Chapter-4
PROPERTIES AND SYNTHESIS OF PASSIVE NETWORKS

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The Driving-Point Function of a Passive Oneport - The Positive Real Function (PRF)

- the driving-point impedance or admittance of a passive network (networks containing resistors, inductors, capacitors, mutual inductances, and ideal transformers) must be *positive real.*
- In addition to being a real rational function, a function F(s) is positive real if

(1) $F(s)$ is real if s is real.

(2) $\mathrm{Re}[F(s)] \geq 0$ if $\mathrm{Re}[s] \geq 0$.

❑ properties of positive-real function:

(1) The reciprocal of a positive-real function is another positive-real function.

(2) $F(s)$ can have no right-halfplane poles or zeros.

(3) $\mathrm{Re}[F(j\omega)] \geq 0$.

(4) If $F(s)$ has a pole on the j axis, it must be simple with a real and positive residue. [The residue in a pole at s_i is the coefficient k_i in the term $k_i/(s - s_i)$ of the partial-fraction expansion of $F(s)$].

(5) The Sum of PRF is also PRF but the difference my not be PRF

Check whether the following can be realized or not

$$F(s) = Z(S) = \frac{2s^2+5}{s(s^2+1)}$$

Here,

$$\frac{2(s^2+5)}{s(s^2+1)} = \frac{K_1}{s} + \frac{K_2 s}{s^2+1}$$

$$K_1 = \lim_{s \to 0} \frac{2s^2+5}{(s^2+1)} = 5 \text{ (+ve \& real)}$$

$$K_2 = \lim_{s^2=-1} \frac{2s^2+5}{s} = -\frac{\sqrt{3}}{1} \text{ (-ve \& not real)}$$

Hence $F(s)$ is not PRF.

Check whether the following can be realized or not

$$Z(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}$$

$$= \frac{s^2+6s+8}{s^2+4s+3} = 1 + \frac{2s+5}{s^2+4s+3}$$

$$= 1 + \frac{k_1}{(s+1)} + \frac{k_2}{(s+3)}$$

$$k_1 = \lim_{s \to -1} \frac{2s+5}{s+3} = \frac{3}{2} \quad (+\text{ve real})$$

$$k_2 = \lim_{s \to -3} \frac{2s+5}{(s+1)} = \frac{1}{2} \quad (+\text{ve real})$$

Hence $F(s)$ is PRF.

Driving-Point Function of a Lossless Oneport - The Lossless Function

When a network contains only lossless elements, its driving-point function is a limiting case of the positive-real function - it is a positive-real function for which

$$\mathrm{Re}[F(j\omega)] \equiv 0$$

We shall call these functions *lossless functions.* This term should be thought of as a contraction of *the driving-point function of a lossless network.*

Properties of a lossless function

Each of the following is a necessary condition for a real rational function

$$F(s) = \frac{P(s)}{Q(s)}$$

where $P(s)$ is the numerator polynomial and $Q(s)$ is the denominator polynomial, to be a lossless function [Gu].

Properties

(1) Between $P(s)$ and $Q(s)$, one must be an even polynomial and the other must be an odd polynomial.

i.e. the highest and lowest degree of polynomial at numerator and denominator must not vary by unity.

(2) All poles and zeros lie on the j axis, they are simple, and they alternate.[2]

Their residues must be real and positive because of the positive real conditions mentioned in the previous section.

(3) It must have either a pole or a zero at $s = 0$. Also at $s = \infty$.

(4) $|\deg[P(s)] - \deg[Q(s)]| \equiv 1$.

(5) $P(s)$ and $Q(s)$ have either all even-powered terms or all odd-powered terms, and no intermediate terms may be missing.

(6) The reciprocal of a lossless function is another lossless function.

(7) $P(s) + Q(s)$ is a Hurwitz polynomial - all its zeros lie in the left halfplane.

(8) On the j axis, $F(j\omega) = jX(\omega)$ is purely imaginary. The function $X(\omega)$ is either a reactance function or a susceptance function.[3] It can be shown that

$$\frac{dX(\omega)}{d\omega} > 0$$

Table : Examples of lossless functions

Type	Lossless functions	At $s = 0$	At $s = \infty$
(a)	$\dfrac{(s^2+1)(s^2+3)(s^2+5)}{s(s^2+2)(s^2+4)}$	pole	pole
(b)	$\dfrac{(s^2+2)(s^2+5)}{s(s^2+3)(s^2+7)}$	pole	zero
(c)	$\dfrac{s(s^2+6)(s^2+20)}{(s^2+3)(s^2+9)}$	zero	pole
(d)	$\dfrac{s(s^2+8)(s^2+20)}{(s^2+4)(s^2+10)(s^2+30)}$	zero	zero

Find are these lossless or not?

1 $Z(s) = \dfrac{K(s^2+1)(s^2+5)}{(s^2+2)(s^2+4)}$ → not coz ratio not even to odd or vice versa.

2 $Z(s) = \dfrac{s^5+4s^3+5s}{4s^4+8s^2}$ → No, coz multiple poles at origin.

$= \dfrac{s^4+4s^2+5}{4s^3+8s}$ zeros has real part.

3 $Z(s) = \dfrac{5s(s^2+4)}{(s^2+1)(s^2+3)}$ → Not, [poles zeros do not alternate]

4 $Z(s) = \dfrac{2s^4+20s^2+18}{s^3+4s}$

$= \dfrac{2(s^2+1)(s^2+9)}{s(s^2+4)}$ ⇒ yes.

Do you Know?

$$X_L = 2\pi f L = Ls$$ ← Inductor

$$X_C = \frac{1}{2\pi f C} = \frac{1}{Cs}$$ ← Capacitor

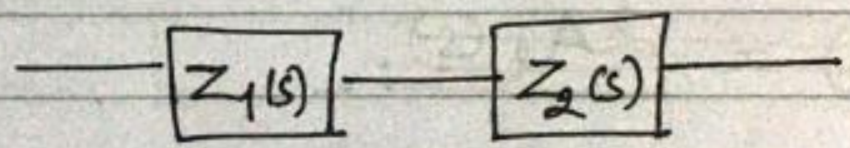
$$Z(s) = \underline{Z_1(s)} + \underline{Z_2(s)}.$$

<u>if</u>

$Z_1(s) = Ls \leftarrow$ Represents inductor

$Z_2(s) = \frac{1}{Cs} \leftarrow$ Represents Capacitor.

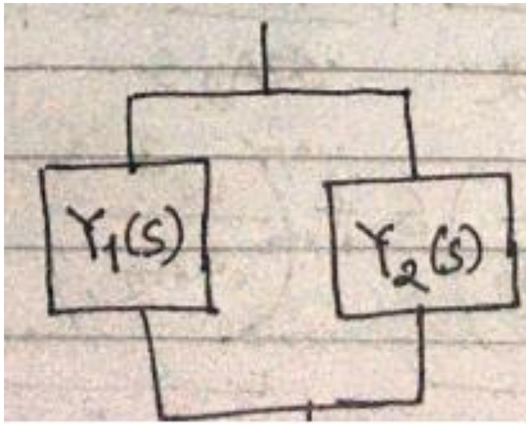
So fig,



$Z_1(s)$ & $Z_2(s)$ are in series.

Do you Know?

$$Y(s) = Y_1(s) + Y_2(s)$$



$Y_1(s)$ & $Y_2(s)$ are in parallel.

$Y_1(s) = KS$ ← It represents Capacitor and it's value is K

$Y_2(s) = A_0/S$ ← It represents Inductor & it's value is $L = \frac{1}{A_0}$

Do you Know?

$$Z(s) = \frac{2A_2 s}{s^2 + \omega^2} \quad \text{or,}$$

then,

$$Y(s) = \frac{s^2 + w^2}{2A_2 s}$$

$$= \frac{s}{2A_2} + \frac{w^2}{2A_2 s} = Y_1(s) + Y_2(s)$$

$$= \underbrace{\left(\frac{1}{2A_2}\right)}_{\text{Capacitor}} s + \underbrace{\left(\frac{w^2}{2A_2}\right) \frac{1}{s}}_{\text{Inductor}}$$

$$\frac{1}{2A_2} \qquad\qquad \frac{2A_2}{w^2} \text{ as } L = \frac{1}{A_0}$$

As $C = K$

Here these L and C will be in parallel form

Do you Know?

$$Y(s) = \frac{2A_2 s}{s^2 + w^2}$$

then

$$Z(s) = \frac{s^2 + w^2}{2A_2 s}$$

$$= \frac{s}{2A_2} + \frac{w^2}{2A_2 s}$$

$$= \underbrace{\left(\frac{1}{2A_2}\right) s}_{\text{Inductor}} + \underbrace{\left(\frac{w^2}{2A_2}\right) \frac{1}{s}}_{\text{Capacitor}}$$

$$L = \frac{1}{2A_2} \qquad = \frac{2A_2}{w^2}$$

Here these L and C will be in Series form

Lets See an Example

The driving point impedance of a n/w is given by

$$z(s) = \frac{s^3+4s}{s^2+2}$$ Realize the n/w.

Soln:-

step1 Since the degree of Numerator is one higher than denominator it is evident that the z(s) pole at $s=\infty$ indicating presence of series inductance.

$$s^2+2)\ s^3+4s\ (s$$
$$\underline{s^3+2s}$$
$$2s$$

$$\therefore z(s) = s + \frac{2s}{s^2+2} = z_1(s) + z_2(s).$$

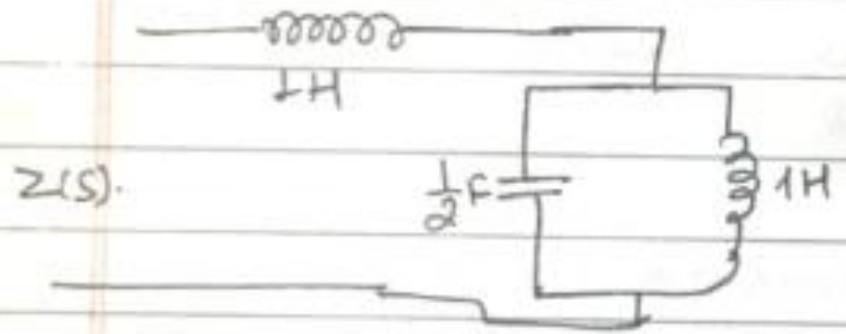
Thus $z_1(s) = s$ [1·s] indicates that the Series inductance will have the value 1H.

Step 2 Since $Z_2(s) = \frac{2s}{s^2+2} \Rightarrow Y_2(s) = \frac{s^2+2}{2s}$

Presence of pole at $s=\infty$ is evident as the degree of Numerator is still one higher. For admittance function $Y_2(s)$, presence of pole at $s=\infty$ indicates a parallel capacitance whose value can be determined by breaking

$$Y_2(s) = \frac{s}{2} + \frac{1}{s} \Rightarrow Y_3(s) + Y_4(s).$$

$Y_3(s) = \frac{1}{2}s$ indicates value of capacitance to be $\frac{1}{2}$ F in parallel to $Y_4(s) = \frac{1}{s}$ which is evidently an inductor of L = 1H.



LC Network Synthesis

Any LC network can be synthesized in the form of

1. Foster (I and II)form
2. Cauer (I and II)form

Foster's expansion of a lossless function

Foster I

$$F(s) = Z(s)$$

$$= \frac{A_0}{s} + Hs + \frac{2A_2 s}{\omega_2^2 + s^2} + \frac{2A_4 s}{\omega_4^2 + s^2} + \cdots$$

$$= Z_1(s) + Z_2(s) + Z_3(s) + \cdots$$

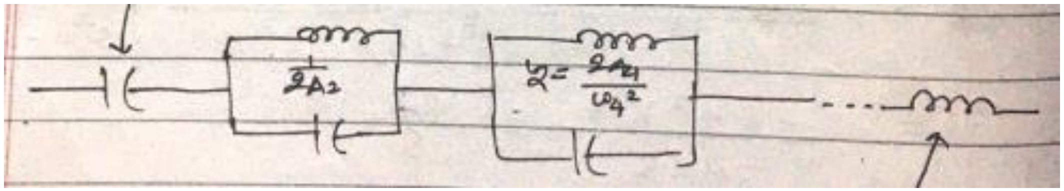
Here,

$Z_1(s) = \frac{A_0}{s} = \frac{1}{A_0 s}$ → Represents Capacitor, $C_0 = \frac{1}{A}$

$Z_2(s) = Hs$ → represents inductor $L_2 = H$

$Z_3(s) = \frac{2A_2 s}{s^2 + \omega_2^2}$ or $Y(s) = \frac{s^2 + \omega_2^2}{2A_2 s}$.

$$= \underbrace{s/2A_2}_{\text{Capacitor}} + \underbrace{\frac{\omega_2^2}{2A_2 s}}_{\text{Inductor}}$$



Foster II

$$F(s) = Y(s) = \frac{H(s^2+\omega_1^2)(s^2+\omega_3^2)}{s(s^2+\omega_2^2)(s^2+\omega_4^2)}$$

$$Y(s) = \frac{A_0}{s} + H(s) + \frac{2A_2 s}{s^2+\omega_2^2} + \frac{2A_4 s}{s^2+\omega_4^2} + \cdots$$

$$= Y_1(s) + Y_2(s) + Y_3(s) + \cdots$$

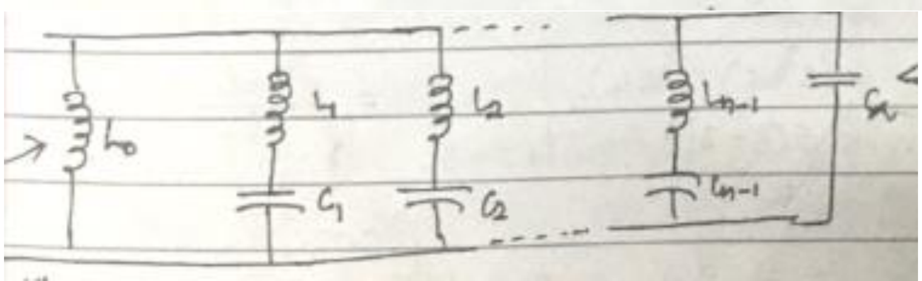
Here,

$$Y_1(s) = \frac{A_0}{s} \Rightarrow L_0 = \frac{1}{A_0}$$

$$Y_2(s) = H(s) \Rightarrow C_n = H$$

$$Y_3(s) = \frac{2A_2 s}{s^2+\omega_2^2} \Rightarrow Z_3(s) = \frac{s^2+\omega_2^2}{2A_2 s}$$

$$= \frac{s}{2A_2} + \frac{\omega_2^2}{2A_2 s}$$



Foster’s expansion of a lossless function

IF it is Z(s) it is foster I and if it is Y(s) then it is foster II, when it is F(s) then you must do both if possible

A lossless function written in partial-fraction form will be

$$F(s) = \frac{k_0}{s} + k_\infty s + \sum_i \left[\frac{c_i}{s + j\omega_i} + \frac{c_i}{s - j\omega_i}\right]$$

It is more convenient to combine each pair of such terms in above equation will then read

$$F(s) = \frac{k_0}{s} + k_\infty s + \sum_i \frac{k_i s}{s^2 + \omega_i^2}$$

It is clear that

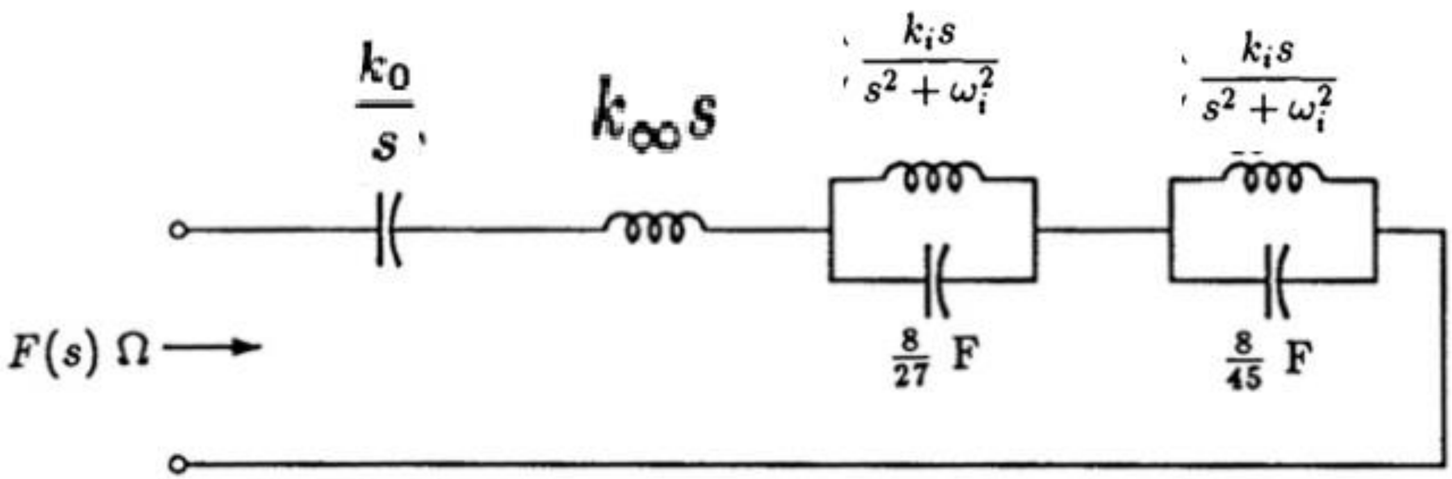
$$k_0 = sF(s)\big|_{s=0}$$

$$k_\infty = \frac{F(s)}{s}\bigg|_{s=\infty}$$

To obtain k_i we multiply by $(s^2+\omega_i^2)/s$ and then set $s^2=-\omega_i^2$ to get

$$k_i = \left[\frac{(s^2+\omega_i^2)}{s}F(s)\right]_{s^2=-\omega_i^2}$$

$$F(s) = \frac{k_0}{s} + k_\infty s + \sum_i \frac{k_i s}{s^2+\omega_i^2}$$



EXAMPLE: Obtain the Foster's expansion of

FIRSTLY WE WILL REALIZE TO OBTAIN FOSTER -I

$$F(s) = \frac{(s^2+1)(s^2+5)(s^2+20)}{s(s^2+2)(s^2+10)} = \frac{s^6+26s^4+125s^2+100}{s^5+12s^3+20s}$$

SOLUTION Foster's expansion is given by formula $F(s) = \frac{k_0}{s} + k_\infty s + \sum_i \frac{k_i s}{s^2+\omega_i^2}$

$$k_0 = sF(s)\big|_{s=0} \qquad k_0 = \frac{1\times 5\times 20}{2\times 10} = 5$$

$$k_\infty = \frac{F(s)}{s}\bigg|_{s=\infty} \qquad k_\infty = 1$$

$$k_i = \left[\frac{(s^2+\omega_i^2)}{s}F(s)\right]_{s^2=-\omega_i^2}$$

$$k_1 = \frac{(s^2+1)(s^2+5)(s^2+20)}{s^2(s^2+10)}\bigg|_{s^2=-2} = \frac{(-1)(3)(18)}{(-2)(8)} = \frac{27}{8}$$

$$k_2 = \frac{(s^2+1)(s^2+5)(s^2+20)}{s^2(s^2+2)}\bigg|_{s^2=-10} = \frac{(-9)(-5)(10)}{(-10)(-8)} = \frac{45}{8}$$

Hence.

$$F(s) = \frac{5}{s} + s + \frac{\frac{27}{8}s}{s^2+2} + \frac{\frac{45}{8}s}{s^2+10}$$

Further $$F(s) = \frac{5}{s} + s + \frac{\frac{27}{8}s}{s^2+2} + \frac{\frac{45}{8}s}{s^2+10}$$

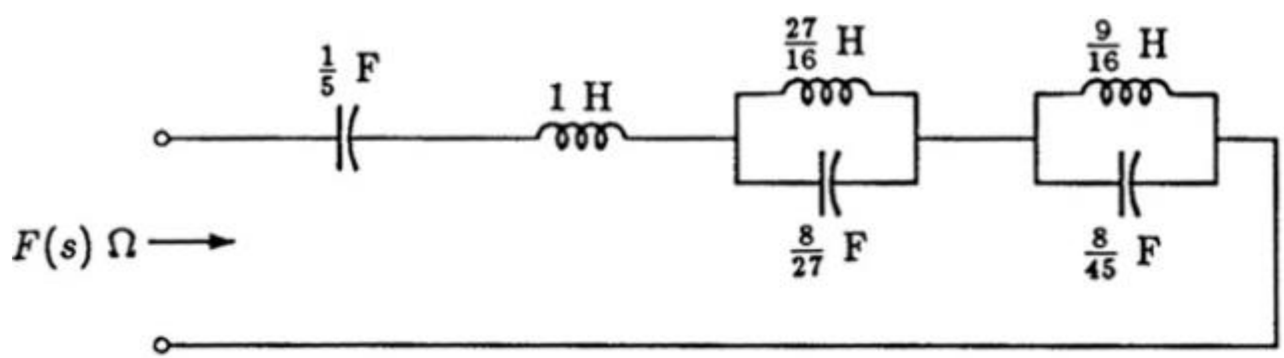
the first term is an impedance of 5/s ohms, which is the impedance of a capacitor of 1/5 farad. The second term is an impedance of 1s ohms, which is the impedance of an inductance of 1 henry. The third term corresponds to a circuit of

$$\frac{\frac{27}{8}s}{s^2+2}\ \Omega \qquad \text{or} \qquad \frac{8}{27}s + \frac{16}{27s}\ \mho$$

which may be identified with the parallel combination of an $\frac{8}{27}$-F capacitor and a $\frac{27}{16}$-H inductor.

The fourth term corresponds to a circuit of $$\frac{\frac{45}{8}s}{s^2+10}\ \Omega \qquad \text{or} \qquad \frac{8}{45}s + \frac{16}{9s}\ \mho$$

which corresponds to the parallel combination of an $\frac{8}{45}$-F capacitor and a $\frac{9}{16}$-H inductor.



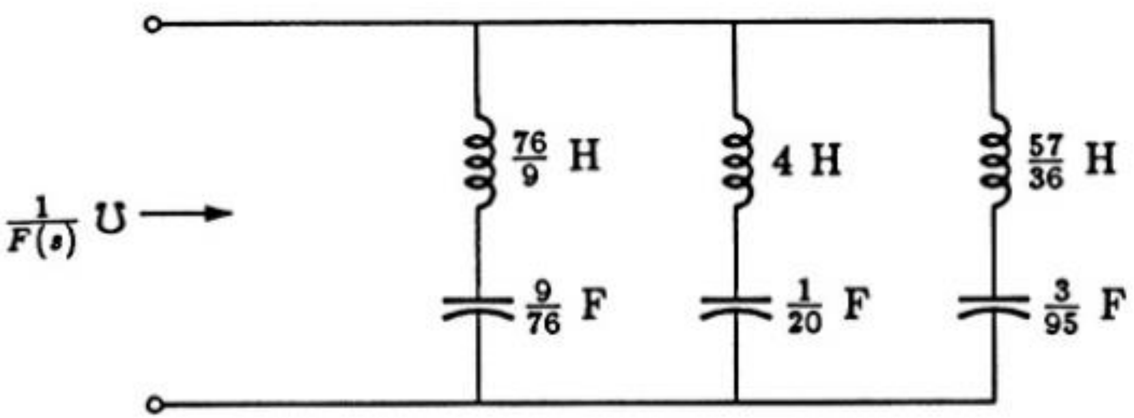
Alternatively, we may take the reciprocals of and realize it as an admittance (FOSTER -II). We have

$$Y(s) = \frac{1}{F(s)} = \frac{\frac{9}{76}s}{s^2+1} + \frac{\frac{1}{4}s}{s^2+5} + \frac{\frac{12}{19}s}{s^2+20} \ \mho$$

Each term can be identified as a series LC branch. For instance, the first term is equal to

$$\frac{\frac{9}{76}s}{s^2+1} \ \mho \qquad \text{or} \qquad \frac{76}{9}s + \frac{76}{9s} \ \Omega$$

which is a 76/9 inductor in series with a 9/76 capacitor. Similar interpretation of the other two terms and the parallel combination of the three branches leads to the network



Removal of poles at infinity

If a lossless function F(s) has a pole at infinity, the pole can be removed by subtraction. The remainder is

$$F_1(s) = F(s) - k_\infty s$$

The order of F1(s) is one lower than that of F(s). F1(s) is another lossless function that does not have a pole at infinity.

For example, if

$$Z(s) = \frac{(s^2+4)(s^2+16)}{s(s^2+9)}\ \Omega$$

then

$$Z_1(s) = Z(s) - s = \frac{11s^2+64}{s(s^2+9)}\ \Omega$$

Since $Z_1(s)$ has no pole at infinity, it must have a zero there. Hence $Y_1(s) = 1/Z_1(s)$ has a pole there. We can remove the pole there again.

$$Y_2(s) = Y_1(s) - \frac{1}{11}s = \frac{\frac{35}{121}s}{s^2 + \frac{64}{11}} \ \mho$$

We can again remove the pole at infinity from $1/Y_2(s)$ to get

$$Z_3(s) = \frac{1}{Y_2(s)} - \frac{121}{35}s = \frac{704}{35s} \ \Omega$$

It can be observed that each removal of a pole at infinity is really the reduction of an improper fraction into a proper fraction. For example, this above equation may be written as

$$\frac{s^4 + 20s^2 + 64}{s^3 + 9s} = s + \frac{11s^2 + 64}{s^3 + 9s}$$

which can also be accomplished by long division. The entire sequence of steps can be duplicated by continued long division as shown below.

$$
\begin{array}{r|l}
 & s\ \Omega \\
\hline
s^3+9s & s^4+20s^2+64 \\
 & s^4+9s^2 \\
\hline
 & 11s^2+64
\end{array}
\qquad
\begin{array}{r|l}
 & \frac{1}{11}s\ \mho \\
\hline
11s^2+64 & s^3+9s \\
 & s^3+\frac{64}{11}s \\
\hline
 & \frac{35}{11}s
\end{array}
\qquad
\begin{array}{r|l}
 & \frac{121}{35}s\ \Omega \\
\hline
\frac{35}{11}s & 11s^2+64 \\
 & 11s^2 \\
\hline
 & 64
\end{array}
\qquad
\begin{array}{r|l}
 & \frac{35}{704}s\ \mho \\
\hline
64 & \frac{35}{11}s \\
 & \frac{35}{11}s
\end{array}
$$

Alternatively, we may use a continued fraction to summarize the above results, namely

$$
F(s) = s + \cfrac{1}{\cfrac{1}{11}s + \cfrac{1}{\cfrac{121}{35}s + \cfrac{1}{\cfrac{35}{704}s}}}
$$

The realization by continued removal of poles at infinity is known as the ***Cauer 1 realization.***

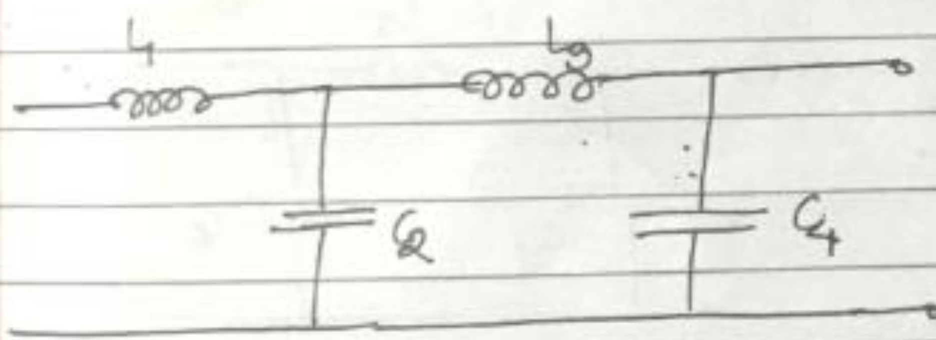
Cauer form (continue fraction form)

Let $Z(s) = \frac{M(s)}{N(s)}$

1) If order of M is greater than N $(M > N)$

$$Z(s) = Z_1(s) + \cfrac{1}{Y_2(s) + \cfrac{1}{Z_3(s) + \cfrac{1}{Y_4(s) + \cdots}}}$$

$$= L_1(s) + \cfrac{1}{C_2 s + \cfrac{1}{L_3 s + \cfrac{1}{C_4 s + \text{—}}}}$$

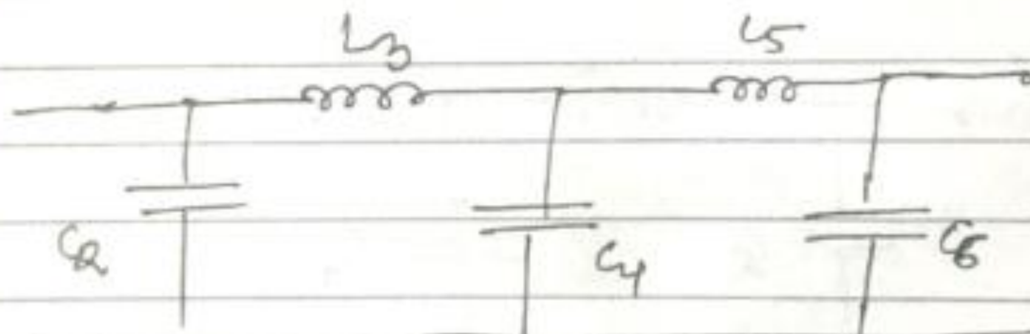


Causer first form.

If $m<n$, first invert $z(s)$ then proceed.
(Removal of pole at zero).

$$z(s) = \cfrac{1}{y_2(s) + \cfrac{1}{z_3(s) + \cfrac{1}{y_4(s) + \cfrac{1}{z_5(s) + \cfrac{1}{y_6(s) + \cdots}}}}}$$

$$= \cfrac{1}{C_2 s + \cfrac{1}{L_3 s + \cfrac{1}{C_4 s + \cfrac{1}{L_5 s + \cdots}}}}$$



A driving point impedance is given by

$$z(s) = \frac{s(s^2+4)(s^2+6)}{(s^2+1)(s^2+5)}$$

obtain the first form of cauer network.

$$z(s) = \frac{s^5 + 10s^3 + 24s}{s^4 + 6s^2 + 5}$$

$$s^4+6s^2+5 \,\big)\, s^5 + 10s^3 + 24s \,\big(\, s$$

$$\underline{s^5 + 6s^3 + 5s}$$

$$4s^3 + 19s \,\big)\, s^4 + 6s^2 + 5 \,\big(\, 5/4$$

$$\underline{s^4 + \tfrac{19}{4}s^2}$$

$$\tfrac{5}{4}s^2 + 5 \,\big)\, 4s^3 + 19s \,\big(\, \tfrac{16}{5}s$$

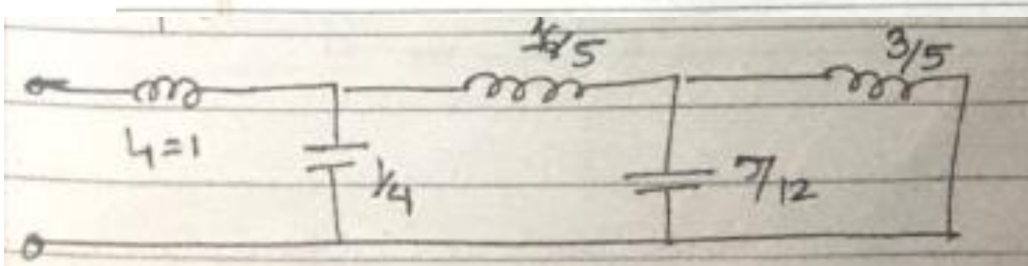
$$\underline{4s^3 + 16s}$$

$$3s \,\big)\, \tfrac{5}{4}s^2 + 5 \,\big(\, \tfrac{5}{12}$$

$$\underline{\tfrac{5}{4}s^2}$$

$$5 \,\big)\, 3s \,\big(\, \tfrac{3}{5}s$$

$$3s$$



Synthesize the following in Cauer-2 form.

$$z(s) = \frac{8s^3 + 10s}{5 + 6s^2 + s^4}$$

Soln:- We should rearrange the function as below to obtain the Cauer 2nd form.

$$z(s) = \frac{10s + 8s^3}{5 + 6s^2 + s^4} = \frac{1}{\frac{5 + 6s^2 + s^4}{10s + 8s^3}}$$

$$10s + 8s^3 \,\big)\, 5 + 6s^2 + s^4 \,\big(\, \frac{1}{2s} \Rightarrow y_2$$

$$5 + 4s^2$$

$$2s^2 + s^4 \,\big)\, 10s + 8s^3 \,\big(\, \frac{5}{s} \Rightarrow z_3 = \frac{1}{Cs}$$

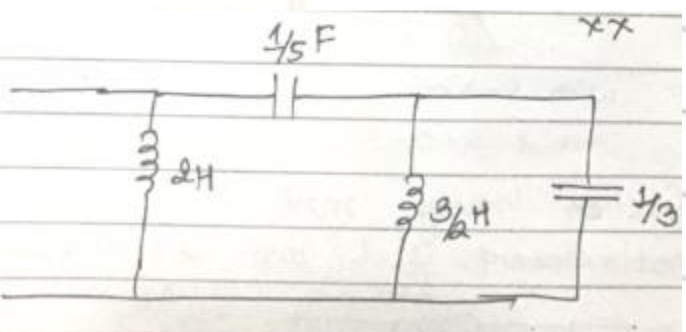
$$10s + 5s^3$$

$$3s^3 \,\big)\, 2s^2 + s^4 \,\big(\, \frac{2}{3s} \Leftarrow y_4$$

$$2s^2$$

$$s^4 \,\big)\, 3s^3 \,\big(\, \frac{3}{s} \Leftarrow z_5$$

$$3s^3$$



Example 3

Find the first form of Cauer network

$$z(s) = \frac{6s^3 + 2s}{9s^4 + 4s^2 + 1/6}$$

Solution:-

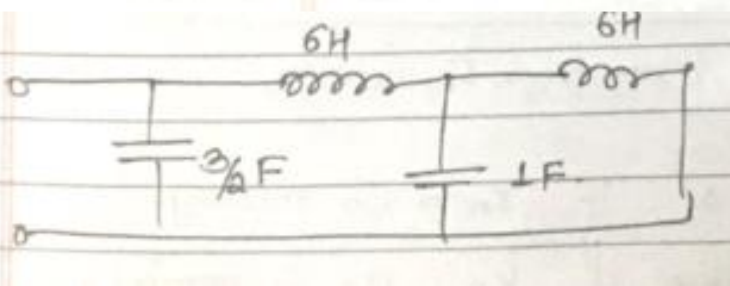
Since the order of polynomial of denominator is higher than the Numerator, Here we invert the f^n and proceed. It may be observed that zero formed at $s \Rightarrow \infty$ and indicates absence of first element.

$$6s^3 + 2s \,)\, 9s^4 + 4s^2 + 1/6 \,(\, 3/2\, s \Leftarrow y_2(s)$$
$$9s^3 + 3s^2$$

$$s^2 + 1/6 \,)\, 6s^3 + 2s \,(\, 6s \Leftarrow z_2(s)$$
$$6s^3 + s$$

$$s \,)\, s^2 + 1/6 \,(\, s \Leftarrow y_4(s)$$
$$s^2$$

$$1/6 \,)\, s \,(\, 6s \Leftarrow z_4(s).$$
$$s$$



Removal of poles at the origin

If a lossless function has a pole at the origin, it can be removed by substraction. The remainder is

$$F_2(s) = F(s) - \frac{k_0}{s}$$

The order of F2(s) is one lower than that of F(s) . F2(s) is another lossless function with no pole at the origin. It must have a zero there. We reciprocate it. The reciprocal must have a pole there. We can remove it by subtraction. we can reciprocate it and remove another pole there.

We continue this process until the remainder is trivial

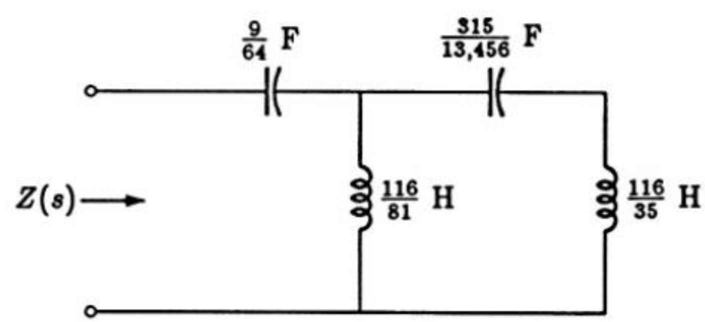
$$Z(s) = \frac{(s^2+4)(s^2+16)}{s(s^2+9)}\ \Omega \qquad = \qquad \frac{s^4+20s^2+64}{s^3+9s}$$

Let's use the impedance function of

$$Z_4(s) = Z(s) - \frac{\frac{64}{9}}{s} = \frac{s\left(s^2+\frac{116}{9}\right)}{s^2+9}$$

$$Y_5(s) = \frac{1}{Z_4(s)} - \frac{81}{116s} = \frac{\frac{35}{116}s}{s^2+\frac{116}{9}}$$

Identifying each removal as the extraction of a network element also results in a network that realizes the given lossless function. The network is realizes the impedance function.



The sequence of removal of poles at the origin can also be effected by long division as illustrated for this example in the following

$$
\begin{array}{rlllll}
 & \frac{64}{9}/s\ \Omega & & & & \\ \cline{2-2}
9s+s^3 & \multicolumn{1}{|l}{64+20s^2+s^4} & & & & \\
 & 64+\frac{64}{9}s^2 & \frac{81}{116}/s\ \mho & & & \\ \cline{2-3}
 & \frac{116}{9}s^2+s^4 & \multicolumn{1}{|l}{9s+s^3} & & & \\
 & & 9s+\frac{81}{116}s^3 & \frac{13456}{315}/s\ \Omega & & \\ \cline{3-4}
 & & \frac{35}{116}s^3 & \multicolumn{1}{|l}{\frac{116}{9}s^2+s^4} & & \\
 & & & \frac{116}{9}s^2 & \frac{35}{116}/s\ \mho & \\ \cline{4-5}
 & & & s^4 & \multicolumn{1}{|l}{\frac{35}{116}s^3} & \\
 & & & & \frac{35}{116}s^3 & \\ \cline{5-5}
\end{array}
$$

We could also use the continued-fraction notation to summarize the results of the long division

$$F(s) = \frac{64}{9s} + \cfrac{1}{\cfrac{81}{116s} + \cfrac{1}{\cfrac{13,456}{315s} + \cfrac{1}{\cfrac{35}{116s}}}}$$

RC Oneport Network

properties of RC

(i) Poles and zeros lies on negative real axis

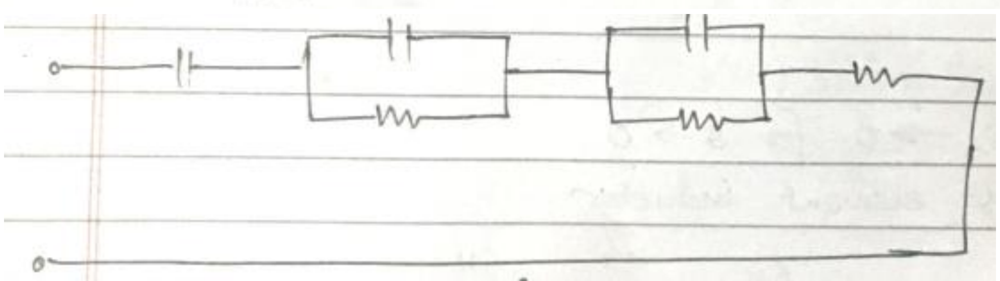
(ii) Poles and zeros alternate on negative real axis.

(iii) The residue of poles must be real and +ve.

(iv) The slope of $z(s)$ along the negetive real axis $(-\sigma)$ is less than or equal to zero.

(v) At two critical frequencies i.e. at $s=0$ & $s=\infty$ $z(s)$ is given by.

$$z(0) = \begin{cases} \infty & \text{When } K_0 \text{ is present} \quad \text{— (I)} \\ \sum_{i=1}^{M} R_i & \text{When } K_0 \text{ is missing} \quad \text{— (II)} \end{cases}$$



Check given T.F. are RC or not.

Date ______
Page ______

$$Z(s) = \frac{(s+2)(s+4)}{(s+1)}$$ not : do not alternate

$$Z(s) = \frac{(s+1)(s+4)(s+9)}{s(s+2)(s+5)}$$ yes.

Foster Series expansion

Foster series expansion of RC n/w is -

$$Z(s) = \frac{A_0}{s} + \frac{A_1}{s+\sigma_1} + \frac{A_2}{s+\sigma_2} + \dots + A_\infty = Z_0 + Z_1 + \dots + Z_\infty$$

$\frac{A_0}{s}$ represents Capacitor of $C_0 = \frac{1}{A_0}$

$Z_i(s) = \frac{A_i}{s+\sigma_i}$ represents Capacitor

$C_i = \frac{1}{A_i}$ in parallel with resistor $R_i = \frac{A_i}{\sigma_i}$

Foster second form :

$$Y(s) = A_0 + \frac{A_1 s}{s+\sigma_1} + \frac{A_2 s}{s+\sigma_2} + \cdots + Hs$$

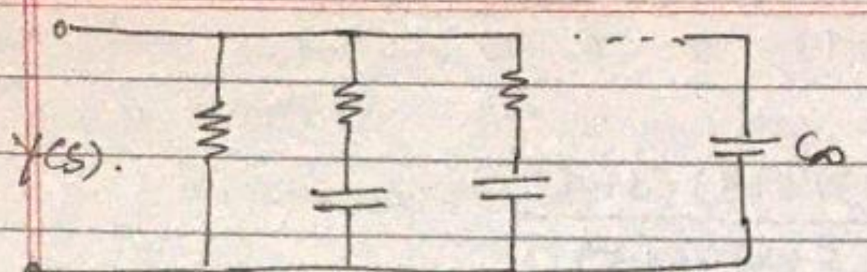
$$= y_0(s) + y_1(s) + y_2(s) + \cdots + y_\infty(s).$$

Here,

$y_0(s) = A_0$ represents Register $R_0 = \frac{1}{A_0}$

$y_i(s) = \frac{A_i s}{s+\sigma_i}$ represent register $R_i = \frac{1}{A_i}$ with Capacitor $C_i = A_i/\sigma_i$

and $y_\infty(s) = Hs$ represents Capacitor $C_\infty = H$



2nd form of foster RC n/w.

Example

An impedance function is given by -

$$Z(s) = \frac{s^2+5s+4}{s^2+2s}$$

Realize in <u>both</u> foster forms.

Solution.

$$Z(s) = \frac{s^2+5s+4}{s^2+2s} = \frac{(s+1)(s+4)}{s(s+2)},$$

$$Z(s) = 1 + \frac{3s+4}{s(s+2)} = 1 + \frac{K_1}{s} + \frac{K_2}{s+2}$$

where,

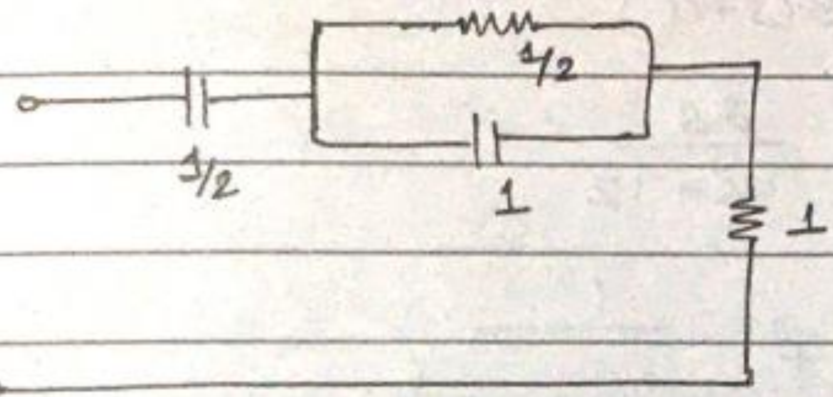
$$K_1 = \left.\frac{3s+4}{s+2}\right|_{s=0} = 2$$

$$K_2 = \left.\frac{3s+4}{s}\right|_{s=-2} = 1$$

Thus $Z(s) = 1 + 2/s + \frac{1}{s+2}$

Here residue are +ve and real in expression $Z(s)$.
zeros are at $-1, -4$, while poles are at $s = 0$ and -2.

thus poles and zeros are alternate.



To obtain foster II invert the given function.

$$Y(s) = \frac{s^2+2s}{s^2+5s+4} = 1 - \frac{3s+4}{(s+1)(s+4)}$$

$$= 1 - \frac{k_1}{s+1} - \frac{k_2}{s+4}$$

Be careful

Since negative coefficient appear, hence we take $y(s)/s$

$$\therefore \frac{y(s)}{s} = \frac{s(s+2)}{s(s+1)(s+4)} = \frac{s+2}{(s+1)(s+4)}$$

$$= \frac{k_1}{s+1} + \frac{k_2}{s+4}$$

$$k_1 = \left.\frac{s+2}{s+4}\right|_{s=-1} = 1/3$$

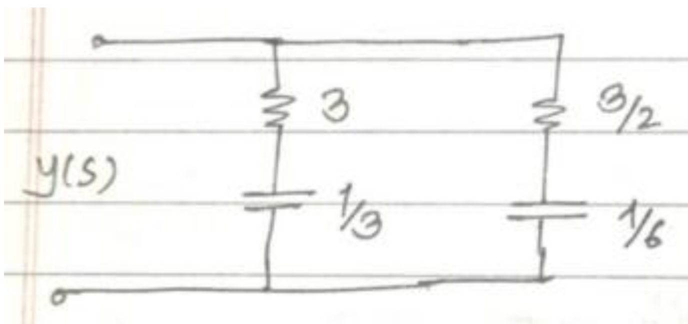
$$k_2 = \left.\frac{s+2}{s+1}\right|_{s=-4} = 2/3$$

$$\therefore \frac{y(s)}{s} = \frac{1/3}{s+1} + \frac{2/3}{s+4}$$

$$y(s) = \frac{s}{3(s+1)} + \frac{2s}{3(s+4)}$$

$$= \frac{s}{3s+3} + \frac{2s}{3s+12}$$

$$= \frac{1}{3 + \frac{1}{\frac{1}{3}s}} + \frac{1}{\frac{3}{2} + \frac{1}{\frac{1}{6}s}}$$



Continue Fraction Method

$$z(s) = \frac{6(s+2)(s+4)}{s^2+3s} = \frac{6s^2+36s+48}{s^2+3s}$$

$$s^2+3s \,\big)\, 6s^2+36s+48 \,\big(\, 6\,\Omega$$

$$6s^2+18s$$

$$18s+48 \,\big)\, s^2+3s \,\big(\, \tfrac{1}{18}s\ \mho$$

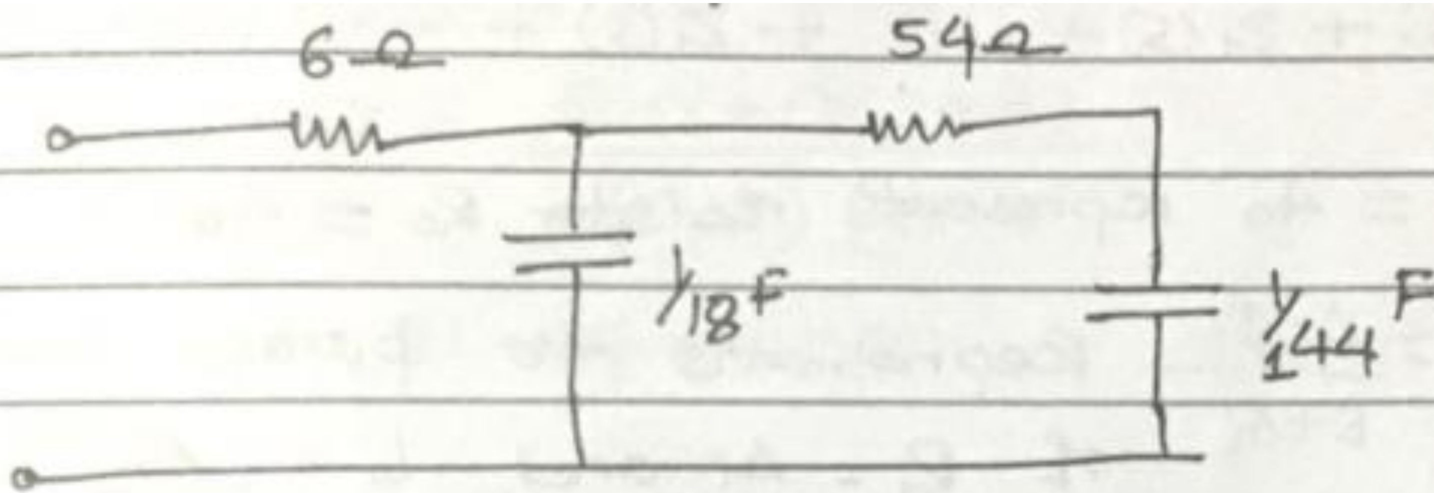
$$s^2+\tfrac{8}{3}s$$

$$\tfrac{1}{3}s \,\big)\, 18s+48 \,\big(\, 54\,\Omega$$

$$-18s$$

$$48 \,\big)\, \tfrac{1}{3}s \,\big(\, \frac{s}{144}$$

$$\tfrac{1}{3}s$$



6Ω
54Ω
1/18 F
1/144 F

RL Network

Proporties of RL impedance function.

1. Poles and zeros of RL impedance function are located on -ve real axis and they alternate.
2. The critical frequency nearest the origin is zero and critical frequency near to $\sigma = \infty$ must be a pole.
3. The residue of poles must be real and negetive for $Z(S)$.

$$Z(S) = \frac{H(S+\sigma_1)(S+\sigma_2)\cdots}{(S+\sigma_3)(S+\sigma_4)\cdots}$$

Foster Expansion of RL network.

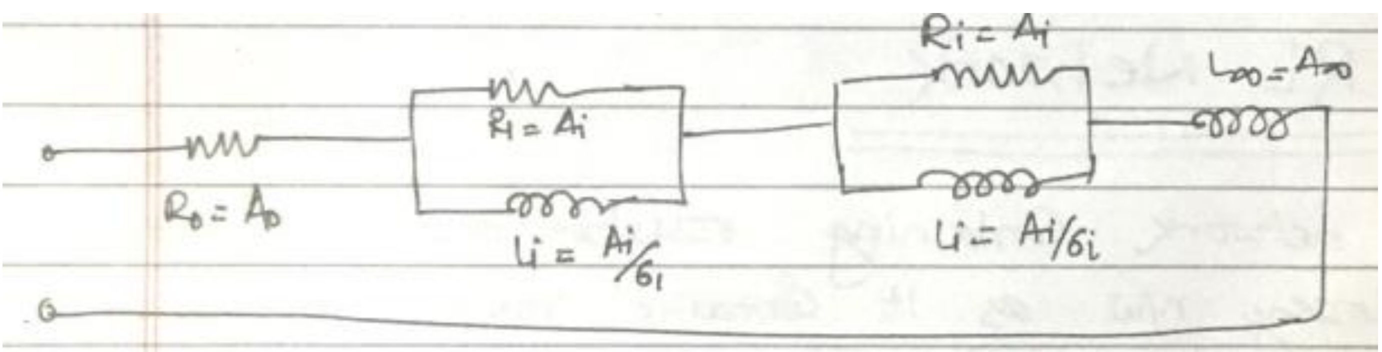
Foster I The impedance of a RL network in Foster's form can be represented as –

$$Z(s) = A_0 + \frac{A_1 s}{s+\sigma_1} + \cdots + \frac{A_i s}{s+\sigma_i} + \cdots + A_\infty s$$

$$= Z_0(s) + Z_1(s) + \cdots + Z_i(s) + \cdots + Z_\infty(s).$$

Here, $Z_0(s) = A_0$ represents resistor $R_0 = A_0$

$Z_i(s) = \dfrac{A_i s}{s+\sigma_i}$ Represents the parallel combination of $R_i = A_i$ and $L_i = \dfrac{A_i}{\sigma_i}$



Foster II form

The driving point admittance of RL n/w is

$$y(s) = \frac{A_0}{s} + \frac{A_1}{s+\sigma_1} + \cdots + \frac{A_i}{s+\sigma_i} + \cdots + H(s)$$

$$= y_0(s) + y_1(s) + \cdots + y_i(s) + \cdots + y_\infty(s).$$

Here, $y_0(s) = \frac{A_0}{s}$ represents Inductor $L_0 = \frac{1}{A_0}$

$y_i(s) = \frac{A_i}{s+\sigma_i}$ represents series combination of inductor $L_i = \frac{1}{A_i}$ and resistor $R_i = \sigma_i / A_i$

$y_\infty(s) = H$ represents $R_\infty = H$.

Example

An impedance function is given by

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+4)}$$

Find the RL representation of foster first form of n/w.

Soln:-

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+4)} = 2 - \frac{1}{s+1} - \frac{3}{s+2}$$

Thus residue are real but negative and poles and zeros alternate on negative real axis.

$$\therefore \frac{Z(s)}{s} = \frac{2(s+1)(s+3)}{s(s+2)(s+4)}$$

$$= \frac{k_1}{s} + \frac{K_2}{s+2} + \frac{k_3}{s+4}$$

$$K_1 = \left.\frac{2(s+1)(s+3)}{(s+2)(s+4)}\right|_{s=0} = 3/4$$

$$K_2 = \left.\frac{2(s+1)(s+3)}{s(s+4)}\right|_{s=-2} = -2/-4 = 1/2$$

$$K_3 = \left.\frac{2(s+1)(s+3)}{s(s+2)}\right|_{s=-4} = \frac{6}{8} = 3/4$$

$$\therefore \frac{z(s)}{s} = \frac{3}{4s} + \frac{1}{2(s+2)} + \frac{3}{4(s+4)}$$

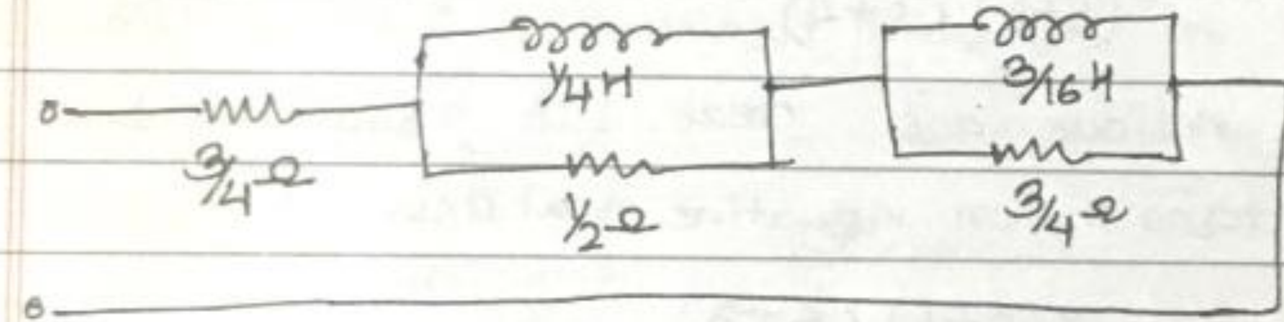
$$\therefore z(s) = \frac{3}{4} + \frac{s}{2(s+2)} + \frac{3s}{4(s+4)}$$

$$A_0 = R_0 = 3/4$$

$$Z(s) = \frac{3}{4} + \frac{1}{2 + \frac{4}{s}} + \frac{1}{\frac{4}{3} + \frac{16}{3s}}$$

$R_1 = \frac{1}{2}\,\Omega$ & $L_1 = \frac{1}{4}\,H$

$R_2 = 3/4\,\Omega$ & $L_2 = 3/16\,H$



Q. Find foster-II of $y(s) = \frac{(s+4)(s+6)}{(s+3)(s+5)}$

Soln:- $y(s) = \frac{(s+4)(s+6)}{(s+3)(s+5)} = \frac{s^2+10s+24}{s^2+8s+15} = 1 + \frac{2s+9}{(s+3)(s+5)}$

$$y(s) = 1 + \frac{A_1}{s+3} + \frac{A_2}{s+5}$$

$\Rightarrow$ $A_1(s+5) + A_2(s+3) = 2s+9$

On solving we get, $A_1 = 3/2$ & $A_2 = 1/2$

$$\therefore Y(s) = 1 + \frac{3}{2(s+3)} + \frac{1}{2(s+5)}$$

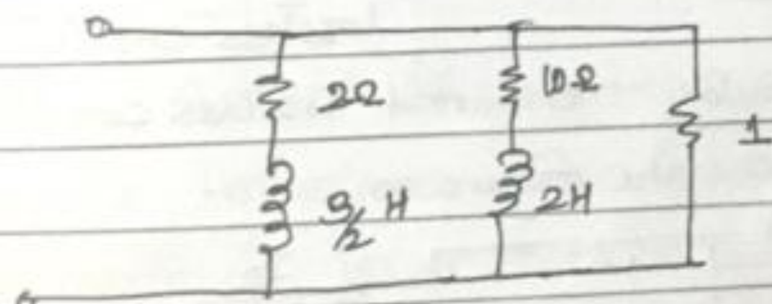
$$= y_1(s) + y_2(s) + y_3(s)$$

Comparing with $Y(s) = \frac{B_0}{s} + \frac{B_1}{s+\sigma_1} + \cdots + H.$

$$y_1(s) = 1 = R_\infty$$

$$y_2(s) = \frac{3}{2(s+3)} = \frac{1}{\frac{2}{3}s + 2}$$ i.e. $R_1 = 2$ & $L_1 = 2/3$

$$y_3(s) = \frac{1}{2(s+5)} = \frac{1}{2s+10}$$ i.e. $R_2 = 10$ $L_2 = 2$



LC ladder with equal termination

Steps:

In doubly terminated form, let $R_1 = R_2 = 1$ (normalized value) Then steps for realization:

1. obtain $|\rho(j\omega)|^2$ form given $T(j\omega)$ or $t(j\omega)$

$$|T|^2 = \frac{1}{1+\omega^{2n}} = \frac{1}{D(s)} \Rightarrow |\rho(j\omega)|^2 = 1 - t(j\omega)^2$$

$$\therefore \rho(s) = \frac{s^n}{D(s)}$$

2. Calculate $Z_{11}(s)$ using equation

$$Z_{11} = \frac{R_1[1+\rho(s)]}{[1-\rho(s)]} = \frac{1+\rho(s)}{1-\rho(s)}.$$

3. Finally realize Z_{11} as a lossless two port network terminated in a resistance.

Question Realize the third-order doubly terminted Butterworth lowpass filter with R1=R2=1ohm.

Solution:-

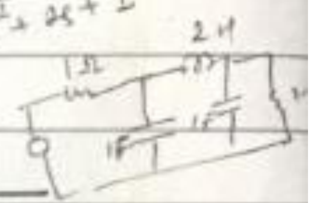
$$T(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} = \frac{1}{D(s)}$$

$$|T(j\omega)|^2 = \frac{1}{1+\omega^6}$$

$$\boxed{\rho(s) = \frac{s^n}{D(s)}}$$

$$\therefore \rho(s) = \frac{s^3}{s^3 + 2s^2 + 2s + 1}$$

$$Y_{in} = \frac{2s^3 + 2s^2 + 2s + 1}{2s^2 + 2s + 1}$$



We have,

$$Z_{11} = \begin{cases} \dfrac{1-\rho(s)}{1+\rho(s)} = \dfrac{2s^2 + 2s + 1}{2s^3 + 2s^2 + 2s + 1} \\[2ex] \dfrac{1+\rho(s)}{1-\rho(s)} = \dfrac{2s^3 + 2s^2 + 2s + 1}{2s^2 + 2s + 1}. \end{cases}$$

$$2s^2+2s+1 \,\big)\, 2s^3+2s^2+2s+1 \,\big(\, 1s \quad (Z_1)$$

$$2s^3+2s^2+s$$

$$s+1 \,\big)\, 2s^2+2s+1 \,\big(\, 2s \quad (Y_1)$$

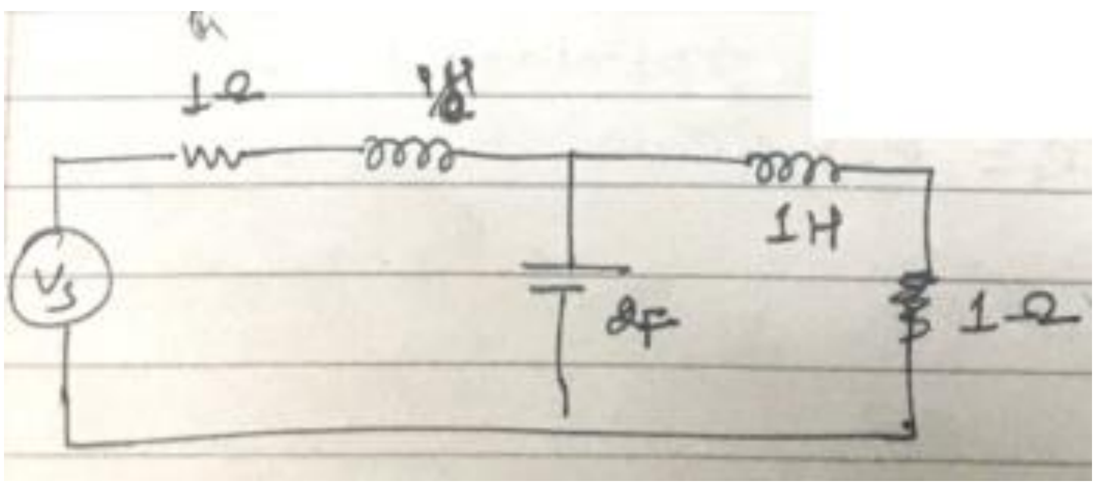
$$2s^2+2s$$

$$1 \,\big)\, s+1 \,\big(\, s \quad (Z_2)$$

$$s$$

$$1 \,\big)\, 1 \,\big(\, 1 \quad (Y_2)$$

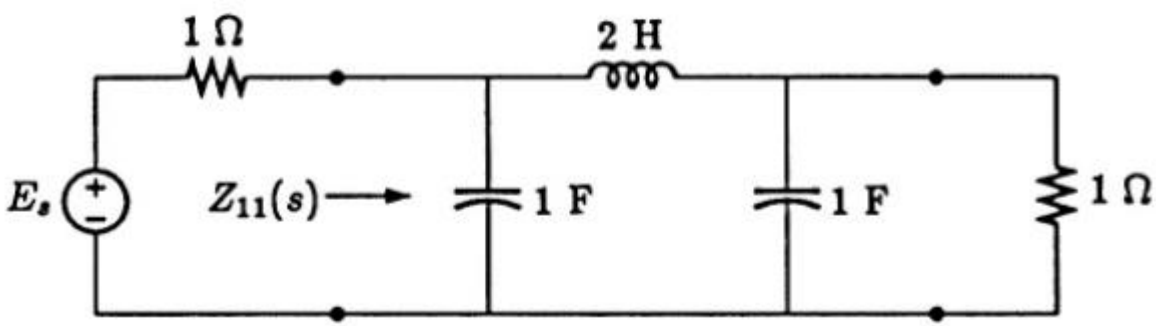
$$1$$



In the same above question If we had chosen the lower signs i.e

$$Z_{11}(s) = \frac{1-\rho(s)}{1+\rho(s)} = \frac{2s^2+2s+1}{2s^3+2s^2+2s+1}$$

The Circuit would be



LC Ladders with Unequal Terminations

EXAMPLE Realize a third-order Butterworth lowpass filter for $R_1 = 1\ \Omega$ and $R_2 = 4\ \Omega$.

SOLUTION

$$t^2(0) = \frac{\dfrac{|E_s|^2 R_2}{(R_1 + R_2)^2}}{\dfrac{|E_s|^2}{4R_1}} = \frac{4R_1R_2}{(R_1 + R_2)^2} \leq 1$$

$$t^2(0) = \frac{4 \times 1 \times 4}{(1+4)^2} = 0.64$$

We let

$$|t(j\omega)|^2 = \frac{0.64}{1+\omega^6}$$

And

$$|\rho(j\omega)|^2 = 1 - |t(j\omega)|^2$$

$$= 1 - \frac{\omega^6}{1+\omega^6}$$

$$= \frac{\omega^6 + 0.36}{\omega^6 + 1}$$

The Third order butterworth filter is

$$\rho(s) = \frac{s^3}{s^3 + 2s^2 + 2s + 1}$$

$$Z_{11}(s) = \frac{1 + \rho(s)}{1 - \rho(s)}$$

We can make

$$\rho(s) = \frac{s^3 + 0.6}{s^3 + 2s^2 + 2s + 1}$$

and choose

$$Z_{11} = \frac{1 + \rho(s)}{1 - \rho(s)} = \frac{2s^3 + 2s^2 + 2s + 1.6}{2s^2 + 2s + 0.4}$$

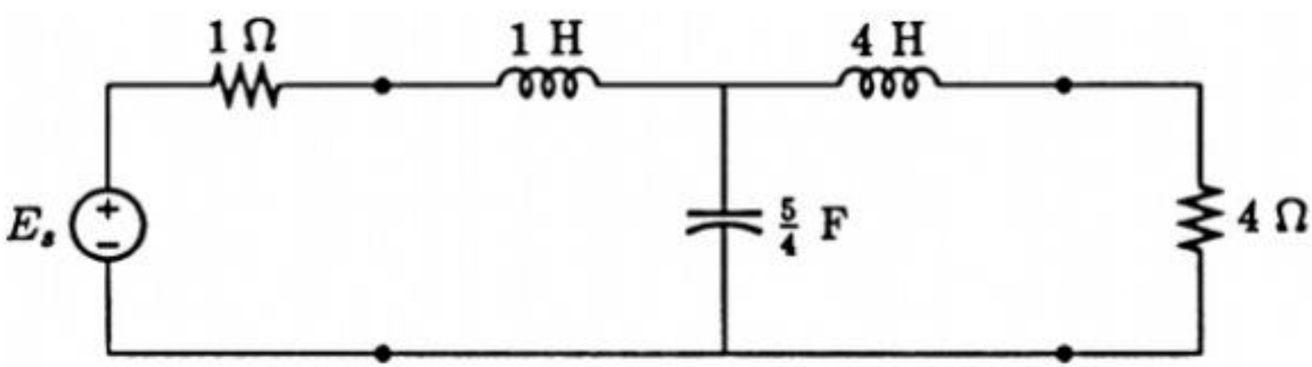
Applying Foster’s preamble results in the following long division

$$
\begin{array}{r|l}
 & s\ \Omega \\
\hline
2s^2+2s+0.4 & 2s^3+2s^2+2s+1.6 \\
 & 2s^3+2s^2+0.4s \\
\hline
 & 1.6s+1.6
\end{array}
$$

$$
\begin{array}{r|l}
 & \tfrac{5}{4}s\ \mho \\
\hline
1.6s+1.6 & 2s^2+2s+0.4 \\
 & 2s^2+2s \\
\hline
 & 0.4
\end{array}
$$

$$
\begin{array}{r|l}
 & 4s+4\ \Omega \\
\hline
0.4 & 1.6s+1.6 \\
 & 1.6s+1.6 \\
\hline
\end{array}
$$

Gathering the results of the long division, we obtain the circuit



Transmission zeros at the origin and infinity

Example: Realize the third-order Butterworth lowpass filter for the singly-terminated arrangement

SOLUTION : **We have** $$Z_{21}(s) = \frac{K}{s^3 + 2s^2 + 2s + 1}$$

Since the numerator is even, we divide the numerator and the denominator by the odd part of the denominator and write

$$Z_{21}(s) = \frac{\dfrac{K}{s^3 + 2s}}{1 + \dfrac{2s^2 + 1}{s^3 + 2s}}$$

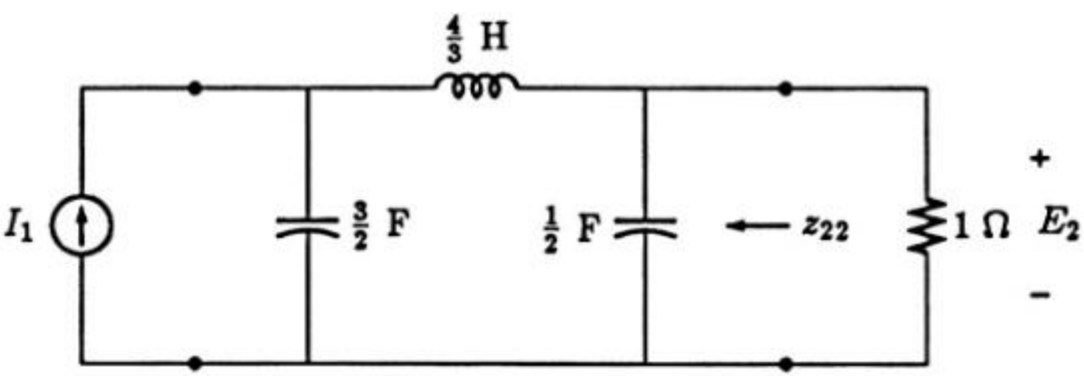
There are three transmission zeros, all at s=α Since each Cauer 1 step removes a pole at infinity completely, we apply Cauer 1 steps three times to

$$z_{22} = \frac{2s^2 + 1}{s^3 + 2s}$$

We now perform the long division

$$
\begin{array}{r|l}
 & \tfrac{1}{2}s\ \mho \\
\hline
2s^2+1 & s^3+2s \\
 & s^3+\tfrac{1}{2}s \\
\hline
\end{array}
\qquad
\begin{array}{r|l}
 & \tfrac{4}{3}s\ \Omega \\
\hline
\tfrac{3}{2}s & 2s^2+1 \\
 & 2s^2 \\
\hline
\end{array}
\qquad
\begin{array}{r|l}
 & \tfrac{3}{2}s\ \mho \\
\hline
1 & \tfrac{3}{2}s \\
 & \tfrac{3}{2}s \\
\hline
\end{array}
$$

and the network of Fig. is obtained



To evaluate K, we see that at $s = 0$,

$$\frac{E_2}{I_1} = 1 = Z_{21}(0) = K$$

Hence $K = 1$.

Thank you
End of Chapter-4