Nozzle, Steam Nozzle and Steam Turbine

STAGNATION, SONIC PROPERTIES AND ISENTROPIC EXPANSION IN NOZZLE

The stagnation values are useful reference conditions in a compressible flow. Suppose the properties of a flow (such as T, p, ρ etc.) are known at a point. The stagnation properties at a point are defined as those which are to be obtained if the local flow were imagined to cease to zero velocity isentropically. The stagnation values are denoted by a subscript zero. Thus, the stagnation enthalpy is defined as

$$h_0 = h + \frac{1}{2}V^2$$

For a calorically perfect gas, this yields,

$$c_p T_0 = c_p T + \frac{1}{2}V^2 \qquad (18.1)$$

which defines the stagnation temperature. It is meaningful to express the ratio of (T_0 / T) in the form

$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_p T} = 1 + \frac{\gamma - 1}{2} \cdot \frac{V^2}{\gamma RT}$$

or, $$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} Ma^2 \qquad (18.2)$$

If we know the local temperature (T) and Mach number (Ma), we can fine out the stagnation temperature T_0. Consequently, isentropic relations can be used to obtain stagnation pressure and stagnation density as.

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left[1 + \frac{\gamma - 1}{2} Ma^2\right]^{\frac{\gamma}{\gamma\ 1}} \qquad (18.3)$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \left[1 + \frac{\gamma - 1}{2} Ma^2\right]^{\frac{\gamma}{\gamma\ 1}} \qquad (18.4)$$

In general, the stagnation properties can vary throughout the flow field.

However, if the flow is adiabatic, then $h + \frac{V^2}{2}$ is constant throughout the flow. It follows that the h_0, T_0 and a_0 are constant throughout an adiabatic flow, even in the presence of friction. Here a is the speed of sound and the suffix signifies the stagnation condition. It is understood that all stagnation properties are constant along an isentropic flow. If such a flow starts from a large reservoir where the fluid is practically at rest, then the properties in the reservoir are equal to the stagnation properties everywhere in the flow (Fig. 18.1).

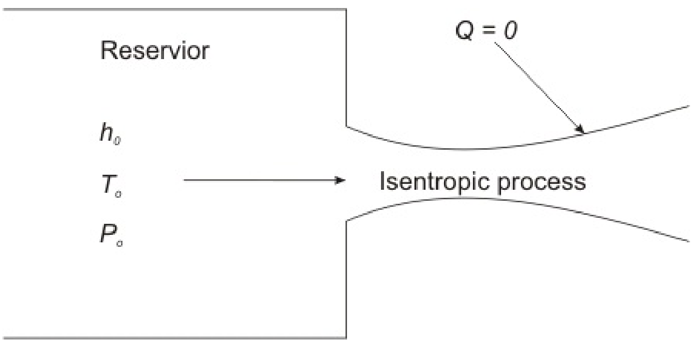

Fig 18.1 An isentropic process starting from a reservoir

There is another set of conditions of comparable usefulness where the flow is sonic, Ma=1.0. These sonic, or critical properties are denoted by asterisks: p^*, ρ^*, a^*, and. T^*. These properties are attained if the local fluid is imagined to expand or compress isentropically until it reachers Ma=1.

We have already discussed that the total enthalpy, hence T_0, is conserved so long the process is adiabatic, irrespective of frictional effects. In contrast, the stagnation pressure p_0 and density ρ_0 decrease if there is friction.

From Eq.(18.1), we note that

$$V^2 = 2c_p (T_0 - T)$$

or, $$V = \left[\frac{2\gamma R}{\gamma - 1}(T_0 - T)\right]^{\frac{1}{2}} \qquad (18.5a)$$

is the relationship between the fluid velocity and local temperature (*T*), in an adiabatic flow. The flow can attain a maximum velocity of

$$V_{max} = [\frac{2\gamma R T_0}{\gamma - 1}]^{\frac{1}{2}} \quad (18.5b)$$

As it has already been stated, the unity Mach number, Ma=1, condition is of special significance in compressible flow, and we can now write from Eq.(18.2), (18.3) and (18.4).

$$\frac{T_0}{T^*} = \frac{1+\gamma}{2} \quad (18.6a)$$

$$\frac{p_0}{p^*} = (\frac{1+\gamma}{2})^{\frac{\gamma}{\gamma-1}} \quad (18.6b)$$

$$\frac{\rho_0}{\rho^*} = (\frac{1+\gamma}{2})^{\frac{\gamma}{\gamma-1}} \quad (18.6c)$$

For diatomic gases, like air $\gamma = 1.4$, the numerical values are

$$\frac{T^*}{T_0} = 0.8333, \quad \frac{p^*}{p_0} = 0.5282, \quad and \quad \frac{\rho^*}{\rho_0} = 0.6339$$

The fluid velocity and acoustic speed are equal at sonic condition and is

$$V^* = a^* = [\gamma R T^*]^{1/2} \quad (18.7a)$$

or, $$V^* = [\frac{2\gamma}{\gamma + 1} R T_0]^{\frac{1}{2}} \quad (18.7b)$$

We shall employ both stagnation conditions and critical conditions as reference conditions in a variety of one dimensional compressible flows.