Similarity and Dimensional Analysis

or, with another arrangement of the π terms,

$$\phi\left[\eta_h, \frac{gH}{N^2D^2}, Ma, \frac{P}{\rho N^3 D^5}\right] = 0 \qquad (3.2)$$

If data obtained from tests on model machine, are plotted so as to show the variation of dimensionless parameters $\frac{Q}{ND^3}, \frac{gH}{N^2D^2}, Ma, \frac{P}{\rho N^3 D^5}$ with one another, then the graphs are applicable to any machine in the same homologous series. The curves for other homologous series would naturally be different.

Specific Speed

The performance or operating conditions for a turbine handling a particular fluid are usually expressed by the values of N, P and H, and for a pump by N, Q and H. It is important to know the range of these operating parameters covered by a machine of a particular shape (homologous series) at high efficiency. Such information enables us to select the type of machine best suited to a particular application, and thus serves as a starting point in its design. Therefore a parameter independent of the size of the machine D is required which will be the characteristic of all the machines of a homologous series. A parameter involving N, P and H but not D is obtained by dividing $(\pi_4)^{1/2}$ by $(\pi_2)^{5/4}$. Let this parameter be designated by K_{s_T} as

$$K_{s_T} = \frac{(P/\rho N^3 D^5)^{1/2}}{(gH/N^2D^2)^{5/4}} = \frac{NP^{1/2}}{\rho^{1/2}(gH)^{5/4}} \qquad (3.3)$$

Similarly, a parameter involving N, Q and H but not D is obtained by divining $(\pi_1)^{1/2}$ by $(\pi_2)^{3/4}$ and is represented by K_{s_P} as

$$K_{s_P} = \frac{(Q/ND^3)^{1/2}}{(gH/N^2D^2)^{3/4}} = \frac{NQ^{1/2}}{(gH)^{3/4}} \qquad (3.4)$$

Since the dimensionless parameters K_{s_T} and K_{s_P} are found as a combination of basic π terms, they must remain same for complete similarity of flow in machines of a homologous series. Therefore, a particular value of K_{s_T} or K_{s_P} relates all the combinations of N, P and H or N, Q and H for which the flow conditions are similar in the machines of that homologous series. Interest naturally centers on the conditions for which the efficiency is a maximum. For turbines, the values of N, P and H, and for pumps and compressors, the values of N, Q and H are usually quoted for which the machines run at maximum efficiency.

The machines of particular homologous series, that is, of a particular shape, correspond to a particular value of K_s for their maximum efficient operation. Machines of different shapes have, in general, different values of K_s. Thus the parameter $K_s(K_{s_T}\ or\ K_{s_P})$ is referred to as the *shape factor* of the machines. Considering the fluids used by the machines to be incompressible, (for hydraulic turbines and pumps), and since the acceleration due to gravity dose not vary under this situation, the terms g and ρ are taken out from the expressions of K_{s_T} and K_{s_P}. The portions left as $NP^{1/2}/H^{5/4}$ and $NQ^{1/2}/H^{3/4}$ are termed, for the practical purposes, as the *specific* speed N_s for turbines or pumps. Therefore, we can write,

$$N_{s_T} \text{ (specific speed for turbines)} = NP^{1/2}/H^{5/4} \qquad (3.5)$$

$$N_{s_P} \text{ (specific speed for turbines)} = NQ^{1/2}/H^{3/4} \qquad (3.6)$$

The name specific speed for these expressions has a little justification. However a meaning can be attributed from the concept of a hypothetical machine. For a turbine, N_{s_T} is the speed of a member of the same homologous series as the actual turbine, so reduced in size as to generate unit power under a unit head of the fluid. Similarly, for a pump, N_{s_P} is speed of a hypothetical pump with reduced size but representing a homologous series so that it delivers unit flow rate at a unit head. The specific speed N_s is, therefore, not a dimensionless quantity.

The dimension of N_s can be found from their expressions given by Eqs. (3.5) and (3.6). The dimensional formula and the unit of specific speed are given as follows:

Specific speed	Dimensional formula	Unit (SI)
N_{s_T} (turbine)	$M^{1/2}\ T^{-5/2}\ L^{-1/4}$	$kg^{1/2}/\ s^{5/2}\ m^{1/4}$
N_{s_P} (pump)	$L^{3/4}\ T^{-3/2}$	$m^{3/4}\ /\ s^{3/2}$

The dimensionless parameter K_s is often known as the dimensionless specific speed to distinguish it from N_s.