STUDY OF MAX-CUT PROBLEM BY USING QUANTUM APPROXIMATE OPTIMIZATION ALGORITHM

A PROJECT WORK SUBMITTED TO THE
DEPARTMENT OF PHYSICS CENTRAL CAMPUS OF TECHNOLOGY
INSTITUTE OF SCIENCE AND TECHNOLOGY
TRIBHUVAN UNIVERSITY
NEPAL

FOR THE AWARD OF
BACHELOR OF SCIENCE (B.Sc.) IN PHYSICS

By
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MAY-6,2022

RECOMMENDATION

This is to recommend that **RAJU RAI** ,has carried out project work entitled **"STUDY OF MAX-CUT PROBLEM BY USING QUANTUM APPROXIMATE OPTIMIZATION ALGORITHM"** for the requirement to the project work in Bachelor of Science in Physics under my/our supervision in the Department of Physics, Central Campus of Technology, Institute of Science and Technology(IoST), Tribhuvan University(T.U), Nepal.
To our knowledge, this work has not been submitted for any other degree.

He has fulfilled all the requirements laid down by the Institute of Science and Technology

(IoST),Tribhuvan University (T.U.), Nepal for the submission of the project work for the partial fulfillment of Bachelor of Science (B.Sc.) degree.

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DECLARATION

This project work entitled "**STUDY OF MAX-CUT PROBLEM BY USING QUANTUM APPROXIMATE OPTIMIZATION ALGORITHM**" is being submitted to the Department of Physics, Central Campus of Technology, Institute of Science and Technology (IoST), Tribhuvan University (T.U.), Nepal for the partial fulfillment of the requirement to the project work in Bachelor of Science (B.Sc.) degree in Physics. This project work is carried out by me under the supervision of Prem Sagar Dahal and co-supervision of Dibakar Sigdel in the Department of Physics,Central Campus of Technology, Institute of Science and Technology (IoST), Tribhuvan University (T.U.), Nepal.

This work is original and has not been submitted earlier in part or full in this or any other form to any university or institute, here or elsewhere, for the award of any degree.

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LETTER OF FORWARD

Date:

On the recommendation of PREM SAGAR DAHAL and Dr.DIBAKAR SIGDEl, this project work is submitted by Raju Rai,Symbol No. ,T.U. Registration No, entitled "STUDY OF MAX-CUT PROBLEM BY USING QUANTUM APPROXIMATE OPTIMIZATION ALGORITHM" is forwarded by the Department of Physics, Central Campus of Technology, for the approval to the Evaluation Committee, Institute of Science and Technology (IoST), Tribhuvan University (T.U.), Nepal
He has fulfilled all the requirements laid down by the Institute of Science and Technology (IoST),Tribhuvan University (T.U.), Nepal for the project work.

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BOARD OF EXAMINATION AND CERTIFICATE OF APPROVAL

This project work (PRO-406) entitled "STUDY OF MAX-CUT PROBLEM BY USING QUANTUM APPROXIMATE OPTIMIZATION ALGORITHM" by RAJU RAI ,under the supervision of PREM SAGAR DAHAL and co-supervision of Dr.DIBAKAR SIGDEL in the Department of Physics, Central Campus of Technology, Institute of Science and Technology (IoST),Tribhuvan University (T.U.), is hereby submitted for the partial fulfillment of the Bachelor of Science(B.Sc.) degree in Physics.This report has been accepted and forwarded to the Controller of Examination, Institute of Science and Technology, Tribhuvan University, Nepal for the legal procedure.

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ABSTRACT

The max-cut problem is also known as the NP-hard problem which means the cut the graph with maximum edges and two groups of vertices. This problem can be solved by the classical approach with less accuracy results so, to solve the problem we can use the quantum approach. Quantum computing is a computation of problems by using the quantum approach. In quantum computing there are various kinds of algorithms are available but to solve the max-cut problem, we can implement the QAOA(Quantum Approximate Optimization Algorithm). QAOA is one of the masterpiece algorithm which provides us with approximate results of the max-cut problem. In my project, I have taken two graphs having 2 vertices and 5 vertices. I have done the analytical part of the graphs. Where I used the Quantum computation machine, of QISKIT. In this project, I have explained quantum gates and how Hamiltonian works in quantum computation.

सोधसार

अधिकतम(कट समस्यालाई ल्ए(हार्ड समस्याको रूपमा पनि चिनिन्छ जसको अर्थ अधिकतम किनाराहरू र शीर्षहरूका दुई समूहहरू भएको ग्राफ काट्नु हो। यो समस्या शास्त्रीय दृष्टिकोण द्वारा कम सटीकता परिणाम संग हल गर्न सकिन्छ त्यसैले, समस्या को समाधान गर्न को लागी हामी क्वान्टम दृष्टिकोण को उपयोग गर्न सक्छौं। क्वान्टम कम्प्युटिङ क्वान्टम दृष्टिकोण प्रयोग गरेर समस्याहरूको गणना हो। क्वान्टम कम्प्युटिङमा विभिन्न प्रकारका एल्गोरिदमहरू उपलब्ध छन् तर अधिकतम(कट समस्या समाधान गर्न, हामी त्तब्इब् ९क्वान्टम अनुमानित अप्टिमाइजेसन एल्गोरिदम० लागू गर्न सक्छौं। त्तब्इब् उत्कृष्ट कृति एल्गोरिदम मध्ये एक हो जसले हामीलाई अधिकतम(कट समस्याको अनुमानित परिणामहरू प्रदान गर्दछ। मेरो परियोजनामा, मैले २ वटा ठाडो र ५ वटा ठाडो भएका दुईवटा ग्राफ लिएको छु। मैले ग्राफको विश्लेषणात्मक भाग गरेको छु। जहाँ मैले त्तक्ष्क्ष्क्ष्त् को क्वान्टम गणना मेसिन प्रयोग गरें। यस परियोजनामा, मैले क्वान्टम गेटहरू र ह्यामिलटोनियनले क्वान्टम गणनामा कसरी काम गर्छ भनेर व्याख्या गरेको छु।

LIST OF ACRONYMS AND ABBREVIATIONS

QAOA	Quantum Approximate Optimization Algorithm
VQA	Variational Quantum Algorithms
MAX-CUT	Maximum Cut

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1 INTRODUCTION

1.1 General introduction:

Quantum computing is generally viewed as a field of study centered on developing computer technology based on the principles of quantum theory. In Quantum Computing computers will have a special kind of feature called speed. The computer will get high accuracy as well as supreme speed.The quantum concept is valuable because it allows for more than just two states. It relates to interactions between particles at a tiny scale, so small that the rules of physics as we normally experience them no longer apply. However, it is possible that some of the unusual – and counterintuitive - phenomena occurring at this level could be used to overcome the limitations of traditional computing.[9]

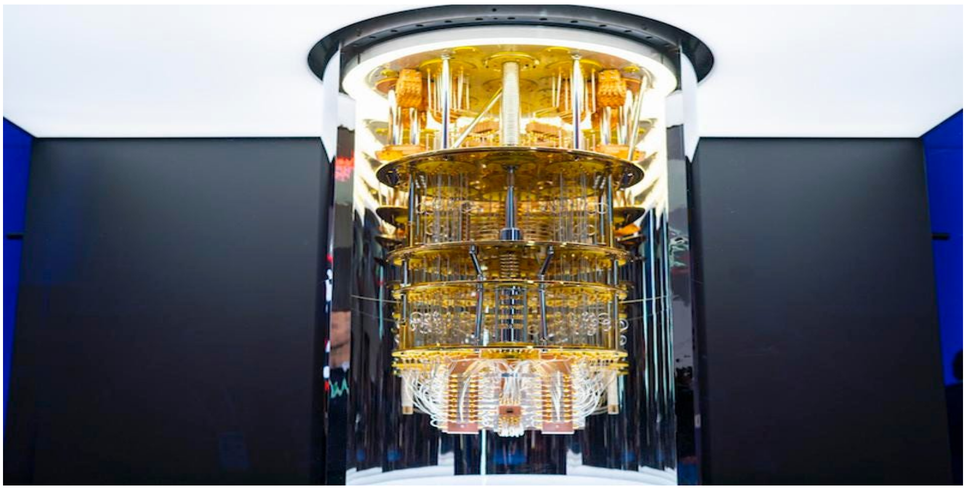

Figure 1: Picture of Quantum Computer by IBM

Quantum computers are based upon the quantum theory, like superposition, interference, and entanglement. Quantum interference allows an object to coexist in states of “being” and “non-being” at a given time. Superposition is the ability of a quantum system to be in multiple states at the same time until it is measured. Quantum entanglement is one vital long-distance relationship, In Quantum entanglement two particles link together in a certain way no matter how far apart they are in space(Spooky Action Of Distance).[8] As we know in classical computers, a ”Bit” is the smallest unit of data in a computer.

A bit has a single binary value, either 0 or 1. Although computers usually provide instructions that can test and manipulate bits and bits store the data. Similarly, in quantum computing, a qubit or quantum bit(denoted by $|0\rangle$ and $|1\rangle$) is the basic unit of quantum information—the quantum version of the classic binary bit physically realized with a two-state device[3] .[3]qubits are represented by a superposition state vector in 2n dimensional Hilbert space.(for example, $|0\rangle and|1\rangle$)

The building blocks of a digital circuit are logic gates, which execute numerous logical operations that are required by any digital circuit. These can take two or more inputs but only produce one output. for example, classical gates are, AND gate, OR gate, NOT gate. These gates are also called the basic gates. Other gates are NAND, NOR, X-OR, and X-NOR . a quantum logic gate (or simply quantum gate) is a basic quantum circuit operating on a small number of qubits.[10] They are the building blocks of quantum circuits like classical logic gates for conventional digital circuits. Quantum gates are unitary operators and are described as unitary matrices relative to some basis. Examples of Quantum gates are Hadamard gate, X-gate, Y-gate, Z-gate, Toffoli (CCNOT) gate, Swap gate, etc. Quantum Fourier transform is a linear transformation on quantum bits and is the

quantum analog of the discrete Fourier transform. Quantum Fourier Transform (QFT), is a key step for quantum algorithms that exhibit exponential speed up. It is used, for order finding and period finding in many algorithms.[1] The quantum approximate optimization

algorithm (QAOA) is a general technique that can be used to find approximate solutions to combinatorial optimization problems. like, max-cut problems(NP-hard problems).QAOA can be regarded as an application of VQE. Where The variational quantum eigensolver (or VQE) uses the variational principle to compute the ground state energy of a Hamiltonian.[5]

1.2 Quantum Computing:

Quantum computing is a rapidly-emerging technology that harnesses the laws of quantum mechanics to solve problems too complex for classical computers. Today, IBM Quantum makes real quantum hardware – a tool scientists only began to imagine three decades ago – available to thousands of developers. Our engineers deliver ever-more-powerful superconducting quantum processors at regular intervals, building toward the quantum computing speed and capacity necessary to change the world. These machines are very different from the classical computers that have been around for more than half a century. Here's a primer on this change technology. When scientists and engineers encounter

difficult problems, they turn to supercomputers. These are very large classical computers, often with thousands of classical CPU and GPU cores. However, even supercomputers struggle to solve certain kinds of problems. If a supercomputer gets stuck, that's probably because the big classical machine was asked to solve a problem with a high degree of complexity. When classical computers fails, it's often due to complexity of problems.Complex problems are problems with lots of variables interacting in complicated ways. Modeling the behavior of individual atoms in a molecule is a complex problem, because of all the different electrons interacting with one another. Sorting out the ideal routes for a few hundred tankers in a global shipping network is complex too.

1.3 Quantum Gates

Quantum gates is the basic building blocks of circuits which helps to create the complex and simple circuit in quantum computers. in classical computer we use the AND, OR, NOT gates similarly, we use the X-gate , Y-gate , Z-gate and Hadamard gates in quantum computers. They have their own functions, according to arrangement. These gates are made on the principle of pauli matrix operations.

Gate	Notation	Matrix
NOT (Pauli-X)	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Z	$-\boxed{Z}-$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard	$-\boxed{H}-$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
CNOT (Controlled NOT)	$\begin{array}{c} \bullet \\ \vert \\ \oplus \end{array}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Figure 2: Representation of Quantum gates

1.4 Max-Cut:

In an undirected graph with edge weights, the MAX-CUT problem consists in finding a partition of the nodes into two subsets, such that the sum of the weights of the edges having endpoints in different subsets is maximized. It is a well-known NP-hard problem with applications in several fields, including VLSI design and statistical physics. In this article, a greedy randomized adaptive search procedure (GRASP), a variable neighborhood search (VNS), and a path-relinking (PR) intensification heuristic for MAX-CUT are proposed.[5]
The give graph is the four vertices graph where we can do the quantum operations and find its max-cut solution. There are so many algorithms we have but one algorithm known as the greedy algorithm is very useful for the max-cut problem, which includes, Considering an Undirected Graph(G) formed by Vertices(V) Edges(E). We start ordering on the Vertices(V) as V1, V2,...Vn and also we have two empty bins or sets S, T (Where these will be having the vertices set once after the cut is maximized). Pick the first vertex V1 and place it in S. For each subsequent vertex we place it in the bin or set such that —E(S, T)— is maximized.[7]
The given below picture the graph having 4 vertices and 4 edges, when we create the quantum circuit, and simulates the circuit in quantum simulator, this gives the max-cut values.

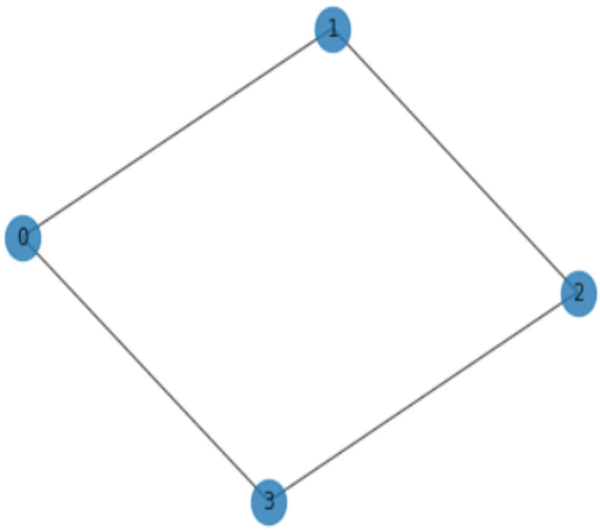

Figure 3: The graph having 4 vertices

1.5 Quantum Approximate Optimization Algorithm:

There are many loopholes in the greedy algorithm, so there is one algorithm that works based on the quantum operation. Quantum Approximate Optimization Algorithm (QAOA), a hybrid quantum-classical algorithm, compares its computational cost with a state-of-the-art classical solver for the NP-hard problem called Max-Cut. The solution of Max-Cut, even if approximate, has practical application in machine scheduling, image recognition, or for the layout of electronic circuits. In our simulations, we include both decoherence and dissipation to verify that the quantum algorithm is robust to realistic levels of noise and consider the compilation of the algorithm in terms of one- and two-qubit gates satisfying the hardware connectivity of a square grid with the corresponding routing overhead. Multiple instances at different problem sizes are solved to estimate the absolute time required to run QAOA and extrapolate its scaling cost given below is the circuit that shows the circuit of QAOA.[4]

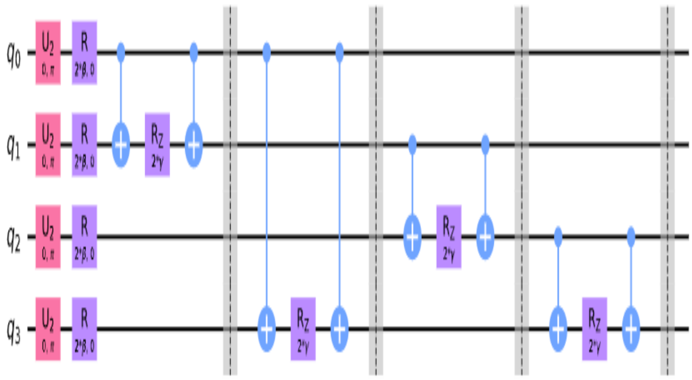

Figure 4: Quantum Circuit of Quantum Approximate Optimization Algorithm

1.6 Graph and Circuit:

The mathematical representation of a network it describes the relationship between lines and points is called a graph. a path between two or more points is called a circuit.

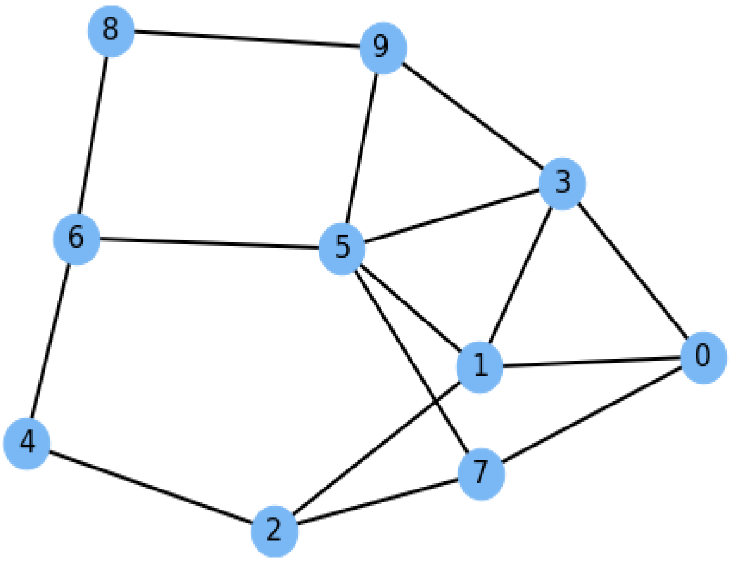

Figure 5: The graph having 10 vertices

1.6.1 Optimization:

Optimization deals with finding the best solution to a problem of graphs. This problem is very important in mathematics, for example, the max-cut problem.Quantum optimization algorithms are quantum algorithms that are used to solve optimization problems. Mathematical optimization deals with finding the best solution to a problem (according to some criteria) from a set of possible solutions. Mostly, the optimization problem is formulated as a minimization problem, where one tries to minimize an error which depends on the solution: the optimal solution has the minimal error. Different optimization techniques are applied in various fields such as mechanics, economics and engineering, and as the complexity and amount of data involved rise, more efficient ways of solving optimization problems are needed.[5] The power of quantum computing may allow problems which are not practically feasible on classical computers to be solved, or suggest a considerable speed up with respect to the best known classical algorithm.

1.6.2 Rationale:

This paper is a review of the study on the solution of the max-cut problem(optimization problem) using the Quantum Approximate Optimization Algorithm.
The basic reason for this project is to present the information about how the QAOA works to solve the max-cut problem with a higher accurate approach.
The basic reason for this project is to present the information about how the QAOA works to solve the max-cut problem with a higher accurate approach.
The study of max-cut problems can open a new dimension in the field of mathematics and physics which helps in the field of programming too. Also, we can get enough knowledge about the quantum system and algorithms like QAOA. which also tells about how quantum and classical approaches are different.

1.6.3 Objectives:

The max-cut problem is one of the basic optimization problems of mathematics. in the classical approach, we could not get the result so, approximately. so to overcome this problem of the classical approach we applied the QAOA(Quantum approximate Optimization Algorithm). This approach can solve the max-cut problem approximately with high accuracy and with high speed. In this project, I used two different graphs with vertices 2, 5.

1.6.4 General Objectives

1)To study the what is the max-cut problem also called the NP-hard problem. and how the quantum phenomenon will be important in the upcoming future.and detail study of quantum computing and linear algebra.
2)To understand how quantum computing and classical computing are different. To know how quantum gates and programming are important in Quantum computing. And to study the how QAOA works to solve the Optimization problem.

1.6.5 Specific Objectives

1)To solve the max-cut(NP-hard) problem using the Quantum Approximate algorithm(QAOA).
2)To know how the programming knowledge and software knowledge are important in the quantum computing.

2 LITERATURE REVIEW

As the MaxCut problem is NP-hard, there exists no polynomial-time algorithm to solveit exactly, unless P = NP, which most likely is not true. Therefore, a vast majority ofdeveloped algorithms to solve MaxCut are approximation algorithms The first of such algorithms

with a ratio of 0.5 was presented in 1976 by Sahni and Gonzales. It is a simple greedy algorithm that iterates through the nodes and on each step decides which subset assigns a vertex vi, based on which what maximizes the size of the cut. Since then some research were done, presenting algorithms with slightly improved performance guarantees of 1 is the number of vertices, m — the number of edges.

In 1994, Goemans and Williamson(GW) developed an algorithm that achieves an approximation

ratio of 0.878 and has a time complexity of O(n2log(n)[4]. It is based on randomly rounding of a solution to semidefinite programs and is still considered the state-of-the-art algorithm for the MaxCut problem with a proven approximation ratio.

Many further pieces of research proposed some improvements for solving the MaxCut

problem. Homer and Peinado gave a parallelized version of GW[6]. Kim and Moon developed a hybrid genetic algorithm(GA) for the MaxCut problem, which gives promising results, outperforming GW in many tests. Kim, Yoon, and Geem used a harmony search algorithm for solving MaxCut, which produced even greater cut sizes than GA.

Apart from that, there have been good results achieved for certain types of graphs. Feige,

Karpinski, and Langberg considered enhancing the GW algorithm with an additional local step that moves misplaced vertices from one side of the partition to the other, until no such vertices are left. They showed that their algorithm obtains an approximation ratio of at least 0.921 for graphs of maximal degree 3, and 3-regular graphs the approximation ratio is at least 0.924[11]

3 MATERIALS AND METHODS

3.1 Materials

Based on experimentally determined first we have to python code and how to use the IBM quantum computer. And also knowledge of basic jupyter , and basic knowledge of python package(Matplotlib and pandas).In this project first we have done the quantum circuits of respective graph and the different probability values are determined. which gives the probability of eigen state, of the problem. And we got the visualizer of quantum bits for each vertices of graphs, known as Bloch Sphere. The picture of bloch sphere is given below. Rest of material are Visual studio code, which helped me to do python code,

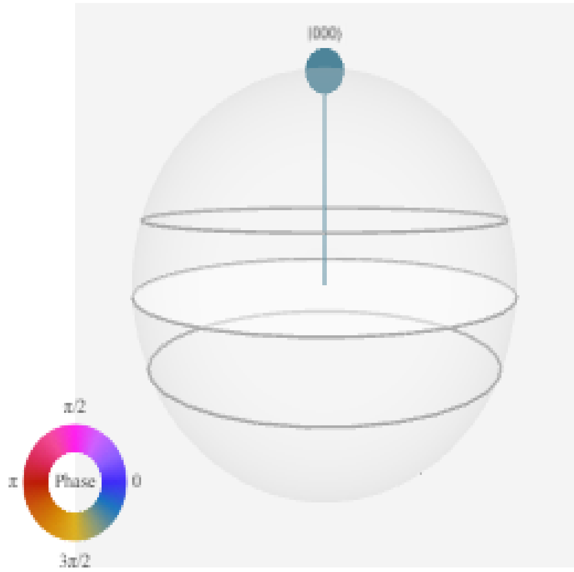

Figure 6: Bloch sphere for the visualize the quantum bits.

also commonly referred to as VS Code, is a source-code editor made by Microsoft for Windows, Linux and macOS. Features include support for debugging, syntax highlighting, intelligent code completion, snippets, code refactoring.

3.2 Methods

3.2.1 Mapping max-cut to ising model

To solve an optimization problem on a quantum computer, we need to turn it into a problem of measurement of a quantum Hamiltonian, which is the system's total energy. Finding the maximum cat of the graph can be easily mapped to a problem of maximizing the Hamiltonian of an Ising model[2]. The Ising model is an example of a lattice spin model. In each lattice site k, we consider a spin k with two possible states: k = ±1 (corresponding to — ↑i and — ↓i). Nearest sites interact with each other with an energy J. If there is no external field interacting with the lattice, the full energy of a spin configuration is a Hamiltonian.

$$\mathrm{H}(\sigma) = -\Sigma(\sigma_i \sigma_j)$$

3.2.2 Quantum Approximate Optimization Algorithm

The algorithm chosen for this study, namely the Quantum Approximate Optimization Algorithm (QAOA), belongs to the class of hybrid algorithms and requires, in addition to the execution of shallow quantum circuits, a classical optimization process to improve the quantum circuit itself. Since its proposal7, QAOA has attracted considerable interest as an application of pre-error-corrected quantum devices12,13,14,15,16,17. The goal of

QAOA is to solve, in an approximate way and by using a shallow quantum circuit, combinatorial problems like constraint-satisfaction (SAT) problems in their "maximization" formulation. In general, SAT problems are characterized by a set of constraints and by the question of whether it is possible to satisfy all constraints at the same time or not. Their "max" form asks what is the maximum number of constraints that can be satisfied at the same time. The latter question is at least as hard as the former one. What differentiates it from other variational algorithms is that the quantum circuit is

determined by the problem instance with very few adjustable parameters, and it is believed that shallow circuits suffice to achieve good approximate solutions. We apply QAOA to solve Max-Cut, a graph partitioning problem defined as follows. Given a graph, assign one of two colors (e.g., black or white) to each vertex. An edge can be "cut" when it connects two vertices of different color. The problem is to assign the colors so that as many edges as possible can be cut at the same time. Max-Cut belongs to the complexity class called NP-hard, and closely related problems have been used as benchmarks in the context of Adiabatic Quantum Optimization and, in particular, with D-Wave Inc. devices18. The objective function corresponds to the hermitian operator:

We are going now to dive deep into the Quantum Approximation Optimization Algorithm, introduced by Farhi [4]. It is a quantum algorithm that produces approximate solutions for combinatorial optimization problems.
For MaxCut the core idea of QAOA is to find the highest energy state of the system, which is a state with the highest energy eigenvalue E.
$E\psi = H\psi$
Using QAOA we try to guess what this state looks like if we can't solve the Hamiltonian for eigenstates and eigenvalues exactly. For that, we can construct a "trial" (also called "guess") state, which is a linear combination of different Hamiltonian eigenstates

3.2.3 Trial state for max-cut

The trial state of max-cut is the constructing phase of max-cut where we convert the problem into characterized hamiltonian. and a find an approximate solution to the Max-Cut problem by maximizing the expected value of the Hamiltonian given as

h H$_i = h\alpha, \beta(H)\alpha, \beta_i$
where

h=plank's constant whose values is 6.626 070 15 x 10-34 J Hz-1.

H$_i = Hamiltonianoperator.$

$\alpha = Itistheoperator.$

$\beta = Itistheoperator.$

3.3 For two vertex of graph

Following experiments were done on a QASM simulator, provided by Qiskit, that emulates the execution of quantum circuits on a real quantum computer and returns measurement counts.for demonstration, I took the 2 vertices graph. for that first we have to deal with hamiltonian.

$$H(\sigma) = -\Sigma(\sigma_i \sigma_j)$$

for maximum Hamiltonian , we should take:
$\sigma_i = 1$ and $\sigma_j = -1$

After getting the maximum Hamiltonian, now Solving the Max-Cut problem for this graph is trivial but we can show how QAOA works, I have written mathematical form in words. A superposition of all the eigenstates of the Hamiltonian and among them there is one or more state which is the solution to the MaxCut problem. After the application of the operator on the eigenstates, we can get the probabilities of the system. for that we can apply the formula given below: the given symbols ,and has usual meaning.And other symbols indicate the quantum states.

3.4 mathematical treatment

The expression given below is the probability of superposition of all the eigenstates of the Hamiltonian which are the solution to the MaxCut problem (— ↑↓i and — ↓↑i in case of our graph.

$|A_{\uparrow\uparrow}|^2 = |A_{\downarrow\downarrow}|^2 = 1/4(1+\sin 2\alpha)\sin 4\beta$

$|A_{\uparrow\uparrow}|^2 = |A_{\uparrow\downarrow}|^2 = 1/4(1-\sin 2\alpha)\sin 4\beta$

if $\alpha = \pi/4 and \beta = \pi/4$.

$|A_{\uparrow\uparrow}|^2 = |A_{\downarrow\downarrow}|^2 = 0$

and

$|A_{\uparrow\uparrow}|^2 = |A_{\uparrow\downarrow}|^2 = 0.5$

which gives the values 0 and 0.5 , for same directed state and 0.5 for the opposite directed state. The picture shown in below is also the mathematical form which mentioned above. where α and β are two construct a parameter dependent trial states.

The picture which is shown below is bloch sphere can be used to,visual representation of qubits, where θ and ϕ.These are the parameters of bloch sphere

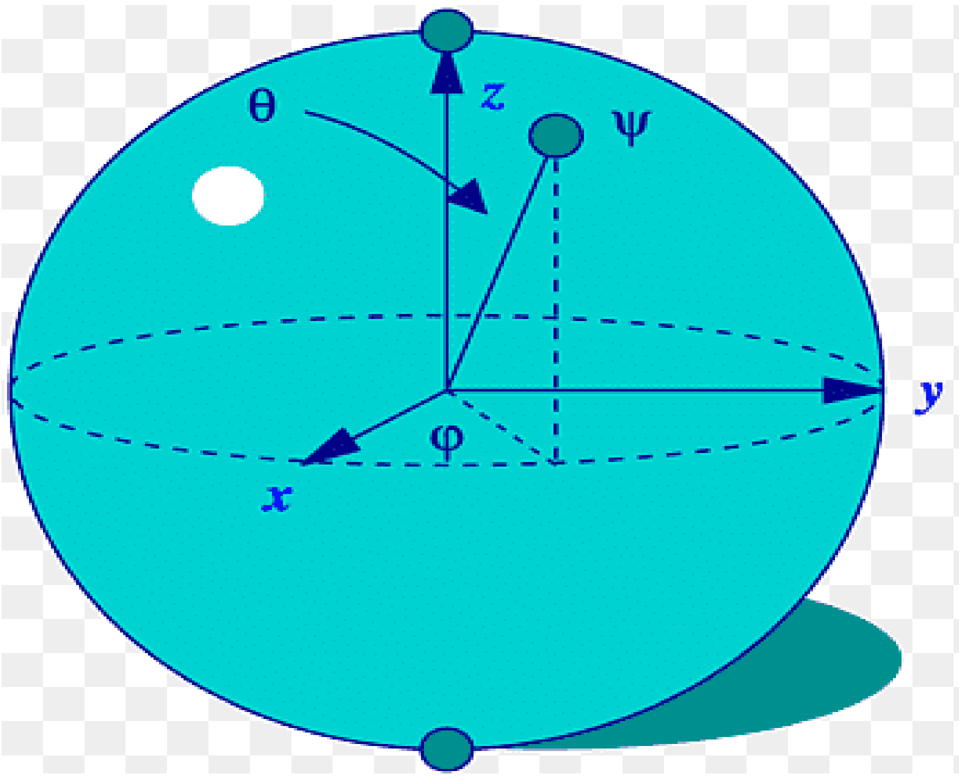

Figure 7: Bloch sphere

3.4.1 Quantum Circuit of 2 vertices graph for random parameters

The circuit is a path, similarly 2-solve max-cut problem of 2-vertices graph. I have created the 2-vertices-based quantum circuit, where I have not used the Hadamard gate, Observer, and Toffoli gate. that the initial state has to be a uniform superposition over all qubits. This can be achieved by without applying the Hadamard gate to each qubit , this kind of circuits doesn't give the exact solutions.

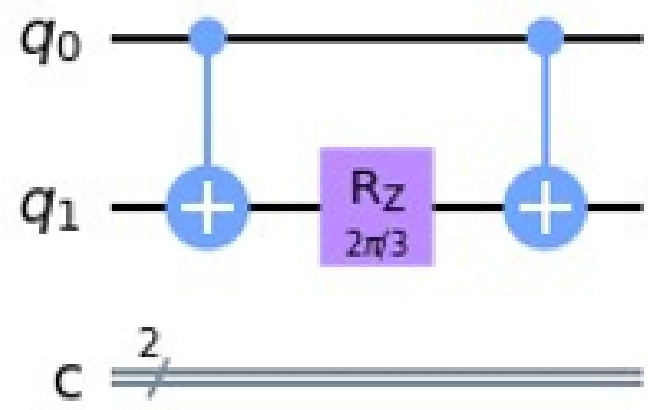

Figure 8: Quantum Circuit for 2 vertices graph with random parameters

3.5 Code of 2 vertices based quantum circuit for random parameters

The 2 vertices based quantum circuit can be create by using following code, in jupyter notebook
from qiskit import QuantumRegister, ClassicalRegister, QuantumCircuit
from numpy import pi

qreg$_q = QuantumRegister(3,'q')$
$creg_c = ClassicalRegister(3,'c')$
$circuit = QuantumCircuit(qreg_q, creg_c)$

circuit.cx(qreg$_q[0], qreg_q[1])$
$circuit.rz(pi/2, qreg_q[1])$
$circuit.cx(qreg_q[0], qreg_q[1])$

```
In [ ]: from qiskit import QuantumRegister, ClassicalRegister, QuantumCircuit
        from numpy import pi

        qreg_q = QuantumRegister(3, 'q')
        creg_c = ClassicalRegister(3, 'c')
        circuit = QuantumCircuit(qreg_q, creg_c)

        circuit.cx(qreg_q[0], qreg_q[1])
        circuit.rz(pi/2, qreg_q[1])
        circuit.cx(qreg_q[0], qreg_q[1])
```

Figure 9: The code for 2 vertices quantum circuit for random parameters in jupyter

3.5.1 Quantum Circuit of 2 vertices graph for optimal parameters

The circuit is a path, similarly 2-solve max-cut problem of 2-vertices graph. I have created the 2-vertices-based quantum circuit, where I have used the Hadamard gate, Observer, and Toffoli gate. that the initial state has to be a uniform superposition over all qubits. This can be achieved by applying the Hadamard gate to each qubit. After applying all the gates, the measurements have to be done.

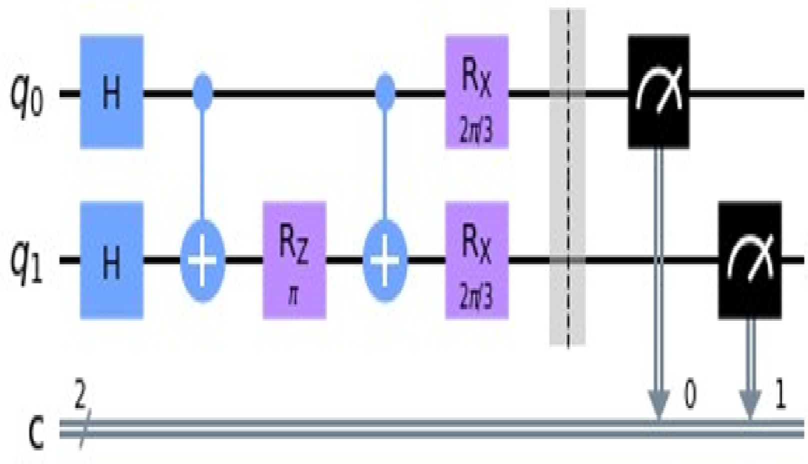

Figure 10: Quantum Circuit for 2 vertices graph with optimal parameters

3.6 Code of 2 vertices based quantum circuit for optimal parameters

The 2 vertices based quantum circuit can be create by using following code, in jupyter notebook

import numpy as np

import networkx as nx

import matplotlib.pyplot as plt

from qiskit import QuantumCircuit, execute, Aer

from qiskit.visualization import plot$_h istogram$

$fromscipy.optimizeimportminimize$

$backend = Aer.get_backend('qasm_simulator')$

$defC_operator_gates(G, alpha)$:

$n = G.number_o f_nodes()$

$qc = QuantumCircuit(n, n)$

for i, j in G.edges():

qc.cx(i, j)

qc.rz(alpha * 2, j)

qc.cx(i, j)

return qc

qc = C$_operator_gates(G, np.pi/2)$

$qc.draw("mpl")$

3.6.1 Quantum circuit of 5-vertices graph for random parameters

The circuit is a path, similarly 2-solve max-cut problem of 5-vertices graph. I have created the 5-vertices-based quantum circuit, where I have not used the Hadamard gate, Observer, and Toffoli gate. that the initial state has not to be a uniform superposition over all qubits. This can be achieved by without applying the Hadamard gates to each qubits. After applying all the gates.

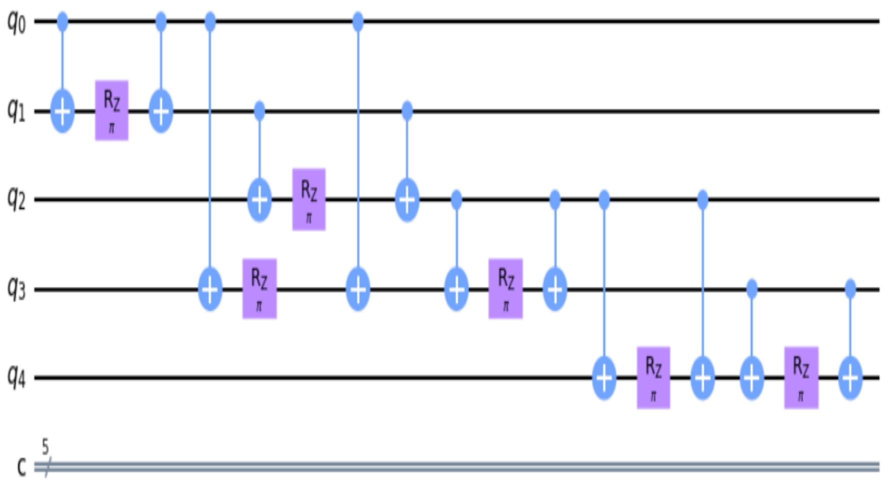

Figure 12: Quantum circuit of 5-vertices graph for random parameters

3.7 Code of 5-vertices based quantum circuit for random parameters

The 5 vertices based quantum circuit can be create by using following code, in jupyter notebook

import numpy as np

import networkx as nx

import matplotlib.pyplot as plt

from qiskit import QuantumCircuit, execute, Aer

from qiskit.visualization import plot$_{h}istogram$

from scipy.optimize import minimize

backend = Aer.get$_{b}ackend('qasm_{s}imulator')$

def C$_{o}perator_{g}ates(G, alpha):$

n = G.number$_{o}f_{n}odes()$

qc = QuantumCircuit(n,n)

for i, j in G.edges():

qc.cx(i, j)

qc.rz(alpha * 2, j)

qc.cx(i, j)

return qc

qc = C operator gates(G, np.pi / 2)

qc.draw(”mpl”)

The pictorial of the code is given below,which had run in jupyter notebook.

```
In [4]: def C_operator_gates(G, alpha):
            n = G.number_of_nodes()
            qc = QuantumCircuit(n,n)

            for i, j in G.edges():
                    qc.cx(i, j)
                    qc.rz(alpha * 2, j)
                    qc.cx(i, j)

            return qc
```

Figure 13: Code for circuit of 5-vertices graph for random parameters

3.7.1 Quantum circuit of 5-vertices graph for optimal parameters

The circuit is a path, similarly 2-solve max-cut problem of 5-vertices graph. I have created the 5-vertices-based quantum circuit, where I have used the Hadamard gate, Observer, and Toffoli gate. that the initial state has to be a uniform superposition over all qubits. This can be achieved by applying the Hadamard gates to each qubit. After applying all the gates.

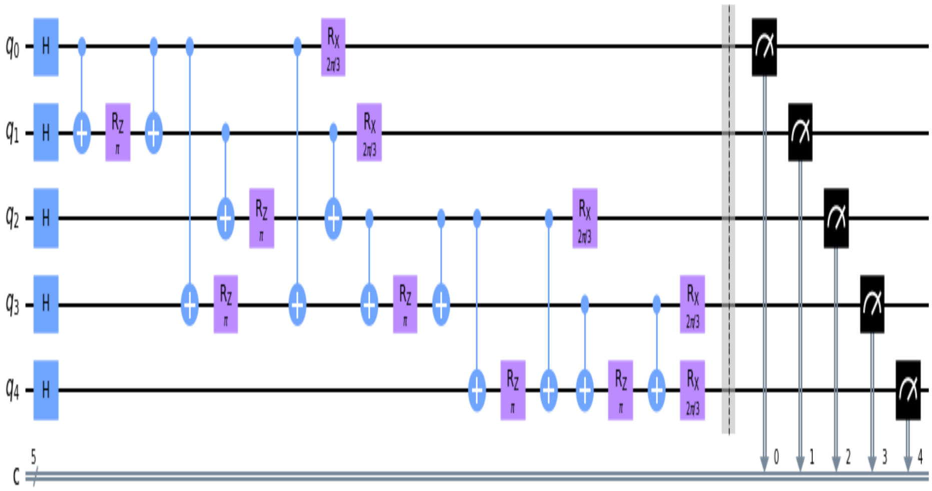

Figure 14: Quantum circuit of 5-vertices graph for optimal parameters

3.8 Code of 5-vertices based quantum circuit for optimal parameters

The 2 vertices based quantum circuit can be create by using following code,$injupyternotebook$

$importnumpyasnp$

$importnetworkxasnx$

$importmatplotlib.pyplotasplt$

$fromqiskitimportQuantumCircuit, execute, Aer$

$fromqiskit.visualizationimportplot_histogram$

$fromscipy.optimizeimportminimize$

$backend = Aer.get_backend('qasm_simulator')$

$defcreate_full_circuit(G, alpha, beta$

$n = G.number_of_nodes()$

$qc = QuantumCircuit(n, n)$

$qc.h(range(n))$

$qc+ = C_operator_gates(G, alpha)$

$qc+ = B_operator_gates(G, beta)$

$qc.barrier(range(n))$

$qc.measure(range(n),range(n))$

$returnqc$

$qc_5 = create_full_circuit(G,np.pi/2,np.pi/3)$

$qc_5.draw("mpl")$

The pictorial of the code is given below,which had run in jupyter notebook.

```
In [7]: def create_full_circuit(G, alpha, beta):
            n = G.number_of_nodes()
            qc = QuantumCircuit(n,n)

            qc.h(range(n))

            qc += C_operator_gates(G, alpha)
            qc += B_operator_gates(G, beta)

            qc.barrier(range(n))
            qc.measure(range(n), range(n))

            return qc
```

```
In [8]: qc_5 = create_full_circuit(G, np.pi/2, np.pi/3)
        qc_5.draw("mpl")
```

Figure 15: Code for quantum circuit of 5-vertices graph for optimal parameters

4 RESULTS AND DISCUSSION

The experiments were done on a QASM simulator, provided by Qiskit, that simulates the execution of quantum circuits on a real quantum computer and returns measurement counts.The whole experiments results are given below.

4.0.1 For 2 vertices graph

The graph having 2 vertices, where 0 indicates first vertex and 1 indicates the second vertex. During the execution, in quantum computer , computer should assign the 2 quantum bits $|0\rangle$ and $|1\rangle$ in both vertices.The given graph is the unsolved graph.Now, we are going to solve the max-cut of the graph using QAOA, for that we have already constructed the quantum circuit in (fig-10) and (fig-8) in previous chapter.

Figure 16: 2 vertices graph

4.0.2 Distribution of states for the 2-vertices graph with random parameters

Now we can run the circuit which, we constructed in the previous chapter(fig-6) for a 2-vertices graph. After 1024 iterations of running the circuit with the random parameters, we received the following distribution of states.

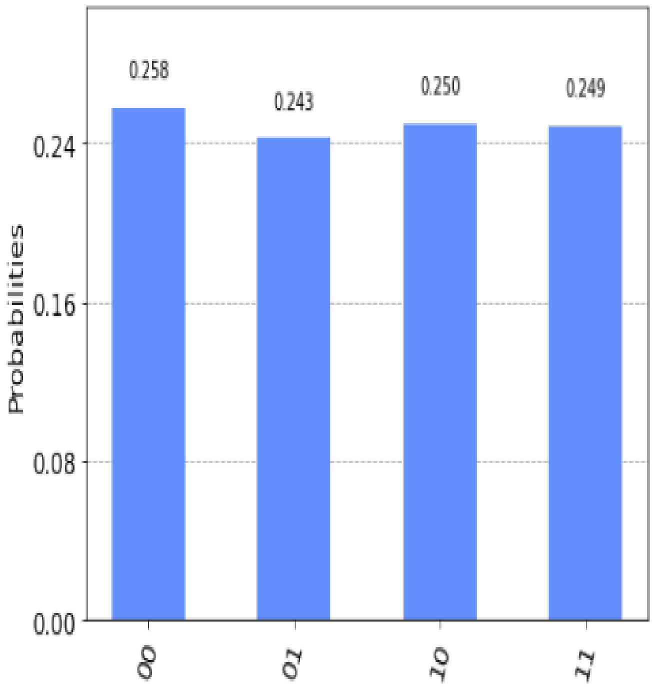

Figure 17: Distribution of states for the 2-vertices graph with random parameters

This is, obviously, not the correct solution to the problem. Recall that we now have to maximize the objective function, which is the Hamiltonian in the case of the Max-Cut problem to get the optimal parameters. After that one can run the circuit with the optimal parameters.

4.1 Distribution of states for the 2-vertices graph with optimal parameters

Running the circuit with optimal parameters gives either state 01 or 10 which are the exact solutions for the Max-cut problem for the given graph. These results match with those we received when calculating the probabilities manually in (3.5). where,the mathematical results are 0.5 and our distributed probabilities are approximately equally and likely. which means the the max cut solution is done. max-cut value is 1 and also QAOA cut is also 1. in this case the value of max-cut and average value of max-cut are equals which indicates, the solution is perfect, and significantly accurate.

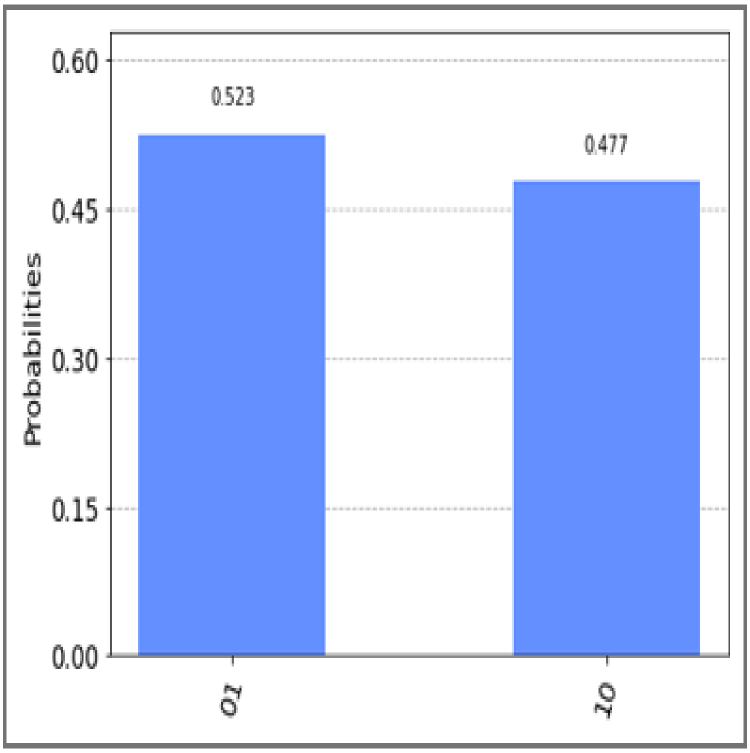

Figure 18: Distribution of states for the 2-vertices graph with optimal parameters

4.2 Solved 2 vertices graph having max-cut

This is the graph , which solved and blue indicates first vertex and red indicates second vertex.The max cut of graph =1, where Exact max cu=1 , QAOA cut=1 . from this graph, we can say that number some times number vertices is more than number of cut. This graph is made by using python based program in jupyter note boo. 2 nodes, as each node can be assigned to either the ”red” or ”blue” sets, there are 2^2=4 possible assignment, out of which we have to find one that gives maximum number of edges between the sets ”red” and ”blue”. The number of such edges between two sets in the figure, as we go from left to right 1 and 1. We can see, after enumerating all possible assignments,Hence if we encode ”red” as 1 and ”blue” as 0, the bitstrings ”01” and ”10” that represent the assignment of nodes to either set are the solutions.

Figure 19: Solved the 2-vertices graph

4.3 5 vertices graph

Graph is the combinatorial optimization problem, which can be minimized its paths for different aspect, for example for flow and for cut the graph. The initial unsolved graph of 5 vertices graph is given below. where 1,2,3,4,and 5 indicates the vertices of the graph. and in graph, total six edges are available. This kind of graph is little bit complex than 2 vertices graph and useful in field of deep learning.

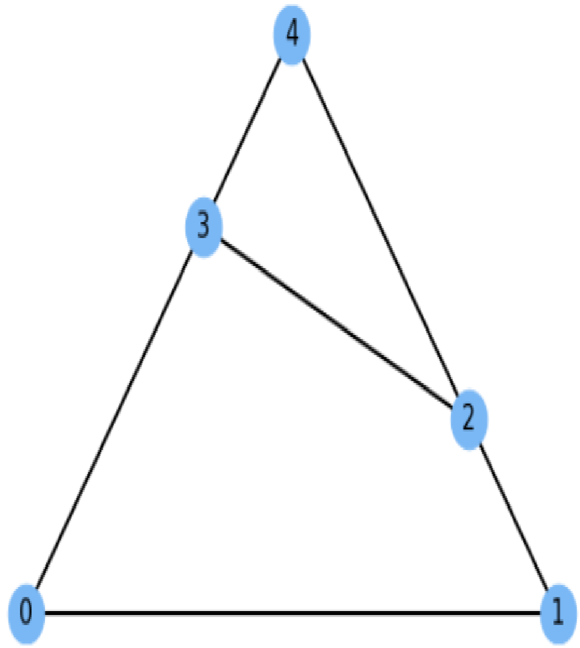

Figure 20: Unsolved 5 vertices graph

4.4 Distribution of states for the 5-vertices graph with optimal parameters

After maximization of the Hamiltonian, running the circuit(fig-7) with optimal parameters gives the distribution of states as on.that, the maximum cut here is the cut for the most frequent state (10101). the max-cut=5.

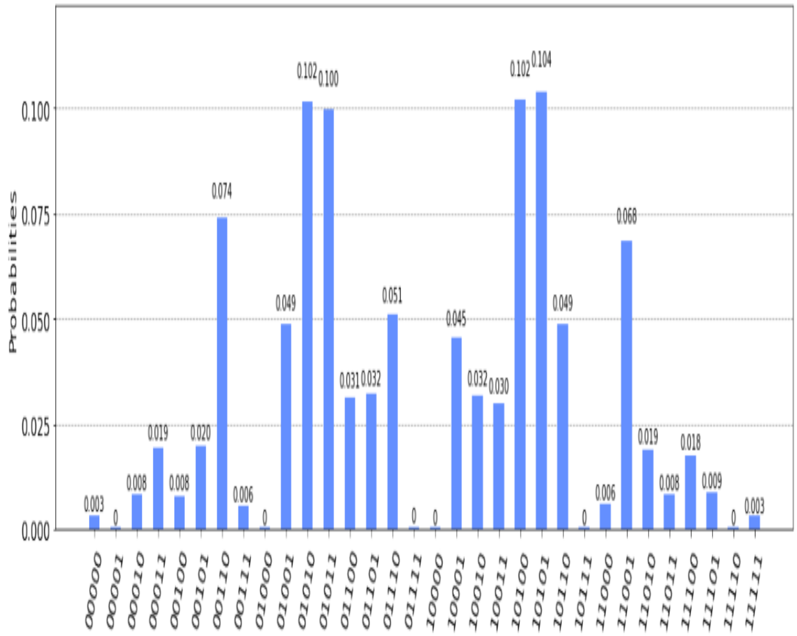

Figure 21: Distribution of states for the 5-vertices graph with optimal parameters

4.5 Maximum cut solution for the 5-vertices graph

This graph is 5 vertices graph. where two sets of vertices are created (1,3,4) and (0,2) and max-cut 5 . to get this graph , i used the python based coding in jupyter note book. according to graph , we can say that number of cut is less than number of edge.

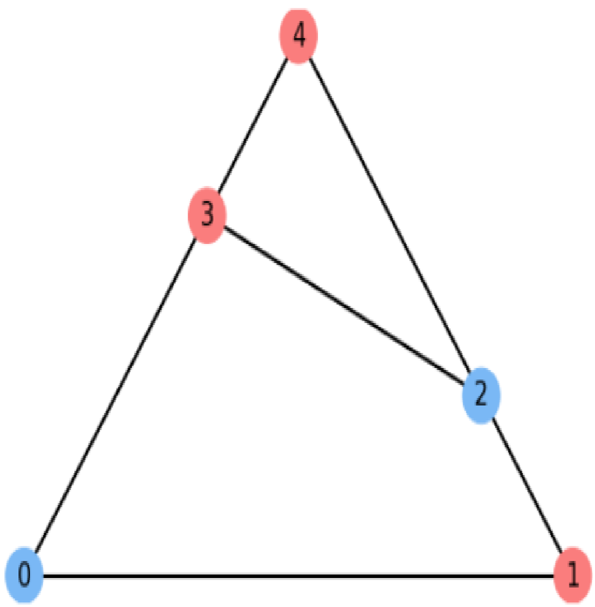

Figure 22: Solved 5 vertices graph

4.6 Distribution of 5 vertices graph in Real Quantum Computer

The circuit which , constructed previous chapter, now implemented in IBM quantum computer, which provided the distribution , given below. in this distribution frequent states are the same for both cases and they are the exact solutions to the max-cut problem. from graph we can say 10101 bit-string provides the max-cut solution.in diagram we can see bits string and x-direction and frequency are Y-direction.This distribution is much more optimized than previous distribution.

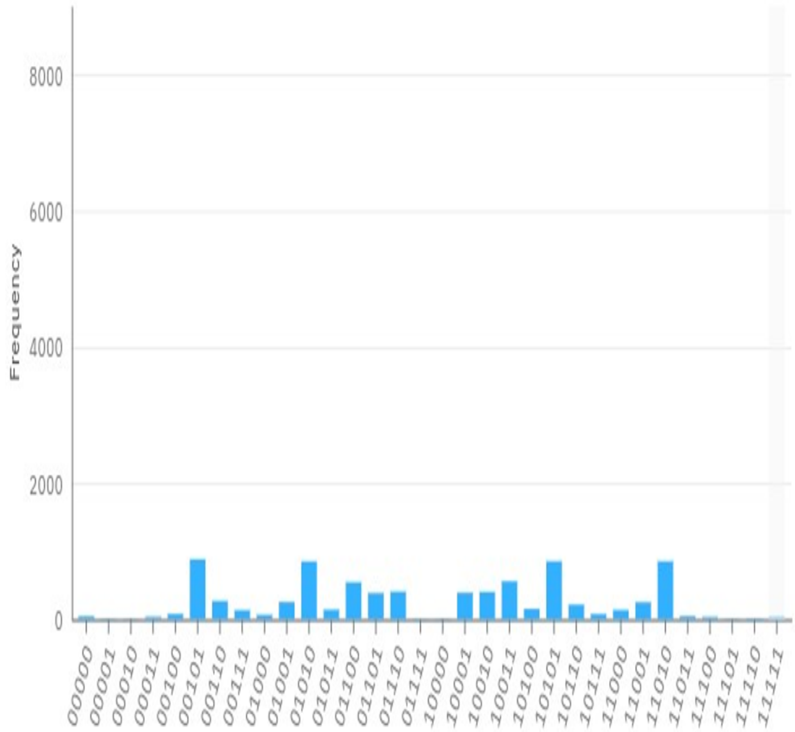

Figure 23: Distribution of states for the 2-vertices graph with random parameters

4.7 The results of the experiments

The tabulated results are the,outcome of 2-vertices and 5 vertices graph's cut numbers. Here for 2 vertices, i got the exact cut 1, QAOA cut 1 and average cut 1. similarly, the outcome of 5 vertices graphs are exact cut 5, QAOA cut 5 and QAOA average cut 4. which is shown in table 1.

Table 1: Experiments results for the 2 and 5-vertices graphs

Number of vertices	Exact max cut	QAOA max cut	QAOA average cut
2	1	1	1
5	5	5	4

4.8 Experimental result and mathematical result

Hence, the result,which obtained from quantum simulator, with optimal parameters gives either state 01 or 10 for 2 vertex graph(fig-19), which are the exact solutions for the Max-Cut problem for the 2 vertex graph. These results match with calculated in (3.5), when calculating the probabilities manually, that is mathematical we got symmetry values 0.5 and 0.5 and from experimental,0.523 and 0.477, which are approximately same result. similarly experimental results or the 5 vertex graph are shown in fig-22. There is also a relatively high probability to receive states 00110 and 11001, for which the maximum cut value is 4. It is easy to verify that the lower is a probability to receive a state — the lower is a cut size value for it.

5 Conclusion and Remarks

5.1 Conclusion

This thesis has investigated the Quantum Approximation Optimization
Algorithm for solving the unweighted 2-MaxCut problem. I showed in detail how it theoretically works on the 2-vertices graph. For this graph and executed it on the quantum computer simulator. The results received, coincide with our calculations. In this work, I also held experiments for the 5 vertices graphs. For all the graphs, the maximum cut size received after running the QAOA is the exact solution for the MaxCut problem. I got the max cut for 2 vertices based graph is 1 and max-cut of 5 vertices graph is 5, this means after increment of vertices cut also increases. This is a satisfactory result for an approximation algorithm, proving that QAOA has great potential in solving the MaxCut problem.

5.2 Novelty and National Prosperity aspect of Project work

This project present the max-cut problem of graph having vertices 2 and 5 by using QAOA.This project is revised project which is previously done on relative topic, and present the result accordingly.A part from past work,this project presents max-cut results in probability and bitstings, also project, provides how the increments of vertices reduces the efficiency of QAOA.
In real-world, optimization of graph is used in the internet field, Google maps/ Yahoo maps, social media, web Page searching, City Planning, Traffic Control, Transportation Navigation, Travelling Salesman Problem, GSM mobile phone networks, Map colouring, time table scheduling . further, in upcoming future it is used for the neural system, for robotics and advancement in the medical field.
if we implement this project in advanced ways, then we can get the a lot development in context of AI(Artificial Intelligence) and which is going to be helpful for all over the world for advanced life.

5.3 Limitation of the work

The major limitation of this project is the computational unavailability, which i had faced and took a long time for simple calculation. i have done all circuitary work in IBM QUANTUM SIMULATOR, which can provide Qiskit based codes and QASM based codes, but if we put wrong code or wrong orientation of gates provides the wrong results[9].

5.4 Recommendation for the further work

we can further advanced the project to next level, by optimizing 10 ,20 ,60 vertices based graphs. we can do AI based projects to solve the deep leaning, neural systems. in neural network max-cut graphs can be implemented as, to classify information, cluster data.[3]

References

[1] Adriano Barenco, Artur Ekert, Kalle-Antti Suominen, and Päivi Törmä. Approximate quantum fourier transform and decoherence. *Physical Review A*, 54(1):139, 1996.

[2] Barry A Cipra. An introduction to the ising model. *The American Mathematical Monthly*, 94(10):937–959, 1987.

[3] Franklin de Lima Marquezino, Renato Portugal, and Carlile Lavor. Bits and qubits. In *A Primer on Quantum Computing*, pages 7–34. Springer, 2019.

[4] Michel X Goemans and David P Williamson. Improved approximation algorithms for maximum cut and satisfiability problems using semidefinite programming. *Journal of the ACM (JACM)*, 42(6):1115–1145, 1995.

[5] Rebekah Herrman, Lorna Treffert, James Ostrowski, Phillip C Lotshaw, Travis S Humble, and George Siopsis. Impact of graph structures for qaoa on maxcut. *Quantum Information Processing*, 20(9):1–21, 2021.

[6] S Homer and M Peinado. Two distributed memory parallel approximation algorithms for max-cut. *J. of Parallel and Distributed Computing*, 1:48–61, 1997.

[7] Claire Mathieu and Warren Schudy. Yet another algorithm for dense max cut: go greedy. In *Proceedings of the nineteenth annual ACM-SIAM symposium on Discrete algorithms*, pages 176–182. Citeseer, 2008.

[8] Chandralekha Singh. Helping students learn quantum mechanics for quantum computing. In *AIP Conference Proceedings*, volume 883, pages 42–45. American Institute of Physics, 2007.

[9] Andrew Steane. Quantum computing. *Reports on Progress in Physics*, 61(2):117, 1998.

[10] Robert S Sutor. *Dancing with Qubits: How quantum computing works and how it can change the world.* Packt Publishing Ltd, 2019.

[11] Sophia Zhyrovetska. Solving maxcut problem with quantunum approximate optimization algorithm. 2021.