Effect of Area Variation on Flow Properties in Isentropic Flow

In considering the effect of area variation on flow properties in isentropic flow, we shall concern ourselves primarily with the velocity and pressure. We shall determine the effect of change in area, *A*, on the velocity *V*, and the pressure *p*.

From Bernoulli's equation, we can write

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) = 0$$

$$\text{or,} \quad dp = -\rho V\, dV$$

Dividing by ρV^2, we obtain

$$\frac{dp}{\rho V^2} = -\frac{dV}{V} \qquad (19.1)$$

A convenient differential form of the continuity equation can be obtained from Eq. (14.50) as

$$\frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho}$$

Substituting from Eq. (19.1),

$$\frac{dA}{A} = \frac{dp}{\rho V^2} - \frac{d\rho}{\rho}$$

$$\text{or,} \quad \frac{dA}{A} = \frac{dp}{\rho V^2}\left[1 - \frac{V^2}{dp/d\rho}\right] \qquad (19.2)$$

Invoking the relation ($a^2 = \dfrac{dp}{d\rho}$) for isentropic process in Eq. (19.2), we get

$$\frac{dA}{A} = \frac{dp}{\rho V^2}\left[1 - \frac{V^2}{\alpha^2}\right] = \frac{dp}{\rho V^2}\left[1 - Ma^2\right] \qquad (19.3)$$

From Eq. (19.3), we see that for Ma<1 an area change causes a pressure change of the same sign, i.e. positive d*A* means positive d*p* for Ma<1. For Ma>1, an area change causes a pressure change of opposite sign.

Again, substituting from Eq.(19.1) into Eq. (19.3), we obtain

$$\frac{dA}{A} = -\frac{dV}{V}\left[1 - Ma^2\right] \qquad (19.4)$$

From Eq. (19.4), we see that Ma<1 an area change causes a velocity change of opposite sign, i.e. positive d*A* means negative d*V* for Ma<1. For Ma>1, an area change causes a velocity change of same sign.

These results are summarized in Fig.19.1, and the relations (19.3) and (19.4) lead to the following important conclusions about compressible flows:

1. At subsonic speeds (Ma<1) a decrease in area increases the speed of flow. A subsonic nozzle should have a convergent profile and a subsonic diffuser should possess a divergent profile. The flow behaviour in the regime of Ma<1 is therefore qualitatively the same as in incompressible flows.
2. In supersonic flows (Ma>1), the effect of area changes are different. According to Eq. (19.4), a supersonic nozzle must be built with an increasing area in the flow direction. A supersonic diffuser must be a converging channel. Divergent nozzles are used to produce supersonic flow in missiles and launch vehicles.

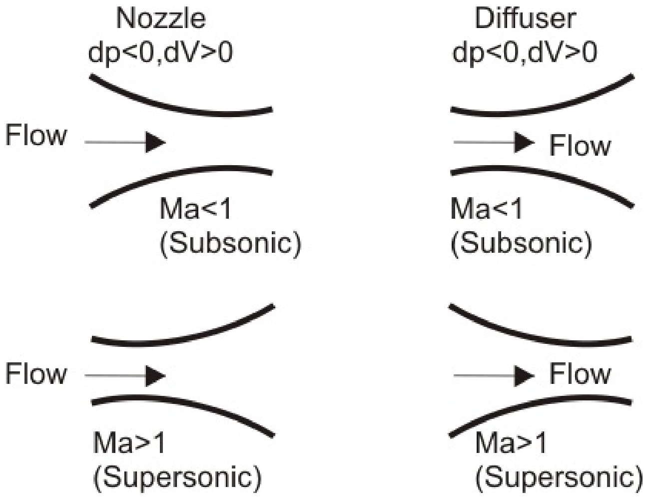

Fig 19.1 Shapes of nozzles and diffusersin subsonic and supersonic regimes

Suppose a nozzle is used to obtain a supersonic stream staring from low speeds at the inlet (Fig.19.2). Then the Mach number should increase from Ma=0 near the inlet to Ma>1 at the exit. It is clear that the nozzle must converge in the subsonic portion and diverge in the supersonic portion. Such a nozzle is called a *convergent-divergent nozzle.* A convergent-divergent nozzle is also called a *de Laval nozzle*, after Carl G.P. de Laval who first used such a configuration in his steam turbines in late nineteenth century (this has already been mentioned in the introductory note). From Fig.**19.2** it is clear that the Mach number must be unity at the throat, where the area is neither increasing nor decreasing. This is consistent with Eq. (19.4) which shows that d*V* can be non-zero at the throat only if Ma=1. It also follows that the sonic velocity can be achieved only at the throat of a nozzle or a diffuser.

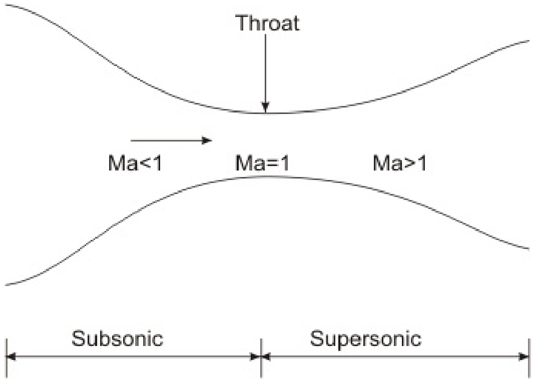

Fig 19.2 A convergent-divergent nozzle

The condition, however, does not restrict that *Ma* must necessarily be unity at the throat, According to Eq. (19.4), a situation is possible where $Ma \neq 1$ at the throat if d*V*=0 there. For an example, the flow in a convergent-divergent duct may be subsonic everywhere with *Ma* increasing in the convergent portion and decreasing in the divergent portion with $Ma \neq 1$ at the throat (see Fig.19.3). The first part of the duct is acting as a nozzle, whereas the second part is acting as a diffuser. Alternatively, we may have a convergent-divergent duct in which the flow is supersonic everywhere with *Ma* decreasing in the convergent part and increasing in the divergent part and again $Ma \neq 1$ at the throat (see Fig. 19.4).

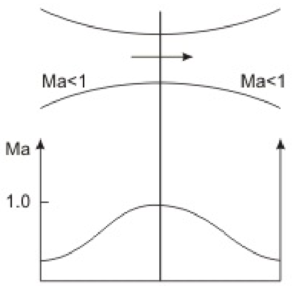

Fig 19.3 Convergent-divergent duct with $Ma \neq 1$ at throat

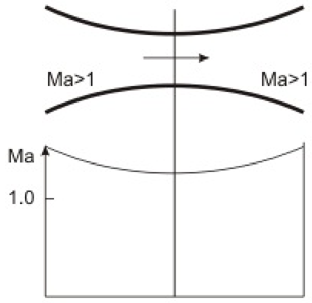

Fig 19.4 Convergent-divergent duct with $Ma \neq 1$ at throat