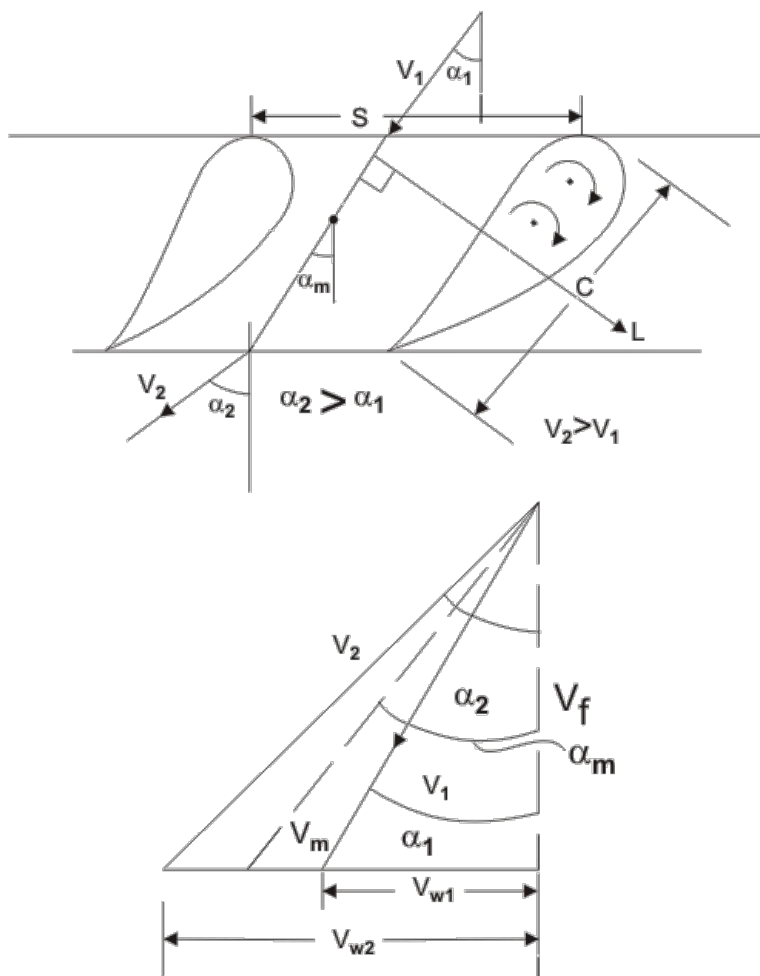S, C -depend on the design of the cascade

α_1, α_2 - flow angles at the inlet and outlet

Lift is perpendicular to α_m line

Turbine Cascade: Static pressure will drop across the turbine cascade, i.e $p_2 < p_1$

$$L = \rho V_m \Gamma$$

Lift coefficient, $C_L = \dfrac{L}{\frac{1}{2} \rho V_m^2 \, C} = 2 \dfrac{S}{C} (\tan \alpha_2 - \tan \alpha_1) \cos \alpha_m$

Above discussion is based on Kutta-Joukowski theorem

Assumption - Inviscid flow

In reality, we face viscous flow together with formation of wakes. Thus, the viscous flow is the cause of drag which in turn affect the lift .

Effect of viscous flow

As the fluid passes through the cascade, there will be a decrease in total pressure between the inlet to the cascade and at a section well downstream of the cascade due to frictional effect on aerofoils and also losses due to mixing of blade wades (i.e. the effect of viscous flow). Since at up and downstream, the flow is uniform and kinematics of flow remains unchanged, the dynamic pressure at up and downstream remains the same. Thus loss in total pressure is the same as that of static pressure.

The loss in total pressure $\bar{\omega} = p_{0_1} - p_{0_2}$ consists of two components:

(1) Frictional loss due to the formation of boundary layer on blades.

(2) Mixing of the blade wades.

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Uniform flow

Wake zone

Uniform flow after mixing
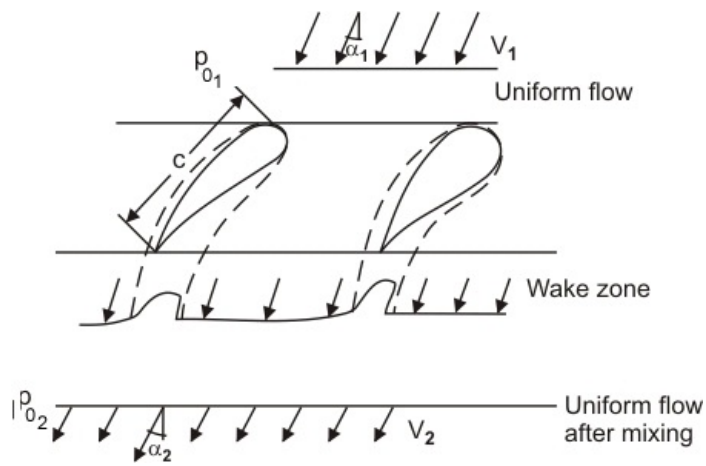
Compressor Cascade (Viscous Case)

In compressor cascade, due to losses in total pressure $(\tilde{w})$, there will be an axial force $S\tilde{w}$ as shown in figure below. Thus the drag, which is perpendicular to the lift, is defined as

$$D = \tilde{w}S \cos \alpha_m$$

ϖS = Force
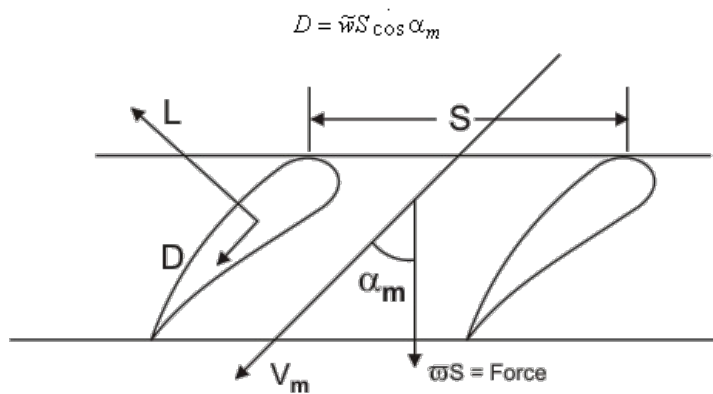
The lift will be reduced due to the effect of drag which can be expressed as:

Effective lift = $\bar{L} = L - \bar{\varpi}S \sin \alpha_m = \rho V_m \Gamma - \bar{\varpi}S \sin \alpha_m$

The lift has decreased due to viscosity, $\bar{L} = \rho V_m \Gamma - D \tan \alpha_m$

Actual lift coefficient $C_L = 2\dfrac{S}{C}(\tan \alpha_1 - \tan \alpha_2)\cos \alpha_m - C_D \tan \alpha_m$

where, C_D = drag coefficient, $C_D = \dfrac{D}{\frac{1}{2}\rho V_m^2 C}$

In the case of turbine, drag will contribute to work (and is considered as useful).