

■ ■

Welcome to the ■

■

new session. In this session, we are going ■  
to focus on the relationship between price ■

■

and demand through a curve called demand response ■  
curve. So, what is the demand response curve? So, ■

■

generally the retailers are going to offer a price ■  
for a particular product or service in the market. ■

■

Now, the market is going to react ■  
by realizing a particular demand ■

■

at that particular price point. ■

So, the demand response curve essentially tells us ■

■

what is the realized demand at a price that is ■  
offered for a particular product or service. ■

■

Now, why is this a business analytics topic? ■

Well, the demand response curve itself ■

■

is a to two variable plot price on the x axis ■  
and demand on the y axis price on the x axis ■

■

demand on the y axis. So at a particular point, ■  
we keep changing the price and we keep realizing ■

■

the demand at that particular price. ■

So, today's session or a couple of sessions ■

■

are going to focus on how we estimate ■  
this demand response curve. So, we are ■

■

going to discuss various aspects of various ■  
relationship types between price and demand. ■

■

And in general, the basic mechanism of this ■  
price and demand relationship through this ■

■

demand response curve. ■

So, typically, this is what a ■

■

demand response curve is going to look like. ■

So, we have this particular curve, a rough ■

■

curve which is done like this, this is what we ■  
are going to call the demand response curve. ■

■

So, what does it say? So, at a particular price ■  
 let us say  $P_1$  this is the price offered and you ■  
 ■  
 hit the curve you look at this left and this ■  
 $Q_1$  is the quantity demanded by the market ■  
 ■  
 at that particular price point. ■  
 Now, we know that we are going to adjust ■  
 ■  
 our demand based on the prices that are offered ■  
 in the market. For example, if the price is  $P_3$ , ■  
 ■  
 the quantity that is going to be demanded by the ■  
 market is going to be  $Q_3$ . So, every retailer has ■  
 ■  
 to decide what is the best price and let us call ■  
 this best price as  $P^*$  what is the best price ■  
 ■  
 to be offered for that particular product or ■  
 service. At that particular optimal let us say ■  
 ■  
 optimal price point there is going to be some ■  
 optimal demand realized in the market. ■  
 ■  
 ■  
 Now, let us say that we make a mistake in ■  
 calculating this optimal price and we end up ■  
 ■  
 pricing more. There is always a potential for the ■  
 retailer to reduce the price which means go left ■  
 ■  
 to reduce the price and capture more demand ■  
 because at  $P^*$  the demand is going to be ■  
 ■  
 $Q^*$ , if we reduce the price if we reduce ■  
 the price from  $P^*$  to  $P_1$ , the demand ■  
 ■  
 is going to jump up from  $Q^*$  to  $Q_1$ . ■  
 So, this is that there is always a latent demand ■  
 ■  
 which is realized by this part of the region. This ■  
 region represents the latent demand. How do we ■  
 ■  
 capture the latent demand? By reducing the price, ■  
 by reducing the price we can capture more and more ■  
 ■  
 demand. Now, this portion of the plot represents ■  
 what is called consumer surplus. Now, consumer ■  
 ■  
 surplus essentially is the benefit that the ■  
 consumer gets by paying less and less price. ■

■  
■  
So, if we are actually increasing the price if ■  
we are actually increasing the price from P star ■

■  
to P3, let us say the consumer surplus is going ■  
to be eaten away that much. So, this light blue ■

■  
shaded region is going to be the reduction in the ■  
consumer surplus that the consumer is getting, ■

■  
because the price is at P star and not at P3. ■  
If the price are if the prices are increased ■

■  
from P star to P3, the consumer surplus is ■  
going to consumer surplus is going to go down ■

■  
by this much amount by the ■  
light blue region. ■

■  
■  
So, this is how the region is going to be ■  
read, this is how the plot has to be read ■

■  
as I was saying. So, this optimal price P ■  
star has to be very carefully chosen. So, ■

■  
if we reduce the price, there is always a scope to ■  
capture more demand, if we increase the price the ■

■  
consumer surplus may get affected we have I mean ■  
just like we are talking about consumer surplus ■

■  
we can talk about the producer surplus I ■  
mean I am pointing you to the basic economics ■

■  
course which is the slightly outside the ■  
current discussion on forecasting the curve, but ■

■  
nevertheless important for the discussion. ■  
Another important question then is if this is the ■

■  
price rate, if this is the demand response curve, ■  
how should the retailer decide this optimal price? ■

■  
What should be the objective of finalizing this ■  
price? Now, there are a couple of ways that we ■

■  
can go about. One is what is called as revenue ■  
maximizing price a revenue maximizing price ■

■

every retailer wants to maximize the revenue the money that they collect from the customers of the product and service. So, that should always be a concern.

However, we have to realize that maximizing revenue may not be the same as maximizing profit.

So, there is a profit maximizing

price, the price at which the profit is maximized, the price at which the revenue is maximized and

we have to keep in mind that these two may be different objectives. Sorry, these are different

objectives. We have to keep in mind that the optimal prices when we are maximizing revenue

may be different from the optimal prices when we are maximizing the profit.

How do we go about with these two objectives? Let us discuss that later. So, this is the basics of

the demand response curve which is the blue curve which is pointed here. Let

us understand the properties of this. So, this is essentially a function that describes

how the demand varies as a function of price very similar to the demand supply curve in economics,

however, this is for a single seller in a single market whereas, the demand supply curve aggregates

various supply in the market and aggregates various demands in the market.

So, this is slightly different from that scenario, where we are considering a single seller at

a single time point in a single market. If you notice, there are four important things

that you notice from this demand response curve. First of all, the demand for a product is always

going to be non-negative, which means that it is always going to lie on the positive side of the y axis. You cannot have demand going negative. You cannot have demand going less than 0 does not make sense. Demand cannot be negative. So, the demand response curve is always going to be on the positive side. Similarly, you cannot offer negative prices, I mean, let us not get into the details, but negative prices would mean that the retailer gives money to the customer for using the product and services, which generally does not happen in the market. Therefore, even the prices are going to be on the positive side.

So, you do not expect prices to go on the negative side. The lowest price is going to be 0. We are not going to look at a case where negative prices are even possible. So, the curve is going to be non-negative in both senses. The demand is going to be positive even if the prices offered in the market are going to be positive. It is going to be a continuous curve. So this is going to be a nice smooth continuous curve without any breakages without any breakages.

So, for example, this is the price and this is the demand. You do not expect something like this, which means that you do not know what the demand is going to be between these two price points.

So, what is going to be the demand? So, are we saying that there is no demand, but that also means that demand is 0. So, there is not going to be a discontinuity in the curve, there is not

going to be a discontinuity in the curve. ■

Very similarly, the curve is going to be very ■

■

smooth and differentiable which means that ■

tangent is always possible tangent is always ■

■

possible at all the price points tangents is ■

always possible at all the price points. So, ■

■

the curve is also going to be differentiable. ■

And unless we are talking about very specialized ■

■

goods, that curve is generally going to be ■

downward sloping. What does it mean? ■

■

■

It means that as the prices ■

increase the demand for the product ■

■

reduces. There are certain products for ■

example you can always think of Giffen goods. ■

■

Examples could be luxury items ■

for example, Rolex watches, ■

■

Rolex watches sometimes if you increase the prices ■

if you increase the prices, the demand for the ■

■

product may actually go up because the exclusivity ■

appeal for the product may go up. ■

■

■

So, but generally speaking, we ■

are going to look at goods whereas ■

■

the prices are increasing, the demand is actually ■

going to come down. So if the prices increase, ■

■

the demand is going to come down and therefore, ■

the curve is going to be a downward sloping curve, ■

■

downward sloping curve. So, these are ■

the four important properties once again, ■

■

the quantity is going to be non-negative, ■

the prices are going to be non-negative, ■

■

the prices are going to be non-negative. ■

So, it is first quadrant curve, the curve is cost ■

■

and cost nice, which means that it is continuous ■

it is differentiable and more importantly, ■

■  
we are going to look at a scenario where the curve ■  
is downward sloping the curve is downward sloping, ■

■  
which means that as the prices go up, the ■  
demand goes down, the demand goes down. ■

■  
So, these four these four properties are ■  
important. As I said, sometimes some goods ■

■  
may or may not hold but we are right now going ■  
to not worry about those kinds of goods. Right ■

■  
we are going to look at goods where the demand ■  
response curve is a downward sloping curve.■

■  
[Music]■

■  
so■

■  
two ways of looking at how how we■

■  
measure the price sensitivity uh how how■

■  
sensitive is the is the demand to the■

■  
price■

■  
right■

■  
so two ways of looking at it one is■

■  
simply one is simple slope which is■

■  
how much the demand changes with■

■  
response to the change in price■

■  
uh so it is essentially the change in■

■  
demand demand at uh price point  $p_2$  minus■

■  
demand at a price point  $p_1$  minus  $p_1$  ah■

■  
divided by  $p_2$  minus  $p_1$  so it's a it's■

■  
so essentially■

■  
the delta in the demand divided by delta■

■  
in the price right so very very uh ■  
■  
simple uh uh ratio of change in uh uh ■  
■  
change in demand divided by change in ■  
■  
price right so ■  
■  
as we know ■  
■  
since the curve is downward sloping ■  
■  
downward sloping ■  
■  
if  $p_1$  is greater than  $p_2$  this means that ■  
■  
the demand at  $p_1$  is lesser than demand ■  
■  
at  $p_2$  what does that mean ■  
■  
we go back ■  
■  
right we go back let us go back ■  
■  
so if ■  
■  
so let us look at ■  
■  
let us look at what is this point here ■  
■  
this point here is demand at  $p_3$  ■  
■  
and this point here is demand at  $p_1$  ■  
■  
now we know that  $p_3$  is greater than  $p_1$  ■  
■  
the price  $p_3$  is greater than price  $p_1$  ■  
■  
and we can also see that demand at ■  
■  
 $p_3$  demand at  $p_3$  is here which is  $q_3$  ■  
■  
so this is  $q_3$  which is always lesser ■  
■  
than demand at  $p_1$  which is  $q_1$  right so ■  
■  
 $q_1$  is greater than  $q_3$  ■  
■  
whereas  $p_3$  is greater than  $p_1$  so we know ■  
■



that right■

■

which means that this slope this slope■

■

if you define the slope it is going to■

■

be a negative■

■

negative value the slope is going to be■

■

negative value■

■

so we can we can we can■

■

look at slope as a local local estimator■

■

of change in demand for a very small■

■

change in price■

■

so very simple ah just calculate the■

■

calculate the slope of the slope of the■

■

tangent at that particular point and we■

■

will get the ah we will get the■

■

ah sensitivity■

■

ah■

■

for that particular price■

■

there is other way to calculate price■

■

sensitivity and that is called demand■

■

elasticity okay that is called demand■

■

elasticity■

■

so it is the ratio of percentage change■

■

in demand to the percentage change in■

■

price notice the difference notice the■

■

difference right■

■

notice the difference so essentially it■

■ is this is change in demand with change ■  
 ■ in price this is percentage change in ■  
 ■ demand with percentage change in price ■  
 ■ so you divide then the ■  
 ■ numerator by demand at p1 you divide the ■  
 ■ denominator by price p1 right so ■  
 ■ so essentially the numerator becomes a ■  
 ■ unitless quantity because you are ■  
 ■ dividing demand by demand so this the ■  
 ■ units cancel out ■  
 ■ the denominator becomes a unitless ■  
 ■ quantity because it is price divided by ■  
 ■ price so units cancel out so essentially ■  
 ■ unlike slope ■  
 ■ elasticity is unit less quantity it's a ■  
 ■ unit less quantity ■  
 ■ right uh so it is remember i let us ■  
 ■ define it again percentage change in ■  
 ■ demand to the percentage change in price ■  
 ■ right so percentage change in demand to ■  
 ■ the percentage change in price ■  
 ■ so for example elasticity of 2 would ■  
 ■ mean that a 10 percent reduction in ■  
 ■ price ■  
 ■ uh is essentially increases the demand ■  
 ■

by 20 percent right that that that's the

meaning of this

two so 20 percent increase so 10 percent

reduction in price increases the demand

by 20

right so we are we are talking

percentages we are talking percentages

in elasticity unlike slope where we are

talking about change in demand to change

in price

elasticity is percentage change in

demand to percentage change in price

so that's the difference between

elasticity and slope of the demand

response curve

now uh what do what what is the what is

the

general interpretation of elasticity

there are certain goods which are

supposed to be less elastic

for example

ah let us take the example of common

salt that we use in our food

now

salt is required without salt the food

is just not going to taste which means

■  
that even if the prices of salt go up i■  
■  
don't expect our consumption of salt to■  
■  
to change that much because salt is■  
■  
needed salt is essential quantity right■  
■  
without salt the the the food is not■  
■  
going to taste■  
■  
so i would expect the elasticity of■  
■  
something like salt to be much lesser■  
■  
right■  
■  
otherwise uh go to the other extreme ah■  
■  
think about uh think about■  
■  
a service like a holiday■  
■  
right a service like a holiday now uh uh■  
■  
holiday uh■  
■  
if the if the holiday is going to cost■  
■  
us too much there is a very high■  
■  
probability that we may change our plan■  
■  
right most of us may want to change our■  
■  
plan we may still go to our holiday but■  
■  
we may probably■  
■  
choose a different service■  
■  
ah reduce the number of days or do■  
■  
something but essentially react to that■  
■  
change in price■  
■  
so■  
■

i would say that a holiday is a service■

■  
where the elasticity is generally quite■

■  
high right■

■  
so here uh there is another■

■  
thing that we have to look at is■

■  
elasticity may also depend on time■

■  
elasticity may also depend on time so■

■  
there is a short term elasticity and■

■  
there may be a long term elasticity so■

■  
here here is here are few examples right■

■  
so for example uh here are few examples■

■  
of as i said salt■

■  
for a product like salt for a product■

■  
like salt uh in short term i need salt i■

■  
just need salt right i mean there is■

■  
absolutely no■

■  
there is no alternative to that■

■  
so for a for a product like salt uh the■

■  
elasticity may be■

■  
i mean yeah zero right uh i i just i■

■  
just need salt that's it i just need■

■  
salt■

■  
uh for example on the other hand uh a■

■  
two wheeler a two wheeler right uh a two■

■  
wheeler i will say that elasticity is■

■  
quite large because uh uh if the■

■  
two-wheeler is going to cost me too much■  
■  
uh■  
■  
i will say ah■  
■  
let let me take a bus today let me take■  
■  
a bus today and not buy this two wheeler■  
■  
right so i i may i may i may look for■  
■  
alternate modes of transport at least in■  
■  
the short term at least in the short■  
■  
term■  
■  
i may i may look for alternates uh■  
■  
alternatives to buying a two wheeler■  
■  
so in that sense the the the short term■  
■  
elasticity of two wheeler may be much■  
■  
much higher than a short term elasticity■  
■  
for sure for salt■  
■  
now let us say that there is some■  
■  
emergency meeting that i have to attend■  
■  
ah there may be some emergency meeting■  
■  
that i have to attend and i just have to■  
■  
take a flight i mean there is no■  
■  
alternative i just have to take a flight■  
■  
i have to go attend a meeting■  
■  
i have to go attend to a personal thing■  
■  
uh i just need to travel■  
■  
so■  
■

in the short term■  
■  
in case of emergency for example we may■  
■  
argue that■  
■  
even if the prices are higher■  
■  
ah if the if the demand requires i mean■  
■  
if the if the situation requires that i■  
■  
travel i will have to travel i will have■  
■  
to travel■  
■  
so pop the airline travel may have a■  
■  
very very uh low short-term elasticity■  
■  
uh movies right movies uh if the movie■  
■  
tickets are expensive i may postpone■  
■  
i may say let me watch tv at home today■  
■  
instead of buying a 500 rupee■  
■  
movie ticket■  
■  
or something like that let let me go to■  
■  
my friend's house■  
■  
right but■  
■  
i i may want to postpone■  
■  
that that purchase because i may want to■  
■  
find alternatives whereas for a salt■  
■  
there is actually no alternative■  
■  
right so■  
■  
in those cases i may expect a larger■  
■  
short-term elasticity but as i said■  
■  
elasticity may also have a time axis■

■  
which means that the long term ■  
■  
elasticity may be different ■  
■  
for example for air travel ■  
■  
now if i have emergency requirement i ■  
■  
must reach there faster and therefore ■  
■  
whatever is the price i may want to pay ■  
■  
and catch that flight and reach my ■  
■  
destination ■  
■  
however in the long term in the long ■  
■  
term if i have if i can plan ■  
■  
and if the price tickets if the flight ■  
■  
tickets are really ■  
■  
expensive ■  
■  
i may still want to find alternate mode ■  
■  
of transportation and therefore in the ■  
■  
long term airline travel ■  
■  
may have a large value for elasticity so ■  
■  
in the short term emergency cases i ■  
■  
don't care if the price of the flight is ■  
■  
uh too much i have to travel therefore i ■  
■  
have to travel elasticity may be low in ■  
■  
the long term if you allow me to plan my ■  
■  
trip carefully ■  
■  
ah if the if the ■  
■  
price points are just not acceptable to ■  
■



me■

■  
i may find alternate modes of■

■  
transportation and therefore■

■  
the elasticity goes up significantly in■

■  
the long term■

■  
for salt■

■  
the elasticity may not change that much■

■  
right because as i said salt is■

■  
essential commodity salt is essential■

■  
commodity■

■  
so salt is essential commemoration it■

■  
may go up little bit■

■  
maybe i i will i will say that anyway■

■  
eating up■

■  
too much of salt is bad for my health i■

■  
may cut down on the quantity of salt i■

■  
eat■

■  
however the the the margin that i have■

■  
is is quite less therefore the■

■  
elasticity may not change drastically■

■  
as it would change for airline travel■

■  
ah look at uh look at two wheeler on the■

■  
other hand for a two wheeler uh today i■

■  
go to the showroom and i say the prices■

■  
of two wheeler is too much and therefore■

■  
let today at least at least for a week■

time let me manage with alternate modes

of transportation uh but in the long term if there is a demand for two wheeler there is a demand for two wheeler i mean really in the long term you really can't avoid so there are certain goods just wanted to show you that there are certain goods where the elasticity may come down over a period of time there are certain goods where the elasticity may go up drastically over a period of time a petrol is the example of later ah two wheeler as an example of former right so for movies movies uh uh i mean in general if the ticket prices are expensive i i may find alternate modes of alternate modes of transportation uh alternate modes of entertainment alternate modes of entertainment but uh i may still go for a movie in the long term right uh if you allow me to plan longer time maybe i'll buy subscription

to one of the ott and i never have to go  
for uh  
movie theater right  
so the elasticity may go up drastically  
over a longer period of time  
right so that that's how elasticity  
changes  
ah depending on the goods  
right depending on the goods and  
depending on the time frame depending on  
the time frame right sometimes it  
changes drastically like 0.1 to 2.4  
these are by the way examples we are not  
saying that airline travel has a  
elasticity of 2.4  
in the long term right  
it may depend on  
it may depend on  
general consumer behavior right for some  
people airline travel  
may not be that elastic because even in  
long term uh i i i may not want to  
prefer alternate modes of transportation  
uh if i want to go for a movie i i once  
again 3.7 is not going to be elasticity  
for everyone in the long term right so

■  
these are just ■  
■  
ah these are just ■  
■  
representative numbers they are not ■  
■  
elasticity values for everybody in the ■  
■  
short term or for the long term but i ■  
■  
hope i have conveyed the concept of ■  
■  
elasticity ■  
■  
depending on the product and services or ■  
■  
depending on the time frame depending on ■  
■  
the time access ■

■  
all right so ■

■  
just to recap we have looked at price ■

■  
sensitivity using two methods one is ■

■  
calculating the slope the other one is ■

■  
calculating the elasticity ■

■  
and we have interpreted both of them ■

■  
So, let us come to the different relationships ■  
between the price and demand. ■

■  
Different relationship. ■

■  
So, this this blue curve should the blue curve ■  
be a straight downward sloping line or should ■

■  
it be a nonlinear downward sloping, a downward ■  
sloping curve. ■

■  
How should the relationship be that that will ■  
also matter. ■

■  
The simplest possible example is a linear ■  
demand response curve. ■

■  
So, demand response curve. ■

■  
So, demand at a particular price point, demand  
at a particular price point  $p$  may have some

■  
initial value  $D_0$ , slope is given by  $m$  and  
 $p$  is the price.■

■  
So, what is the demand at a particular price?■

■  
So, this is the price and  $D_0$  minus  $m$  into  
 $p$  will be the demand at that particular price.■

■  
This  $D_0$  is called the demand at price 0.■

■  
When  $p$  is 0, what is the demand?■

■  
Demand is  $D_0$ .■

■  
So, this is the demanded price equal to 0.■

■  
This can also be called as the market size,■  
because look at the interpretation.■

■  
Price is 0, you are offering the product for  
free.■

■  
So, when you are offering the product for  
free, what, how many people actually demand■

■  
this product will really tell you what is  
the market size.■

■  
What is the complete market size?■

■  
What is the total market size?■

■  
Because you really cannot do anything better.■

■  
You are offering the product for free.■

■  
So, the demand that, that particular price  
point which is actually 0, you cannot really■

■  
use anything lesser than that.■

■  
Anything less than that actually means that  
you are actually giving money to the consumers■

■  
to use your product.■

■  
So, as we said, we are not going to consider■

that case, we are going to look at only non-negative

values.

So, price equal to 0 is really the minimum price.

At that price, what is the demand, that that is called the market size and  $m$  is called

the slope.

So, mathematically what is  $D_0$ ?

$D_0$  will be the y intercept.

We are talking about a linear response curve, so we are going to draw a straight line.

Let me change the color, so, let us say that this is the demand response curve.

As I said earlier, what is on the x, always remember what is on the x axis, x axis is

always the price and y axis is always the demand.

Demand responds to a particular price.

This guy here is  $D_0$  because the price is 0.

At price equal to 0, what is the total market size?

To be precisely correct, I should not have drawn this portion, I should simply start

here, and end here because as it is negative or non-negative.

So, I should not extend it to the negative region I should not extend it below the x

axis.

So, this is called  $D_0$  this is called the market size.

Now the price at which demand is equal to 0 is called satiating price.

■  
Let us represent that by a new color.■

■  
This is, this price is called satiating price.■

■  
Satiating price means, the demand is, the market is satiated.■

■  
That is it.■

■  
If you offer, if you increase the price beyond PS.■

■  
If you increase the price beyond PS, demand is expected to be 0.■

■  
This is this is really the highest price that you can charge.■

■  
This is really the highest price that you can charge.■

■  
So, that price is given by when the demand is equal to 0, so set this equal to 0 and■

■  
you are going to get the satiating price at as  $D_0$  divided by  $m$ , so that is called the■

■  
satiating price.■

■  
So, we have looked at the y intercept, which is the market size, we have looked at the■

■  
x intercept, which is called the satiating price.■

■  
The elasticity of this curve is given by this expression.■

■  
And you can verify from the definition of elasticity that this really is the elasticity■

■  
of this curve.■

■  
What we should notice what is important to notice is, this explanation you can derive■

■  
using the definition that we had a couple of slides ago.■

■  
What you should be more attentive towards■

is to realize that this elasticity depends■

on the price, which means that, the elasticity■  
does not remain constant in this region.■

Elasticity keeps changing.■

How does it change?■

When price is 0, the elasticity is 0, price■  
is 0 elasticity is 0.■

Whereas, when the price approaches  $P_s$  really■  
cannot charge more than  $P_s$  because at  $P_s$  the■

demand becomes 0, what is the point of charging■  
more than  $P_s$ , the demand has already become■

0.■

So, as the price approaches  $P_s$ , the elasticity■  
goes to infinity.■

So, elasticity can be as low as 0, it can■  
be as high as infinity when the price is approaching■

$P_s$ , when the price is approaching  $P_s$ .■

So, for a linear response curve elasticity■  
is not constant.■

elasticity can keep changing, alright.■

Take a moment to digest.■

So, this is ■  
a linear demand response curve, and therefore,■

it is a straight line downward sloping, positive,■  
continuous and differentiable and these are■

the interpretations.■

Interpretation of market size, interpretation■  
of satiating price and the elasticity interpretation.■

Now, the relationship between price and demand■  
need not always be linear, it may be nonlinear.■

So, there is another curve called constant■  
elasticity curve.■



■  
So, here we saw the elasticity keeps changing  
from 0 to infinity.■

■  
Now, what if we want to keep the elasticity  
constant?■

■  
That curve is given by this expression this  
expression demand is equal to  $C$  into  $p$  to■

■  
the power of negative, epsilon, epsilon is  
the elasticity.  $C$  is a constant  $C$  is the demand■

■  
when price is equal to 1, so this nonlinear  
relationship will represent a curve where■

■  
the elasticity does not change.■

■  
Elasticity is constant at a particular value  
of epsilon.■

■  
Now, you fix up the value of epsilon and the  
curve will change.■

■  
So, essentially, this is your price x axis  
is always the price y axis is always the demand,■

■  
and there may be a nonlinear relationship  
once again downward sloping, non-negative,■

■  
continuous and differentiable, but the shape  
of the curve is nonlinear this time.■

■  
What is the property the property is, we want  
to keep the elasticity constant.■

■  
Now, you cannot guarantee that the demand  
may be either finite or the demand even may■

■  
be satiated.■

■  
The market may be satiated.■

■  
Because you realize that as  $p$  approaches 0  
as you reduce the price if you reduce the■

■  
price, the demand keeps spiking, demand keeps  
growing.■

■  
So, demand may even approach infinity, as  
the prices are reducing, so you cannot say■

■  
that demand is finite, because demand may  
spike up, demand may spike up as you are closing

■  
towards 0 from the right, and you realize,  
that the demand may never touch the x axis.

■  
The demand may never touch the x axis.

■  
It may be asymptotic on the x axis.

■  
So, for any value of  $p$  the demand may or may  
not even become 0.

■  
There is always some demand, there is always  
some customer who is trying to buy the product

■  
at a price that you may offer.

■  
So, we cannot guarantee that our demand may  
be finite, we cannot guarantee that demand

■  
curve will ever hit the x axis, we cannot  
guarantee.

■  
Revenue for the previous curve also for this  
curve also revenue is always given by price

■  
multiplied by the demand.

■  
So, the total demand for the product multiplied  
by the price charged for each product will

■  
tell you the revenue.

■  
For the constant elasticity curve the revenue  
will be  $C$  into  $p$  into  $C$  into  $p$  to the power

■  
of  $1 - \epsilon$ .

■  
As I said earlier, the price may be optimized  
for maximizing the revenue.

■  
In that case you will maximize this curve  
and then find  $p^*$  or we may maximize profit

■  
and therefore, get an optimal value of price.

■  
So, if we are maximizing revenue, this is  
the function that we are trying to maximize.

■

Revenue is simply price multiplied by the demand.

You can do the same thing even for the linear demand response curve.

I hope things are clear so far.

So, let us move ahead.

But before we move ahead, so essentially, if the elasticity less than 1, what does it

mean?

It means that it is a product like salt, it is a product like salt where the product is

actually quite inelastic, which means that the demand does not change even if the prices

fluctuate.

As I said, we need salt that is it, we need salt cannot do without salt.

So, in that case, there is always a way to increase revenue simply by increasing the

prices.

Because revenues are prices multiplied by the demand.

Even if you increase the price, the demand is not going to change much and therefore,

revenues can be increased simply by increasing the prices.

However, if we are dealing with a product which is highly elastic, which means that

the elasticity is more than 1, then revenue making revenue probably can be increased by

setting up very, very low prices, setting up very, very low prices.

And therefore, essentially, mass selling therefore, we sell a lot and therefore generate more

■  
revenue.■

■  
Price of each unit may be quite small, but■  
that is compensated by huge demand because■

■  
we saw the curve as the prices are lesser■  
the demand may be shooting upwards and that■

■  
is how there is a mechanism to increase the■  
revenue.■

■  
But there may be other mechanisms to get more■  
profit, but right now, we are talking about■

■  
increasing the revenue because we saw the■  
revenue functions.■

■  
So, the strategies available for inelastic■  
product may be different from the strategies■

■  
available for a highly elastic product, highly■  
elastic demand.■

■  
Let us pause here for a moment and recap what■  
we have done.■

■  
So, we have looked at what is called as a■  
demand response curve.■

■  
Demand response curve is the relationship■  
between price and demand.■

■  
We have seen four important properties.■

■  
It is usually downward sloping, except for■  
few goods.■

■  
Luxury goods are good example where the downward■  
sloping may get violated.■

■  
Always non-negative, considering the case■  
that we are offering 0 as the minimum price■

■  
and it has nice properties that it is continuous■  
and differentiable.■

■  
We need continued continuity, we need differentiability■  
because we should be able to take derivative.■

■  
Because we want to maximize revenue, we may■

want to maximize profit, therefore, finding  
optimal prices, we may need these properties  
of continuity and differentiability.

So, these are the four nice properties of  
a demand response curve.

Demand sensitivity is measured using slope,  
which is just change in demand to change in  
prices.

Slope is generally negative or we can measure  
price sensitivity using what is called as  
elasticity.

Elasticity is percentage change in demand  
to percentage change in price, the ratio of  
these two.

So, usually unitless quantity a different  
interpretation from slope.

And one important property of elasticity is,  
it may have a time axis the elasticity value  
in short term may be different than the elasticity  
value in the long term.

Different relationship types between price  
and demand we can think of a very simple linear  
relationship between price and demand.

So,  $D_0$  minus  $m$  into  $p$  where  $m$  is the slope  
 $D_0$  is the  $y$  intercept,  $y$  intercept is also

called the market size, that is the demand  
when price is 0.

When the demand is 0, that price is called  
satiating price, elasticity of a linear demand  
response curve actually is not constant.

It keeps changing, it may be as low as 0,  
it may be very close to infinity when the

prices are fairly close to the satiating price.■

■

So that is a very simple linear demand response■  
curves.■

■

We can think about our nonlinear response■  
curve.■

■

For example, we can think about a curve where■  
the elasticity remains constant, and that■

■

curve is given by demand is equal to  $C$  into■  
 $p$  to the power of negative  $\epsilon$  where  $\epsilon$ ■

■

is the elasticity.  $C$  is a constant, because■  
it represents demand when price is equal to■

■

1.■

■

In such a curve, the basic properties are■  
not still violated, it is still a non-negative■

■

curve.■

■

It is still a downward sloping curve.■

■

It has continuity, it is differentiable.■

■

So, all those properties hold, but in this■  
kind of a curve, we cannot guarantee that■

■

the demand may be finite, we cannot guarantee■  
that the demand is always satiated.■

■

So, the revenue is simply price multiplied■  
by demand.■

■

And if we are dealing with a product, which■  
is complete, which is very inelastic the there■

■

is a scope to increase revenue by simply increasing■  
price.■

■

If we are dealing with a very, very elastic■  
product, the revenue can be increased by reducing■

■

the price setting the price fairly close to■

0.■

■

So, let us end the session here.■

■

And we will continue with this.■

■  
As I said the objective of this topic is to■  
be able to estimate this demand response curve,■

■  
but we will come to that in the next session.■

■  
So, let us pause and come back for the next■  
session of this later.■

■  
[Music]■

■  
okay welcome back to the session on■

■  
demand response curve we were looking at■

■  
we were looking at the relationship■

■  
between price and demand in the last■

■  
session we saw the relationship■

■  
to have four properties non-negativity■

■  
downward sloping continuity and■

■  
differentiability■

■  
we saw a slope and elasticity to be the■

■  
way to measure price sensitivity■

■  
and we saw two types of relationship one■

■  
was a simple linear relationship■

■  
and the other one was uh■

■  
relationship which maintains the■

■  
elasticity in the in a in the linear■

■  
demand response curve■

■  
ah the elasticity may not be constant■

■  
however in a constant elasticity■

■  
relation relationship the elasticity■

■  
remains constant■

■  
but the relationship becomes non-linear■  
■  
we saw that ah we saw that the■  
■  
mechanisms for increasing the revenue■  
■  
may be different whether we are dealing■  
■  
with a different for■  
■  
ah inelastic product and it may be■  
■  
different for a elastic product■  
■  
now the now let us continue with the■  
■  
further question what is the what is the■  
■  
what is the■  
■  
analytics problem here■  
■  
the analytics problem here■  
■  
is to be able to estimate a demand■  
■  
response curve■  
■  
right■  
■  
so we can we can think of conducting an■  
■  
experiment in the market where we offer■  
■  
different prices■  
■  
and at those different prices we check■  
■  
what is the realized demand at that■  
■  
particular time so for example■  
■  
ah for for example let me change this■  
■  
thing■  
■  
so i may offer a price i may offer a■  
■  
price■  
■



and actually measure the  
quantity demanded in the market so let  
us say that for a particular product let  
us not even  
give a specific product  
let us say that i offer a price of three  
three rupees what is the realized demand  
if i offer a price of six rupees what is  
the demand if i offer a price of two  
rupees what is the demand if i offer a  
price of 10 rupees what is the demand  
right now i don't expect the values to  
become negative because we are dealing  
with non-negative  
relationships right non-negative  
values  
so even if i say that the price is 50 i  
will say ah probably the demand comes  
down to 0 may not come down to 0 but  
but 0 is the  
the  
smallest value that i am going to  
observe  
so  
essentially we want to collect this kind  
of a data through an experiment

■  
so essentially we will have prices ■  
■  
offered and corresponding realized ■  
■  
demand values ■  
■  
so essentially  $a_h$  ■  
■  
demand can be considered to be the ■  
■  
dependent variable demand  $a_h$  is a ■  
■  
reaction of the market demand is a ■  
■  
reaction of the market ■  
■  
prices are the triggers for that ■  
■  
reaction therefore prices can be ■  
■  
considered to be an explanatory variable ■  
■  
prices explain the difference in the ■  
■  
demand values ■  
■  
so prices can be considered to be ■  
■  
explanatory variable demand can be ■  
■  
considered to be dependent variable and ■  
■  
let us say that from some experiment ■  
■  
that we have conducted in the ■  
■  
marketplace at a particular time ■  
■  
i have this data available now from this ■  
■  
i want to estimate what may be the slope ■  
■  
i want to estimate what may be the ■  
■  
elasticity ■  
■  
 $a_h$  ■  
■  
the elasticity may not be constant ■  
■

however if my relationship looks like a  
relationship where  
i have  
i have  $d$  is equal to  $c$  into  $p$  to the  
power of negative epsilon i may think of  
having a constant elasticity right uh  
slope  
may come in only when i am talking about  
a linear relationship  
so pop  
soap  
linear relationship so essentially first  
of all i may have to think about what  
kind of relationship i want to fit if i  
want to fit a linear relationship  
between price and demand i may be  
interested in finding the slope  
if i am thinking of  
if i am thinking of  
fitting a relationship where the  
elasticity may be constant  
i may i may want to calculate the  
elasticity i may want to estimate the  
elasticity from the data that i may have  
right  
so the value of slope i am going to get

is only going to be an estimate of slope

from the data that I may have

may not have the entire population

values I may have collected a small

sample value of prices and the

corresponding demand at those particular

prices

So, what does this problem look like? Any guesses. To me, this problem looks like a

simple linear regression problem.

So, a Simple Linear Regression, SLR, Simple Linear Regression, may help us calculate the

y-intercept. Which are the market size and the slope. So, SLR will tell us if the linear

relationship is a good fit for the data available from the market experiment.

Now, what do we do when we are thinking about a non-linear relationship where we are, we

want to keep the elasticity constant, what do we do with that? Can we use SLR in those

cases? What is the guess? Can we use SLR in those cases? What is the answer? Probably,

yes. And we will see how that can, how SLR can even help in that case. So, what I have

is sample data and let us build a corresponding simple linear relationship, a simple linear

regression model for that data.

So, let me stop the presentation and go

to the Excel sheet. So, let us say, let me increase the zoom here. So, let us say that

■ these are the various prices offered and these ■  
were the corresponding demand at those particular ■  
■ prices, various demands at these particular ■  
prices. When the price was 9 rupees, the demand ■  
■ realized was 2,891, when the demand was 32, ■  
the price, when the price was 32, the demand ■  
■ was 370, when the price was 26, the demand ■  
was 2,946. So, these are the different combinations ■  
■ of price and demand at that particular price. ■

■ So, let us see if the linear relationship ■  
fits better. So, what are we going to do? ■

■ Let us understand the plot. So, what I am ■  
going to do is draw a scatterplot. So, the ■

■ x-axis is the price, the y-axis is the demand. ■  
This is a typical demand response curve. So, ■

■ these are the realized points. So, when the ■  
price was 5 rupees, the realized demand was ■

■ 6,707. When the price was 39, the realized ■  
demand was 297. When the price was 35, the ■

■ realized demand was 484. When the price was ■  
40, the realized demand was 193. ■

■ So, now let me try and fit a linear relationship ■  
between price and demand. Let us try to fit ■

■ that. So, what I will do? I will go, add a ■  
trend line. I will add a linear trend line. ■

■ And I will ask Excel to print the equation ■  
and the r square for the relationship. Do ■

■ we know how to interpret this equation? And ■  
do we know how to interpret this r square? ■

■ Do you recall your DM course, probably it ■  
was discussed there? Anyway, does not matter. ■

■ Let us read this value. ■

■ So, what is this? This is, what is the equation? ■

The equation is  $y$  is equal to 5842.8 minus

157.7 into  $x$ . What is  $y$ ?  $Y$  is demand. What is on the  $y$ -axis?  $Y$ -axis has demand. Demand

is 5843. Let me write, let me round this up to point 8, I will write this as 5843 minus,

I will round this up and I will say 158 into the price because the  $x$ -axis is the price.

So, how do you interpret this 5843? Compare that with a linear relationship which is  $D$

is equal to  $D_0$  minus  $m$  into  $p$ , so this is your  $D_0$ . This is the demand when the price

is equal to 0. Even if you offer the product for free, what is going to be the demand?

The demand is going to be 5,800 odd.

Now, for a unit change in price, per unit change in price how much do you expect the

demand to change, that is given by the slope. So, for a unit change in demand, for a unit

change in price, the demand is going to go down by 158 units, a negative slope. You increase

the price by 1 unit, the demand is going to go down by 158 units. That is what the predicted

equation says, prediction equation says.

Now, what is this  $R$  square of 0.733? What does that mean? Do you recall? For that let

us run a simple linear regression model. How do you run a simple linear regression model

in Excel? So, I go to data. I have a chance to show you the Excel add-in. There is no

data analysis toolpak here. So, let me add a data analysis toolpak. So, we will go to

options, go to add-ins and I will ask Excel to add analysis toolpak, data analysis toolpak.

I will say go. So, I will add an analysis toolpak. Let me not add solver add-in right

now. Let us add that.

Now, you see one more option available here called data analysis. Let me use that data

analysis. Let me go to the option called regression. What are my y values? My y values, my response

variable, my dependent variable is demand. So, I will say this is my variable. What is

the explanatory variable, x values, the x-axis is the price? So, these are the various price

values. I will tell Excel, this is where the prices are. Labels, yes, I have labels in

the first row, yes. Output range, let me ask Excel to print the output here and let us

print it. So, this is how Excel runs regression.

Now, let us understand, let us interpret these values. So, this 0.85, is multiple R. What

is multiple R? If you recall, Excel had an option called regression. What we are running

is what is called simple linear regression. Excel does not differentiate between simple

linear regression and multiple linear regression. So, Excel simply says regression. Therefore,

the R that is run, the R that is calculated is called multiple R, more suited for MLR,

Multiple Linear Regression, but this R represents the correlation coefficient between the price

and the demand.

Do you recall what the correlation coefficient is? You must recall it from the DM course.

So, how do you, independently how would you calculate a correlation coefficient? So, there

is a function in Excel called CORREL, correlation, and you say that these are, this is my array

number one, you do not have to have the label, and this is my array number two, it is negative

0.85. Negative relationship, because as the prices are going up, the demand is coming

down. So, this is the correlation coefficient between prices and demand.

Now, it should not matter whether I put the demand array first or the pricing, the prices

array first. So, here I have put the demand array first and the prices array later, does

not matter. Correlation is, does not depend on which variable is entered first. So, even

if I entered this first, this later, the value should not change. So, it does not matter.

It is just the relationship, it is just the correlation coefficient between two arrays

of data.

So, what does this 0.855 represent? 0.855 represent a correlation coefficient between

price and demand. The negative value represents the sign of correlation. And the value 0.85

represents the strength of correlation. The value is closer to 1 stronger than the association

between the variables and the negative or positive value only tells us the direction

of the association. As the prices go up the demand goes down, and therefore, that is represented

by this negative value.

Now, what is the R square? R square is simply the square of the R. If I square this correlation

coefficient, if I do this, this square, I am going to get R square. R square is the



■ square of the correlation coefficient. Now, ■  
there is one more interpretation of the R ■

■ square. Let us do that little later. Now, ■  
what is that adjusted R square? Let us leave ■

■ the adjusted R square discussion also to a ■  
case when we are discussing multiple linear ■

■ regression. Adjusted R square only makes sense ■  
when we have more than one explanatory variable. ■

■ So, let us go back to PPT. Now, I have explained ■  
the important things. I have explained the ■

■ prediction equation. How do I predict demand? ■  
So, I have predicted the y-intercept which ■

■ is 5843, the predicted value of the slope ■  
is, the estimated value of the slope is 158, ■

■ and we have explained R square which is the ■  
square of the correlation term. Let me go ■

■ back to PPT and explain a few more details ■  
about the simple linear regression. ■

■ So, what is simple linear regression? Simple ■  
linear regression describes how the conditional ■

■ mean of Y changes with different values of ■  
X. So, what is conditional mean? What do you ■

■ mean by conditional mean? You give me a value ■  
of X, I will tell you what is the value of ■

■ Y. what is the value of Y? Value of Y is going ■  
to be beta naught plus beta 1 x. What is this ■

■ beta naught? Beta naught is D0 for us. What ■  
is beta 1? Beta 1 is m for us. So, this is ■

■ called the conditional mean of y with respect ■  
to x. ■

■ So, the simple linear regression model, simple ■  
regression model, simple linear regression ■

■ model essentially tells us that it is going ■  
to be a line with an intercept of B0 and a ■

■  
slope of  $B_1$ . The intercept was the y-intercept ■  
which is  $D_0$ , which is the market size. This ■

■  
is, however, the expected value of  $Y$ . This ■  
is the expected value of  $Y$ . So, however, the ■

■  
points that we saw, the points may, the observed ■  
points may be away from them, away from this ■

■  
line. So, the gap at any price point this ■  
gap between what is the expected value of ■

■  
 $Y$  and what is the actual value of  $Y$  this gap ■  
is essentially the error in the regression ■

■  
model. ■

■  
So, the deviation from the mean is called ■  
error. Errors are usually represented by epsilon. ■

■  
I should not have confused that with elasticity. ■  
But let us also denote error by epsilon and ■

■  
this is not to be confused with elasticity. ■  
Elasticity is also epsilon, the error is also ■

■  
epsilon, but they have no correlation, only ■  
the same notation. So, on average, we do not ■

■  
expect any errors. Therefore, the error terms ■  
are expected to have a value of 0 on average. ■

■  
So, what do I mean by, once again, what is ■  
this deviation? So, this deviation is, this ■

■  
is the prediction line. Where is this prediction ■  
line coming from? This prediction line is ■

■  
coming from the data that we have collected. ■  
So, this may be data ■

■  
and this is the production line. So, what ■  
are we saying? We are saying that at this ■

■  
particular price, let us say  $P_1$ , the x-axis ■  
is always price, the y-axis is always demand. ■

■  
This is my predicted value of  $y$ . And how is ■  
that given? That is calculated using this ■

■

expression,  $\beta_0$  plus  $\beta_1 x$ . That will tell me what is the expected value of

demand at that particular price.

However, the realized value is somewhere here, the realized value was somewhere here. So,

this gap, the vertical gap between the actual value of demand realized at that particular

price point and the predicted value of  $y$  coming from the simple linear regression this gap

is called error. Now, on average, I do not expect this error. Therefore, on average,

the expected value of error is 0. So, what is, what are errors? Errors are deviations

of responses from the predicted value of  $y$ . Where is the predicted value of  $y$ ? This is

the predicted value of  $y$ . This is the actual value of  $y$ . At this particular price point,

let us say  $P_3$ , the predicted value is here, the actual value is here, so this is the  $y$ ,

this is the error.

So, now then how am I supposed to draw this line. I am supposed to draw this line in such

a way that these errors are minimized, but you realize the problem with that. Sometimes

the error may be positive. Let us say if I calculate this epsilon as  $y$  minus  $\hat{y}$ ,

$\hat{y}$  is the predicted value of  $y$ , let us say, and  $y$  is the observed value. So, the predicted

value comes from this  $\beta_0$  plus  $\beta_1 x$  minus  $\hat{y}$ ,  $\hat{y}$  is the actual observed

value. Now, sometimes  $y$  minus  $\hat{y}$  may be positive, which is in this case,  $y$  minus  $\hat{y}$

turns out to be positive, sometimes the  $\hat{y}$  may, sometimes the epsilon may turn

out to be negative,  $y$  minus  $\hat{y}$  may be negative.

Now, if I say that, let us say this is my error one and this is my error three and this

is my error seven at different values of price, so if I simply add up the errors,  $e_1$  plus

$e_2$  plus  $e_3$  all the way to  $e_n$ , I have  $n$  values in my sample. And if I say that minimizing

this, it may not work out because some of the positive errors may cancel out some of

the negative errors. And I may in general underestimate the total error in the model.

Therefore, even though the objective is to minimize the errors, we usually do not minimize

the sum of errors.

So, what do we do? What is the method of nullifying the effect of positive or negative error?

Usually, we square the error terms, we usually square the error terms, and only then minimize

the summation. So, in general, what are we minimizing? We are minimizing the sum of square

errors, sum because it is all summed up, the sum of square errors. That is the objective

with which we run the regression model. So, what is this? This is a mean square error

line. This is the line where the mean sum of square errors is minimized. That is how

we generate the line.

What are the properties of this error term? We are making essentially three assumptions

about the error term. We want the error term to be independent. What do I mean by that?

We do not want the error term even to be dependent on error term  $e_2$ , we do not want error three

■  
to be dependent on  $e_1$  and  $e_2$ , we do not, in  
general, want these error terms to be independent.■

■  
We want these error terms to have equal variances.■  
The variances of all the error terms should■

■  
be  $\sigma^2$ . And we want these■  
error terms to be normally distributed. We■

■  
want the error terms to be normally distributed.■  
These are the three assumptions about the■

■  
error terms.■

■  
Now, even before we interpret our simple linear■  
regression model, we should confirm that these■

■  
assumptions hold. If the error terms are not■  
independent, if the error terms do not have■

■  
equal variances, if the error terms do not■  
have normality, we are going to say that the■

■  
assumptions are violated, and therefore, we■  
should be very, very careful in interpreting■

■  
the results of our regression model.■

■  
How do we check the assumptions? We will do■  
that little later. Right now, let us start■

■  
by saying that we assume that these three■  
hold. We assume that the errors are independent.■

■  
We assume that the errors have equal variances.■  
We assume that the error terms have a normal■

■  
distribution. But these assumptions have ■  
to be checked.■

■  
So, with these error terms, what is the observed■  
value of  $y$ ? The observed value of  $y$  is the■

■  
predicted value of  $y$  plus the error terms,■  
where error terms have a normal distribution.■

■  
Why do I say that the error terms of normal■  
distribution, because that is my assumption?■

■  
On average this is the mean of the normal■

distribution, this is the variance of the

normal distribution. On average, we had anyway said that the expected value of error terms

is 0. On average, we expect the error terms to have 0 value. So, the mean is 0 and the

variance is  $\sigma^2$ . So, that is how we write this. It is a normally distributed

error term with a mean 0 and variance  $\sigma^2$ .

Once again, because the error terms have these three assumptions because we want the error

terms to have these three properties, we are going to now say that the observations are

independent of each other, the observations are independent of each other, they have

equal variances around the regression line and they are normally distributed around the

regression line. What do I mean by that?

So, essentially, if we say, these are the value of  $x$ , this is the value of  $y$ , and let

us say that this is a regression line, our regression line was downward sloping. So,

let me draw this as a downward sloping line. Now, at a particular  $x$ , at a particular value

of  $x$ , let us say  $x_1$ , this is the predicted value of  $y$ , this is the estimated value of

$y$ . This is what the regression line is going to give me  $\beta_0 + \beta_1 x$ . That

is what the line is going to give me. The observation, however, could be anywhere.

So, what is the, where is the observation? The observation we are going to say is normally

distributed. It can be anywhere on this line. It could be, the observed value of  $y$  could

be here, could be here, could be here, could  
be here, could be anywhere. The expected value

is going to be here. Similarly, pick a different  
value of  $x$ , let us pick it sufficiently away

so that I can draw this properly, let us pick  
a different value of  $x$ . This is going to be

the point on the  $y$ , a point on the green curve  
is going to be the predicted value of  $y$ . So,

this is the predicted value of  $y$ .

However, where can the  $y$ , actual  $y$  be? Actual  
 $y$ , so the actual  $y$

could be anywhere. So, it could be here, it  
could be here, it could be here for that particular

value of  $x$ . So, because of this error term,  
the observed value of  $y$  could be anywhere

because of the error terms. So, we can keep  
drawing this normal distribution curve for

each value of  $x$ . We can keep doing this.

Let us stop and go back to the Excel sheet.  
Let me stop the presentation. So, what is

the standard error now? We have seen this  
is essentially your sigma epsilon, the standard

deviation of the error term. This is not a  
square value, this is a standard error. So,

this is the standard deviation, not the squared  
value.

So, now, we said we are going to defer the  
interpretation of 0.724 later. When we take

up an example of MLR we had described this,  
we had described this, now we are describing

this, which is the standard deviation of the  
error terms. A number of observations, there

are 31 observations. You can notice there

are 31 observations. The first row is the

label. So, there are 31 observations. So,  $n$  is 31. So, this is that  $n$  31.

So, how do you interpret this ANOVA table, first of all, degrees of freedom? Why do we

have one degree of freedom for regression? Because what is the prediction equation. The

prediction equation is  $y$  is equal to  $\beta_0$  plus  $\beta_1 x$ . So, how many values

are we estimating? We are estimating the value of  $\beta_0$ , which is the  $y$ -intercept,

we are estimating the value of slope  $\beta_1$ . So, we are estimating two parameters. So,

degrees of freedom will be 2 minus 1, this is 1.

Why is the total degree of freedom 30? If you recall, remember your regression discussion

in the previous course, why is this 30? This is always  $n$  minus 1. I had 31 observations,

31 minus 1 is 30. Why do I get 29 here? So, the total degree of freedom is 30. So, this

is 30 total degree of freedom minus the regression degree of freedom is 1, so 30 minus 1 is 29.

That is how I get my degrees of freedom. Why do I get 1 here? Recall your discussion of

regression from the previous courses. It is two parameters being estimated  $\beta_0$

and  $\beta_1$ , 2 minus 1 is 1. Why is this 30? Total observations are 31, sorry, 31, total

observations are 31, 31 minus 1 is 30. Why is this 29? Total degrees of freedom is 30,

regression degrees of freedom is 1, so 30 minus 1 is 29.

Now, the sum of squares, I am not going to



get into details of the sum of squares. I

am going to rely on your previous course.

Otherwise, I will put up a primer on how do

you interpret the sum of squares for regression

and the sum of squares for residual. The Sum

of squares for residual and sum of squares

for regression essentially comes from how

do you calculate the sum of squares for regression

and the sum of squares for error. So, what

is this value? What is this large value? This

large value is your SSE, the sum of squares

of the errors, residuals are also errors.

So, this is your SSE.

This is called SSR or SSM, the sum of squares

of the model or the sum of squares of the

regression. Usually, if you call it SSR, you

have to be careful because is it SS for regression

or is it SS for residuals you have to be careful.

So, you can always call this SSM, the sum

of squares for the model, sum of squares for

error to avoid any confusion. How is mean

square calculated? Mean square is calculated

as the sum of squares divided by degrees of

freedom. So, this 133 is calculated as the

sum of squares divided by 1. How is this mean

square error calculated? The mean square of

error is calculated as the sum of squares

of the error divided by degrees of freedom.

So, if you want to verify this, how will you

get this? So, this is calculated as the sum

of squares divided by degrees of freedom

and that is how you get your mean sum of the

square. Now, Mean Sum of Squares, this is

Mean Sum of Squares, this is still the sum

of squares. So, this is MSE, Mean Sum of Squares, but this is still the sum of squares. Now,

if you take a square root of this, what are you supposed to get? You will get your standard

error, because we just standard deviation, you take a square root. The square root of

MSE will give you the standard error. So, it is that value.

So, let us get rid of these. We do not want this. We only want to interpret the Excel

output. How do you calculate the F test statistic? First of all, what is the F test statistic?

This is a test statistic. So, this is a test statistic. What is the test statistic? The

test statistic is usually for a hypothesis test. So, what hypothesis are we testing?

We are testing a hypothesis that the regression model is significant or not significant,  $H_0$

and  $H_1$ . So, here we are saying the regression model is not significant. The null

hypothesis is regression model is not significant.

Now, what is the regression model? The regression model is  $\beta_0 + \beta_1 x$ . When will

the regression model not be significant? When will we say that  $x$  has no bearing on the values

of  $y$ ? When will we say that? When this  $\beta_1$  value is 0, if this  $\beta_1$  value is 0, then

$x$  will have no impact on  $y$  and then your entire regression model will collapse. So, what is

the other way of saying that the regression model is not significant? You can say that

this  $\beta_1$  is 0. Once  $\beta_1$  is 0, the regression model will not be significant against the

alternate hypothesis that regression is significant.■

So, the overall significance of the regression■

■

is tested using this F test statistic.■

■

Now, let us not get into the distribution.■

It has an F distribution which has two degrees■

■

of freedom, one is numerator degrees of freedom,■

one is denominator degrees of freedom. Let■

■

us not look at the shape of the F distribution.■

Let us say that this is the test statistic■

■

value. Now, you usually look at the test statistic■

value and decide about the rejection of the■

■

null hypothesis by looking at the P-value.■

This is the P-value. So, this is the P-value■

■

for this hypothesis test.■

■

Now, how do you decide about the hypothesis,■

you compare the P-value with the significant■

■

value which is alpha, alpha is usually 5 percent,■

so 0.05, and if P is less than alpha, you■

■

are generally going to reject the null hypothesis.■

What is the P-value here? P-value is, of the■

■

order of 10 to the power of negative 10 which■

is anyway going to be lesser than 0.05. Therefore,■

■

we are going to reject this null hypothesis.■

What was the null hypothesis?■

■

The null hypothesis was that the regression■

model is not significant. So, we are going■

■

to reject this null hypothesis. And therefore,■

conclude that the regression model is significant.■

■

So, we say that the alternate hypothesis is■

good, a regression model is significant. So,■

■

that is the overall significance of the regression■

model is tested using this P-value. This is■

■

the overall significance test of a regression■

model. P-value is 7.8, 10 to the power of■

negative 10 anything of that order is going to be lesser than 0.05 and therefore we can

safely reject the null hypothesis. So, that is how you interpret the upper block, ANOVA

block of the regression output of excel.

Take a minute to digest it. Let us hope that you have understood. If you have not, you

can playback the video and go back, more importantly, go back to your regression discussion in the

previous courses that should help you understand and interpret this ANOVA table of the Excel

regression output. That is out of the way and we have confirmed that the regression

is significant, let us interpret, let us see what is the estimated value of the y-intercept

and the slope. So, y-intercept was 5842, we knew that 5842, 5842.8 was the estimate of

the y-intercept, which is the total market size. What was the slope estimate? The slope

estimate was negative 157, we knew that from the excel output earlier. So, these are your

coefficients.

So, what are we saying? We are saying that the estimate of beta naught, we cannot know

the value of beta naught, we are saying that the estimate of beta naught, let us call it

beta,  $b_0$ , let us say that that is  $b_0$ , that value is 5842, an estimate of beta 1, which

is  $b_1$ , what value is negative 157.7. And anyway, this is an estimation. This is the population

value, this is the sample value, sample value from the 31 observations that you have. This

is the sample value. This is how you are going

to estimate the population value from the

sample value, so these are sample values.

So, there is always going to be some error.

So, this is the standard error for estimating

of intercept, this is a standard error for

estimating the slope.

Now, we can individually test whether this

beta naught is 0 or beta 1 is 0 by running

a localized hypothesis test. This was the

overall hypothesis test whether the regression

was significant, now let us run a local hypothesis

test. What is this test statistic for? This

is the test statistic for a hypothesis. What

is the null hypothesis? We are checking whether

beta 1 is 0, alternate hypothesis beta 1 not

0.

What is the P-value? P-value is anyway very,

very small. And therefore, we reject this

null hypothesis and say that beta 1 is not

0. What about this, what about this? This

is essentially checking a hypothesis that

beta naught is 0 against the alternate hypothesis

that beta naught is not 0. What is the P-value?

P-value is of the order of 10 to the power

of negative 15. And therefore, we can safely

reject even this null hypothesis, and therefore,

confirm that beta naught is not 0.

Now, can you see a similarity? Look at this

hypothesis test and look at this hypothesis

test, more particularly focusing on the P-value.

P-value here was 7.8, 10 to the power of negative

10. P-value here was the same. Why did that

happen? This is happening because we are running

essentially a simple linear regression model.■

In a simple linear regression model, what■

■

is the overall significance of regression?■

Regression will not be significant if beta■

■

naught is 0.■

■

So, when we are checking the overall regression■

significance hypothesis, we are essentially■

■

checking whether beta naught is 0. Here we■

are directly checking whether beta naught■

■

is 0. So, even though the test statistic use■

was different, here it was a t statistic and■

■

here it was an F statistic, the overall test■

is the same. We are checking whether beta■

■

naught is 0 or not. If beta naught is 0, your■

regression collapses. And therefore, it is■

■

not very surprising that the P-value is the■

same.■

■

So, if you rejected the null hypothesis of■

overall regression insignificance and therefore■

■

concluded that the regression was significant,■

you are going to pretty much conclude the■

■

same thing here by saying that beta 1 is not■

0. Excel also prints the confidence interval,■

■

95 percent confidence interval of beta 1.■

It says that beta 1 is going to be anywhere■

■

from 193 negative to 121 negatives. And what■

is the expected value? The expected value■

■

is negative 157. Similarly, for beta naught,■

beta naught is expected to be anywhere from■

■

5023 to 6662 and the expected value is 5842.■

That is how you interpret the y-intercept■

■

and the slope reported by Excel regression■

output. Remember that this is only an estimate.■

■

You never, you are going to know the actual■

population value  $\beta_0$  and  $\beta_1$ . What

you have is some estimate  $b_0$  and  $b_1$ .

So, that is all you can get from regression

because you only have 31 values, 31-odd values, 31 exactly values.

Let me conclude this regression output by telling you one more way of interpreting this

R square. Earlier what did I, what did we say, R square was just the square of the correlation

coefficient. Now, you can think of R square as the ability of the regression to explain

the variability in  $y$ . So, one interpretation of R square is that the prices, these prices,

different prices are helping me explain the variability of  $y$ . There is variability in

the demand values.

So, this regression model, what are, how can we interpret this 0.73, we can say that using

price as an explanatory variable, how much explanatory power does it have, we can say

that it can explain 73 percent variability in  $y$ . Let me say that again. Using prices

as an explanatory variable can explain 73 percent variability in  $y$ . That is the interpretation

of this R square. This is called the coefficient of determination. This is

only a recall. This must have been done in the previous courses. We are only going to

recall.

How do you calculate this 0.733? So, what is the sum of the square, because of regression

it is this value. What is the total sum of squares? It is this value. So, if I simply

do this, this is the amount explained by the regression and this is the total variability

that should be 0.733. So, because of regression, I can explain this much amount of variation

out of the total variation in  $y$ . So, the  $R^2$  square is called the coefficient of determination.

It tells me the amount of variability explained because of the regression model. Here, it

is a simple regression model. I have only used price as an explanatory variable. So,

what does that do to our demand response curve?

Now, going back to the PPT, let me go back to PPT. So, here

I estimated a linear demand response curve. I estimated the  $y$ -intercept. I estimated the

slope. And therefore, I am going to fit a linear demand response relationship to my

data. And what was the data? This was the data. And that is how we have used simple

linear regression to predict our market size. Market size is expected to be 5842, total

demand is supposed to be 5842. Even if you give away the product for free, we expect

about 5842 demand value to be realized.

Every increase in the prices, every unit increase in the prices is going to reduce the demand

by 157 units. So, that is how we estimate the demand response curve. And let us pause

here and end this session with an explanation of SLR applied to the price demand relationship.

So, let us pause here, continue with this in the next session. So, I am pausing the

video now.