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Assignment - 1

(Name - Rajpreet Singh

Ques 1 $y = Ae^{x} + Be^{-x} + x^2 \Rightarrow Ae^{x} + Bx^{-2} = y - x^2$ — ①

$$\frac{dy}{dx} = Ae^{x} - Be^{-x} + 2x$$

$$\frac{d^2y}{dx^2} = Ae^{x} + Be^{-x} + 2$$

$$\frac{d^2y}{dx^2} = y - x^2 + 2 \quad \text{(using ①)}$$

or $\boxed{y'' - y + x^2 - 2 = 0}$

Ques 2 $\frac{y}{x} \cdot \frac{dy}{dx} + \frac{x^2+y^2-1}{2(x^2+y^2)+1} = 0$

Let $x^2 + y^2 = t \Rightarrow 2x + 2\cancel{y}\frac{dy}{dx} = \frac{dt}{dx}$

$$\Rightarrow 1 + \frac{y}{x}\frac{dy}{dx} = \frac{dt}{2dx} \cdot \frac{1}{x}$$

$$\Rightarrow \frac{1}{2x}\frac{dt}{dx} - 1 + \frac{t-1}{2t+1} = 0$$

$$\Rightarrow \frac{1}{2x}\frac{dt}{dx} = 1 - \left[\frac{t-1}{2t+1}\right] = \frac{2t+1-t+1}{2t+1} = \frac{t+2}{2t+1}$$

$$\Rightarrow \int \frac{2t+1}{2t+4}\, dt = \int x \cdot dx$$

$$\rightarrow \int \frac{2t+4-3}{2t+4} \cdot dt = \int x \cdot dx$$

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$$\int dt - \frac{3}{2}\int \frac{dt}{t+2} = \int x \cdot dx$$

$$\Rightarrow t - \frac{3}{2}\log|t+2| = \frac{x^2}{2} + c$$

$$\Rightarrow 2(x^2+y^2) - 3|x^2+y^2+2| = x^2 + c$$

$$\Rightarrow \boxed{x^2+2y^2 - 3\log|x^2+y^2+2| = c}$$

(3) $(1+e^{x/y})dx + e^{x/y}(1-x/y)dy = 0$

$$\Rightarrow (1+e^{x/y})\frac{dx}{dy} + e^{x/y}\left(1-\frac{x}{y}\right) = 0$$

$$\Rightarrow (1+e^t)\left[y\cdot\frac{dt}{dy} + t\right] + e^t(1-t) = 0$$

$\frac{x}{y} = t$

$dx = yt$

$\Rightarrow \frac{dx}{dy} = y\frac{dt}{dy} + t$

$$\Rightarrow (1+e^t)y\frac{dt}{dy} + t + \cancel{te^t} + e^t - \cancel{e^t \cdot t}$$

$$\Rightarrow (1+e^t)y\cdot\frac{dt}{dy} + t + \cancel{te^t} + e^t - \cancel{te^t} = 0$$

$$\Rightarrow (1+e^t)y\cdot\frac{dt}{dy} = -(t+e^t)$$

$$\Rightarrow \int\frac{(1+e^t)\cdot dt}{-(t+e^t)} = \int\frac{dy}{y}$$

$t+e^t = z$

$(1+e^t)\,dt = dz$

$$\Rightarrow \int\frac{dz}{-z} = \int\frac{dy}{y}$$

$$\Rightarrow -\log z = \log y + c$$

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$\Rightarrow$ $-\log(t+e^t) = \log y + c$

$-\log\left(\frac{x}{y} + e^{x/y}\right) = \log y + c$

$-\log\left[\frac{x + y e^{x/y}}{y}\right] = \log y + c$

$-\log[x + ye^{x/y}] + \cancel{\log y} = \cancel{\log y} + c$

$\Rightarrow$ $x + ye^{x/y} = \cancel{x} e^{-c}$

$\Rightarrow$ $\boxed{x + ye^{x/y} = c}$

(4) $(y \log y) \cdot dx + (\cancel{d}x - \log y)\, dy = 0$

$y \log y . dx + \cancel{x}x . dy - \log y\, dy = 0$ $\qquad \log y = t$

$dx + \frac{x}{x \log y} . dy \; 1 - \frac{1}{y} . dy = 0$ $\qquad \frac{1}{y} dy = dt$

$\frac{dx}{\cancel{dy}} + \left[\frac{1}{y \log y} . dy\right] - \frac{1}{y} dy = 0 \Rightarrow \frac{dx}{dy} + \left(\frac{1}{y \log y}\right) x = \frac{1}{y}$

$\Rightarrow$ $\frac{dx}{dy} + Px = Q$ $\leftarrow$ using

$IF = e^{\int P . dy} = e^{\int \frac{dy}{y \log y}} = e^{\log(\log y)} = \log y$

$\Rightarrow$ $x \log y = \int \frac{1}{y} \log y . dy$

$\Rightarrow$ $x \log y = \int t . dt$

$\Rightarrow$ $dx \log y = \frac{t^2}{2} + c$

$\Rightarrow$ $\boxed{dx \log y = \frac{(\log y)^2}{2} + c}$

⑤ $(x^2y^2 + \cancel{x}xy + 1)y \cdot dx + (x^2y^2 - \cancel{x}y + 1)\, x \cdot dy = 0$

$t = xy \Rightarrow y = t/x$

$$dy = \frac{dt\,x - t}{x^2}$$

$$\Rightarrow (t^2+t+1)\frac{t}{x}\,dx + (t^2-t+1)\left[\frac{x\,dt - t\,dx}{x^2}\right]\cancel{x} = 0$$

$$\left[\frac{t}{x}(t^2+t+1) - \frac{t}{x}(t^2-t+1)\right] \cdot dx = (-t^2+t-1)\,dt$$

$$\Rightarrow \left[\cancel{\frac{t^3}{x}} + \frac{t^2}{x} + \cancel{\frac{t}{x}} - \cancel{\frac{t^3}{x}} + \frac{t^2}{x} - \cancel{\frac{t}{x}}\right] dx = (-t^2+t+1)\,dt$$

$$\Rightarrow \frac{2t^2}{x} \cdot dx = (-t^2+t-1)\,dt$$

$$\Rightarrow \frac{dx}{x} = (-t^2+t-1)\,dt$$

$$\Rightarrow \int\frac{dx}{x} = \int\left[\frac{-1}{2} + \frac{1}{2t} - \frac{1}{2t^2}\right] dt$$

$$\Rightarrow \log x = \frac{-t}{2} + \frac{\log t}{2} + \frac{1}{2t} + C$$

$$\Rightarrow \boxed{\log x = -\frac{xy}{2} + \frac{\log(xy)}{2} + \frac{1}{2xy} + C}$$

⑥ $\frac{dy}{dx} - y = y^2(\sin x + \cos x)$

$\Rightarrow$ Multiply By " $1/y^2$ " on Both Sides.

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$$\Rightarrow \frac{1}{y^2}\cdot\frac{dy}{dx} - \frac{1}{y} = (\sin x + \cos x)$$

$$\left[-\frac{1}{y} = t \Rightarrow \frac{1}{y^2}\frac{dt}{dx} = \frac{dt}{dx} \right]$$

$$\Rightarrow \frac{dt}{dx} + t = (\sin x + \cos x) \qquad \left\{ \frac{dy}{dx} + Py = Q(x) \quad IF = e^{\int P\,dx} \right\}$$

$$IF = e^{\int 1\cdot dx} = e^x$$

$$\Rightarrow e^x\cdot t = \int e^x(\sin x + \cos x)\cdot dx$$

$$\Rightarrow e^x t = \int e^x\cdot\sin x\cdot dx + \int e^x\cos x\cdot dx$$

$$\Rightarrow e^x\cdot t = \sin x\cdot e^x - \cancel{\int \cos x\cdot e^x\cdot dx} + \cancel{\int e^x\cos x\cdot dx} + c$$

$$\Rightarrow e^x\cdot t = \sin x\cdot e^x + c$$

$$\boxed{-\frac{e^x}{y} = \sin x\cdot e^x + c}$$

(7) $\log x, \log x^2, \log x^3$

$$W(x) = \begin{vmatrix} \log x & 2\log x & 3\log x \\ 1/x & 2/x & 3/x \\ -1/x^2 & -2/x^2 & -3/x^2 \end{vmatrix}$$

$$\Rightarrow \log x\left(\cancel{\frac{-6}{x^3}} + \cancel{\frac{6}{x^3}}\right) - 2\log x\left[\cancel{\frac{-3}{x^3}} + \cancel{\frac{3}{x^3}}\right] + 3\log x \left(\cancel{\frac{-2}{x^3}} + \cancel{\frac{2}{x^3}}\right) = 0$$

$\therefore$ Therefore its is not linearly dependent.

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(8) $9y'' + 6y' + y = 0$, $y(0) = 0$, $y'(0) = 1$

$SF = [9D^2 + 6D + 1]y = 0$

$AE = 9m^2 + 6m + 1 = 0$

$(3m+1)^2 = 0 \Rightarrow m = -\frac{1}{3}, \frac{-1}{3}$

$\Rightarrow y = e^{-x/3}[c_1 + c_2 x] = c_1 e^{-x/3} + c_2 x e^{-x/3}$

$y(0) = 0$

$\Rightarrow 0 = 1(c_1) \Rightarrow c_1 = 0$

$y' = c_1 e^{-x/3}\left(-\frac{1}{3}\right) + c_2\left(x \cdot e^{-x/3}\left(\frac{-1}{3}\right) + e^{-x/3}\right)$

$y'(0) = 1$

$\Rightarrow 1 = 0 + c_2[0+1] \Rightarrow 1 = c_2$

$CS \Rightarrow y = e^{-x/3}[0 + x]$

$\Rightarrow \boxed{y = xe^{-x/3}}$

(9) Given, $y = e^{-x}$

$\Rightarrow \frac{dy}{dx} = -e^{-x} \Rightarrow \frac{dy}{dx} = -y$

$\Rightarrow \boxed{\frac{dy}{dx} + y = 0}$

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$y = xe^{-x}$

$\frac{dy}{dx} = -xe^{-x} + e^{-x}$

$\frac{d^2y}{dx^2} = -[-xe^{-x} + e^{-x}] - e^{-x} = xe^{-x} - 2e^{-x}$

$\frac{d^2y}{dx^2} + y = 2 \cdot \frac{dy}{dx} \Rightarrow \boxed{\frac{d^2y}{dx^2} + 2 \cdot \frac{dy}{dx} + y = 0}$

(10) $y'' - 2y' + y = xe^x \sin x$

$\Rightarrow (D^2 - 2D + 1)y = SF$

$AE = m^2 - 2m + 1 = 0 \rightarrow \cancel{2}(m-1)^2 = 0$

$\Rightarrow m = +1, +1$

$CF = e^x (C_1 + C_2 x)$

$PI = \frac{1}{D^2 - 2D + 1} xe^x \sin x = \frac{1}{(D-1)^2} e^x (x \sin x)$

$\left[\begin{array}{l} e^x(\sin x) \\ e^{ax} \times V \\ a = 1 \\ \begin{cases} D \rightarrow D + a \\ D \rightarrow D + 1 \end{cases} \end{array}\right]$

$PI = e^x \frac{1}{D^2} x \sin x$

$PI = e^x \frac{1}{D} \int x \sin x = e^x \frac{1}{D} \left[-x \cos x + \int \cos x \right]$

$PI = e^x \frac{1}{D} \left[\cancel{x} \cos x + \sin x \right]$

$\Rightarrow PI = e^x \left[\int -x \cos x + \int \sin x \right]$

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$\Rightarrow e^x[-x\sin x + \cos x] - \cos x]$

$\Rightarrow e^x[-x\sin x - 2\cos x]$

$$\boxed{CS = e^x(C_1 + C_2 x) + e^x[-x\sin x - 2\cos x]}$$

(11) $y'' + a^2 y = \sec ax$

$\Rightarrow (D^2 + a^2)y \Rightarrow m^2 + a^2 = 0 \Rightarrow m = \pm ai$

$CF = c_1 \cos ax + c_2 \sin ax \quad (CF = C_1 y_1 + C_2 y_2)$

$$\boxed{PI = uy_1 + vy_2}$$

$\Rightarrow PI = u\cos ax + v\sin ax$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos ax & \sin ax \\ -a\sin ax & a\cos ax \end{vmatrix}$$

$= a\cos^2 ax + a\sin^2 ax = a$

$u = \int \frac{y_2}{W} X\,dx = -\int \frac{\sin ax}{a} \sec ax = -\int \frac{\tan ax}{a}$

$u = -\frac{1}{a^2} \log(\sec ax)$

$v = \int \frac{y_1}{W} X\,dx = \int \frac{\cos ax}{a} \sec ax \cdot dx = \frac{x}{a}$

$$\boxed{v = \frac{x}{a}}$$

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$$PI = -\frac{1}{a^2}\log(\sec ax).\cos ax + \frac{x}{a}\sin ax$$

$$y = CF + PI$$

$$\boxed{y = C_1\cos ax + C_2\sin ax - \frac{1}{a^2}\log(\sec ax).\cos ax + \frac{x}{a}\sin ax}$$

12 $$y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$$

$$SF = (D^2 - 6D + 9)y \Rightarrow AE = m^2 - 6m + 9 = 0$$

$$AE = (m-3)^2 = 0 \Rightarrow m = 3, 3$$

$$CF = e^{3x}(C_1 + C_2x) = C_1e^{3x} + C_2xe^{3x}$$

$$y_1 = e^{3x}, \quad y_2 = xe^{3x}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & 3xe^{3x} + e^{3x} \end{vmatrix}$$

$$\{e^{3x} = t\}$$

$$\Rightarrow \begin{vmatrix} t & xt \\ 3t & 3xt + t \end{vmatrix} = 3xt^2 + t^2 - 3xt^2 = e^{6x}$$

$$u = -\int \frac{y_2}{W}X\,dx = -\int \frac{xe^{3x}}{e^{6x}}\cdot\frac{e^{3x}}{x^2}\cdot dx = -\int \frac{1}{x}dx$$

$$\boxed{u = -\log x}$$

$$v = \int \frac{y_1}{W}X\,dx = \int \frac{e^{3x}}{e^{6x}}\cdot\frac{e^{3x}}{x^2}\cdot dx = \frac{-1}{x}$$

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$PI = -\log x e^{3x} - \frac{1}{x} x e^{3x}$

$PI = -\log x e^{3x} - e^{3x} \Rightarrow -e^{3x}(\log x + 1)$

$CS = CF + PI$

$y = e^{3x}(C_1 + C_2 x) - e^{3x}(\log x + 1)$

(13) $y'' - 3y' + 2y = x^2 + e^x$

$SF = (D^2 - 3D + 2)y = x^2 + e^x$

$AE \Rightarrow m^2 - 3m + 2 = 0$

$m^2 - 2m - m + 2 = 0 \Rightarrow (m-1)(m-2) = 0 \Rightarrow m = 1, 2$

$CF = c_1 e^x + c_2 e^{2x}$

Trial sol^n $PI = a_1 x^2 + a_2 x + a_3 + a_4 \cancel{x} e^x = y$

$y = a_1 x^2 + a_2 x + a_3 + a_4 e^x \cdot x$

$y' = 2a_1 x + a_2 + a_4 [x e^x + e^x]$

$y'' = 2a_1 + a_4 (x e^x + 2e^x)$

Given, $y'' - 3y' + 2y = x^2 + e^x$

$2a_1 + a_4 x e^x + 2a_4 e^x - 6a_1 x - 3a_2 - 3a_4 (x e^x + e^x)$
$+ 2a_1 x^2 + 2a_2 x + 2a_3 + 2a_4 x \cdot e^x = x^2 + e^x$

Comparing.

$2a_1 = 1 \Rightarrow \boxed{a_1 = 1/2}$ $2a_4 - 3a_4 = 1 \Rightarrow \boxed{a_4 = -1}$

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$-6a_1 + 2a_2 = 0$
$2a_2 = 6a_1$
$a_2 = 3a_1$
$\boxed{a_2 = 3/2}$

$2a_1 - 3a_2 + 2a_3 = 6$
$3a_2 - 2a_3 = 1$
$\frac{a}{2} - 1 = 2a_3$
$\boxed{a_3 = 7/4}$

$PI = \frac{x^2}{2} + \frac{3x}{2} + \frac{7}{4} + xe^x$
$= \frac{1}{4}(2x^2 + 6x + 7 - 4xe^x)$

$CS = PI + CF$

$$\boxed{y = c_1e^x + c_2e^{2x} + \frac{1}{4}(2x^2 + 6x + 7 - 4xe^x)}$$

(T4) $\cancel{2}x^2y'' + 4xy' + 2y = e^x$

$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$

$\boxed{x = e^z}$
$\Rightarrow x^2D^2 = \theta(\theta - 1)$
$\Rightarrow xD = \theta$

$(x^2D^2 + 4xD + 2)y = e^x$

$(\theta(\theta - 1) + 4\theta + 2)y = e^x$

$AE = m^2 + 3m + 2 = 0$
$\Rightarrow m^2 + 2m + m + 2 = 0 \Rightarrow m(m+2) + 1(m+2) = 0$
$\Rightarrow (m+1)(m+2) = 0$
$\Rightarrow m = -1, -2$

$e^z = t$
$e^z dz = dt$

$CF = c_1e^{-z} + c_2e^{-2z}$

$PI = \frac{e^x}{\theta^2 + 3\theta + 2} = \frac{e^{e^z}}{\theta^2 + 3\theta + 2} = \frac{e^{e^z}}{(\theta+1)(\theta+2)}$

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$$= \left[\frac{1}{\theta+1} - \frac{1}{\theta+2}\right] e^{e^z} = \frac{1}{\theta+1} e^{e^z} - \frac{1}{\theta+2} e^{2z}$$

$$\frac{1}{D-a} X = e^{ax}\int X\cdot e^{-ax}\, dx$$

$$\Rightarrow e^{-z}\int e^{z} e^{e^z}\, dz - e^{-2z}\int e^{2z} e^{e^z}\, dz$$

$$\Rightarrow e^{-z}\int e^t\, dt - e^{-2z}\int e^t\cdot t\, dt$$

$$\Rightarrow e^{-z} e^{e^t} - e^{-2z}\left[te^t - \int e^t\right]$$

$$\Rightarrow e^{-z}\cdot e^{e^t} - e^{-2t}\left[e^t\cdot e^{e^t} - e^{e^t}\right]$$

$$\Rightarrow e^{-z}\cdot e^{e^z} - e^{-2z} e^{e^z}\left[e^z - 1\right]$$

$$= e^{e^z}\left[\cancel{e^{-z}} - \cancel{e^{-z}} + e^{-2z}\right] = e^{e^z} e^{-2z}$$

$$y = c_1 e^{-z} + c_2 e^{-2z} + e^{e^z} e^{-2z}$$

$$y = \frac{c_1}{x} + \frac{c_2}{x^2} + \frac{e^x}{x^2}$$

(15) $(3x+2)^2 \dfrac{d^2y}{dx^2} + 5(3x+2)\dfrac{dy}{dx} - 3y = x^2 + x + 1$

$\Rightarrow \cancel{(2+3x)^2\, d^2y}$

$$\Rightarrow \left[(2+3x)^2 D^2 + 5(2+3x)D - 3\right] y = x^2 + x + 1 \quad \text{— (1)}$$

Put, $2+3x = e^z \Rightarrow z = \log(2+3x)$

$(2+3x)D = 3\theta$

$(2+3x)^2 D^2 = 3(\theta-1)\theta\times 3$] Put in (1)

$$\Rightarrow \left[9\theta(\theta-1) + 5(3\theta) - 3\right] y = \left(\frac{e^z-2}{3}\right)^2 + \left(\frac{e^z-2}{3}\right) + 1$$

$$\Rightarrow (9\theta^2 + 6\theta - 3)\, y = \frac{1}{9}\left(e^{2z} - e^z + 7\right)$$

$$(3\theta^2 + 2\theta - 1)\, y = \frac{1}{27}\left(e^{2z} - e^z + 7\right)$$

$AE \Rightarrow 3m^2 + 2m - 1 = 0$

$(3m-1)(m+1) = 0 \Rightarrow m = 1/3, -1$

$CF = c_1 e^{-z} + c_2 e^{z/3} = c_1 (3x+2)^{-1} + c_2 (3x+2)^{1/3}$

$PI = \dfrac{1}{3\theta^2 + 2\theta + 1} \left[\dfrac{1}{27} (e^{2z} - e^{z} + 7) \right]$

$\Rightarrow \dfrac{1}{27} \left(\dfrac{e^{2z}}{3\theta^2 + 2\theta - 1} - \dfrac{e^{z}}{3\theta^2 + 2\theta - 1} + \dfrac{7e^{0z}}{3\theta^2 + 2\theta - 1} \right]$

$\Rightarrow \dfrac{1}{27} \left(\dfrac{e^{2z}}{15} - \dfrac{e^{z}}{4} - 7 \right) = \dfrac{1}{27} \left(\dfrac{(3x+2)^2}{15} - \dfrac{(3x+2)}{4} - 7 \right]$

$CS = CF + PI$

$\Rightarrow y = c_1 (3x+2)^{-1} + c_2 (3x+2)^{1/3} + \dfrac{1}{27} \left[\dfrac{(3x+2)^2}{15} - \dfrac{(3x+2)}{4} - 7 \right]$

(16) $a_0 = \dfrac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot dx = \dfrac{1}{\pi} \left[\int_{-\pi}^{0} -\pi \cdot dx + \int_{0}^{\pi} x \cdot dx \right]$

$\Rightarrow \dfrac{1}{\pi} \left[(-\pi x)_{-\pi}^{0} + \dfrac{x^2}{2} \Big|_{0}^{\pi} \right] = \dfrac{1}{\pi} \left[0 - \pi^2 + \dfrac{\pi^2}{2} \right]$

$= \dfrac{1}{\pi} \left(\dfrac{-\pi^2}{2} \right) = -\pi/2$

$a_n = \dfrac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(nx)\, dx$

$= \dfrac{1}{\pi} \left[\int_{-\pi}^{0} \pi \cdot \cos(\pi x)\, dx + \int_{0}^{\pi} dx \cdot \cos(\pi x) \cdot dx \right]$

$$= \frac{1}{\pi}\left[\left(\frac{-\pi \sin nx}{n}\right)\Big|_{-\pi}^{0} + \left(\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right)\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[0 + 0 + 0 + \frac{(-1)^n}{n^2} + \frac{(-1)}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \, dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi \cdot \sin(nx) \cdot dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi \cdot \sin(nx)\, dx + \int_{0}^{\pi} x \cdot \sin nx \cdot dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{\pi\cos nx}{n}\right)_{-\pi}^{0} + \left(\frac{-x\cos nx}{n} + \frac{\sin(nx)}{n^2}\right)_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n} - \frac{2\pi(-1)^n}{n}\right] = \frac{1-2(-1)^n}{n}$$

$$f(x) = \frac{-\pi}{4} + \frac{1}{\pi}\left[\frac{-2\cos x}{1^2} - \frac{2\cos 3x}{3^2} - \cdots\right]$$

$$+ \left[3\sin x - \frac{1}{2}\sin 2x + 3\,\frac{\sin 3x}{2} - \frac{\sin 4x}{4} + \cdots\right]$$

$$f(x) = \frac{x-\pi}{2} \Rightarrow x = 0$$

$$f(x) = -\pi/2$$

$$\Rightarrow \frac{-\pi}{2} = \frac{-\pi}{4} + \frac{1}{\pi}\left[(-2)\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]\right]$$

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$$\Rightarrow \frac{\pi}{4} - \frac{\pi}{2} = \frac{-2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \cdots\right]$$

$$\boxed{\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \cdots}$$

(17) $f(t) = 1 - t^2$ for $-1 \le t \le 1$

$f(t) = 1 - t^2 \Rightarrow$ (Even $f(t)$) $\Rightarrow b_n = 0$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{2}{1}\int_0^1 (1-t^2)\,dt$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\cdot\cos\left(\frac{n\pi x}{l}\right) = \frac{2}{1}\int_0^1 (1-t^2)\cos n\pi t\,dt$$

$$2\left[\int_0^1 \cos(n\pi t) - \int_0^1 t^2 \cos(n\pi t)\right]$$

$$\Rightarrow 2\left[\left(\frac{\sin n\pi t}{n\pi}\right) - 2\left[\frac{-t\cos n\pi t}{n^2\pi^2} + \int \frac{\cos n\pi t}{n^2\pi^2}\right]\right]$$

$$\Rightarrow 2\left[-\left(\frac{t^2\sin(n\pi t)}{n\pi}\right) - 2\left[\frac{-t\cos n\pi t}{n^2\pi^2} + \frac{\sin(n\pi t)}{n^3\pi^3}\right]\right]_0^1$$

$$\Rightarrow 2\left[-\left(\frac{t^2\sin n\pi t}{n\pi} + \frac{2t\cos n\pi t}{n^2\pi^2} - \frac{2\sin(n\pi t)}{n^3\pi^3}\right)\right]_0^1$$

$$\Rightarrow 2\left[\left(0 - \frac{2(-1)^n}{n^2\pi^2} - 0\right) - (0-0-0)\right]$$

$$= \frac{-4(-1)^n}{n^2\pi^2}$$

Fourier Series

$$f(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{-4(-1)^n}{n^2\pi^2}\cos n\pi t$$

$$f(t) = \frac{2}{3} - \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos(n\pi t)$$

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(18) $$f(x) = \begin{cases} 1+\frac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$

$$a_0 = \int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0}\left(1+\frac{2x}{\pi}\right)dx + \int_{0}^{\pi}\left(1-\frac{2x}{\pi}\right)dx\right]$$

$$\Rightarrow \frac{1}{\pi}\left[\left(\cancel{2}\cdot x+\frac{x^2}{\pi}\right)_{-\pi}^{0} + \left(x-\frac{x^2}{\pi}\right)_{0}^{\pi}\right]$$

$$a_n \Rightarrow \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \cdot dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}\left(1+\frac{2x}{\pi}\right)\cos nx \cdot dx + \int_{0}^{\pi}\left(1-\frac{2x}{\pi}\right)\cos nx \cdot dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}\left(\cos nx + \frac{2x\cos nx}{\pi}\right)dx + \int_{0}^{\pi}\left(\cos nx - \frac{2x\cos nx}{\pi}\right)dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{2}{\pi n^2} - \frac{2(-1)^n}{n\pi^2}\right) + \left[\frac{-2(-1)^n}{\pi n^2} + \frac{2}{\pi n^2}\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{2}{\pi n^2} - \frac{2(-1)^n}{\pi\pi^2} - \frac{2(-1)^n}{\pi n^2} + \frac{2}{\pi n^2}\right] = \frac{4\left(1-(-1)^n\right)}{(\pi n)^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}\left(1+\frac{2x}{\pi}\right)\sin nx\,dx + \int_{0}^{\pi}\left(1-\frac{2x}{\pi}\right)\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}\left(\sin(nx) + \frac{2x\sin(nx)}{\pi}\right)dx + \int_{0}^{\pi}\left(\sin nx - \frac{2x\sin nx}{\pi}\right)dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{\cos nx}{n} - \frac{2\sin(nx)}{\pi n^2} + \frac{2x\cos nx}{\pi n}\right]_{0}^{\pi}$$

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$$\frac{1}{\pi}\left[\frac{-1}{n}-\left(-\frac{(-1)^n}{n}+\frac{2\pi(-1)^n}{\pi n}\right)\right]+\left(\frac{-(-1)^n}{n}+\frac{2\pi(-1)^n}{\pi n}-\left(\frac{-1}{n}\right)\right)$$

$= 0$

$$\Rightarrow f(x)=\sum_{n=1}^{\infty} \frac{4(1-(-1)^n)}{\pi^2 n^2}\cos nx = \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^n \cos nx}{n^2}$$

$$f(x) = \frac{4}{\pi^2}\left[\frac{2\cos x}{1^2}+\frac{2\cos 3x}{9}+\frac{2\cos 5x}{5^2}+\dots\right]$$

$$f(x) = \frac{1+\frac{2x}{\pi}+1-\frac{2x}{\pi}}{2} = \cancel{2}/2 = 1 \quad \boxed{\begin{matrix}\text{Put } x=0\\ \cos 0 = 1\end{matrix}}$$

$$1 = \frac{4}{\pi^2}\left(2\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\right)\right]$$

$$\Rightarrow \frac{8}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\right]=1$$

$$\Rightarrow \boxed{\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\right)=\pi^2/8}$$

(19) Half range consine series $= \begin{cases} kx & 0 \le x \le l/2 \\ k(l-x) & l/2 \le x \le l \end{cases}$

$$f(x) = \frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left[\frac{n\pi x}{l}\right]$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\cdot dx$$

$$a_0 = \frac{2}{l}\int_0^{l/2} kx\, dx + \int_{l/2}^{l}(kl-kx)\,dx$$

$$a_0 = \frac{2}{l}\left(\frac{kl^2}{8}+\cancel{kl^2}-\frac{\cancel{2}kl^2}{\cancel{2}}+\frac{kl^2}{8}\right)$$

$$a_0 = \frac{2}{l}\left(\frac{kl^2}{4}\right)=\frac{kl}{2}$$

$$a_n = \frac{2}{l}\int_0 \frac{kx}{} \cos\left(\frac{n\pi x}{l}\right) dx + \int_{l/2}^{l} k(l-x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_0 = \frac{2k}{l}\left[\frac{lx \sin\left(\frac{n\pi x}{l}\right)}{n\pi} + \frac{l^2}{n^2\pi^2}\cos\left(\frac{n\pi x}{l}\right)\right]_0^{l/2}$$

$$+\left[\frac{l^2 \sin\left(\frac{n\pi x}{l}\right)}{n\pi}\right]_{l/2}^{l} - \left(\frac{lx\sin\left(\frac{n\pi x}{l}\right)}{n\pi}\right.$$

$$\left.+\frac{l^2}{n^2\pi^2}\cos\left(\frac{n\pi x}{l}\right)\right]_{l/2}^{l}\Bigg]$$

$$\Rightarrow a_n = \frac{2k}{l}\left[\frac{l^2(-1)^n}{2n\pi} - \frac{l^2}{n^2\pi^2} - \frac{l^2(-1)^n}{n\pi} - \frac{l^2(-1)^n}{n^2\pi^2} + \frac{l^2(-1)^n}{2n\pi}\right]$$

$$\Rightarrow a_n = \frac{-2kl}{n^2\pi^2}\left[1+(-1)^n\right]$$

$$f(x) = \frac{kl}{4} + \sum_{n=1}^{\infty}\left[\frac{-2kl}{n^2\pi^2}\right]\left(1+(-1)^n\right]\cos\left(\frac{n\pi x}{l}\right)$$

$$\boxed{f(x) = \frac{kl}{4} - \frac{2kl}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{1+(-1)^n}{n^2}\right]\cos\left[\frac{n\pi l}{l}\right]}$$

(20) $f(x) = \cos ax, \quad -\pi < x < \pi$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{l}}$$

$$C_n = \frac{1}{2l}\int_{-l}^{l} f(x) e^{\frac{-in\pi l}{l}} dx$$

$$\int e^{ax}\cos bx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right] + c$$

Comparing, here $\underline{a = in}$, $\underline{b = a}$ $\Rightarrow a^2+b^2 = \underline{-n^2+a^2}$

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$$\Rightarrow C_n = \frac{1}{2\pi}\left[\frac{e^{inx}}{a^2-n^2}\left[-in\cos ax + a\sin ax\right]\right]_{-\pi}^{\pi}$$

$$C_n = \frac{1}{2\pi}\left[\frac{e^{in\pi}}{a^2-n^2}(-in\cos a\pi + a\sin a\pi)\right] - \frac{1}{2\pi}\left[\frac{e^{in\pi}}{a^2-n^2}\right.$$

$$\left.(-in\cos a\pi - a\sin a\pi)\right]$$

$$\left\{\begin{array}{ll} e^{-in\pi} = e^{in\pi} = (-1)^n & e^{i\theta} = \cos\theta + i\sin\theta \\ e^{-i\theta} = \cos\theta + i\sin\theta \quad (\theta = n\pi) & \Rightarrow e^{in\pi} = \cos n\pi = (-1)^n \\ \text{here } \theta = n\pi \Rightarrow e^{-in\pi} = \cos n\pi - i\sin \pi n & \\ \Rightarrow e^{-inx} = (-1)^n & \end{array}\right\}$$

$$C_n = \frac{1}{2\pi}\left[\frac{(-1)^n}{a^2-n^2}\left(-in(-1)^a + a\sin a\pi\right)\right] - \frac{1}{2\pi}\left[\frac{(-1)^n}{a^2-n^2}\right.$$

$$\left.[-in(-1)^a - a\sin a\pi)\right]$$

$$C_n = \frac{1}{2\pi}\cdot\frac{(-1)^n}{a^2-n^2}\left[-in(-1)^a + a\sin a\pi + in(-1)^a + a\sin a\pi\right]$$

$$C_n = \frac{1}{\pi}\,\frac{(-1)^n}{a^2-n^2}\left[a\sin a\pi\right]$$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{l}}$$

$$\boxed{\cos ax = \frac{a\sin a\pi}{\pi}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{a^2-n^2}e^{inx}}$$

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