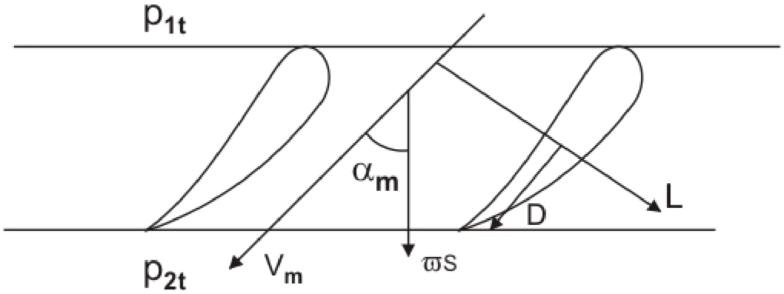
Turbine Cascade (Viscous case)



Drag = $D = \varpi S \cos \alpha_m$

Effective lift = $L + \varpi S \sin \alpha_m = \rho V_m \Gamma + \varpi S \sin \alpha_m$

Actual lift coefficient, $C_L = 2\dfrac{S}{C}(\tan \alpha_2 - \tan \alpha_1)\cos \alpha_m + C_D \tan \alpha_m$

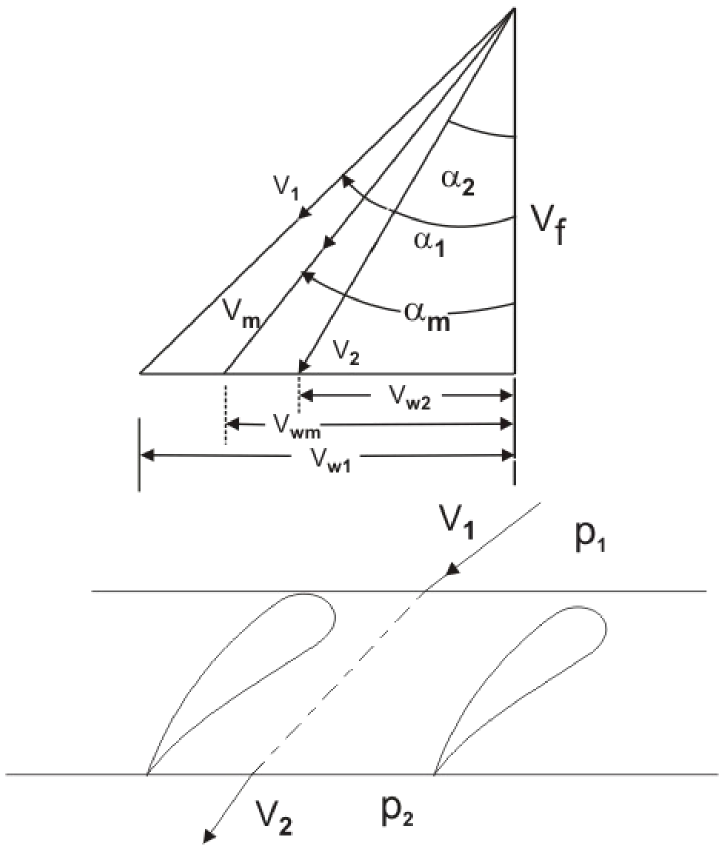
The drag increases the lift. Thus, thedrag is an useful component for work.

Blade efficiency (or diffusion efficiency)

For a compressor cascade, the blade efficiency is defined as:

$$\eta_b = \frac{\text{Actual rise in static pressure}}{\text{Ideal static pressure rise}}$$

Due to viscous effect, static pressure rise is reduced



$$\eta_b = \frac{(p_2 - p_1)_{\text{ideal}} - \text{loss}}{(p_2 - p_1)_{\text{ideal}}}$$

$$\eta_b = \frac{\frac{\rho}{2}(V_1^2 - V_2^2) - \varpi}{\frac{\rho}{2}(V_1^2 - V_2^2)} = 1 - \frac{\varpi}{\frac{\rho}{2}(V_1^2 - V_2^2)}$$

from velocity triangle: $V_1^2 = V_{w1}^2 + V_f^2,\ V_2^2 = V_{w2}^2 + V_f^2$

$$V_1^2 - V_2^2 = (V_{w1} + V_{w2})(V_{w1} - V_{w2})$$

$$\eta_b = 1 - \frac{\bar{\varpi}}{\frac{\rho}{2}(V_{w1}+V_{w2})(V_{w1}-V_{w2})}$$

Also we get
$$\frac{V_{w1}+V_{w2}}{2} = V_{wm}$$
$$= V_m \sin \alpha_m$$

$$\eta_b = 1 - \frac{\bar{\varpi}}{\rho V_m \sin \alpha_m} \frac{(\cos \alpha_m) S}{(V_{w1}-V_{w2}) S} \cdot \frac{1}{\cos \alpha_m}$$

$$= 1 - \frac{D}{\rho V_m \Gamma \sin \alpha_m \cos \alpha_m} = 1 - \frac{D}{L \sin \alpha_m \cos \alpha_m}$$

$$= 1 - \frac{2D}{L \sin 2\alpha_m}$$

[Approximation: $L \simeq \rho \Gamma V_m$
i.e.in the expression for lift, the effect of drag is ignored]

$$(\eta_b)_{\text{comp cascade}} = 1 - \frac{2C_D}{C_L \sin 2\alpha_m}$$

η -maximum, if $\dfrac{d\eta_b}{d\alpha_m} = 0 \Rightarrow \cos 2\alpha_m = 0$

$$\alpha_m = 45°$$

The value of α_m for which efficiency is maximum, $\alpha_m = 45°$