

### Parametric Calculations

The mass flow rate through the impeller is given by

$$\dot{m} = \rho_1 Q_1 = \rho_2 Q_2 \quad (39.1)$$

The areas of cross sections normal to the radial velocity components  $V_{f1}$  and  $V_{f2}$  are  $A_1 = \pi D_1 b_1$  and  $A_2 = \pi D_2 b_2$

$$m = \rho_1 V_{f1} (\pi D_1 b_1) = \rho_2 V_{f2} (\pi D_2 b_2) \quad (39.2)$$

The radial component of velocities at the impeller entry and exit depend on its width at these sections. For small pressure rise through the impeller stage, the density change in the flow is negligible and the flow can be assumed to be almost incompressible. For constant radial velocity

$$V_{f1} = V_{f2} = V_f \quad (39.3)$$

Eqs. (39.2) and (39.3) give

$$b_1 / b_2 = D_2 / D_1 \quad (39.4)$$

### Work

The work done is given by Euler's Equation (refer to Module-1) as

$$w = U_2 V_{w2} - U_1 V_{w1} \quad (39.5)$$

It is reasonable to assume zero whirl at the entry. This condition gives

$$\alpha_1 = 90^\circ, \quad V_{w1} = 0 \quad \text{and hence, } U_1 V_{w1} = 0$$

Therefore we can write,

$$V_1 = V_{f1} = V_{f2} = U_1 \tan \beta_1 \quad (39.6)$$

Equation (39.5) gives

$$w = U_2 V_{w2} = U_2^2 \left( \frac{V_{w2}}{U_2} \right) \quad (39.7)$$

For any of the exit velocity triangles (Figure 39.3)

$$\begin{aligned} U_2 - V_{w2} &= V_{f2} \cot \beta_2 \\ \frac{V_{w2}}{U_2} &= \left[ 1 - \frac{V_{f2} \cot \beta_2}{U_2} \right] \end{aligned} \quad (39.8)$$

Eq. (39.7) and (39.8)

$$w = U_2^2 [1 - \phi \cot \beta_2] \quad (39.9)$$

where  $\phi = (V_{f2} / U_2)$  is known as flow coefficient

$$\text{Head developed in meters of air } H_a = \frac{U_2 V_{w2}}{g} \quad (39.10)$$

$$\text{Equivalent head in meters of water} = \frac{\rho_a H_a}{\rho_w} \quad (39.11)$$

where  $\rho_a$  and  $\rho_w$  are the densities of air and water respectively.

Assuming that the flow fully obeys the geometry of the impeller blades, the specific work done in an isentropic process is given by

$$(\Delta h_0) = U_2 (1 - \phi \cot \beta_2) \quad (39.12)$$

The power required to drive the fan is

$$P = m (\Delta h_0) = m U_2 V_{w2} = m U_2^2 (1 - \phi \cot \beta_2)$$

$$= mc_p(\Delta T_0)$$

(39.13)