Optimization of Apodization Profile of Chirped Fiber Bragg Grating for Chromatic Dispersion Compensation

Dispersion Compensation Using Chirped Apodized FBG

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Abstract—**In this paper, we have presented the optimization of *hyperbolic-tangent* profile for fiber Bragg grating (FBG) as dispersion compensator in optical fiber communication system. We have shown that the optimized profile can compensate for chromatic dispersion up to 2237 ps/nm at 1550 nm. The traditional dispersion compensating fiber (DCF) based compensation system can be replaced by this FBG based model which offers many advantages.**

Keywords—FBG; coupled mode theory; dispersion compensation; optical fiber communication

I. Introduction

Optical fiber is a major role-player in optical communications as a wave guiding medium. Fiber Bragg grating (FBG) is playing significant role in optical fiber communication as filter, stabilizer, gain flattening filter, dispersion compensator, optical router etc. Furthermore, it is also used as sensor for sensing temperature, pressure and strain etc. The FBG is a special form of optical fiber where the refractive index of the core is variable. As a result, the wavelength response of the fiber changes and various applications emerge. The FBG is written inside the core of a photosensitive optical fiber using ultraviolet rays. The periodicity of FBG can be of two kinds e.g. mechanical like variation of the core diameter and optical like variation of the refractive index of the core. Two identical counter propagating modes, forward and backward, get coupled in FBG and the energy is transferred from the forward traveling to the backward traveling mode. As a result, we get reflection of the modal energy which is strongly wavelength dependent. The most noticeable feature of FBG is the flexibility of desired spectral characteristics. Numerous physical parameters can be varied including induced index change, length, apodization, period chirp, fringe tilt and whether the grating supports counter propagating or co-propagating coupling at a desired wavelength. These parameters enable FBG to ensure the desired applications stated above.

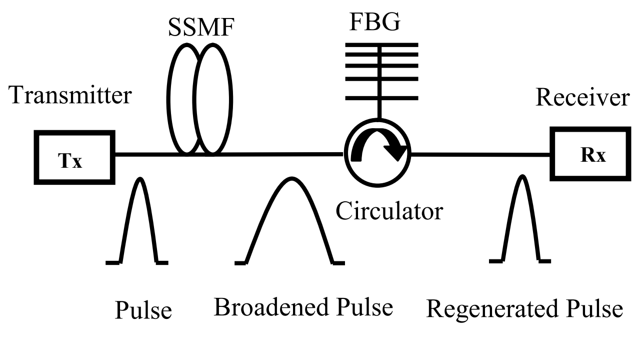

Fig 1: Model for Dispersion compensation

Chromatic dispersion is our primary concern. It broadens the signals and creates information loss. Currently dispersion compensating fibers (DCF) are being used to reduce and manage dispersion in optical fiber transmission lines. In high speed and ultra-high speed optical fiber communication, dispersion impaired distortions are much dominant and detrimental. For high speed communication, FBG is much prospective as dispersion compensator and it has many advantages compared to DCF like almost lossless, compact, easily tunable and negligible nonlinearity. FBGs are found to be much better as dispersion compensators than DCFs [1]. That's why, FBG based dispersion compensation has drawn appreciable research interest [2-4]. Some previous works regarding FGBs and gratings have shown that apodization strength variation can effectively compensate dispersion and improve the spectral characteristics of the FBG [5]. Different profiles have been used in FBG to create optimum dispersion compensation. So far a *hyperbolic tangent* profile with wide flat region at the center and non-abrupt decaying slope has been reported as optimum [6-7]. It has broader reflectivity, linear group delay response and maximum dispersion compensation. Besides, thermal chirping can create adjustable dispersion and adjustable center wavelength which can enable FBGs to adapt network conditions automatically [8].

In this paper, we have shown that an appropriate selection of the apodized optimized *hyperbolic-tangent* profile can play a more significant role in dispersion compensation. We have varied the profile parameters as well as apodization factor parameters and tabulated the data. This analysis has enabled us to choose the appropriate *hyperbolic-tangent* profile to achieve as low as −2237 ps/nm dispersion at 1550 nm. As far as we know, this is the minimum value obtained from a single FBG. Moreover, by optimizing the profile, a moderate range of wavelength band (1548 nm to 1551 nm) can be achieved with −3660 ps/nm to −482 ps/nm dispersion, which might be suitable for dense WDM systems.

II. THEORETICAL ANALYSIS

The model under investigation is shown in Fig. 1. This model is straight-forward. It consists of a lossless standard single-mode fiber (SSMF) followed by an FBG with a circulator. The FBG, which is in reflective mode, will reflect the desired wavelengths and omit the unwanted one. There is no amplifier as the fiber as well as FBG and circulator are considered as lossless.

Coupled-mode theory can be easily evaluated for uniform grating. For non-uniform grating we have to use some evaluation methods. One of them is transfer matrix method. The coupled-mode equations are the main basis of analyzing the FBG and are solved using transfer matrix method. The coupled-mode equations are given as [9]

$$\frac{dB}{dz}+i\left[\kappa_{dc+}\,{}^{1}\!/_{2}\left(\Delta\beta-\frac{d\varphi(z)}{dz}\right)\right]B=-i\kappa_{ac}^{*}F \quad (1)$$

$$\frac{dF}{dz}-i\left[\kappa_{dc+}\,{}^{1}\!/_{2}\left(\Delta\beta-\frac{d\varphi(z)}{dz}\right)\right]F=i\kappa_{ac}B \quad (2)$$

Where,

$$B=A_1(z)e^{-({}^{i}\!/_{2})[\Delta\beta z-\varphi(z)]} \quad (3)$$

and

$$F=A_2(z)e^{({}^{i}\!/_{2})[\Delta\beta z-\varphi(z)]} \quad (4)$$

In these equations, F is forward and B is backward travelling waves, A_1 and A_2 are the amplitudes of forward and backward travelling waves, respectively, z is the transmission distance, κ_{ac} is the ac coupling coefficient and is defined as

$$\kappa_{ac}(z)=\frac{\pi}{\lambda}\nu K(z) \quad (5)$$

Where $\nu K(z)$ is defined as maximum ac index change, ν is the fringe visibility of the index change and K(z) is coupling coefficient defined at the end of this section. κ_{dc} is the dc coupling coefficient, which is expressed as

$$\kappa_{dc}=\Delta\beta-{}^{1}\!/_{2}\frac{d\varphi}{dz} \quad (6)$$

Here ${}^{1}\!/_{2}\frac{d\varphi}{dz}$ is the phase term for a linear chirp, is defined as

$${}^{1}\!/_{2}\frac{d\varphi}{dz}=-\frac{4\pi\eta_{eff}z}{\lambda_D^2}\;\frac{d\lambda_D}{dz} \quad (7)$$

Where, η_{eff} is the effective refractive index, λ_D is the Bragg wavelength and $\frac{d\lambda_D}{dz}$ is the 'chirp' which is a measure of the rate of change of the design wavelength with position of the grating.

$\Delta\beta$ is defined as the detuning which is independent of z,

$$\Delta\beta\equiv\beta-\frac{\pi}{\Lambda}$$

$$\Delta\beta=2\pi\eta_{eff}({}^{1}\!/_{\lambda}-{}^{1}\!/_{\lambda_D}) \quad (8)$$

Λ is the period of the grating and β is wave number.

The z dependent refractive index of the fiber becomes,

$$\delta n(z)=\Delta n\left\{1+{}^{\nu}\!/_{2}\left(e^{i\left[\left(\frac{2\pi N}{\Lambda}\right)z+\varphi(z)\right]}\right)\right\} \quad (9)$$

Where, Δn is the averaged refractive index change and N is an integer which signifies harmonic order.

The grating phase $\varphi(z)$is defined by [10],

$$\varphi(z)=Fz^2/2L_g^2 \quad (10)$$

Where L_g is grating length and F is chirp parameter defined by,

$$F=8\pi\eta_{eff}^2L_g^2/\lambda_D^2cD_fL_f \quad (11)$$

Where D_f is dispersion parameter of the fiber and L_f is the fiber link length.

The coupled-mode equations for uniform grating are straightforward. The reflectivity, delay and dispersion can be found directly from the equation. For non-uniform apodized profile coupled-mode equations can be simplified using Ricatti differential equation. Different methods can also be used for this purpose. In this paper, we have used transfer matrix method. Transfer matrix method provides a fast and highly accurate modeling in frequency domain.

The transfer function of the whole grating is [11]

$$\begin{bmatrix}B\left(-\frac{L}{2}\right)\\ F\left(-\frac{L}{2}\right)\end{bmatrix}=[M]\begin{bmatrix}B\left(\frac{L}{2}\right)\\ F\left(\frac{L}{2}\right)\end{bmatrix} \quad (12)$$

Here matrix [M] is defined at the bottom of this page. Where, δl_j is the length between two consecutive values of z and γ is defined by

$$\gamma=\sqrt{\kappa_{ac}^2-\kappa_{dc}^2} \quad (13)$$

The grating length is defined by,

$$L_g=\frac{L_g^{min}}{\alpha_{eff}} \quad (14)$$

Where, L_g^{min} = Minimum grating length and

$$[\mathbf{M}]=\begin{bmatrix}\cosh(\gamma\delta l_j)-(i\delta\sinh(\gamma\delta lj))\,/\,\gamma & (-\kappa_{ac}\sinh(\gamma\delta lj))/\gamma\\ \kappa_{ac}\sinh\,(\gamma\delta lj))/\gamma & \cosh(\gamma\delta l_j)+(i\delta\sinh(\gamma\delta lj))\,/\,\gamma\end{bmatrix}$$

α_{eff}= Apodization parameter, which determines whether the profile is tighter or weak, α_{eff} is defined as

$$\alpha_{eff} = \frac{\int_0^{L_g} |z| K(z) dz}{\int_0^{L_g} |z|\, dz} \tag{15}$$

$K(z)$ is coupling coefficient of the coupled-mode equations. We will vary this parameter and observe the reflectivity, group delay and dispersion characteristics of FBG which are expressed as follows, reflectivity, $r(z) = \frac{B(z)}{F(z)}$, delay $\tau = \frac{\lambda^2}{2\pi c}\frac{d\theta_r}{d\lambda}$ and dispersion, $D = \frac{d\tau}{d\lambda}$; where θ_r is phase angle of reflection coefficient $r(z)$. Our proposed equation for $K(z)$ is

$$K(z) = i + \tanh\left[\tau\left(1 - 4\left(\frac{z}{L_g}\right)^{\alpha}\right)\right] \tag{16}$$

Where, i is an integer and i = 0, 1, 2, 3 etc. For our model, τ = 4 and α = 3 are taken in order to attain broader bandwidth of reflectivity. We have varied i and have optimized the value to realize the desired dispersion. Our proposed profile is given by

$$K(z) = 4 + \tanh\left[4\left(1 - 4\left(\frac{z}{L_g}\right)^{3}\right)\right] \tag{17}$$

This profile has the highest negative dispersion value at 1550 nm. The parameters used for numerical calculations are listed in Table I.

TABLE I. Different parameters and their values used to generate the figures.

Parameters	*Values*
Chirp, $\frac{d\lambda_D}{dz}$	−1 nm/cm
Effective refractive index, η_{eff}	1.46
Bragg wavelength, λ_D	1550 nm
Grating length, L_g	2 cm
Fringe visibility, ν	5×10^{-4}

III. OPTIMIZATION AND RESULTS

After choosing the profile we have investigated the spectral characteristics of the FBG. For this purpose we evaluated its reflectivity, group delay and dispersion. We have chosen eight different variations for *hyperbolic-tangent* profile and checked them to find the best suitable profile.

The reflectivity of the chosen tangent profiles are shown in Fig. 2. According to the figure, only the profile defined as $K(z) = \tanh[4(1\text{-}4(z/L_g)^3)]$ has reflectivity lower than unity. Other profiles have broad bandwidth along with gradual slope decay. We can also observe that the bandwidth increases with the increase of i define in (16). For better results broader bandwidth with slower slop decay is expected.

Fig. 3 shows the relationship of group delay versus wavelength. Group delay is decreasing as we are increasing the i of $K(z)$. At 1550 nm wavelength, the group delay values are tabulated in Table II. We have also shown that, group delay decreases almost linearly from 1547 nm to 1550 nm for most of the profiles.

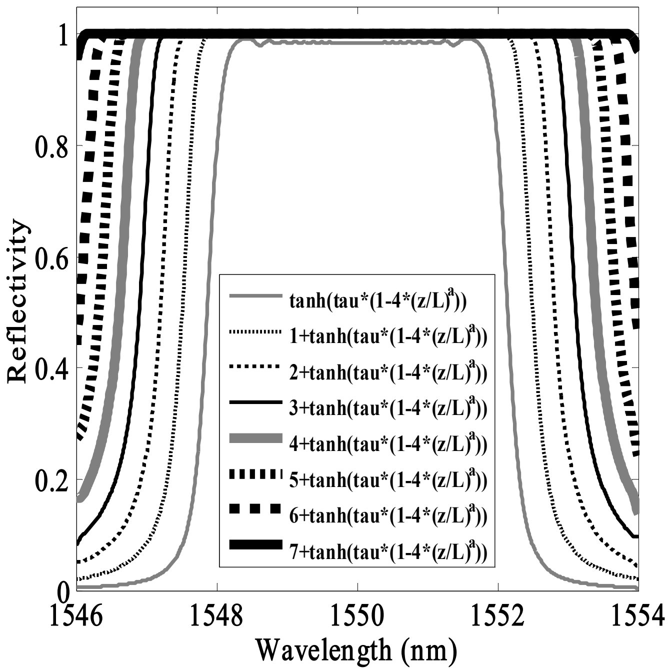

Fig 2: Reflection versus wavelength for eight hyperbolic tangent profiles.

Fig. 4 depicts the dispersion against wavelength for different values of i for *hyperbolic-tangent* profile. As shown in Fig. 4, the dispersion in various profiles changes differently. When the values of the integer i remain within 4, the dispersion increases from almost −650 ps/nm to −2200 ps/nm. When we increase i from 5 to 7, dispersion decreases and gets to as low as −632.6ps/nm for 7. According to the Table II, It is clearly evident that when the value of i is 4 we get maximum dispersion of −2237 ps/nm at 1550 nm and the group delay is also fairly acceptable of 73.32 ps. But, this value of dispersion is existent at a shorter bandwidth for this i. Again, when the value of i is 2 the profile is suitable for dense WDM systems, as it has a moderate range of wavelength band from 1548 nm to 1551 nm with −3660 ps/nm to −482 ps/nm dispersion. However, it has a dispersion of −1739 ps/nm at 1550 nm, which is lower than the maximum dispersion we have achieved for i = 4. We can adjust (not automatic) the profile to achieve desired characteristics from FBG.

In case of periodic in-line dispersion compensation using DCF, the span length for SSMF based transmission line is typically 40 km to 50 km. We can show that FBGs with the proposed profile can be utilized to enhance the span length up to 140 km. This implies that the costing will be reduced considerably. Besides, we will achieve lower signal loss which will increase amplifier spacing and nonlinearity will be much low. Consequently the overall system with FBG based dispersion compensation will be much cost effective.

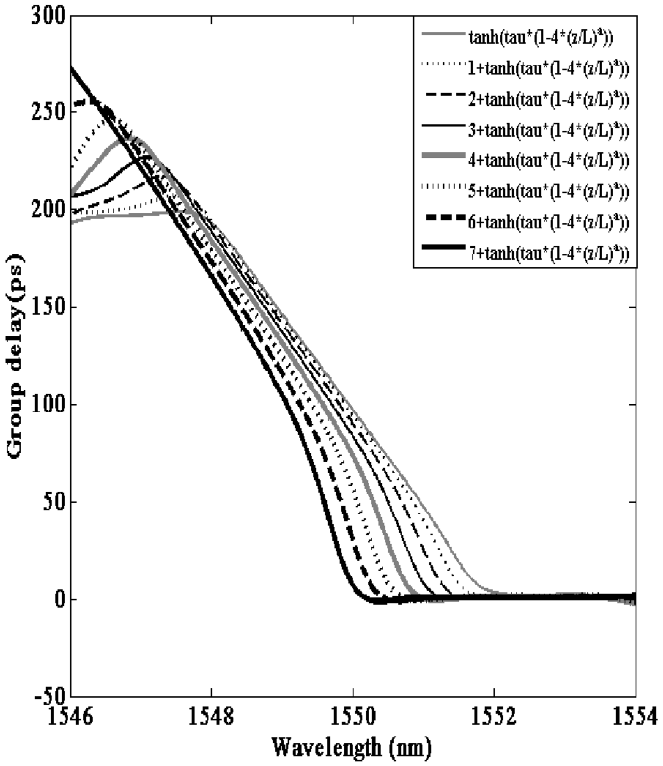

Fig 3: Group delay versus wavelength for hyperbolic tangent.

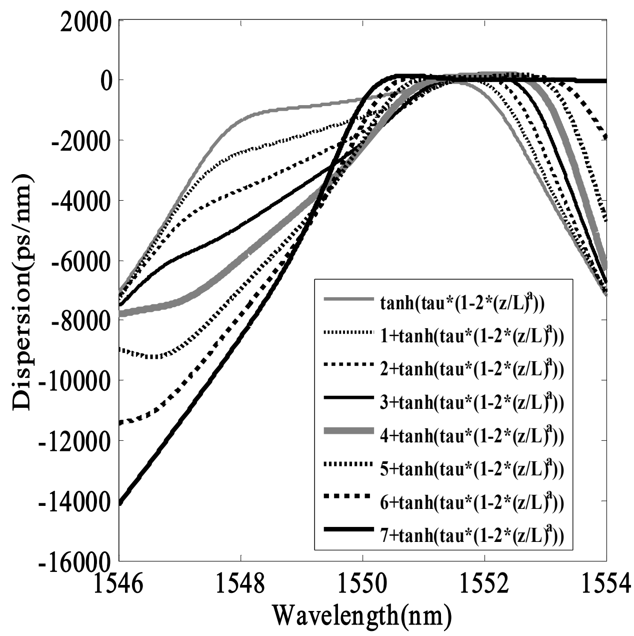

Fig 4: Dispersion versus wavelength for hyperbolic tangent.

According to (15) the apodization factor can be calculated. We have plotted grating length against apodization factor for various fiber lengths as demonstrated in Fig. 5 and found that we need longer grating to support the FBG at 140 km. Tighter profile will need longer grating length while weak gratings will need shorter grating length. Longer grating length may increase the cost, thus creating an inverse relationship with the apodization factor. So, we need to choose our apodization factor wisely, which depends on τ and α (16). We have found an approximate apodization factor of 0.826 suitable for our proposed profiles (15). However, the cost of FBG will not be high compared to DCF for particular amount of compensation.

TABLE II. Group Delay and Dispersion For Various Values Of Integer *i*

$K(z) = i + \tanh\left[4\left(1 - 4\left(\frac{z}{L_g}\right)^3\right)\right]$		
i	***Group delay (ps)***	***Dispersion (ps/nm)***
0	96.94	-748.7
1	94.36	-1226
2	90	-1739
3	83.47	-2106
4	73.32	-2237
5	56.24	-2031
6	30.26	-1471
7	7.14	-632.6

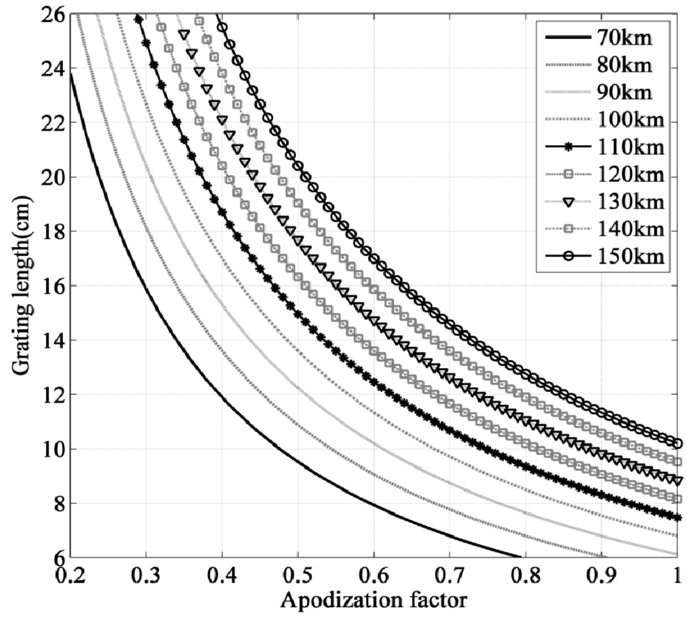

Fig 5: Grating length versus Apodization factor for different fiber link

IV. CONCLUSION

In this paper, we have shown an optimization routine for dispersion compensation using FBG. We have found that, by varying the coupling profile, the huge accumulated dispersion of SSMFs in optical fiber transmission line can be managed efficiently. An FBG with proposed profile of *hyperbolic-tangent* can compensate for dispersion up to −2237 ps/nm dispersion at 1550 nm. This profile can be used with a span length of 140 km of SSMF based transmission lines. Another modified profile, as we have reported, can be used in DWDM systems from 1548 nm to 1551 nm wavelength range with −3660 ps/nm to −482 ps/nm dispersion with a slightly lower dispersion value (−1739 ps/nm) at 1550nm. This will reduce the overall cost of FBG based compensation technology.

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