1) $x = S, y = V$

$\frac{\partial S}{\partial x} = 1, \frac{\partial V}{\partial y} = 1$

$\frac{\partial S}{\partial y} = 0, \frac{\partial V}{\partial x} = 0$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V \longrightarrow \text{MAXWELL EQN 1.}$$

2) $x = T, y = V$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \longrightarrow \text{MAXWELL EQN 2}$$

3) $x = S, y = P$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P \longrightarrow \text{MAXWELL EQN 3}$$

4) $x = T, y = P$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \longrightarrow \text{MAXWELL EQN 4.}$$

Read Maxwell Relations Notes. (LMS)

CONDUCTION: Transfer of heat from more energetic particles to [less] energetic particles. Observed in solids, liquids & gases.

1) Fluids - diffusion, collision.
2) Solids - free vibrations.

- requires material medium · geometry of medium · temperature

$$\dot{Q} = \frac{KA\Delta T}{\Delta x}$$

$\Delta x \to 0$ $\boxed{\dot{Q} = KA\frac{dT}{dx}}$ Fourier's Law of Heat Conduction.

K: Thermal conductivity - rate of heat transfer ~~per unit area~~ through a unit thickness of the material per unit area per unit temperature difference.

Units : $\frac{W}{(m)°C}$

Thermal conductivity of gases $\propto \frac{\sqrt{T}}{\sqrt{M}}$ T: Temperature
M: Molar mass.

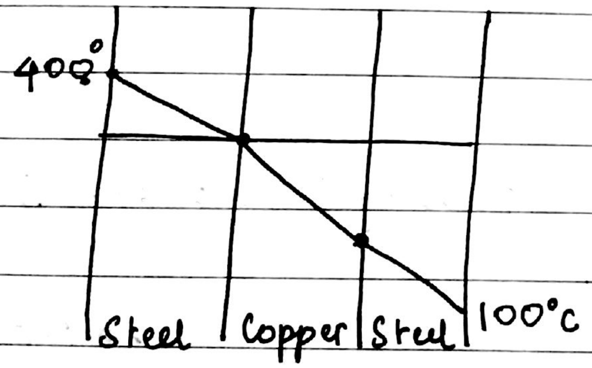
<u>Thermal Diffusion</u>:

<u>One 'D' heat diffusion equation</u>:

$$\dot{Q} = KA\frac{dT}{dx}$$

$$Q_{net} = \frac{du}{dt}$$

If the Fourier law is applied



$$Q_{net} = -KA\frac{\partial^2 T}{\partial x^2}\delta x$$

$$~~Q_{net}~~ -Q_{net} = \frac{du}{dt} = \rho c A \frac{d(T - T_{net})}{dt}$$

$$= \rho C A \frac{dT}{dt} \delta x$$

ρ: density
C: Specific heat capacity
A: Area.

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho C}{K} \frac{\partial T}{\partial t} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

where : $\alpha = \frac{K}{\rho C}$

α : Thermal diffusivity.
Units of α : m^2/s

$$\boxed{\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}}$$

CONVECTION :
Transfer of heat by movement of particles.
It involves combined effects of conduction and fluid motion.
i) Forced convection
ii) Natural/free convection

i) fluid is forced to move over the surface by an external agent.
ii) fluid moves because of buoyancy.

Convective pla...
- Earth's Core
- Photosphere of Stars.

Equation :
$\dot{Q}_{conv} = h A (T_s - T_\infty)$ W.
h : convective heat transfer coefficient. W/m^2
T_s: Surface temperature
T_∞: Temperature sufficiently far away from the fluid.

Another equation for convection:

$$\frac{\partial T}{\partial t} + (V \cdot \nabla) T = K \nabla^2 T$$

$V \cdot \nabla T$: Advection due to temperature change.
$K \nabla^2 T$: How heat is diffused
K : Thermal diffusivity.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \qquad T(x,t) = T_\infty \text{ when } x = L.$$

Biot number :

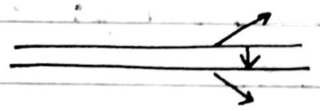
$$Bi = \frac{hL}{K}$$

→ used to understand what sort of problem we are solving

Convection equation used to understand entropy of the universe.
(Entropy problem in Notes)

<u>RADIATION</u>:

$$1 = \alpha_\lambda + \rho_\lambda + \tau_\lambda$$



ρ_λ: Reflection
α_λ: Absorption
τ_λ : Transmission.

<u>Law</u> : Stefan Boltzmann's Law.
The amt. of energy is proportional to T^4

$$e(T) = \sigma T^4$$

e : energy radiated.

$$e_\lambda(\lambda, T) = \int_0^\lambda e_\lambda(\lambda, T)\, d\lambda$$

11-02-16

Statistical Mechanics.

Bulk properties.

Microstate : A state of a system where all parameters of the constituent particles are specified.
e.g. : velocity, collision time, ~~etc~~ momenta, position etc.

In quantum mechanics, one can specify energies of particles.
Macrostate is the most stable state which contains an overwhelming no. of microstates.

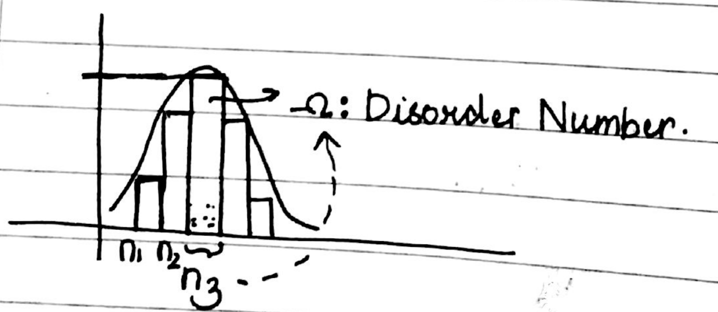
Ensembles

i) Microcanonical Ensemble : isolated system with fixed energy and number of particles is 'N'.

ii) Canonical Ensemble : the system is in a heat bath at constant temperature and number of particles is 'N'.

iii) Grand canonical Ensemble : No. of particles, temperature and energy not fixed.

Ω : Disorder Number is defined as the number of microstates available to a macrostate.



Particles with maximum entropy.

classmate

Date

Page

For isolated systems, volume could be fixed.

$$N = \sum_i n_i$$

$$E = \sum_i n_i E_i$$

Computing most probable macrostate distribution

1) Enumerate Ω

$$n_1 = \frac{N!}{n_1!(N-n_1)!}$$

$$n_2 = \frac{(N-n_1)!}{n_2!(N-n_1-n_2)!}$$

$\vdots$

$$n_j = \frac{N!}{\prod_j n_j!}$$

n_j has degeneracy of g_j

g_j Number of ways the degeneracy of $= g_j^{n_j}$

For all levels : $\prod_j \left(g_j^{n_j}\right)$

Total $\Omega = N! \prod_j \frac{g_j^{n_j}}{n_j!}$

2) Maximize Ω, with E & N fixed.

$$\sum_j n_j = N, \quad \sum_j n_j E_j = E$$

At maximum, $d\Omega = 0$

$$d(\ln \Omega) = \frac{1}{\Omega} d\Omega = 0$$

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$$\ln \Omega = \ln N! + \sum_j n_j \ln g_j - \sum_j \ln (n_j!)$$

Striling's approximation:

$$\ln N! = N \ln N - N$$

$$\ln \Omega = N \ln N - N + \sum_j n_j \ln g_j - n_j \ln (n_j) + n_j$$

$$= N \ln N + \sum_j n_j \ln g_j - n_j \ln (n_j)$$

$$= N \ln N + \sum_j n_j \ln \left(\frac{g_j}{n_j} \right)$$

$$d(\ln \Omega) = \sum_j \cancel{\ln \left(\frac{g_j}{n_j} \right) dn_j - dn_j} \ln \left(\frac{g_j}{n_j} \right) dn_j - dn_j$$

If we assume n_j is large $-\frac{g_j}{n_j^2} \cdot \frac{n_j}{g_j}$

$\sum_j dn_j = 0$ for fixed N

and $\sum_j E_j \, dn_j = 0$ for fixed E

$$\sum_j \ln \left(\frac{g_j}{n_j} \right) dn_j = 0$$

Constraints:

$\alpha \sum_j dn_j = 0$ & $-\beta \sum \epsilon_j dn_j = 0$

$$\Rightarrow \sum_j \left(\ln \left(\frac{g_j}{n_j} \right) + \alpha - \beta \epsilon_j \right) dn_j = 0$$

α & β are so chosen that $\ln \left(\frac{g_j}{n_j} \right) + \alpha - \beta \epsilon_j = 0$

then $n_j = g_j \, e^{\alpha} \cdot e^{-\beta \epsilon_j}$

$$n_j = g_j \, e^{\alpha} \cdot e^{-\beta \epsilon_j}$$

$$= \frac{N g_j \cancel{e^{-\beta \epsilon_j}}\, e^{-\beta \epsilon_j}}{\sum_j g_j e^{-\beta \epsilon_j}}$$

$$= \frac{N g_j}{Z} e^{-\beta \epsilon_j}$$ Maxwell Boltzmann Distribution

Partition function

$$\beta = \frac{1}{KT}$$

K: Boltzmann constant

T: absolute temperature.

$T = 0$, $n_j = 0$ except for n_1

For $T > 0$,

$$\Delta \varepsilon \approx KT$$

Lasers : Population inversion

$$\frac{n_2}{n_1} = \frac{g_2 e^{-\varepsilon_2/KT}}{g_1 e^{-\varepsilon_1/KT}} > 1$$

for $g_1 = g_2$

$$e^{-\varepsilon_2/KT} > e^{-\varepsilon_1/KT}$$

$$-\frac{\varepsilon_2}{KT} > -\frac{\varepsilon_1}{KT}$$

$$\Rightarrow -\frac{\varepsilon_2}{T} > -\frac{\varepsilon_1}{T} \qquad T < 0$$

$$\Rightarrow \varepsilon_2 > \varepsilon_1$$

System cools.

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Problems with this derivative:

1) n_j is not always large
2) There could be other macrostates with same energy E.
Will the disorder be Ω or is it small so that it can be elimin...

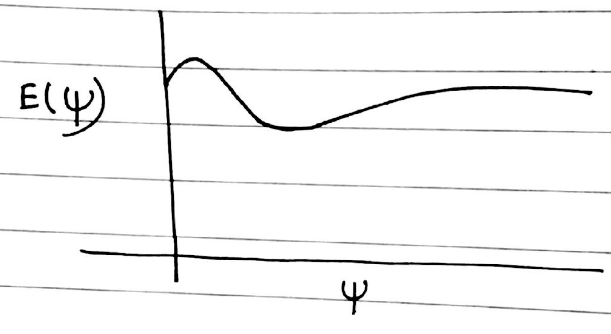
$\tilde{n}_j$ should be weighted against all possible Ω

$$\tilde{n}_j = \frac{\sum_j n_j \Omega_j(n_j)}{\sum_j \Omega_j(n_j)}$$

$$\text{So the } \tilde{n}_j = \frac{N g_j e^{-\epsilon_j/KT}}{\sum g_j e^{-\epsilon_j/KT}}$$

If n_j is not always large,
$\sum n_j \ln n_j - n_j$

$\boxed{\ln \sum \Omega = \ln \Omega(\tilde{n}_j)}$ DERIVATION IN NOTES.



If the wave fn is same for any two particles, the particles indistinguishable. (Bosons)
If the wave fn is different & antisymmetric, the particles distinguishable. (Fermions).

Paramagnetism } understood using Bosons & fermions.
Bands in metals }

Bose-Einstein Distribution for Bosons:

i) Boson : Objects with symmetric wave functions and integral spins.
ii) Fermions : Antisymmetric wave functions and half integral spins.

$\Omega = 1$

All particles are indistinguishable

Macrostates = Microstates.

Unless $g \gg 1$ for some levels, we can count microstates $\Omega \gg 1$

Because the states in a particular level are distinguishable.

To put m_i particles in g_i states

m_i identical particles in $g_i - 1$ barriers on a straight line.

$\circ\circ\circ|\circ\circ| \quad |\circ\ \circ\circ|\circ\circ\circ\circ$

(g_i-1) 1's
~~$m_i + g_i - 1$~~ C_{m_i}

$$\text{Number of ways} = \frac{(m_i + g_i - 1)!}{(m_i)!\,(g_i - 1)!} \qquad \sum_i m_i = N$$

$$\Omega = \frac{(m_i + g_i - 1)!}{(m_i)!\,(g_i - 1)!} \qquad \sum_i m_i \varepsilon_i = E$$

$$\ln \Omega = \sum [\ln (m_i + g_i - 1)! - \ln (m_i)! - \ln (g_i - 1)!]$$

$$= \sum_i [(m_i + g_i - 1)\ln(m_i + g_i - 1) - (m_i + g_i - 1) - m_i \ln(m_i) + m_i - (g_i - 1)\ln(g_i - 1) + g_i - 1]$$

$$= \sum_i [(m_i + g_i)\ln(m_i + g_i) - m_i \ln(m_i) - g_i \ln(g_i)]$$

for $g_i, m_i \gg 1$

$$d(\ln \Omega) = \sum_i dm_i \ln(m_i + g_i) + dm_i - dm_i \ln(m_i) - dm_i = 0$$

(or)

$$\sum_i \ln \frac{(m_i + g_i)}{m_i} dm_i = 0$$

Again $\alpha \sum dm_i = 0$ and $-\beta \sum \varepsilon_i dm_i = 0$

$$\Rightarrow \sum_i \ln \frac{(m_i + g_i)}{m_i} + (\alpha - \beta \varepsilon_i) dm_i = 0$$

$\Rightarrow$

$$\frac{m_i + g_i}{m_i} = e^{-\alpha} . e^{\beta \varepsilon_i}$$

$$m_i = \frac{g_i}{e^{-\alpha} e^{\beta \varepsilon_i} - 1}$$

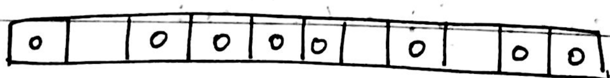
$$\beta = \frac{1}{KT}$$

Average over the g_i levels in each clump

$$n_i = \frac{1}{e^{-\alpha} e^{\beta \varepsilon_i} - 1}$$ (Bose-Einstein distribution)

α is ~~called~~ ~~as~~ related to chemical potential by $\alpha = \frac{\psi}{KT}$ where ψ is chemical potential.

Fermi-Dirac:



Either a particle exists or no particle is there.

m_i boxes out of g_i $= \frac{g_i!}{m_i!(g_i - m_i)!}$

$$\Omega_i = \prod_i \frac{g_i}{m_i!(g_i - m_i)!}$$

Constraints:

$\sum m_i = N$, $\sum m_i \varepsilon_i = E$

$\ln \Omega = \cancel{\sum_i g_i} \; \sum_i g_i \ln g_i - m_i \ln m_i - (g_i - m_i)\ln(g_i - m_i)$

where $g_i \gg 1$, $m_i \gg 1$

$d(\ln \Omega) = \sum -dm_i \ln m_i - dm_i + dm_i(\ln(g_i - m_i))$

$= \sum \ln\left(\frac{g_i - m_i}{m_i}\right) dm_i = 0$

$\alpha \sum dm_i = 0$

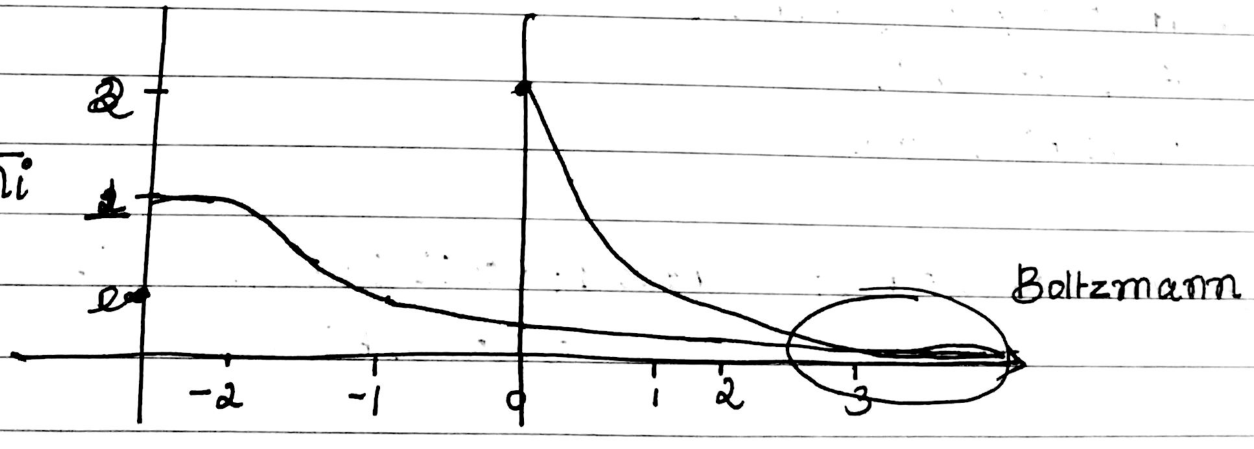
$-\beta \sum \varepsilon_i dm_i = 0$

So, $\sum_i \ln \frac{(g_i - m_i)}{m_i} + (\alpha - \beta \varepsilon_i) dm_i = 0$

(or) $\frac{g_i - m_i}{m_i} = e^{-\alpha} e^{\beta \varepsilon_i}$

$m_i = \frac{g_i}{e^{-\alpha} e^{\beta \varepsilon_i} + 1}$

$g_i \Rightarrow \boxed{\eta = \frac{1}{e^{-\alpha} e^{\beta \varepsilon_i} + 1}}$ Fermi-Dirac.



$(\varepsilon_i - \mu)$ in units of KT

$\underline{\beta :}$

$$\beta \approx \frac{d \ln \Omega}{E}$$

Also given by : $\frac{1}{k_B} \frac{dS}{dE}$

comparing

Thermodynamic sense yields $\frac{dS}{dE} = \frac{1}{T}$

$$P = \frac{1}{K_B T} \approx \frac{1}{KT}$$

~~Bo~~ Partition fn for both Bose Einstein & Fermi-Dirac

$$Z_{FD, BE} = \prod_k \left(1 \pm \lambda e^{-\beta \epsilon_i}\right)^{\pm 1}$$

+ sign for FD

− sign for BE

Applications of statistics:

1) Heat capacity of solids
2) Free $e^{\ominus}$ model
3) Radiation law.

Comparison table : Chapter 9 : Statistical Mechanics

-Arthur Beiser.

1) HEAT CAPACITY OF SOLIDS:

Equipartition states that all terms in a Hamiltonian with squared coordinates x, p_i will contribute ~~half~~ $\frac{1}{2}KT$ to average energy provided quantization is not important

Dulong-Petit Empirical Law:

$C_V = 3R/\text{mole}$

$R = N_A \cdot K$

$U = 3NKT$

→ Valid at high temperatures but at low temperatures significant deviations were found.

Let a crystal lattice structure of a solid comprising of 'N' atoms be treated as an assembly of 3N distinguishable one dimensional oscillators.

Internal energy of a solid made of 'N' atoms is:

$$\boxed{U = 3NK\,\theta_E\left(\frac{1}{2} + \frac{1}{e^{\theta_E/T} - 1}\right)}$$

θ_E : Einstein Temperature. $\approx \dfrac{h\nu}{K}$

Heat capacity $C_V = \left(\dfrac{\partial U}{\partial T}\right)_V = 3NK\left(\dfrac{\theta_E}{T}\right)^2 \dfrac{e^{\theta_E/T}}{\left(e^{\theta_E/T} - 1\right)^2}$

If 'T' is very large $C_V = 3NK = 3nR$

n : No of moles.

$\dfrac{\theta_E}{T} \gg 1 \qquad C_V = 3NK\left(\dfrac{\theta_E}{T}\right)^2 e^{-\theta_E/T}$

as $T \to 0$, $C_V \to 0$

At moderate high temperatures

If an element has large θ_E, θ_E/T will be large.

$T \gg 0$

C_V is small.

$$\Theta_E = \frac{h\nu}{K}$$

ν should be very large

$\nu = \frac{1}{2\pi}\sqrt{\frac{K_F}{\mu}}$ 		K_F : Force const.
μ : reduced mass.

large ν → large K_F or small μ.
Example : Diamond
$C_v = 3NK$ at 1450 K

However "Lead"

2) Free electron model.
Valence electrons in metals are losely bound - low ionization potential.
For perfect lattice, there is free propogation and infinite conductivity of $e^{\ominus}$s however, lattice imperfections, lattice vibrations and scattering by other $e^{\ominus}$s leads to resistance of a crystal lattice.

Phonons.

Free $e^{\ominus}$ model → Fermi-Dirac statistics are applied
1) Electron spins = $\pm \frac{1}{2}$

2) We neglect lattice potential.
3) Electrons are confined to a box.
4) Neglect mutual interruptions.

Single particle state (n_x, n_y, n_z, σ), the energy is given by

$$\epsilon_{n_x, n_y, n_z, \sigma} \approx \frac{\hbar^2 \pi^2}{2mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

$$\approx \frac{\hbar^2}{2m} k^2$$

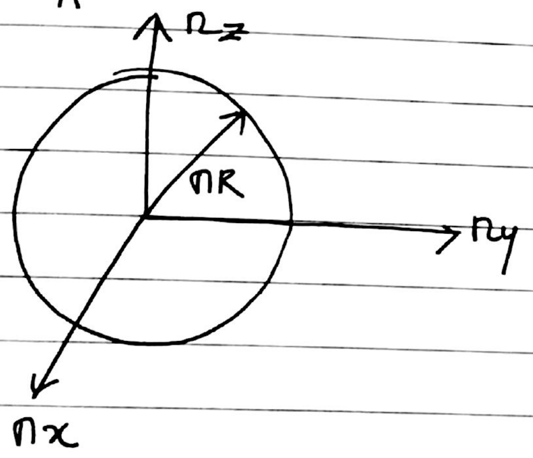
where $K^2 = \frac{\pi^2}{L^2}(n_x^2 + n_y^2 + n_z^2)$

$k = \frac{2\pi}{\lambda}$ the wave vector.

Number of states b/w k and $k+dk$ is given by:

$$\frac{1}{8} \times 4\pi k^2 dk \times 2 \times \left(\frac{L}{\pi}\right)^3$$

$$= \frac{V}{\pi^2} k^2 dk$$



$\frac{2\pi}{\lambda}$
2π

The no. of states from $k=0$ to $k=k$ are

$$\frac{1}{8} \times \frac{4}{3}\pi (n_R)^3 \times 2$$

No of states between energy E and $E + dE$ is

$$g(E)\, dE = \frac{V}{\pi^2} \cdot \frac{2\pi \epsilon}{\hbar^2} \frac{dk}{dE} \cdot dE$$

$$= \frac{V}{\pi^2} \frac{2m^2 E}{\hbar^4} \sqrt{\frac{\hbar^2}{2mE}}\, dE$$

$$= \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^3}\right)^{3/2} E^{1/2}\, dE$$

$$= V C E^{1/2} dE$$

$$\hbar = \frac{h}{2\pi}$$

$g(E)$

$$N = \sum_j n_j$$

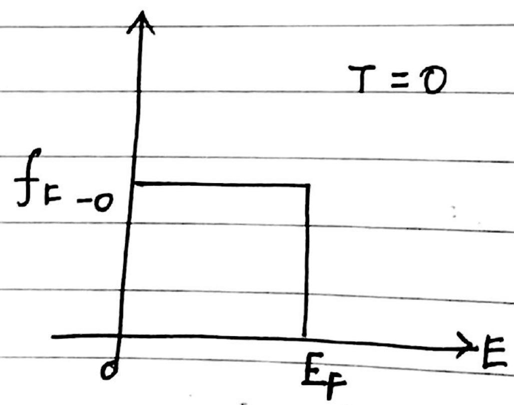
$$= \sum \frac{g_j}{e^{\beta(\epsilon_j - \mu)} + 1} = \int_0^\infty \frac{g(E)\, dE}{e^{\beta(\epsilon_j - \mu)} + 1}$$

$$E = \sum_j n_j E_j = \int_0^\infty \frac{E\, g(E)\, dE}{e^{\beta(\epsilon_j - \mu)} + 1}$$

$\mu(N, T, V)$

at $T = 0$

$$f_{FD}(E) = \frac{1}{e^{\beta(\epsilon_i - \mu)} + 1}$$



$\mu(T = 0) = E_F$ Fermi Energy level.

$f_{FD}(\epsilon) = 0$ for $E > E_F$

$= 1$ for $E < E_F$

$$N = \int_0^{\infty} g(E)\, f_{F-D}(E)\, dE$$

$$= \int_0^{E_F} g(E)\, dE$$

$$= Vc \int_0^{E_F} E^{1/2}\, dE$$

$$= Vc\, \frac{2}{3} (E_F)^{3/2}$$

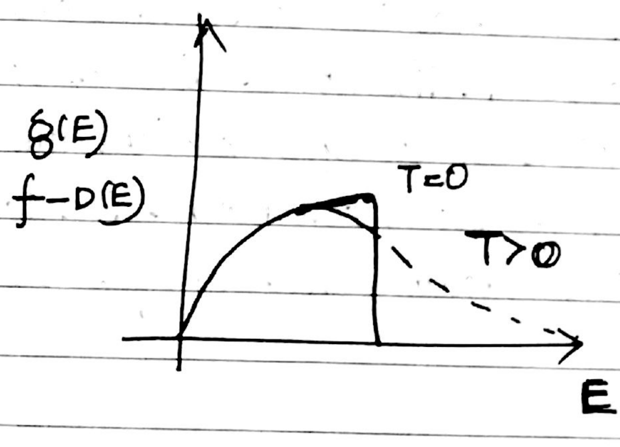
$$E_F = \left\{ \frac{3N}{2V} \frac{2\pi^2 k^3}{(2m)^{3/2}} \right\}^{2/3} = \left(\frac{3N\pi^2}{V} \right)^{2/3} \frac{\hbar^2}{2m}$$

$$= \frac{\hbar^2}{2m} \cdot k_F^2 .$$

k_F : characteristic Fermi wave vector.

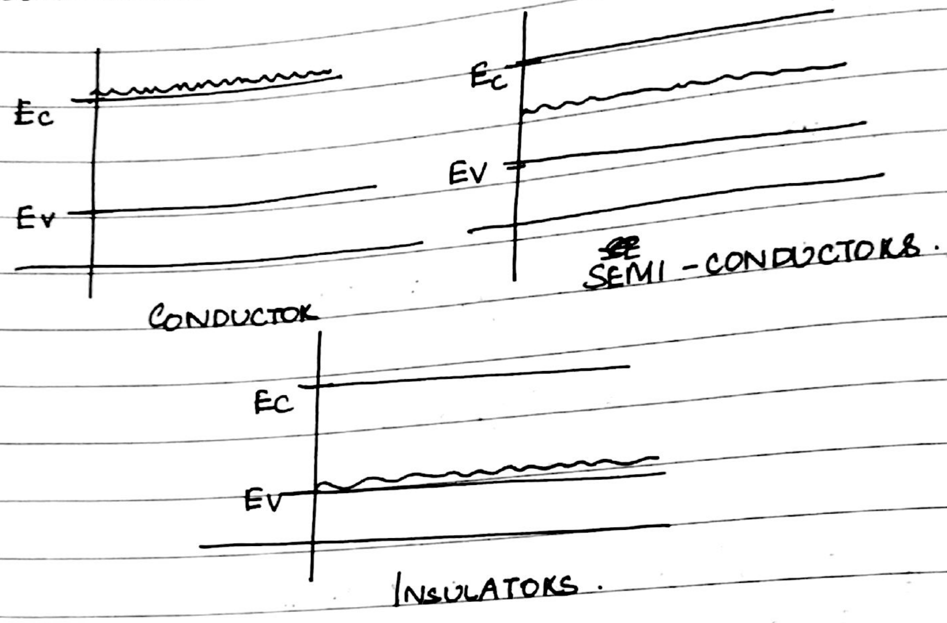
$$E = Vc \int_0^{E_F} E^{3/2}\, dE$$

$$= Vc \left(\frac{2}{5} \right) E^{5/2} = \cancel{\frac{3}{5} NEK} \quad \frac{3}{5} N E_F$$



C_V for a Fermi gas $\propto T$.
At $T>0$, we will except $e^{\ominus}$s to gain an energy of KT. However, most $e^{\ominus}$s cannot gain KT as the states above them are already occupied. Hence, ~~it~~ $e^{\ominus}$s near E_f can gain KT to move to unoccupied states. This results in a round off cliff.

Applications:
We can understand the behaviour of conductors, insulators and semi-conductors using Fermi-Dirac statistics.



Fermi-level: Level occupied at $T=0$.

BLACK BODY RADIATION:

Blackbody: The radiation is enclosed in a vessel and all the walls are at a uniform temperature T. The atoms in the wall will emit & absorb em radiation & at equilibrium the distribution of photons will follow a temperature The body has an infinitesimal hole which radiates. This body is a black body.

Assumptions:

(i) Photons have spin 1

(ii) These photons do not interact with each other.

(iii) They have zero rest mass and travel with speed c.

(iv) $\omega = ck$ } dispersion relation.
$\varepsilon = \hbar\omega = \hbar ck$

(v) Blackbody is enclosed in a chamber.
Properties:
(v) Black body will absorb & emit radiations until thermal eq. is achieved.
(vi) The no. of photons are not conserved because of absorption & re-emission.

α and η are 0. ~~Because~~ Heat capacity of this black body is assumed to be 0.

$E = \sum_{g_i} n_i \varepsilon_i = \text{constant}.$

$$\Omega = \prod_j \cancel{(n_j + g_j)} \frac{(n_j + g_j - 1)!}{(n_j)!(g_j - 1)!}$$

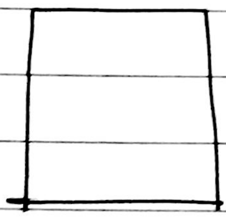
Minimize Ω subject to $E = \text{constant}$.

$\ln \Omega$, $d(\ln \Omega)$

$$d(\ln \Omega) = \sum \ln\left(\frac{n_j + g_j - 1}{n_j}\right) dn_j = 0$$

$$\frac{1}{e^{\beta \varepsilon_i} - 1} = n_j$$

$$n_j = \frac{1}{e^{\beta \varepsilon_i} - 1}$$



Consider a box of volume L^3. There is a standing wave.

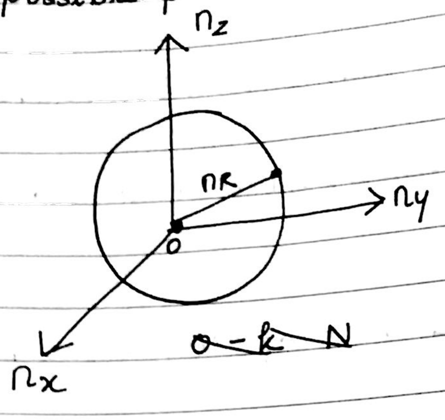
$$\frac{n}{2}\lambda = L \quad \Rightarrow \cancel{k_x, k_y, k_z = n_x}$$

$$k_{x,y,z} = \frac{n_{x,y,z}\pi}{L}$$

For any photon $\bar{k} = (k_x, k_y, k_z)$

$$\frac{\omega^2}{c^2} = k^2 = k_x^2 + k_y^2 + k_z^2$$

"2" possible polarizations — linear, circular.



No of photon states from $o - k = \frac{1}{8} \times \frac{4\pi n_K^3}{3} \times 2$

$$n_K^2 = \frac{L^2}{\pi^2} k^2$$

No of photon states b/w k & ~~dk~~ k + dk,

$$\frac{V}{\pi^2} k^2 dk$$

Photon states b/w ω and ω+ $d\omega$ $= \frac{V}{\pi^2} \frac{\omega^2}{c^2} \frac{d\omega}{c}$

No of photons b/w ω and $\omega + d\omega$ are : $N(\omega) d\omega$

$N(\omega) d\omega$ = Density of states x Probability of occupation (B.E.)

$$= \frac{V\omega^2 d\omega}{\pi^2 c^3 (e^{\beta\hbar\omega} - 1)}$$

Energy of the photons $= \hbar\nu\, N(\omega) d\omega$

$$u(\omega, T)\, d\omega = \frac{V\hbar}{\pi^2 c^3} \frac{\omega^3 d\omega}{e^{\beta\hbar\omega} - 1}$$

Per unit volume $u(\omega, T)\, d\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3 d\omega}{e^{\beta\hbar\omega} - 1}$

T_1, T_2

If the spectrum shifts left → blue shift.

$T_2 > T_1$:

Maximum of $n(\omega, T)$ is given by :

$$\frac{e^{\beta\hbar\omega} 3\omega^2 - \omega^2 \beta\hbar e^{\beta\hbar\omega}}{e^{\beta\hbar\omega} - 1} = 0$$

$$3(e^x - 1) = 0 \quad \text{where } x = \beta\hbar\omega$$

↓

Solved numerically

Accurate value of x obtained is 2.822

$$\frac{\hbar \omega_{max}}{kT} = 2.822$$

$$\omega_{max} = \frac{2.822k}{\hbar} T \quad \text{(Wien's displacement law)}$$

$$\omega = \frac{2\pi c}{\lambda_{max}} = \frac{2.822k}{\hbar} T$$

(or) $\lambda_{max} = \frac{hc}{2.822k} \frac{1}{T}$ (Blue shift).

$$u(T) = \int_0^\infty u(\omega, T)\, d\omega$$

$$= \frac{\hbar}{\pi^2 c^3} \int_0^\infty \frac{\omega^3 d\omega}{e^{\beta\hbar\omega} - 1}$$

$$= \frac{\hbar}{\pi^2 c^3} \frac{1}{\beta^4 \hbar^4} \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

$$u(T) = \frac{k^4 T^4}{\pi^2 c^3 \hbar^3} \cdot \frac{\pi^4}{15} = \frac{k^4 T^4 \pi^4}{15 \pi^2 c^3 \hbar^3} = \sigma T^4$$

$$\sigma = \frac{\pi^2 k^4}{15 c^3 \hbar^3}$$

$3\omega^2(e^{\beta\hbar\omega} -$

$- \beta\hbar\omega$
$(e^{\beta\hbar\omega})$

Scanned by CamScanner

Radiation emitted by Black body:

ΔA - area of the hole.

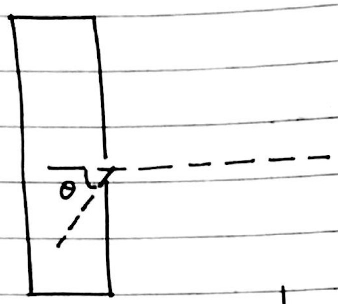
photons are coming out in 2 directions

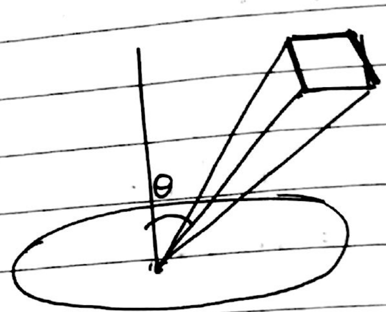
$\theta \rightarrow \theta + d\theta \quad , \phi \rightarrow \phi + d\phi$

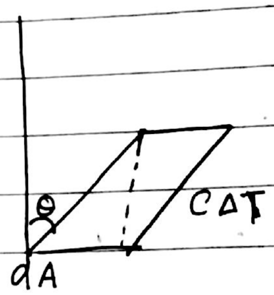
$$d\Omega = \frac{r \sin\theta \, d\phi \, r d\theta}{r^2}$$

$$= \sin\theta \, d\theta \, d\phi$$

Total solid angle $= 4\pi$.







The photons will pass through in ΔT are located in a parallelopiped of length $c\Delta T$.

Volume of parallelopiped $= dA \times c\underset{\delta t}{\Delta T} \times \cos\theta$.

No. of photons / unit time from θ to $d\theta$ and ϕ to $\phi + d\phi$ with ω to $\omega + d\omega$ $= \frac{\text{Volume of parallelopiped}}{\Delta t} \times$ fractional solid angle $\times$ number density in the freq. interval.

$$= c\,dA \cos\theta \frac{\times d\Omega}{4\pi} \times \frac{N(\omega)\,d\omega}{V}$$

Total energy radiated b/w ω to $\omega + d\omega$ $= \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \sin\theta \frac{dA\,c\cos\theta}{4\pi} \frac{\hbar\omega N(\omega)\,d\omega}{V}$

$$= \frac{1}{4} c\hbar\omega \, dA \, \frac{N(\omega)\,d\omega}{V}$$

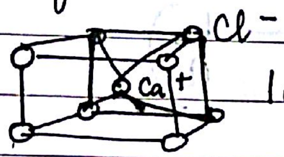
$$= \frac{1}{4} c \, u(\omega, T) \, dA \, d\omega$$

Total power radiated $= \frac{1}{4} c \int_0^\infty u(\omega, T)\, d\omega$

$$= \cancel{\frac{1}{2} c\, \omega\, T^4} \quad \sigma T^4$$

$$\sigma = \frac{\pi^2 k^4}{60\, c^3 \hbar^3}$$

Ionic crystals.



Ionic ~~chr~~ crystal of CaCl

Na^+ $6 - Cl^-$

Ca^+ Cl^- 8

$$u_1 = -\frac{6e^2}{4\pi\varepsilon_0 r}$$

12 Na^+ atoms. $\sqrt{2}\, r$

$$u_2 = +\frac{12 e^2}{4\pi\varepsilon_0 (\sqrt{2}\, r)}$$

$$u_{coulomb} = \frac{-e^2}{4\pi\varepsilon_0 r}\left[6 - \frac{12}{\sqrt{2}} + \dots\right]$$

$$= -1.748 \frac{e^2}{4\pi\varepsilon_0 r}$$

$$= -\alpha \frac{e^2}{4\pi\varepsilon_0 r}$$

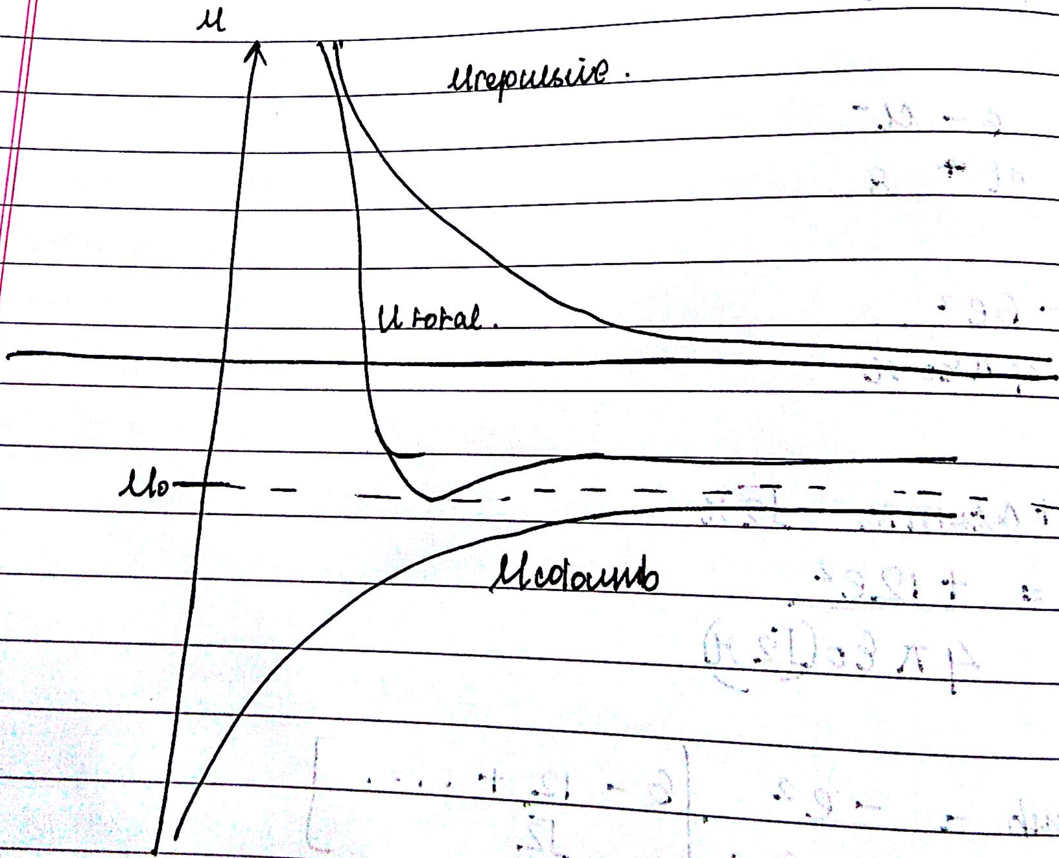
$$U_{repulsive} = \frac{B}{r^n}$$

$$U_{total} = U_{coloumb} + U_{repulsive}$$

$\frac{dU}{dr} = 0$ where $r = r_0$
r_0 is the equilibrium separation of i...

$$B = \frac{\alpha e^2}{4\pi\varepsilon_0 n r^{n-1}}$$

Potential Energy $U = \frac{-\alpha e^2}{4\pi\varepsilon_0 r_0}\left(1 - \frac{1}{n}\right)$.



Weifman - Franz Law:

Ratio of thermal to electrical resistivity

$$\frac{K}{\sigma} = \frac{3k^2}{2e^2}$$ · using Maxwell Boltzmann. ✗

$$\frac{K}{\sigma} = \frac{\pi^2 k^2}{3e^2}$$ using FD. ✓

L-J Potential: (Lennard-Jones)

→ is an empirical formula

→ interaction between neutral atoms or molecules.

$$V_{LJ} = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right] = \varepsilon\left[\left(\frac{r_m}{r}\right)^{12} - 2\left(\frac{r_m}{r}\right)^{6}\right]$$

Inter particle potential is zero.

At r_m, potential reaches a minimum.

Quantum Mechanics.

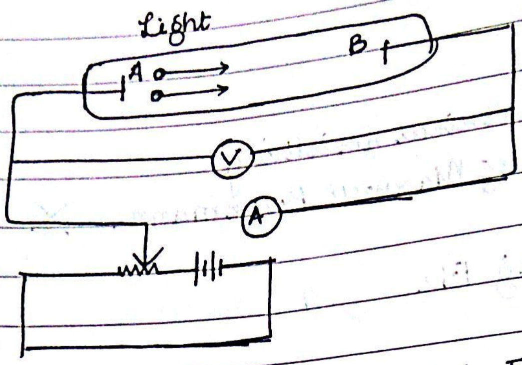
Rayleigh-Jean

$$~~u(\nu)d~~ \; u(\nu)\,d\nu = \frac{8\pi kT}{c^3}\nu^2\,d\nu$$

as $\nu \to \infty$ $\quad u(\nu) \to \infty$

Spectrum observations $\longrightarrow 0$.

$$u(\nu)d\nu = \cancel{8\pi k}\,\frac{8\pi h}{c^3}\,\frac{\nu^3\,d\nu}{e^{h\nu/kT} - 1}$$



Light emitted as quanta. (photons)

1) Atomic spectrum — H_2
Bohr explained the atomic spectrum & energy levels using quantum mechanics.
2) Schrödinger waves
DeBroglie. Wave particle duality.
3) Heisenberg uncertainity principle.

Wave approach - Schrödinger
Matrix approach - Heisenberg.
Operator approach - Dirac.
} Approaches employed to study QM.

Applications of QM:

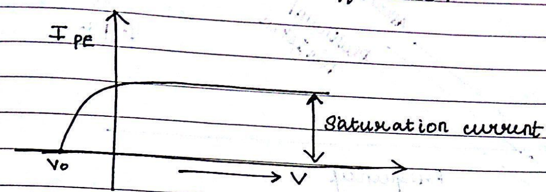
- MRI (spin)
- Scanning Tunnel Microscope (STM) (photograph one atom of gold)
- Quantum computing Q-bit.

Shor's algorithm (Quantum entanglement)

PHOTOELECTRIC EFFECT.

Characteristics of photoelectrons:

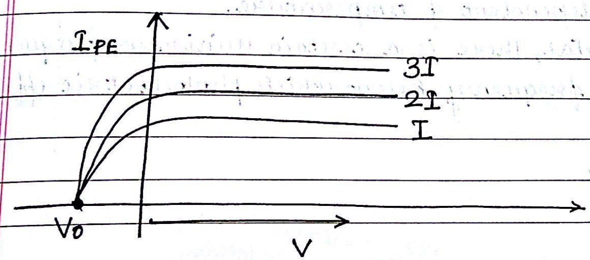
(i) Effect of applied potential difference.



V_0 - Stopping potential.

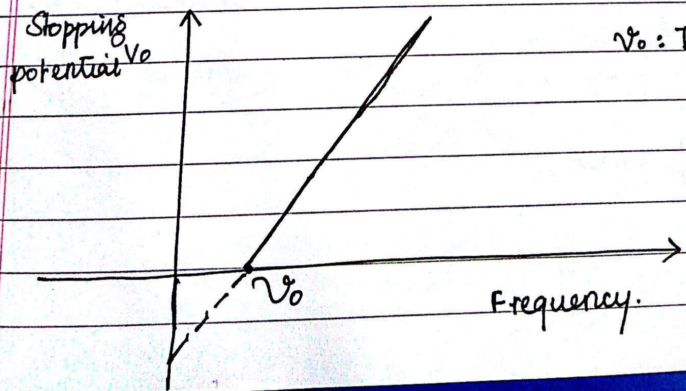
The negative potential of the plate B at which the photoelectric current becomes 0 is the cut off potential/stopping potential.

(ii) Effect of incident radiation.



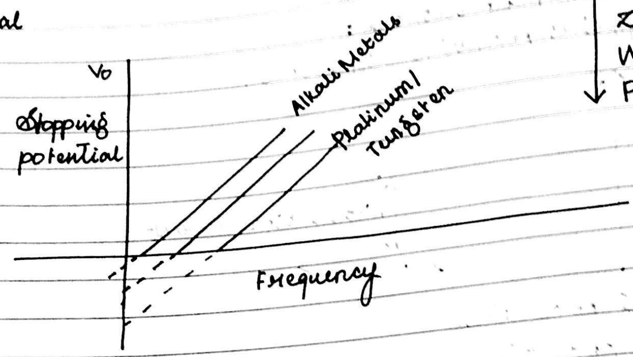
Saturation current $\propto$ incident radiation intensity.

(iii) Effect of frequency.



ν_0 : Threshold Frequency.

4) Metal



K
Na
Zn
W
Pt

<u>Theory</u> :
The no of $e^{\ominus}$s emitted per second $\propto$ intensity of incident radiation.
The max. velocity or the KE of photons depends on the frequency of the radiation and is independent of temperature.
~~There is~~ For every metal, there is a certain minimum frequency called the threshold frequency below which photoelectric effect is not observed.

$$h\nu = \underbrace{W_0} + \frac{1}{2}mv^2$$

W_0 : Work function
→ A property of the material used.
$W_0 = h\nu_0$ $\qquad \nu_0$: threshold frequency.
λ_0 : long wavelength limit.
$\lambda < \lambda_0$

$$\lambda_0 = \frac{c}{\nu_0} = \frac{ch}{W_0}$$

$$\Rightarrow \lambda_0 = \frac{12400}{W_0} \text{Å}$$

λ_0

$$h\nu = h\nu_0 + \frac{1}{2}mv^2$$

(or) $\boxed{\frac{1}{2}mv^2 = h(\nu - \nu_0)}$ (or) $\boxed{\frac{1}{2}mv^2 = h\nu - W_0}$

$$\frac{1}{2}mv^2 \propto h\nu$$

$$v^2 \propto \nu$$

We know that V_0 is the stopping potential:

$$eV_0 = h\nu - h\nu_0$$

$$V_0 = \frac{h\nu}{e} - \frac{h\nu_0}{e}$$

Compton effect or Compton Scattering:

Before collision:	After collision.
1) Energy of incident photon = $h\nu$.	Energy of scattered photon = $h\nu'$
2) Momentum of incident photon = $\frac{h\nu}{c}$	Momentum of scattered photon = $\frac{h\nu'}{c}$
3) Rest ~~mass~~ energy of $e^\ominus$ = m_0c^2	Energy of $e^\ominus$ = mc^2 (velocity 'v')
4) Momentum = 0	Momentum = mv

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Energy of the system before collision = $h\nu + m_0c^2$

after collision = $h\nu' + mc^2$

Horizontal Momentum before collision $= \frac{h\nu}{c}$

after collision $= \frac{h\nu'}{c}\cos\theta + mv\cos\phi$

Vertical Momentum before collision $= 0$

after collision $= \frac{h\nu'}{c}\sin\theta - mv\sin\phi$

$$\therefore \frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + mv\cos\phi$$

$$0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi$$

$$mvc\cos\phi = h\nu - h\nu'\cancel{\cos\phi}\cos\theta$$
$$mvc\sin\phi = h\nu'\sin\theta$$

$$m^2v^2c^2 = h^2[\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta]$$

$$h\nu + m_0c^2 = h\nu' + mc^2$$
$$mc^2 = h(\nu - \nu') + m_0c^2$$

Square

$$m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

$$m^2c^2v^2 = h^2[\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta)$$

Subtract:

$$m^2c^4 - m^2v^2c^2 = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$m^2c^2(c^2 - v^2) = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

(or) $$\frac{m_0^2c^2(c^2 - v^2)}{\left(1 - \frac{v^2}{c^2}\right)} = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$m_0^2c^4 = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$\Rightarrow 2h(\nu-\nu')m_0c^2 = 2h^2\nu\nu'(1-\cos\theta)$$

$$\Rightarrow \frac{(\nu-\nu')}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\boxed{\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)}$$

Multiply by c,

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)}$$

$$\boxed{\Delta\lambda = \frac{2h}{m_0c}\sin^2\frac{\theta}{2}}$$

$$\alpha = \frac{h\nu}{m_0c^2}$$

$$\tan\phi = \frac{\sin\theta}{(1-\cos\theta + 2\alpha\sin^2\frac{\theta}{2})}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} + 2\alpha\sin^2\frac{\theta}{2}}$$

$$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}(1+\alpha)}$$

$$= \frac{\cancel{\cos}\cot\frac{\theta}{2}}{1+\alpha}$$

KE:

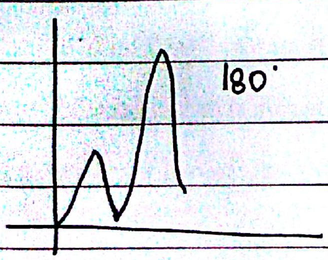
$$K = (m - m_0)c^2 = h\nu - h\nu'$$

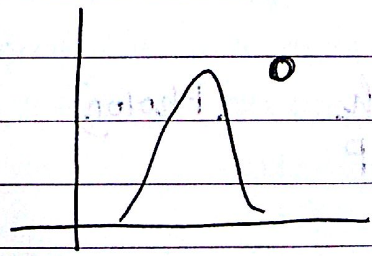
$$\nu' = \frac{\nu}{1 + \alpha(1-\cos\theta)}$$

$$K = h\nu - \frac{h\nu}{1+\alpha(1-\cos\theta)}$$

$$= h\nu\left[1 - \frac{1}{1+\alpha(1-\cos\theta)}\right]$$

$$= h\nu\left[\frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}\right]$$





Wave particle duality and uncertainity:

Matter	Wave
• has mass	frequency
• has velocity	wavelength
• has momentum	phase velocity
• has energy	amplitude
	Intensity
	Energy

Light → Wave phenomenon – Interference
– Diffraction

Light → Particle phenomenon – Photoelectric effect
– Blackbody radiation
– Compton effect.

Wave and particle nature do not exist simultaneously.

De Broglie's wavelength:

$$\lambda = \frac{h}{p}$$

$$E = \cancel{m} h\nu$$

$$= \frac{hc}{\lambda}$$

$$E = mc^2$$

$$mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc} = \frac{h}{p} \quad \text{(Photon)}$$

For a particle moving with velocity 'v',

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\frac{1}{2}m_0v^2 = eV$$

$$v = \sqrt{\frac{2eV}{m_0}}$$

$$\lambda = \frac{h}{m_0v} = \frac{h}{m_0\sqrt{\frac{2eV}{m_0}}}$$

$$= \frac{h}{\sqrt{2m_0eV}} = \frac{12.26}{\sqrt{V}} \mathring{A}$$

If $V = 100V$

$\lambda = 1.226 \mathring{A}$

The properties of matter waves

i) Lighter the particle, greater the λ.

ii) Smaller the vel. of the particle, greater the wavelength.

iii) If $v = 0$, $\lambda \rightarrow \infty$ } Matter waves are generated by motion of the

iv) If $v = \infty$, $\lambda = 0$ } particles.

~~And~~, Matter waves are independent of charge unlike em waves

The velocity of matter waves depends on the velocity of the particles involved. The velocity of em waves is constant.

The velocity of a matter wave is greater than velocity of light.

Matter waves velocity

- Mechanical motion of the particle 'v'
- The propogation of the wave 'w'

$$E = h\nu , mc^2$$
$$h\nu = mc^2$$
$$\nu = \frac{mc^2}{h}$$

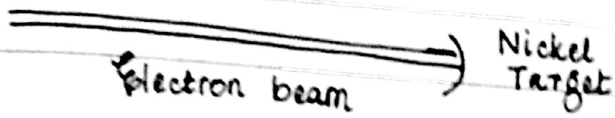
$$w = \nu \times \lambda$$
$$= \frac{mc^2}{h} \times \frac{h}{mv}$$
$$= \frac{c^2}{V}$$

The particle velocity 'v' cannot exceed the velocity of light
∴ Wave velocity 'w' will be greater than velocity of light.

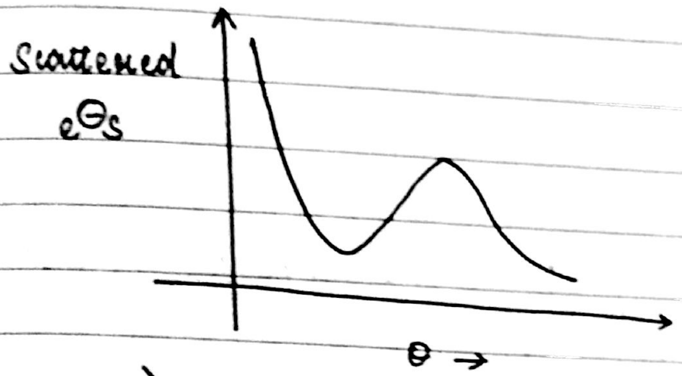
Wave velocity is called phase velocity.
Phase velocity can

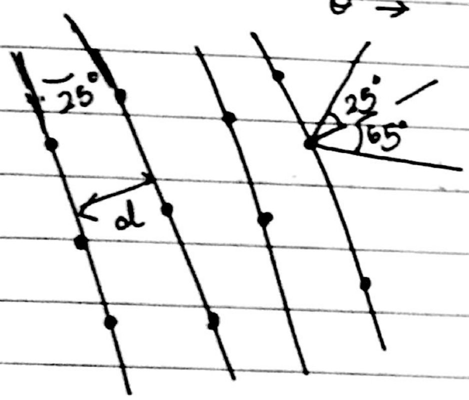
The wave & particle aspects of a ~~given~~ moving body cannot appear at the ~~Same time. Waves exhi~~ together in the same exp. Waves have particle like properties and particles have wave like properties. Matter wa are a symbolic representation to understand these phenomena. The wave nature of the matter Introduces uncertainity in location the position of the particle. However, if the wave is strong, there is a good chance of finding the particle and vice-versa.

Davisson - Germer experiment :
Electron diffraction experiment



Interatomic distance of Ni is known.
The incident light was dull red → thermionic emission.



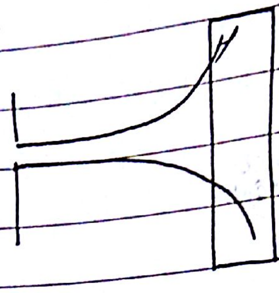


$d_{inter} = 2.15\,Å$
$d = 2.15 \sin 25°$
$= 0.09\,Å$

$2d\sin\theta = \lambda$
$\theta = 65°$
$\lambda = 1.65\,Å$
Voltage applied = 54V
$\lambda = \frac{12.26}{\sqrt{54}} = 1.67\,Å$

Thompson.

10,000V



Group velocity:

$y = A\sin(\omega t - kx)$

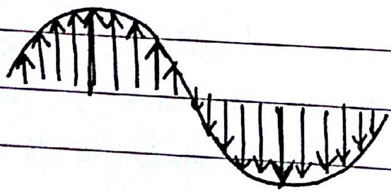
Phase velocity $= \frac{\omega}{k}$

$$\frac{dx}{dt} = \frac{\omega}{k} = V_p$$

Phase velocity is the velocity with which the planes of constant phase advance ~~in~~ through the medium.

Group velocity

Monochromatic wave consisting of pulses.

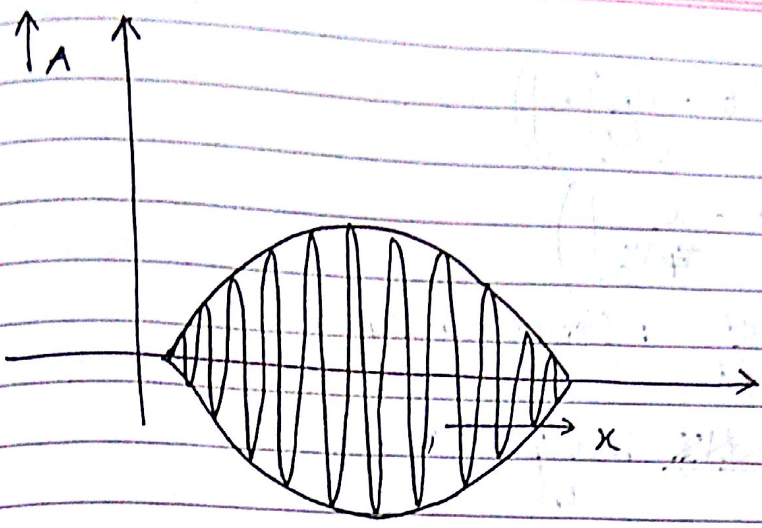


Group vel. is the vel. with which the energy in the group is transmitted.

~~Wave packet~~

Wave packet :-

A pulse.



A wave pkt comprises of group of waves slightly differing in va vel. & λ with phases & amplitudes ~~in~~ such ~~as~~ that they interfere constructively over a small region of space where the particle can be located and outside this space they interfere destructively so that the amplitude reduces to 0.

The vel. of the wave pkt is the group vel.

Expression for group velocity :

$$y_1 = a\sin(\omega t - kx)$$

$$y_2 = a\sin(\omega' t - k'x)$$

$$y = y_1 + y_2 = \underbrace{\boxed{2a\cos\left(\frac{\omega-\omega'}{2}t - \left(\frac{k-k'}{2}\right)x\right)}}_{K_g}\sin\left(\frac{\omega+\omega'}{2}t - \left(\frac{k+k'}{2}\right)x\right)$$

$$\frac{\omega+\omega'}{2} = \omega \, , \quad \frac{k+k'}{2} = k \quad \text{(approximation)}$$

$$y = K_g \sin(\omega t - kx)$$

Phase velocity $= \dfrac{\omega}{k}$

$$\omega - \omega' = d\omega$$

$$k - k' = dk$$

$$2a\cos\left(\frac{d\omega}{2}t - \frac{dk}{2}x\right) \Rightarrow \text{Amplitude.}$$

$$= 2a\cos\left(\frac{d\omega}{2}\left[t - \left(\frac{dk}{d\omega}\right)x\right]\right)$$

$$= 2a\cos\left(\frac{d\omega}{2}\left[t - \frac{x}{v_g}\right]\right)$$

$$\frac{d\omega}{dk} = v_g = \frac{\omega - \omega'}{k - k'}$$: group velocity.

$$v_g = v_p - \cancel{\lambda\, dv_g}\ \lambda\frac{dv_p}{d\lambda}$$

$$\cancel{v_g = v_p}$$

$$\omega = v_p k$$

$$d\omega = dv_p . k + v_p\, dk$$

$$\frac{d\omega}{dk} = v_p + k\frac{dv_p}{dk}$$

$$v_g = v_p + k\frac{dv_p}{dk}$$

$$= v_p + k\frac{dv_p}{d\lambda}.\frac{d\lambda}{dk}$$

$$\cancel{= v_p +}$$

$$k = \frac{2\pi}{\lambda}, \quad \frac{d\lambda}{dk} = \frac{-2\pi}{k^2}$$

$$v_g = v_p + k.\frac{dv_p}{d\lambda}.\frac{-2\pi}{k^2}$$

$$= v_p + \frac{dv_p}{d\lambda}\left(\frac{-2\pi\lambda}{2\pi}\right)$$

$$= v_p - \lambda\frac{dv_p}{d\lambda}$$

Prove that group vel. is the particle vel.

$$V_g = \cancel{V_g}\, V_p - \lambda \frac{dV_p}{d\lambda}$$

$$= \lambda^2 \left[\frac{V_p}{\lambda^2} - \frac{1}{\lambda} \frac{dV_p}{d\lambda} \right]$$

$$= -\lambda^2 \frac{d}{d\lambda}\left(\frac{V_p}{\lambda}\right)$$

$$= -\lambda^2 \frac{d\nu}{d\lambda}$$

$$\frac{1}{V_g} = -\frac{1}{\lambda^2} \frac{d\lambda}{d\nu}$$

$$= \frac{d}{d\nu}\left(\frac{1}{\lambda}\right)$$

$$\boxed{\frac{1}{V_g} = \frac{d}{d\nu}\left(\frac{1}{\lambda}\right)}$$

E : Total energy

V : Potential energy

$$\frac{1}{2} m v^2 = E - V$$

$$v = \sqrt{\frac{2(E-V)}{m}}$$

$$\lambda = \frac{h}{mv}$$

$$\frac{1}{\lambda} = \frac{mv}{h}$$

$$= \frac{m\sqrt{\frac{2(E-V)}{m}}}{h}$$

$$= \frac{\sqrt{2m(E-V)}}{h}$$

$$\frac{1}{v_g} = \frac{d}{d\nu}\left(\frac{m}{h}\sqrt{\frac{2(E-V)}{m}}\right)$$

$$E = h\nu$$

$$\frac{1}{v_g} = \frac{d}{d\nu}\left(\frac{m}{h}\sqrt{\frac{2(h\nu-V)}{m}}\right)$$

$$= \frac{1}{h}\frac{d}{d\nu}\left(\sqrt{2m(h\nu-V)}\right)$$

$$= \frac{1}{h}\cdot\frac{1}{2}\left(2m(h\nu-V)\right)^{-1/2}\cdot 2mh$$

$$= \frac{m}{\left(2m(E-V)\right)^{1/2}}$$

$$= \sqrt{\frac{m}{\cancel{2m}\,2(E-V)}}$$

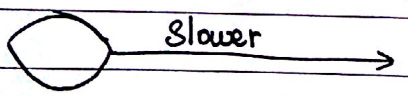
$$\therefore v_g = \sqrt{\frac{2(E-V)}{m}} = v$$

Group velocity = velocity of the particle.

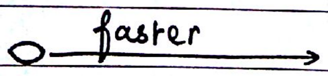
~~Guiding wave~~

The wave pkt and ~~particle~~ guiding wave possesses the properties of the particle moving with vel. v_g.

Heisenberg's Uncertainty Principle



velocity can be determined but position cannot be determined



velocity may not be determinable but can determine the posn.

Heisenbergs uncertainity principle states the limit of measurability

$$\frac{h}{4\pi} = \frac{\hbar}{2} \qquad \hbar = \frac{h}{2\pi}$$

According to Heisenberg uncertainty, it is impossible to measure both position and momentum of a particle simultaneously to any desired degree of accuracy.

It also states it is impossible to specify precisely and simultaneously the values of both members of particular pair of physical variables that describe the behaviour of an atomic system. The order of magnitude of the uncertainties in the knowledge of two variables must be atleast Planck's constant h

$\Delta x, \Delta p$

$\Delta E, \Delta t$

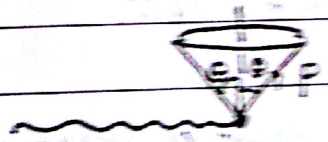
$\Delta J, \Delta \theta$

$$\Delta p \cdot \Delta x \geq \frac{h}{4\pi}$$

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

$$\Delta J \cdot \Delta \theta \geq \frac{h}{4\pi}$$

$$\Delta x \approx \frac{\lambda}{2\sin\theta}$$



Photon strikes on $e^{\ominus}$ and is scattered.

$$\Delta p = p\sin\theta - (-p\sin\theta)$$

$$= 2p\sin\theta$$

$$= \frac{2h\sin\theta}{\lambda}$$

$$\Delta p \cdot \Delta x = 2\frac{h}{\lambda}\sin\theta \cdot \frac{\lambda}{2\sin\theta}$$

$$\approx h$$

Applications of the uncertainity principle:

i) Non-existence of $e^{\ominus}$s in a nucleus.

ii) Fix the Bohr's first orbit radiis.

iii) Binding energy of the $e^{\ominus}$s in an atom.

iv) Radiation of light from an excited atom.

① Electrons cannot exist in nucleus.

If $e^{\ominus}$s were confined to nucleus:

$$\Delta x \approx 2 \times 10^{-14}\ m$$

Radius of the nucleus is of the order of $10^{-14}\ m$.

$$\Delta P > \frac{h}{2\pi \Delta x} \geq \frac{1.055 \times 10^{-31}}{2 \times 10^{-14}}$$

$$\approx 10^{-20}\ Ns$$

$$E^2 = p^2c^2 + m_0c^2$$

$$= 10 MeV$$

But electrons are emitted at 3 to 4 MeV.

∴ The uncertainity in position must be greater than 10^{-4} m.

~~Pho~~ protons/neutrons ≈ 50MeV.

②

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classmate
Date
Page

Q. Calculating the Bohr radius.

$$\Delta x . \Delta p \geq \frac{h}{4\pi}$$

$$\Delta p \geq \frac{h}{\Delta x \cdot 4\pi}$$

K.E: $$T \geq \frac{h^2}{(4\pi)^2 \, 2m(\Delta x)^2}$$

V: Potential Energy.

$$V = \frac{Ze^2}{r}$$

$$\Delta V > \frac{-Ze^2}{\Delta x}$$

$$\Delta E > \frac{h^2}{16\pi^2(2m)\Delta x^2} - \frac{Ze^2}{\Delta x}$$

Uncertainity in energy will be minimum if:

$$\frac{d\Delta E}{d\Delta x} = 0$$

and second derivative is +ve.

$$\cancel{\frac{h^2}{16\pi^2}} \quad \frac{h^2}{16\pi^2 m(\Delta x)^3} = \frac{Ze^2}{(\Delta x)^2}$$

$$\Delta x \approx \frac{h^2}{16\pi^2 m Z e^2}$$

Schrodinger's Wave Equation:

$\psi(x, y, z, t) \rightarrow$ Amplitude of matter waves.

$$\frac{\partial^2\psi}{\partial t^2} = V^2\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = V^2\nabla^2\psi$$

$$\boxed{\frac{\partial^2\psi}{\partial t^2} = V^2\nabla^2\psi}$$

$$\psi = \psi_0 \sin \omega t$$
$$= \psi_0 \sin(2\pi \nu t)$$
$$\frac{\partial \psi}{\partial t} = \psi_0 . 2\pi\nu \cos(2\pi\nu t)$$

$$\frac{\partial^2 \psi}{\partial t^2} = -4\pi^2\nu^2 \psi_0 \sin(2\pi\nu t)$$

$$= -4\pi^2\nu^2 \psi$$
$$= -\frac{4\pi^2 v^2}{\lambda^2} \psi$$

$$v^2 \nabla^2 \psi = -\frac{4\pi^2 v^2}{-\lambda^2} \psi$$

$$\cancel{\nabla^2 \psi =}$$

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0$$ (Time Independent equation)

$$\lambda = \frac{h}{mv}$$

E = Total energy

'V' = Potential energy

$$\frac{1}{2} m v^2 = E - V$$

$$\cancel{K =}$$

$$m^2 v^2 = 2m(E - V)$$

$$\nabla^2 \psi + \frac{4\pi^2}{h^2} 2m(E - V) \psi = 0$$

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

$$\hbar = \frac{h}{2\pi}$$

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

For a free particle,

$V = 0$ (potential $= 0$)

$$\nabla^2 \psi + \frac{2m}{\hbar^2} E \psi = 0$$

Time dependent equation:

Eliminate 'E'

$$\psi(x, y, z, t) = \psi_0(x, y, z) e^{-i\omega t}$$

$$\psi = \psi_0 e^{-i\omega t}$$

$$\frac{d\psi}{dt} = \psi_0(-i\omega) e^{-i\omega t}$$

$$\left(\begin{aligned} \omega &= 2\pi\nu \\ E &= h\nu \\ \nu &= \frac{E}{h} \end{aligned} \right)$$

$$\frac{d\psi}{dt} = \psi_0 (-i2\pi\nu) e^{-i\omega t}$$

$$= -2\pi i \frac{E}{h} \psi$$

$$= \frac{-iE}{\hbar} \psi \quad \text{(or)} \quad E\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\nabla^2 \psi + \frac{2m}{\hbar^2} \left(i\hbar \frac{\partial \psi}{\partial t} - V\psi \right) = 0$$

$$\nabla^2 \psi = -\frac{2m}{\hbar^2} \left[i\hbar \frac{\partial \psi}{\partial t} - V\psi \right]$$

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\underbrace{\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right)}_{\text{Hamiltonian } H} \psi = \underbrace{i\hbar \frac{\partial \psi}{\partial t}}_{E}$$

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$\psi \quad \phi$

$$\boxed{\hat{H}\psi = \hat{E}\psi}$$

$$\hat{H} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$$

$$\hat{E} = i\hbar\frac{\partial}{\partial t}$$

ψ, ψ^*

$\psi^2 = \psi \cdot \psi^*$

General solution for Schrodinger's wave equation:

$$\left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial \psi}{\partial t}$$

$$\iiint (\psi)^2 \, dz \, dx \, dy = 1.$$

'V' is independent of time

Suggested solutions are 'standing waves' like in nature.

$$\psi = f(x,y,z)\,\phi(t)$$

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(x,y,z)\right] f(x,y,z)\,\phi(t) = i\hbar\frac{\partial}{\partial t}\left(f(x,y,z)\,\phi(t)\right)$$

Divide both sides by a fn $f(x,y,z)\,\phi(t)$

$$\frac{1}{f(x,y,z)}\left[\left(\frac{-\hbar^2}{2m}\nabla^2 + V(x,y,z)\right) f(x,y,z)\right] = \frac{i\hbar}{\phi(t)}\frac{\partial \phi(t)}{\partial t}$$

$$\frac{1}{f(x,y,z)}\left[\left(\frac{-\hbar^2}{2m}\nabla^2 + V(x,y,z)\right) f(x,y,z)\right] = E$$

$$\frac{i\hbar}{\phi(t)}\frac{\partial \phi(t)}{\partial t} = E$$

~~where E is a constant of~~

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

(or) $$\underbrace{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

Time independent eqn.

$$i\hbar \frac{1}{\phi(t)} \frac{\partial \phi(t)}{\partial t} = E$$

$$\Rightarrow \frac{\partial \phi(t)}{\phi(t)} = \frac{1}{i\hbar} E \partial t$$

$$\log \phi(t) = \frac{Et}{i\hbar}$$

$$\therefore \phi(t) = e^{-iEt/\hbar}$$

$$\psi = f(x, y, z) e^{-iEt/\hbar}$$

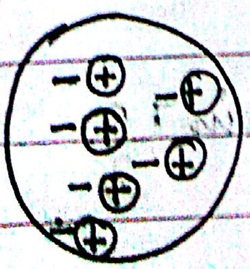
When the particle is confined to a finite region, then for a single valued cont. & finite soln.s of this eqn, there can be only certain values of E & those values of E are said to characteristic/proper/eigen values. Corresponding to each eigen value, there exists only 1 fn ψ & that fn ψ is called characteristic fn/wave fn/eigen fn.

MODELS OF ATOMS:

1867 → Cathode ray tubes.

JJ Thomson model:

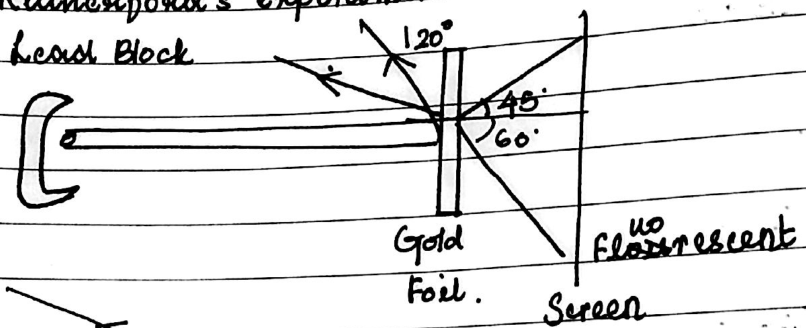
Plum pudding model.

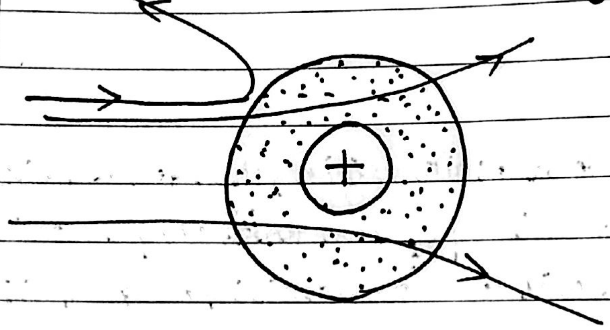


Electrons should emit every possible radiation which is not true

Drawbacks:
Acc. to em theory if an $e^{\ominus}$ is vibrating it should radiate energy and the freq. of the emitted spectral line should be the same as that of an $e^{\ominus}$. In case of H atom, Thomson observed 1300Å line only. But, several spectral lines were observed. It could not explain scattering of α particles.

Rutherford's experiment and Rutherford's model of atom





Observations:

i) Most of the α particles went straight through the gold fo
ii) Some of α-particles collided with gold atoms and were scatter by some angle.
iii) Few particles turned toward the source

Rutherford's scattering formula:

$$N = \frac{t n Q (2Ze^2)^2}{4 (4\pi\varepsilon_0)^2 r^2 (m v_0^2)^2 \sin^4 \frac{\theta}{2}}$$

classmate
Date
Page

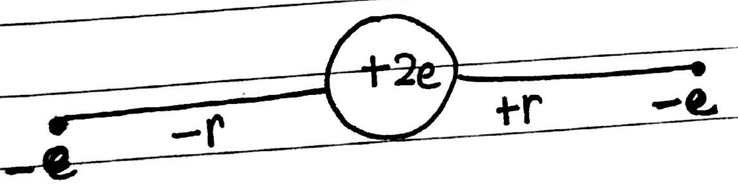
t: thickness of foil
n: no. of atoms ~~pu~~ per unit area
N: ~~Avg~~ No of ~~scatterers observed on the screen~~ particles that scatter through an angle θ in a distance r.
Q: total no. of α-particles that strike unit area of the scatterer.
Potential energy of α particles : $\frac{2Ze^2}{4\pi\epsilon}$

Conserved total energy $= mv_0^2$
θ : Angle of scattering.

(i) No of scatter particles $\propto \cancel{\frac{1}{(\theta)^4}}$ $\frac{1}{(\text{half angle})^4}$
$\propto$ thickness
$\propto$ atomic no. of the target
$\propto \frac{1}{\text{KE of particle}}$

(i) Acc. to Rutherford's model, the +ve charge of the atom instead of being uniformly distributed throughout the sphere of atomic dim. is concentrated in a very small vol. of the order of 10^{-13} cm in diameter at its centre. This central core is now called nucleus. It is surrounded by a cloud of e⊖s which makes the atom neutral. The force b/w e⊖s and α particles is ignored.

Drawbacks:
i) Stability of the atom.



Force of attraction $= \frac{e \times 2e}{4\pi \varepsilon_0 r^2}$

Force of repulsion b/w $e^{\ominus}s = \frac{e \times e}{4\pi\varepsilon_0 \cdot (2r)^2}$

There is an unbalanced force, which implies that $e^{\ominus}s$ should fall into the nucleus.

~~So~~. Rutherford suggested that $e^{\ominus}s$ should be moving in planetary orbits.

$$\frac{e \times 2e}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

Revolving $e^{\ominus}s$ should radiate energy continuously. But atomic spectra is discrete in nature.

Bohr's theory :

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

$$mvr = \frac{nh}{2\pi}$$ (Angular momentum is quantized)

Atomic spectrum

Lyman & Balmer

Balmer, Rydberg formula:

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

$$= R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \rightarrow \text{Bohr.}$$

Quantization

Bohr model

(i) An $e^{\ominus}$ in an atom moves in a circular orbit around the nucleus under the influence of Coloumb's force of attraction.

(ii) As the atom on the whole is stable, this attraction force is balanced by centrifugal force.

(iii) Only those orbits are possible for which the orbital angular momentum of the $e^{\ominus}$ = an integral multiple of $h/2\pi$.

The $e^{\ominus}$ moving in such allowed orbit does not emit radiation. Thus, the total energy of the $e^{\ominus}$ in any of the stationary orbits remains constant.

Em radiation is emitted only if $e^{\ominus}$ jumps from one stationary orbit to another.

$E_2 - E_1 = h\nu$ (emitted)

$E_1 - E_2 = -h\nu$ (absorbed)

(a) **Radius of orbits:**

$$v = \frac{nh}{2\pi m r}$$

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m}{r}\left(\frac{nh}{2\pi m r}\right)^2$$

$$\Rightarrow r_n = \frac{n^2h^2\varepsilon_0}{\pi m e^2}$$

$r_1 = 0.529\,\text{Å}$

Atom is about 10^{-10} m in diameter.

(b) **Velocity of the $e^{\ominus}$s:**

$$mv\left(\frac{n^2h^2\varepsilon_0}{\pi m e^2}\right) = \frac{nh}{2\pi}$$

$$v = \frac{e^2}{2nh\varepsilon_0}$$

which means as $n \uparrow$, velocity $\downarrow$

c) ν (Frequency of ~~excited~~ $e^{\ominus}$s)

$$\frac{1}{T} = \frac{v}{2\pi r}$$

$$= \frac{me^4}{4\varepsilon_0^2 n^3 h^3}$$

d) Energy of $e^{\ominus}$s :

$$KE = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \frac{m e^4}{(2nh\varepsilon_0)^2}$$

$$= \frac{me^4}{8n^2h^2\varepsilon_0^2}$$

$$P = \frac{e}{4\pi\varepsilon_0 r}$$ (Potential at a distance r from the nucleus)

$$PE = V \times (-e) = -e^2/4\pi\varepsilon_0 r$$

$$= -\frac{me^4}{4n^2h^2\varepsilon_0^2}$$

$$E = KE + PE$$

$$= -\frac{me^4}{8n^2h^2\varepsilon_0^2}$$

e) Frequency of emitted radiation:

$$h\nu = E_{n_2} - E_{n_1}$$

$$= -\frac{me^4}{8n_2^2h^2\varepsilon_0^2} + \frac{me^4}{8n_1^2h^2\varepsilon_0^2}$$

$$= \frac{me^4}{8h^2\varepsilon_0^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\cancel{\therefore R = \frac{me^4}{8h^2\varepsilon_0^2}}$$

$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2h^3c}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$R = \frac{me^4}{8\varepsilon_0^2h^3c}$$

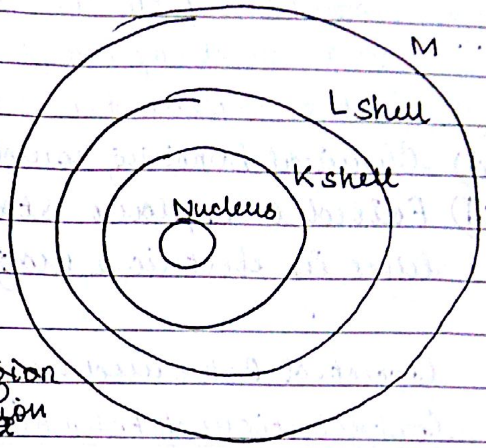
Series:

$E_1 = -13.6$ eV K shell

$E_2 = -3.4$ eV L shell

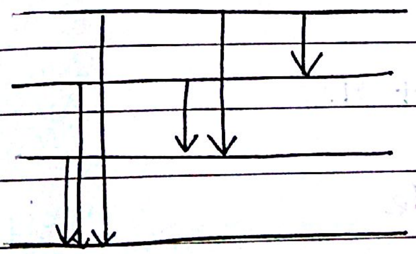
$E_3 = -1.51$ eV M shell

⋮



$E_\infty = 0$ eV $n = \infty$

	n_1	n_2	~~Region~~ Region
Lyman	1	2, ...	UV
Balmer	2	3, ...	Visible
Paschen	3	4, ...	IR
Bracket	4	5, ...	IR
Pfund	5	6, ...	IR



$$\lambda = \frac{h}{mv}$$

From the perspective of De Broglie hypothesis. Stationary orbits are those in which orbital circumference $2\pi r$ are an integral multiple of ~~the~~ de-Broglies wavelength.

$$2\pi r = \frac{nh}{mv} \Rightarrow mvr = \frac{nh}{2\pi}.$$

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Drawbacks:

(i) Could not explain spectra of complex atoms other than hydrogen.
(ii) It does not tell about the distribution of e⊖s in the atom.
(iii) It does not tell about intensity of spectral lines.
(iv) Transitions from one level to another cannot be understood which involve selection principles.
(v) fails to explain fine structure of spectral lines.
(vi) Chemical bonding could not be explained.
(vii) Failed to explain Stark & Zeeman effect i.e splitting of spectral lines in electric & magnetic field respectively.

Corrected Bohr model by Sommerfield using relativistic correction.

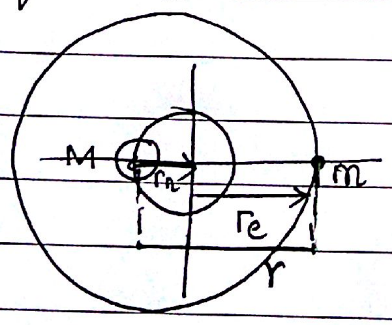
Contributions of Bohr model:

(i) 'n' - principal quantum number.
(ii) Number of possible e⊖s in a particular shell.

$n=1$	$n=2$	$n=3$	$n=4$
K	L	M	N
2	8	18	32

$2n^2$

(iii) Mass of nucleus was determined.



$$r = r_n + r_e$$

$$\frac{M}{m} = \frac{r_e}{r - r_e}$$

$$r_n = \frac{rm}{m+M}$$

$$V = r_e \omega$$

$$V_n = r_n \omega$$

$$T = \frac{1}{2} m r_e^2 \omega^2 + \frac{1}{2} m r_n^2 \omega^2 = \frac{1}{2} \mu r^2 \omega^2 \quad \left(\mu = \frac{mM}{m+M}\right)$$

$L = \mu r^2 \omega$

$\mu r^2 \omega = \frac{nh}{2\pi}$

If nucleus is not moving

$m r^2 \omega = \frac{nh}{2\pi}$

Importance of R (Rydberg constant)

'μ'

$$E = \frac{\mu e^4}{8 n^2 h^2 \varepsilon_0^2}$$

$$= \frac{\mu e^4}{8 h^2 \varepsilon_0^2} \frac{1}{n^2}$$

$$\frac{1}{\lambda} = R' \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$R_H = \frac{\mu e^4}{8 h^3 \varepsilon_0^2 c}$$

The wavelengths obtained (using 'μ') are slightly greater than those corresponding to an infinitely heavy nucleus. some detectable change in values of λ.

Effect of finite mass on R:

Wave number $$\bar{\nu} = \frac{e^4}{8 \varepsilon_0^2 c h^3} \left(\frac{mM}{m+M} \right) \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

(or)

$$\bar{\nu} = \underbrace{\frac{m e^4}{8 \varepsilon_0^2 c h^3}}_{R_H} \left(\frac{1}{1 + \frac{m}{M}} \right) \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$R' = \frac{R_H}{1 + \frac{m}{M}}$$

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Bohr model can be used to understand hydrogen-like atoms like He^+, Li^{2+} ....
Heavier nucleus, shorter wavelength.

Series for helium:
Fowler
Pickering $\rightarrow$ $R_{He}\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ $n = 3 \ldots\ldots$

$$R_{He} = 4R_H$$

R_H and R_{He} are known.

$$\frac{R_H}{R_{He}} = \frac{1 + \frac{m}{M_{He}}}{1 + \frac{m}{M_H}}$$

$$M_{He} = 3.917 M_H$$

$$\frac{R_H}{R_{He}} = \cancel{1.9967}\ \frac{199677.75}{109722.73} = K$$

$$K = \frac{1 + \frac{m}{3.917 M_H}}{1 + \frac{m}{M_H}}$$

$$\frac{M_H}{m} \cong 1840$$

$$\text{Ionization potential} = \frac{1}{2}\frac{m}{\hbar^2}\left(\frac{Ze^2}{4\pi\epsilon_0}\right)^2$$

$$= 13.6 Z^2 \text{ eV}.$$

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Elliptical orbits: (Sommerfield)

r, p.

Two components of velocity:

Along the radial vector → radial velocity

Perpendicular to the radius → transverse velocity.

Two momenta: radial momentum, azimuthal momentum } Quantized

ϕ: azimuthal angle.

Principal quantum number = n_R (Radial quantum no.) + n_ϕ (Azimuthal quantum number)

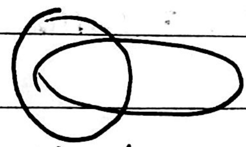
$n = 1$ $\quad n_R, n_\phi = 0\ 1$

$\qquad\qquad\qquad\quad 1\ 0$

$n_\phi \neq 0$

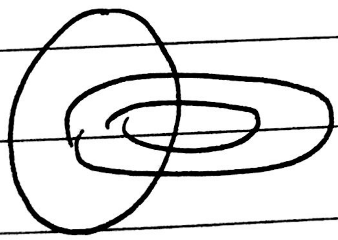
$n = 2$ $\quad n_R, n_\phi = 0, 2$

$\qquad\qquad\qquad\quad 1, 1$



one circular, one elliptical.

$n = 3$ $\quad$ 3 possible orbits : 1 circular, two elliptical



$$\oint P_r \, dr = n_r h$$

$$\oint P_\phi \, d\phi = n_\phi h$$

Allowed elliptical orbits :

$\phi : 0$ to 2π

$$\int_0^{2\pi} P_\phi \, d\phi = P_\phi \int_0^{2\pi} d\phi = P_\phi . 2\pi = n_\phi h$$

$$P_\phi = \frac{n_\phi h}{2\pi}$$

$$\oint P_r \, dr = n_r h$$

$$P_r = m \frac{dr}{dt}$$

$$= m \frac{dr}{d\phi} \cdot \frac{d\phi}{dt}$$

$$= m \frac{dr}{d\phi} \cdot \cancel{mr^2} \frac{P_\phi}{mr^2} \qquad \left(\because P_\phi = mr^2 \frac{d\phi}{dt} \right)$$

$$= \frac{P_\phi}{r^2} \frac{dr}{d\phi}$$

$$\oint \frac{P_\phi}{r^2} \frac{dr}{d\phi} \, dr = n_r h$$

(or)

$$\oint \frac{P_\phi}{r^2} \left(\frac{dr}{d\phi}\right) \left(\frac{dr}{d\phi}\right) d\phi = n_r h$$

$$\Rightarrow P_\phi \oint \left(\frac{1}{r} \frac{dr}{d\phi}\right)^2 d\phi = n_r h$$

$$\frac{1}{r} = \frac{1 + \varepsilon \cos\phi}{a(1-\varepsilon)^2}$$

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$$\frac{1}{r^2}\frac{dr}{d\phi} = \frac{\varepsilon\sin\phi}{a(1-\varepsilon)^2}$$

$$\frac{1}{r}\frac{dr}{d\phi} = \frac{\varepsilon\sin\phi}{\cancel{a(1-\varepsilon^2)}\; 1+\varepsilon\cos\phi}$$

$$P\phi\int_0^{2\pi}\left(\frac{\varepsilon\sin\phi}{1+\varepsilon\cos\phi}\right)^2 d\phi = n_r h$$

$$u = \varepsilon\sin\phi,\ v = \frac{1}{1+\varepsilon\cos\phi}$$

$$du = \varepsilon\cos\phi\, d\phi$$

$$dv = \frac{\varepsilon\sin\phi\, d\phi}{(1+\varepsilon\cos\phi)^2}$$

$$\int u\,dv = uv - \int v\,du$$

$$P\phi\left(\left[\frac{\varepsilon\sin\phi}{1+\varepsilon\cos\phi}\right]_0^{2\pi} - \int_0^{2\pi}\frac{\varepsilon\cos\phi\, d\phi}{1+\varepsilon\cos\phi}\right) = n_r h$$

$$P\phi\left(0 - \int_0^{2\pi}\left(\frac{1}{1+\varepsilon\cos\phi} - 1\right)d\phi\right) = n_r h$$

$$P\phi\left(\frac{2\pi}{(1-\varepsilon^2)^{1/2}} - 2\pi\right) = n_r h$$

$$\Rightarrow \frac{2\pi P\phi}{(1-\varepsilon^2)^{1/2}} - 2\pi P\phi = n_r h$$

$$P\phi = \frac{n_\phi h}{2\pi}$$

$$\Rightarrow \frac{n_\phi h}{(1-\varepsilon^2)^{1/2}} - n_\phi h = n_r h$$

$$\therefore n_r = n_\phi\left(\frac{1}{(1-\varepsilon^2)^{1/2}} - 1\right)$$

$$n_r = \frac{n_\phi}{\sqrt{1-\varepsilon^2}} - n_\phi$$

(or)

$$n_r + n_\phi = \frac{n_\phi}{\sqrt{1-\varepsilon^2}}$$

$$\frac{n}{n_\phi} = \frac{1}{\sqrt{1-\varepsilon^2}}$$

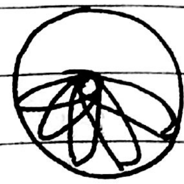
$$\left(\frac{n_\phi}{n}\right)^2 = 1-\varepsilon^2$$

$$= \frac{b^2}{a^2}$$

$$\frac{n_\phi}{n} = \frac{b}{a}$$

Energy for this system remains the same as the one obtained using Bohr model.

$$E = \frac{-me^4Z^2}{8\varepsilon_0^2h^2} \cdot \frac{1}{n^2}$$



Relativistic correction for the velocity of $e^\ominus$:
Innermost Bohr orbit velocity is $0.007c$
Mass of $e^\ominus$ varies.

$$E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2n^2h^2} \times \left[1 + \frac{\alpha^2Z^2}{n}\left(\frac{1}{n_\phi} - \frac{3}{4n}\right)\right]$$

$$\alpha = \frac{e^2}{2\varepsilon_0 ch} \approx \frac{1}{317}$$ (Sommerfield structure constant)

$n = 3$

n_R	n_ϕ
0	3
1	2
2	1

Explains the fine structure of the spectral lines.

Transitions possible :

$n = 3 \quad n_\phi = 3$ to $n = 2 \quad n_\phi = 2$ ✓

$n = 3 \quad n_\phi = 3$ to $n = 2 \quad n_\phi = 1$

$n = 3 \quad n_\phi = 2$ to $n = 2 \quad n_\phi = 2$

$n = 3 \quad n_\phi = 2$ to $n = 2 \quad n_\phi = 1$ ✓

$n = 3 \quad n_\phi = 1$ to $n = 2 \quad n_\phi = 2$ ✓

$n = 3 \quad n_\phi = 1$ to $n = 2 \quad n_\phi = 1$

6 possible transitions.

Only 3 lines are observed.

Selection rule : ~~n_ϕ values~~ Only those e⊖s jumps from upper sub-level to lower sub-level are allowed for which n_ϕ changes by unity. i.e.

$\Delta n_\phi = \pm 1$

OPERATORS :

Quantity	Operator
Position, x	x
Linear momentum, p	$\dfrac{\hbar}{i}\dfrac{\partial}{\partial x}$
Potential energy, $U(x)$	$U(x)$
Kinetic energy $= \dfrac{p^2}{2m}$	$-\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}$
Total energy, E	$i\hbar\dfrac{\partial}{\partial t}$
~~Total energy, H~~	
Total energy, H	$-\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2} + U(x)$

Expectation values :

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^* \hat{p}\, \psi \, dx$$

$$= \frac{\hbar}{i} \int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial x} dx$$

$$\langle E \rangle = \int^{\infty} \psi^* \hat{E} \psi \, dx$$

$$= i\hbar \int^{\infty} \psi^* \frac{\partial \psi}{\partial t} dx$$

In general :

$$\langle G \rangle = \int_{-\infty}^{\infty} \psi^* \hat{G} \psi \, dx$$

Eigenvalues and operators :

$\hat{G} \psi_n = G_n \psi_n$

$\hat{G}$: operator for 'G'.

G_n : real number

ψ_n : nth eigenvalue.

$\hat{H} \psi_n = E_n \psi_n$

Hilbert Space 'H'

Real (or) complex

Inner product and is also a complete metric space.

w.r.t. distance function, induced by inner product.

$\langle x, y \rangle$

Let x & y be complex.

If x & y belong to Hilbert space, then

$\cancel{\langle x, y \rangle}\ \langle y, x \rangle = \langle x, y \rangle$

$\langle a x_1 + b x_2, y \rangle = a \langle x_1, y \rangle + b \langle x_2, y \rangle$

(and) $\langle x, x\rangle \geq 0$ ~~if x~~ 0 when $x = 0$.

$$\int_{-\infty}^{\infty} \psi^*(x)\phi(x)\,dx = \langle \psi | \phi \rangle$$

$$\int_{-\infty}^{\infty} \psi_n^*(x)\,\psi_m(x)\,dx = \langle \psi_n | \psi_m \rangle = \langle n | m \rangle$$

$$\langle A \rangle = \int_{-\infty}^{\infty} \psi^*(x)\,\hat{A}\,\psi(x)\,dx = \langle \psi | \tilde{A} | \psi \rangle$$

$\langle \psi | \phi \rangle^* = \langle \phi | \psi \rangle$

$\psi(x) = |\psi\rangle$ (Notation for $\psi(x)$)
Ket

$\psi^*(x) = \langle \psi^* |$

$\langle \psi^* | |\psi\rangle$ inner product

$|\psi\rangle\langle\psi^*|$ outer product

PURE STATE — MIXED STATE

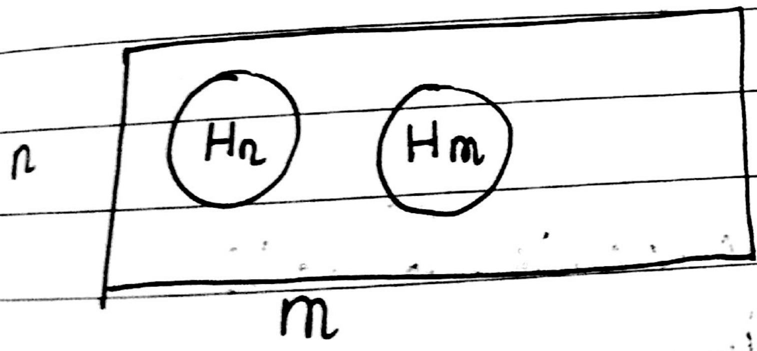
$|\psi\rangle = \alpha_1 |u_1\rangle + \alpha_2 |u_2\rangle + \ldots + \alpha_n |u_n\rangle$ * Superposition.

$u_1, u_2 \ldots u_n$: basis vector elements.

$$\sum_{i=1}^{n} |\alpha_i|^2 = 1$$

Entanglement : A pure state $|\psi\rangle \in H_{nm}^{\,x}$ is called separable $\Leftrightarrow$

$|\psi\rangle = \sum_{i=1}^{n} \alpha_i |u_i\rangle \in H_n$, $|\phi\rangle = \sum_{j=1}^{m} \beta_i |u_i\rangle \in H_m$



$|x\rangle$ is called separable &
if : $|x\rangle = |\psi\rangle \otimes |\phi\rangle$
$\downarrow$
Outer product

$= |\psi\rangle\langle\phi|$

If a pure state cannot be expressed in terms of other states then it is called an entanglement.

$|0\rangle, |1\rangle$: 0 state / ground state & , 1 state / 1st excited state

2 Q-bit system.

$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$

$|\phi_1\rangle = \cancel{\alpha\langle 0| + \beta\langle 1|}\ \alpha|0\rangle + \beta|1\rangle$

$|\phi_2\rangle = \gamma|0\rangle + \delta|1\rangle$

$|\alpha|^2 + |\beta|^2 = 1$

$|\gamma|^2 + |\delta|^2 = 1$

Assume $|\psi^+\rangle$ is separable,

$|\psi^+\rangle = |\phi_1\rangle \otimes |\phi_2\rangle$

$= \cancel{\langle\alpha|0\rangle +}\ (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle)$

$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \left(\alpha|0\rangle + \beta|1\rangle\right) \otimes \left(\gamma|0\rangle + \delta|1\rangle\right)$

There are no values of $\alpha, \beta, \gamma, \delta$ st.,

$\alpha\beta = \beta\delta = \frac{1}{\sqrt{2}}$ and $\alpha\delta = \beta\gamma = 0$.

This state is an entangled state.

$|\psi^+\rangle$ is an entangled state.

Q-bits - spin of $e^{\ominus}$s.

$|\uparrow\rangle$ or $|\downarrow\rangle$ Another notation for Q-bits

- lesser space } quantum chips.
- faster.

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Postulates of Quantum Mechanics:

1) The state of the system is completely defined by wave fn $\psi(x,t)$.

2) All measurable quantities/observables are described by Hermitian linear operators.

3) The only values that are obtained in a measurement of an observable 'A' are eigen values 'an' of the corresponding operator '$\hat{A}$'.

4) The measurement changes the state of the system to the eigenfunction of $\hat{A}$ with eigenvalue 'an'.

5) If a system is described by a normalized wave fn ψ, then the average value of an observable corresponding to $\hat{A}$ is $\langle a \rangle$
$\langle a \rangle = \int \psi^* \hat{A} \psi \, d\tau$.

6) The wave fn/ state fn of a system evolves with in time according to time dependent Schrodinger wave equation.

7) The eigenfunctions of operators corresponding to observables form a complete set.

When a system includes several identical particles, only certain wave functions can describe its physical states. → Bosons & Fermions.

For $e^{\ominus}$s the wave fn must change sign, when coordinates of 2 $e^{\ominus}$s are interchanged. No 2 $e^{\ominus}$s can have the same spin. (Pauli's exclusion principle).

1) State
2) Measurables / measurement.
3) Time evolution

First Postulate $\psi(x,t)$

$|\psi|^2$

$\psi^*(x,t)\,\psi(x,t) = |\psi(x,t)|^2$ (Probability density at time t & position x)

$\int \psi^* \psi \, d\tau = 1$

$\psi(x,t)$ should be well-behaved.

→ single-valued

→ ψ & ψ^* should be continuous

→ finite. (ψ & ψ^*)

Second Postulate:

operator

$$p_x = -i\hbar \frac{d}{dx}$$

↳ Linear operator i.e. it satisfies.

$$\hat{A}[f(x) + g(x)] = \hat{A}(f(x)) + \hat{A}(g(x))$$

and $\hat{A}[c f(x)] = c\hat{A}[f(x)]$

↳ Hermetian

$$\int \psi_1^* \hat{A} \psi_2 \, d\tau = \int \psi_2 (\hat{A}\psi_1)^* \, d\tau$$

Eigen functions of Hermitian operators are orthogonal i.e.

$$\int \psi_m^* \psi_n \, d\psi = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \end{cases}$$

↳ Similar to δ function

Third Postulate:

$$\psi = c_1\phi_1 + c_2\phi_2$$

$$\hat{A}\phi_1 = a_1\phi_1$$

$$\hat{A}\phi_2 = a_2\phi_2$$

If ψ is not an eigen fn of the operator,

$A \longrightarrow a_1, a_2$ with probabilites $c_1{}^2, c_2{}^2$ respectively.

	value	probability
$A \rightarrow$	a_1	c_1^2
	a_2	c_2^2

ψ_n $\hat{A}$ ψ_n is eigen function.

$\hat{A}\psi_n = a_n\psi_n$

$\langle a \rangle = \int \psi_n^* \hat{A} \psi_n \, d\tau$

$= a_n \int \psi_n^* \psi_n \, d\tau = a_n$

$\langle a \rangle = a_n$

If ψ is an eigen function of the operator:

$\langle a \rangle = \int \psi^* \hat{A} \psi \, d\tau$

$= \int (c_1\phi_1 + c_2\phi_2)^* \hat{A} (c_1\phi_1 + c_2\phi_2)$

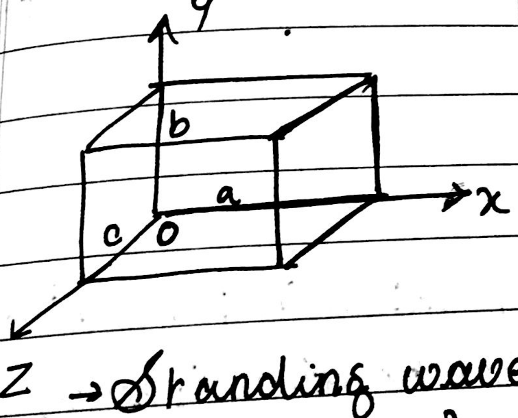
$= c_1^2 a_1 + c_2^2 a_2$

Average possible values weighted by their probabilities $= \langle a \rangle$

Solutions of Schrödinger Wave equation:

Time independent equation:

PARTICLE IN A BOX:



Probability of finding a particle in the box using SWE.

Determine the potential.

$\rightarrow$ Standing wave solutions

$$V(x,y,z) = 0 \quad \begin{cases} 0 < x < a \\ 0 < y < b \\ 0 < z < c \end{cases}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$\cancel{\psi(x)}$ $\psi(x,y,z) = X(x)Y(y)Z(z)$

$= XYZ$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = K_x$$

$$-\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} - \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} - \frac{2mE}{\hbar^2} = \cancel{K_y}\ K_x$$

(or)

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} - \frac{2mE}{\hbar^2} - K_x$$

$$\cancel{\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = K_y} \qquad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = K_y$$

$$-\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} - \frac{2mE}{\hbar^2} - K_x = K_y$$

$$\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -\frac{2mE}{\hbar^2} - K_x - K_y = K_z$$

$$-\frac{2mE}{\hbar^2} = K_x + K_y + K_z$$

$$K_{x,y,z} = -\frac{2mE_{x,y,z}}{\hbar^2} \qquad \left(K_x = -\frac{2mE_x}{\hbar^2},\ K_y = -\frac{2mE_y}{\hbar^2},\ K_z = -\frac{2mE_z}{\hbar^2} \right)$$

$$\frac{\partial^2 X_{x,y,z}}{\partial x^2} + \frac{2mE_{x,y,z}}{\hbar^2} = 0$$

$$X(x) = A\sin(Bx + c)$$

$|X(x)|^2$: Probability of finding the particle in the x direction.

At the walls, probability of finding the particle is zero,

$|X(x)|^2 = 0$ when $x = 0$ and $x = a$

$X(x) = 0$ " " " "

$$0 = A\sin(0 + c) \qquad A \neq 0$$

$$\sin c = 0 \quad \text{and} \quad 0 = A\sin(Ba + c)$$

$$\sin Ba = 0$$

$$Ba = n_x\pi$$

$$B = \frac{n_x\pi}{a}$$

$$X(x) = A\sin\left(\frac{n_x \pi}{a} x\right)$$

$x = 0$ and $x = a$

Normalize,

$$\int_0^a |X(x)|^2 dx = 1$$

$$\int_0^a \left(A \sin \frac{n_x \pi x}{a}\right)^2 dx = 1$$

$$A^2 \int_0^a \sin^2\left(\frac{n_x \pi x}{a}\right) dx = 1$$

$$\frac{A^2 a}{2} = 1$$

$$A = \sqrt{\frac{2}{a}}$$

$$X(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n_x \pi}{a} x\right)$$

$$Y(y) = \sqrt{\frac{2}{b}} \sin\left(\frac{n_y \pi}{b} y\right)$$

$$Z(z) = \sqrt{\frac{2}{c}} \sin\left(\frac{n_z \pi}{c} z\right)$$

$$\cancel{\psi} = \psi_{n_x, n_y, n_z}(x, y, z) = \frac{2\sqrt{2}}{\sqrt{abc}} \sin\left(\frac{n_x x \pi}{a}\right) \sin\left(\frac{n_y \pi y}{b}\right) \sin\left(\frac{n_z \pi z}{c}\right)$$

Energy E:

$$\frac{\partial^2 X}{\partial x^2} = -\left(\frac{n_x \pi}{a}\right)^2 \sqrt{\frac{2}{a}} \sin\left(\frac{n_x \pi}{a} x\right)$$

$$= -\left(\frac{n_x \pi}{a}\right)^2 \cdot X(x)$$

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$$-\left(\frac{n_x\pi}{a}\right)^2 X(x) + \frac{2mE_x}{\hbar^2} X(x) = 0$$

(or) $$E_x = \frac{1}{2m}\left(\frac{n_x \pi \hbar}{a}\right)^2$$

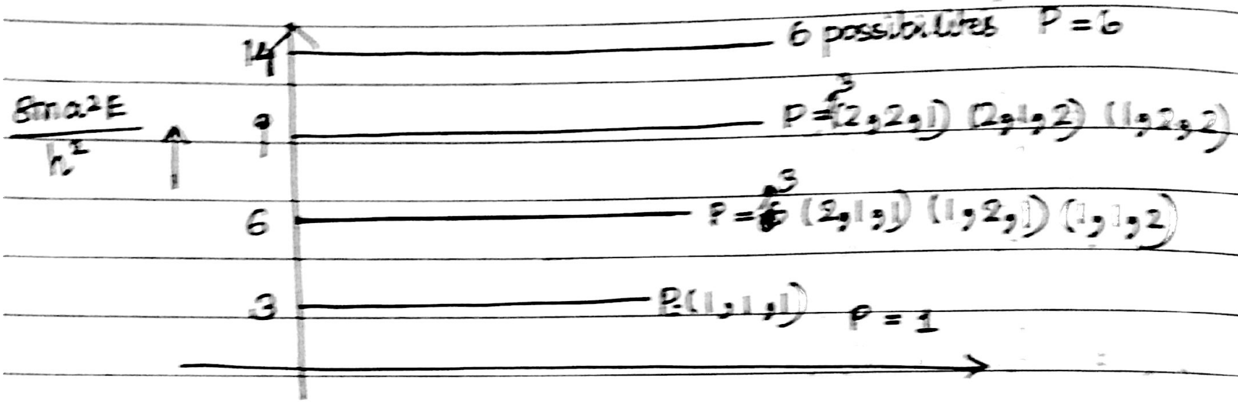
$$= \frac{n_x^2 h^2}{8ma^2}$$

$$E = E_x + E_y + E_z = \frac{h^2}{8m}\left[\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right]$$

$a = b = c$

$$E = \frac{h^2}{8ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

n_x, n_y, n_z are +ve integers



P: degree of degeneracy

Momentum:

$\psi^* = \psi$

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^* p \psi \, dx$$

$$p = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

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$x = L$

$$\langle p \rangle = \frac{\hbar}{i} \cdot \frac{2}{L} \cdot \frac{n\pi}{L} \int_0^L \sin\left(\frac{n_x \pi x}{L}\right) \cos\left(\frac{n_x \pi x}{L}\right) dx$$

$$= \frac{\hbar}{i} \cdot \frac{2}{L} \cdot \frac{n_x \pi}{L} \cdot \frac{1}{2p} \sin^2 px$$

$$\langle p \rangle = \frac{\hbar}{iL} \left[\sin^2 \frac{n\pi x}{L} \right]_0^L$$

$\sin^2 0 = \sin^2 n\pi = 0$

$\cos 2n\pi x$

~~$p_n = \pm\sqrt{2mE_n} = \pm \frac{n\pi\hbar}{L}$ Average value of $p = 0$~~

$\cancel{\frac{p^2}{2m}} \neq 0$

$$E = \frac{p^2}{2m} \qquad p = \pm\sqrt{2mE}$$

$$= \pm \frac{n\pi\hbar}{L} = \pm \frac{n_x h}{2L}$$

Eigenvalues for 'p'

$\hat{p}\psi_n = p_n \psi_n$ Characteristic equation

$$\frac{\hbar}{i} \frac{\partial}{\partial x} \sqrt{\frac{2}{L}} \sin\left(\frac{n_x \pi x}{L}\right) = \frac{\hbar}{i} \frac{n\pi}{L} \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right) \neq p_n \psi_n$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$= \frac{1}{2i} e^{i\varphi} - \frac{1}{2i} e^{-i\varphi}$$

$$\psi_n^+ = \frac{1}{2L} \sqrt{\frac{2}{L}}\, e^{in\pi x/L}$$

$$\psi_n^- = \frac{1}{2L} \sqrt{\frac{2}{L}}\, e^{-in\pi x/L}$$

$$\hat{p}\psi_n^+ = p_n^+\psi_n^+$$

$$\frac{\hbar}{i}\frac{d}{dx}\psi_n^+ = \frac{n\pi\hbar}{L}\psi_n^+$$

Similarly,

$$\frac{\hbar}{i}\frac{d}{dx}\psi_n^- = \frac{n\pi\hbar}{L}\psi_n^-$$

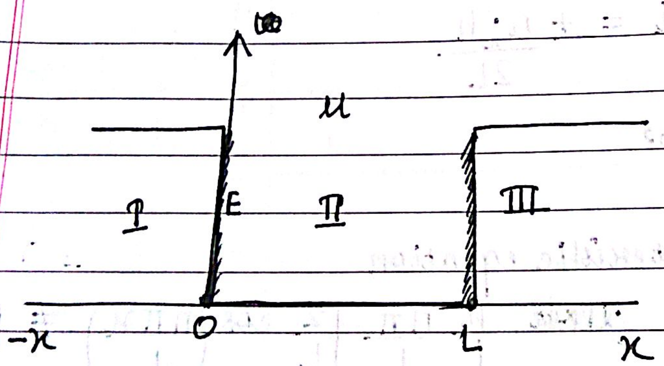
Eigenfunctions : ψ_n^+, ψ_n^-

The state ψ is a superposition of two states : ψ_n^+, ψ_n^-.

ψ_n^+, ψ_n^- are the basis vectors.

↳ also called momentum eigenfunctions of particle in a box.

<u>Potential Well</u> :



One-dimension:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

$$a = \frac{\sqrt{2m(U-E)}}{\hbar}$$

$$\frac{d^2\psi}{dx^2} - a^2\psi = 0 \qquad x<0$$

$$x>L$$

$$\psi_I = Ce^{ax} + De^{-ax}$$

$$\psi_{III} = Fe^{ax} + Ge^{-ax}$$

ψ_{I}, ψ_{III} finite everywhere

$e^{-ax} \rightarrow -\infty$ $\qquad$ $e^{ax} \rightarrow \infty$

$x \rightarrow -\infty$ $\qquad$ $x \rightarrow \infty$

D, F should be 0

$\psi_{I} = Ce^{ax}$

$\psi_{III} = Ge^{-ax}$

ψ exponentially decreases within the barriers at the side of the well and inside the well the solutions are:

$$\psi_{II} = A\sin\sqrt{\frac{2mE}{\hbar}}x + B\cos\sqrt{\frac{2mE}{\hbar}}x$$

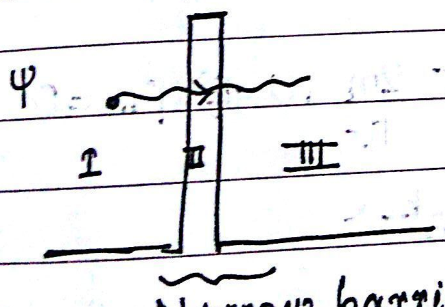
If the well is infinitely high, B = 0.

$\psi = 0 \longrightarrow x = 0, x = L$

$\psi_{II} = C$ at $x = 0$ and $\psi_{II} = G$ at $x = L$

For either solution, both ψ and dψ must be continuous at $x = 0$ and $x = L$. The wave functions inside and outside each side of the well must not only have the same value where they join but also the same slopes. Then only, energy quantization holds. Also note that the energy ~~values~~ levels obtained for this system ~~is less than that of an~~ are lower for each n than that of a particle in an infinite well.

Tunneling effect:



Narrow barrier

The wave can tunnel through the potential barrier rather than jumping across the barrier.

$$\frac{d^2\psi_I}{dx^2} + \frac{2m}{\hbar^2}E\psi_I = 0$$

$$\frac{d^2\psi_{III}}{dx^2} + \frac{2m}{\hbar^2}E\psi_{III} = 0$$

$$\psi_I = Ae^{ik_1x} + Be^{-ik_1x}$$

$$\psi_{III} = Fe^{ik_1x} + Ge^{-ik_1x}$$

$$k_1 = \frac{\sqrt{2mE}}{\hbar} = \frac{p}{\hbar} = \frac{2\pi}{\lambda}$$

Incoming wave $\psi_{I+} = Ae^{ik_1x}$

$|\psi_{I+}|^2$

Let group velocity be V_+, then the flux of particles arriving at the barrier be

$$S = |\psi_{I+}|^2 V_+$$

Reflected wave

$\psi_{I-} = Be^{-ik_1x}$

Note: $\psi_I = \psi_{I+} + \psi_{I-}$

Transmitted wave

$\psi_{III+} = Fe^{ik_1x}$

for $x > L$

$\psi_{III} = \psi_{III+} = Fe^{ik_1x}$

$$T = \frac{|\psi_{III+}|^2 V_{III+}}{|\psi_{I+}|^2 V_{I+}} \quad \text{(Transmission probability)}$$

$$= \frac{F\,F^* V_{III+}}{AA^* V_{I+}}$$

$$\frac{d^2\psi_{II}}{dx^2} + \frac{2m}{\hbar^2}(E-U)\psi_{II} = \frac{d^2\psi_{II}}{dx^2} - \frac{2m}{\hbar^2}(U-E)\psi_{II} = 0$$

Since $U > E$, $\psi_{II} = Ce^{-k_2x} + De^{k_2x}$

$$k_2 = \frac{\sqrt{2m(U-E)}}{\hbar}$$

Real solutions

The solution does not represent a moving particle. The particles are not oscillating. But this does not imply that $|\psi|^2 = 0$.

$\therefore |\psi_{II}|^2 \neq 0$

there is a finite probability of finding a particle within the barrier.

Apply boundary conditions,

At $x=0$, $\psi_I = \psi_{II}$

$$\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx}$$

At $x=L$, $\psi_{II} = \psi_{III}$

$$\frac{d\psi_{II}}{dx} = \frac{d\psi_{III}}{dx}$$

$A + B = C + D$ $\quad (\psi_{II} = \psi_I$ at $x=0)$

$ik_1 A - ik_1 B = k_2 C + k_2 D$

$Ce^{-k_2 L} + De^{k_2 L} = Fe^{ik_1 L}$ $\quad (\psi_{II} = \psi_{III}$ at $x=L)$

$-k_2 Ce^{-k_2 L} + k_2 De^{k_2 L} = ik_1 Fe^{ik_1 L}$ at $x=L$.

$$\frac{A}{F} = \left[\frac{1}{2} + \frac{i}{4}\left[\frac{k_2}{k_1} - \frac{k_1}{k_2}\right]\right] e^{(ik_1 + k_2)L} + \left[\frac{1}{2} - \frac{i}{4}\left[\frac{k_2}{k_1} - \frac{k_1}{k_2}\right)\right] e^{(ik_1 - k_2)L}$$

Assumption: All Potential barrier is high relative to E of incident particles, then

$$\frac{k_2}{k_1} > \frac{k_1}{k_2}$$

and so

$$\frac{k_2}{k_1} - \frac{k_1}{k_2} \approx \frac{k_2}{k_1}$$

also let the width of the barrier be such that ψ_{II} is severely weakened between $x=0$ to $x=L$.

$K_2 L \gg 1$

$e^{k_2 L} \gg e^{-k_2 L}$

$$\frac{A}{F} = \left(\frac{1}{2} + \frac{ik_2}{4k_1}\right) e^{(ik_1 + k_2)L}$$

Quantum Mechanics - Arthur Beiser. (Problems)
Examples 5.3, 5.4, 5.5, 5.6.

$$\left(\frac{A}{F}\right)^* = \left(\frac{1}{2} - \frac{ik_2}{4k_1}\right) e^{(ik_1 + k_2)L}$$

$$\frac{AA^*}{FF^*} = \left(\frac{1}{4} + \frac{k_2^2}{16k_1^2}\right) e^{2k_2L}$$

Assuming that velocity of particles has survived,

$$V_{III+} = V_{I+}$$

$$\left|\frac{V_{III+}}{V_{I+}}\right| = 1$$

$$T = \frac{16}{4 + \left(\frac{k_2}{k_1}\right)^2} e^{-2k_2L}$$

$$\left(\frac{k_2}{k_1}\right)^2 = \frac{\frac{2m(U-E)}{\hbar^2}}{\frac{2mE}{\hbar^2}}$$

$$= \frac{U}{E} - 1$$

$$T = \frac{16}{\left(4 + \left(\frac{U}{E} - 1\right)\right)} e^{-2k_2L}$$

$$= \frac{16}{\cancel{4} + \left(\frac{U}{E} - 1\right)} e^{-2\sqrt{2m(U-E)}/\hbar \, L}$$

Factors affecting the transmission probability:

(i) ~~Barg~~ Width of the barrier

(ii) Ratio $\frac{U}{E}$

Tunneling: used in scanning tunnel microscopes

Harmonic Oscillators:

Equation of simple harmonic oscillation:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \rightarrow \text{Solution: } x = A\cos(2\pi\nu t + \phi)$$

where $\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$F = -kx$$

~~F(x)~~ For small displacements, $F(x) = \left(\frac{dF}{dx}\right)_{x=0} x$

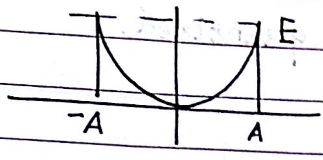
$u(x) = \frac{1}{2}kx^2$

Quantization:

(i) The allowed energies will not form a continuous spectrum, but instead a discrete spectrum of certain specific energy values

(ii) The lowest level cannot be 0. It has to be some value E_0

(iii) If the potential well is.



There is a finite probability that the particle will cross the potential well and go beyond A & -A.

$u = \frac{1}{2}kx^2$; Solve Schrodinger wave equation:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - \frac{1}{2}kx^2\right)\psi = 0$$

$$y = \left(\frac{1}{\hbar}\sqrt{km}\right)^{1/2} x = \sqrt{\frac{2\pi m\nu}{\hbar}}\, x \quad \text{and} \quad \alpha = \frac{2E}{\hbar}\sqrt{\frac{m}{k}} = \frac{2E}{h\nu}$$

$$\frac{d^2\psi}{dy^2} + (\alpha - y^2)\psi = 0$$ $\psi \to 0$ if $y \to \infty$ in order to satisfy $\int_{-\infty}^{\infty} |\psi|^2 dy = 1$

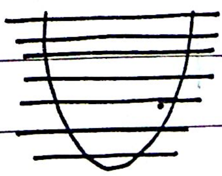
$\int_{-\infty}^{\infty} |\psi|^2 dy = 1$ is only true if $\alpha = 2n+1$

or $E_n = \left(n + \frac{1}{2}\right)h\nu$

The wave functions: $\psi_n = \left(\frac{2m\nu}{\hbar}\right)^{1/4} (2^n n!)^{-1/2} H_n(y) e^{-y^2/2}$

H_n: Hermite polynomials. ~~Hn~~ ~~Hn~~ E_n

Potential well



Fermions/Bosons are oscillating particles.

HYDROGEN ATOM

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2m}{\hbar^2}(E - U)\psi = 0$$

Electrical Potential Energy $U = \frac{-e^2}{4\pi\varepsilon_0 r}$

In spherical coordinates,

$$\frac{1}{r}\sin^2\theta \frac{\partial}{\partial r}\left(r^2 \frac{\partial\psi}{\partial r}\right) + \sin\theta \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\psi}{\partial\theta}\right) + \frac{\partial^2\psi}{\partial\phi^2} + \frac{2mr^2\sin^2\theta}{\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0 r} + E\right)\psi = 0$$

$(r, \theta, \phi) \rightarrow$ Spherical coordinate variables.

$\psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$

$\psi = R\Theta\Phi$

Solve using variable separation method:

Separating the variables:

$$\frac{d^2\Phi}{d\phi^2} + m_l^2\Phi = 0$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + \left[l(l+1) - \frac{m_l^2}{\sin^2\theta}\right]\Theta = 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \left[\frac{2m}{\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0 r} + E\right) - \frac{l(l+1)}{r^2}\right]R = 0$$

$$m_l^2 = -\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}$$

1) $\Phi(\phi) = Ae^{im_l\phi}$

m_l : magnetic quantum number

$Ae^{im_l\phi} = Ae^{im_l(\phi + 2\pi)}$

$m_l = 0, \pm Z$ (any integer)

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$l \rightarrow$ orbital quantum number.
Solve for E using equation of R:

$$E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\left(\frac{1}{n^2}\right)$$

$$= \frac{E_1}{n^2}$$

$n = 1, 2 \ldots$
E_1: energy of the first Bohr orbit.
n: Principal quantum number.
Constraint on n: Atleast be $l+1$
$l = 0, \ldots, n-1$

$\Theta(\theta)$: Constraint on l obtained by solving for $\Theta(\theta)$
l is an integer $\geq |m_l|$.

1) $m_l = 0, \pm Z \rightarrow \phi \quad m_l = 0, \pm 1, \pm 2 \ldots \pm l$
2) $l \geq |m_l| \rightarrow \Theta \quad l = 0, 1, 2 \ldots, n-1$
3) $n \geq l+1 \rightarrow R \quad n = 1, 2, 3 \ldots$

Physical significance of the quantum numbers.
1) n: Quantization of energy
2) l: Quantization of angular momentum (magnitude)

$$R \rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \left(\frac{2m}{\hbar^2}\left[\frac{e^2}{4\pi\varepsilon_0 r} + E\right] - \frac{l(l+1)}{r^2}\right)R = 0$$

E = ~~KE~~ Radial KE + Orbital KE + U

$$KE_{orbit} = \frac{\hbar^2 \, l(l+1)}{2mr^2}$$

(KE radial when

$$KE_{orbital} = \frac{1}{2}mv^2_{orbital}$$

KEorbital vanishes.

$$KE_{orbital} = \frac{L^2}{2mr^2}$$

L : $mV_{orbital}$ (Momentum)

$$\frac{L^2}{2mr^2} = \frac{\hbar^2\, l(l+1)}{2mr^2}$$

$$L = \sqrt{l(l+1)}\ \hbar$$

$l = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$\quad\ \ s \quad p \quad d \quad f \quad g \quad h \quad i$

Quantization of

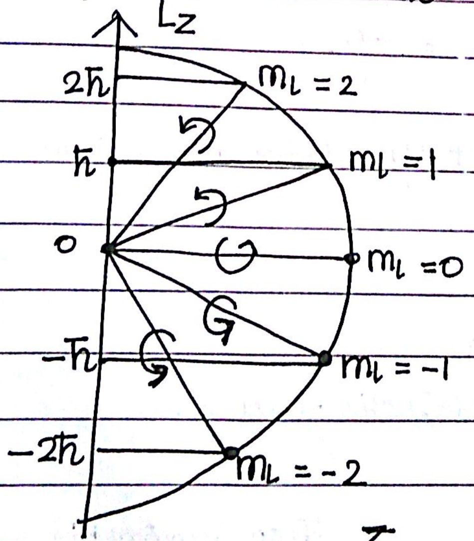
~~Direction of angular momentum.~~

3) m_l : Quantization of direction of angular momentum.

$$L = \sqrt{l(l+1)}\ \hbar$$

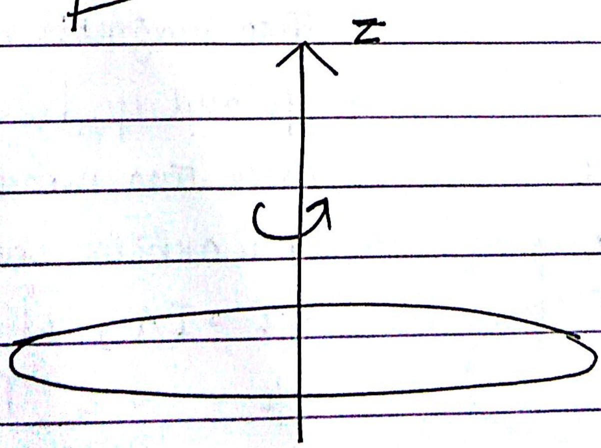
$$L_z = m_l \hbar$$

$$m_l = 0,\ \pm 1,\ \pm 2 \ldots \pm l$$



$L = \sqrt{l(l+1)}\ \hbar$

$= \sqrt{6}\ \hbar \ (l = 2)$



$|L| > |L_z|$

Spin quantum number 'S'

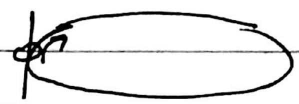
Intrinsic angular momentum of an electron.

Fine structure (spectral lines) caused due to spin.

Splitting of spectral lines under magnetic field - Zeeman effect.

$\cancel{S = \pm\frac{1}{2}}$ $\quad S = \frac{1}{2} \quad m_s = \pm\frac{1}{2}$

Spin angular momentum $S = \sqrt{s(s+1)}\,\hbar$



Coupling of angular momentum L & S.

If magnetic field is applied in z-direction,

$S_z = m_s \hbar \qquad m_s = \pm\frac{1}{2}$

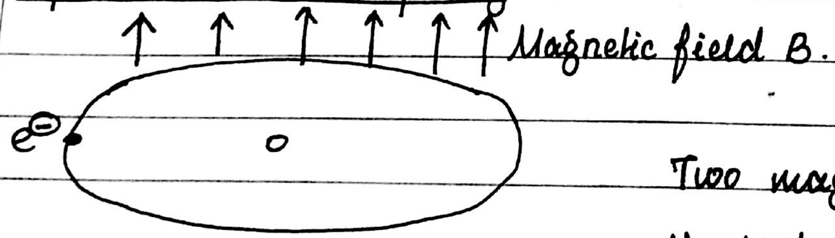
m_s : spin magnetic quantum number

determines the direction of $e^{\ominus}$ spin.

Spin magnetic moment $\mu_s = \frac{-e}{m} S$

$$\mu_{sz} = \frac{\pm e\hbar}{2m} = \pm \mu_B \text{ (Bohr magneton)}$$

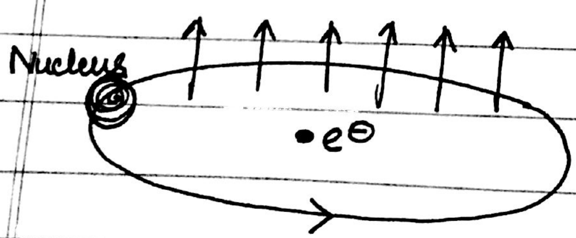
Spin-orbital coupling :



Two magnetic moments:

μ_L and μ_s. Interaction between these two moments creates spin orbital coupling.

Electron frame :



$\mu_l = IA, \; |\vec{L}| = |\vec{r} \times \vec{p}|$

$\mu_l = \frac{-e}{2m_0} \vec{l}$

When $l = \hbar$, $\mu_l = \mu_B$ (Bohr magneton)

Interaction b/w $e^{\ominus}$ spin magnetic moment and magnetic field leads to spin-orbit coupling.

$\mu_m = \pm \mu_B B$

B : magnitude of the magnetic field

Depending on the orientation of 'S', the energy of an $e^{\ominus}$ will be greater or smaller by an amount μ_m.

Every quantum state except S are split into two substates

$s = \frac{1}{2}$

$S = 2s + 1$ (Possible orientations for S)

<u>Total Angular Momentum</u>

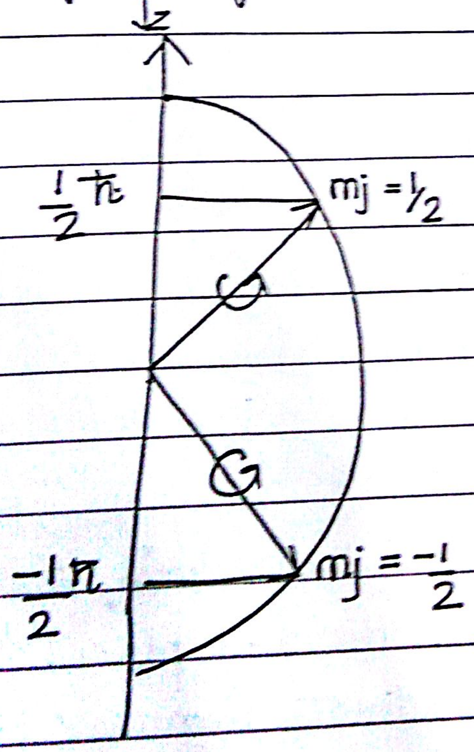
$$J = L + S$$

$$= \sqrt{j(j+1)}\,\hbar$$

$$J_z = m_j \hbar$$

$$j = l \pm s$$

$$m_j = -j, \ldots, j$$



$$J = \sqrt{j(j+1)}\,\hbar$$

$$= \sqrt{\frac{3}{2}}\,\hbar \quad \frac{\sqrt{3}\,\hbar}{2}$$

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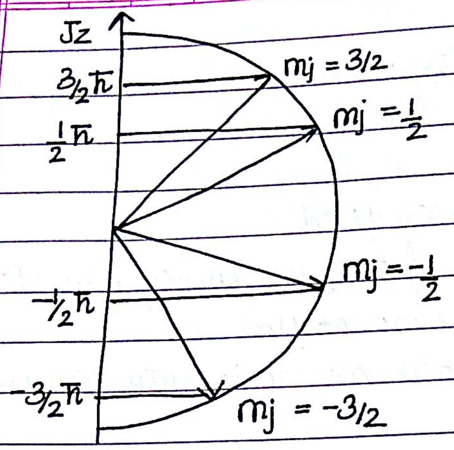


$J = \frac{\sqrt{15}\hbar}{2}$

LS coupling:

$\vec{L} = \Sigma L_i$

$\vec{S} = \Sigma S_i$

$\vec{J} = L + S$

then magnitudes of L, S and J, z are given by:

$L = \sqrt{\vec{L}(\vec{L}+1)}\hbar$

$L_z = M_L\hbar$

$S = \sqrt{\vec{S}(\vec{S}+1)}\hbar$

$S_z = M_S\hbar$

$J = \sqrt{\vec{J}(\vec{J}+1)}\hbar$

$J_z = M_J\hbar$

If $L > S$, $J \Rightarrow$ ~~2L+1~~ $2S+1$ values

If $L < S$, $J \Rightarrow 2L+1$ values

Hyperfine structure

$\rightarrow |L| > |L_z|$

If L could exactly point along the z axis, then the $e^{\ominus}$ orbit will be confined only to the x-y plane.

The uncertainty in the z coordinate becomes 0.

<u>Spin orbital coupling:</u>

$|\vec{L}| = \hbar\sqrt{l(l+1)}$

$|\mu_l| = \mu_B\sqrt{l(l+1)}$

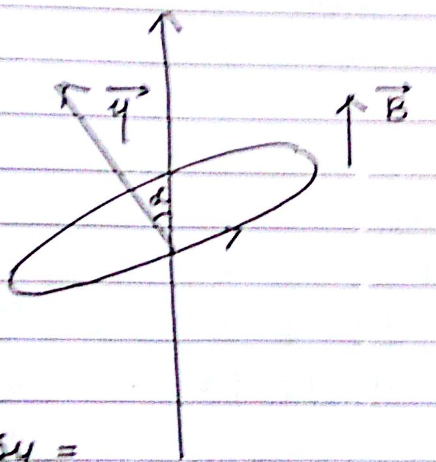
$\mu_{lz} = -\mu_B m_l \qquad m_l = 0, \pm 1, \ldots, \pm l$

$|\vec{S}| = \sqrt{s(s+1)}\,\hbar$

$S_z = m_s \hbar$

$m_s = \pm\frac{1}{2}$

$$\mu_s = -\underbrace{g_s}_{\text{Lande factor}} \frac{e}{2m_0}\vec{S}$$



$\tau = \mu \times B$

Potential energy =

$-\mu . B$ (Magnitude of potential energy)

If $\alpha = 0$, potential energy is minimum ~~and~~

μ_s is parallel to B or antiparallel. This leads to a doublet in spectrum.

Therefore, fine spectra observed.

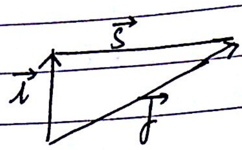
Electron spin can also interact with spin of the nucleus.

$$V_{l.s} = \frac{a}{\hbar^2}\vec{L}.\vec{S}$$

$$j = \vec{L} + \vec{S}$$

$$V_{ls} = \frac{a}{2\hbar^2}\left(|\vec{J}|^2 - |\vec{l}|^2 - |\vec{S}|^2\right)$$

$\hat{j}^2, \hat{l}^2, \hat{s}^2$: three operators.



$\underline{\underline{j}}$

$$\hat{j}^2\psi = \hbar^2 j(j+1)\psi$$
$$\hat{j}_z\psi = \hbar m_j \psi$$

$$V_{ls} = \frac{a}{2}\left(j(j+1) - l(l+1) - s(s+1)\right)$$

Split in the energy level.

Explains fine spectrum

$j = l \pm s$

<u>Commutator relations:</u>

$$L = r \times p = \det\begin{vmatrix} i & j & k \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$L_x = yp_z - zp_y$$
$$L_y = zp_x - xp_z$$
$$L_z = xp_y - yp_x$$

$$\hat{L}_x = i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$
$$\hat{L}_y = i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$
$$\hat{L}_z = i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

ϕ

In spherical coordinates,

$$\hat{L}_x = i\hbar[-\sin\phi \frac{\partial}{\partial\theta} - \text{ctg}\,\theta \cos\phi \frac{\partial}{\partial\phi})$$

$$\hat{L}_y = i\hbar[\cos\phi \frac{\partial}{\partial\theta} - \text{ctg}\,\theta \sin\phi \frac{\partial}{\partial\phi})$$

$$\hat{L}_z = i\hbar \frac{\partial}{\partial\phi}$$

$$[\hat{L}_x, \hat{L}_y] f(x,y,z) = \hat{L}_x \hat{L}_y f(xyz) - \hat{L}_y \hat{L}_x f(xyz) = -i\hbar L_z$$

$$[\hat{L}_y, \hat{L}_z] = -i\hbar \hat{L}_x$$

$$[\hat{L}_x, \hat{L}_z] = -i\hbar \hat{L}_y$$

$$[\hat{L}_x, \hat{L}_x] = 0 \quad = [L^2, L_x] = [L^2, L_y] = [L^2, L_z]$$

$$[\hat{L}_y, \hat{L}_y] = 0$$

$$[\hat{L}_z, \hat{L}_z] = 0$$

~~$[x, \hat{p}_{xy}]$~~ Commutator relations between x and p:

$$\left.\begin{array}{l} [x, \hat{p}_y], \ [x, \hat{p}_z] \\ [y, \hat{p}_x], \ [y, \hat{p}_z] \\ [z, \hat{p}_{\cancel{y}x}], \ [z, \hat{p}_y] \end{array}\right\} = 0$$

$[x, p_x], [y, p_y], [z, p_z] \neq 0$ (due to uncertainty)
$= i\hbar$

$[A, B] = -[B, A]$ Antisymmetry

$[c_1 A + c_2 B, C] = c_1[A, C] + c_2[B, C]$ Linearity.

$[AB, C] = A[B, C] + [A, C]B$ → Leibnitz rule

$[[A,B], C] + [[B, C], A] + [[C,A], B] = 0$ → Jacobi identity.

Hydrogen Atom:

$P(r) = \int R^*(r) R(r)\, dR$

$P(\theta) = \int \Theta^*(\theta)\Theta(\theta) \sin\theta\, d\theta$

$P(\phi) = \int \phi^*(\phi)\phi(\phi)\, d\phi$

Possibility of finding an $e^\ominus$ in a kernel where these three probabilities intersect.

$\psi = a\psi_n + b\psi_m$

$\langle\psi\rangle = a^2\int \psi_n^* x \psi_n dx$

$\frac{E_m - E_n}{h}$

$+ b^2\int \psi_m^* x \psi_m dx + 2a * b \cos(2\pi f t)\int \psi_n^* x \psi_m dx$

If $e^\ominus$ is in state n, $b=0$

If $e^\ominus$ is in state m, $a=0$

If it is transitive both integrals contribute.

The cosine ~~tim~~ term gives the frequency of emitted radiation.

Electron does not emit dipole radiation. The third integral is non-zero when

$\Delta l = \pm 1$ and $\Delta m_l = 0, \pm 1$

When $\Delta m_l = 0$, there is no change of angular momentum along z axis and the photon is emitted along x-y plane.

When $\Delta m_l = \pm 1$, it is emitted along $\pm z$ axis.

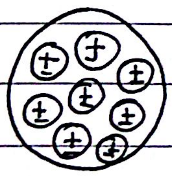
Similarly, if $\Delta l = \pm 1$, the angular momentum of atom goes through a transition. The difference in angular momentum is carried away by photon.

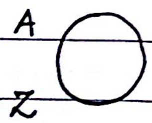
NUCLEAR PHYSICS

Origins:
→ Rutherford's α-scattering experiment.
→ Discovery of radioactive elements.

Types of forces:
Strong force – holds the nucleus together. Short range force.
~~Weak force~~ → Range: 10^{-15} m, Energy: 0.1 GeV
Weak force – 10^{-5} times weaker than strong force. Range: 10^{-17} m.
Particles: Neutrinos, Bosons.
Electromagnetic – $1/137$
Gravity – 10^{-39}

Strong forces/Nuclear physics:
Nucleus





$A = Z + N$

Isotopes: ${}^{14}_{6}C$: half life: 5730 yrs.

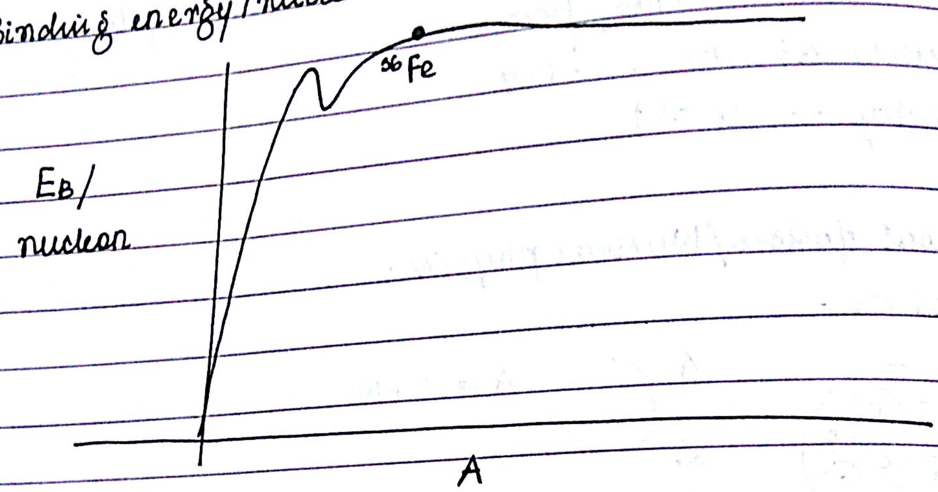
Radius of nucleus:
$R = R_0 A^{1/3}$
$R_0 = 1.2 \times 10^{-15}$ m
~~A~~ $m = Au$, m: mass of the nucleus. $u = 1.66 \times 10^{-27}$ kg.
All nuclei will have same density (10^{+17} kg/m^3)

Binding Energy: total rest energy of the nucleus and it is ~~greater~~ less than the rest energy of the separate nucleons.

Binding energy $E_B = \left(ZM_H + Nm_N - {}^{A}_{Z}M\right)c^2$

M_H : Mass of hydrogen atom
${}^{A}_{Z}M$: Mass of neutral atom
m_N : Mass of neutron
Z : Atomic number
N : Number of neutrons.

Binding energy/nucleon vs A curve:



Spin and magnetic moment

$S = \sqrt{s(s+1)}\,\hbar = \sqrt{\frac{3}{2}}\,\hbar$

$m_s = \pm\frac{1}{2}$

μ_N (Nuclear magneton) $= \frac{e\hbar}{2m_p} = 3.152 \times 10^{-6}$ eV/T

$\mu_{pe} = \pm 2.793\,\mu_N$ (Magnetic moment of proton)
$\mu_{nz} = \mp 1.913\,\mu_N$ (Magnetic moment of neutron)
$U_m = -\mu_z B$

μ_z: total magnetic moment

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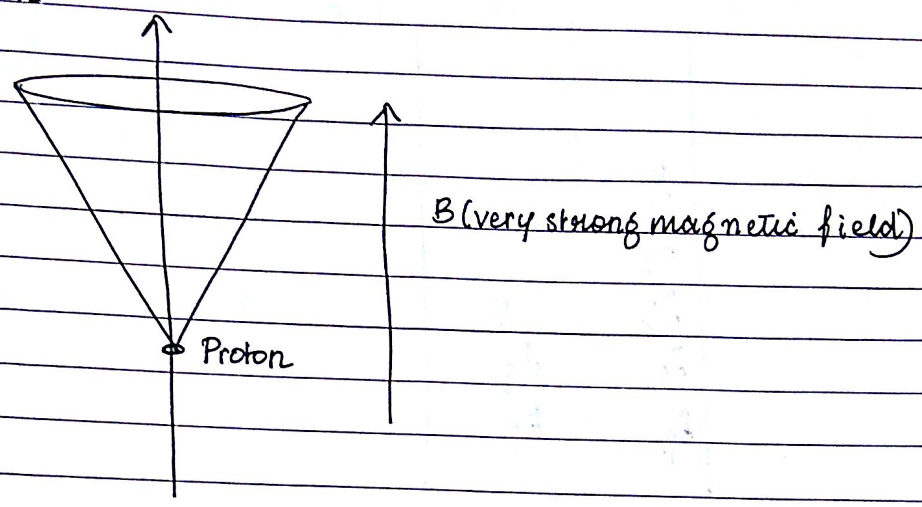
Larmor frequency for protons:

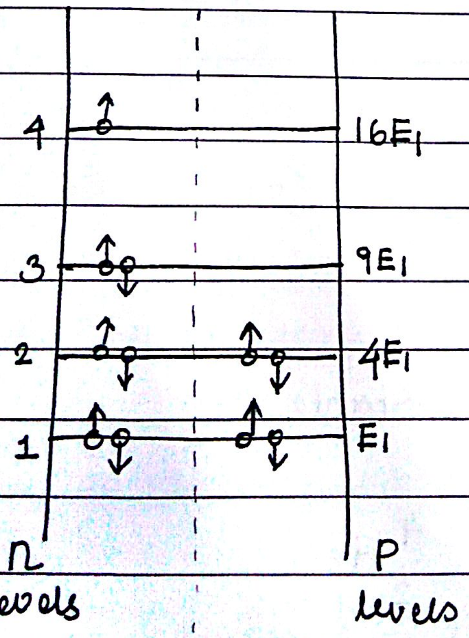
$$\nu_L = \frac{2 \mu_{pz} B}{h}$$



Nature of nuclear physics:

1) It does not depend on charge
2) Short range force $10^{-15} m$
3) Stronger than electric force
4) It is saturated force
5) It favors formation of pairs of nucleons with opposite spins.

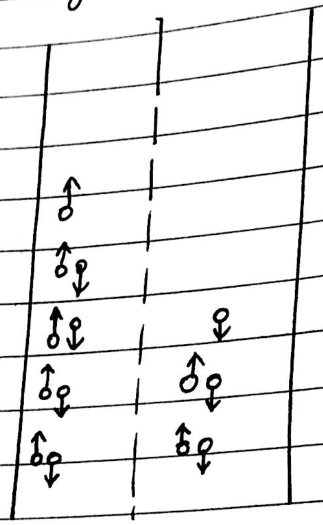
No particular equation to describe the nuclear force.



Stable nucleus.

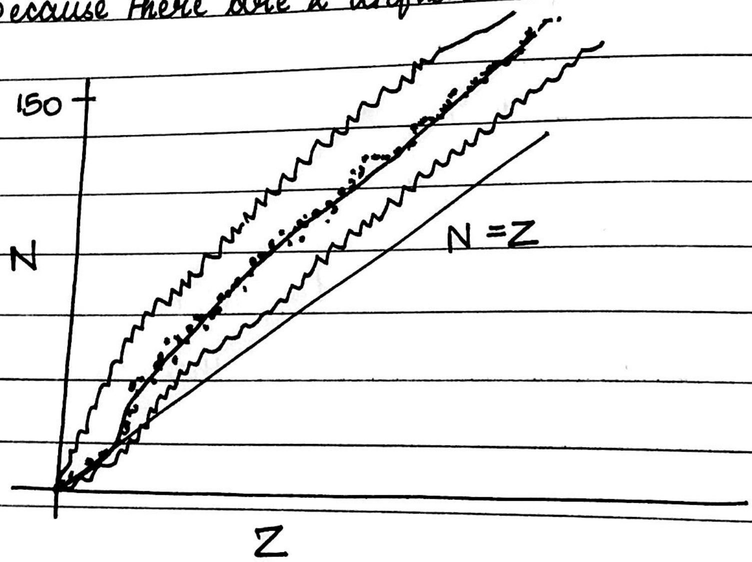
Stable if $Z \cong N$ for small A

For large A values, $N \geq Z$



Unstable

because there are 2 unfilled states.



$Z > 83$

$N > 126$

$A > 209$

Unstable nucleus $\xrightarrow{\text{decay}}$ Stable

<u>Alpha decay:</u>

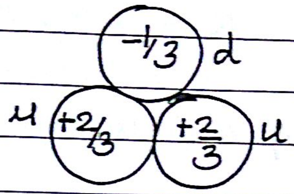
$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

Beta decay:

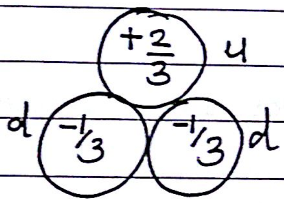
$n^0 \rightarrow p^+ + e^- + \bar{\nu}$ (Antineutrino)

$p^+ \rightarrow n^0 + e^+$ (positron) $+ \nu$ (neutrino)

Proton:



Neutron:



Gamma decay:

$p^+ + e^- \longrightarrow n^0 + \nu$

$\frac{1}{2} = e^{-\lambda t}$

$\lambda t = \ln(2)$

$N(t) = N_0 e^{-\lambda t}$

$T_{1/2} = \cancel{\ln\left(\frac{2}{\lambda}\right)} \ \frac{\ln 2}{\lambda}$

Fission and Fusion:

Two low mass nuclei fuse to form high mass nucleus - fusion.

High mass nuclei decay to low mass nuclei - fission.

When nuclear reactions happen, they conserve:

(i) charge

(ii) momentum & angular momentum

(iii) Energy.

(iv) Total number of nucleons.

$M_A + M_B \rightarrow M_C + M_D$

$Q = (\underbrace{M_A + M_B}_{F_1} - \underbrace{(M_C + M_D)}_{F_2}) c^2$

$F_1 > F_2$: exoenergic reaction $Q>0$

$F_1 < F_2$: endoenergic reaction $Q<0$

Cross-section of reaction: very difficult to estimate.

3 dimensional SWE TIE:

Shell model:

Nucleus is a spherical well. Here, the neutrons and protons behave as Fermi gas.

If the nucleus deforms due to any interactions, the potential energy changes.

If there ~~is~~ are any deviations from the average interactions b/w the nucleons, there are perturbations.

$f(x) \rightarrow f(x+\Delta x)$

→ Stability of low mass number nucleus.

→ Magic numbers - Nuclei are highly stable

$\underbrace{2, 8, 20, 28, 50, 82, 126}$

large BE/nucleon than neighboring nuclei.

If both N and Z are magic nos, then such nuclei are specially stable.

$\psi(r,t) = R_n(r) Y_{lm}(\theta, \phi)$

$E_{nl} = \left(2n + l + \frac{3}{2}\right)\hbar\omega. \quad n = 0, 1, 2, 3 \ldots$

$l = 0, 1, 2 \ldots$

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$$E_{n_x, n_y, n_z} = E_{nl} = \left(n_x + n_y + n_z + \frac{3}{2}\right)\hbar\omega$$

This model fails for large atonic magic nos as it neglects spin orbit energy ~~numbers~~ which splits the levels and causes an increasing orbital quantum numbers.

Liquid drop model:

Nuclear force is attractive unless the nucleons are squeezed together. Nuclear properties like binding energy can be understood in terms of an empirical relation.

$$B(Z,N) = c_1 A - c_2 A^{2/3} - c_3 Z(Z-1)A^{1/3} - c_4 \frac{(N-Z)^2}{A}$$

c_1, c_2, c_3, c_4 determined experimentally.

~~1st term~~ $c_1 A$: volume term; represents volume energy

$c_2 A^{2/3}$: surface term; responsible for the decrease in BE. nucleons near the surface of the nucleus are less deep in the potential well. The surface term is important for BE of small nuclei.

$c_3 Z(Z-1)A^{1/3}$: coloumb interactions; each pair of protons contribute equally to the repulsive force.

$c_4 \frac{(N-Z)^2}{A}$: If a proton transforms into a neutron or vice-versa the BE decreases.

No nuclei with $Z > 120$ can exist because the coloumb force becomes too large for such nuclei.

Does not explain spin.

Collective model – explains ~~both~~ nuclear properties using both shell and liquid drop model.

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Bieser - Pg 421.

α - decay:

O ⟶ O + ^4_2He *(Quantum tunneling)

parent nucleus — Daughter nucleus

The α-particle typically ~~⊕~~ knocks the walls 10^{24} times But it may take 10^{10} years for one α particle to come out of the nucleus.

$$\log_{10}\lambda = \log_{10}\left(\frac{V}{2R_0}\right) + 1.29\, Z^{1/2} R_0^{1/2} - 1.72\, Z E^{-1/2}.$$

V: velocity of α-particle.

Z: atomic no. of daughter nucleus.

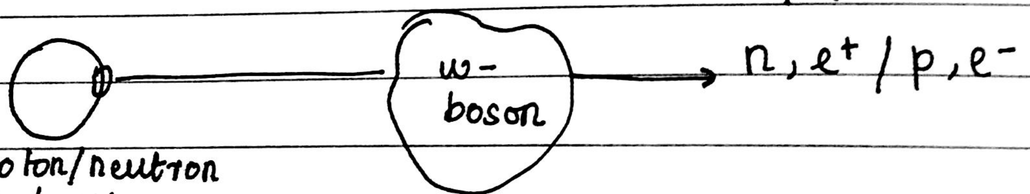
E: energy in MeV.

β - decay:

$n \rightarrow p + e^- + \bar{\nu}$ → Probability of this rxn is very low. ∴ detection of e^+ is difficult.

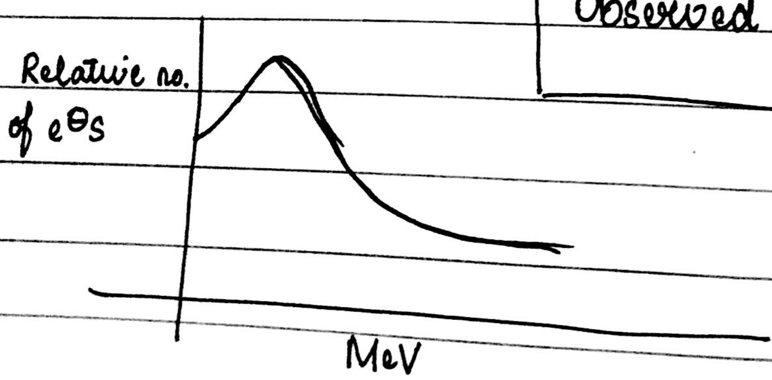
$p + \bar{\nu} \rightarrow n + e^+$ } Inverse β decay

$\underbrace{n + \nu \rightarrow p + e^-}$ → Probability of this rxn is ~~the weakest~~ high



proton/neutron captures ~~an~~ a ~~neut~~ anti-neutrino/neutrino

$$\boxed{\begin{array}{c} p + e^- \rightarrow n + \nu \\ \text{Observed in high Z nucleus.} \end{array}}$$



γ - decay:

$^{87}_{38}Sr^* \longrightarrow Sr + \gamma +$ high energy.

Internal conversions happen in high Z nuclei. The nucleus ~~captures an e⊖~~. Sometimes, it may give its energy to one of the atomic e⊖s usually K shell e⊖s. This e⊖ is knocked out and emits an X-ray. Energy = BE + its own PE.

Fundamental particles:

i) Strong Force	Quarks, Hardons	Gluons, Mesons
ii) EM	charged particles	Photons
iii) Weak	Quarks, leptons	Intermediate bosons.
iv) Gravitational	All	gravitons

Antiparticles: e^+, e^-. Every particle has an antiparticle.

1). Leptons - fermions. unaffected by strong interactions.

~~Electron~~ ~~e^-~~ ~~e^+~~

Particle	charge	Antiparticle	Stability	Spin
Electron	e^-	e^+	Stable	1/2
e-neutrino	ν_e	$\overline{\nu_e}$	Stable	1/2
muon	μ^-	μ^+	2.2×10^{-16}	1/2
μ-neutrino	ν_μ	$\overline{\nu_\mu}$	Stable	1/2
Tau	τ^-	τ^+	2.9×10^{-21}	1/2
τ-neutrino	ν_τ	$\overline{\nu_\tau}$	Stable	1/2

~~Every~~ Every lepton has its associated neutrino.

Neutrino oscillations. - neutrinos change from one flavour to another.

μ – muon

$\pi^+ \longrightarrow \mu^+ + \nu_e$

$\pi^- \longrightarrow \mu^- + \overline{\nu_\mu}$

$\underbrace{\pi}_{\text{pion}} \longrightarrow \underbrace{\gamma + \gamma}_{\text{gamma rays.}}$

π – pion.

Pions are supposed to exist in the nucleus and help to keep the nucleus in balance.

$t=0$ (p) $\longleftrightarrow$ (n)

(n) $\pi^+ \xrightarrow[c]{}$ (n)   Proton emits a pion

(n)   (p)   The other neutron captures the pion and becomes a proton.

2) Hardons: subject to strong interactions.

- Mesons – Bosons
- ~~Bary~~ Baryons – Fermions.

Bieser -495. Table 13.3.

S – Strangeness number.

Ex

$$p^+ + p^+ \Longrightarrow \Lambda^0 + K^0 + p^+ + \pi^+$$

$$S: \quad 0 \quad 0 \qquad -1 \quad +1 \quad 0 \quad 0$$

Baryon number (B) and Lepton number. (L_μ, L_e, L_τ)

~~These~~ Lepton numbers and Baryon numbers are always conserved in any reaction.

DATE / / PAGE

Neutron decay:

	$n^0 \rightarrow p^+ + e^- + \overline{\nu_e}$
Lepton number	$0 \rightarrow 0 + 1 - 1$
Baryon no:	$1 \rightarrow 1 + 0 + 0$

All antibaryons : B = -1

All baryons : B = 1

Quarks:

Table 13.4

What gives particles mass?

→ Quantum field theory

↙ Space field

↘ Force field.

If excited, seen as particles.

Force field hypothesis created singularities & divergence of solution.

To overcome this divergence, coupling of the two fields was introduced.

Three kinds of spins:

the second spin removes the divergence.

One particle that remains spinless is the massive Higgs Boson. It interacts with any other particle and gives that particle mass.

Notes

1) Postulates of QM
2) Derive relations using postulates.
3) Particle in a Box.
   Get the solution of ψ using a given potential. u or E.
4) Tunneling effect. Explain tunneling using wave functions.
5) Hydrogen Atom:
   Explain the quantum numbers and their importance.
6) Operators / Commutor relations. L_x, L_y, L_z ...
7) Write a note on hyperfine structure / fine structure.
8) Explain hydrogen series from the ideas of wave functions and probabilities.

Q:

A particle is confined to box of length 'L'.

$$\psi = A\sin\left(\frac{\pi x}{L}\right) \qquad 0 < x < L$$

$$= 0 \qquad \text{otherwise}$$

Calculate A.

Band theory of solids — Nuclear Physics.