$$\text{stage efficiency} = \eta_s = \frac{\textit{Work done by the rotor}}{\textit{Isentropic enthalpy drop}} \qquad (23.1)$$

$$\eta_s = \frac{\dot{m}U\Delta V_w}{\dot{m}(\Delta H)_{isen}} = \frac{\dot{m}U\Delta V_w}{\dot{m}\left(\frac{V_1^2}{2}\right)} \cdot \frac{\dot{m}(V_1^2/2)}{\dot{m}(\Delta H)_{isen}} \qquad (23.2)$$

or,

or, $\eta_s = \eta_b \times \eta_n \quad [\eta_n = \textit{Nozzle efficiency}]$ (23.3)

Optimum blade speed of a single stage turbine

$$\Delta V_w = V_{r1}\cos\beta_1 + V_{r2}\cos\beta_2$$

$$= V_{r1}\cos\beta_1 + \left(1 + \frac{V_{r2}}{V_{r1}} \cdot \frac{\cos\beta_2}{\cos\beta_1}\right)$$

$$= (V_1\cos\alpha_1 - U) + (1 + kc) \qquad (23.4)$$

where, $k = (V_{r2}/V_{r1})$ = friction coefficient

$$c = (\cos\beta_2/\cos\beta_1)$$

$$\eta_b = \frac{2U\Delta V_w}{V_1^2} = 2\frac{U}{V_1}\left(\cos\alpha_1 - \frac{U}{V_1}\right)(1+kc)$$

$$\rho = \frac{U}{V_1} = \frac{\text{Blade speed}}{\text{Fluid velocity at the blade inlet}} \text{ = Blade speed ratio} \qquad (23.5)$$

η_b is maximum when $\frac{d\eta_b}{d\rho} = 0$ also $\frac{d^2\eta_b}{d\rho} = -4(1+kc)$

or, $$\frac{d}{d\rho}\{2(\rho\cos\alpha_1 - \rho^2)(1+kc)\} = 0$$

or, $$\rho = \frac{\cos\alpha_1}{2} \qquad (23.6)$$

α_1 is of the order of 18^0 to 22^0

Now, $(\rho)_{opt} = \left(\frac{U}{V_1}\right)_{opt} = \frac{\cos\alpha_1}{2}$ (For single stage impulse turbine)

∴ The maximum value of blade efficiency

$$(\eta_b)_{\max} = 2(\rho\cos\alpha_1 - \rho^2)(1+kc)$$

$$= \frac{\cos^2\alpha_1}{2}(1+kc) \qquad (23.7)$$

For equiangular blades,

$$(\eta_b)_{\max} = \frac{\cos^2\alpha_1}{2}(1+k) \qquad (23.8)$$

If the friction over blade surface is neglected

$$(\eta_b)_{\max} = \cos^2\alpha_1 \qquad (23.9)$$