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2. For a positive real number L, consider an arc–length parametrized smooth curve $\alpha : [0, L] \to \mathbb{R}^3$. You are given that α is without singular points of orders 0 and 1. If the the torsion of α is identically zero, show that the trace of α is contained in a plane.

$|\alpha'(t)| = 1 \quad ; \quad \alpha(t) \neq 0 \quad ; \quad \alpha'(t) \neq 0$

to mai kya kru bhai ?? 1 1 1 1

$\cancel{\text{Torsion}(\alpha) = 0 \Rightarrow \|\alpha''(t)\| = 0}$

$\Rightarrow \cancel{\alpha''(t) =}$

$\text{Torsion}(\alpha) = 0 \Rightarrow \|\vec{b}\| = 0 \Rightarrow \|\vec{t} \times \vec{n}\| = 0$

$\Rightarrow \vec{t} \times \vec{n} = 0 \text{ or } \vec{b} = 0$

$$\begin{pmatrix} t \\ n \\ b \end{pmatrix}' = \begin{bmatrix} 0 & K & 0 \\ -K & 0 & -\tau \\ 0 & \tau & 0 \end{bmatrix} \begin{pmatrix} t \\ n \\ b \end{pmatrix}$$

$t' = Kn \quad ; \quad n' = -Kt - \tau b \quad ; \quad b' = \tau n$

$\Rightarrow n' = -Kt$

$(\vec{t} \times \vec{n})' = t' \times n + t \times n' = K(n \times n) + -K(t \times t) = 0$

mai to kr hi rha hu 1 1 1 1

$\cancel{\Rightarrow t' \times n = -(t \times n')}$ $\boxed{t \times n' = 0}$ and $\boxed{t' \times n = }$

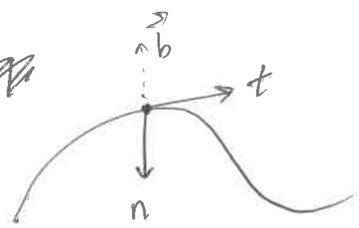
$\cancel{n' = -t}$

$\cancel{|n'| = |-t| = 1}$ ~~(arc length parame~~

$|n'| \neq 1$

At every point, the direction of $\vec{t}$ is parallel $|t|$ to the direction of change of $\vec{n}$

jai ho bhai 1 1 1 1



Since $\vec{b}$ is identically zero, there is no change in the direction or magnitude of. This is possible only if the trace(α) is contained in a plane, then a plane. If the trace is not contained in a plane, then magnitude may still remain constant but direction will change as # contradiction as torsion is identically zero