$$V_{w_2} = -(V_{r_2} \cos \beta_2 - U)$$

where V_{r_1} and V_{r_2} are the velocities of the jet relative to the bucket at its inlet and outlet and β_2 is the outlet angle of the bucket.

From the Eq. (1.2) (the Euler's equation for hydraulic machines), the energy delivered by the fluid per unit mass to the rotor can be written as

$$E/m = [V_{w_1} - V_{w_2}]\,U$$

$$= [V_{r_1} + V_{r_2} \cos \beta_2]\,U \qquad (26.1)$$

(since, in the present situation, $U_1 = U_2 = U$)

The relative velocity V_{r_2} becomes slightly less than V_{r_1} mainly because of the friction in the bucket. Some additional loss is also inevitable as the fluid strikes the splitter ridge, because the ridge cannot have zero thickness. These losses are however kept to a minimum by making the inner surface of the bucket polished and reducing the thickness of the splitter ridge. The relative velocity at outlet V_{r_2} is usually expressed as $V_{r_2} = KV_{r_1}$ where, K is a factor with a value less than 1. However in an ideal case (in absence of friction between the fluid and blade surface) K=1. Therefore, we can write Eq.(26.1)

$$E/m = V_{r_1}[1 + K \cos \beta_2]\,U \qquad (26.2)$$

If Q is the volume flow rate of the jet, then the power transmitted by the fluid to the wheel can be written as

$$P = \rho\, Q\, V_{r_1}[1 + K \cos \beta_2]U$$

$$= \rho\, Q[1 + K \cos \beta_2](V_1 - U)U \qquad (26.3)$$

The power input to the wheel is found from the kinetic energy of the jet arriving at the wheel and is given by $\frac{1}{2}\rho Q v_1^2$. Therefore the wheel efficiency of a pelton turbine can be written as

$$\eta_w = \frac{2\rho Q[1 + K \cos \beta_2](V_1 - U)U}{\rho Q V_1^2}$$

$$= 2\,[1 + K \cos \beta_2]\left[1 - \frac{U}{V_1}\right]\frac{U}{V_1} \qquad (26.4)$$

It is found that the efficiency η_w depends on K, β_2 and U/V_1. For a given design of the bucket, i.e. for constant values of β_2 and K, the efficiency η_w becomes a function of U/V_1 only, and we can determine the condition given by U/V_1 at which η_w becomes maximum.

For η_w to be maximum,

$$\frac{d\eta_w}{d(U/V_1)} = 2[1 + K \cos \beta_2][1 - 2\frac{U}{V_1}] = 0$$

or, $$U/V_1 = \frac{1}{2} \qquad (26.5)$$

$d^2\eta_w / d(U/V_1)^2$ is always negative.

Therefore, the maximum wheel efficiency can be written after substituting the relation given by eqn.(26.5) in eqn.(26.4) as

$$\eta_{w\,\max} = 2(1 - K \cos \beta_2)/2 \qquad (26.6)$$