik et al [31] proposed a set-up for measuring the difference between Abraham's and Minkowski's predictions in optics. They showed that when a train of short laser pulses is sent through a fiber wound up on a cylindrical drum, the Abraham theory predicts a torque which, by inserting realistic parameters, is found to be detectable. Indeed, the same torque when calculated with the Minkowski tensor takes the opposite sign. This clearly shows that with time variation both force and torque could show us the difference between Abraham and Minkowski momentum.

IV. Conclusion

In conclusion, we have studied the difference between Abraham and Minkowski force density in time domain for a Mie particle in both absorbing and gain media. The time-averaging effect diminishes the difference between Abraham and Minkowski momenta have been clear from the calculation above. The incongruity is not generally apparent due to the cancellation of time dependent part when we take time average. The findings may be helpful in manipulating optical force and torque. It may also be used to match with the specific practical phenomena observed, when need arises.

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