Red Black Gauss Seidel Solver for Poisson Equation

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1 Introduction

This report contains answers of the theoretical parts of assignment2.

2 Answers of Theoretical Section

2.1 Discretization Matrix M

$$M = \begin{pmatrix} D & E & & & & \\ E & D & E & & & \\ & E & D & E & & \\ & & E & D & E & \\ & & & E & D & E \\ & & & & \ddots & \ddots & \ddots \end{pmatrix}, E = \begin{pmatrix} -1/h_y^2 & & & \\ & -1/h_y^2 & & \\ & & -1/h_y^2 & \\ & & & \ddots \end{pmatrix}$$

and

$$D = \begin{pmatrix} 2/h_x^2 + 2/h_y^2 + k^2 & -1/h_x^2 & & & \\ -1/h_x^2 & 2/h_x^2 + 2/h_y^2 + k^2 & -1/h_x^2 & & \\ & -1/h_x^2 & 2/h_x^2 + 2/h_y^2 + k^2 & -1/h_x^2 & \\ & & & & \\ & & \ddots & \ddots & \ddots \end{pmatrix}$$

In above matrix M, in each row the diagonal entry is $2/(h_x^2)+2/(h_y^2)+k^2$ which is more then absolute sum of all the column for that row and it happens for all the rows. Maximum absolute sum of columns for any row is $2/(h_x^2) + 2/(h_y^2)$ which is less then absolute value of the diagonal entry. Hence the matrix M is strictly diagonal dominant.

2.2 Performance Graphs

The figures below depicts the time taken(in sec) for parallel Red Black Gauss Seidel Solver against various number of threads.

For small grid points(32 and 33) if we increase the number of threads we do not see any improvement, rather we see performance inefficiency due to thread overhead, but for large grid points, we see improvement in the performance.

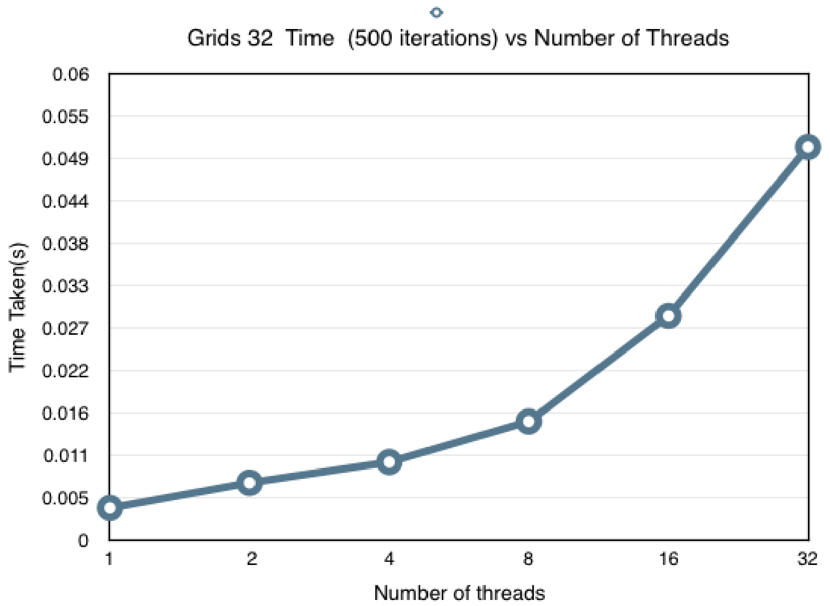

Figure 1: Time taken (in sec) vs number of threads for $32 * 32$ grid points.

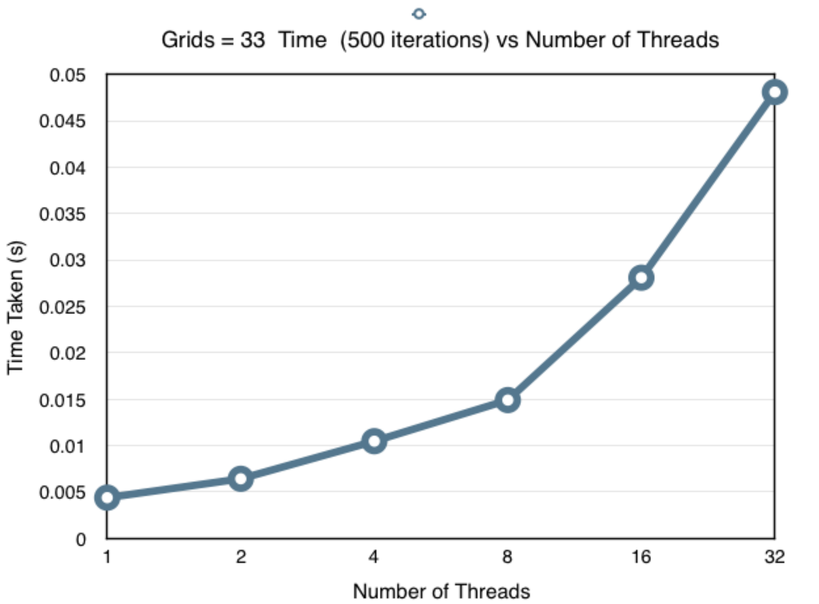

Figure 2: Time taken (in sec) vs number of threads for $33 * 33$ grid points.

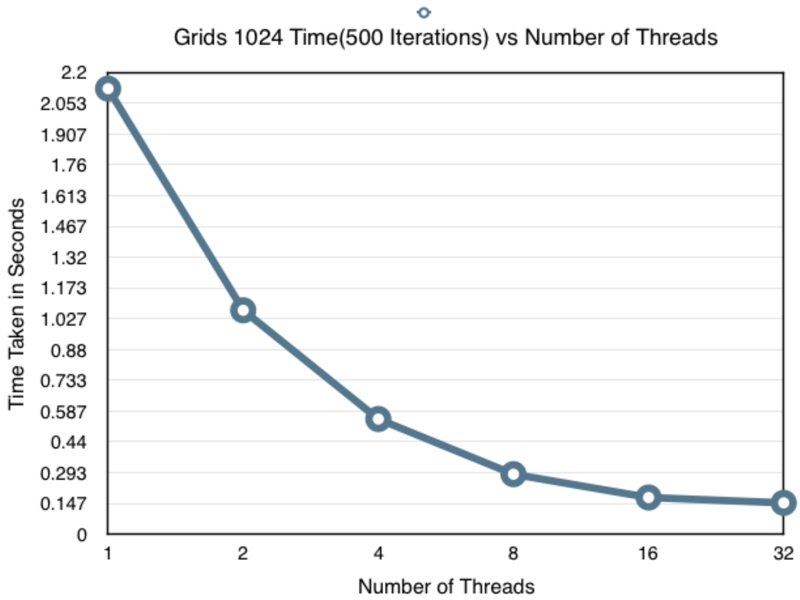

Figure 3: Time taken (in sec) vs number of threads for $1024 * 1024$ grid points.

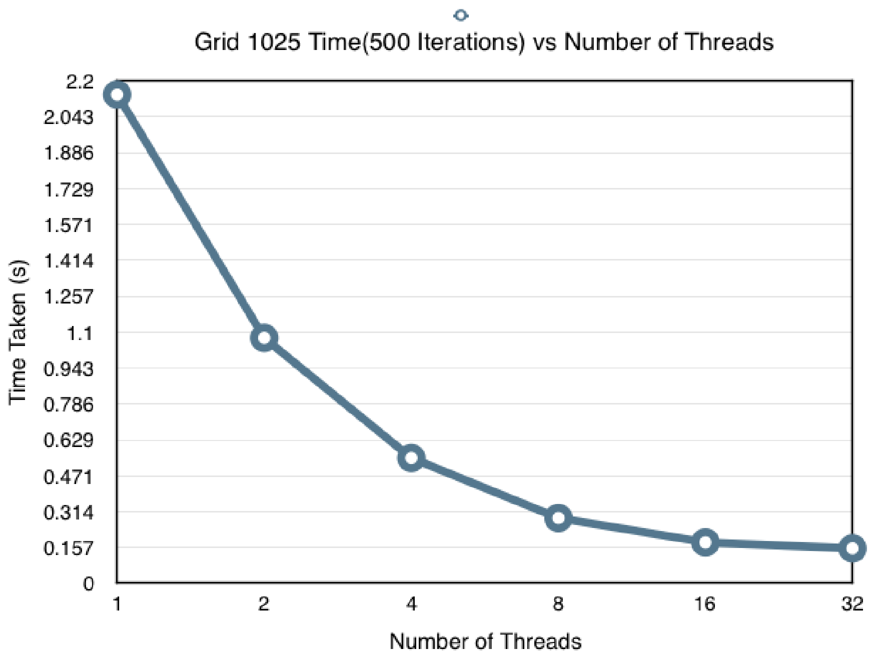

Figure 4: Time taken (in sec) vs number of threads for $1025 * 1025$ grid points.

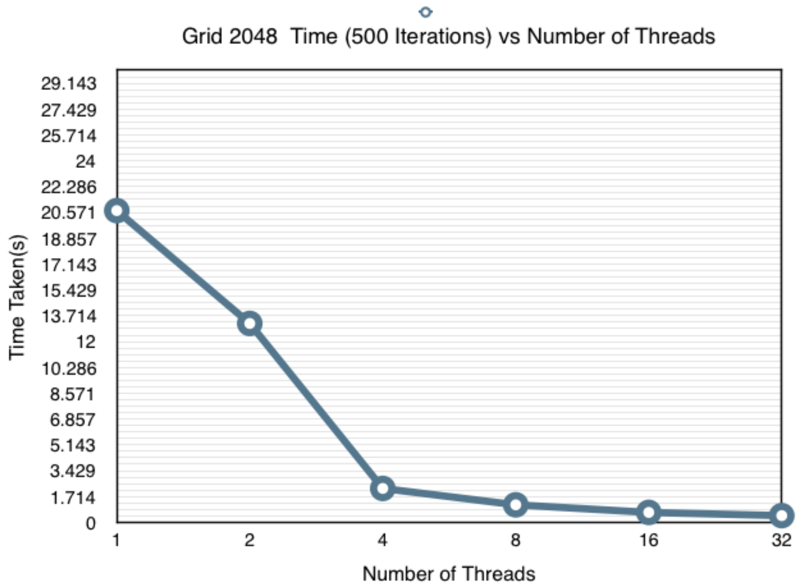

Figure 5: Time taken (in sec) vs number of threads for $2048 * 2048$ grid points.

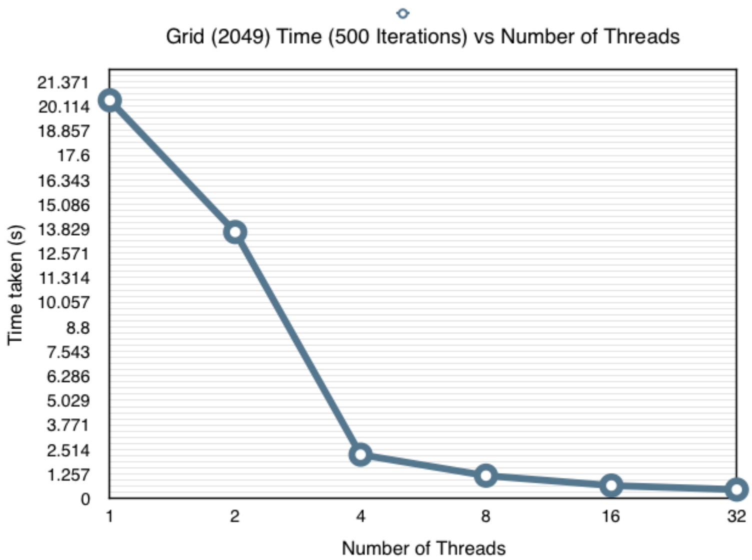

Figure 6: Time taken (in sec) vs number of threads for $2049 * 2049$ grid points.

2.3 Convergence of Jacobi Method

The iteration matrix
$C : D^{-1}(L + U)$,
where D , L and U are diagonal, lower and upper triangular part of discretization matrix M respectively.
Spectral radius ($\rho(C)$) is less than 1(refer equation), therefore the Jacobi method is convergent.

$$\rho(C) <= \|C\|_\infty = \frac{2/h_x^2 + 2/h_y^2}{2/h_x^2 + 2/h_y^2 + k^2} <= 1 \tag{1}$$