Lagrange Multipliers

Wiki { – is a strategy for finding the local maxima & minima of a function subject to Equality constraints. The great advantage of this method is

Vector field → of a function say $f(x, y) = \begin{bmatrix} x^2 + y \\ 2x - y^2 \end{bmatrix}$

~~Gradient~~

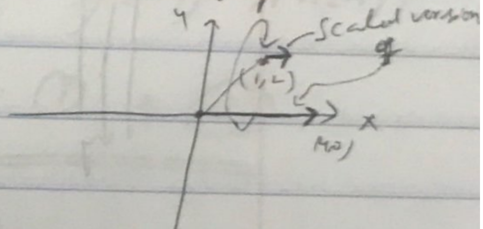
f has 2D input
& has 2 dimensional vector

Vector fields can help you here. { Hard to imagine the graph.

→ Take points on the xy coordinate system & add the $f(x,y)$ value to x, y points

Ex: $(x, y) = 1, 2$

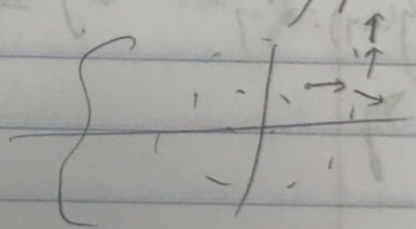
$f(x,y) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$



$1,2 + 3 \; -2$

$= 4, 0$

So at Every point



[Khan Academy] { Is called Vector field.

Gradient $\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$ (Ex: of $f(x,y)$)

↳ The vector field of ∇f points in the direction of steepest ascent.

→ Why gradient is the direction of steepest ascent?

P.T.O

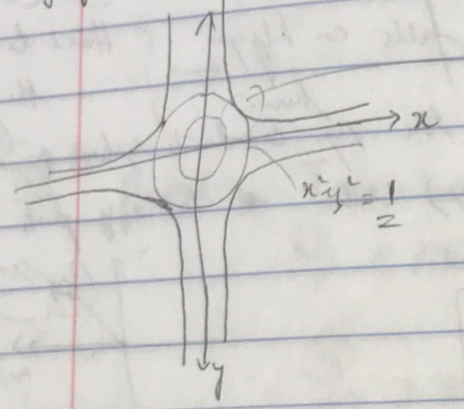
$$f(x,y) = x^2 + y^2$$

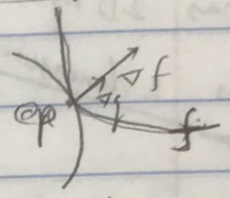
$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \partial f/\partial y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

Gradient Continued later (set to study)

Lagrange Continued

Maximize $f(x,y) = x^2 y$ on the set $x^2+y^2=1 \rightarrow (1)$





∇f is perpendicular to contour f

∇g is " to " g

$g(x,y) = x^2+y^2$

∇g

By Observation $f(x,y)$ on the $x^2+y^2=1$ is maximum @ P_1 to 8

Here they are tangent.

$$\nabla f(x_p, y_p) = \lambda \nabla g(x_p, y_p) \rightarrow (2)$$

Gradients are parallel but not necessary same magnitude

λ is called Lagrange Multiplier

$\Rightarrow 2xy = \lambda 2x ; \; x^2 = \lambda 2y ; \; x^2+y^2=1$

Solve

$\rightarrow$ If $\mathcal{L}(x,y,\lambda) = f(x,y) - \lambda(\underbrace{g(x,y)-1}_{h(x,y)})$

& solve $\nabla_{x,y,\lambda} \mathcal{L}(x,y,\lambda) = 0$

$\rightarrow$ 3 Equations 3 unknowns

p. 7.0

$\cancel{\nabla_x L(x,y,}$

$$\nabla_\lambda L(x,y,\lambda) = 0 - (g(x,y) - 1) = 0$$

$$\boxed{\Rightarrow g(x,y) = 1}$$

i.e. $x^2+y^2=1$ our constraint.

$$\nabla_{x,y} L(x,y,\lambda) = \nabla_{x,y} f(x,y) - \lambda \nabla_{x,y} g(x,y) = 0$$

$$\Rightarrow \boxed{\nabla f(x,y) = \lambda \nabla g(x,y)}$$

Solving 3 equations
x,y might have
more than one
sol^n, since we
are maximizing
take x,y that
maximizes f(x,y)

So the auxillary function we assumed

$\cancel{L = f(x,y) < \lambda g(x,y)}$ $L = f(x,y) - \lambda h(x,y)$

satisfies condition 1 + 2

$\rightarrow$

$$W_{LN} = \underset{w}{\arg\min}\ w^T w \quad \text{subject to } Xw = y$$

$$L =$$