Similarity and Dimensional Analysis

Thus, π_1 represents the condition for kinematic similarity, and is known as *capacity coefficient* or *discharge coefficient* The second π term π_2 is known as the *head coefficient* since it expresses the head *H* in dimensionless form. Considering the fact that *ND* $\propto$ rotor velocity, the term π_2 becomes gH/U^2 , and can be interpreted as the ratio of fluid head to kinetic energy of the rotor, Dividing π_2 by the square of π_1 we get

$$\frac{\pi_2}{\pi_1^2} = \frac{gH}{(Q/D^2)^2} \propto \frac{total\ fluid\ energy\ per\ unit\ mass}{kinetic\ energy\ of\ the\ fluid\ per\ unit\ mass}$$

The term π_3 can be expressed as $\rho(ND)D/\mu$ and thus represents the Reynolds number with rotor velocity as the characteristic velocity. Again, if we make the product of π_1 and π_3, it becomes $\rho(Q/D^2)D/\mu$ which represents the Reynolds's number based on fluid velocity. Therefore, if π_1 is kept same to obtain kinematic similarity, π_3 becomes proportional to the Reynolds number based on fluid velocity.

The term π_4 expresses the power *P* in dimensionless form and is therefore known as *power coefficient* . Combination of π_4 , π_1 and π_2 in the form of $\pi_4/\pi_1\pi_2$ gives $P/\rho QgH$. The term 'PQgH' represents the rate of total energy given up by the fluid, in case of turbine, and gained by the fluid in case of pump or compressor. Since *P* is the power transferred to or from the rotor. Therefore $\pi_4/\pi_1\pi_2$ becomes the hydraulic efficiency η_h for a turbine and $1/\eta_h$ for a pump or a compressor. From the fifth π term, we get

$$\frac{1}{\sqrt{\pi_5}} = \frac{ND}{\sqrt{E/\rho}}$$

Multiplying π_1, on both sides, we get

$$\frac{\pi_1}{\sqrt{\pi_5}} = \frac{Q/D^2}{\sqrt{E/\rho}} \propto \frac{fluid\ velocity}{lacal\ asoustic\ velocity}$$

Therefore, we find that $\pi_1/\sqrt{\pi_5}$ represents the well known *Mach number* , Ma.

For a fluid machine, handling incompressible fluid, the term π_5 can be dropped. The effect of liquid viscosity on the performance of fluid machines is neglected or regarded as secondary, (which is often sufficiently true for certain cases or over a limited range).Therefore the term π_3 can also be dropped.The general relationship between the different dimensionless variables (π terms) can be expressed as

$$f\left[\frac{Q}{ND^3}, \frac{gH}{N^2D^2}, \frac{E/\rho}{N^2D^2}, \frac{P}{\rho N^3D^5}\right] = 0 \qquad (3.1)$$

Therefore one set of relationship or curves of the π terms would be sufficient to describe the performance of all the members of one series.