Simulation and Scientific Computing

Raju Ram, Sagar Dolas,Seyedehelmira Birang Oskoue

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1 Introduction

This document contains answers to theoretical section of assignment 3.

2 Finite Difference Laplace Operator

By applying laplace operator on $v_{\nu,\mu}^{i,j}$ we get

$$\frac{4v_{\nu,\mu}^{i,j}}{h^2} - \frac{1}{h^2}(v_{\nu,\mu}^{i+1,j} + v_{\nu,\mu}^{i-1,j} + v_{\nu,\mu}^{i,j+1} + v_{\nu,\mu}^{i,j-1}) = \lambda_{\nu,\mu} v_{\nu,\mu}^{i,j} \tag{1}$$

When we plug in $v_{\nu,\mu}^{i+1,j}$ values in equation 1 LHS of the equation becomes

$$\frac{4}{h^2}(sin(\pi\nu x_i)sin(\pi\mu y_j)) - \frac{1}{h^2}(sin(\pi\nu x_{i+1})sin(\pi\mu y_j) + sin(\pi\nu x_{i-1})sin(\pi\mu y_j) + sin(\pi\nu x_i)sin(\pi\mu y_{j+1}) + sin(\pi\nu x_i)sin(\pi\mu y_{j-1})) \tag{2}$$

Replacing x_i by ih and y_i by jh, equation 2 becomes

$$\frac{4}{h^2}(sin(\pi\nu ih)sin(\pi\mu jh)) - \frac{1}{h^2}(sin(\pi\nu(i+1)h)sin(\pi\mu jh) + sin(\pi\nu(i-1)h)sin(\pi\mu jh) + sin(\pi\nu ih)sin(\pi\mu(j+1)h) + sin(\pi\nu ih)sin(\pi\mu(j-1)h)) \tag{3}$$

Using

$$sin(A)sin(B) = \frac{1}{2}(cos(A-B) - cos(A+B))$$

equation 3 can be written as

$$\begin{aligned}
&\frac{4}{h^2}(sin(\pi\nu ih)sin(\pi\mu jh))+ \\
&\quad \frac{-1}{2h^2}(cos(\pi\nu(i+1)h - \pi\mu jh) - cos(\pi\nu(i+1)h + \pi\mu jh))+ \\
&\quad \frac{-1}{2h^2}(cos(\pi\nu(i-1)h - \pi\mu jh) - cos(\pi\nu(i-1)h + \pi\mu jh))+ \\
&\quad \frac{-1}{2h^2}(cos(\pi\nu ih - \pi\mu(j+1)h) - cos(\pi\nu ih + \pi\mu(j+1)h))+ \\
&\quad \frac{-1}{2h^2}(cos(\pi\nu ih - \pi\mu(j-1)h) - cos(\pi\nu ih + \pi\mu(j-1)h))
\end{aligned} \tag{4}$$

From equation 4 sum of the last 4 terms can be written as

$$\frac{-1}{h^2}(cos(\pi\nu ih - \pi\mu jh)(cos(\pi\nu h) + cos(\pi\mu h)) - \\ cos(\pi\nu ih + \pi\mu jh)(cos(\pi\nu h) + cos(\pi\mu h))) \\ = \frac{-2}{h^2} sin(\nu\pi ih) sin(\mu\pi jh)(cos(\nu\pi h) + cos(\mu\pi h)) \quad (5)$$

Now equation 4 becomes

$$sin(\nu\pi ih) sin(\mu\pi jh)\frac{2}{h^2}(2 - 2cos(\nu\pi h) - 2cos(\mu\pi h)) = \\ v_{\nu,\mu}(4/h^2)(sin^2(\pi\nu h/2) + sin^2(\pi\mu h/2)) = v_{\nu,\mu} * \lambda_{\nu,\mu} \quad (6)$$

3 Parallel Overhead

latency = α
time taken to send k elements = k/β, but since the bandwidth is divided in 2 parts, one for send and one for received so time taken will be k/β,
Time taken to update elements in k iterations :- $X * k(k-1)$

Thus Total time taken per iteration per processor is:

$$\frac{1}{k}(\alpha + 2k/\beta + X * k(k-1)) = (\alpha/k + 2/\beta + X * (k-1))$$

in order to minimize this we need to differentiate w.r.t k and set it to 0, so $k = sqrt(\alpha/X)$, for α = 2ms and X = 0.2 ms, k=sqrt(10), This leads to a minimal overhead value for k=3.