Principle of Similarity and Dimensional Analysis

The principle of similarity is a consequence of nature for any physical phenomenon. By making use of this principle, it becomes possible to predict the performance of one machine from the results of tests on a geometrically similar machine, and also to predict the performance of the same machine under conditions different from the test conditions. For fluid machine, geometrical similarity must apply to all significant parts of the system viz., the rotor, the entrance and discharge passages and so on. Machines which are geometrically similar form a homologous series. Therefore, the member of such a series, having a common shape are simply enlargements or reductions of each other. If two machines are kinematically similar, the velocity vector diagrams at inlet and outlet of the rotor of one machine must be similar to those of the other. Geometrical similarity of the inlet and outlet velocity diagrams is, therefore, a necessary condition for dynamic similarity.

Let us now apply dimensional analysis to determine the dimensionless parameters, i.e., the π terms as the criteria of similarity for flows through fluid machines. For a machine of a given shape, and handling compressible fluid, the relevant variables are given in Table 3.1

Table 3.1 Variable Physical Parameters of Fluid Machine

Variable physical parameters	Dimensional formula
D = any physical dimension of the machine as a measure of the machine's size, usually the rotor diameter	L
Q = volume flow rate through the machine	$L^3 T^{-1}$
N = rotational speed (rev/min.)	T^{-1}
H = difference in head (energy per unit weight) across the machine. This may be either gained or given by the fluid depending upon whether the machine is a pump or a turbine respectively	L
ρ = density of fluid	ML^{-3}
μ = viscosity of fluid	$ML^{-1} T^{-1}$
E = coefficient of elasticity of fluid	$ML^{-1} T^{-2}$
g = acceleration due to gravity	LT^{-2}
P = power transferred between fluid and rotor (the difference between P and H is taken care of by the hydraulic efficiency η_h	$ML^2 T^{-3}$

In almost all fluid machines flow with a free surface does not occur, and the effect of gravitational force is negligible. Therefore, it is more logical to consider the energy per unit mass gH as the variable rather than H alone so that acceleration due to gravity does not appear as a separate variable. Therefore, the number of separate variables becomes eight: $D, Q, N, gH, \rho, \mu, E$ and P. Since the number of fundamental dimensions required to express these variable are three, the number of independent π terms (dimensionless terms), becomes five. Using Buckingham's π theorem with D, N and ρ as the repeating variables, the expression for the terms are obtained as,

$$\pi_1 = \frac{Q}{ND^3}\ ,\quad \pi_2 = \frac{gH}{N^2 D^2}\ ,\ \pi_3 = \frac{\rho N D^2}{\mu}\ ,\ \pi_4 = \frac{P}{\rho N^3 D^5}\ ,\ \pi_5 = \frac{E/\rho}{N^2 D^2}$$

We shall now discuss the physical significance and usual terminologies of the different π terms. All lengths of the machine are proportional to D , and all areas to D^2. Therefore, the average flow velocity at any section in the machine is proportional to Q/D^2 . Again, the peripheral velocity of the rotor is proportional to the product ND . The first π term can be expressed as

$$\pi_1 = \frac{Q}{ND^3} = \frac{Q/D^2}{ND} \propto \frac{\text{fluid velocity}\,V}{\text{rotor velocity}\,U}$$