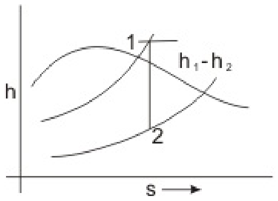
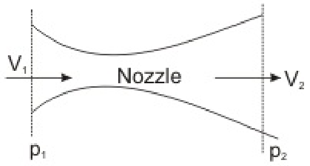
Iscentropic Flow of a vapor or gas through a nozzle





First law of thermodynamics:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$V_2 \approx \sqrt{2(h_1 - h_2)} \qquad \text{(if } V_1 \ll V_2\text{)}$$

Where $(h_1 - h_2)$ is enthalpy drop across the nozzle

Again we know, Tds = dh - νdp

For the isentropic flow, dh = νdp

$$\text{or,} \quad \int_1^2 dh = \int_1^2 vdp$$

$$\text{or,} \quad (h_1 - h_2) = -\int_1^2 vdp \qquad (20.1)$$

Assuming that the pressure and volume of steam during expansion obey the law $p\nu^n$ = constant, where n is the isentropic index

$$-\int_1^2 vdp = -\int_1^2 (p_1 v_1{}^n)^{\frac{1}{n}} p^{-\frac{1}{n}} dp = -\int_1^2 (p_2 v_2^n)^{\frac{1}{n}} p^{-\frac{1}{n}} dp$$

$$= -\left\{ p_2^{\frac{1}{n}} v_2 \left[\frac{p^{1-\frac{1}{n}}}{1-\frac{1}{n}} \right]_1^2 \right\}$$

$$= -\frac{n}{n-1} \left\{ p_2^{\frac{1}{n}} v^2 \left[p_2^{\frac{n-1}{n}} - p_1^{\frac{n-1}{n}} \right] \right\}$$

$$= -\frac{n}{n-1} \{ p_2 v_2 - p_1^{1/2} v_1 p_1^{\frac{n-1}{n}} \}$$

$$= \frac{n}{n-1} \{ p_1 v_1 - p_2 v_2 \} \qquad (20.2)$$

Now, mass flow rate

$$\dot{m} = \rho_2 \; A_2 \; V_2$$

$$\frac{\dot{m}}{A_2} = \rho_2 \; V_2 = \frac{V_2}{v_2}$$

Therefore, the mass flow rate at the exit of the nozzle

$$\frac{\dot{m}}{A_2} = \frac{1}{v_2} \sqrt{\frac{2n}{n-1} p_1 v_1 \{1 - (\frac{p_2}{p_1})^{\frac{n-1}{n}} \}}$$

$$= \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} [(\frac{p_2}{p_1})^{\frac{2}{n}} - (\frac{p_2}{p_1})^{\frac{n+1}{n}} \}} \qquad (20.3)$$

The exit pressure, p_2 determines the $\dot{m}$ for a given inlet condition. The mass flow rate is maximum when,

$$\frac{d}{dy}[y^{\frac{2}{n}} - y^{\frac{n+1}{n}}] = 0 \quad ; \quad y = \frac{p_2}{p_1}$$

$$y = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$$

For maximum $\dot{m}$,

$$\frac{p_{or}}{p_1} = \frac{p^*}{p_1} = \frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}} \qquad (20.4)$$

n =	$\gamma = 1.4,$	for diatomic gases
	$= 1.3,$	for super saturated steam
	$= 1.135,$	for dry saturated steam
	$= 1.035 + 0.1x,$	for wet steam with dryness fraction x

For , $n = 1.4,$ $p^* = 0.528 p_1$ (50%drop in inlet pressure)

$n = 1.3,$ $p^* = 0.546 p_1$

If we compare this with the results of sonic properties, as described in the earlier section, we shall observe that the critical pressure occurs at the throat for Ma = 1. The critical pressure ratio is defined as the ratio of pressure at the throat to the inlet pressure, for checked flow when Ma = 1