Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Class 1 - Introduction

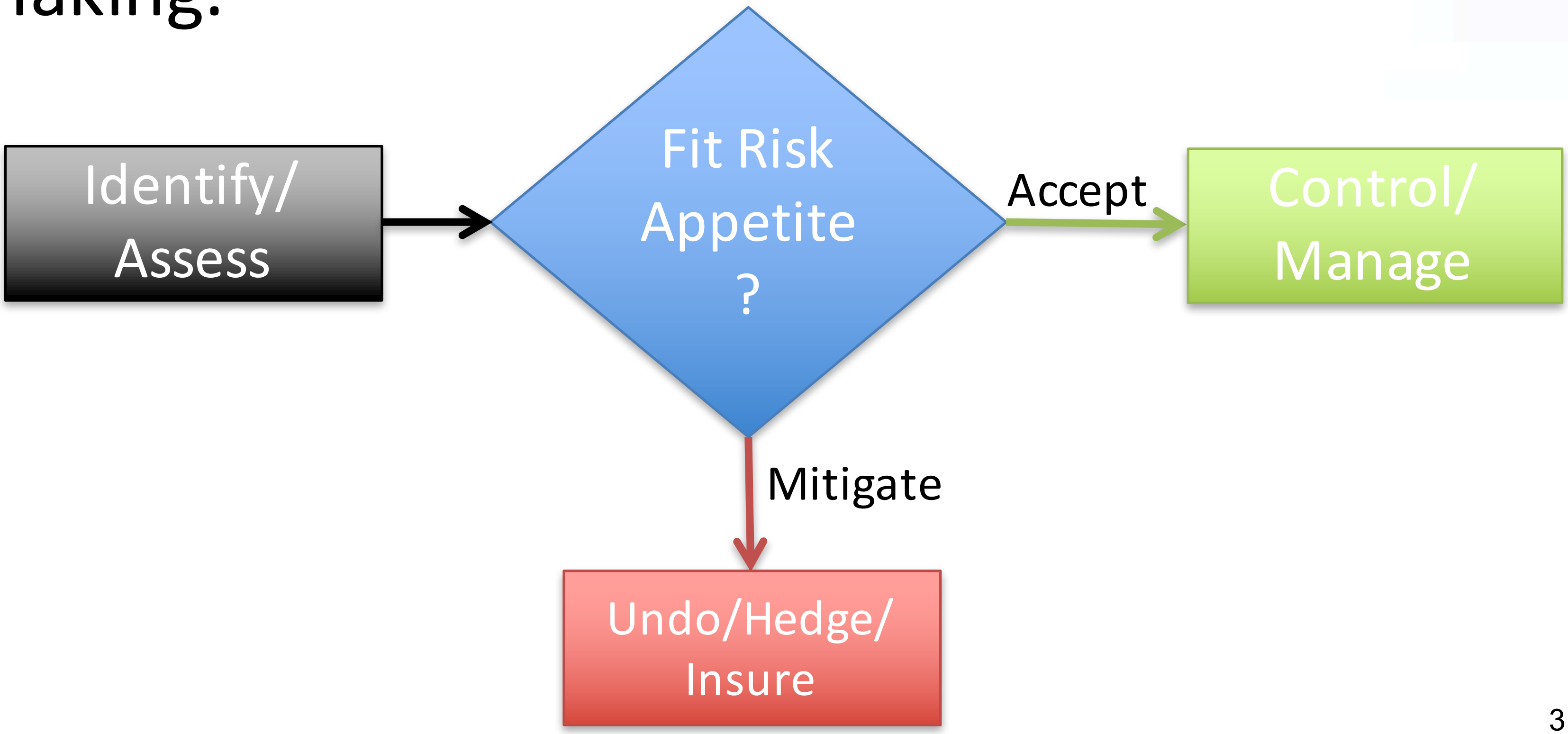
UCLA Anderson
THINK IN THE NEXT

Agenda

- What is risk management?
- Risk Appetite Framework
- Syllabus
- Course Topics

Risk Management

A process for identification, assessment, control, or mitigation of risks in business decision making.



Risk Appetite Framework

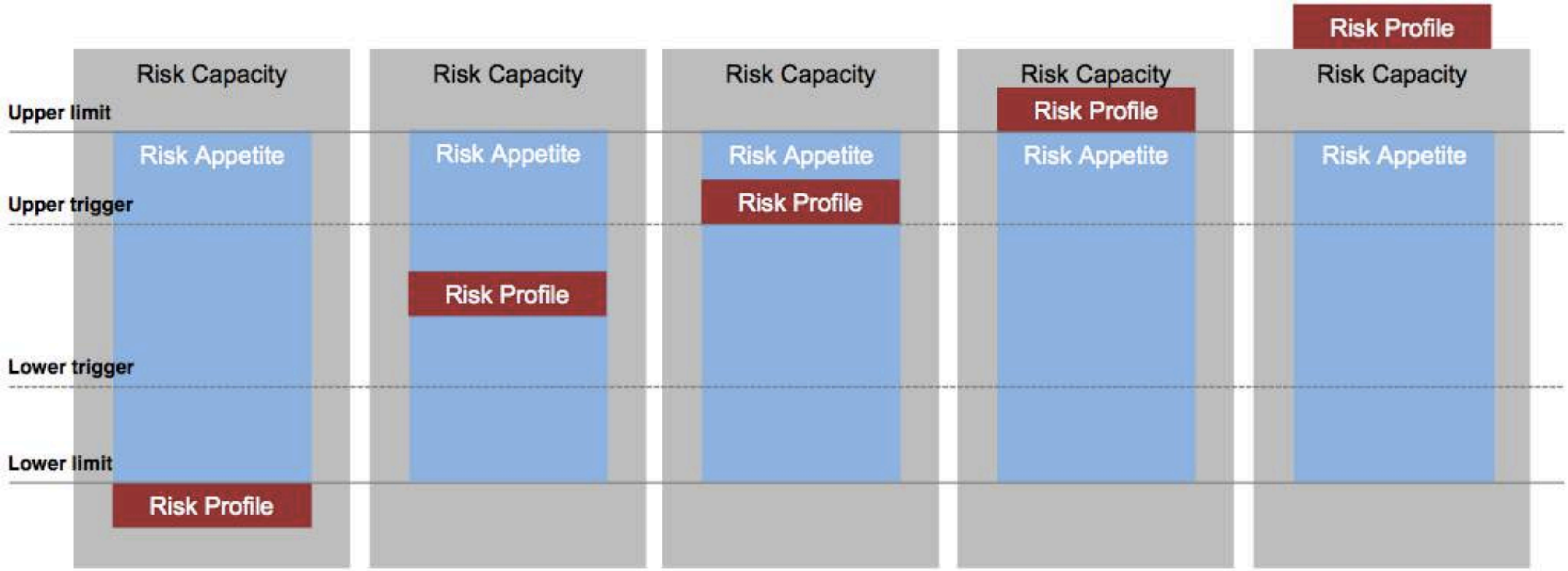
- Derived from the firm’s business strategy, culture and governance.
- Allows to decide whether to accept a specific risk and control it, or to mitigate it.
- Defines a language, which allows to communicate among all stakeholders, internal and external.
- Includes targets, preferences, escalation thresholds, limits and capacities.
- For most part, quantitative, but also qualitative.

Example of a Risk Appetite Statement

Parameter	Types of Risk Appetite Statements	Risk Dimension
Earnings at Risk	"We will tolerate a maximum reduction of X% in projected earnings every Y years"	Profitability
Concentration Risk	"We will maintain a balanced portfolio considering both geography, products and customer types"	Profitability
Rating Ambition	"We will target an Aa level rating for senior unsecured long-term debt "	Capital
CET1 vs ECap	"We will have a common equity Tier 1 capital of minimum x% of economic capital"	Capital
LCR	"We will target a liquidity coverage ration of x% "	Liquidity
Reputational Risks	"We will not be associated with operations which may harm its reputation"	Other

Adapted from "Risk as a Growth Enabler" – Adem Yilmaz – CRO, Hyundai Capital America

Monitoring Risk Profile vs Risk Appetite



Objective under threat	Desired range	Escalation	Objective under threat	Firm is unviable
Risk profile is less than the lower limit. Corrective action must be taken.	Risk profile is between the upper and lower thresholds.	Risk profile is between the upper threshold and limit. Escalation to consider corrective action.	Risk profile exceeds the upper limit. Corrective action must be taken.	Risk profile exceeds risk capacity. The firm must enact its recovery and resolution plan.

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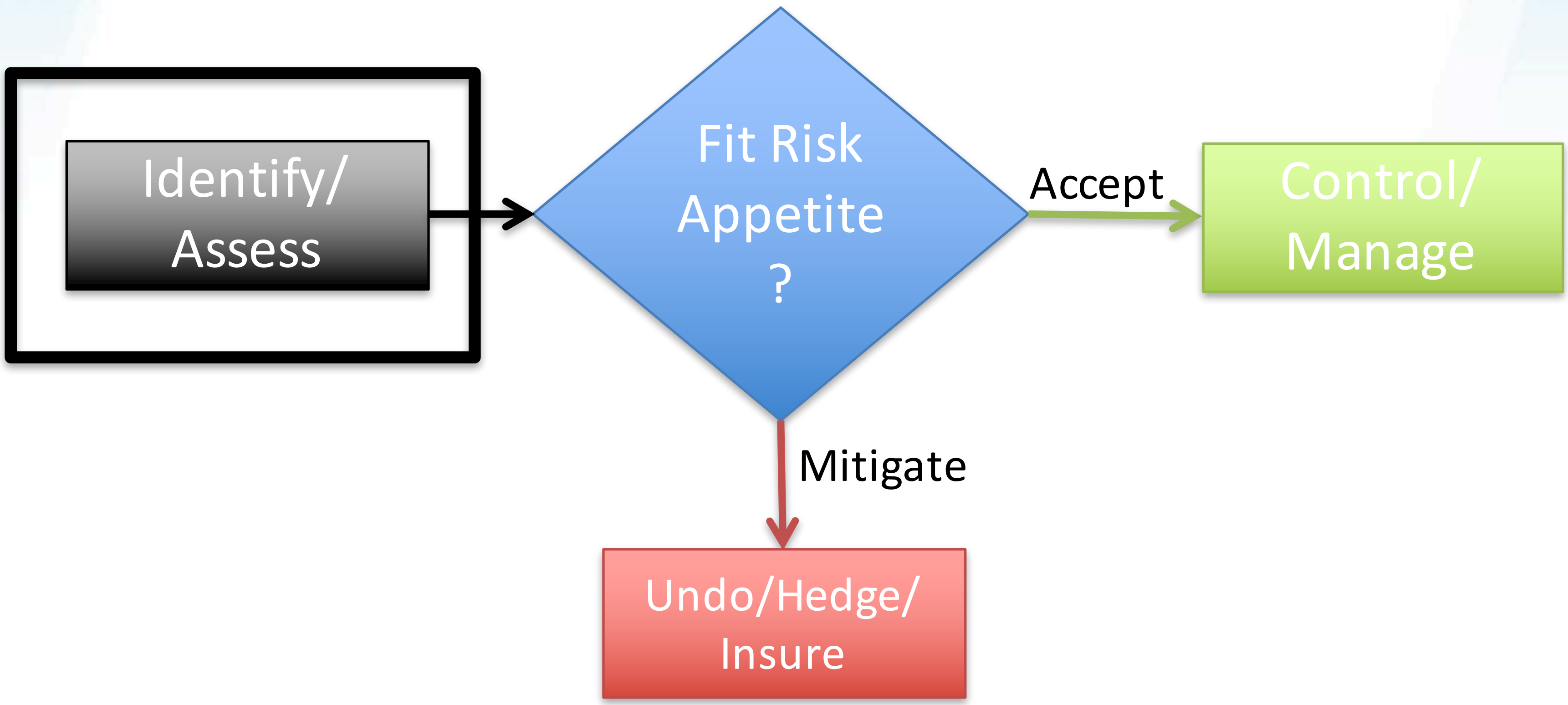
Risk Management Process



Risk Assessment

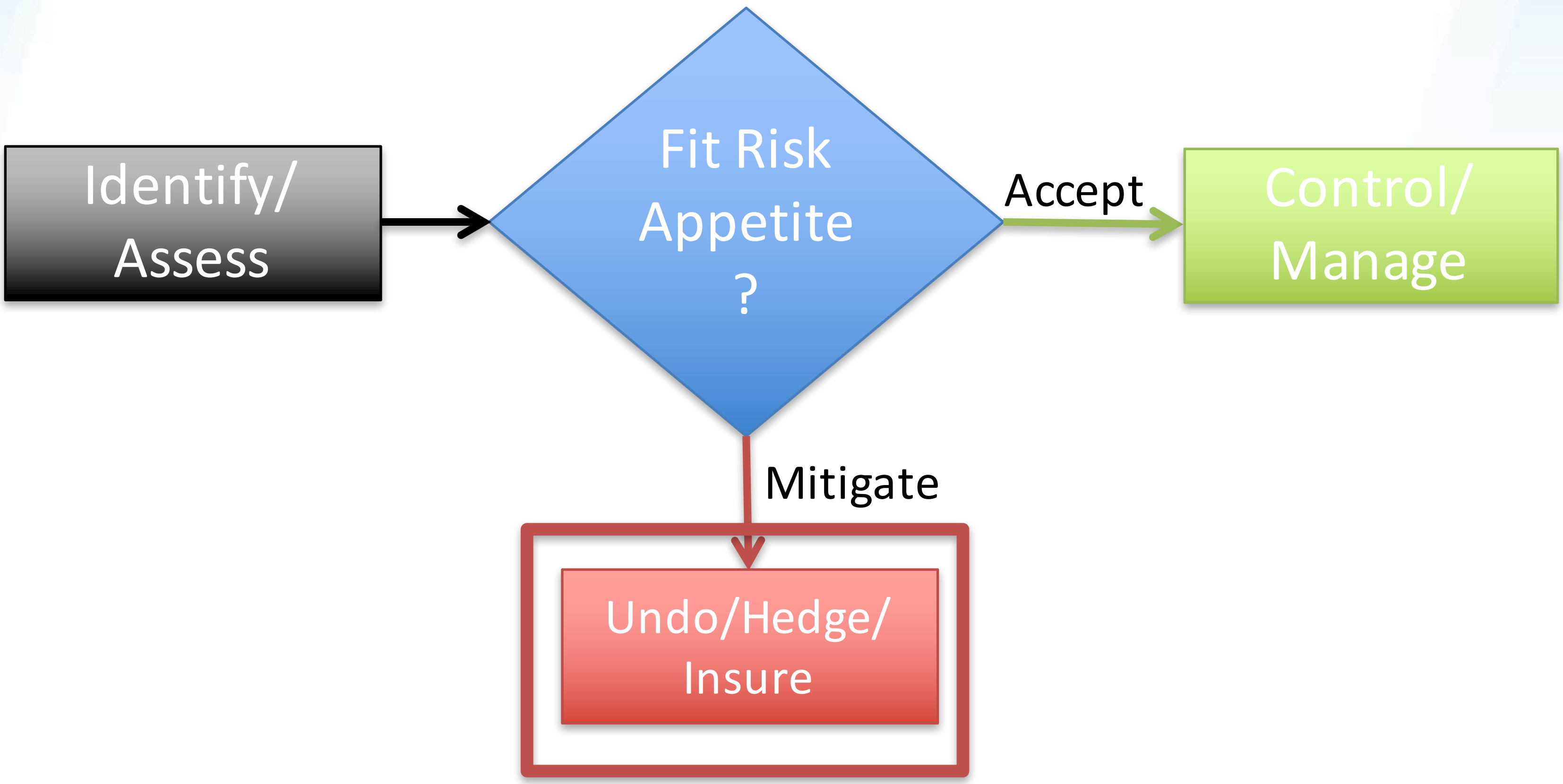
Common ways to assess the nature and severity of risks:

1. Risk statistics
   – Moments of return distribution: standard deviation, skewness, kurtosis
   – Sensitivity to underlying: Greeks, Duration
2. “What if?” analysis:
   – Scenario analysis: shock a single underlying factor and compute potential losses
   – Stress testing: portfolio behavior given a stressful economic scenario
3. Compute the “worst-case” scenario at a certain confidence level (i.e. probability)
   – Value at Risk (VaR)
   – Expected Shortfall (ES), Conditional VaR (CVaR)

Risk Assessment (Cont.)

- Risk Assessment and Control are performed on multiple levels:
- Single Exposure
  - Credit Scoring/Rating
  - Interest Rate Sensitivity
  - Risk-Return Metrics
- Portfolio Risk
  - Economic Capital / VaR
  - Stress Testing
  - Diversification

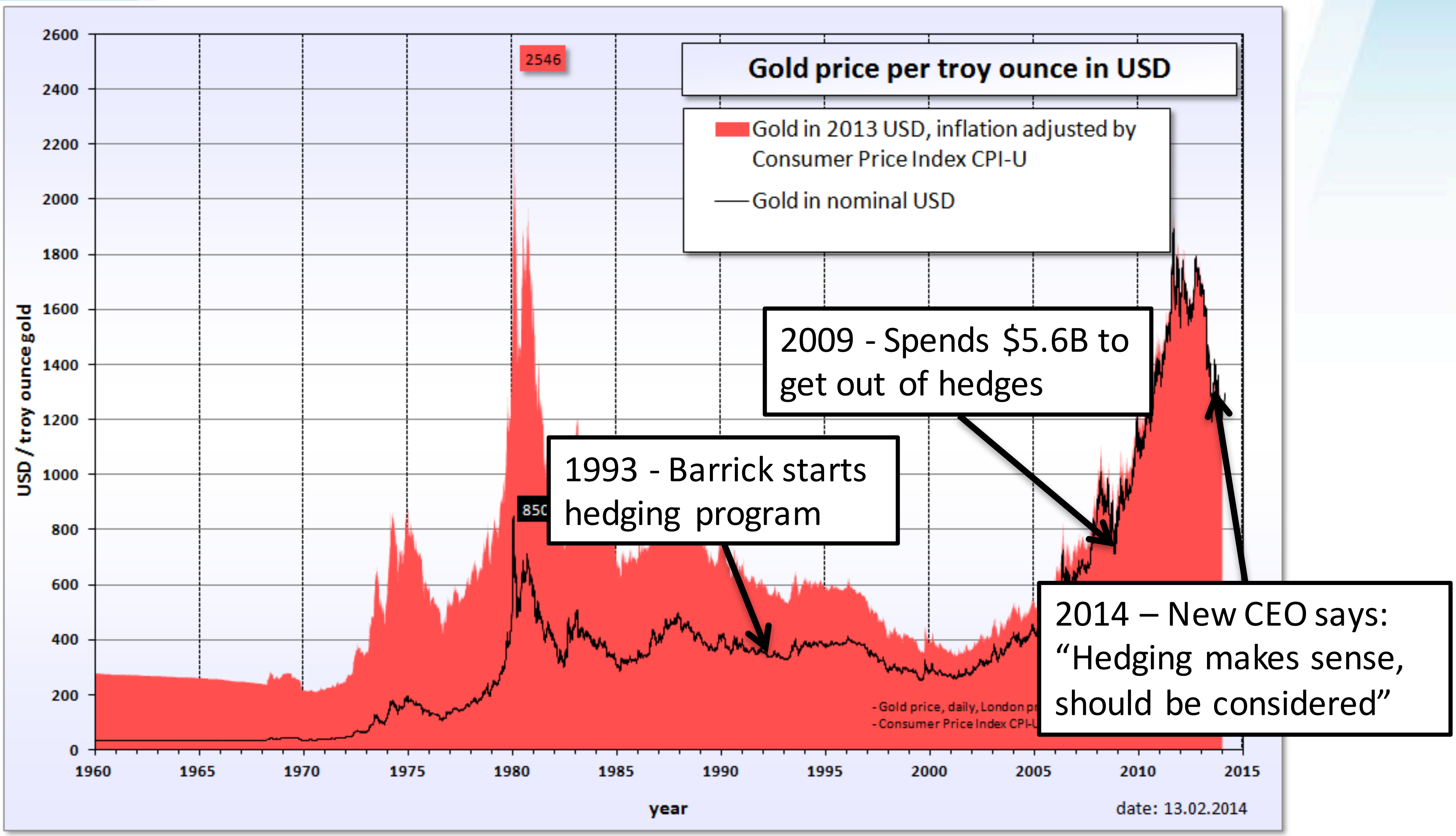
Risk Management Process



Risk Mitigation

- Hedging – eliminating both upside and downside
- Insurance – protecting against unfavorable events and keeping potential upside; typically involves paying a premium
- Diversification – holding a large collection of imperfectly-correlated assets to minimize idiosyncratic risks

Risk Mitigation or Speculation?



Why Hedge?

- If the firm has “unfair” advantage over investors:
  - Expertise and cost advantages in industry-specific markets
  - Firm has more accurate information about current exposures and future prospects
- If cash flow variability can affect firm’s liquidity or interrupt its long-term investment activity.
- Reduce earnings volatility and dividend volatility, as signals of management quality to shareholders

Why Not Hedge?

- It may increase risk to hedge when competitors do not.
- Explaining a situation where there is a loss on the hedge and a gain on the underlying can be difficult.
- Bid-Ask spread is larger for futures and options than spot.
- Employee costs and the cost of aligning their incentives to prevent speculation.

Risk Aware Business Management

- Modern Risk Management supports the organization above and beyond assessment/control/mitigation:
- Risk adjusted profitability and measurement
- Risk-based pricing
- Risk-minded culture and incentives

Syllabus

<u>**Required Material**</u>

Hull, John C. ***Risk Management and Financial Institutions,*** 4th Edition, John Wiley & Sons, 2015 (Hull)

<u>**Additional Texts**</u>

1) McNeil, Frey and Embrechts, ***Qunatitative Risk Management, Revised Edition,*** Princeton University Press, 2015 (MFE)

2) Jorion, Philippe ***Financial Risk Manager Handbook FRM Part I/Part II,*** 6th Edition, John Wiley & Sons, 2011 (Jorion)

- Grading: Problem Sets 30% , Midterm 30%, Final 40%
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- TA: Yuji Sakurai

Foundations

Market Risk

Credit Risk

Enterprise RM

Week	Lecture Topics	Hull	MFE	Jorion
1	Introduction to Class and Risk Management	1.6	1.1, 1.4	1.1,1.5, 27
	Volatility Models	10	4	5.4
2	MLE Estimation		A3	
	VaR I	12.1-12.6	2.2, 2.3	
3	VaR II	12.7-12.10	8, 9.3	
	Interest Rate Risk: Duration and Convexity.	9		13, 14.1-14.2, 6.2-6.5
4	Market VaR I: Model Building, Delta-Gamma Method, Monte Carlo Simulation	14	9.1, 9.2	14.3, 16.3.1, 16.3.3
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	Midterm			
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	Counterparty Credit Risk	18, 20	17.2	22, 24.2
8	Copulas and Dependence I	11	7	16.2
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10	Stress Testing	22		12.5
	Financial Regulation	15, 16, 17	1.3	28

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Volatility Models

- The most common risk measure
  - Especially in trading market risk management
  - Serves as building block for VaR calculations
- Estimation:
  - Historical, i.i.d. assumption
  - Model-based, serial correlation: EWMA and GARCH
  - Implied, from option prices
- Sensitivity and Scenario Analysis for changes in volatility

Maximum Likelihood Estimation

- Statistical Framework for fitting models and testing hypotheses
- We use it throughout the course:
  - Estimate volatility models
  - Fit credit risk models
  - Framework for backtesting VaR
  - Fit copula models

Value at Risk

- Answers "How bad can things get?" in one parameter:
  - "We are *X* percent certain that we will not lose more than *Y* dollars in time *T*."
- We cover:
  - Basic models (discrete and continuous)
  - Desirable and undesirable properties of tail measures
  - Alternative tail measures (Expected Shortfall)
  - Aggregation, Attribution and Allocation
  - Confidence Intervals
  - Back-testing

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Market Risk

- Uncertainty concerning changes in market prices and rates (including interest rates, equity prices, fx rates and commodity prices), the correlations among them and their levels of volatility.
- Examples:
  - Investment portfolio facing equity price risk
  - Company expecting a payment in Foreign Currency
  - Effect of exchange rate on competitiveness in foreign markets
  - Farmer intending to sell wheat
  - Homebuyer taking Adjustable Rate Mortgage (ARM)

Interest Rate Risk

- The basic interest rate risk:
  - Bond value is equal to the discounted value of all future payments.
  - If rates increase, the value of the bond decreases.
- Risk Tools:
  - Sensitivity is measured by Duration and Convexity
  - Scenario analysis: what is the effect of change in one interest rate, or shifts in term structure?
  - Set up the tools to find the “worst-case” scenario of a bond portfolio.
- Consider additional interest rate risks: basis risk, term-structure shape, behavioral options (prepayment risk), etc.

Market Risk VaR

- We analyze the distribution of future returns, with focus on the tails.
- Two main issues:
  - Should we make distributional / parametric assumptions, or make a non-parametric evaluation?
  - How can we analyze the return distribution of a portfolio that has many positions, some that are time consuming to re-price?
- Modeling heavy tails – Extreme Value Theory

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Credit Risk

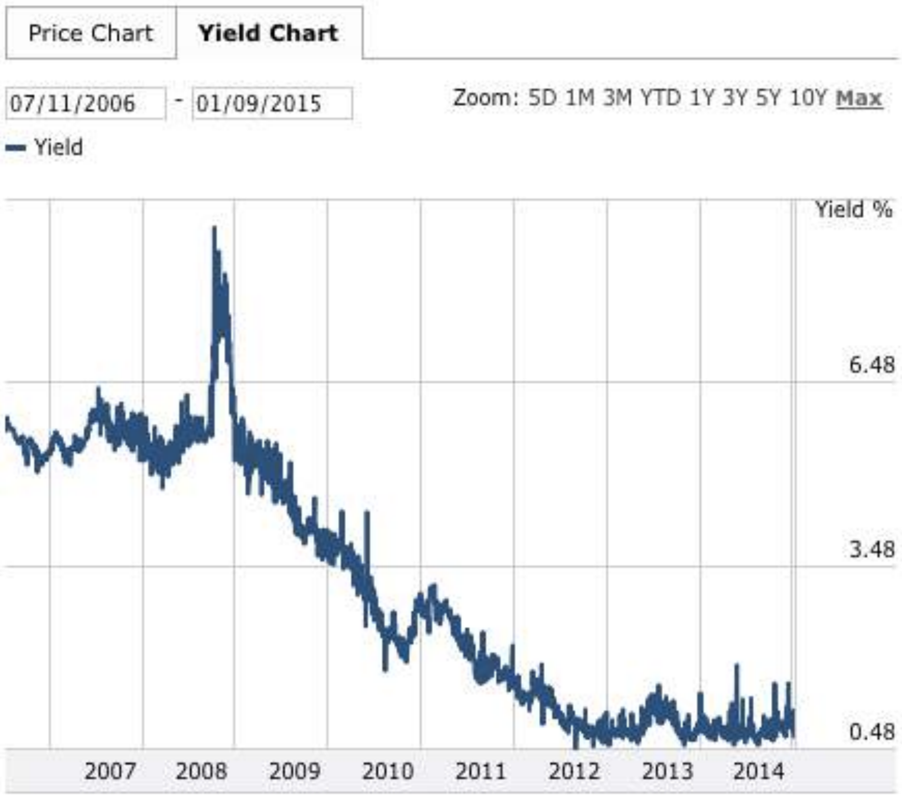
- Risk of default by
  - Obligor/counterparty/borrower/debt issuer
  - which leads to failure to meet contractually obligated payments
  - in relation to actual, contingent or potential claims.
- Examples
  - Loans: Mortgages, Commercial & Industrial, Commercial Real Estate
  - Lines of Credit, Guarantees
  - Trade Credit
  - Counterparty Credit Risk
  - Credit Default Swap

Topics in Credit Risk (1)

- Probability of Default – PD
- What is the chance that a borrower will not pay back the loan, or make payment on a bond?
- We will look at how we deduce this probability from:
  - Historical information about defaults (How to build a model and how to validate it).
  - Market prices.

What does price tell us about PD?

TARGET CORP.
Coupon 5.875%
Maturity 07/15/2016



Topics in Credit Risk (2)

- Loss Given Default – LGD
- Suppose the borrower defaults, what portion of the debt will we lose?
- LGD = 1 – Recovery Rate
- How is this correlated with the probability of default?

Topics in Credit Risk (3)

- Exposure At Default – EAD
- What will be our exposure to the obligor if it defaults?
- In what cases might this be unknown at time zero?

Counterparty Credit Risk

- Suppose we agree to buy 1000bbl of crude forward at $80 from JP Morgan, what is our exposure if JP Morgan defaults tomorrow?
  - If the price of oil is $80? $90? $60?
- What will be our exposure if JP Morgan defaults in a week? What if both sides post collateral every day?
- What is our credit exposure to JP Morgan if we sold (wrote) it a put option on oil?

Dependency of Credit Defaults

- How do we model a degree of dependency between credit exposures or bonds?
- Example:
  - An insurer is selling protection on two bonds that will pay out if **both** bonds default
  - Is the insurance worth more if the bonds are independent or if their perfectly dependent?
  - What if the insurance pays out if **either** of the bonds defaults?

Credit VaR

- It is harder to limit downside of credit portfolios by diversification than equity portfolios:
  - Discrete nature of each credit
  - Bulkiness of credits
  - Contagion and dependence effects
- Credit portfolios usually have many more loans/bonds than stock portfolios
- How do we model the dependency structure among thousands of loans?

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Stress Testing

- Assessing tail risk through scenarios (stories) rather than statistical VaR.
- How do we translate stories into portfolio losses?
- How do we know the probability of the scenario? Is it severe enough?
- Big advantage: Can point us to the cause of the problem, and help us think about a mitigation plan.

Financial Regulation

- Focused on guaranteeing bank survival:
  – Solvency – Using Capital as a cushion for losses
  – Liquidity – Examine funding under stress scenarios
- Basel frameworks historically focused on capital adequacy (leverage).
  – Following the crisis, attention to liquidity has increased.
- Dodd-Frank Act – Regulating bank activities to limit systemic risks.

Operational Risk

- Operational Risk – Potential of failure in relation to employees, contractual specifications and documentation, technology, infrastructure failure and disasters, external influences and customer relationships.
- Examples:
  - Fire, Earthquake, Mudslide
  - Internal/external fraud
  - Teller/dealer mistakes
  - Cyber-attack

Liquidity Risk

- Liquidity Trading Risk, Liquidity Market Risk – Inability to sell an asset in a short time in the required quantity without affecting price.
  - A type of Market Risk
- Liquidity Funding Risk – Inability to meet all payment obligations when they come due, or only being able to meet these obligations at excessive costs
  - Typically a result of other risks
  - A major cause of many financial failures

Other Risks

- Business or Strategic Risk – Risk due to potential changes in general business conditions, such as market environment, client behavior and technological progress
- Compliance or Legal Risk – Risks that the firm will be exposed to regulatory actions, be fined, or incur increased legal expenses (sometimes included in Operational Risk)
- Reputational Risk – Risk that bad publicity will impact the public's trust in the organization
- Insurance Risks – Risks affecting size and timing of insurance and pension obligations, such as longevity and mortality risks

Interaction of Risks

- Market Risk causing Credit Risk
  - Russian default in August 1998
  - Savings & Loan Debacle of 1980s
- Operational Risk causing Market Risk
  - Barings collapse due to unauthorized trading and high volatility in Japanese stock index
- Operational Risk causing Credit Risk
  - Mortgage Loan Buybacks
- Credit Risk causing Market Risk
  - AIG Credit downgrade
  - Credit availability on October 1987
- Wrong-way risk - occurs when there is a correlation between the counterparty's credit risk and the exposure to that counterparty.

Thanks

Financial Risk Management

Spring 2016

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Volatility Models

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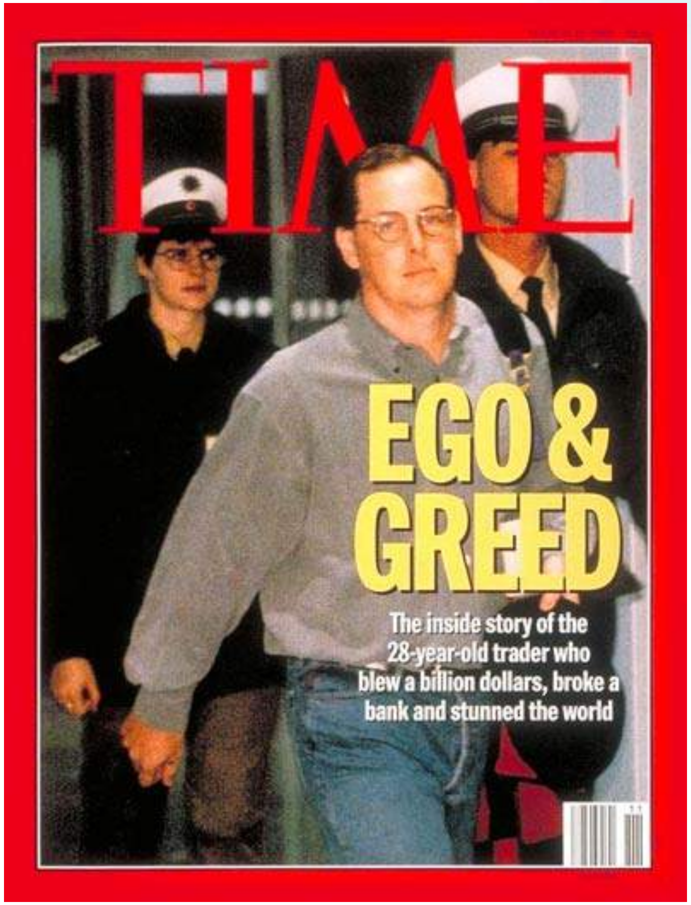
Agenda

- Managing Traded Market Risk
- Volatility Models
  - Exponentially Weighted Moving Average
  - GARCH (1,1)
- Forecasting Volatility
- Scenario Analysis - Volatility Shocks

Rogue Trader

- Barings was UK's oldest merchant bank.
- Leeson managed the Singapore Trading Desk.
- In 1992, he made 10% of Barings profit.
- By 1995, He made total losses of $1.4B
- Chief trader and responsible for settling the trades.



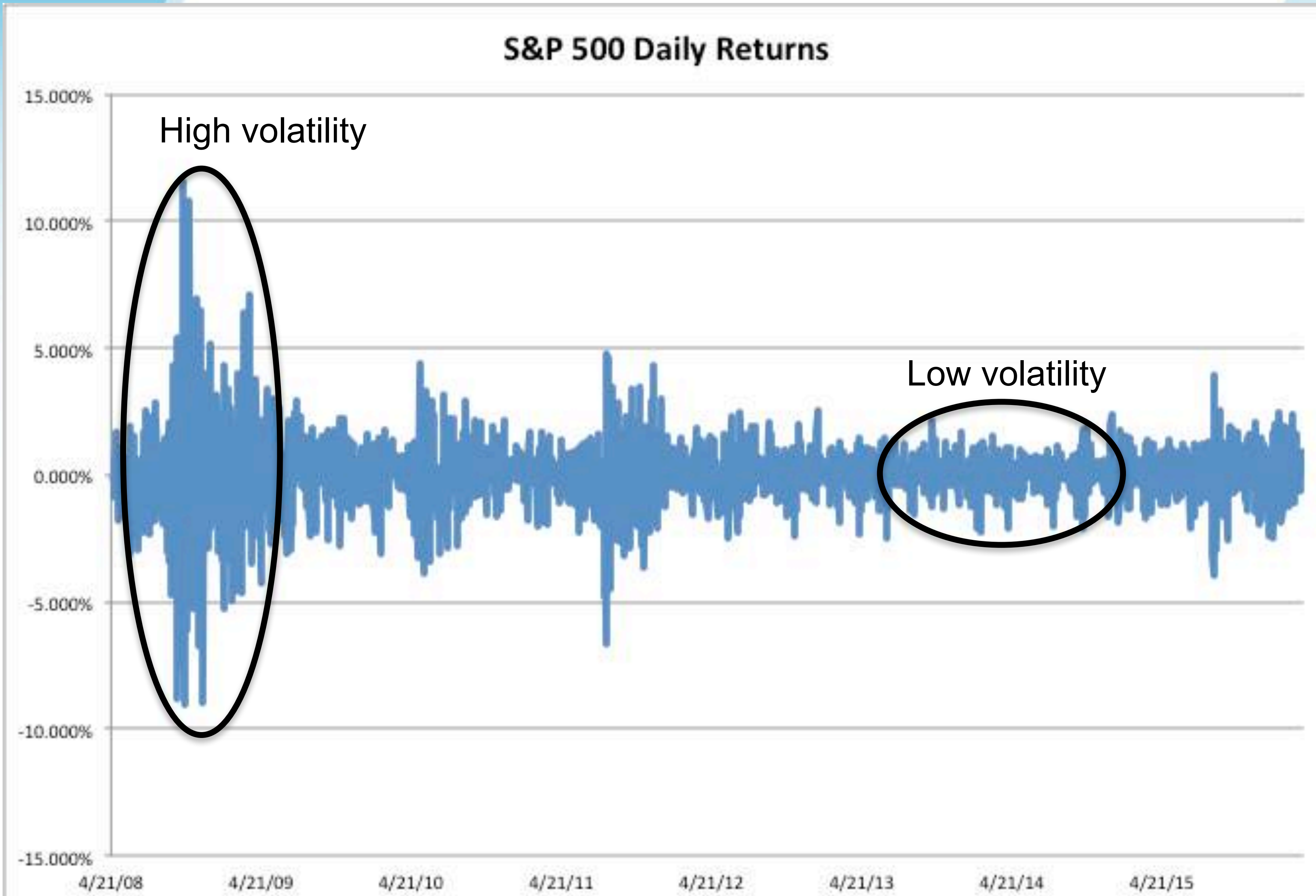
Risk Management of Trading in Financial Institutions

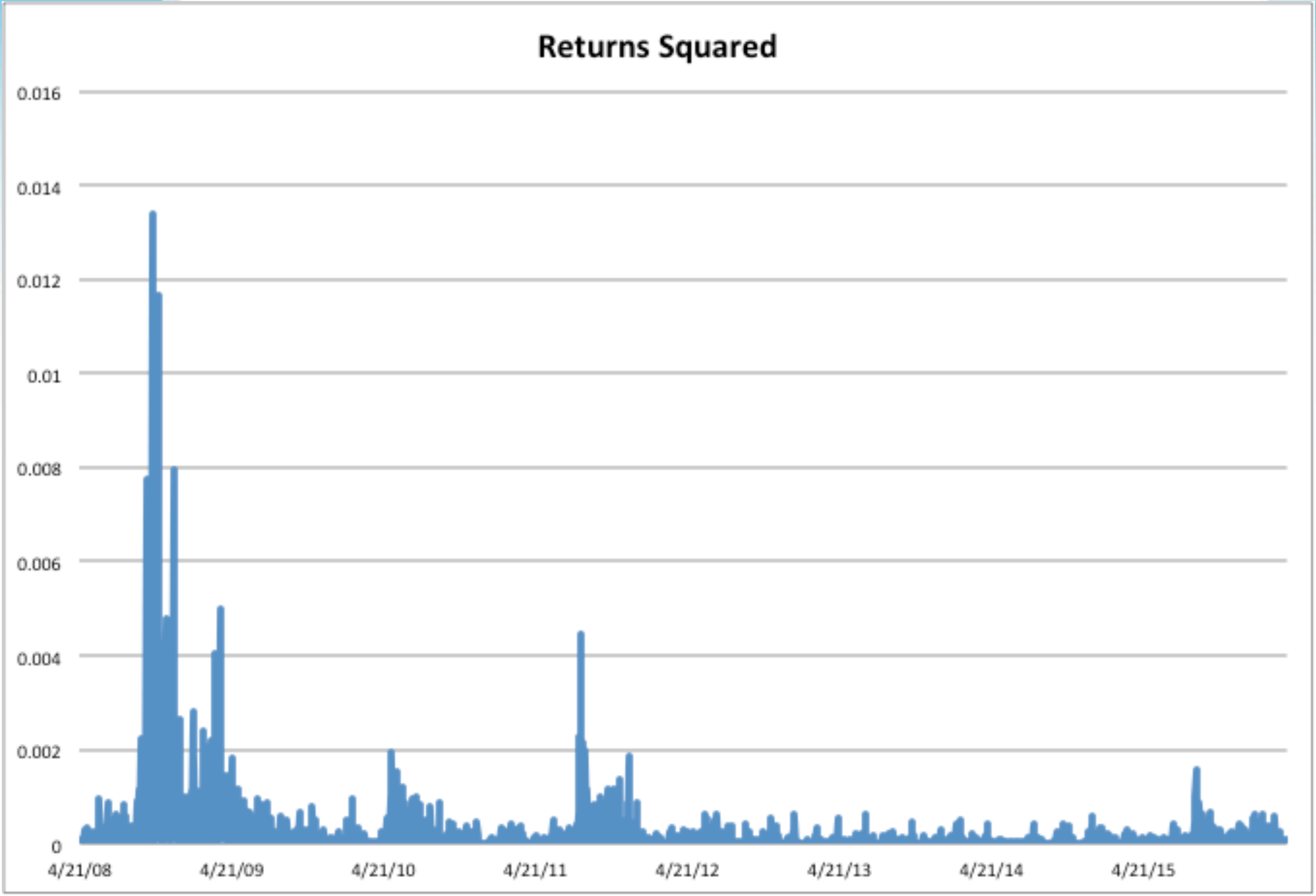
- Front Office – Take on risk as Market Makers or as Proprietary Traders
  - Take risk according to view and within risk limits
- Middle Office – Manages market and operational risk of the trading floor
  - Aggregate risk
  - Control risk limits: internal and regulatory
- Back Office – Record keeping, manage execution and control operational risks

Trading Market Risk

- Trading market risk managers typically look at <u>daily</u> returns.
  - Define u_i as $(S_i - S_{i-1})/S_{i-1}$
  - Assume that the mean value of u_i is zero
  - *Returns* and *log-returns* are very close
- We are interested in determining the expected volatility for the next day.
  - If returns are i.i.d, use m last days to estimate volatility:

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2$$


S&P 500 Daily Returns
High volatility
Low volatility
15.000%
10.000%
5.000%
0.000%
-5.000%
-10.000%
-15.000%
4/21/08
4/21/09
4/21/10
4/21/11
4/21/12
4/21/13
4/21/14
4/21/15



Returns Squared
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
4/21/08
4/21/09
4/21/10
4/21/11
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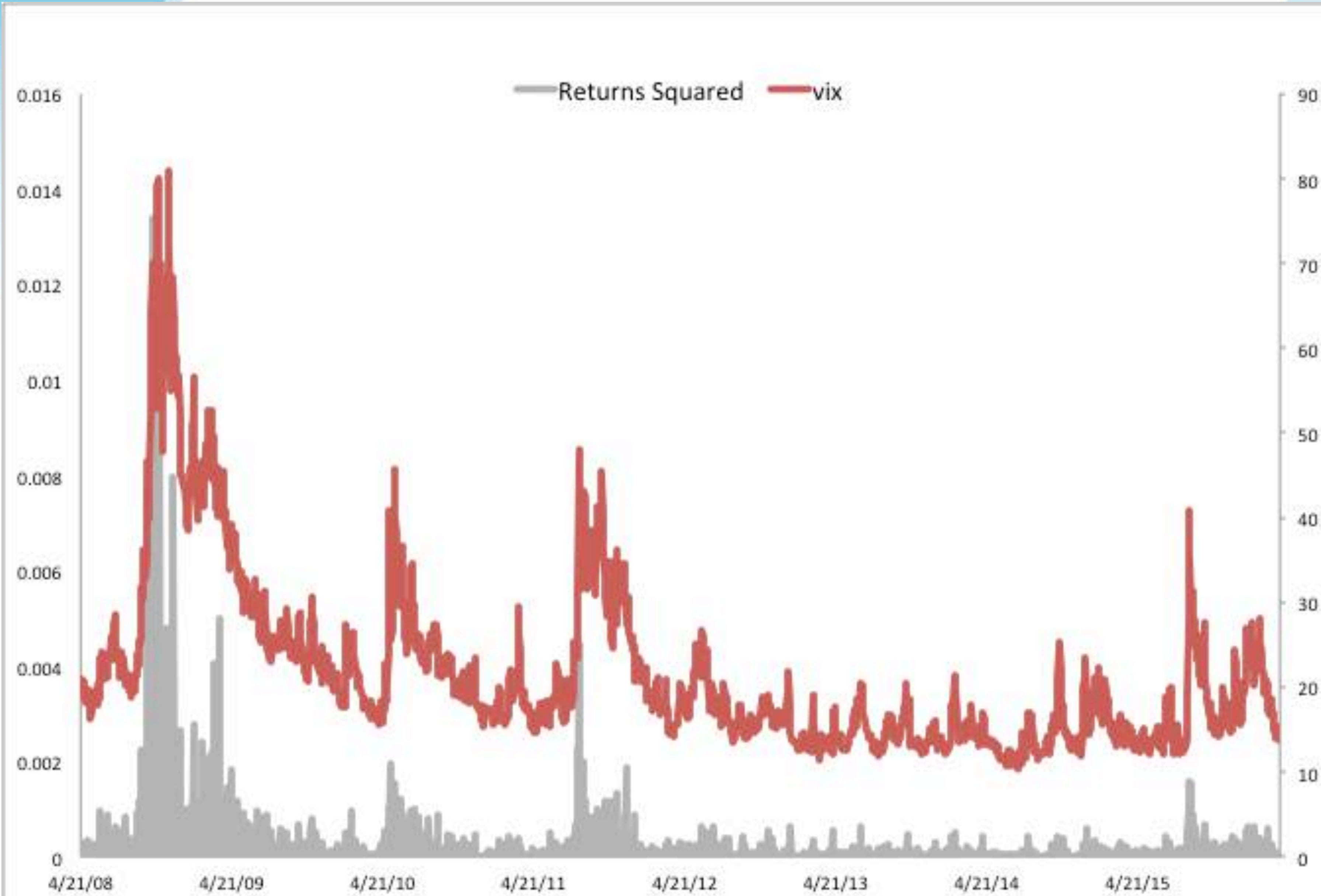
4/21/14
4/21/15



Returns Squared
vix
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
90
80
70
60
50
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20
10
0
4/21/08
4/21/09
4/21/10
4/21/11
4/21/12
4/21/13
4/21/14
4/21/15

Some Stylized Facts

- Returns are not i.i.d. but show very little serial correlation.
- Squared returns, or volatilities, show high serial correlation.
- Return series is heavy tailed.
- High volatility, and extreme events, appear in clusters.

EWMA Model

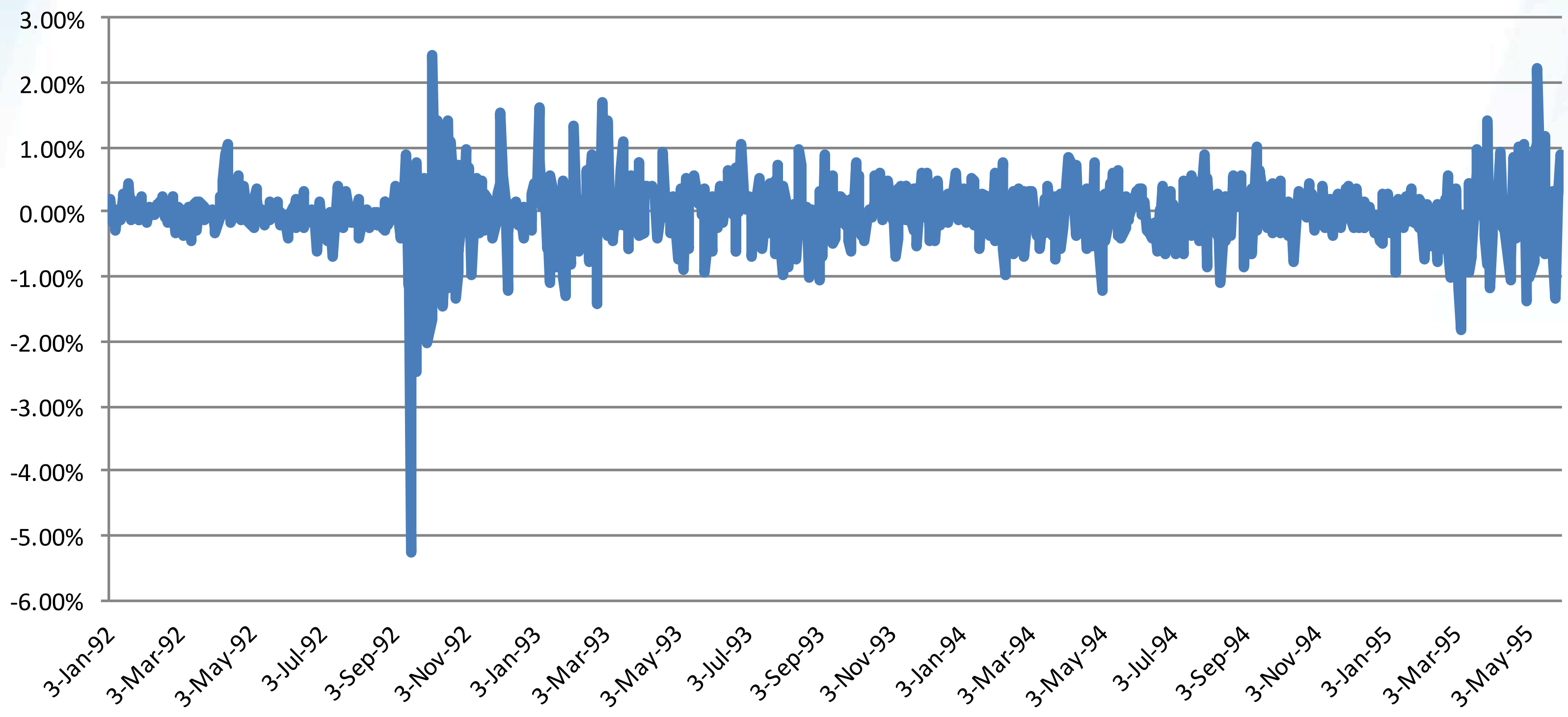
- In an exponentially weighted moving average model, the weights assigned to the u^2 decline exponentially as we move back through time
- This leads to $\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$
- RiskMetrics uses λ = 0.94
- Tracks volatility changes
- With large *m* this leads to a weighted average of squared returns with weights decreasing by λ.

$$\sigma_n^2 = (1-\lambda)\sum_{i=1}^{m} \lambda^{i-1} u_{n-i}^2 + \lambda^m \sigma_{n-m}^2$$

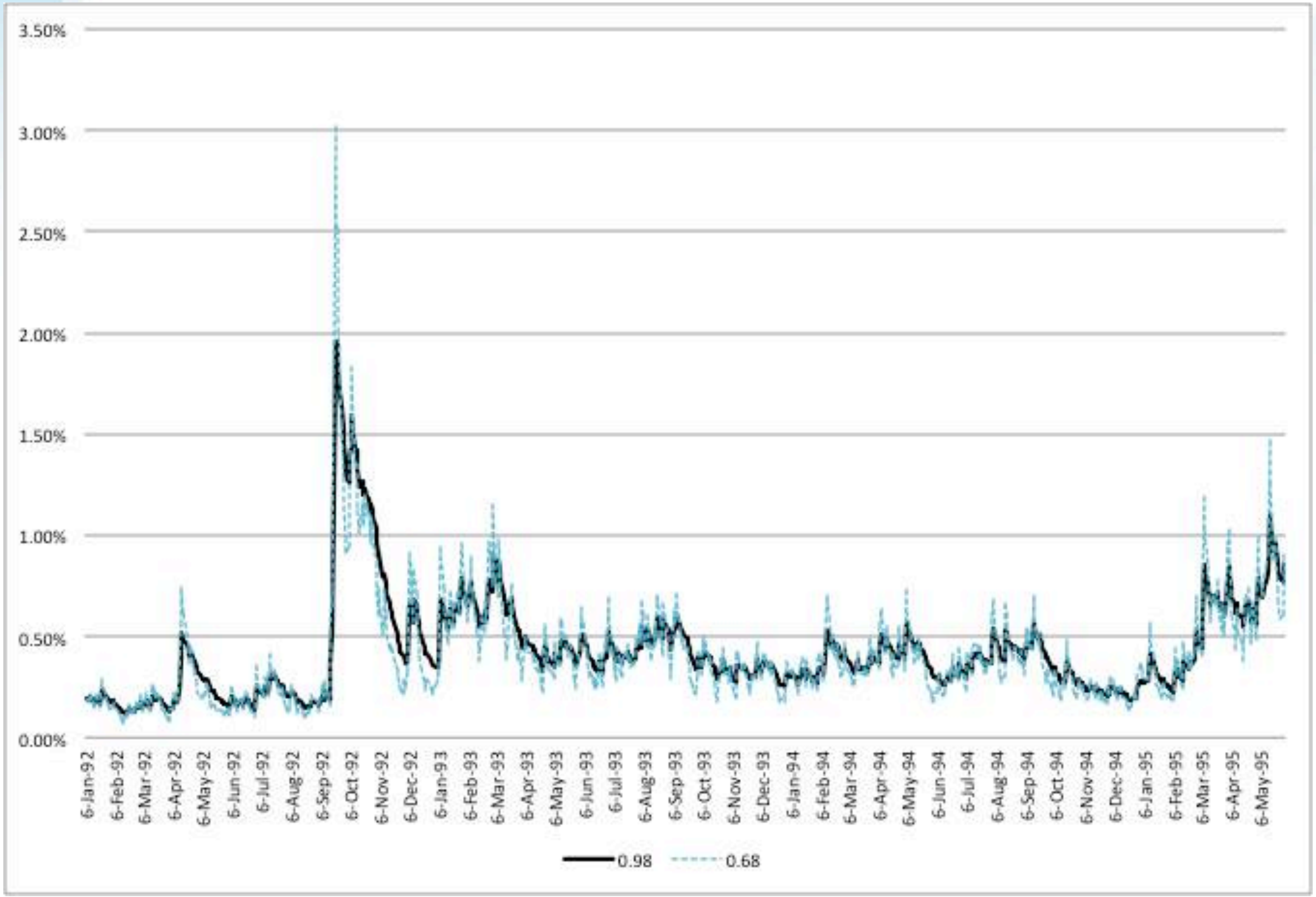
EWMA Model (cont)

- Relatively little data needs to be stored. We need only remember the current estimate of the variance rate and the most recent observation on the market variable
- Low λ leads to more weight on recent returns and therefore volatile estimates of volatility
- High λ leads to less weight on recent returns and therefore slow response to changing volatility.

Daily Returns GBP/USD



EWMA Estimates of Daily Volatility



GARCH (1,1)

In GARCH (1,1) we let the variance revert to a long-run average variance rate

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

Weights sum to 1: $\gamma + \alpha + \beta = 1$

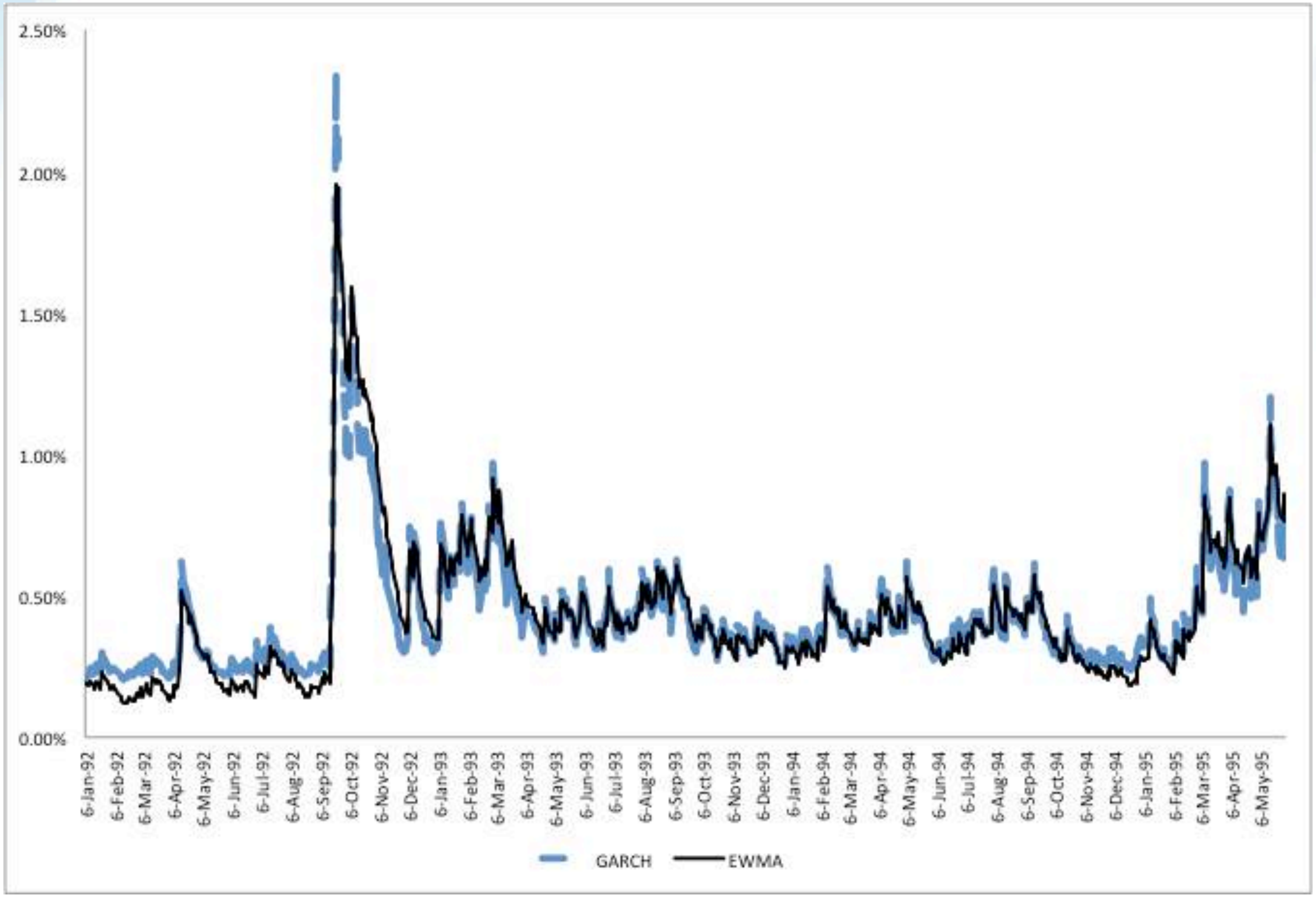
Setting $\omega = \gamma V_L$, we can write:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

And:

$$V_L = \frac{\omega}{1 - \alpha - \beta}$$

GARCH Estimates vs EWMA



Forecasting Future Volatility

- We now ask: What is our prediction for volatility in *t* days.
- If we assume returns are i.i.d. then: $E[\sigma_{n+t}^2] = \sigma_n^2$
- If we assume EWMA: $\sigma_{n+1}^2 = \lambda\sigma_n^2 + (1-\lambda)u_n^2$
- Since: $E[u_n^2] = \sigma_n^2$
- We get the same result:

$E[\sigma_{n+t}^2] = \sigma_n^2$ for *t=1*, and then for all *t*.

Forecasting Future Volatility with GARCH (1,1)

Since $\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$

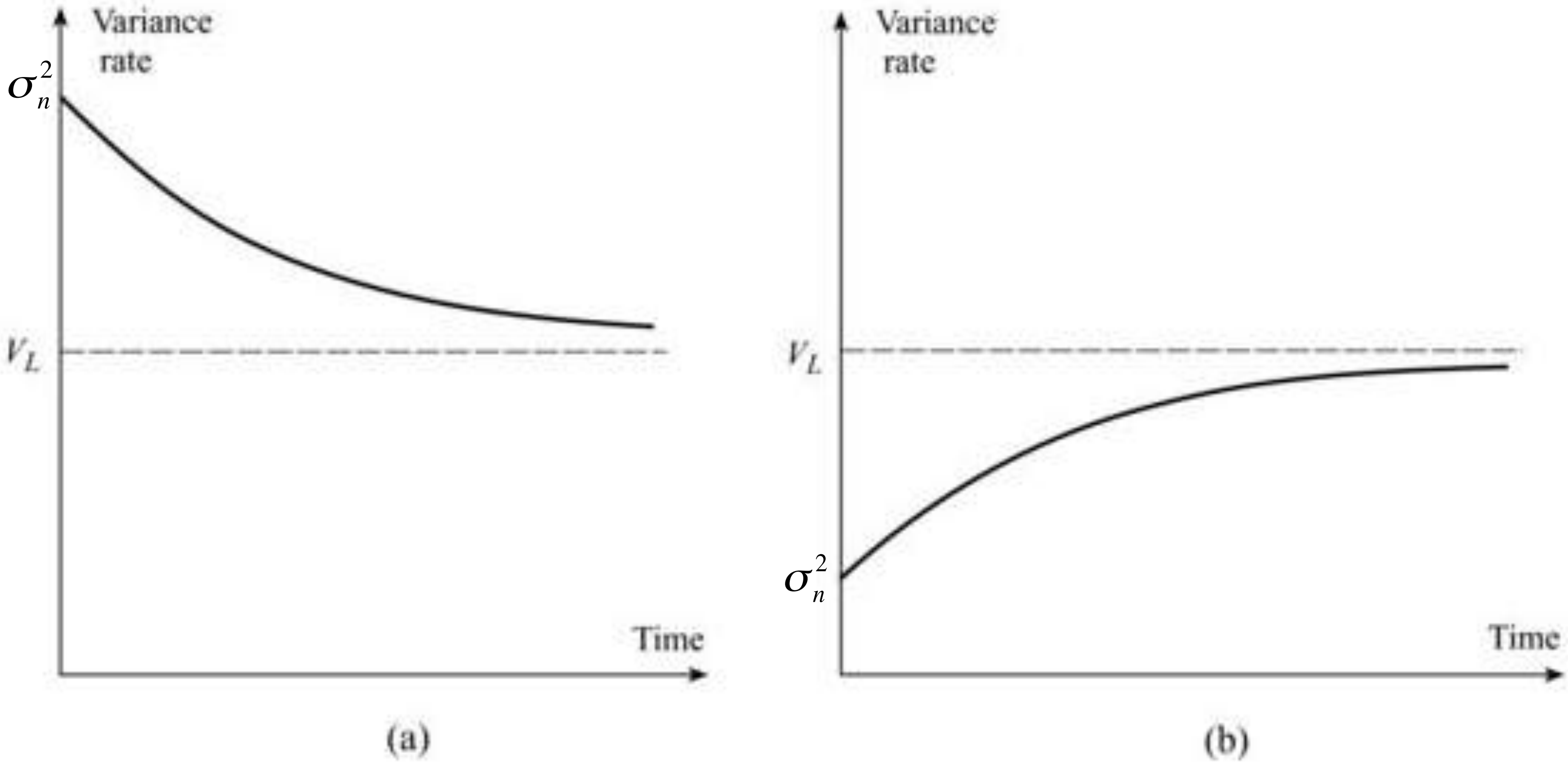
And $E[u_{n+1}^2] = \sigma_{n+1}^2$

We get by iteration that the expected future daily variance is:

$$E[\sigma_{n+t}^2] = V_L + (\alpha + \beta)^t (\sigma_n^2 - V_L)$$

If $\alpha + \beta < 1$ the daily volatility will be reverting to the long run mean

Forecasting Future Volatility cont



$$E[\sigma_{n+t}^2] = V_L + (\alpha+\beta)^t(\sigma_n^2 - V_L)$$

Volatility Over T Days

- We would like to know the volatility over the next T days.
- First, assume that the daily returns on the index are i.i.d. with mean zero and annualized standard deviation of 20%.
- What is the volatility of 10-day return?

$$\sqrt{\frac{10}{252}} \cdot 20\% = 3.98\%$$

Volatility Over T Days - GARCH

- Now, suppose volatility follows GARCH(1,1) with long term annualized volatility of 15%, $\alpha=0.0603$, $\beta=0.9001$, what is the volatility of 10-day return? Current volatility estimate is 20% p.a.

Volatility Over T Days – GARCH

- Convert the variances to daily:

$$V_L = \frac{0.15^2}{252} = 0.000089 \quad \sigma_n^2 = \frac{0.2^2}{252} = 0.000159$$

- Compute the expected future variances, using:

$$E\left[\sigma_{n+t}^2\right] = V_L + (\alpha + \beta)^t \left[\sigma_n^2 - V_L\right]$$

t	0	1	2	3	4	5	6	7	8	9
Variance	0.000159	0.000156	0.000153	0.000151	0.000148	0.000146	0.000144	0.000142	0.000140	0.000138

- Sum the variances: 0.001476
- 10-day volatility is only 3.84%

Average Variance Rate

- Instead of summing up discretely, we can integrate over time: $\int_0^T V_L + (\alpha+\beta)^t \left(\sigma_n^2 - V_L\right) dt$
- Variance over T days is:

$$V_L \cdot T + \frac{1}{a}\left(1 - e^{-aT}\right)\left(\sigma_n^2 - V_L\right) \qquad a = \ln\left(\frac{1}{\alpha+\beta}\right)$$

- The average daily variance is:

$$V_L + \frac{1}{aT}\left(1 - e^{-aT}\right)\left(\sigma_n^2 - V_L\right)$$

Annualized Average Variance Rate

- For pricing an option with T days to maturity we use the annualized average variance over the period.
- The variance per year for an option lasting T days is: $$\sigma(T)^2 = 252\left[V_L + \frac{1-e^{-aT}}{aT}\left(\sigma_n^2 - V_L\right)\right]$$
- σ_n^2 and V_L are daily. $\sigma(T)$ is annualized. T in days.
- Note: $\sigma(0)^2 = 252 \cdot \sigma_n^2$

Example – Option on S&P

- Suppose the S&P 500 is currently at 2,020. What is the value of a European option on the index, with 30 days to expiration, K=2,000, r=3%, σ=20%, q=3%?
  - **Black Scholes value is $65.66.**
- Now, suppose that we estimated a GARCH(1,1) process for the index and found that the long term volatility is 15%, α=0.0603, β=0.9001. What is our estimate of the option's value?

Example – Option on S&P (2)

- First, compute the average annualized volatility:

$$V_L = \frac{0.15^2}{252} = 0.000089 \quad \sigma_n^2 = \frac{0.2^2}{252} = 0.000159$$

$$a = \ln \frac{1}{0.0603+0.9001} = 0.0404$$

$$\sigma(T) = \sqrt{252\left\{V_L + \frac{1-e^{-aT}}{aT}\left[\sigma_n^2 - V_L\right]\right\}} = 18.07\%$$

- The Black-Scholes value is now: \$60.40.
  – \$5.26 less than with the i.i.d. assumption.

Scenario Analysis

- Scenarios may be used to measure the sensitivity of the position value to shocks in underlying parameters.
- For example: To estimate the sensitivity of the option to volatility, we can look at the change in value position for increase/decrease of 1% in volatility.
- Regulators often require banks to consider what would happen to their portfolio if market volatility increased dramatically overnight.

Scenario Analysis - Example

- We want to evaluate the change in value of the option from before, if volatility goes up from 20% to 21%
  1. Assuming returns are i.i.d.
  2. Using GARCH with long term volatility = 15% as before.

Scenario Analysis - Example

1. At σ=20% the price was \$65.66.
   – Recalculating using σ=21%, price is \$68.39.
     The value has gone up by 4.15%
2. Using GARCH: average annualized volatility (aav) is 18.07%, and price is \$60.40.
   – Recalculating using σ=21%, aav is 18.71%, price is \$62.16. The value has gone up only by 2.91%

Sensitivity to Volatility

- Differentiating the Average Annualized Volatility formula for GARCH (1,1) gives us the sensitivity of average annualized volatility to changes in $\sigma(0)$.
- When $\sigma(0)$ changes by $\Delta\sigma(0)$, GARCH (1,1) predicts that $\sigma(T)$ changes by

$$\Delta\sigma(T) \approx \frac{1-e^{-aT}}{aT}\frac{\sigma(0)}{\sigma(T)}\Delta\sigma(0)$$

Thanks

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Maximum Likelihood Estimation

UCLA Anderson
THINK IN THE NEXT

Agenda

- Likelihood function
- Maximum Likelihood Estimation
- Applying MLE to Volatility Models
- Confidence Intervals
- Likelihood Ratio Tests

Likelihood Function - Example

- Suppose we draw one number from a normal distribution, what is the probability density function, if we know μ=1 and σ=3?

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\cdot 3} e^{-\frac{(x-1)^2}{2\cdot 3^2}}$$

- Suppose we don’t know μ and σ, but the number we draw is 1. What is the likelihood?

$$L(\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(1-\mu)^2}{2\sigma^2}}$$

Maximum Likelihood Estimator

- Suppose our data: $X = (x_1, x_2, \cdots, x_n)$ are a realization from a joint probability density . $f(X;\theta)$

  θ is the vector of parameters of the density function.
- The likelihood is: $L(\theta;X) = f(X;\theta)$

  i.e. a function of θ where the observed data are known.
- In maximum likelihood methods, we find the parameters that maximize the likelihood of the observed sample.

Log Likelihood for iid Data

- Typically, we prefer to maximize log-likelihood rather than likelihood.
- If the data are i.i.d. then we have:

$$l(\theta;X) = \log L(\theta;X) = \log \prod_{i=1}^{n} f(x_i;\theta) = \sum_{i=1}^{n} \log L(\theta;x_i)$$

- The parameters that maximize this function can be shown to be good estimators of the true parameter.

Likelihood for iid data example

- Suppose we draw 3 numbers from a Normal distribution: 1, -2, 3.
- Suppose we know mean = 0. The free parameter is v, the variance.
- What is the log likelihood?

$$L(\nu;x_i)=\frac{1}{\sqrt{2\pi\nu}}e^{-\frac{(x_i-0)^2}{2\nu}} \Rightarrow l\left(\nu;x_i\right)=-\frac{1}{2}\ln\left(2\pi\right)-\frac{1}{2}\left[\ln\left(\nu\right)+\frac{x_i^2}{\nu}\right]$$

$$l\left(\nu;x_1,x_2,x_3\right)=\sum l\left(\nu\right)=-\frac{3}{2}\ln\left(2\pi\right)-\frac{3}{2}\ln\left(\nu\right)-\frac{1}{2}\left[\frac{(1)^2}{\nu}+\frac{(-2)^2}{\nu}+\frac{(3)^2}{\nu}\right]$$

- MLE will be *v* that maximizes this expression.

Simple MLE Example

- We observe that a coin falls on heads one time in ten trials. What is our estimate of the probability, *p*, of a coin falling on heads?
- The likelihood of the outcome is:

$$L(p) = 10p(1-p)^9$$

- Let's look at the first order condition to find the maximum.

$$\frac{\partial L}{\partial p} \propto (1-p)^9 - 9p(1-p)^8 = 0$$

$$1 - p = 9p$$

$$p = \frac{1}{10}$$

Simple MLE Example (cont.)

- Suppose we observed two heads in 10 tosses, what is the MLE?
- The likelihood of the outcome is:

$$L(p) = \binom{10}{2} p^2 (1-p)^8$$

- Let's look at the first order condition to find the maximum.

$$\frac{\partial L}{\partial p} \propto 2p(1-p)^8 - 8p^2(1-p)^7 = 0$$

$$(1-p) - 4p = 0$$

$$p = 1/5$$

MLE for N(0, ν)

Estimate the variance, *v*, from *n* observations, $u_1 \ldots u_n$, drawn from a normal distribution with mean zero:

Likelihood: $$\prod_{i=1}^{n}\left[\frac{1}{\sqrt{2\pi v}}\exp\left(\frac{-u_i^2}{2v}\right)\right]=\left[\frac{1}{2\pi v}\right]^{\frac{n}{2}}\cdot\prod_{i=1}^{n}\left[\exp\left(\frac{-u_i^2}{2v}\right)\right]$$

Log Likelihood: $$\frac{n}{2}\ln\left(\frac{1}{2\pi}\right)-\frac{n}{2}\ln(v)-\sum_{i=1}^{n}\left[\frac{u_i^2}{2v}\right]$$

FOC: $$-\frac{n}{2v}+\frac{1}{2v^2}\sum_{i=1}^{n}u_i^2=0$$

MLE: $$v=\frac{1}{n}\sum_{i=1}^{n}u_i^2$$

MLE and Time Varying Volatility

- Models like GARCH(1,1) and EWMA assume that daily returns are Normal with mean zero and volatility, v_i.
  - Note, this is the distribution conditional on all previous daily returns
  - Later, we will relax the Normal assumption with wider tail distributions
- The likelihood is now:

$$\prod_{i=1}^{n}\left[\frac{1}{\sqrt{2\pi v_i}}\exp\left(\frac{-u_i^2}{2v_i}\right)\right]$$

MLE for N(0, ν_i)

- We choose parameters that maximize, where now volatility is changing each day:

$$\sum_{i=1}^{n}\left[-\ln(v_i)-\frac{u_i^2}{v_i}\right]$$

- For the models we previously discussed:

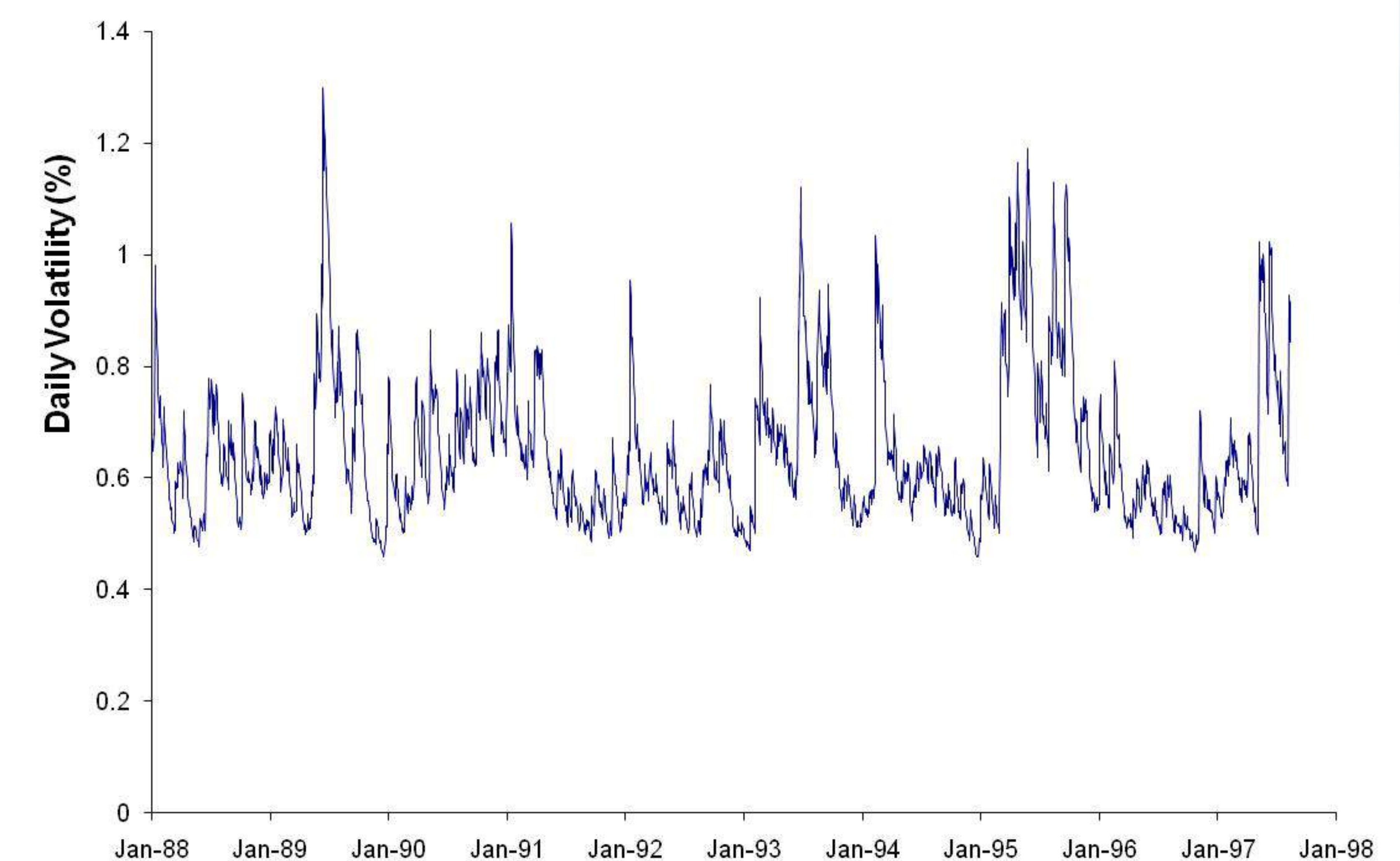
$$EWMA \qquad :\ \nu_i = \lambda\nu_{i-1} + (1-\lambda)u_{i-1}^2$$

$$GARCH(1,1):\ \nu_i = \gamma V_L + \alpha u_{i-1}^2 + \beta\nu_{i-1}$$

MLE for N(0, ν_i) – cont.

- For EWMA – We estimate λ.
- For GARCH(1,1) - We can:
  - Estimate three parameters (α, β, $\omega=\gamma V_L$)
  - Or, assume the the long-run average volatility (V_L) equals to the sample variance and estimate only two parameters using MLE. This is called: Variance Targeting

Daily Volatility of Yen: 1988-1997



Estimating GARCH(1,1)

Day	S_i	u_i	$v_i = \sigma_i^2$	$-\ln v_i - u_i^2/v_i$
1	0.007728			
2	0.007779	0.006599		
3	0.007746	-0.004242	0.00004355	9.6283
4	0.007816	0.009037	0.00004198	8.1329
5	0.007837	0.002687	0.00004455	9.8568
….				
2423	0.008495	0.000144	0.00008417	9.3824
				22,063.5833

1. Assume estimates for alpha, beta and omega.
2. Compute $\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$
3. Compute likelihood and sum across all days.
4. Let optimizer find the parameters to maximize likelihood

Programing MLE Estimator

- Write likelihood function:
  - Takes parameters as inputs
  - Computes the likelihood given the data
- Call a minimization/maximization algorithm such as *optim* in R or *fminunc* in Matlab

$$\boxed{l = \frac{n}{2}\ln\left(\frac{1}{2\pi}\right) + \frac{1}{2}\sum_{i=1}^{n}\left[-\ln(v_i) - \frac{u_i^2}{v_i}\right]}$$

Asymptotic Properties of MLE Estimator

- It can be shown that: $\hat{\theta} \xrightarrow{d} N\left(\theta, \frac{1}{n} I(\theta)^{-1}\right)$
- I(θ) is called the Fisher Information:

$$I(\theta) = E\left[\left(\frac{\partial}{\partial\theta}\ln L(\theta;X)\right)^2\right] = -E\left(\frac{\partial^2}{\partial\theta^2}\ln L(\theta;X)\right)$$

- We use a sample estimate of the Fisher Information:

$$\bar{I}\left(\hat{\theta}\right) = -\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2}{\partial\theta^2}\ln L\left(\hat{\theta};x_i\right)$$

Confidence Interval for MLE Estimator

- This allows us to test hypotheses and build confidence interval around our estimate, if *n* is large enough.
- The standard error of the estimator is:

$$se\left(\hat{\theta}\right) = \sqrt{\frac{1}{n}\bar{I}\left(\hat{\theta}\right)^{-1}}$$

- The *(1-α)* confidence interval for θ is:

$$\hat{\theta} \pm se\left(\hat{\theta}\right)\Phi^{-1}\left(1-\frac{\alpha}{2}\right)$$

Hypothesis Testing using MLE

- We can test the hypothesis at level α:

$$H_0 : \theta = \theta_0$$
$$H_1 : \theta \neq \theta_0$$

- By forming a standard normal: $Z = \dfrac{\hat{\theta} - \theta_0}{se\left(\hat{\theta}\right)} \sim N\left(0,1\right)$

- Reject *H_0* if: $\left|Z\right| \geq \Phi^{-1}\left(1 - \dfrac{\alpha}{2}\right)$

Normal Distribution Example

Suppose that $x_1,\ldots,x_n$ are $i.i.d.\ N\left(\mu,\sigma^2\right)$ with σ^2 known.

The log likelihood: $$\ln L\left(\mu;X\right) = -\frac{n}{2}\left[\ln\left(2\pi\right)+\ln\left(\sigma^2\right)\right]-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(x_i-\mu\right)^2$$

FOC: $$\frac{\partial \ln L\left(\mu;X\right)}{\partial\mu} = \frac{1}{\sigma^2}\left(\sum_{i=1}^{n}x_i - n\mu\right) = 0$$

MLE Estimator: $$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}x_i = \bar{X}$$

Fisher Information: $$\bar{I}\left(\hat{\theta}\right) = -\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2}{\partial\theta^2}\ln L\left(\theta;x_i\right)$$ With: $$\frac{\partial^2 \ln L\left(\mu;x_i\right)}{\partial\mu^2} = -\frac{1}{\sigma^2}$$

$$\Rightarrow \bar{I}\left(\hat{\mu}\right) = -\frac{1}{n}\cdot n\cdot\frac{-1}{\sigma^2} = \frac{1}{\sigma^2}$$

Standard Error: $$se_{\hat{\mu}} = \sqrt{\frac{1}{n}\bar{I}\left(\hat{\mu}\right)^{-1}} = \frac{\sigma}{\sqrt{n}}$$

Normal Distribution – Cont.

$95\% C.I.$ for the mean is:

$$\bar{x} \pm \frac{\sigma}{\sqrt{n}} \Phi^{-1}(0.975) = \bar{x} \pm \frac{\sigma}{\sqrt{n}} 1.96$$

A 2-sided test for the mean different from zero:

$$Z = \frac{\bar{x}}{\sigma / \sqrt{n}}$$

Reject if: $|Z| \geq 1.96$

Homework :

Suppose that $X_1, \ldots, X_n$ are $i.i.d.\ N\left(0, S = \sigma^2\right)$.

$Estimate\ \hat{S}.\ What\ is\ se_{\hat{S}}$?

Likelihood Ratio Test

- Suppose we want to test the hypothesis that some constraints on the parameters hold:
  - For example, μ = 0.
- We can compute the Maximum Likelihood given the constraints, and compare it to the Maximum Likelihood without constraints.
  - What gain in Maximum Likelihood are we giving up by the constraints?
- The following asymptotically holds:
  - *m* is the number of effective constraints.

$$2\left[\max\left\{\ln L(\theta)\right\}-\max\left\{\ln L(\theta_c)\right\}\right]\sim \chi^2_m$$

Normal Distribution – L.R. Test

$$\text{Test for the mean different from zero, when } \sigma \text{ is known:}$$

Unconstrained: $$\max\left\{\ln L(\theta)\right\} = -\frac{n}{2}\left[\ln(2\pi) + \ln\left(\sigma^2\right)\right] - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Constrained: $$\max\left\{\ln L(\theta_c)\right\} = -\frac{n}{2}\left[\ln(2\pi) + \ln\left(\sigma^2\right)\right] - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - 0)^2$$

$$\Downarrow$$

$$LR = 2\left[\max\left\{\ln L(\theta)\right\} - \max\left\{\ln L(\theta_c)\right\}\right] = \frac{1}{\sigma^2}\left[\sum_{i=1}^{n}(x_i)^2 - \sum_{i=1}^{n}(x_i - \bar{x})^2\right] =$$

$$= \frac{1}{\sigma^2}\left[2\bar{x}\sum_{i=1}^{n}x_i - n\bar{x}^2\right] = \frac{n\bar{x}^2}{\sigma^2} \sim \chi_1^2$$

- Reject at 5% if LR>3.841
- Other rejection boundaries: 6.635 at 1%, 2.706 at 10%

Homework

- Daily EUR data from 2/5/14 – 2/5/15
- Estimate EWMA parameter λ using MLE
- Test whether λ is different from 0.96 using Likelihood Ratio Test.

Thanks

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Value at Risk

UCLA Anderson
THINK IN THE NEXT

Agenda

- What is Value at Risk (VaR)?
- Applications of VaR
- Simple Examples
- Expected Shortfall
- Coherent Risk Measures
- Aggregation Through Time
- Choice of VaR Parameters

A Concise Single Measure

- Summarizes multiple aspects of risk in a single number.
  - As opposed to measures like the Greeks and Duration that look at particular sensitivities.
- It is easy to understand and communicate.
- It asks the simple question: "How bad can things get in a given period of time?"

The VaR Statement

"We are α percent certain that we will not lose more than L dollars in time T."

L is the VaR

T is the Time Horizon

α is the Confidence Level

Formal Definition of VaR

- The *VaR* at confidence level, α, is the smallest number *L*, such that the probability that losses exceed *L* is no larger than 1-α.

Or,

$$VaR_\alpha = \min\left\{L : P\left(Loss > L\right) \le 1-\alpha\right\}$$

$$VaR_\alpha = \min\left\{L : P\left(Loss \le L\right) \ge \alpha\right\}$$

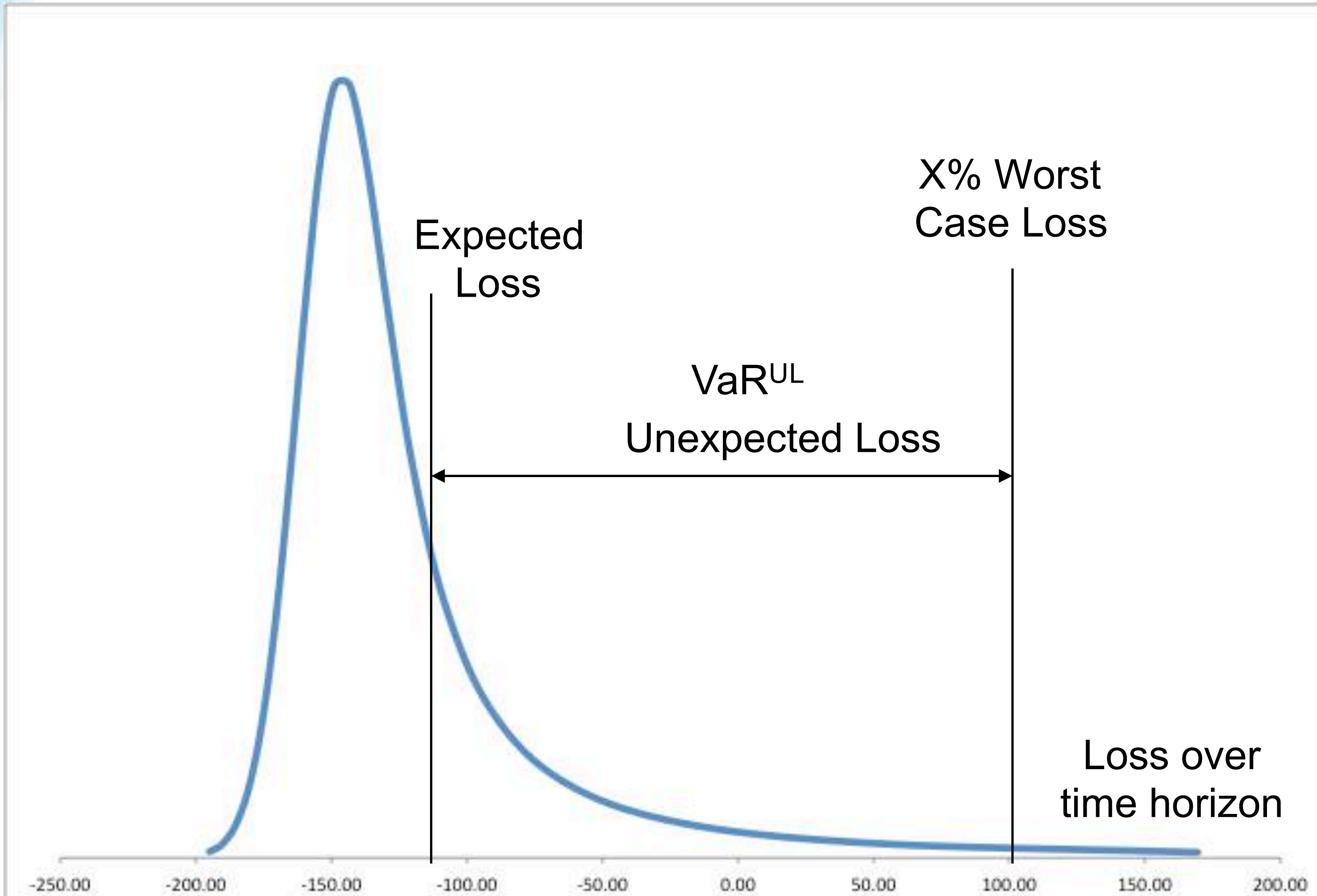
Applications of VaR

- VaR is the industry standard for reporting, both internally and externally.
- It is part of most regulatory frameworks.
- It is used for calculating Capital requirements – How big a cushion do we need to cover losses at a certain probability?
  - Calculations for internal purposes are usually called **Economic Capital**
  - Calculations based on regulatory specifications for reporting purposes are called **Regulatory Capital**.
- It is used for setting limits, predominately on trading floors and portfolios.

Dell, Inc. 10-K

"Based on our foreign currency cash flow hedge instruments …, **we estimate a maximum potential one-day loss in fair value** of … $65 million, … using a Value-at-Risk ("VaR") model. By using market implied rates and incorporating volatility and correlation among the currencies of a portfolio, **the VaR model simulates 3,000 randomly generated market prices and calculates the difference between the fifth percentile and the average as the Value-at-Risk.**"

Expected and Unexpected Loss



Normal Distribution Example

- The daily gains on a portfolio of stocks are distributed normally with mean=0, and standard deviation =$5M
- What is the 95% VaR?

Normsinv(0.05)*σ=-1.645×5 = -8.22mil

$VaR_{95\%}$ = $8.22mil

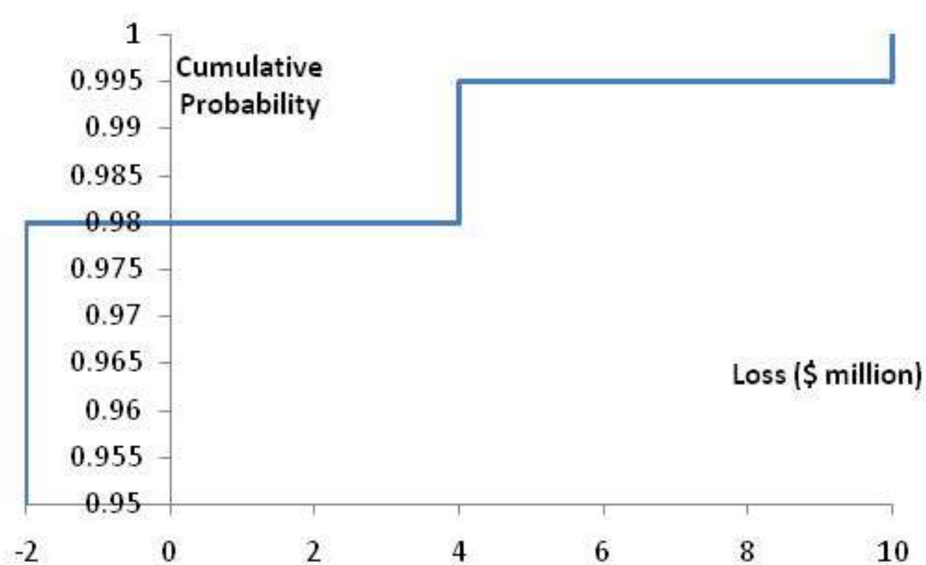
Normal Distribution Example (2)

- The profit from a portfolio over six months is normally distributed with mean \$2 million and standard deviation \$10 million
- The 1% point of the distribution of gains is $2-2.33\times10$ or − \$21.3 million
- The VaR for the portfolio with a six month time horizon and a 99% confidence level is \$21.3M million.
- If we look at Unexpected Losses:
  - Expected Loss = -\$2M
  - Unexpected Loss VaR = \$21.3M – (-\$2M) = \$23.3M

VaR Discrete Example

- A one-year project has a 98% chance of leading to a gain of $2 million, a 1.5% chance of a loss of $4 million, and a 0.5% chance of a loss of $10 million
- What is the VaR at a 99% confidence level?
  - $4 million
- What if the confidence level is 99.9%?
  - $10 million
- What if it is 99.5%?

Cumulative Loss Distribution for Example



$$VaR_{\alpha} = \min\left\{L : P\left(Loss \leq L\right) \geq \alpha\right\}$$

L	Prob.	Cum. Prob	P(Loss>L)
-2	98%	98%	2%
4	1.5%	99.5%	0.5%
10	0.5%	100%	0%

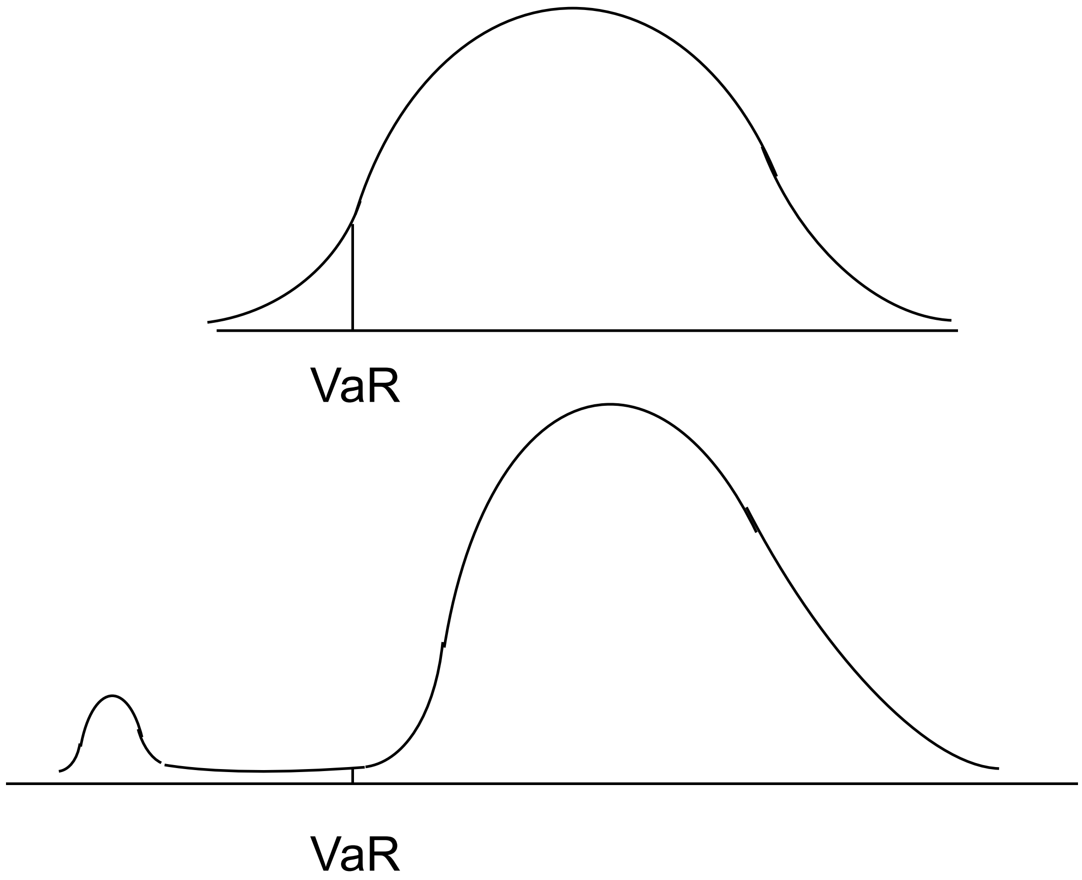
Expected Shortfall

- VaR is the loss level that will not be exceeded with a specified probability, α.
- Expected Shortfall is the average loss over the *1*-α worst cases.
- For continuous distributions: It is the mean loss given that the loss is greater or equal to the VaR level:

$$ES_\alpha = E[L \mid L \geq VaR_\alpha]$$

- Also called: CVaR (Conditional VaR) or Tail Loss
- Two portfolios with the same VaR can have very different expected shortfalls
- Basel has recently moved to using ES rather than VaR

Distributions with the Same VaR but Different Expected Shortfalls



Example: Same VaR, Different ES

- Suppose there are two possible states of the world:
  - A good state with probability = 0.8: Portfolio gains 100
  - A bad state with probability = 0.2: Portfolio loss is randomly drawn from U[50,100].
- What is 90% VaR?
  - Let x be the VaR. It must be between 50 and 100.
  - The probability of loss being less than or equal to x is 0.9:

$$\Pr\left[Loss \le x\right] = 0.8 + 0.2 \times \frac{x - 50}{100 - 50} = 0.9$$

$$x = 75$$

- What is 90% Expected Shortfall?

$$E\left[Loss \mid Loss \ge 75\right] = \frac{75 + 100}{2} = 87.5$$

Example: Same VaR, Different ES (2)

- Suppose the two states are now:
  - A good state with probability = 0.8: Portfolio gains 100
  - A bad state with probability = 0.2: Portfolio loss is randomly drawn from U[0,150].
- What is 90% VaR?
  - Let *x* be the VaR. It must be between 0 and 150.
  - The probability of loss being less than or equal to *x* is 0.9:

$$\Pr\left[Loss \le x\right] = 0.8 + 0.2 \times \frac{x-0}{150-0} = 0.9$$

$$x = 75$$

- What is 90% Expected Shortfall?

$$E\left[Loss \mid Loss \ge 75\right] = \frac{75+150}{2} = 112.5$$

Expected Shortfall - Normal Distribution

- The daily losses on a portfolio of stocks are distributed normally with mean=0, and standard deviation of 5 million
- What is the 95% VaR?
  - VaR = 1.645×5 = 8.22 million
- What is the 95% Expected Shortfall?

$$E[Loss \mid Loss \geq VaR] = \frac{\int_{VaR}^{\infty} L \times \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{L^2}{2\sigma^2}} dL}{P[Loss \geq VaR]} =$$

$$= \frac{\varphi\left(VaR/\sigma\right)}{1-p}\sigma = \frac{\varphi(1.645)}{0.05} \bullet 5 = 10.31$$

Expected Shortfall - Normal Distribution

- For the **Mean Zero Normal Distribution**, Expected Shortfall at a certain confidence level, is like VaR at some higher level
- In our case:

$$\frac{\varphi(1.645)}{0.05} = 2.06 = NORMSINV(0.98)$$

- 95% Expected Shortfall = 98% VaR

Coherent Risk Measures

- **Monotonicity:** If one portfolio always produces a worse outcome than another, its risk measure should be greater
- **Translation Invariance:** If we add an amount of riskless asset paying K to a portfolio its risk measure should go down by K
- **Homogeneity:** Multiplying the size of a portfolio by λ should result in the risk measure being multiplied by λ
- **Sub-additivity:** The risk measures for two portfolios after they have been merged should be no greater than the sum of their risk measures before they were merged = benefit to diversification

VaR vs Expected Shortfall

- VaR satisfies the first three conditions but not the fourth one
  - VaR is not coherent
- Expected shortfall satisfies all four conditions.

Sub-Additivity –Example

- Each of two independent projects has a probability 0.98 of a loss of \$1 million and 0.02 probability of a loss of \$10 million
- What is the 97.5% VaR for each project? \$1M
- What is the 97.5% expected shortfall for each project?
  - Within the 2.5% worst results for the project:
    - Conditional Probability of \$1M = (98-97.5)/2.5=0.5/2.5
      Conditional Probability of 10mil = (100-98)/2.5=2/2.5
  - ES=(0.5/2.5)*1 + (2/2.5)*10 = 8.2

Sub-Additivity –Example (2)

- What is the 97.5% VaR for the portfolio? 11

Loss	Probability	Cum. Probability
1+1=2	0.98*0.98=96.04%	96.04%
10+1=11	2*0.98*0.02=3.92%	99.96%
10+10=20	0.02*0.02=0.04%	100%

- What is the 97.5% expected shortfall for the portfolio?
  - Conditional Prob. of 11 = (99.96-97.5)/2.5, Conditional Prob. of 20 = (100-99.96)/2.5
  - (2.46/2.5)*11 + (0.04/2.5)*20 = 11.144

Sub-Additivity –Example (3)

- This is an example of VaR not satisfying sub-additivity:
  - VaR for 1 project = 1, VaR for 2 projects = 11
  - VaR (2 projects)>2*VaR(1 project)
- Expected Shortfall satisfies sub-additivity
  - ES for 1 project = 8.2, ES for 2 projects = 11.144
  - ES(2 projects)<2*ES(1 project)
  - ES always satisfies sub-additivity

Spectral Risk Measures

- A spectral risk measure assigns weights to quantiles of the loss distribution
- VaR assigns all weight to Xth quantile of the loss distribution
- Expected shortfall assigns equal weight to all quantiles greater than the Xth quantile
- For a coherent risk measure weights must be a non-decreasing function of the quantiles

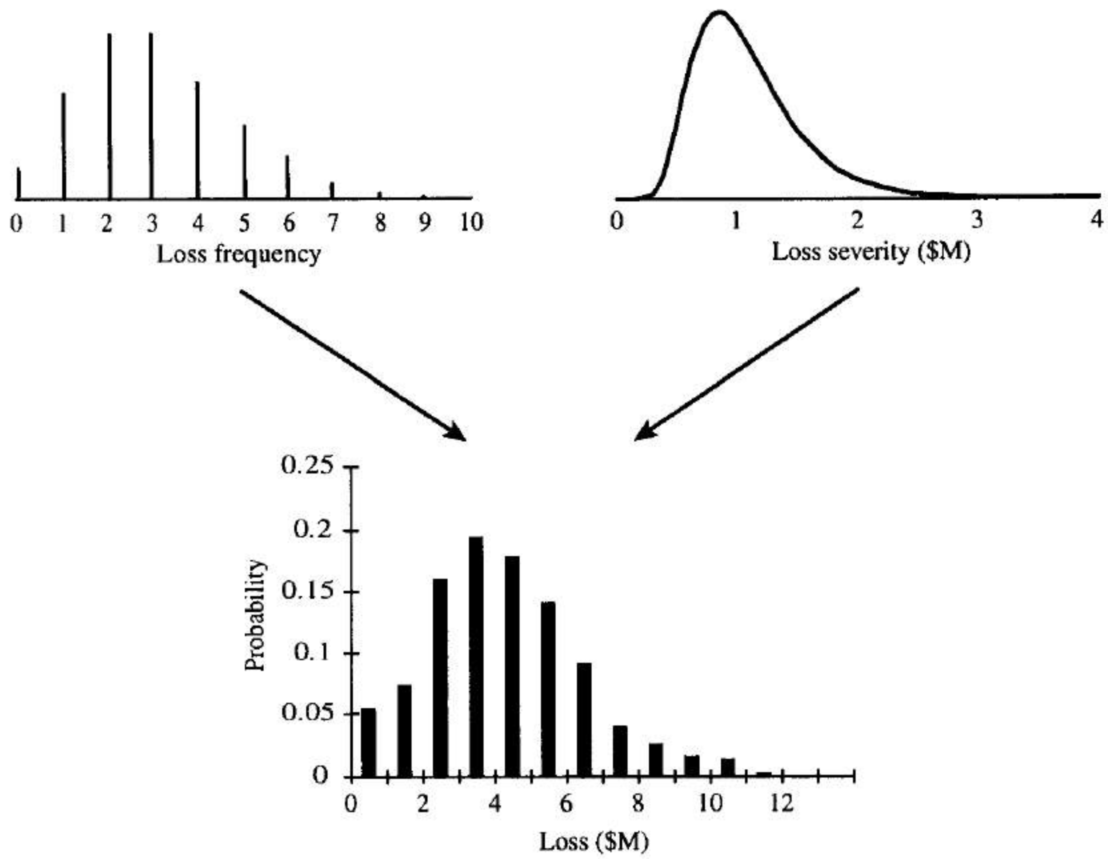
Example: Simulation for Operational Risk VaR

- Assume a bank has figured out that the probability of k events of fraud happening in a year is distributed according to the Poisson distribution:

$$\Pr(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

- The severity of the fraud is distributed lognormally.
- What is the VaR-95% for annual loss due to fraud?

Using Monte Carlo to combine the Distributions



Monte Carlo Simulation

- Create distribution of annual losses, by simulating many trials:
  - Sample from Poisson distribution to determine the number of loss events (=k)
  - Sample k times from the loss severity distribution to determine the loss severity for each loss event
  - Sum loss severities to determine total loss
- Find the 95 percentile of the distribution

Credit VaR for Uncorrelated Loans

- A bank has 100 loans of $15 million each. The probability of default (PD) of each loan is 2.5%. In case of default there is no recovery. Loan defaults are independent of each other.
- What is the Expected Loss (EL) on the portfolio?
  – EL = 100*15*2.5% = $37.5

Credit VaR for Uncorrelated Loans (Example – Cont.)

- What is the $VaR_{96\%}$ of the portfolio?
  - The probability of *K* loans defaulting is given by the Binomial Distribution
  - P(*K* loans default) = BINOMDIST(*K*, 100, 2.5%, 0)
  - P(# defaults $\leq K$) = BINOMDIST(*K*, 100, 2.5%, 1)
- The cumulative Probability is given by:
- There is 96% that 5 loans or less will default.

Num Defaults	Cumulative Prob
0	0.08
1	0.28
2	0.54
3	0.76
4	0.89
5	**0.96**
6	0.99

Credit VaR for Uncorrelated Loans (Example – Cont.)

- $VaR_{96\%}$ is therefore: 5*15 = $75M
- The Unexpected Loss $VaR_{96\%}$ is equal to 75-37.5 = $37.5M
- What if there was 60% recovery on each loan?
  - LGD = 40%
  - VaR-96% = 5*15*0.4 = $30M
- But loan defaults in a portfolio are generally not independent … later in the course.

Aggregating VaR Over Time

- The simplest assumption is that daily gains/losses are normally distributed and independent with mean zero
- The T-day VaR equals $\sqrt{T}$ times the one-day VaR
- If there is positive autocorrelation the T-day VaR will be greater

Independence Assumption in VaR Calculations

- When there is autocorrelation equal to ρ, instead of T-days variance being T times daily variance, the multiplier is:

$$T+2(T-1)\rho+2(T-2)\rho^{2}+2(T-3)\rho^{3}+\ldots 2\rho^{T-1}$$

Impact of Autocorrelation: Ratio of *T*-day VaR to 1-day VaR

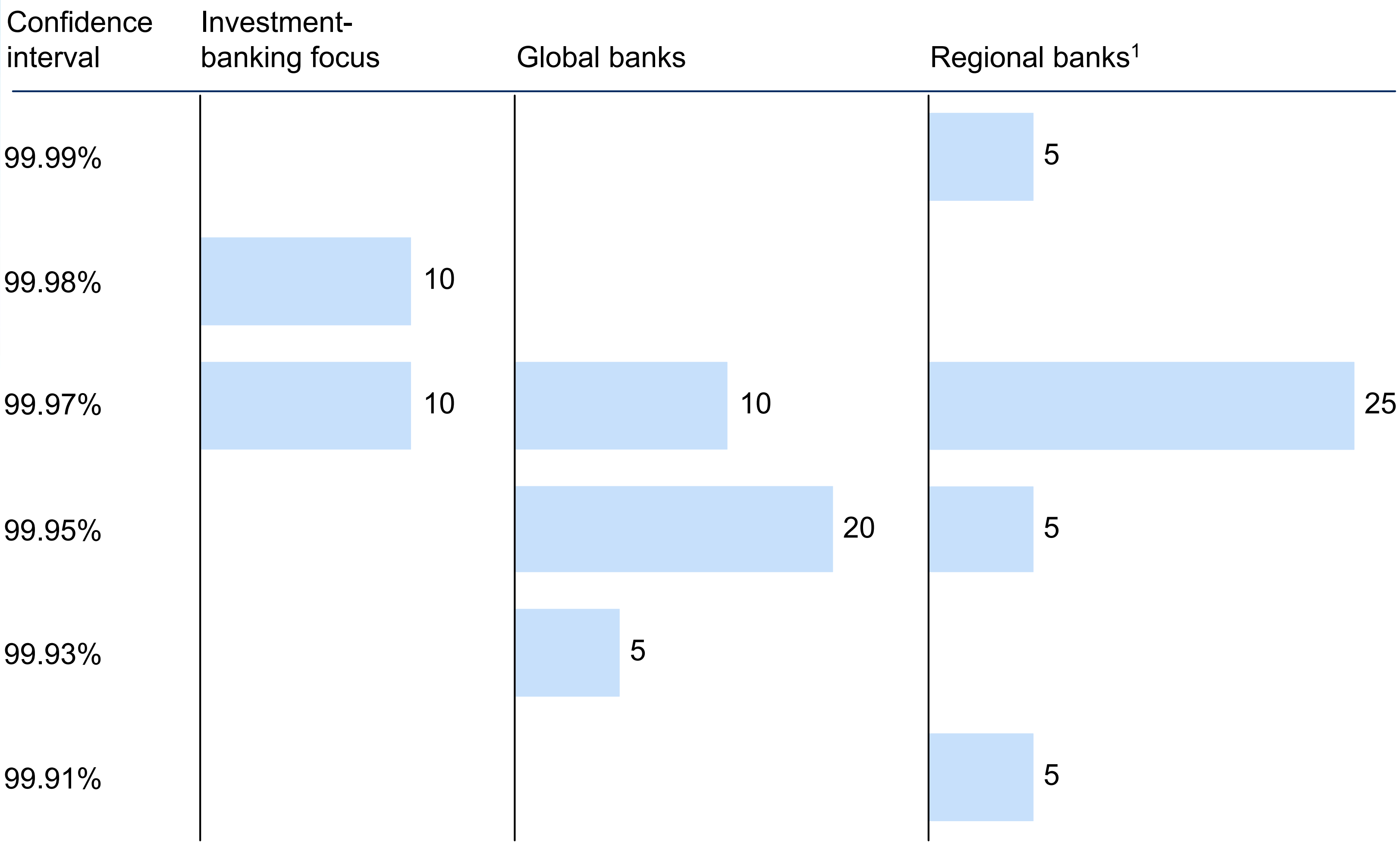
	T=1	T=2	T=5	T=10	T=50	T=250
ρ=0	1.0	1.41	2.24	3.16	7.07	15.81
ρ=0.05	1.0	1.45	2.33	3.31	7.43	16.62
ρ=0.1	1.0	1.48	2.42	3.46	7.80	17.47
ρ=0.2	1.0	1.55	2.62	3.79	8.62	19.35

Choice of VaR Parameters

- Time horizon:
  - Should depend on how quickly portfolio can be unwound.
  - Bank regulators use 1-day for market risk scaled by the square root of time to 10-days, and 1-year for credit/operational risk.
- Confidence level:
  - Depends on objectives. Regulators use 99% for market risk and 99.9% for credit/operational risk.
  - A bank aiming to maintain a AA credit rating might use confidence levels as high as 99.98% for internal Economic Capital calculations.

Exhibit 3 Banks use a range of confidence intervals for economic capital models.

Economic capital-modeling practices at 17 financial institutions, %



1 Some public information of additional regional banks has been included for comparison.
Note: Numbers may not add up to 100 due to rounding.

Source: McKinsey Market Risk Survey and Benchmarking 2011

Thanks

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VaR II

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Allocation and Aggregation of VaR

EADS, Financial Statements

A summary of the VaR position of the Group's financial instruments portfolio at 31 December 2013 and 31 December 2012 is as follows:

(In € million)	Total VaR	Equity price VaR	Currency VaR	Commodity price VaR	Interest rate VaR
31 December 2013					
FX hedges for forecast transactions or firm commitments	577	0	615	0	46
Financing liabilities, financial assets (incl. cash, cash equivalents, securities and related hedges)	156	161	16	0	19
Finance lease receivables and liabilities, foreign currency trade payables and receivables	28	0	4	0	28
Commodity contracts	13	0	1	12	0
Diversification effect	(157)	0	(18)	0	(38)
All financial instruments	**617**	**161**	**618**	**12**	**55**

VaR is used for reporting risk on total portfolio, but also on specific sub-portfolios, and risk-types.

VaR Measures for a Portfolio where an amount x_i is invested in the ith component of the portfolio

- Marginal VaR: $\frac{\partial \text{VaR}}{\partial x_i}$

- Incremental VaR: Incremental effect of the ith component on VaR, i.e. VaR of Portfolio including ith component minus VaR of the Portoflio without it.

- Component VaR: $x_i \frac{\partial \text{VaR}}{\partial x_i}$

Properties of Component VaR

- The total VaR is the sum of the component VaRs (Euler's theorem)

$$VaR_{Total} = VaR\left(\sum_{i=1}^{M} x_i\right) = \sum_{i=1}^{M} \frac{\partial V}{\partial x_i} x_i = \sum_{i=1}^{M} C_i$$

- The component VaR therefore provides a sensible way of allocating VaR to different activities

VaR Attribution Example

- There are two desks on a US Bank's trading floor:
  - Desk A has EUR 100B exposure
  - Desk B has GBP 75B exposure
- Current USD rates are EUR=1.11, GBP=1.53
- The daily volatility of rates are: 0.45%, 0.35% respectively, the correlation is 0.7.
- Assuming Normal daily changes, what is $VaR_{99\%}$ of the portfolio? What are the Incremental $VaR_{99\%}$ and Component $VaR_{99\%}$ of each desk?

VaR Attribution Example

	EUR	GBP	Aggregate
Position in foreign cur.	100	75	
Exchange rate (S_t)	1.11	1.53	
Dollar position (x_i)	111	114.75	
Daily volatility (σ_i)	0.45%	0.35%	
Correlation (ρ)			0.7
Variance (σ_P^2)			0.692
VaR-99%	1.162	0.934	1.935

$$\sigma_P^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2\rho$$

$$VaR_{99\%} = N^{-1}(0.99)\sigma_P$$

VaR Attribution Example

	EUR	GBP	Aggregate
Dollar position (x_i)	111	114.75	
Daily volatility (σ_i)	0.45%	0.35%	
Correlation (ρ)			0.7
Variance (σ_P^2)			0.692
VaR-99%	1.162	0.934	1.935
Incremental VaR	1.000	0.773	
Marginal VaR	0.010	0.007	
Component VaR	1.091	0.844	1.935

From previous slide: $$\frac{\partial VaR}{\partial x_1} = N^{-1}(0.99) \cdot \frac{x_1\sigma_1^2 + x_2\sigma_1\sigma_2\rho}{\sigma_P}$$

Aggregating VaRs

An approximate approach that is used by many companies:

$$\mathrm{VaR}_{\mathrm{total}} = \sqrt{\sum_i \sum_j \mathrm{VaR}_i \mathrm{VaR}_j \rho_{ij}}$$

where VaR_i is the VaR for the *i*th segment, $\mathrm{VaR}_{\mathrm{total}}$ is the total VaR, and ρ_{ij} is the coefficient of correlation between losses from the *i*th and *j*th segments

Big question: How to determine correlation?

Aggregation under Solvency 2

Capital Requirement are based on VaR (or ES) calculations for different risks. Various correlation matrices are then used to aggregate.

Insurance Risks:

	mortality	Longevity	disability	lapse	expenses	revision	CAT
mortality	1						
longevity	-0.25	1					
disability	0.25	0	1				
lapse	0	0.25	0	1			
expenses	0.25	0.25	0.5	0.5	1		
revision	0	0.25	0	0	0.5	1	
CAT	0.25	0	0.25	0.25	0.25	0	1

Market Risks:

	interest rate	equity	property	spread	currency
interest rate	1				
Equity	0.5/0	1			
Property	0.5/0	0.75	1		
Spread	0.5/0	0.75	0.5	1	
Currency	0.5	0.5	0.5	0.5	1

Backtesting

Model Validation

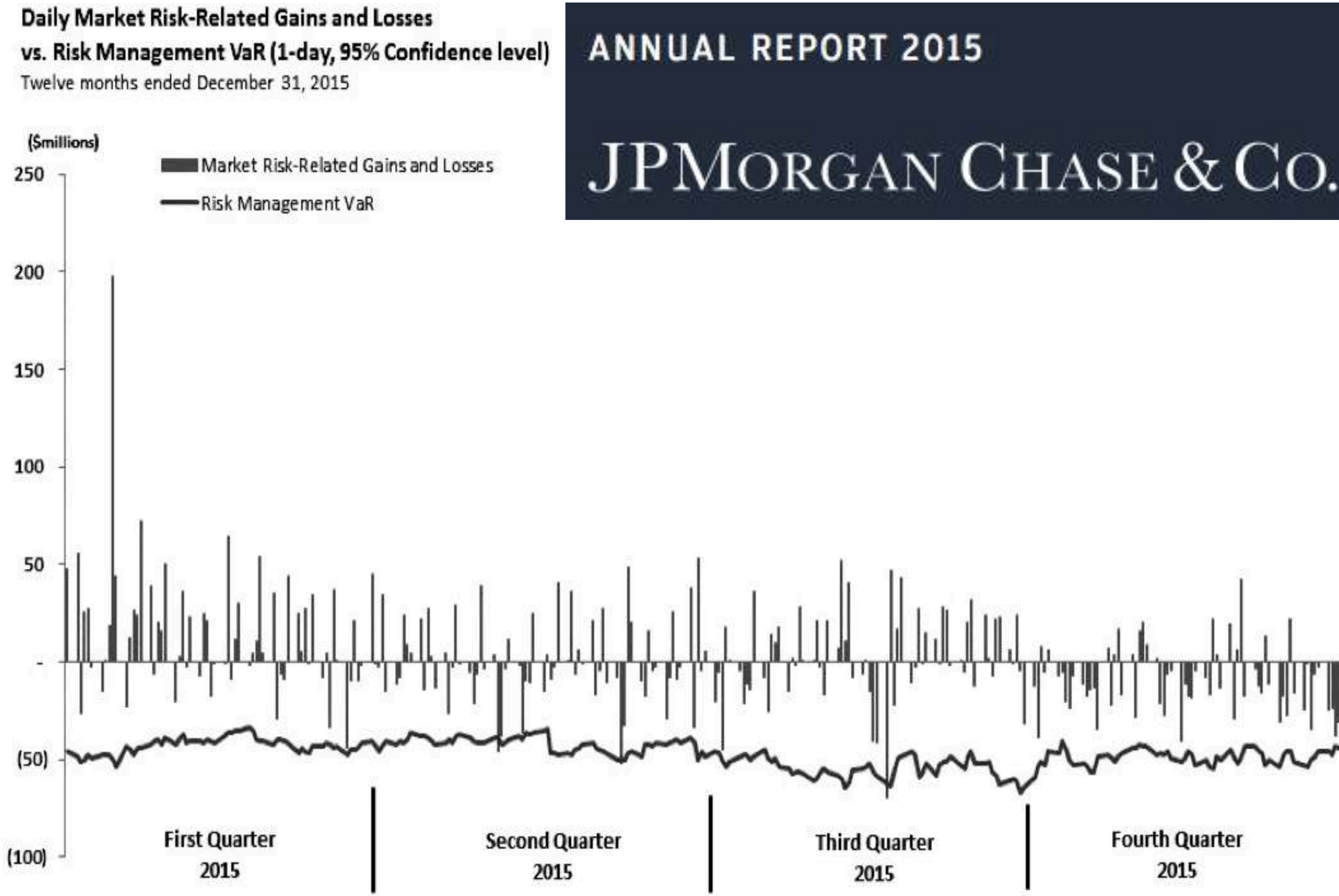
- A process whereby we check whether a model is adequate.
- Validation has garnered a lot of regulatory attention, since models are used for capital and liquidity regulation.
- Can be done with various tools
  - Backtesting
  - Stress testing
  - Independent review and oversight
- Examining for:
  - Faulty assumptions
  - Wrong parameters
  - Inaccurate modeling
  - Errors
- This process also provides ideas and directions for improvement.

Backtesting VaR

- Formal verification that actual losses are in line with those that were projected by the model.
- Compare historically the VaR forecasted for each day and the portfolio return for that day.
- We want to check that the VaR is not underestimating the risk, but we also don’t want it to be too stringent.
- The testing is complicated by the fact that VaR is not a prediction for the daily return, but a statistical statement about its distribution.

ANNUAL REPORT 2015

JPMORGAN CHASE & CO.



Backtesting

- We look at **exceptions**, i.e. days when loss was greater than VaR.
- If the model is valid then the number of exceptions should be in line with the confidence level.
  - For example, for a 99% 1-day VaR, we would like to have exceptions on 1% of the days
  - If exceptions occur on more than 1% of days, then we might be underestimating the risk
  - If exceptions occur on less than 1% of days, then we might be too conservative.

Example

- Suppose we back-test 1-day 99% VaR over 600 days, how many days do we expect the loss to be greater than VaR?
- Should we reject the model if we observed 9 days where losses were greater than VaR? What about 12?
- We are looking for a statistical framework that will tell us how many exceptions are too many or too few.

Actual vs. Hypothetical Returns

- Every day, VaR is computed based on the portfolio at the end of the previous day, therefore it measures the potential losses if the portfolio is “frozen” through the day.
- In fact, portfolios evolve dynamically through the day.
- We observe actual returns, which reflect intraday trades, as well as other profit items.
- Ideally, backtesting is done by comparing VaR to each of two types of returns:
  - Actual returns: Actual profit/loss that was recorded for the portfolio
  - Hypothetical returns: change in portfolio value assuming no change in portfolio composition

Type 1 vs. Type 2 Errors

	Model		
Decision		Correct	Incorrect
	Accept	OK	Type 2 Error
	Reject	Type 1 Error	OK

Type 1 error - What is the probability that we reject a correct model?

- If that probability is very low reject the model.
- Typical hypothesis testing.

Power of a test - What is the probability of rejecting the model when it's really incorrect?

- We need to assume a different model in order to compute this probability.

Statistical Test

To conduct a statistical test we need to determine

- H_0: The value of the parameters if the VaR model holds.
- A **level** for the test (not related to the confidence level of the VaR!!), typically 1% or 5%.
- What is the probability of observing the historical returns/exceptions given H_0. This is the **p-value**.
- If p-value lower than the level of the test then we reject the model.

H_0 Under a Simple Model

- If we assume daily results are serially independent then we have a set of Bernoulli trials.
- If the VaR model holds, then the theoretical probability of an exception is p (=1−α), e.g. for $VaR_{99\%}$: $p=0.01$.
- The probability of m or more exceptions in n days is given by the cumulative binomial distribution:

$$\sum_{k=m}^{n} \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} = 1 - \sum_{k=0}^{m-1} \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

Example – Too many exceptions?

- We back-test a 1-day 99% VaR model over 600 days. There are nine days when the loss was greater than the VaR. Should we reject the model?
  - The probability of <u>9</u> or more losses is:

  p-value = 1 – BINOMDIST(8,600,0.01,TRUE) = 0.152.

  Do not reject at 5% confidence level
- What if there were <u>12</u> exceptions?
  - The probability of 12 or more losses is **0.019**.

  We reject at 5% confidence level

Example - Are we too conservative?

- Suppose a bank uses 99% daily VaR.
- During the last year there have been **no** exceptions, i.e. losses were always less than VaR.
- The CEO claims that the risk department is too conservative. Is she right?
- What is the probability of no exceptions?
  – p-value = BINOMDIST(0,250,0.01,TRUE) = 0.081
  – We cannot reject the model at 5% confidence level
- For this reason, banks like to backtest VaR at lower levels, e.g. 95%, and apply a multiplicative factor.

Binomial converges to Normal

- By the Central Limit Theorem, the Binomial goes to the Normal distribution in the limit.
- If we have T trials with a *p* probability of an exception, then the number of exceptions *(x)* follows the following:

$$E\left[x\right]=pT$$

$$Var\left[x\right]=p(1-p)T$$

$$x\sim N\left(pT,p(1-p)T\right)$$

Z-value test

This leads to a z-value test: $$z = \frac{x - pT}{\sqrt{p(1-p)T}} \sim N(0,1)$$

Consider our previous example: $$z = \frac{12 - 0.01 * 600}{\sqrt{0.01(1-0.01) * 600}} = 2.462$$

Computed to be a p-value of: 0.0069 → Reject the model

Regulatory VaR for Trading Portfolio

The capital required for market risk in the trading portfolio is based on 99% 10-day VaR, which is usually computed by the bank from 1-day VaR:

$$MRC_t = \max\left(VaR_t(0.01), S_t \frac{1}{60}\sum_{i=0}^{59} VaR_{t-i}(0.01)\right) + c$$

The multiplier, S_t, is computed based on N, which is the number of daily exceptions over the last 250 trading days.

$$S_t = \begin{cases} 3.0 & \text{if} \quad N \leq 4 & \text{green} \\ 3 + 0.2(N-4) & \text{if} \quad 5 \leq N \leq 9 & \text{yellow} \\ 4.0 & \text{if} \quad 10 < N & \text{red} \end{cases}$$

Type 1 and Type 2 Errors of Regulatory Test

- What is the probability of a bank with a correct model to be penalized?
  - **1 – BINOMDIST(4,250,0.01,TRUE) = 10.8%**
- What is the probability of an incorrect VaR model, with actual 97% confidence, <u>not</u> being penalized?
  - **BINOMDIST(4,250,0.03,TRUE) = 12.8%**
  - The power is 87.2%. Very low power.

Likelihood Ratio Test

- We can also apply Likelihood Ratio tests
- We test a VaR at confidence level: *1-p*.
- Suppose we observe *x* exceptions over *T* days.
  - MLE estimator of exception probability is $\pi = x/T$
- The (unconditional) likelihood ratio test is:

$$LR_{uc} = 2 \cdot \ln\left[\frac{\left(x/T\right)^{x}\left(1-x/T\right)^{T-x}}{p^{x}(1-p)^{T-x}}\right] \sim \chi^{2}\left(1\right)$$

- Reject at 5% if LR>3.841
- Other rejection boundaries: 6.635 at 1%, 2.706 at 10%

Bunching and conditional coverage

- Bunching occurs when exceptions are not evenly spread throughout the back testing period
  - The serial-independence assumption is invalid
- Statistical tests for bunching are based on extension of the likelihood ratio test.
- To test for independence, we need to satisfy a model for the dependence, which we are trying to reject. The most common one is a Markovian assumption, i.e. the probability of exception today depends on whether there was an exception yesterday (but not farther back than that).

Conditional Coverage – Christoffersen (1998)

	Day Before		Unconditional
Current Day	No Exception	Exception	
No exception	T_{00}	T_{10}	$T\text{-}x$
Exception	T_{01}	T_{11}	x
Total	T_0	T_1	$T=T_0+T_1$

We count the number of exceptions, and classify them according to whether or not the previous day was also an exception.

$\pi = x/T$ – unconditional probability of exception in the sample

$\pi_0 = T_{01}/T_0$ – conditional probability of exception, given no exception the day before

$\pi_1 = T_{11}/T_1$ – conditional probability of exception, given exception the day before

$$LR_{ind} = 2 \cdot \ln\left[\frac{(1-\pi_0)^{T_{00}}(\pi_0)^{T_{01}}(1-\pi_1)^{T_{10}}(\pi_1)^{T_{11}}}{(\pi)^x(1-\pi)^{T-x}}\right] \sim \chi^2(1)$$

Combining Conditional and Unconditional LR Tests

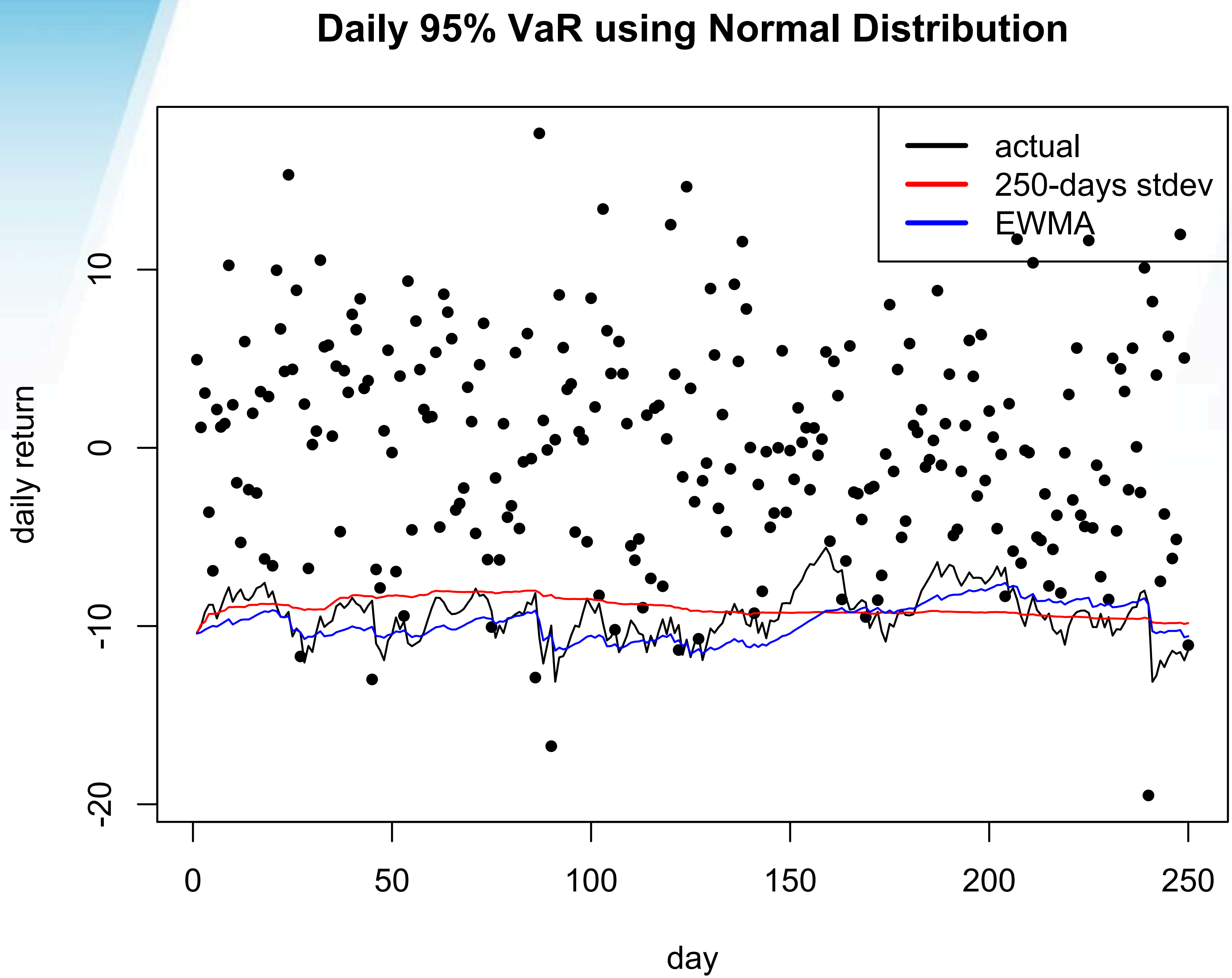
- LR_{ind} is the ratio of likelihood given the Markovian conditional model to the likelihood of the independent observations model.
- We can combine it with LR_{uc} to reject both the independence and the unconditional probability of the model:

$$LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2(2)$$

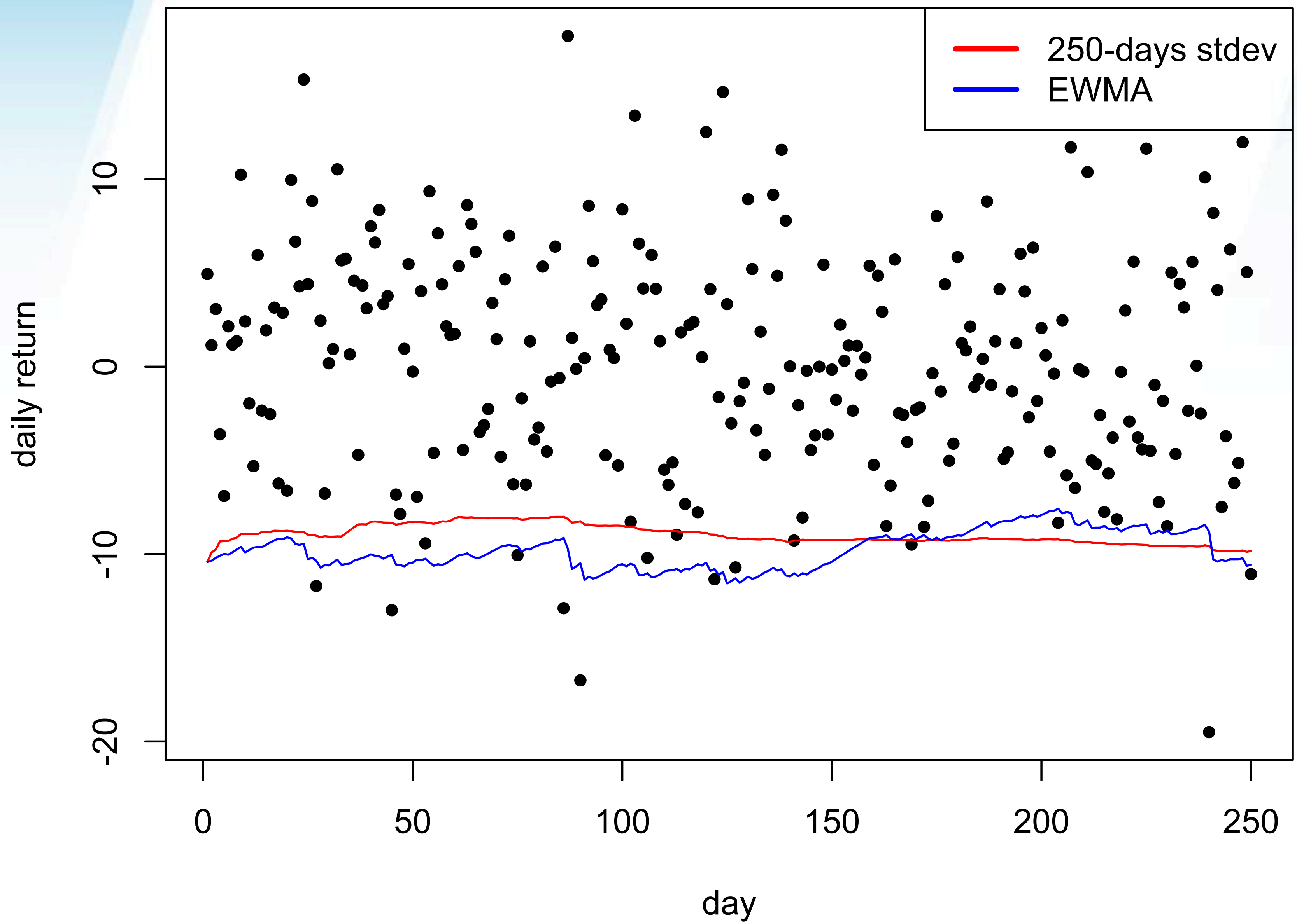
- We reject at 5% if LR > 5.991.

Homework

- Generate 250+250 daily returns, mean=0, and volatility follows EGARCH.
- Estimate daily VaR at 95% assuming: Normal, mean=0, and:
  - Volatility based on last 250 days
  - Volatility based on EWMA, λ=0.97
- Back-test using LR unconditional
- Back-test using conditional coverage.
- Compute power – the probability to reject a false model – i.e. repeat 1000 times and count how often you rejected the false models. How often does the true model get rejected?


Daily 95% VaR using Normal Distribution
actual
250-days stdev
EWMA
daily return
day
10
0
-10
-20
0
50
100
150
200
250

Daily 95% VaR using Normal Distribution



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Financial Risk Management

Spring 2016

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Interest Rate Risk

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Agenda

- Interest Rate Risks
- Bond Prices and Yields
- Duration and Convexity
- Term-structure risk

Interest Rate Risks

- Portfolio Markdown due to change in levels of interest rates
  - A shift of rates at all maturities: duration risk
  - Change in relationship between maturities – yield curve moves
- Cash flow mismatch between assets and liabilities: re-pricing risk
- Change in spreads between different curves: basis risk
  - Liquidity differences
  - Credit spreads
- Interest rate related behavioral options

FRED
3-Month Treasury Bill: Secondary Market Rate
2-Year Treasury Constant Maturity Rate
5-Year Treasury Constant Maturity Rate
10-Year Treasury Constant Maturity Rate
(Percent)
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
1980
1985
1990
1995
2000
2005
2010
2015
Shaded areas indicate US recessions - 2015 research.stlouisfed.org

Orange County – Duration Risk

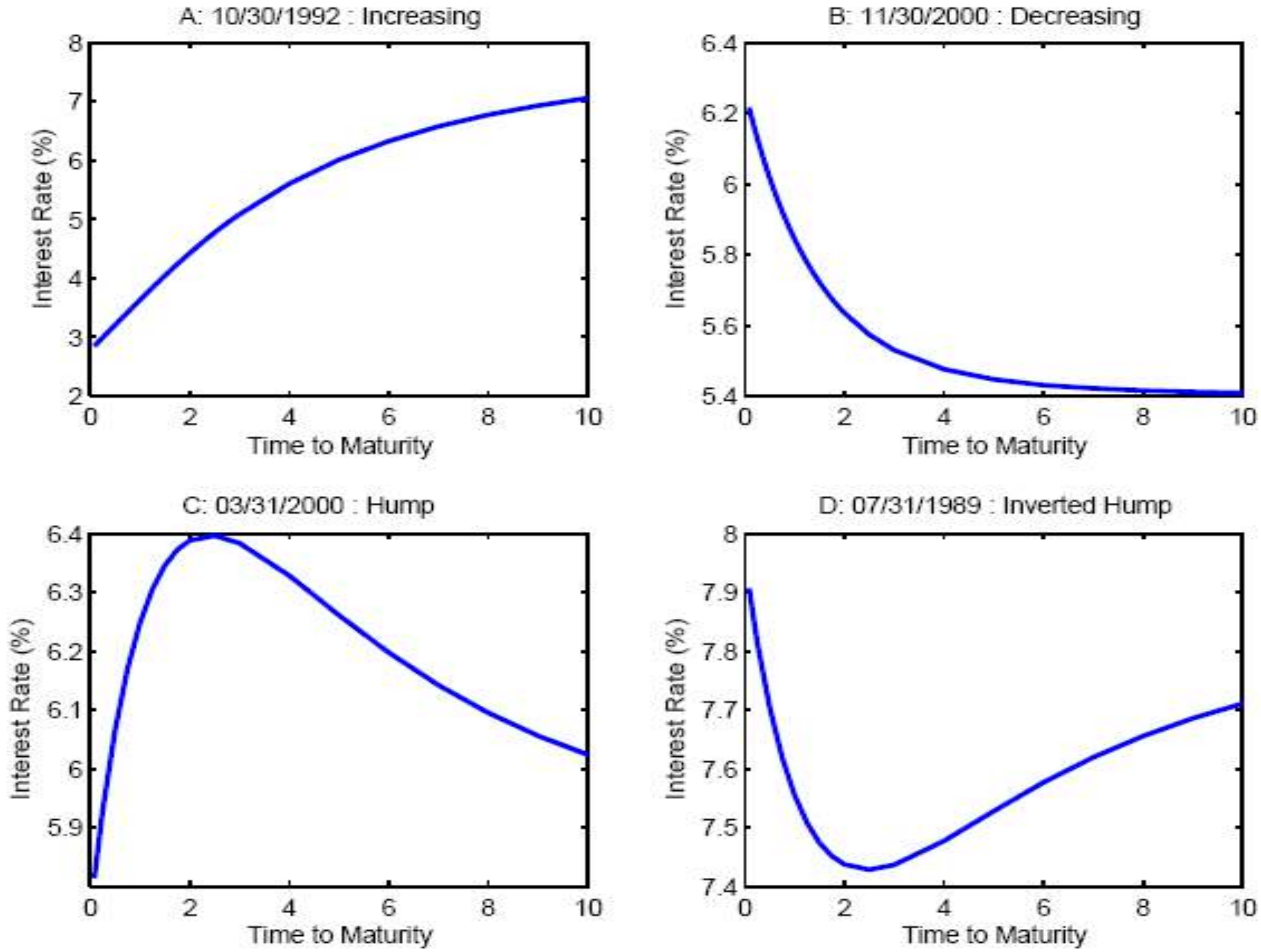
- In 1994, Orange County lost $1.6 billion when the interest rate level suddenly increased from 3% to 5.7%
- This sent the county into bankruptcy
- The county's Treasurer, Bob Citron, had bet, through a mix of structured notes and leverage, that rates would not increase in the future
- The portfolio was too sensitive to changes in interest rates

Term Structure

- The term structure of interest rates, or spot curve, or yield curve, defines the relation between the level of interest rates and their time to maturity
- The term spread is the difference between long term interest rates (e.g. 10 year rate) and the short term interest rates (e.g. 3 month interest rate)
- The term spread depends on many variables: expected future inflation, expected growth of the economy, agents attitude towards risk, etc.
- The term structure varies over time, and may take different shapes

Term Structure of Rates

Figure 2.3 The Shapes of the Term Structure



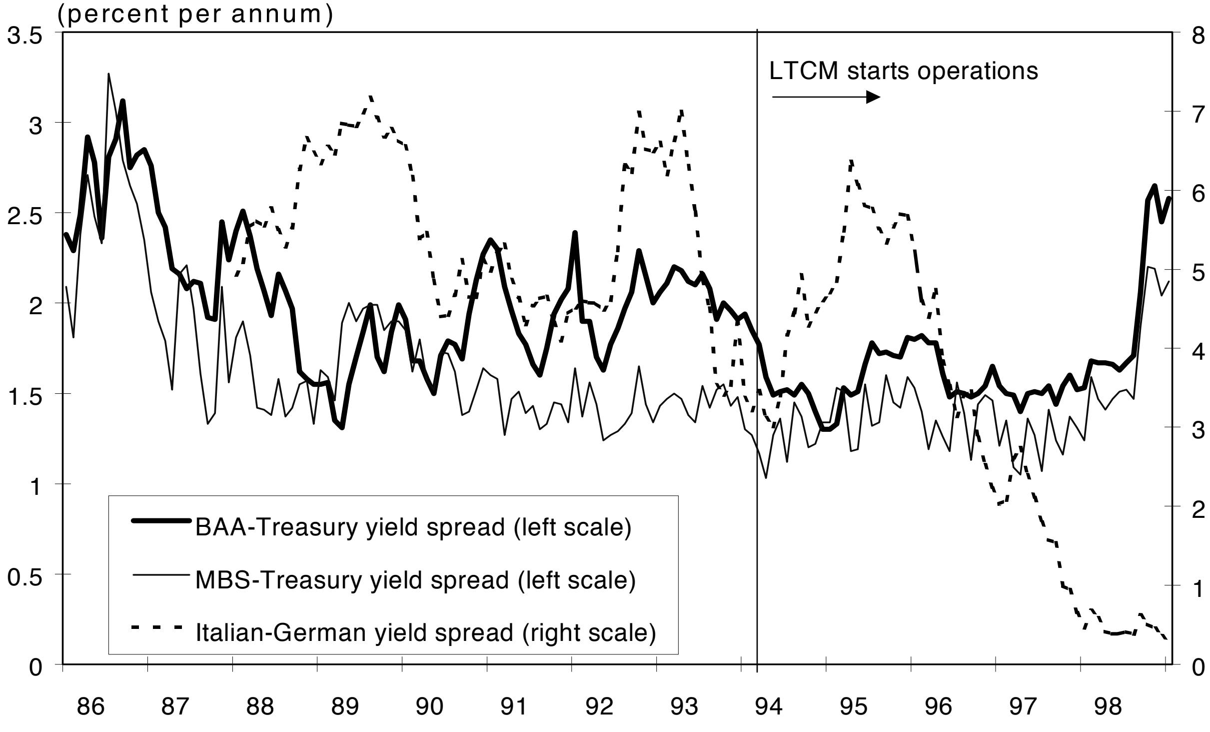
Data Source: Center for Research in Security Prices

Interest Rate Mismatch – Re-pricing Risk

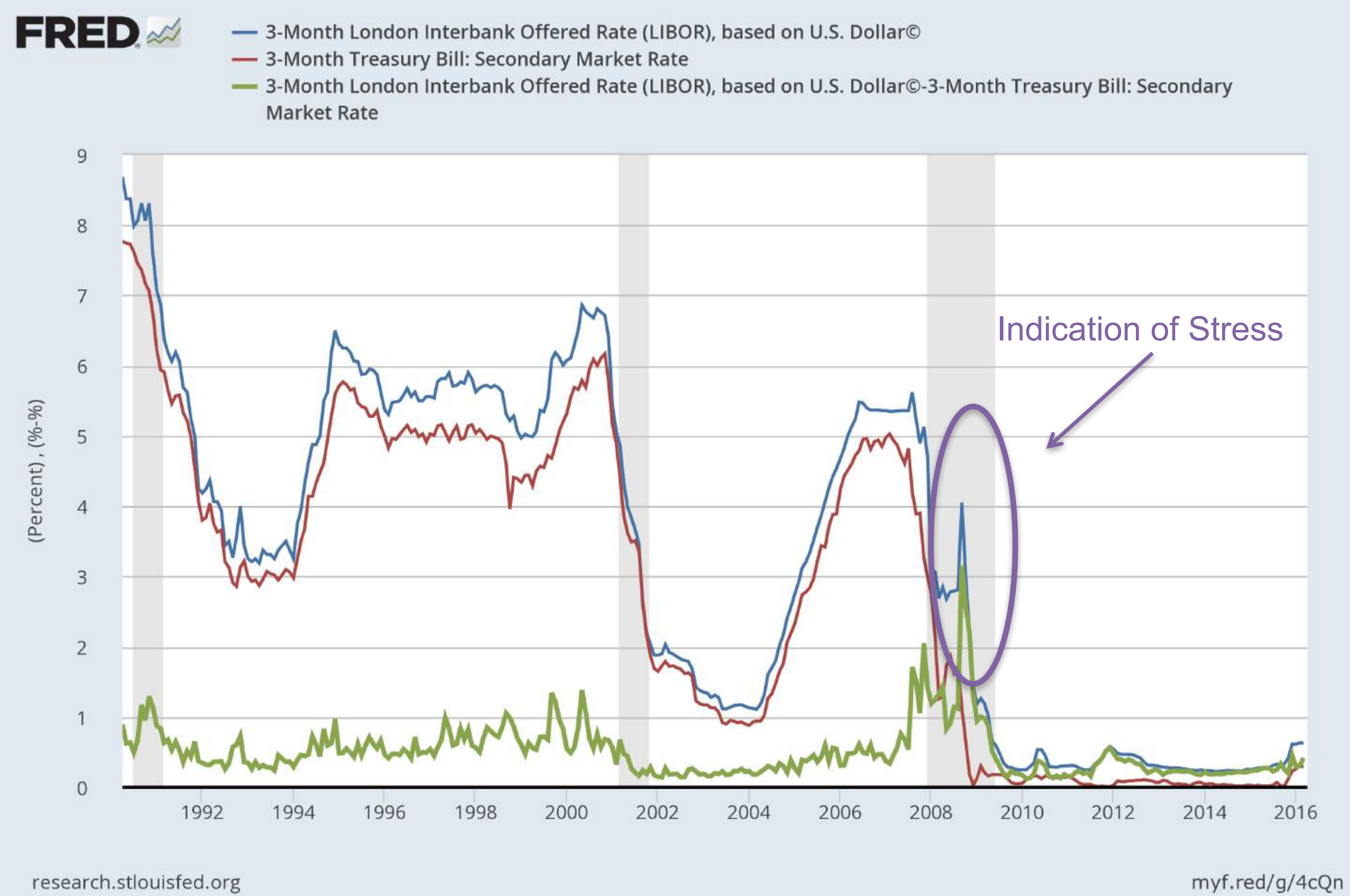
- Savings and Loan (S&L) earned revenue from the difference between long term mortgages (assets) and short term deposits (liabilities)
- Interest rates increased in late 70s and early 80s,
  - S&L received their fixed coupon from mortgages contracted in the past (when rates were low),
  - but now had to pay high interest on new deposits
- This spread sent many S&L out of business

LTCM - Basis Risk

- Long-Term Capital was trading on various, “relative value” trades
- In 1997 some spreads did not converge due to credit and liquidity issues, LTCM ran out of liquidity to fund trades



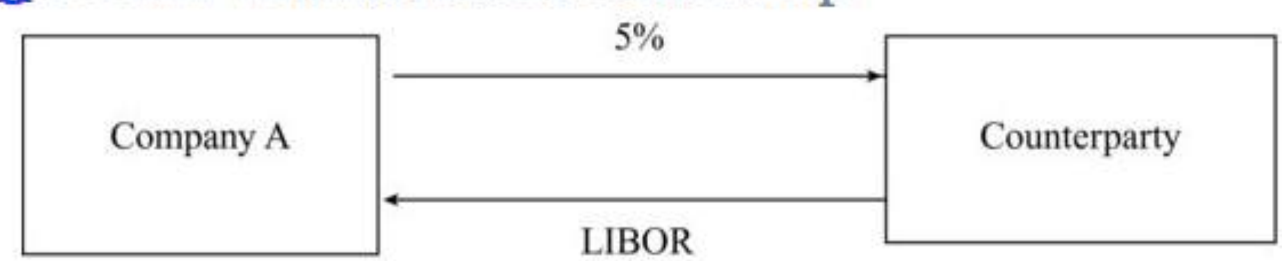
Source: “Risk Management Lessons from Long Term Capital” – P. Jorion 2000


FRED
3-Month London Interbank Offered Rate (LIBOR), based on U.S. Dollar©
3-Month Treasury Bill: Secondary Market Rate
3-Month London Interbank Offered Rate (LIBOR), based on U.S. Dollar©-3-Month Treasury Bill: Secondary Market Rate
(Percent) , (%-%)
9
8
7
6
5
4
3
2
1
0
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
2014
2016
Indication of Stress
research.stlouisfed.org
myf.red/g/4cQn

Interest Rate Swaps

- Swap – an agreement to change cash flows in the future
- Interest Rate Swap – an agreement to change cash flows indexed to a floating rate for a fixed rate.

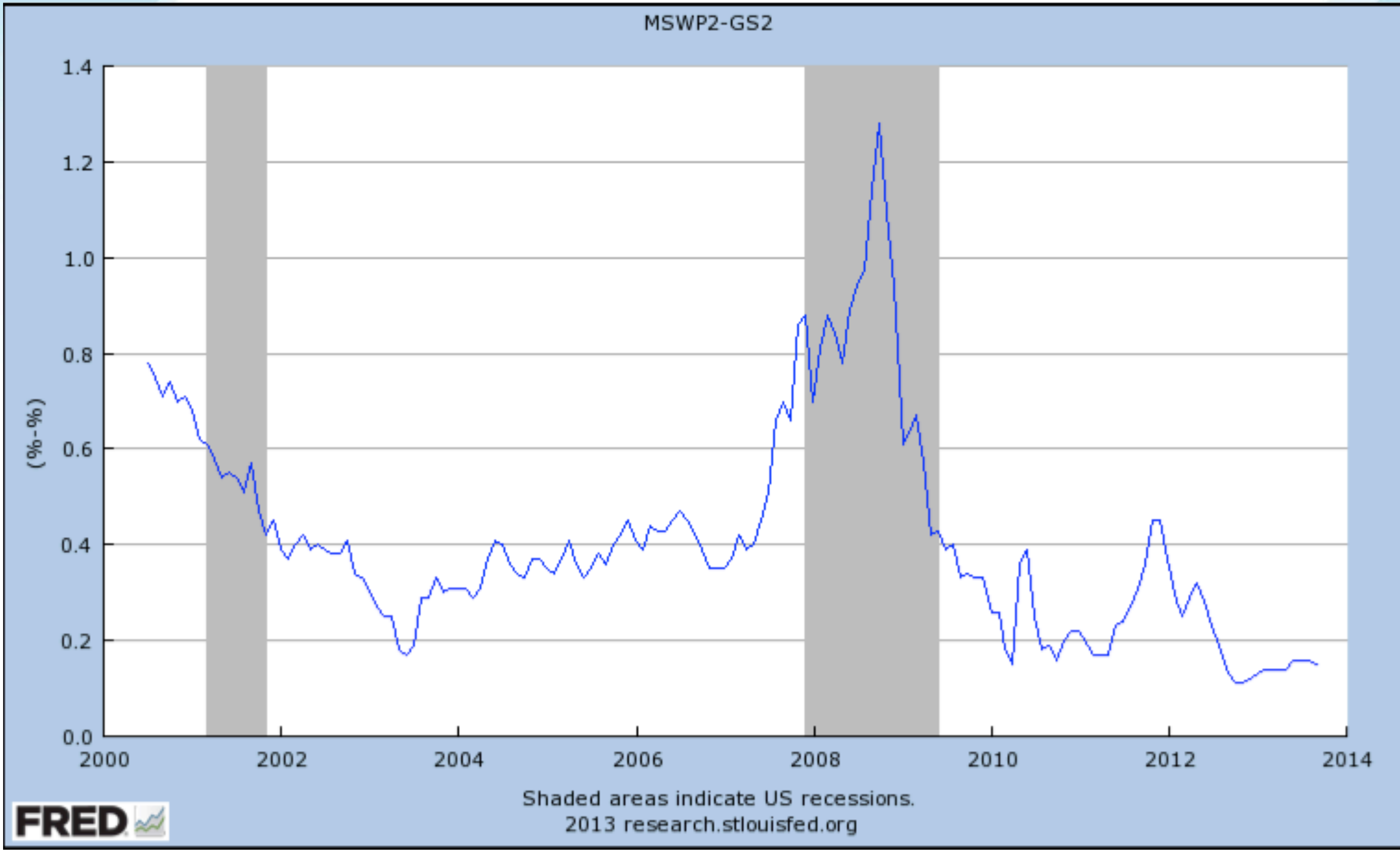
Figure 5.3 A Plain Vanilla Interest Rate Swap



Swap Rates

- Swap rates: Long term fixed rates that are swapped for LIBOR.
- LIBOR/Swap curve: Swap rates are used to extend the LIBOR curve beyond 1 year
- Serve as “risk-free” rates for pricing derivatives in banks, because they reflect AA counterparty risk

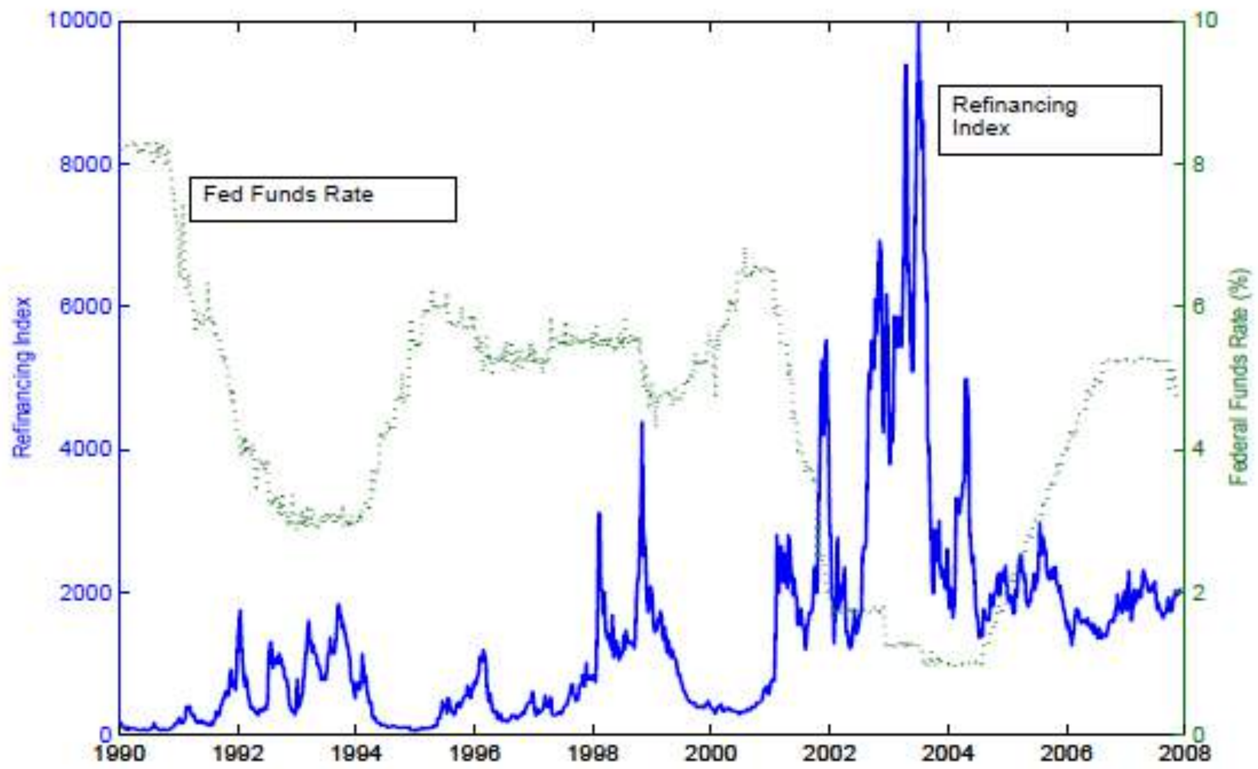
Swap Spread



MSWP2-GS2
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
(%-%)
2000
2002
2004
2006
2008
2010
2012
2014
Shaded areas indicate US recessions.
2013 research.stlouisfed.org
FRED

Prepayment of Mortgages in 2002-2003

Figure 8.4 Refinancing and the Federal Funds Rate



Source: Federal Reserve and Bloomberg.

Bond Prices

- Zero-coupon bonds pay at maturity only.
  - Their price is the value of their Notional, or Face Value, discounted by the relevant spot-rate

$$B = FVe^{-r_i t_i}$$

- Coupon bonds have periodic cash flows, including coupons and notional.
- By arguments of no-arbitrage, a coupon bond Price is equal to the appropriately discounted cash flows:

$$B = \sum_{i=1}^{n} cf_i e^{-r_i t_i} \quad \text{or} \quad B = \sum_{i=1}^{n} \frac{cf_i}{\left(1 + \frac{r_i}{m}\right)^{m \cdot t_i}}$$

Yield To Maturity

- Yield to Maturity is the single rate that will set the present value of cash flows equal to current price:

$$B = \sum_{i=1}^{n} cf_i e^{-y t_i}$$

- The compounding interval affects the yield.
- The yield can be thought of as an average of the interest rates along the different maturities.

Computing Yield - Example

- A semi-annual 10% coupon bond with 3 years to maturity is trading at 94.213, what is the continuously compounded yield on the bond?
- The bond has six more coupon payments (of $5) and a principal payment in 3 years.
- Use Solver to find *y*:

$$5e^{-0.5y} + 5e^{-1.0y} + 5e^{-1.5y} + 5e^{-2.0y} + 5e^{-2.5y} + 105e^{-3.0y} = 94.213$$

$$y = 12\%$$

Duration

- Duration of a bond that provides cash flow cf_i at time t_i is:

$$D = \sum_{i=1}^{n} t_i \left(\frac{cf_i e^{-yt_i}}{B} \right)$$

- Since: $$\frac{\partial B}{\partial y} = \sum_{i=1}^{n} \left(-t_i\right) cf_i e^{-yt_i}$$

- An approximate relationship holds: $$\frac{\Delta B}{B} \approx -D\Delta y$$

Calculation of Duration for a 3-year bond paying a s.a. coupon 10%. Bond yield=12%.

Time (yrs)	Cash Flow ($)	PV ($)	Weight $= \dfrac{cf_i e^{-yt_i}}{B}$	Time × Weight
0.5	5	4.709	0.050	0.025
1.0	5	4.435	0.047	0.047
1.5	5	4.176	0.044	0.066
2.0	5	3.933	0.042	0.083
2.5	5	3.704	0.039	0.098
3.0	105	73.256	0.778	2.333
Total		94.21	1	**2.653**

Using Duration to Estimate Change in Bond Price

- What will be the change in the bond's price if the yield goes up by 10 basis points?

$$\Delta B = -B \cdot D \cdot \Delta y$$

$$\Delta B = -94.213 \cdot 2.653 \cdot 0.001$$

$$\Delta B = -0.25$$

- Verify result by repricing the bond with *y=12.1%*

Modified Duration

- When the yield y is expressed with compounding m times per year

$$\Delta B = -\frac{B \cdot D \cdot \Delta y}{1 + y/m}$$

- The expression

$$\frac{D}{1 + y/m}$$

is referred to as the "modified duration"

Properties of Duration

- Duration is a measure of average time to cash flows.
- **Duration increases with maturity.** The further out the maturity the more sensitive is the bond to yield changes.
- **Duration is lower for higher coupon bond.** The higher the coupons, the larger are the intermediate coupons relative to the last one. Thus the average time of payments gets closer to today.
- **Duration is equal to Maturity for zero coupon bonds.**

Convexity

The convexity of a bond is defined as:

$$C = \frac{1}{B}\frac{d^2B}{dy^2} = \sum_{i=1}^{n} t_i^2 \cdot \frac{cf_i e^{-yt_i}}{B}$$

which leads to

$$\frac{\Delta B}{B} \approx -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

Figure 4.2 Bond Price Approximation with Duration

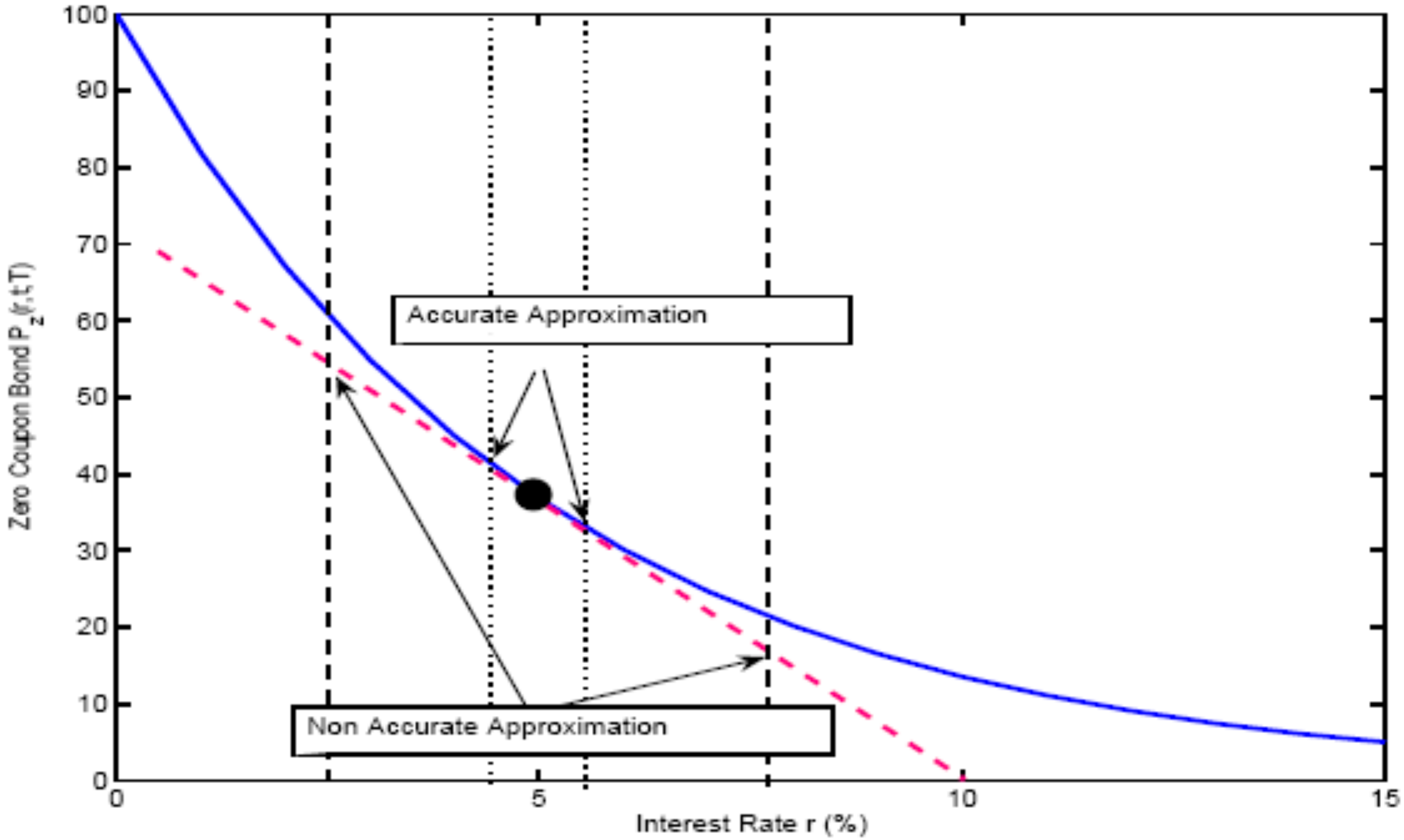
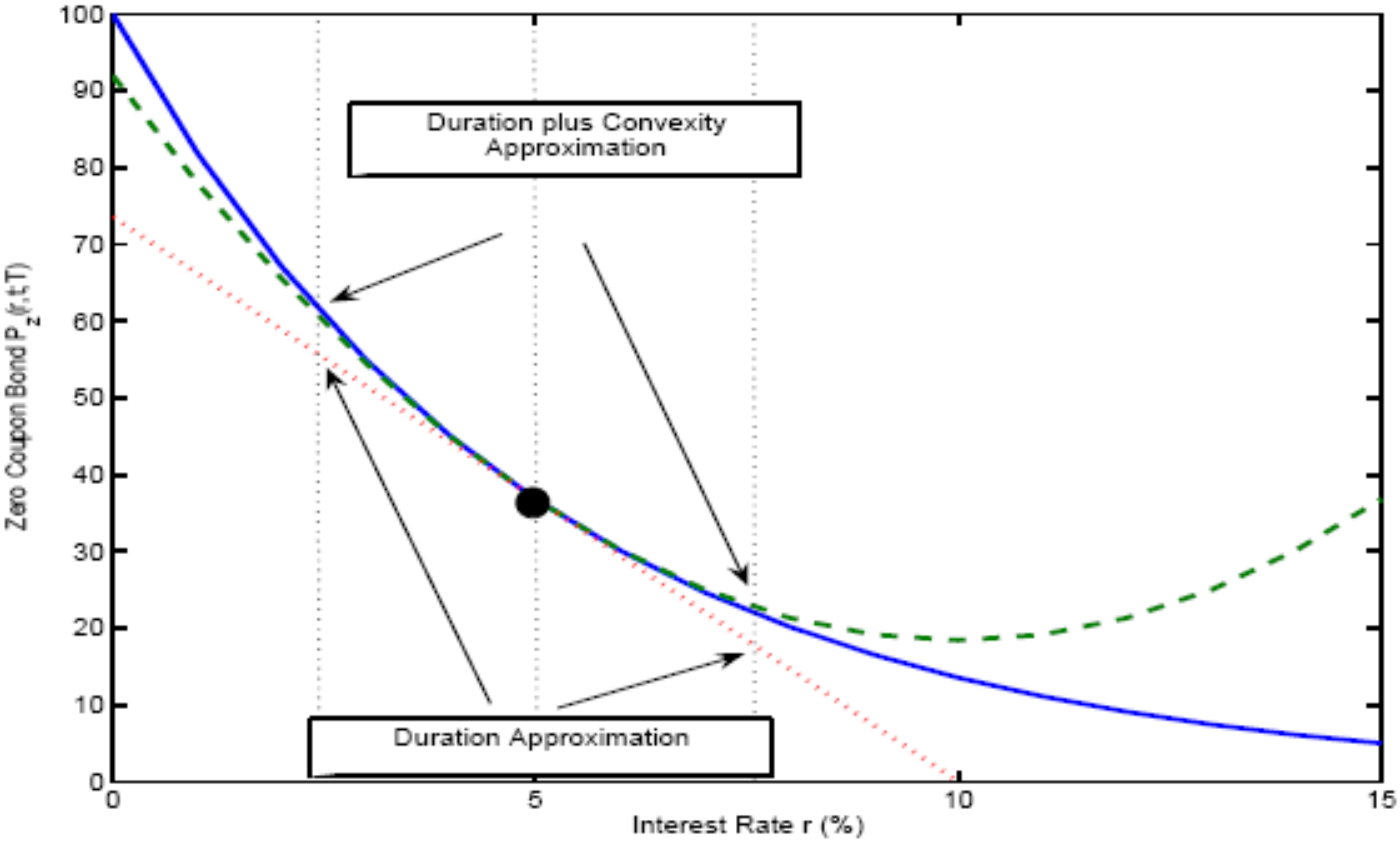


Figure 4.4 Duration plus Convexity Approximation



Portfolios

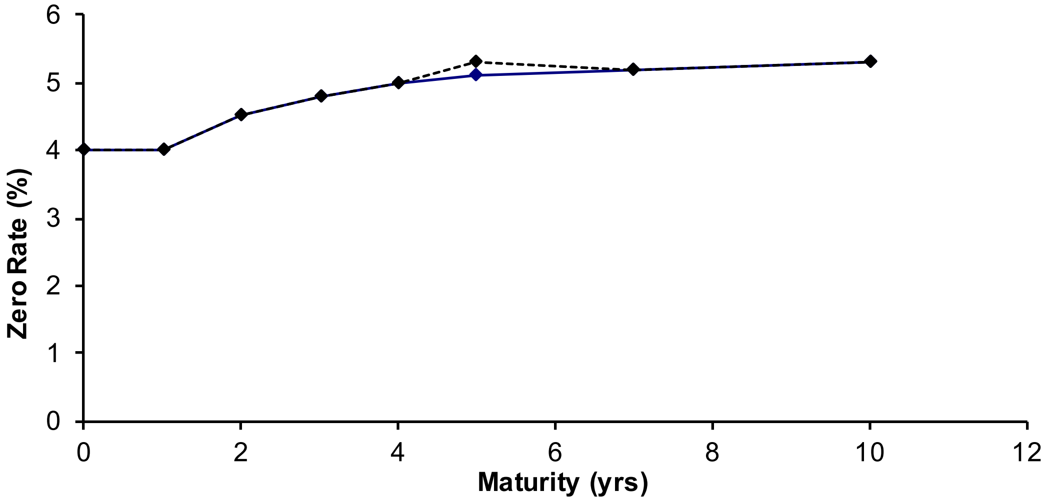
- Duration and convexity can be defined similarly for portfolios of bonds and other interest-rate dependent securities
- The duration of a portfolio is the weighted average of the durations of the components of the portfolio. Similarly for convexity.

Other Measures

- Dollar Duration: Product of the portfolio value and its duration
  - The change in dollar value of the bond for a change in yield
  - The delta of the bond with respect to the yield
- DV01 – Impact of 1 bp parallel shift in all rates
  - Dollar duration * 0.01
- Dollar Convexity: Product of convexity and value of the portfolio

Partial Duration

- Partial Duration – effect on a portfolio of a change to just one point on the zero curve
- Partial Dollar Duration – The dollar change in portfolio value due to change in one rate.



Partial Duration

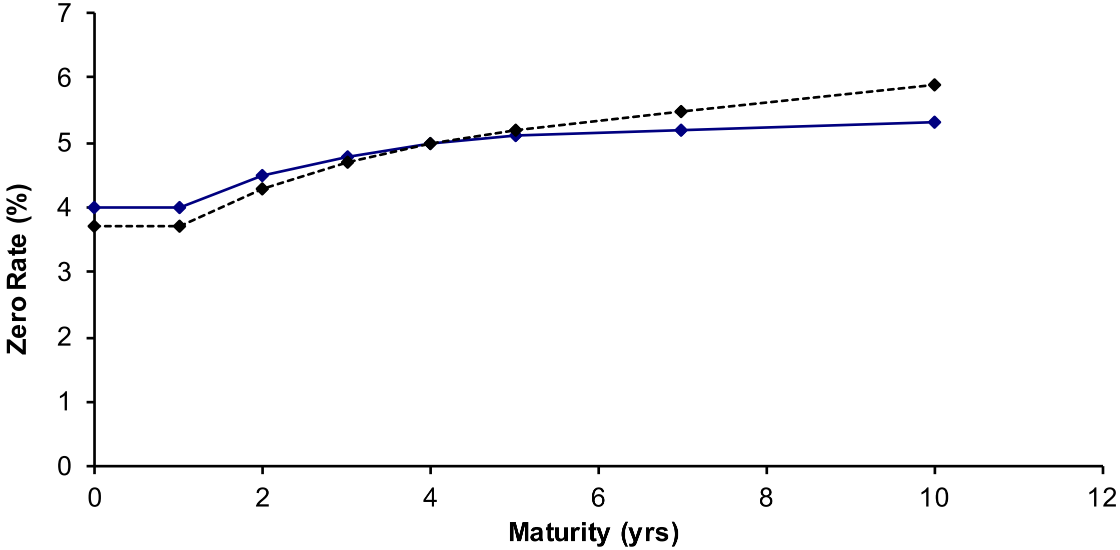
Time (yrs)	Cash Flow ($)	Rate	PV ($) of FV=100	Weight $= \frac{cf_i e^{-r_i t_i}}{B}$	Time *Weight **Duration**	Time * Weight *Price Dollar Duration
0.5	5	12%	4.71	0.050	0.025	2.35
1	5	12%	4.43	0.047	0.047	4.43
1.5	5	12%	4.18	0.044	0.066	6.27
2	5	12%	3.93	0.042	0.083	7.87
2.5	5	12%	3.70	0.039	0.098	9.26
3	105	12%	73.26	0.778	2.333	219.77
Total			94.21	1	**2.65**	249.948

What would happen to the bond's price if 3-year rate went up by 1%?

Partial Durations and Yield Curve Changes

- Any yield curve change can be defined in terms of changes to individual points on the yield curve
- For example, a rotation can be defined by:

Maturity (yrs)	1	2	3	4	5	7	10
Shock	-3ε	-2ε	-ε	0	+ε	+3ε	+6ε



Impact of Rotation

- Suppose we have a portfolio with the following partial durations:

Maturity yrs	1	2	3	4	5	7	10	Total
Partial Duration	0.2	0.6	0.9	1.6	2.0	−2.1	−3.0	0.2

- The impact of the rotation on the proportional change in value of the portfolio:

$$-[0.2\times(-3\varepsilon)+0.6\times(-2\varepsilon)+\ldots+(-3.0)\times(+6\varepsilon)]=25.0\varepsilon$$

Portfolio Sensitivity to Rates

- An investor has the following position
  - Long FV=$1,000 of 1-year zero coupon
  - Short FV = $4,475 of 5-years zero coupon
  - Long FV=$3,000 of 10-years zero coupon
- Interest Rates are 4%, 5%, 6% continuously compounded for 1-, 5-, 10- yrs respectively
  a. Compute the change in portfolio value for 1bp increase in each of the rates.
  b. Is the portfolio sensitive to a parallel shift in interest rate?
  c. Is the portfolio sensitive to flattening of the curve, i.e. 1bp increase in 1-year and 1bp decrease in 10-yr?

Portfolio Sensitivity to Rates (cont)

Time (yrs)	Notional ($)	PV of CF ($)	Dollar Duration (DD)= PV(CF)*Time		a. Change in Value = −DD*0.0001
1	1000	960.79	960.8		-0.096
5	-4475	-3485.13	-17,425.7		1.742
10	3000	1646.43	16,464.3		-1.646

b. Parallel Shift of rates: Total change in price = -0.096+1.742-1.646= 0

c. Increase in 1-year and decrease in 10-year = -0.096+1.646 = 1.550

Portfolio will increase in value if curve flattens

Principal Components Analysis

- Daily changes in the different maturities are correlated.
- Instead of using so many rates it makes sense to use only 2-3 factors.
- Principal Component Analysis is a method to summarize daily movements using the correlation matrix between the different rates.
- The factors generated are by design independent of each other.

Table 8.7 Factor Loadings for Swap Data

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
1-year	0.216	−0.501	0.627	−0.487	0.122	0.237	0.011	−0.034
2-year	0.331	−0.429	0.129	0.354	−0.212	−0.674	−0.100	0.236
3-year	0.372	−0.267	−0.157	0.414	−0.096	0.311	0.413	−0.564
4-year	0.392	−0.110	−0.256	0.174	−0.019	0.551	−0.416	0.512
5-year	0.404	0.019	−0.355	−0.269	0.595	−0.278	−0.316	−0.327
7-year	0.394	0.194	−0.195	−0.336	0.007	−0.100	0.685	0.422
10-year	0.376	0.371	0.068	−0.305	−0.684	−0.039	−0.278	−0.279
30-year	0.305	0.554	0.575	0.398	0.331	0.022	0.007	0.032

$$\Delta y_1 = 0.216\Delta PC_1 - 0.501\Delta PC_2 + 0.627\Delta PC_3 + \ldots$$

$$\Delta y_2 = 0.331\Delta PC_1 - 0.429\Delta PC_2 + 0.129\Delta PC_3 + \ldots$$

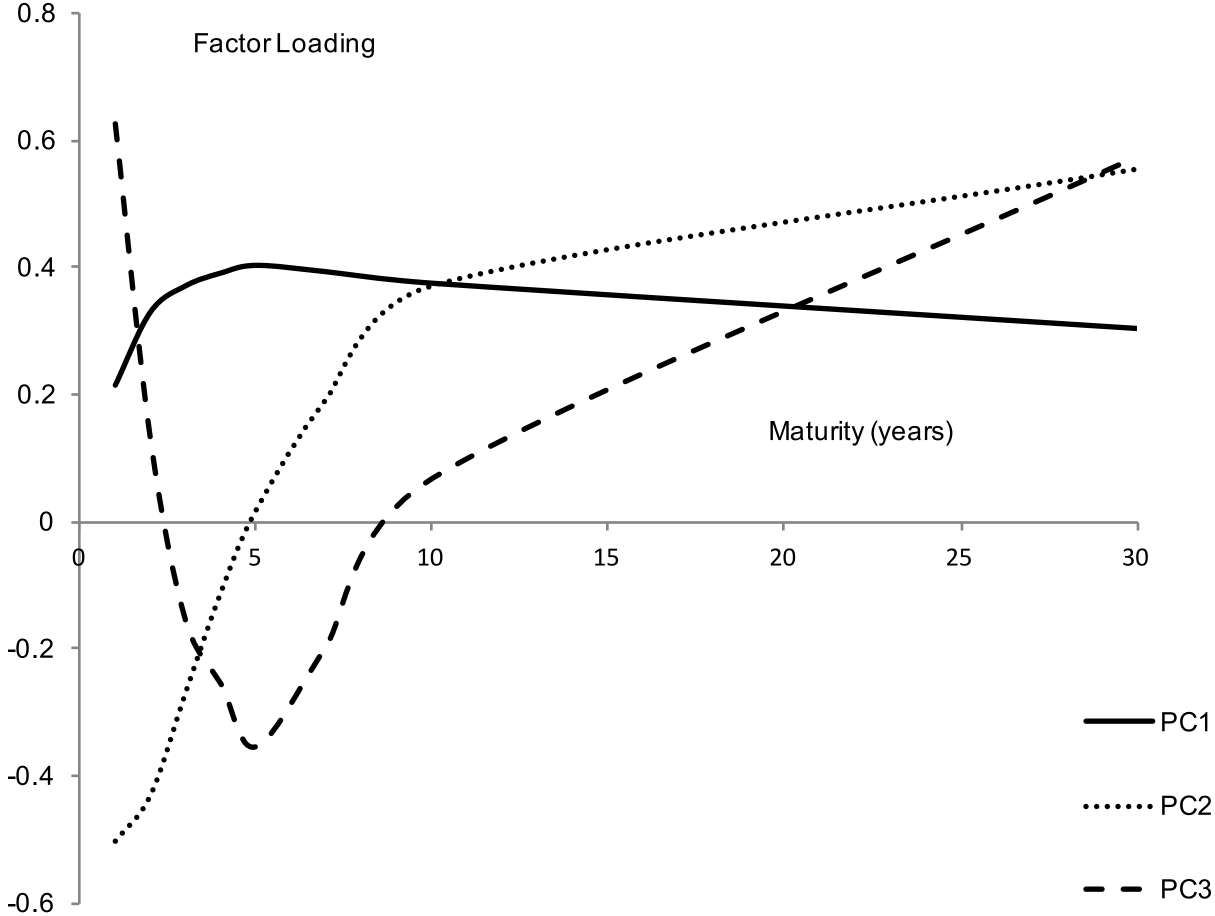
$$\vdots$$

Factor Scores and Variances

- The 8 equations imply that the daily changes in the 8 interest rates can be expressed as daily changes in the factors.
  - These are called daily factor scores.
- We can look at the standard deviation of the factor scores to see how significant is each one:

PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
17.55	4.77	2.08	1.29	0.91	0.73	0.56	0.53

The Three Factors



Parallel Shift

Twist

Bowing

Three Factors Explain Most Interest Rate Moves

- The total variance is the sum of factor score variances: $17.55^2 + 4.77^2 + \cdots + 0.53^2 = 338.8$
- The first factor, parallel shift, explains 90.9% of variance: $17.55^2 / 338.8$
- The second factor, twist, explains 6.8% of variance
- The third factor, bowing, explains 1.3% of variance

Sensitivity to Changes in Yield Curve using Principal Components

Suppose a portfolio has the following sensitivities to 1-basis-point rate moves, in $ millions:

3-Year Rate	4-Year Rate	5-Year Rate	7-Year Rate	10-Year Rate
+10	+4	−8	−7	+2

How sensitive is the portfolio to each one of the factors?

PC1: $10 \times 0.372 + 4 \times 0.392 - 8 \times 0.404 - 7 \times 0.394 + 2 \times 0.376 = -0.05$

PC2: $10 \times (-0.267) + 4 \times (-0.110) - 8 \times 0.019 - 7 \times 0.194 + 2 \times 0.371 = -3.87$

To what risk is it exposed the most?

PC1: -0.05 * 17.55 = -0.88

PC2: -3.87 * 4.77 = -18.46

Thank You

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Market VaR I

UCLA Anderson
THINK IN THE NEXT

Modeling Market VaR

- Parametric Models
  - Delta-Gamma
  - Monte Carlo Simulation
- Non-Parametric Simulation
  - Historical VaR
  - Semi-parametric: time and volatility weighting
- Simulations: Allocation and Confidence Intervals
- We look at 1-day VaR, unless stated otherwise

Model-Building / Parametric Approach

Model-based or Parametric VaR

1. Select a set of market variables or factors that underlie the prices and values of the portfolio
   – E.g. stock indices, interest rates, principal components
2. Assume returns of factors follow certain stochastic processes, i.e. changes in their value in the next day have certain probability distributions
   – E.g. daily stock returns are Normal
3. Estimate parameters for the underlying processes
   – E.g. use GARCH(1,1) to estimate exchange rate volatility
4. Figure out the distribution of daily changes of the portfolio based on the distribution of underlying factors
   – Closed-form or by simulation
5. Find the appropriate percentile of the distribution

Linear-Normal Model Assumptions

- Daily change in the value of a portfolio is linearly related to the daily returns of market variables or factors
- Returns on factors are normally distributed, with mean zero, and a covariance matrix
  - Each factor return, *i*, has variance σ_i
  - Every 2 factor returns, *i* and *j*, have covariance cov_{ij}
- Under these assumptions, returns on the portfolio are also Normal with mean zero.
- To find VaR, we need only find the portfolio Variance

Linear Model / Delta Method

- Define returns on market variables: $\Delta x_i = \dfrac{\Delta S_i}{S_i}$
- And deltas of the portfolio with respect to asset *i*:

$$\delta_i = \frac{\partial P}{\partial S_i}$$

- Then changes in portfolio value are approximated by:

$$\Delta P = \sum_i S_i \delta_i \, \Delta x_i$$

Variance of Portfolio Value

$$\Delta P = \sum_{i=1}^{n} S_i \delta_i \Delta x_i$$

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} S_i \delta_i S_j \delta_j \sigma_i \sigma_j$$

$$\sigma_P^2 = \sum_{i=1}^{n} (S_i \delta_i)^2 \sigma_i^2 + 2 \sum_{i<j} \rho_{ij} S_i \delta_i S_j \delta_j \sigma_i \sigma_j$$

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{cov}_{ij} \, S_i \delta_{ii} S_j \delta_j$$

How should we apply this method to:

1. Portfolio of Options
2. Portfolio of Bonds

Delta Method – Example w/options

1. Consider an investment in options on Microsoft and AT&T. Suppose the stock prices are \$120 and \$30 respectively and the deltas of the portfolio with respect to the two stock prices are 1 and 20 respectively.

- Approximate the change in portfolio value as function of Δx_1 and Δx_2, the returns on the two stocks:

$$\Delta P = 120 \cdot 1 \cdot \Delta x_1 + 30 \cdot 20 \cdot \Delta x_2$$

Delta Method Example - Cont

2. Assume daily return volatility for Microsoft is 2% and that of AT&T is 1%, correlation between the two is 0.3, what is the 5-day 95% VaR?

- The variance of the portfolio is:

$$\sigma_P^2 = (120*0.02)^2 + (600*0.01)^2 + 2*120*0.02*600*0.01*0.3 = 50.40$$

- The five-day 95% VaR is

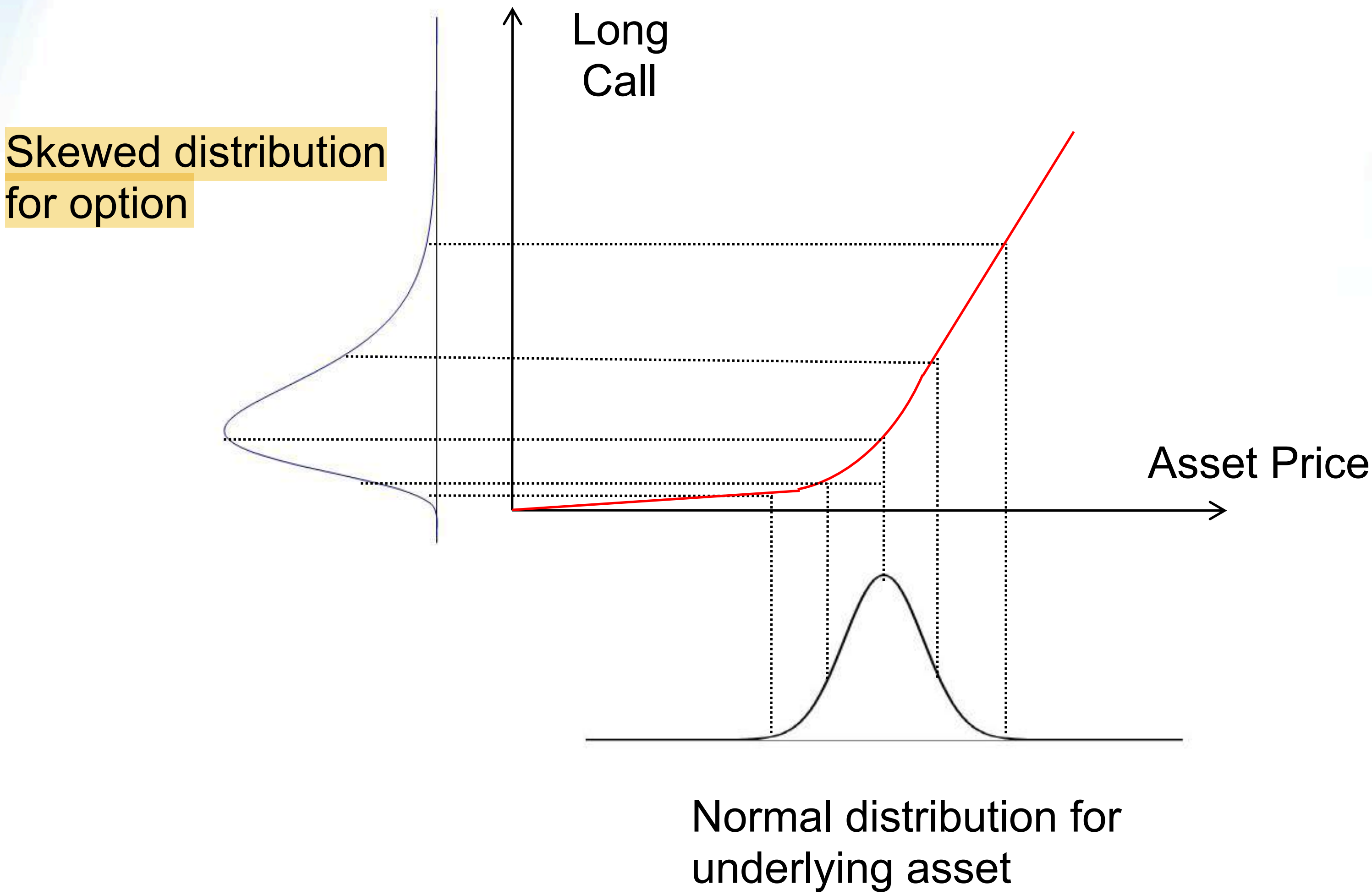
$$VaR = \Phi^{-1}(0.95) \times \sqrt{T} \times \sigma_P = 1.65 \times \sqrt{5} \times \sqrt{50.4} = 26{,}193$$

Delta – Gamma

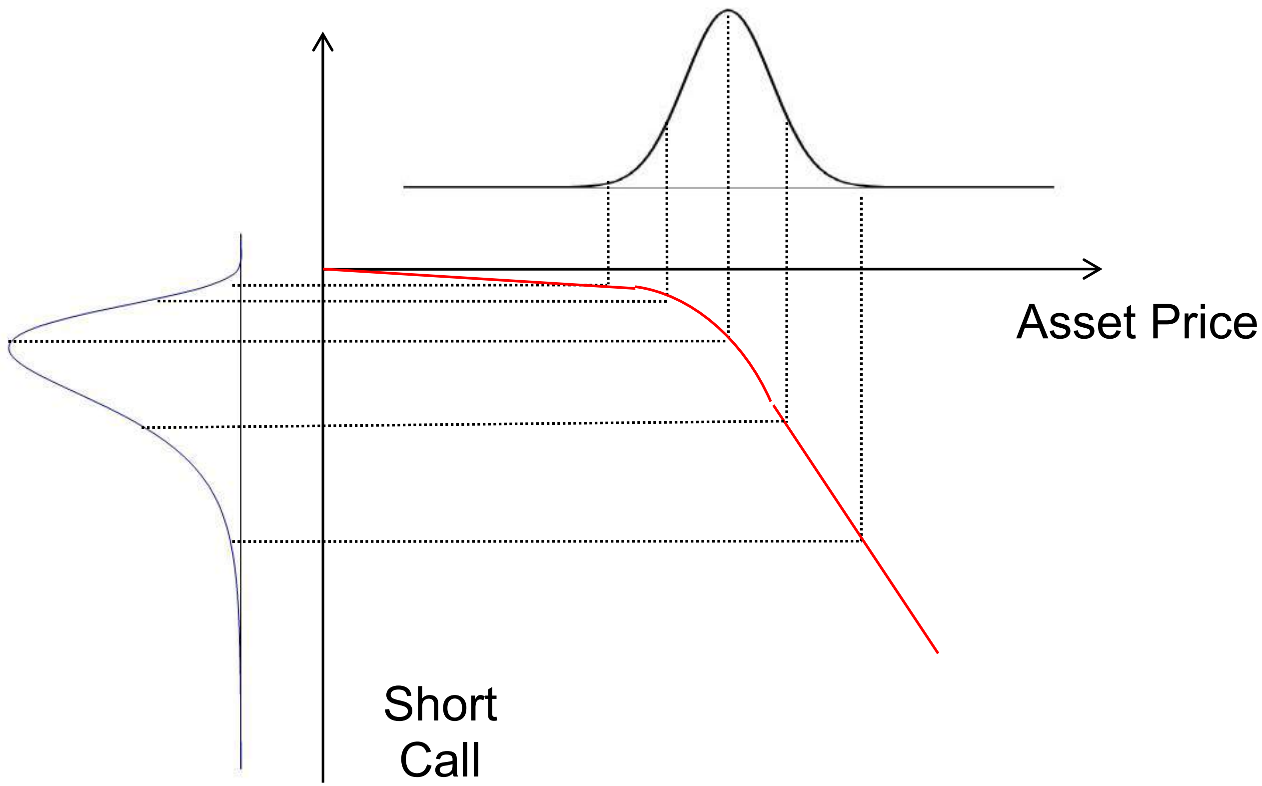
- The linear model will not be accurate because option prices are linear only for small changes in the underlying.
- We can improve our estimation by using gamma as well:

$$\Delta P \approx \delta \cdot \Delta S + \frac{1}{2}\gamma \cdot \left(\Delta S\right)^2$$

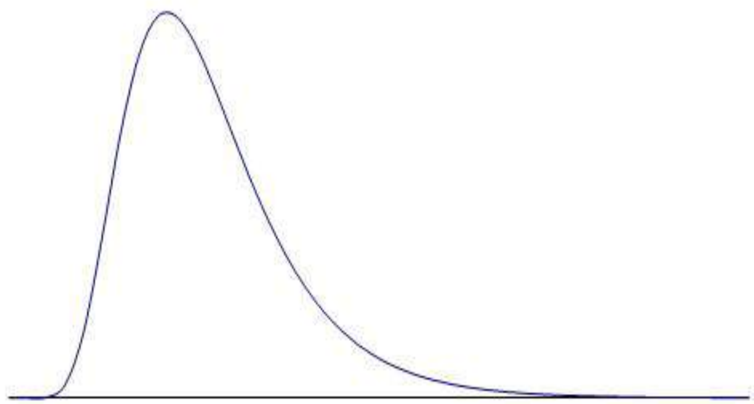
Translation of Asset Price Changes to Price Changes for Long Call



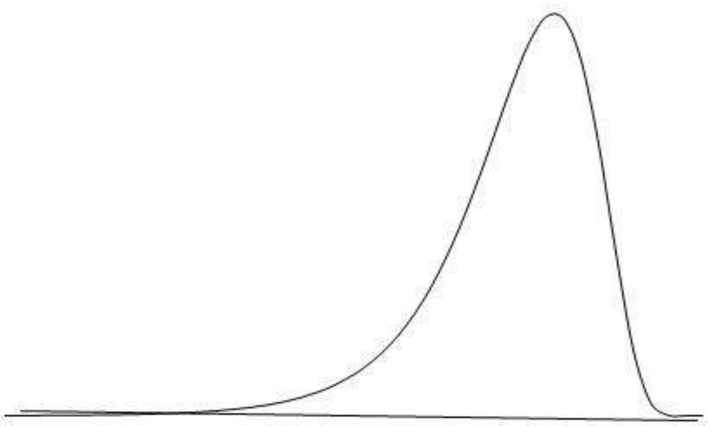
Translation of Asset Price Change to Price Change for Short Call



Impact of Gamma



Positive Gamma



Negative Gamma

Quadratic / Delta-Gamma Model

- For a portfolio dependent on a single asset price it is approximately true that

$$\Delta P = \delta \Delta S + \frac{1}{2}\gamma(\Delta S)^2$$

- so that

$$\Delta P = S\delta\,\Delta x + \frac{1}{2}S^2\gamma\,(\Delta x)^2$$

- Recall $\Delta x \sim N(0,\sigma^2)$, hence:

$$\mu_P = E(\Delta P) = 0.5S^2\gamma\sigma^2$$

$$E(\Delta P^2) = S^2\delta^2\sigma^2 + 0.75S^4\gamma^2\sigma^4$$

$$Var(\Delta P) = E(\Delta P^2) - E(\Delta P)^2 = \delta^2 S^2\sigma^2 + \frac{1}{2}\gamma^2(S^2\sigma^2)^2 = \delta^2 Var(\Delta S) + \frac{1}{2}\gamma^2[Var(\Delta S)]^2$$

$$E(\Delta P^3) = 4.5S^4\delta^2\gamma\sigma^4 + 1.875S^6\gamma^3\sigma^6$$

Quadratic Model

- When there are a small number of underlying market variable moments can be calculated analytically from the delta/gamma approximation
- The Cornish – Fisher expansion can then be used to convert moments to quantiles

Quadratic Model – Estimating Quantiles

- Use Moments to find skewness:

$$\xi_P = \frac{1}{\sigma_P^3} E[(\Delta P - \mu_P)^3] = \frac{E[(\Delta P)^3] - 3E[(\Delta P)^2]\mu_P + 2\mu_P^3}{\sigma_P^3}$$

- Cornish – Fisher: The *q* percentile is $\mu_p + w_q \sigma_p$

where $$w_q = z_q + \frac{1}{6}\left(z_q^2 - 1\right)\xi_p$$

Z_q is the relevant quantile of the standard normal. For example, if we're looking for $VaR_{95\%}$ then it will be $N^{-1}(0.05)=-1.645$.

Delta-Gamma Example

- The current value of a stock index is \$1,500, and its daily volatility is 2%
- A portfolio of options on the index has delta=0.5, gamma=0
- Use the delta-gamma method to estimate the mean, variance and skewness of dollar returns of the portfolio.
- What is 1-day VaR-95%?

Delta-Gamma Example

- Mean=0
- Variance=$\delta^2 S^2 \sigma^2$=(0.5)^2*1500^2*(0.02)^2=225
- Skewness=0
- The 5%-ile is: $0 - 1.645 \cdot \sqrt{225} = -25$
- VaR-95%=|-25|=25

Effect of Gamma on Portfolio and VaR

What if Portfolio Gamma = 0.07? Gamma = -0.07?

Gamma	0.07	0	-0.07
E(ΔP)	31.5	0	-31.5
var(ΔP)	2,210	225	2,210
skewness	2.817	0.000	-2.817
w_q	-0.844	-1.645	-2.446
5%-ile	-8	-25	-146

Modeling Bonds in Linear Model

- Duration Approach: Linear relation between ΔP and Δy (allows parallel shifts only)
- Zero Coupon Bonds: Underlying variables are zero-coupon bond returns with many different maturities
- Principal Components Approach: 2 or 3 independent shifts with their own volatilities, capture most of the variance in term-structure moves

Duration Approach

- Recall $\frac{\Delta P}{P} \sim -Duration \cdot \Delta y$
- Therefore: $\Delta P \approx -P \cdot Duration \cdot \Delta y = -Dollar\, Duration \cdot \Delta y$
- Assume yield follows: $\Delta y \sim N\left(\mu_y, \sigma_y^2\right)$
- Then the price will follow: $\Delta P \sim N\left(\mu_P, \sigma_P^2\right)$

s.t. $\mu_P = -DD \cdot \mu_y \qquad \sigma_P = DD \cdot \sigma_y$

$$\boxed{VaR_{95\%} = -\left(\mu_P - 1.645 \times \sigma_P\right) = DD \cdot \left(\mu_y + 1.645 \times \sigma_y\right)}$$

Duration Approach

- Suppose that the volatility of daily changes in interest rates is 0.1% with mean=0
- Our Portfolio is worth $820 and has duration of 5
- Using the normal-linear approach find the 1-day $VaR_{95\%}$:
  - The dollar duration is 820*5=4,100
    - Portfolio value will drop by $41 for 1% rise in yield.
  - $VaR_{95\%} = 4100*[0+1.645*0.001] = \$6.74$

Duration Approach Caveats

- The duration approximation is for small changes in yield, VaR might involve large changes, where the approximation fails
  – Include convexity and use delta-gamma.
- The portfolio might be affected by non-parallel shifts in the yield curve.

Zero-coupon Bond Returns as Underlying Variables

- We can choose as market variables zero-coupon bond price changes with standard maturities (for example: 1m, 3m, 6m, 1yr, 2yr, 5yr, 7yr, 10yr, 30yr)
- We need to estimate the covariance matrix of all these bond price returns.
- We need to map the portfolio to each of the maturities.
- Suppose we have *n* maturities:

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{cov}_{ij} P_i P_j$$

Bond Portfolio Example

- Consider a portfolio invested $37,397 in 3mm, $331,382 in 6mm and $678,074 in 1-yr bonds
- Rates, vols and correlations for bond prices:

	3-Month	6-Month	1-Year
Zero rate (% with ann. comp.)	5.50	6.00	7.00
Bond price vol (% per day)	0.06	0.10	0.20

	Correlation between daily returns		
	3-Month Bond	6-Month Bond	1-Year Bond
3-month bond	1.0	0.9	0.6
6-month bond	0.9	1.0	0.7
1-year bond	0.6	0.7	1.0

Example – cont.

Portfolio Variance =

$37{,}397^2*(0.06\%)^2+331{,}382^2*(0.10\%)^2+$
$678{,}074^2*(0.20\%)^2+2*37{,}397*331{,}382*(0.06\%)*$
$(0.10\%)*0.9+2*331{,}382*678{,}074*(0.10\%)*(0.20$
$\%)*0.7+2*37{,}397*678{,}074*(0.06\%)*(0.20\%)*0.$
$6 = 2{,}628{,}518 = 1{,}621.3^2$

10-day VaR-99% =

1621.3*2.33*sqrt(10)=$11,946

Zero-Coupon Bond Return Disadvantages

- Requires many underlying variables
- We need to map bond cash flows that arrive at times different than our underlying bonds.

Using PCA to Calculate VaR

- We can use 2 or 3 PCAs as underlying factors.
- It requires:
  - Portfolio sensitivities to those factors
  - Volatilities of the factors
- We estimate less deltas and don't need covariance matrix as PCs are orthogonal

Table 8.8 Standard Deviation of Factor Scores

PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
17.55	4.77	2.08	1.29	0.91	0.73	0.56	0.53

PCA Example

Suppose a portfolio has the following sensitivities to 1-basis-point rate moves, in $ millions:

3-Year Rate	4-Year Rate	5-Year Rate	7-Year Rate	10-Year Rate
+10	+4	−8	−7	+2

What are the portfolio sensitivities to PC1 and PC2?

PC1: $10 \times 0.372 + 4 \times 0.392 - 8 \times 0.404 - 7 \times 0.394 + 2 \times 0.376 = -0.05$

PC2: $10 \times (-0.267) + 4 \times (-0.110) - 8 \times 0.019 - 7 \times 0.194 + 2 \times 0.371 = -3.87$

PCA Example – Cont.

- We get:

$$\Delta P = -0.05 f_1 - 3.87 f_2$$

where f_1 is the first factor and f_2 is the second factor

- If the SD of the factor scores are 17.55 and 4.77 the SD of ΔP is

$$\sqrt{0.05^2 \times 17.55^2 + 3.87^2 \times 4.77^2} = 18.48$$

Thanks

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Market VaR II

MONTE CARLO SIMULATION

Monte Carlo Simulation

To calculate VaR using MC simulation we

- Value portfolio today
- Sample once from the multivariate distributions of the Δx_i
- Use the Δx_i to determine market variables at end of one day
- Revalue the portfolio at the end of day

Monte Carlo Simulation continued

- Calculate ΔP, the change in portfolio value.
- Repeat many times to build up a probability distribution for ΔP
- VaR is the appropriate percentile of the distribution
- For example, with 1,000 trials the 1 percentile is the 10th worst case.

Speeding up Calculations with the Partial Simulation Approach

- In most cases, the positions in the portfolio are too complicated to be fully valued on every iteration
- On every iteration, we use the approximate delta/gamma relationship between ΔP and the Δx_i to calculate the change in value of the portfolio
  - Greeks for the positions are usually already calculated.

Market Risk VaR: Historical Simulation Approach

Historical Simulation

- Collect data on the daily movements in all market variables.
- The first simulation trial assumes that the percentage changes in all market variables are as on the first day
- The second simulation trial assumes that the percentage changes in all market variables are as on the second day
- and so on

Historical Simulation continued

- Suppose we use n days of historical data with today being day n
- Let v_i be the value of a variable on day i
- There are n-1 simulation trials
- The ith trial assumes that the value of the market variable tomorrow (i.e., on day n+1) is: $v_n \frac{v_i}{v_{i-1}}$
- Compute all the values of the market variables for day n+1 for each trial.
- Compute changes in the value of the portfolio.

Example: Portfolio on Sept 25, 2008 (Table 14.1, page 304)

Index	Amount Invested ($000s)	Current Price of Index ($)
DJIA	4,000	11,022.06
FTSE 100	3,000	9,599.90
CAC 40	1,000	6,200.40
Nikkei 225	2,000	112.82
Total	10,000	

U.S. Dollar Value of Stock Indices

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	11,219.38	11,131.84	6,373.89	131.77
1	Aug 8, 2006	11,173.59	11,096.28	6,378.16	134.38
2	Aug 9, 2006	11,076.18	11,185.35	6,474.04	135.94
3	Aug 10, 2006	11,124.37	11,016.71	6,357.49	135.44
….	……	……	…..	……	……
499	Sep 24, 2008	10,825.17	9,438.58	6,033.93	114.26
500	Sep 25, 2008	11,022.06	9,599.90	6,200.40	112.82

501 days → 500 Returns

Scenario #1

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	11,219.38	11,131.84	6,373.89	131.77
1	Aug 8, 2006	11,173.59	11,096.28	6,378.16	134.38
….	……	……	…..	……	……
500	Sep 25, 2008	11,022.06	9,599.90	6,200.40	112.82

Recalculate index levels:

11,022.06*11,173.59 / 11,219.38 = **10,977.08**	9,599.90*11,096.28 / 11,131.84 = **9,569.23**	6,200.40*6,378.16 / 6,373.89 = **6,204.55**	112.82*134.38 / 131.77 = **115.05**

Revalue positions:

4000 * 10,977.08/11,022.06 = **3983.67**	3000 * 9,569.23/9,599.90 = **2990.42**	1000 * 6,204.55/6,200.40 = **1000.67**	2000* 115.05/112.82 = **2039.61**

Portfolio Value = 10,014.334

500 1-day Scenarios

Scenario Number	DJIA	FTSE	CAC	Nikkei	Portfolio Value	Loss
1	10,977.08	9,569.23	6,204.55	115.05	10,014.334	-14.334
2	10,925.97	9,676.96	6,293.60	114.13	10,027.481	-27,481
3	11,070.01	9,455.16	6,088.77	112.40	9,946.736	53,264
….	……	…..	…..	…..	…..	…..
499	10,831.43	9,383.49	6,051.94	113.85	9,857.465	142.535
500	11,222.53	9,763.97	6,371.45	111.40	10,126.439	-126.439

Recall: Current portfolio value: 10,000

Highest Losses

Scenario Number	Loss ($000s)
494	477.841
339	345.435
349	282.204
329	277.041
487	253.385
227	217.974
131	205.256

One-day 99% VaR is the 5th worst loss out of the 500 scenarios=**$253,385**

Using Delta-Gamma Approximation

- To avoid revaluing a complete portfolio 500 times a delta/gamma approximation is sometimes used
- When a derivative depends on only one underlying variable, S

$$\Delta P \approx \delta \Delta S + \frac{1}{2}\gamma(\Delta S)^2$$

Weighting Observations

- Let weights assigned to observations decline exponentially as we go back in time.

$$\frac{\lambda^{n-i}(1-\lambda)}{1-\lambda^n}$$

- Rank observations from worst to best
- Starting at worst observation sum weights until the required quantile is reached

VaR Using Weighted Observations λ=0.995

Scenario Number *(i)*	Loss ($000s)	Weight $\frac{\lambda^{n-i}(1-\lambda)}{1-\lambda^n}$	Cumulative Weight
494	477.841	0.00528	0.00528
339	345.435	0.00243	0.00771
349	282.204	0.00255	0.01027
329	277.041	0.00231	0.01258
487	253.385	0.00510	0.01768
227	217.974	0.00139	0.01906
131	205.256	0.00086	0.01992

One-day 99% VaR=**$282,204**

The lower the λ the more weight will be put on recent observations.

Filtered Historical Simulation / Volatility Updating

- Use a volatility updating scheme, like EWMA or GARCH(1,1) to estimate daily volatilities
- Adjust the percentage change observed on day i for a market variable for the differences between volatility on day i and current volatility
- Value of market variable under ith scenario becomes

$$v_n \frac{v_{i-1} + (v_i - v_{i-1})\sigma_{n+1} / \sigma_i}{v_{i-1}} = v_n \left[1 + \frac{u_i}{\sigma_i} \sigma_{n+1} \right]$$

Volatilities (% per Day) Estimated for Next Day, Using EWMA

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	1.11	1.42	1.40	1.38
1	Aug 8, 2006	1.08	1.38	1.36	1.43
2	Aug 9, 2006	1.07	1.35	1.36	1.41
3	Aug 10, 2006	1.04	1.36	1.39	1.37
….	……	……	……	……	……
499	Sep 24, 2008	2.21	3.28	3.11	1.61
500	Sep 25, 2008	2.19	3.21	3.09	1.59

Scenario #1 with Vol. Update

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	11,219.38	11,131.84	6,373.89	131.77
1	Aug 8, 2006	11,173.59	11,096.28	6,378.16	134.38
….	……	……	…..	……	……
500	Sep 25, 2008	11,022.06	9,599.90	6,200.40	112.82

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	1.11	1.42	1.40	1.38
….	……	……	……	……	……
500	Sep 25, 2008	2.19	3.21	3.09	1.59

DJIA Level for Sep 26, 2008 on scenario #1:

$$11{,}022.06 \times \left[1 + \left[{}^{11{,}173.59}\!\big/_{11{,}219.38} - 1\right]\frac{2.19}{1.11}\right]$$

Volatility Adjusted Losses

Scenario Number	Loss ($000s)
131	1,082.969
494	715.512
227	687.720
98	661.221
329	602.968
339	546.540
74	492.764

McKinsey Working Papers on Risk, Number 32

Managing market risk: Today and tomorrow

May 2012

Market-risk practices at 18 financial institutions, 2011, %

Valuation approach[1]

Simulation approach	Sensitivities	Hybrid	Full revaluation
Historical simulation		35	40
Hybrid	5		5
Monte Carlo	15		

1 Banks are deemed to use the sensitivities approach if they use it exclusively,
hybrid if they use it at least 30 percent of the time, and full revaluation if less than 30 percent.

Source: McKinsey Market Risk Survey and Benchmarking 2011

Exhibit 2 Most banks use equal weighting and look back for one year.

VAR[1] historical-simulation practices at 18 financial institutions, %

	Year				
	1	2	3	4	5
Equal weighting	40	20	5	10	5
Time weighting		10	5		

1 Value at risk.
Note: Numbers may not add up to 100 due to rounding.
Source: McKinsey Market Risk Survey and Benchmarking 2011

Stressed VaR

- Basel requires banks to compute Stressed VaR in conjunction with regular VaR
- Historical simulation based on 250-day period of stressed market conditions
- The one-year chosen should reflect the bank's portfolio, in the sense of worst year for its exposure

ATTRIBUTION IN SIMULATION BASED VAR

Top Losses out of 1000 simulations

Ran k	Loss on A	Loss on B	Loss on C	Total Loss
1	1626	976	734	**3337**
2	2360	440	152	**2952**
3	1792	622	524	**2937**
4	2071	361	19	**2451**
5	1005	336	491	**1832**
6	1170	373	231	**1774**
7	1293	354	66	**1713**
8	1411	38	174	**1623**
9	94	529	969	**1592**
10	1337	61	86	**1484**
11	275	293	831	**1399**
12	186	514	582	**1281**
13	571	253	194	**1018**
14	718	27	225	**970**
15	278	296	258	**832**

$VaR_{99\%}$

Allocating Simulation Based VaR

- Suppose we want to allocate the VaR, i.e. understand how each position is contributing to the total
- If we look at the 10^{th} scenario, which is the VaR 99%, Position A is contributing 90% of the Loss, equal to $1337
- But this is not stable, if we look at the 9^{th} scenario, it is contributing 6%, at $94

Allocating Based on Component Volatility

- One method is to recall our result for Component VaR when VaR only depends on the volatility (e.g. Normal distribution)
- Then allocate the VaR we computed in proportion to the allocation of the volatility.

Allocating Based on Component Volatility

Write the dollar value of the portfolio as: $X = x_i + \sum_{j \neq i} x_j$

Changes in each position are: $\Delta x_i = x_i \cdot R_i, Var\left(R_i\right) = \sigma_i^2$

Changes in portfolio value: $\Delta X = x_i \cdot R_i + \sum_{j \neq i} x_j \cdot R_j$

The variance of portfolio value is:

$$\sigma_P^2 = x_i^2 \sigma_i^2 + 2\sum_{j \neq i} x_i x_j \sigma_i \sigma_j \rho_{ij} + Var\left(\sum_{j \neq i} \Delta x_j\right)$$

Marginal Volatility is therefore: $\dfrac{\partial \sigma_P}{\partial x_i} = \dfrac{2 x_i \sigma_i^2 + 2\sum_{j \neq i} x_j \sigma_i \sigma_j \rho_{ij}}{2\sqrt{\sigma_P^2}}$

$$Component\,vol = x_i \frac{\partial \sigma_P}{\partial x_i} = \frac{\operatorname{cov}\left(\Delta x_i, \Delta X\right)}{\sigma_P}$$

Allocating Based on Component Volatility

- Estimate with sample standard deviation and covariance of simulation results:

$$CoVol_i = x_i \frac{\partial \sigma_P}{\partial x_i} = \frac{1}{\sigma_P} \times \frac{1}{N-1} \sum_{n=1}^{N} (S_i^n - \bar{S}_i)(P^n - \bar{P})$$

σ_P SD of Portfolio Values

S_i^n Value of position i in simulation n

P^n Value of portfolio in simulation n

- Allocate VaR proportionally:

$$CoVaR_i = \frac{VaR}{\sigma_P} \cdot CoVol_i$$

Allocating Based on Expected Shortfall

- We can average the simulations around the VaR to get better results
  - But how many simulations?
- A method preferred by risk managers is to look at the Expected Shortfall:
  - Consider the average losses on each position in the simulations where total loss is at least VaR
  - It is more stable and better reflects the position's contribution to the tail
  - We're guaranteed that the sum of these conditional tails will give us the conditional tail:

$$ES_{99\%} = E\left[L \mid L \geq VaR_{99\%}\right] = E\left[L_1 + L_2 + L_3 \mid L \geq VaR_{99\%}\right] =$$

$$= E\left[L_1 \mid L \geq VaR_{99\%}\right] + E\left[L_2 \mid L \geq VaR_{99\%}\right] + E\left[L_3 \mid L \geq VaR_{99\%}\right]$$

Allocating Based on Expected Shortfall

Rank	Loss on A	Loss on B	Loss on C	Total Loss
1	1626	976	734	**3337**
2	2360	440	152	**2952**
3	1792	622	524	**2937**
4	2071	361	19	**2451**
5	1005	336	491	**1832**
6	1170	373	231	**1774**
7	1293	354	66	**1713**
8	1411	38	174	**1623**
9	94	529	969	**1592**
10	1337	61	86	**1484**
Expected Shortfall	1416	409	345	**2170**
Percentage	65%	19%	16%	
Allocated VaR	969	280	236	**1484**

CONFIDENCE INTERVAL FOR VAR

Bootstrapping for Confidence Intervals

- Suppose we have 500 days of data.
- Sample with replacement from this set to obtain 1000 sets of 500 days of data
- Calculate VaR for each set, by finding the required percentile
- Rank the 1,000 VaRs: The 25th largest and 975th largest form a 95% confidence interval

Parametric Method for Confidence Interval

- We would like to find the Confidence Interval for VaR 99%, that was computed based on *n* daily losses:

1. Estimate a loss distribution (e.g. Normal mean zero) that fits the data
2. x is the qth quantile of the distribution
3. f(x) is the p.d.f. at that point
4. The standard error is: $$\frac{1}{f(x)}\sqrt{\frac{q(1-q)}{n}}$$

Confidence Interval - Example

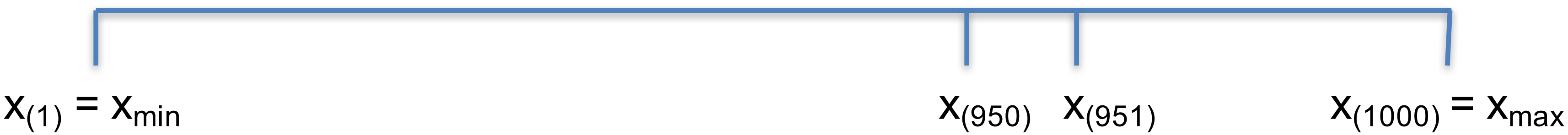
1. We used 500 daily portfolio losses to estimate \$25 as the 99th percentile
2. We find that $N(0,10^2)$ fits the data
3. $2.33*10=23.3$ is the 99th percentile given the normal assumption
4. Using Normal pdf, $f(23.3)=0.0027$
5. The standard error is: $\frac{1}{0.0027}\sqrt{\frac{0.01\times 0.99}{500}}=1.67$

6. 95% C.I. for VaR-99% is:

$$25\pm 1.96\times 1.67=25\pm 3.3$$

CI Using Binomial - Example

- Suppose we create a sample of 1000 x_i, and estimate $VaR_{95\%}$ as $x_{(950)}$.
- Consider a sorted view of the sample:



- What is the probability of observing a sample where true $VaR_{95\%}$ is between $x_{(950)}$ and $x_{(951)}$?
- Same as the probability of 950 draws being less than $VaR_{95\%}$.

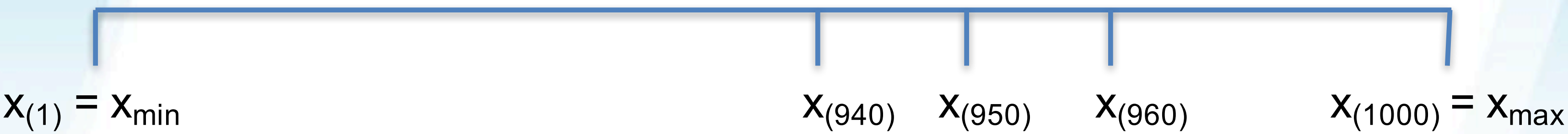
CI Using Binomial - Example

- The probability of exactly 950 draws being less than $VaR_{95\%}$ is binomial, N=1000, p=0.95, x=950:

$$\binom{1000}{950}(0.95)^{950}(0.05)^{50}=0.06$$

- It is actually not that high!
- What is the probability of observing a sample where $VaR_{95\%}$ is between $x_{(940)}$ and $x_{(941)}$?
- Same as the probability of 940 draws being less than $VaR_{95\%}$. $\binom{1000}{940}(0.95)^{940}(0.05)^{60}=0.02$

CI Using Binomial - Example



- To find a confidence interval for $VaR_{95\%}$ add up the probability of observing samples where $VaR_{95\%}$ is in consecutive sub-intervals [$x_{(940)}$ to $x_{(941)}$], [$x_{(941)}$ to $x_{(942)}$] etc…
- Each probability is a binomial similar to the previous one. [$x_{(940)}$ to $x_{(960)}$] is a 85% confidence interval. $\sum_{i=940}^{959} \binom{1000}{i} (0.95)^{i} (0.05)^{n-i} = 0.85$

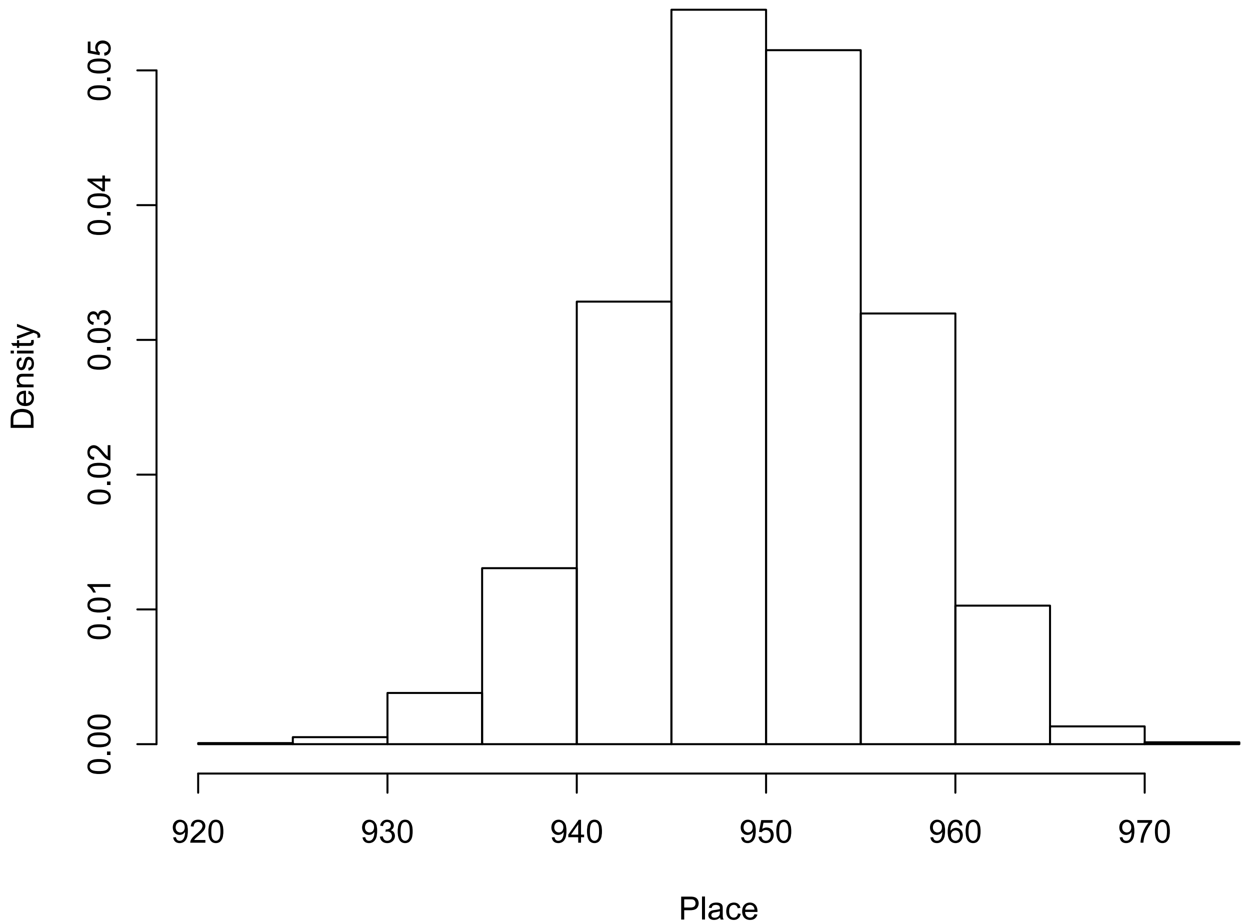
CI Using Binomial - Example

- Expanding the interval to between $x_{(935)}$ and $x_{(965)}$ gives:

$$\sum_{i=935}^{964} \binom{1000}{i} (0.95)^i (0.05)^{n-i} = 0.97$$

- For every sample size, we can find the interval that will give us the required probability.

Location of $N^{-1}(0.95) = 1.645$ in 10,000 samples of 1,000 $x_i \sim N(0,1)$



Thanks

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Heavy Tails and High Confidence Level VaR

UCLA Anderson
THINK IN THE NEXT

Agenda

- Estimating Very High Confidence Level VaRs
- Exponential and Polynomial Tails
- t Distribution
- Power Law

Very High Confidence Level VaRs

- Suppose I have 500 past daily returns to compute historical VaR.
- How do I compute $VaR_{99\%}$?
- What about $VaR_{99.8\%}$? $VaR_{99.9\%}$?
- I have two options for computing VaRs at very high confidence levels:
  - Use a parametric approach to estimate the distribution
  - Use a parametric approach to inflate VaR at a lower confidence level

Results from Historical Simulation

N=500

Scenario Number	Loss ($000s)
494	477.841
339	345.435
349	282.204
329	277.041
487	253.385
227	217.974
131	205.256

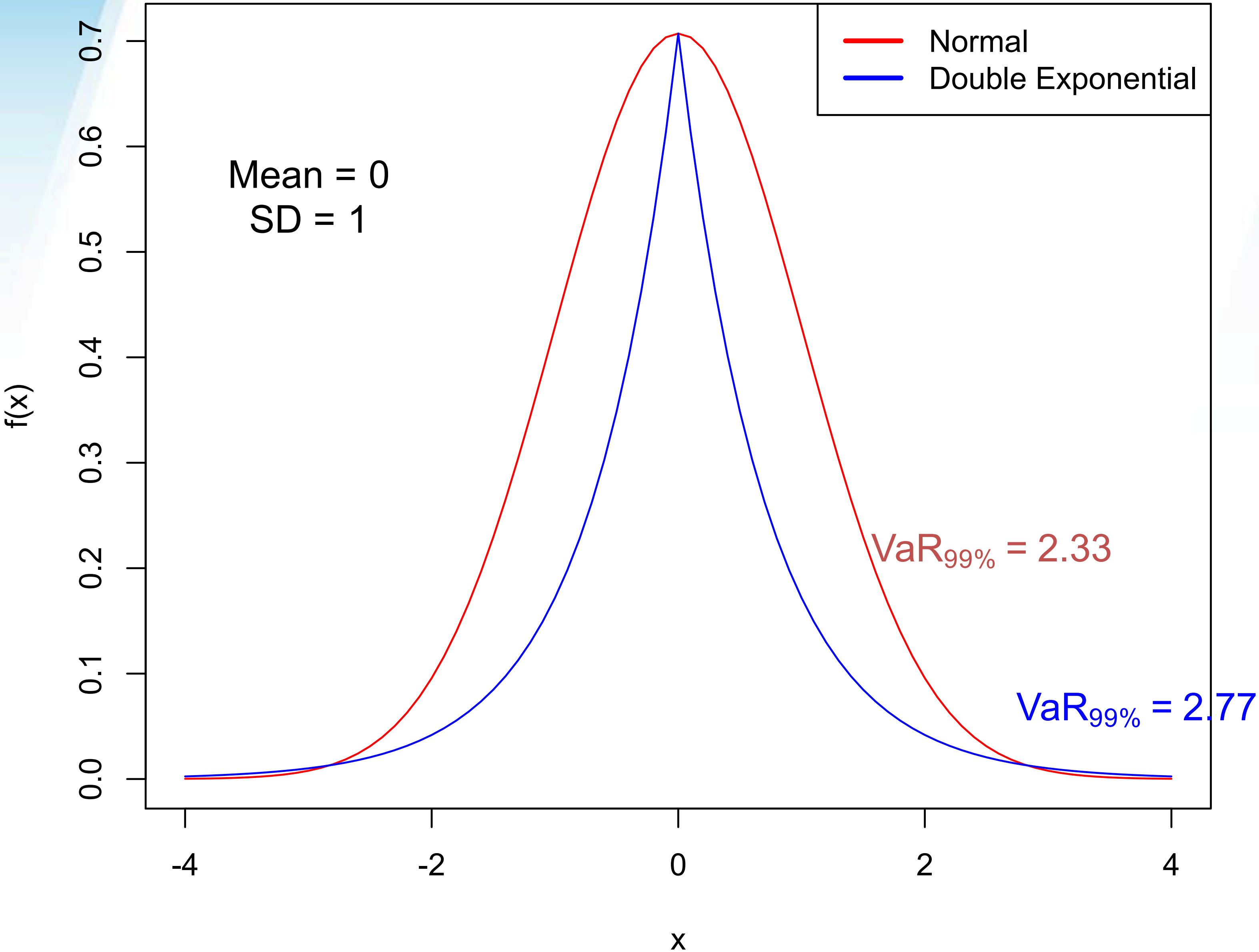
$VaR_{99\%}$? $VaR_{99.8\%}$? $VaR_{99.9\%}$?

Heavy Tails

- Daily exchange rate changes are not normally distributed
  - The distribution has heavier tails than the normal distribution
  - It is more peaked at the center than the normal distribution
- This means that large changes are more likely than the normal distribution would suggest

Exponential Tail

- In order to gauge how heavy is the tail of the distribution, we can look at the density function.
- Consider the density of: $N\left(0,\sigma^2\right)$, $f = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{y^2}{2\sigma^2}}$
- The tail of a distribution with $f \propto e^{-\left|\frac{y}{\theta}\right|}$ will converge slower to 0, and therefore will have a fatter tail.



Normal
Double Exponential
Mean = 0
SD = 1
VaR99% = 2.33
VaR99% = 2.77
f(x)
x
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
-4
-2
0
2
4

Exponential Tail

- The Normal is part of a family of distributions, with exponential rate of convergence to zero:

$$f(y) \propto e^{-\left|\frac{y}{\theta}\right|^{\alpha}}$$

- α is a shape parameter, θ is a scale parameter.
- In case of the Normal α=2 and θ is σ.
- The lower the α the heavier the tail.
- All absolute moments are finite, i.e. $E\left(|Y|^k\right) < \infty$

Polynomial Tail

- To get heavier tails, we have to consider distributions for which the density has polynomial tails, i.e.

$$f(y) \sim A|y|^{-(a+1)} \quad as \quad |y| \to \infty$$

- *a* is called the tail index.
- The k^{th} absolute moment, i.e. $E\left[|y|^k\right]$, exists only if the tail index is larger than *k*.

t - Distribution

- Commonly used way to model polynomial tails.
- The pdf is: $$f_{t,\nu}(y) = \left[\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{(\pi\nu)^{1/2}\,\Gamma\left(\frac{\nu}{2}\right)} \right] \cdot \left[1 + \left(\frac{y^2}{\nu} \right) \right]^{\frac{-(\nu+1)}{2}}$$
- ν is the degrees of freedom
- It is clear that: $f(y) \propto |y|^{-(\nu+1)} \quad as \quad |y| \to \infty$
- Hence, the tail index is ν. The weight of the tail decreases as ν increases.
- It goes to the Standard Normal as ν goes to ∞.

t – Distribution Moments

- The mean exists and equals 0 only if $\nu>1$.
- The variance exists only if $\nu>2$, and is:

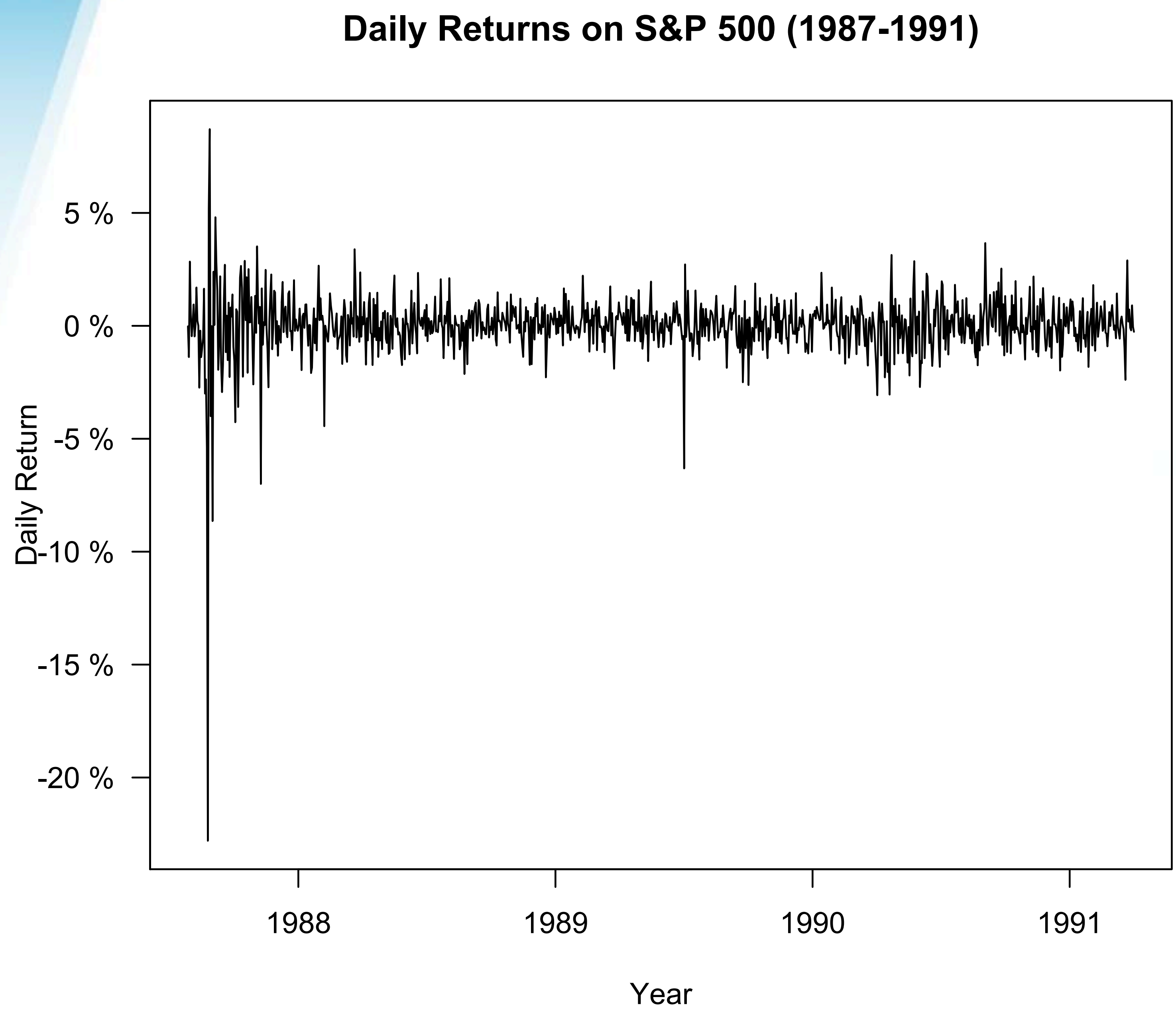
$$\frac{\nu}{\nu-2}$$

- The distribution is symmetric, and its Skewness is zero.
- The Kurtosis exists for $\nu>4$, and is given by:

$$Kurt = 3 + \frac{6}{\nu-4}$$

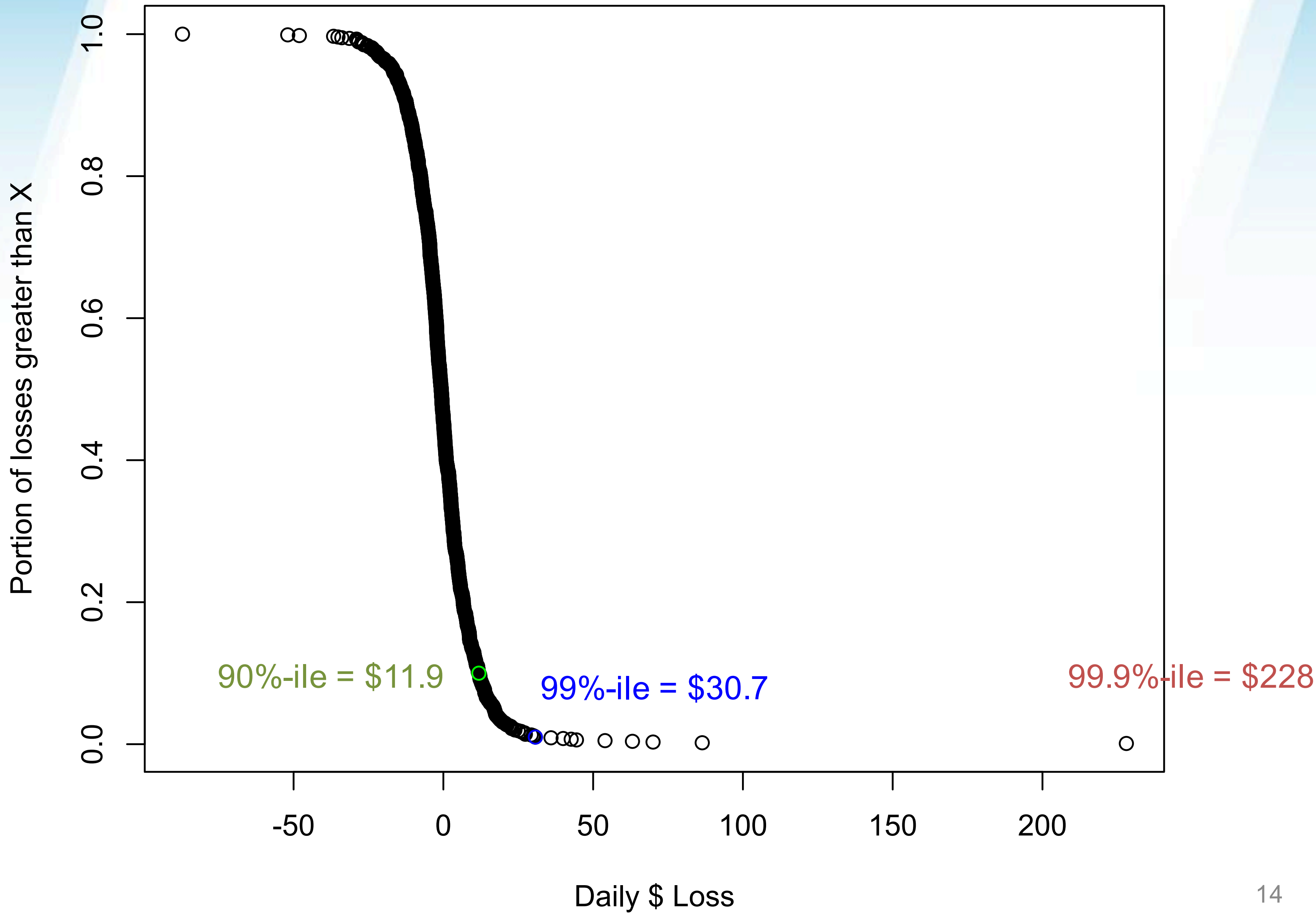
t-Distribution

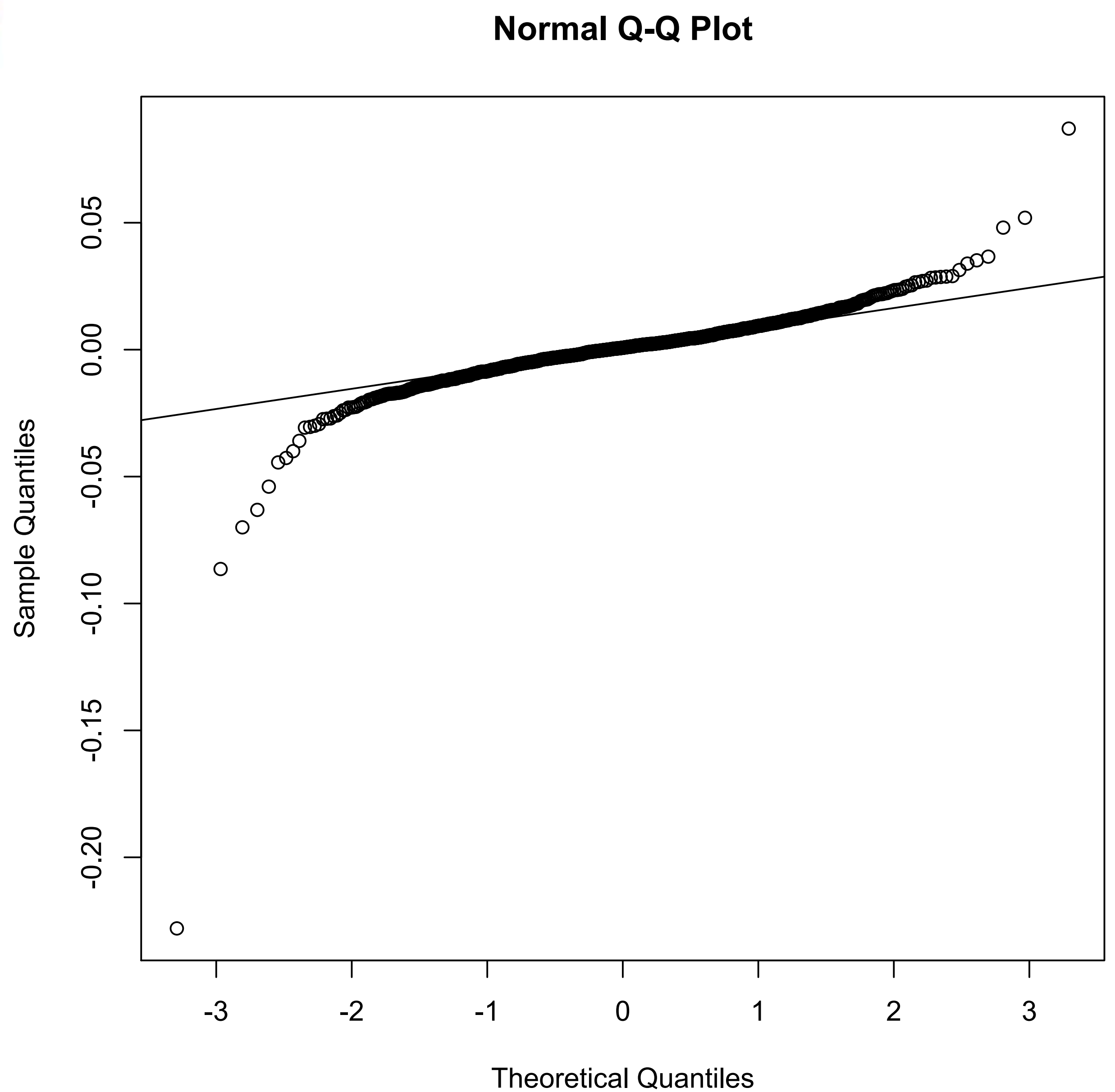
- The classic t-distribution has mean zero, and variance defined by ν
- We can shift and scale it.
- If Y has classic t-distribution with ν degrees of freedom then: $\mu + \lambda Y \sim t_{\nu}\left(\mu, \lambda^{2}\right)$
- μ is the mean, λ is the scale, the variance is equal to: $\lambda^{2}\dfrac{\nu}{\nu-2}$

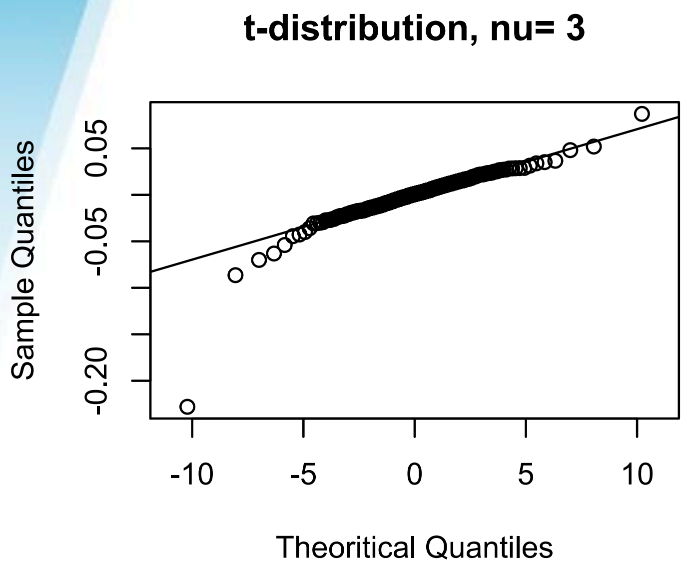


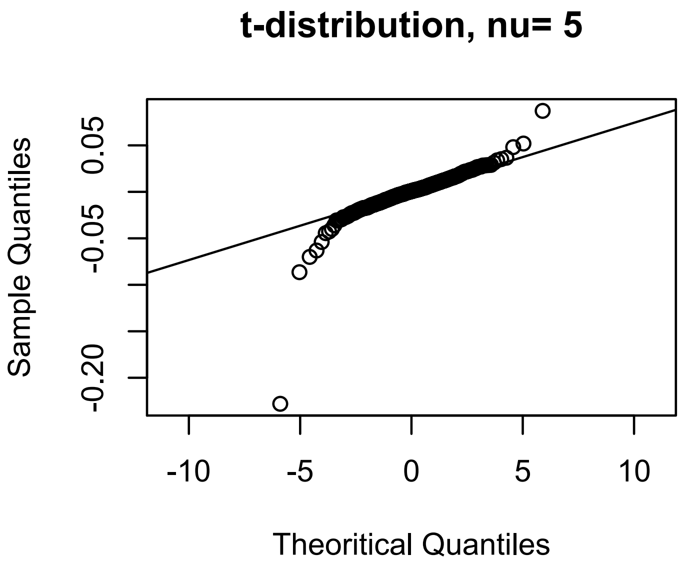
Daily Returns on S&P 500 (1987-1991)
5 %
0 %
-5 %
-10 %
-15 %
-20 %
Daily Return
1988
1989
1990
1991
Year

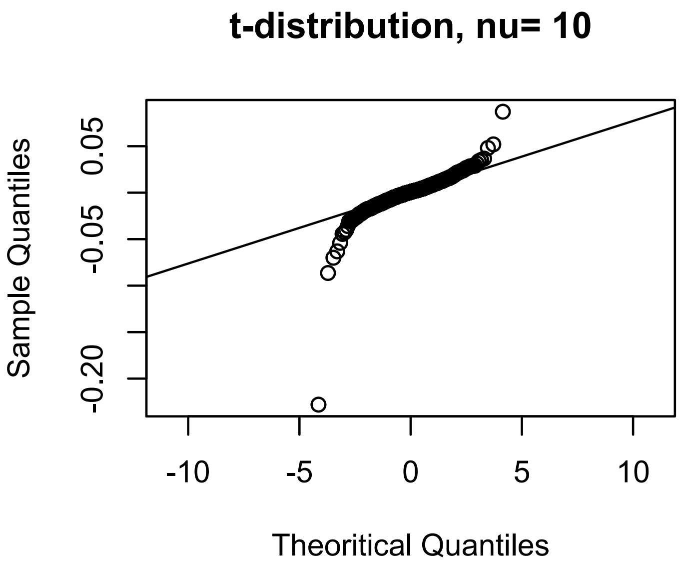
Daily losses on $1000 invested in S&P 500 (1987-1991)

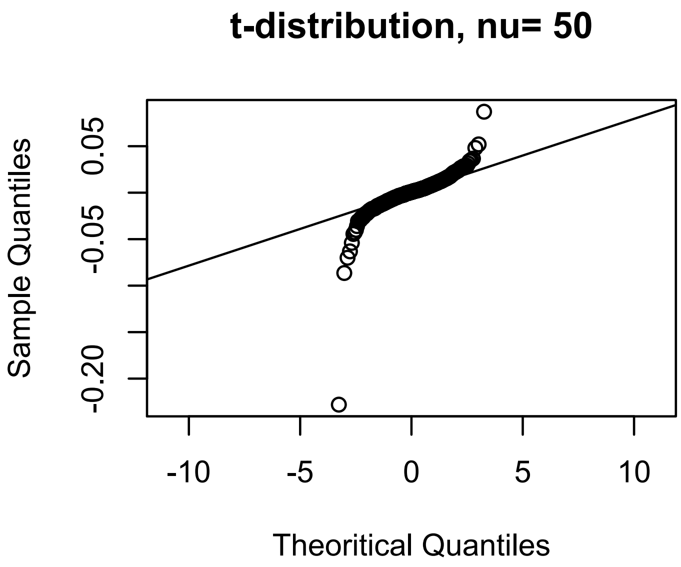



Normal Q-Q Plot
Sample Quantiles
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-3
-2
-1
0
1
2
3
Theoretical Quantiles


t-distribution, nu= 3
Sample Quantiles
0.05
-0.05
-0.20
-10
-5
0
5
10
Theoritical Quantiles


t-distribution, nu= 5
Sample Quantiles
0.05
-0.05
-0.20
-10
-5
0
5
10
Theoritical Quantiles


t-distribution, nu= 10
Sample Quantiles
0.05
-0.05
-0.20
-10
-5
0
5
10
Theoritical Quantiles


t-distribution, nu= 50
Sample Quantiles
0.05
-0.05
-0.20
-10
-5
0
5
10
Theoritical Quantiles

Fitting t-distribution

- To fit a t-distribution, we can use MLE.

$$f_{t,\nu}(X) = \left[\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{(\pi\nu)^{1/2}\,\Gamma\left(\frac{\nu}{2}\right)} \right] \prod_{i=1}^{n} \left[1 + \left(\frac{\left(\frac{x_i-\mu}{\lambda}\right)^2}{\nu} \right) \right]^{\frac{-(\nu+1)}{2}}$$

- Use R:

```
fitt = fitdistr(X,"t")
param = as.numeric(fitt$estimate)
mean = param[1]        lambda = param[2]
nu= param[3]
sd = lambda*sqrt((nu)/(nu-2))
```

Using MLE: **v = 2.98, λ = 7.16**

Estimating VaR 99.9%

- Is \$228 a good estimator for $\text{VaR}_{99.9\%}$?
- The average daily loss/gain is roughly 0.
- If we use the Normal distribution, with the sample standard deviation, σ=13.54:

$$VaR_{99.9\%} = \Phi^{-1}\left(0.999\right) \cdot \hat{\sigma} = \$42$$

- If we use the MLE t-distribution estimates:

$$VaR_{99.9\%} = t_{\nu}^{-1}\left(0.999\right) \cdot \hat{\lambda} = \$74$$

GARCH with t-errors

- We can also fit a GARCH with t-distribution for the errors: $U_t = \sigma_t \varepsilon_t$

$$\sigma_t^2 = \omega + \alpha \cdot U_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$$

- Where: $\varepsilon_t \sim f_{t,\nu}$

The Power Law

- For many variables in practice, it is approximately true that, when *X* is large enough: $\Pr(X > x) = Kx^{-\alpha}$
- *K* and α are parameters to be estimated.
- To find VaR$_{1-p}$: $p = \Pr(Loss > VaR_{1-p}) = K\left[VaR_{1-p}\right]^{-\alpha}$

$$\Rightarrow \left[VaR_{1-p}\right]^{\alpha} = \frac{K}{p} \Rightarrow VaR_{1-p} = \left(\frac{K}{p}\right)^{1/\alpha}$$

- For example: $VaR_{99\%} = \left(\frac{K}{0.01}\right)^{1/\alpha}$

Power Law – Example

- Q: Suppose we know that $\text{VaR}_{95\%}$ is \$10M, and $\alpha = 3$, what is the probability of the loss being greater than \$20M?
- A:

$$VaR_{95\%} = \left(\frac{K}{0.05}\right)^{1/\alpha}$$

$$0.05 = K \cdot 10^{-3}$$

$$K = 50$$

$$p = 50 \cdot 20^{-3} = 0.00625$$

Using the Power Law to Estimate Higher Confidence Level VaR

- Since, $VaR_{1-p} = (K/p)^{1/\alpha}$
- If we feel more confident estimating a lower percentile, we can derive higher percentiles if we know alpha:

$$\frac{VaR_{1-p_1}}{VaR_{1-p_0}} = \left(\frac{p_0}{p_1}\right)^{1/\alpha}$$

- Q: Suppose we know that $VaR_{95\%}$=\$10M, and $\alpha = 3$, what is $VaR_{99\%}$?
- A: $$\frac{VaR_{99\%}}{VaR_{95\%}} = \left(\frac{0.05}{0.01}\right)^{1/3} \Rightarrow VaR_{99\%} = 10 \cdot 5^{1/3} = \$17.10M$$

Compare to Normal: $$\frac{VaR_{99\%}}{VaR_{95\%}} = \frac{N^{-1}(0.99)}{N^{-1}(0.95)} \Rightarrow VaR_{99\%} = 10 \cdot 1.414 = \$14.14M$$

Expected Shortfall for Power Law

Power Law: $\mathrm{P}\left[X > x\right] = Kx^{-\alpha}$

CDF of X: $F\left(x\right) = P\left[X \le x\right] = 1 - Kx^{-\alpha}$

PDF of X: $f\left(x\right) = \alpha K x^{-(\alpha+1)}$

Let d=$VaR_{1\text{-}p}$

Conditional PDF: $f\left(x \mid x \ge d\right) = \dfrac{\alpha K x^{-(\alpha+1)}}{Kd^{-\alpha}} = \alpha d^{\alpha} x^{-(\alpha+1)}$

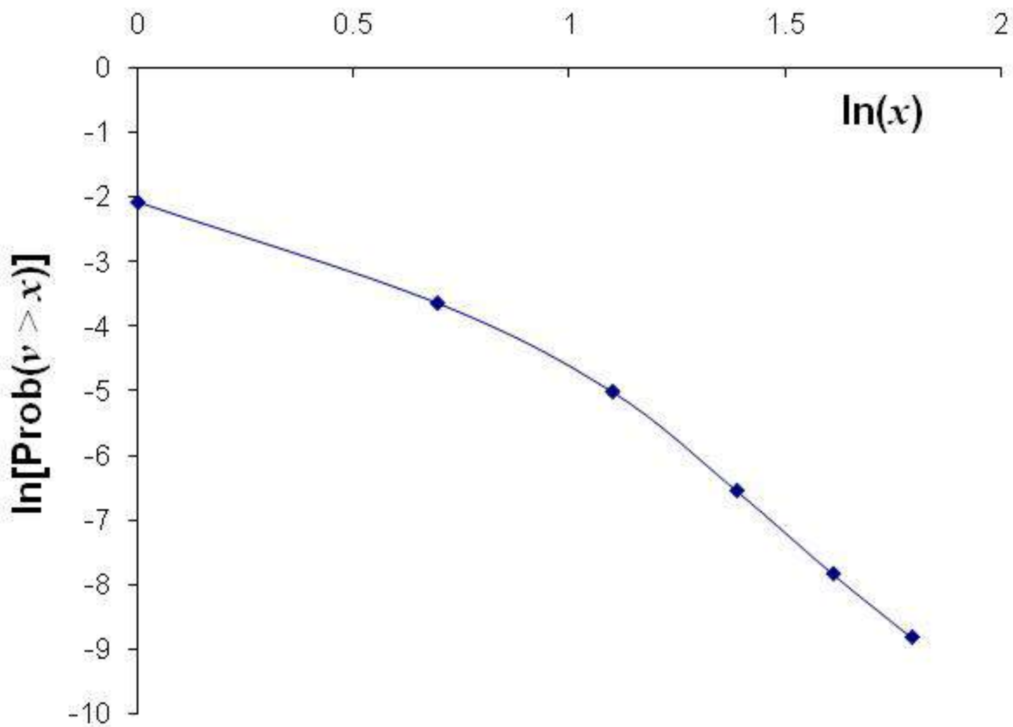
$$E\left[x \mid x \ge d\right] = \int_d^{\infty} \alpha d^{\alpha} x^{-(\alpha+1)} \cdot x dx = \alpha d^{\alpha} \int_d^{\infty} x^{-\alpha} dx$$

$$E\left[x \mid x \ge d\right] = \frac{\alpha}{\alpha - 1} d$$

$$ES_{1-p} = \frac{\alpha}{\alpha - 1} VaR_{1-p}$$

The lower the alpha, the higher the ratio ES/VaR.

Log-Log Plot for Estimating Power Law



Far enough in the tail, we can run a regression to find K and α:

$$\Pr\left(v > x\right) = Kx^{-\alpha} \Rightarrow \text{Ln[Prob}(v > x)] = \ln K - \alpha \ln x$$

Estimating Power Law

Suppose daily returns for S&P are given in *SPreturn* array, with length *n*:

x=sort(SPreturn) #Sort the returns

m=100 #Consider the first 100, i.e. greatest losses

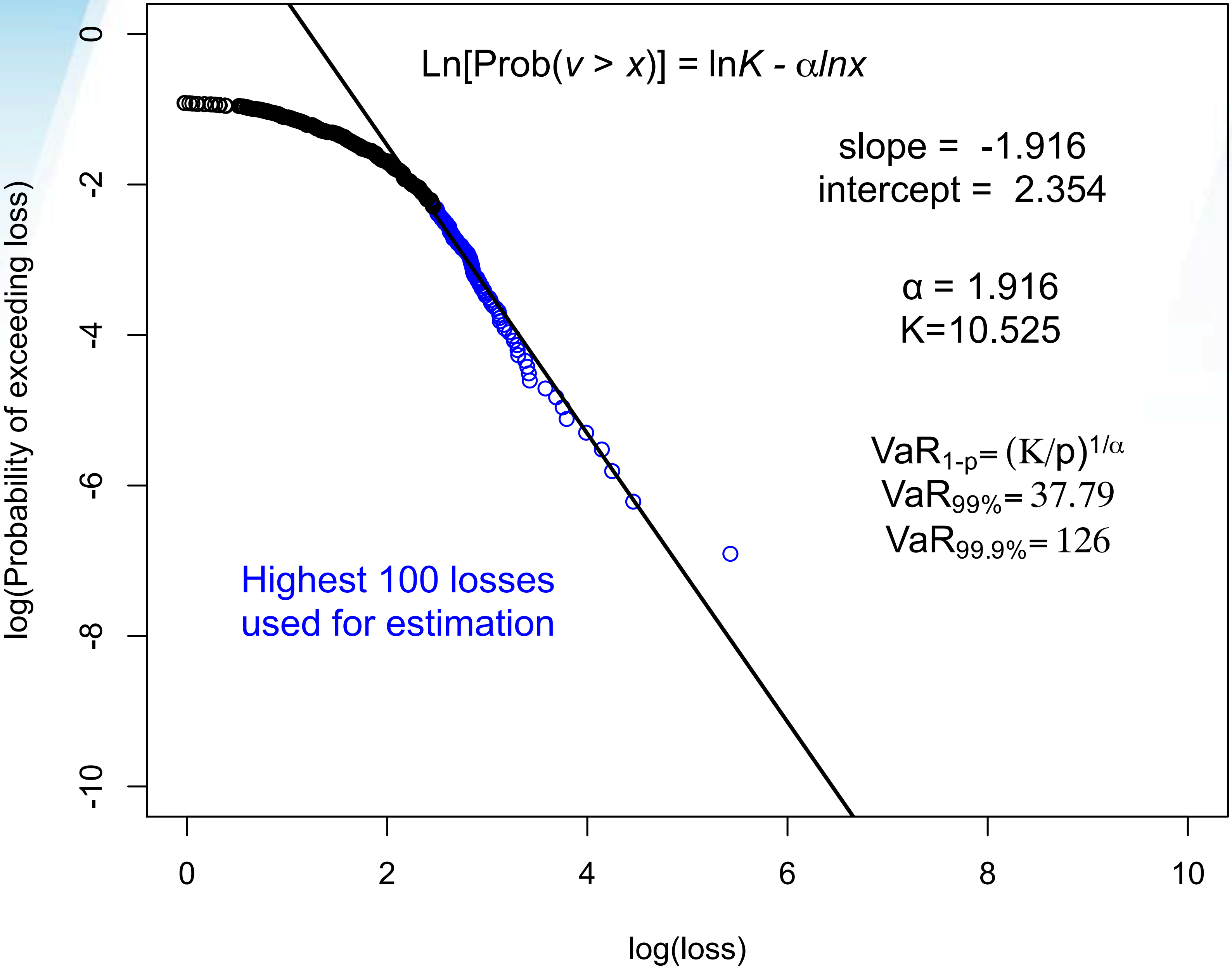
xx=log(-x[1:m]) #Take log of absolute value of losses

yy=log((1:m)/n) #yy[i] is the log of the portion of losses greater than xx[i]

fit =lm(yy~xx) #Estimate the regression

paste("slope = ", fit$coef[2])

Daily returns on $1000 invested in S&P 500 (1987-1991)



Hill's Alpha

- An estimator for alpha.

Conditional distribution: $f\left(x \mid x \geq d\right) = \alpha d^{\alpha} x^{-(\alpha+1)}$

$$E\left[\ln(x) \mid x \geq d\right] = \int_{d}^{\infty} \alpha d^{\alpha} x^{-(\alpha+1)} \cdot \ln(x)dx$$

Integration by parts: $E\left[\ln(x) \mid x \geq d\right] = \ln(d) + \frac{1}{\alpha}$

$$\alpha = \left(E\left[\ln(x) - \ln(d) \mid x \geq d\right]\right)^{-1}$$

- We can use a conditional sample average as an estimator of the conditional expectation.

Hill's Alpha (cont.)

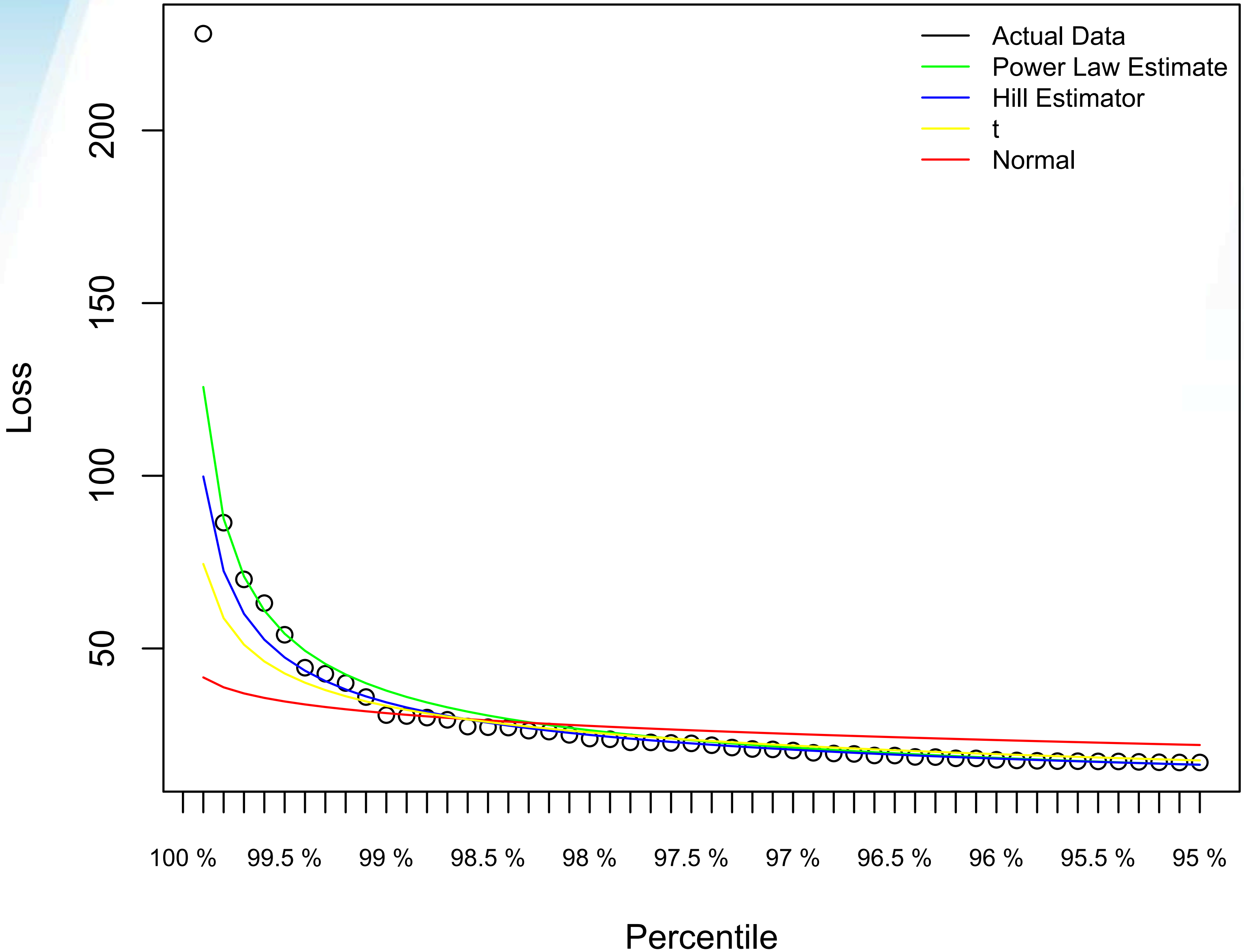
- Select a high loss level, *d*. Consider all the losses that are greater than *d.* Suppose there are *n(d)* such losses, call them $x_{(i)}$ then:

$$\hat{\alpha}^{Hill}\left(d\right)=\frac{n\left(d\right)}{\sum_{i=1}^{n(d)}\ln\left(\frac{x_{(i)}}{d}\right)}$$

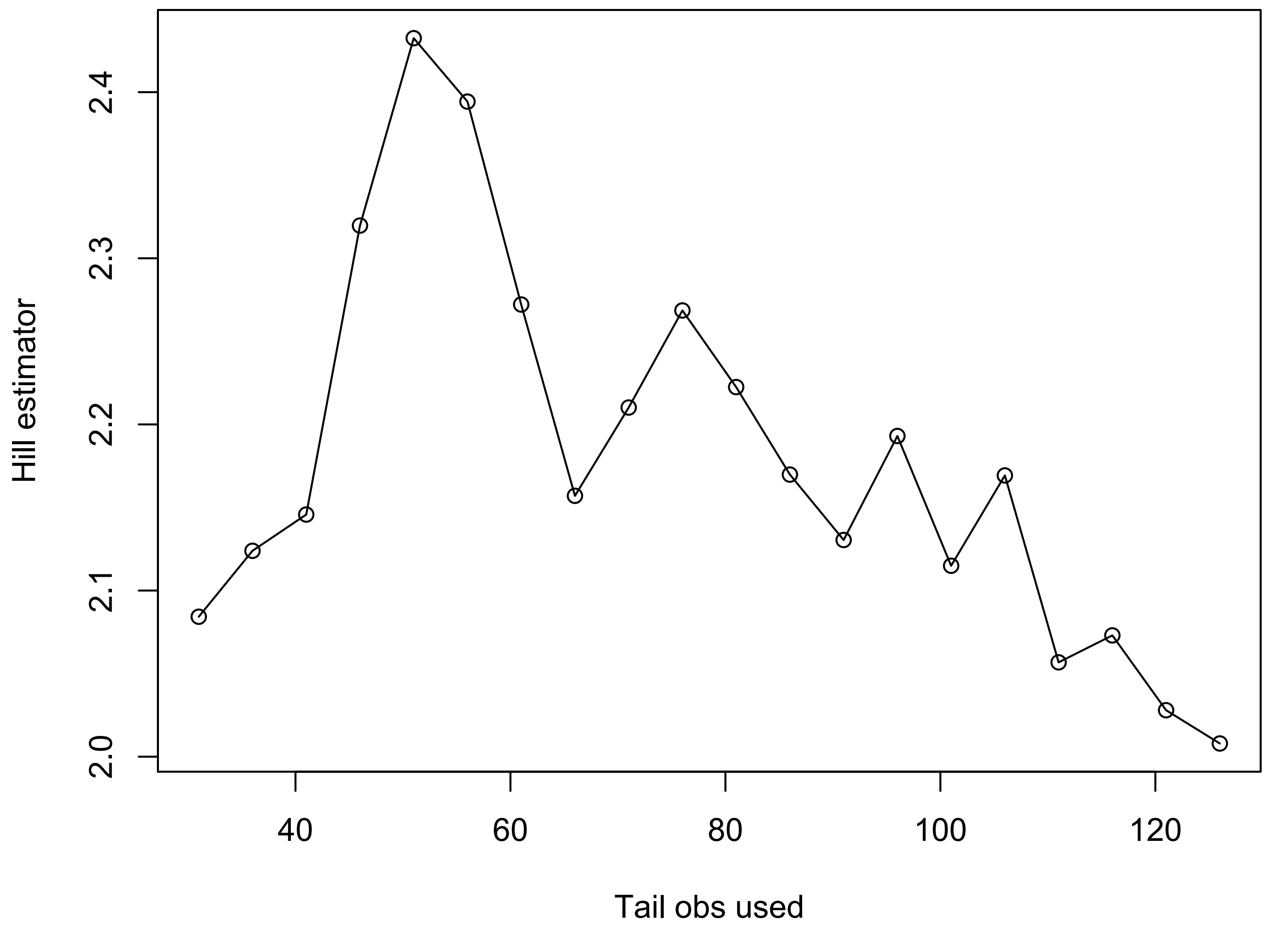
- Extend code to estimate $\hat{\alpha}^{Hill}$ using largest 100 losses: recall: `x=sort(SPreturn) and m=100`

```
hill.alpha = m/sum(log(x[1:m]/x[m]))
```

Top percentiles of daily losses on $1000 S&P position



Hill Estimates based on different cutoffs (d) for 1000 S&P Daily Returns



Thanks

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Credit Risk –

Estimating Default Probabilities

UCLA Anderson
THINK IN THE NEXT

Credit Risk

- Risk of an obligor or counterparty default, which would lead to their failure to meet contractual obligations in relation to actual, contingent or potential claims.
- Examples
  - Loans: Mortgages, C&I, CRE
  - Corporate bonds, EM bonds, Muni bonds
  - Lines of Credit, Guarantees
  - Trade Credit
  - Counterparty Credit Risk
  - Credit Default Swap

Credit Risk Measures

- Probability of Default (PD) – The likelihood that the borrower will fail to make full and timely repayment of its financial obligations
  - Usually measured per year
- Exposure At Default (EAD) – The expected amount of the debt at the time of default
- Loss Given Default (LGD) – The amount of the loss if there is a default, expressed as a percentage of the EAD
  - Equal to (1 – Recovery Rate)
- Expected Loss (EL) = PD*LGD*EAD

Agenda

- Credit Rating and Historical Data
- Conditional vs Unconditional Default Probabilities
- Rating Methods
  - Statistical Methods
  - Expert Judgment and Scorecards
  - Validation of Ratings
- Recovery Rates
- Market Based Methods
  - CDS and bond spreads
  - Structural / Merton’s model
  - Market Implied Signals

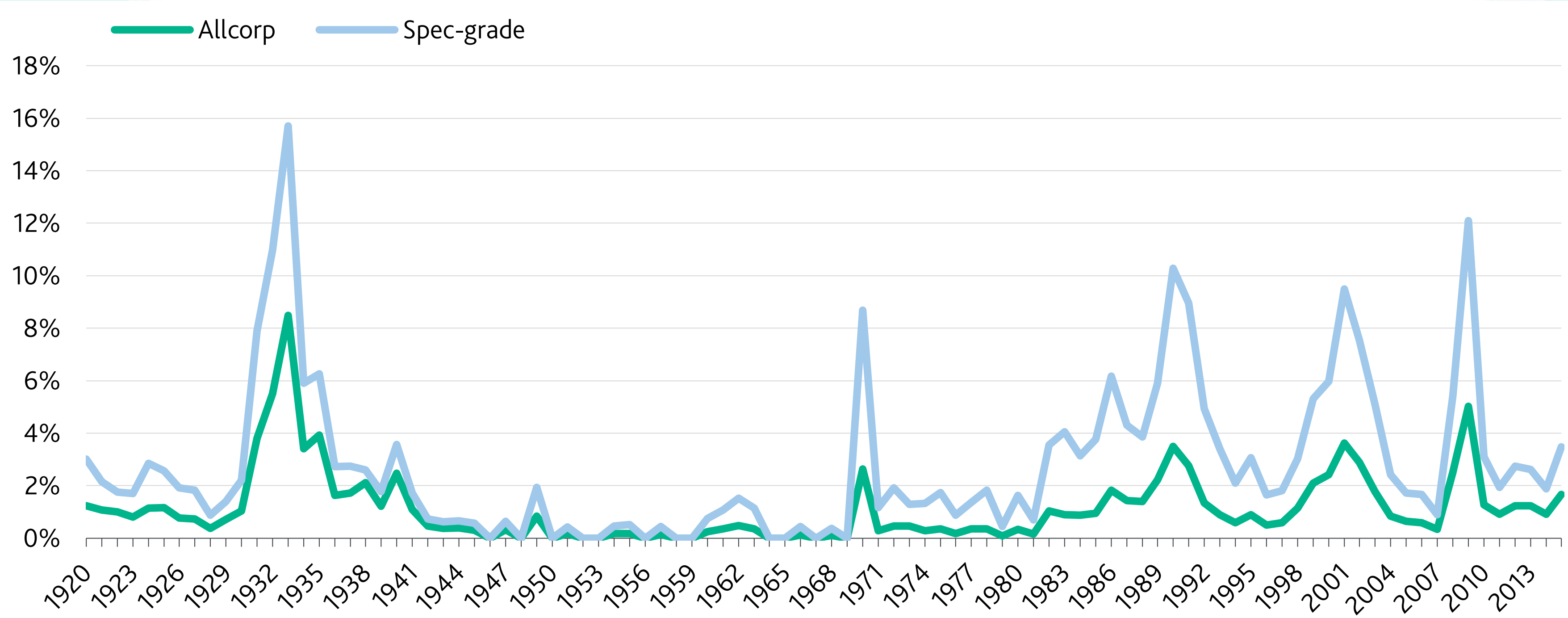
Credit Rating

- We can allocate exposures into rating groups according to their perceived probability of default
- External ratings – performed by rating agencies (Moody’s, S&P, Fitch)
- Internal ratings – performed by the bank/lender/owner of exposure

Moody's Rating Scale

	Long-Term	Short-Term
Investment-Grade	Aaa	
	Aa1	
	Aa2	
	Aa3	Prime-1
	A1	
	A2	
	A3	Prime-2
	Baa1	
	Baa2	Prime-3
	Baa3	
Speculative-Grade	Ba1	
	Ba2	
	Ba3	
	B1	
	B2	
	B3	Not Prime
	Caa1	
	Caa2	
	Caa3	
	Ca	
	C	

Annual Corporate Default Rates



Moody's Annual Default Study, 2016

Annual Corporate Defaults, 1920-2015

	Aaa	Aa	A	Baa	Ba	B	Caa-C
Mean	0.000%	0.059%	0.093%	0.273%	1.032%	3.197%	10.450%
Median	0.000%	0.000%	0.000%	0.000%	0.561%	2.101%	7.699%
SD	0.000%	0.176%	0.264%	0.458%	1.609%	3.819%	11.233%
Min	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Max	0.000%	0.855%	1.639%	1.990%	11.550%	19.444%	50.000%

Moody's 2016

Default Rates by Industry - 2015

Industry	Default Rates*	Industry	Default Rates*
metals & mining	6.5%	fire: insurance	0.8%
energy: oil & gas	6.3%	telecommunications	0.5%
consumer goods: non-durable	4.7%	fire: real estate	0.5%
environmental industries	4.3%	high tech industries	0.5%
forest products & paper	4.2%	utilities: electric	0.3%
media: diversified & production	3.6%	automotive	0.0%
services: consumer	2.8%	capital equipment	0.0%
aerospace & defense	2.6%	chemicals, plastics, & rubber	0.0%
construction & building	2.4%	containers, packaging, & glass	0.0%
hotel, gaming, & leisure	2.3%	energy: electricity	0.0%
services: business	2.3%	fire: finance	0.0%
media: advertising, printing & publishing	2.2%	healthcare & pharmaceuticals	0.0%
Retail	2.1%	sovereign & public finance	0.0%
consumer goods: durable	2.0%	transportation: cargo	0.0%
beverage, food, & tobacco	2.0%	transportation: consumer	0.0%
wholesale	1.7%	utilities: oil & gas	0.0%
Banking	1.5%	utilities: water	0.0%
media: broadcasting & subscription	1.0%		

Moody's 2016

Cumulative Average Default Rates 1983 - 2015

Rating	1	2	3	4	5	6	7
Aaa	0.000	0.013	0.013	0.039	0.068	0.102	0.139
Aa	0.024	0.067	0.123	0.210	0.321	0.419	0.517
A	0.061	0.186	0.394	0.609	0.871	1.159	1.457
Baa	0.200	0.508	0.854	1.266	1.679	2.104	2.503
Ba	0.958	2.663	4.728	6.903	8.812	10.567	12.135
B	3.622	8.564	13.590	18.086	22.184	25.857	29.207
Caa-C	10.578	18.729	25.529	31.021	35.572	38.986	41.637

Moody's 2016

Cumulative Default Chart

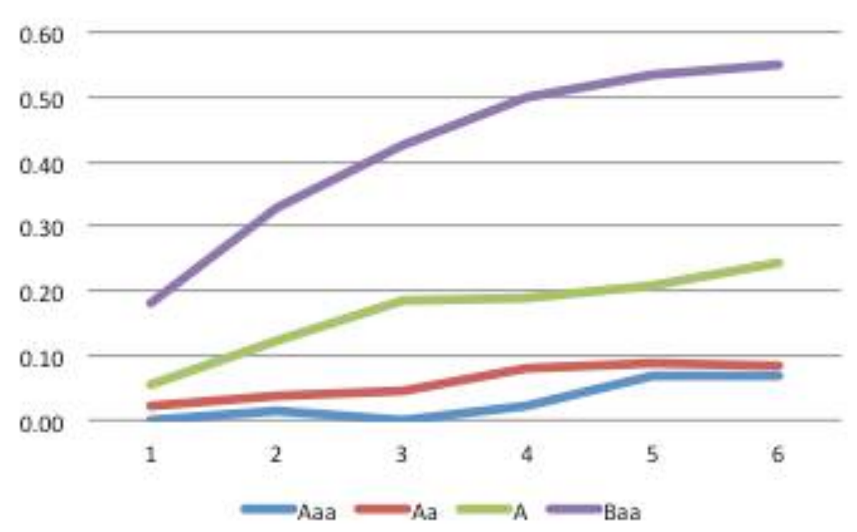
- The table shows the probability of default for companies starting with a particular credit rating
- A company with an initial credit rating of Baa has a probability of 0.200% of defaulting by the end of the first year, 0.508% by the end of the second year, and so on
- It has 2.503% chance of defaulting in 7 years, or 97.497% of not defaulting in 7 years.

Conditional vs. Unconditional Default Probability

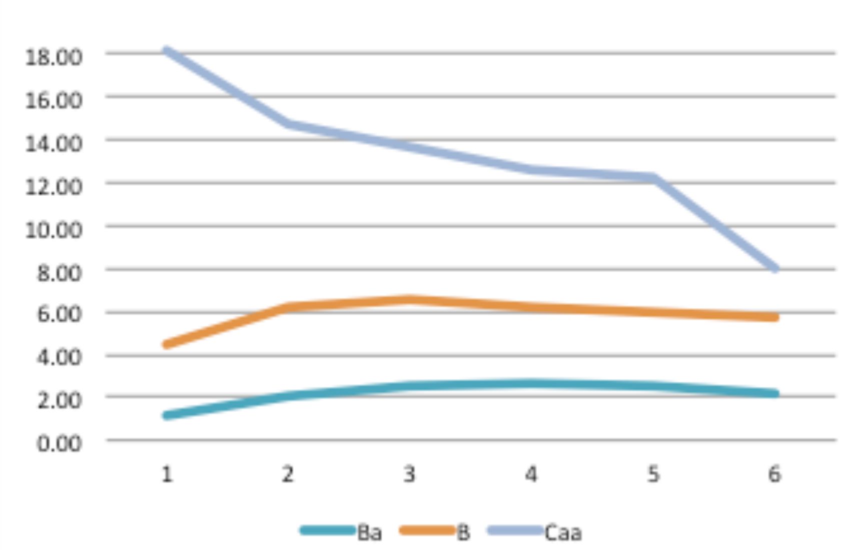
- The unconditional default probability is the probability of default as seen at time zero
  - Example: Baa in year 3: 0.854%-0.508%=0.346%
- The conditional probability of default is the probability of default over a time period conditional on no earlier default
  - Baa in year 3: 0.346%/(100%-0.508%) = 0.348%

Year	1	2	3	4	5	6	7
Cumulative PD	0.200	0.508	0.854	1.266	1.679	2.104	2.503
Unconditional PD	0.200	0.308	0.346	0.412	0.413	0.425	0.399
Conditional PD	0.200	0.309	0.348	0.416	0.418	0.432	0.408

Conditional PD in One Year in the Future



For a company that starts with a good credit rating default probabilities tend to increase with time



For a company that starts with a poor credit rating default probabilities tend to decrease with time

Hazard rates

- Hazard rate is the conditional default probability for unit of time.
- Suppose that $\lambda(t)$ is the hazard rate at time t
- The probability of default between times t and $t+\Delta t$ conditional on no earlier default is $\lambda(t)\Delta t$
- The probability of default by time T is

$$1 - e^{-\bar{\lambda}(T)\cdot T}$$

where $\bar{\lambda}(T)$ is the average hazard rate between time zero and time T. (see next slide)

Probability of Default by time T

V(t) – Probability of Survival till time *t*
Q(T) – Probability of Default by time *T*

The probability of survival till time t+Δt is the probability of survival till time t multiplied by the conditional probability of not defaulting over Δt:

$$V(t+\Delta t) = V(t) * [1-\lambda(t)\Delta t]$$

$$\frac{V(t+\Delta t)-V(t)}{\Delta t} = -\lambda(t)V(t)$$

$$\frac{dV(t)}{dt} = -\lambda(t)V(t) \Rightarrow V(t) = e^{-\int_0^t \lambda(\tau)d\tau}$$

$$V(T) = e^{-\int_0^T \lambda(t)dt} \Rightarrow Q(T) = 1-e^{-\int_0^T \lambda(t)dt} = 1-e^{-\bar{\lambda}(T)\cdot T}$$

One-Year Rating Transition Matrix (% probability, Moody's 1970-2010)

Initial	Rating at year end								
Rating	Aaa	Aa	A	Baa	Ba	B	Caa	Ca-C	Default
Aaa	90.42	8.92	0.62	0.01	0.03	0.00	0.00	0.00	0.00
Aa	1.02	90.12	8.38	0.38	0.05	0.02	0.01	0.00	0.02
A	0.06	2.82	90.88	5.52	0.51	0.11	0.03	0.01	0.06
Baa	0.05	0.19	4.79	89.41	4.35	0.82	0.18	0.02	0.19
Ba	0.01	0.06	0.41	6.22	83.43	7.97	0.59	0.09	1.22
B	0.01	0.04	0.14	0.38	5.32	82.19	6.45	0.74	4.73
Caa	0.00	0.02	0.02	0.16	0.53	9.41	68.43	4.67	16.76
Ca-C	0.00	0.00	0.00	0.00	0.39	2.85	10.66	43.54	42.56
Default	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Internal Rating

EAD of Advanced IRBA Credit Exposures by PD Grade (including Postbank)

	iAAA – iAA 0.00 – 0.04 %	iA 0.04 – 0.11 %	iBBB 0.11 – 0.5 %	iBB 0.5 – 2.27 %	iB 2.27 – 10.22 %	iCCC 10.22 – 99.99 %	Default[1]
Central Governments							
EAD gross in € m.	85,351	4,948	2,804	1,404	732	423	–
EAD net in € m.	93,599	6,227	2,533	583	207	50	–
Average PD in %	–	0.08	0.30	1.40	5.67	13.05	100.00
Average LGD in %	49.24	39.44	42.77	11.04	42.70	48.91	5.00
Average RW in %	0.49	23.16	49.88	25.96	165.01	215.08	62.50
Institutions							
EAD gross in € m.	15,719	31,913	13,132	2,706	2,251	481	166
EAD net in € m.	16,636	32,136	11,890	2,356	2,191	481	166
Average PD in %	0.04	0.07	0.25	1.08	3.00	21.77	100.00
Average LGD in %	31.64	27.03	19.44	21.83	4.59	5.51	13.43
Average RW in %	5.54	11.10	22.18	53.91	16.29	30.79	25.55
Corporates							
EAD gross in € m.	76,225	65,701	66,759	50,632	21,795	5,753	7,598
EAD net in € m.	78,535	64,830	62,096	45,023	18,351	4,993	7,361
Average PD in %	0.03	0.07	0.24	1.17	4.70	23.56	100.00
Average LGD in %	32.63	34.72	30.90	24.84	22.79	16.78	28.19
Average RW in %	9.50	17.86	31.06	49.72	79.28	92.15	24.14
Retail Exposures Secured by Real Estate Property							
EAD gross in € m.	2,766	9,976	45,086	67,241	12,762	5,432	2,680
EAD net in € m.	2,766	9,976	45,078	67,203	12,730	5,410	2,665
Average PD in %	0.03	0.08	0.29	1.05	4.70	21.24	100.00
Average LGD in %	12.13	15.18	10.40	12.21	9.69	8.85	17.99
Average RW in %	1.36	4.88	5.72	16.50	31.73	53.92	14.53

Internal Rating Methods

- Credit Scoring –
  - Statistical methods
  - Typically used for retail credit, credit cards and increasingly for SME
  - Applied in automatic fashion
- Expert Judgment and Scorecards –
  - Combined quantitative and qualitative method
  - Typically used for larger firms, sovereigns and munis
  - Applied on a credit by credit basis

Qualitative Response (QR) Models

- What is the probability of default in the next period, conditional on current market and firm characteristics (typically accounting ratios)?
- The models take the form of:

$$P[D_i = 1 \mid X_i = x] = F(\alpha + \beta x)$$

- Probit Model: $F(z) = N(z)$

- Logit Model: $F(z) = \dfrac{e^z}{1+e^z}$

Fitting QR Models using MLE

- Suppose we observe N companies, with characteristic x_i and an indicator whether they defaulted, D_i
- We can write the likelihood of one observation:

$$L_i = \left[F(\alpha + \beta x_i)\right]^{D_i} \left[1 - F(\alpha + \beta x_i)\right]^{1-D_i}$$

- The log likelihood of observing the data:

$$\log L = \sum_{i=1}^{N} D_i \log\left[F(\alpha + \beta x_i)\right] + (1 - D_i)\log\left[1 - F(\alpha + \beta x_i)\right]$$

Example

In this example, we will analyze the data in the `CreditCard` data set in R's `AER` package. The following variables are included in the data set:

1. `card` = Was the application for a credit card accepted?
2. `reports` = Number of major derogatory reports
3. `income` = Yearly income (in USD 10,000)
4. `age` = Age in years plus 12ths of a year
5. `owner` = Does the individual own his or her home?
6. `dependents` = Number of dependents
7. `months` = Months living at current address
8. `share` = Ratio of monthly credit card expenditure to yearly income
9. `selfemp` = Is the individual self-employed?
10. `majorcards` = Number of major credit cards held
11. `active` = Number of active credit accounts
12. `expenditure` = Average monthly credit card expenditure

Linear Discriminant Analysis

- We suppose that firm characteristics (x) have a multivariate Normal distribution conditional on whether the firm defaults or not.
- Assume they have the same covariance in both cases, but different means:

$$f_1(x \mid D=1) = \frac{1}{(2\pi)^{p/2} \mid \Sigma \mid^{1/2}} e^{-\frac{1}{2}(x-\mu_1)'\Sigma^{-1}(x-\mu_1)}$$

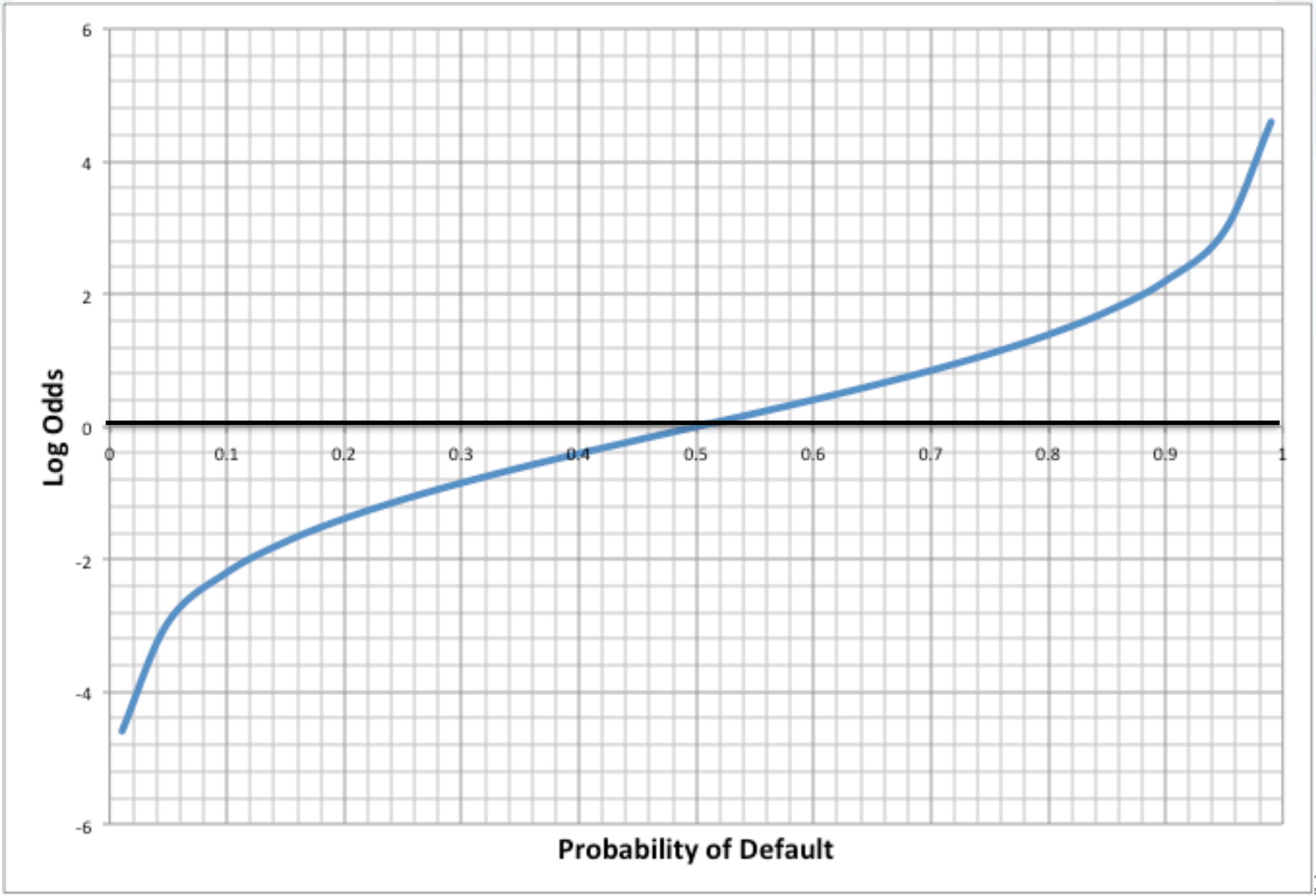
$$f_0(x \mid D=0) = \frac{1}{(2\pi)^{p/2} \mid \Sigma \mid^{1/2}} e^{-\frac{1}{2}(x-\mu_0)'\Sigma^{-1}(x-\mu_0)}$$

- We can estimate the mean and covariance by their sample statistics.

Linear Discriminant Analysis (cont)

- By Bayes Rule: $$P(D=1 \mid X=x) = \frac{f_1(x) \times P(D=1)}{f_1(x) \times P(D=1) + f_0(x) \times P(D=0)}$$ $$P(D=0 \mid X=x) = \frac{f_0(x) \times P(D=0)}{f_1(x) \times P(D=1) + f_0(x) \times P(D=0)}$$
- We estimate P(D=1) by π, the proportion of defaults in the sample. P(D=0) is 1-π.
- Define log odds as: $$\log\left[\frac{P(D=1 \mid x)}{P(D=0 \mid x)}\right]$$
- It is a transformation of the probability of default

Log Odds vs Probability



LDA – Log Odds and Linear Rule

$$\log\left[\frac{P(D=1\,|\,x)}{P(D=0\,|\,x)}\right]=\log\left[\frac{f_1(x)\times P(D=1)}{f_0(x)\times P(D=0)}\right]=$$

$$=-\frac{1}{2}(x-\mu_1)'\Sigma^{-1}(x-\mu_1)+\frac{1}{2}(x-\mu_0)'\Sigma^{-1}(x-\mu_0)+\log\frac{\pi}{1-\pi}=$$

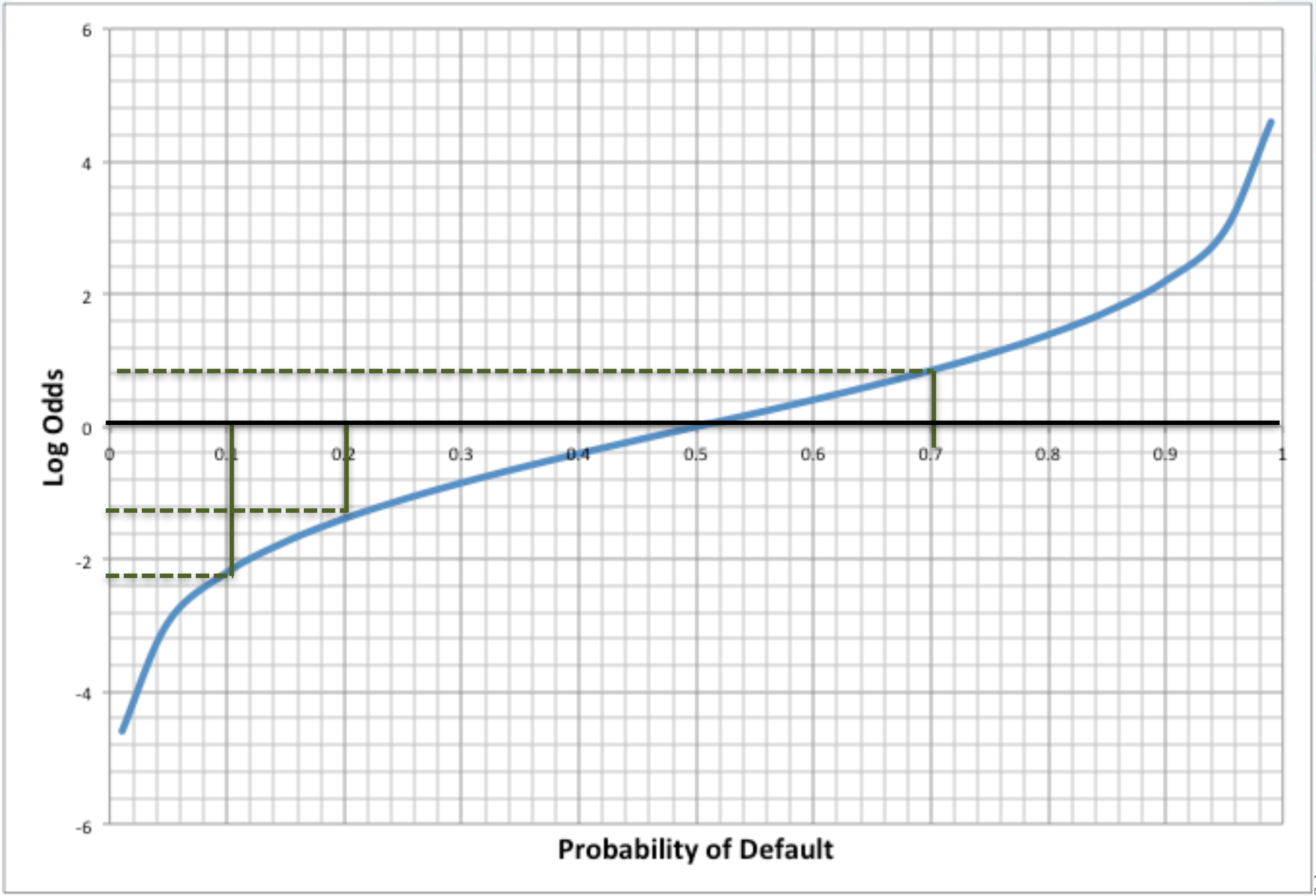
$$=(\mu_1-\mu_0)'\Sigma^{-1}x-\frac{1}{2}(\mu_1+\mu_0)'\Sigma^{-1}(\mu_1-\mu_0)+\log\frac{\pi}{1-\pi}$$

This is a linear combination of the *x*'s: $\log-odds = b_0 + b_1 \cdot x$

$$b_0=\log\frac{\pi}{1-\pi}-\frac{1}{2}(\mu_1+\mu_0)'\Sigma^{-1}(\mu_1-\mu_0)$$

$$b_1=(\mu_1-\mu_0)'\Sigma^{-1}$$

LDA – Multiple Ratings



Altman's Z-score
(Manufacturing companies)

- X_1=Working Capital/Total Assets
- X_2=Retained Earnings/Total Assets
- X_3=EBIT/Total Assets
- X_4=Market Value of Equity/Book Value of Liabilities
- X_5=Sales/Total Assets

What do the ratios mean?

- X1 measures liquidity
- X2 is indicative of cumulative profitability, but also age of firm and leverage
- X3 is a measure of underlying profitability
- X4 measures how much assets can drop in market value before they don’t cover liabilities
- X5 Firm’s ability to compete and generate revenues

Conditional Means in Altman's Population

Variable	Bankrupt Group Mean[n]	Nonbankrupt Group Mean[n]	F Ratio[n]
X_1	-6.1%	41.4%	32.50*
X_2	-62.6%	35.5%	58.86*
X_3	-31.8%	15.4%	26.56*
X_4	40.1%	247.7%	33.26*
X_5	1.5X	1.9X	2.84

N = 33.

To estimate the LDA we use the conditional means of the characteristics conditional on bankruptcy.
There is a significant difference between the means of the variables conditional on whether there was default.

Altman's Z-score
(Manufacturing companies)

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$$

$Z > 3.0$: default is unlikely;

$2.7 < Z < 3.0$: we should be on alert;

$1.8 < Z < 2.7$: moderate chance of default;

$Z < 1.8$: high chance of default

Log Odds and Logit

- The log odds is linear in *x* for the LDA.
- It is also the case for Logit:

$$P(D=1 \mid x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}} \Rightarrow P(D=0 \mid x) = \frac{1}{1+e^{\alpha+\beta x}}$$

$$\log \frac{P(D=1 \mid x)}{P(D=0 \mid x)} = \log \frac{e^{\alpha+\beta x}(1+e^{\alpha+\beta x})}{1+e^{\alpha+\beta x}} = \alpha + \beta x$$

- The two models are not the same since LDA has a restriction on the conditional distribution of x_i given D_i.

Model Estimation

- Probit and Logit models can be estimated in R using glm:

glm(formula = card~income, family = binomial (link="probit"), data=CreditCard)

- Likelihood estimation techniques (Fisher Information and LR tests) can be used to select variables and assess goodness of fit.
- LDA implies computing sample averages and covariance matrix

Scorecards

- Rating agencies and commercial lenders typically use scorecards that incorporate expert judgment with quantitative models
- The scorecards are particular to certain industries, types of projects, or company size
- The results are interpreted as ratings which are then calibrated to PD

Global Packaged Goods Industry

Broad Rating factors	Factor Weightings	Rating Sub Factor	Sub-Factor Weighting
Scale and Diversification	44%	Total Sales (USD Billions)	20%
		Geographic Diversification	12%
		Segmental Diversification	12%
Franchise Strength and Potential	14%	Market Share	7%
		Category Assessment	7%
Profitability	7%	EBIT Margin	7%
Financial Policy	14%	Financial Policy	14%
Leverage and Coverage	21%	Debt/EBITDA	7%
		RCF/Net Debt	7%
		EBIT/Interest Expense	7%
Total	100%	Total	100%

Global Pharmaceutical Industry

Broad Rating Factors	Factor Weighting	Rating Sub-Factors	Sub-Factor Weighting
Scale	25%	Revenue	25%
Business Profile	25%	Product and Therapeutic Diversity	15%
		Geographic Diversity	10%
Patents and Pipeline	16%	Patent exposures	8%
		Pipeline quality	8%
Leverage and Cash Coverage	24%	Debt/EBITDA	9%
		(Cash Flow from Operations)/Debt	9%
		Pharmaceutical Cash Coverage of Debt	6%
Financial Policy	10%	Financial Policy	10%
Total	100%	**Total**	100%

Quantitative Categories

Global Chemical Industry

Broad Rating Factor	Factor Weighting	Rating Sub-Factor	Sub-Factor Weighting
Scale	20%	Revenues	10%
		PP&E (net)	10%
Business Profile	20%	Business Profile	20%
Profitability	10%	EBITDA Margin	5%
		ROA - EBIT/Avg. Assets	5%
Leverage & Coverage	30%	Debt / EBITDA	10%
		EBITDA / Interest Expense	10%
		Retained Cash Flow / Debt	10%
Financial Policy	20%	Financial Policy	20%
Total	**100%**	**Total**	**100%**

Factor 1

Scale (20%)

Sub-factor	Sub-factor Weight	Aaa	Aa	A	Baa	Ba	B	Caa	Ca
Revenues (USD Billions)	10%	≥ \$100	\$50 - \$100	\$15 - \$50	\$5 - \$15	\$1.5 - \$5	\$0.2 - \$1.5	\$0.1 - \$0.2	< \$0.1
PP&E (net) (USD Billions)	10%	≥ \$40	\$20 - \$40	\$8 - \$20	\$3 - \$8	\$0.6 - \$3	\$0.025 - \$0.6	\$0.005 - \$0.025	< \$0.005

Factor 4

Leverage & Coverage (30%)

Sub-factor	Sub-factor Weight	Aaa	Aa	A	Baa	Ba	B	Caa	Ca
Debt / EBITDA	10%	< 0.5x	0.5x - 1.25x	1.25x - 2x	2x - 3x	3x - 4x	4x - 6x	6x - 8x	≥ 8x
EBITDA / Interest Expense	10%	≥ 40x	25x - 40x	15x - 25x	8x - 15x	2x - 8x	1x - 2x	0.5x - 1x	< 0.5x
Retained Cash Flow / Debt	10%	≥ 95%	60% - 95%	30% - 60%	20% - 30%	10% - 20%	5% - 10%	1% - 5%	< 1%

Qualitative Categories

Factor 2

Business Profile (20%)

Sub-factor	Sub-factor Weight	Aaa	Aa	A	Baa
Business Profile	20%	Expected to have highly stable cash flow generation across industry and economic cycles supported by highly diverse specialty product lines with dominant market positions, no concentration of cash flow sources, stable end markets, global leading/low cost operations and structural cost advantages. Technological leadership limits threats to competitive position and supports improving existing market positions and new market opportunities.	Expected to have very stable cash flow generation across industry and economic cycles supported by diverse specialty product lines with leading market positions, low concentration of cash flow sources, stable end markets, global low cost operations and structural cost advantages. Technological leadership results in few threats to competitive position and new market opportunities.	Expected to have stable cash flow generation across industry and economic cycles supported by multiple specialty product lines with large market positions, moderate-to-low concentration of cash flow sources, relatively stable end markets, global predominantly low cost operations and may have structural cost advantages. Technological leadership results in meaningful barriers to entry.	Expected to have moderate volatility of cash flow generation across industry cycles supported by multiple commodity or specialty product lines with significant market positions, moderate concentration of cash flow sources, cyclical end markets, cost competitive operations in more than one region, and limited structural cost advantages. Technology and operating knowhow moderates competitive threats.

		Ba	B	Caa	Ca
		Expected to have cyclical cash flow generation across industry cycles supported by two or more mostly commodity product lines with mid-sized market positions, moderately-high concentration of cash flow sources, cyclical end markets in one region, average cost operations focused on one region, little structural cost advantages. Limited differentiation based on technology and knowhow.	Expected to have highly cyclical cash flow generation, high reliance on a single commodity product line with modest market positions, high concentration of cash flow sources, cyclical end markets in one region, average-to-high cost operations with limited geographic diversity or a single plant site and no structural cost advantages. No real differentiation based on technology and knowhow.	Expected to have highly volatile cash flow generation, a single commodity product line sold to few customers for limited uses; an insignificant market position, concentrated exposure to small cyclical markets, no pricing power, and a single operating site that has an uncompetitive cost structure. Substantial structural and technological disadvantages.	Expected to have highly volatile cash flow generation, a single commodity product line sold to few customers for a single use, an insignificant market position with many large competitors, concentrated exposure to a small cyclical market and uncertain demand, no pricing power, and a single operating site that has an uncompetitive cost structure. Permanent structural and technological disadvantages.

Validation of PD Models

- Assessment of Discriminatory Power – The ability of the rating system to differentiate between borrowers who will default and those who won't
  - "Does a better rating imply lower chance of default?"
- Calibration – Is the difference between estimated PD and observed default rates acceptable
  - Similar to backtesting in market risk,
  - But, with less observations to test since:
    - Annual instead of daily
    - High grade borrowers rarely default

Confusion Matrix

Model Prediction

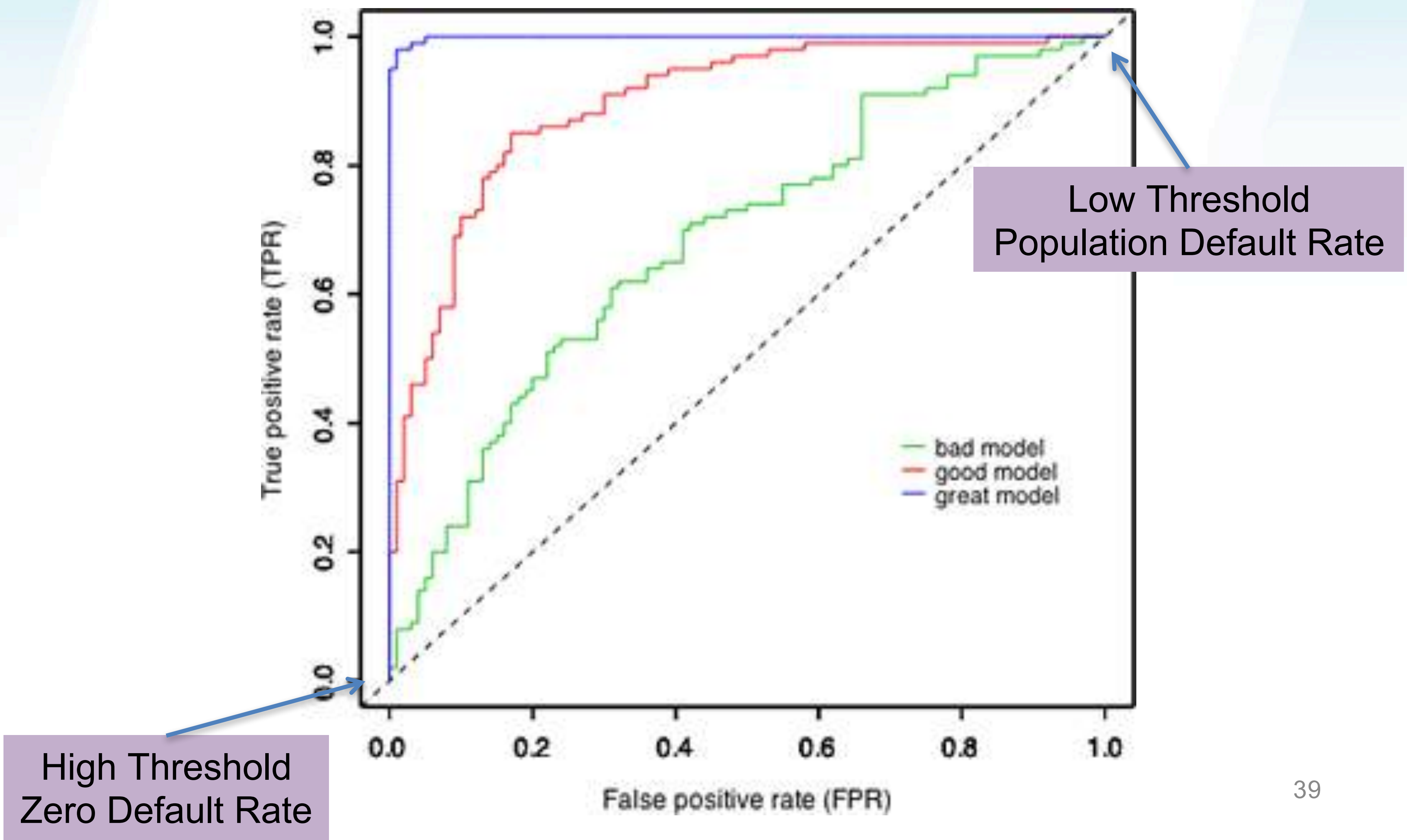
Actual Outcome

	Good - Lend	Bad - Turn Away
Good	True Positive	False Negative
Bad	False Positive	True Negative

True Positive Rate = TP/(TP+FN): How many of "actual goods" did we lend to?
False Positive Rate = FP/(FP+TN): How many of "actual bads" did we lend to?

As we lower the threshold score and loosen our underwriting, both rates go up.

Receiver Operating Characteristic (ROC) Curve



ROC Example

Issuer	Rating	Default	Lend ALL	Lend Above CCC	Lend Above B	Lend Above BB
XYZ	CCC	1	FP	TN	TN	TN
ABC	CCC	1	FP	TN	TN	TN
…	CCC	0	TP	FN	FN	FN
	B	1	FP	FP	TN	TN
	B	0	TP	TP	FN	FN
	B	1	FP	FP	TN	TN
	B	0	TP	TP	FN	FN
	BB	1	FP	FP	FP	TN
	BB	0	TP	TP	TP	FN
	BB	0	TP	TP	TP	FN
TPR			1	0.8	0.4	0
FPR			1	0.6	0.2	0

Threshold Determination

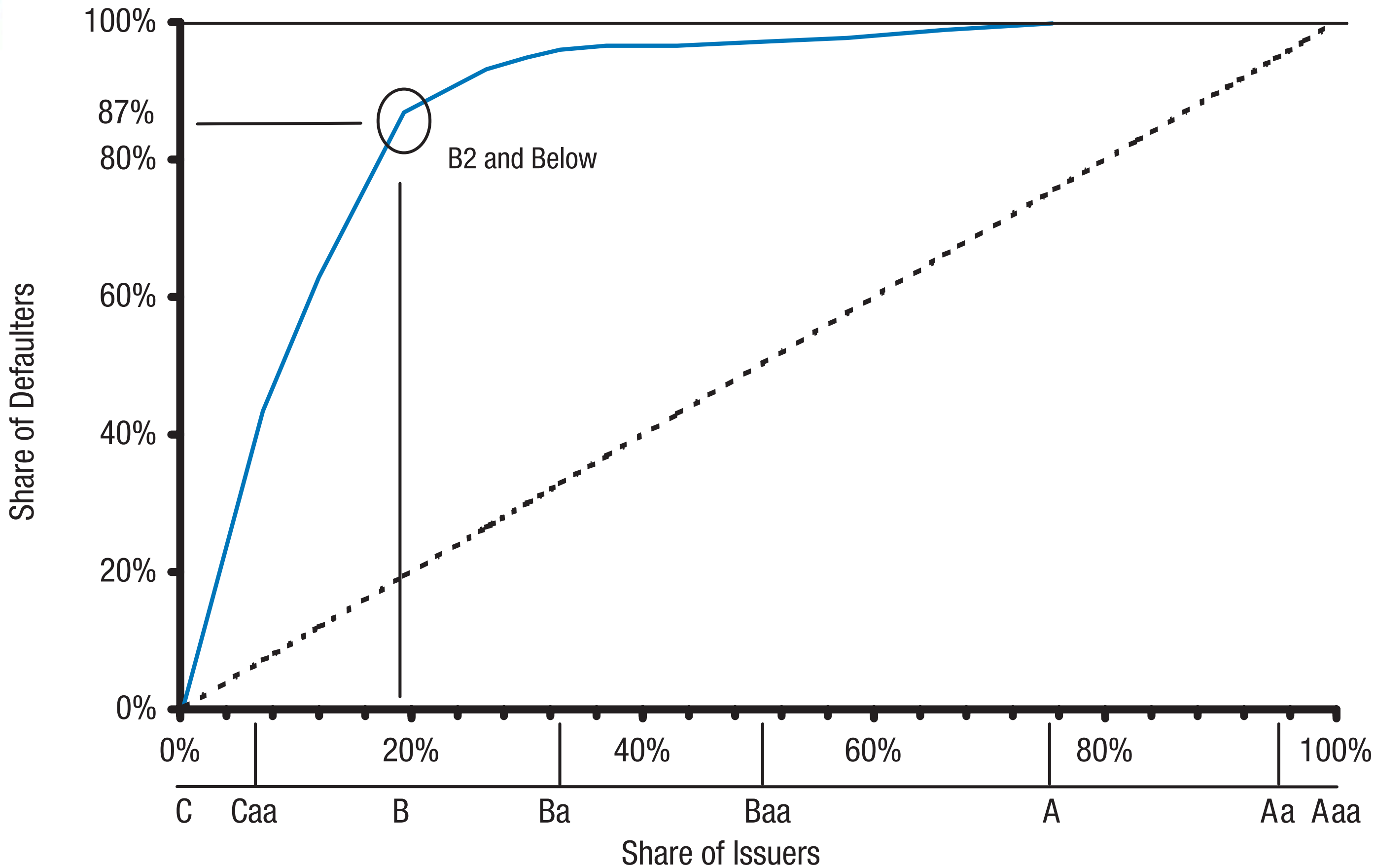
- Suppose that the life time value of a customer is LTV, the credit line is D, and we are able to recover R percent from a defaulted account.
- The expected profit for a given threshold is:

  Profit = TP*LTV – FP*D*(1-R)

- We maximize profit where the slope of the ROC is:

  D*(1-R)/LTV

Cumulative Accuracy Profile (CAP)



Rank by Model score

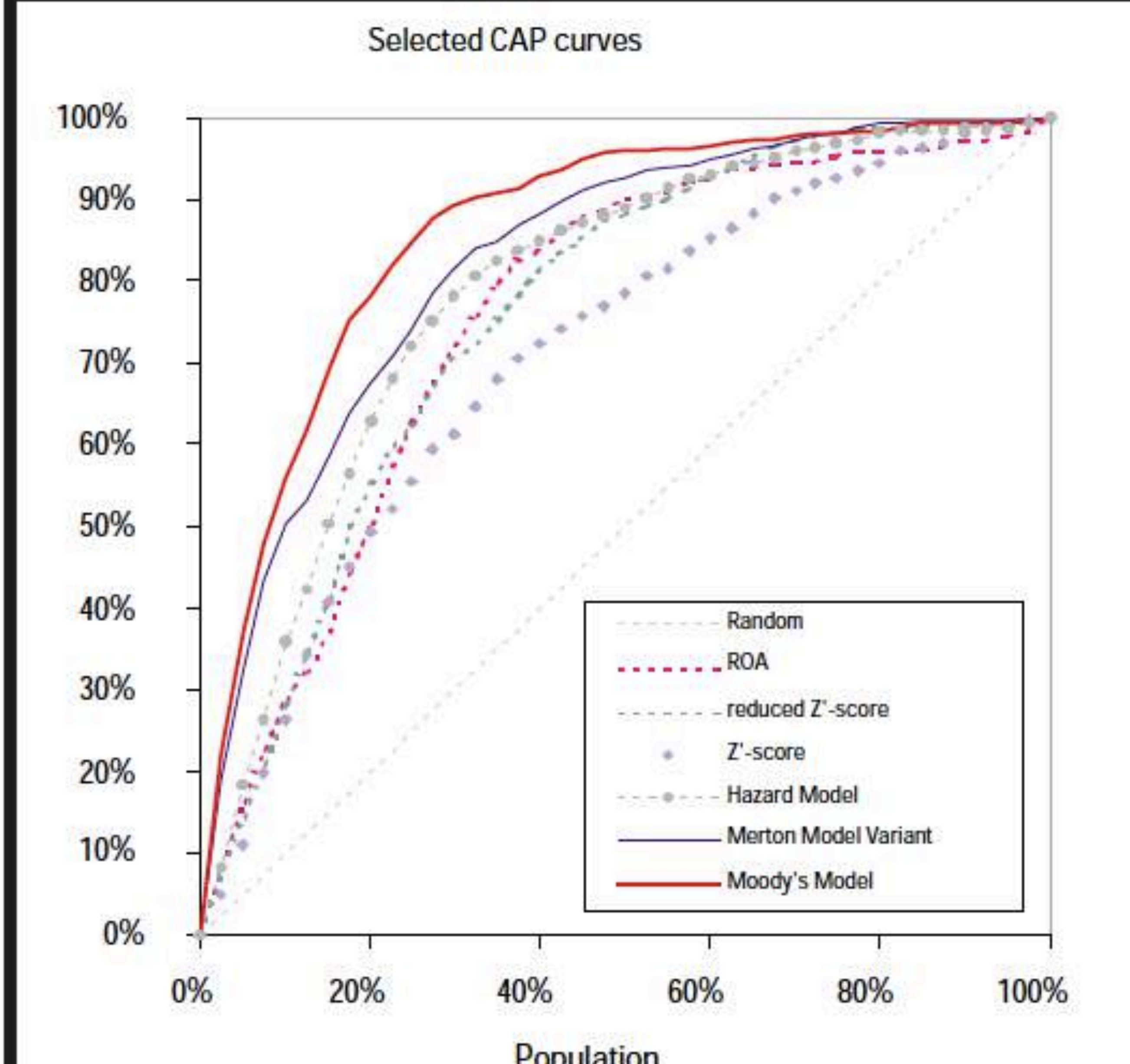
Measuring the performance of corporate bond ratings – Moody's 2003

CAP Example

Issuer	SORT BY Rating	Default	Cum Share of Issuers	Cum Share of Defaults
XYZ	CCC	1	10%	20%
ABC	CCC	1	20%	40%
…	CCC	0	30%	40%
	B	1	40%	60%
	B	0	50%	60%
	B	1	60%	80%
	B	0	70%	80%
	BB	1	80%	100%
	BB	0	90%	100%
	BB	0	100%	100%

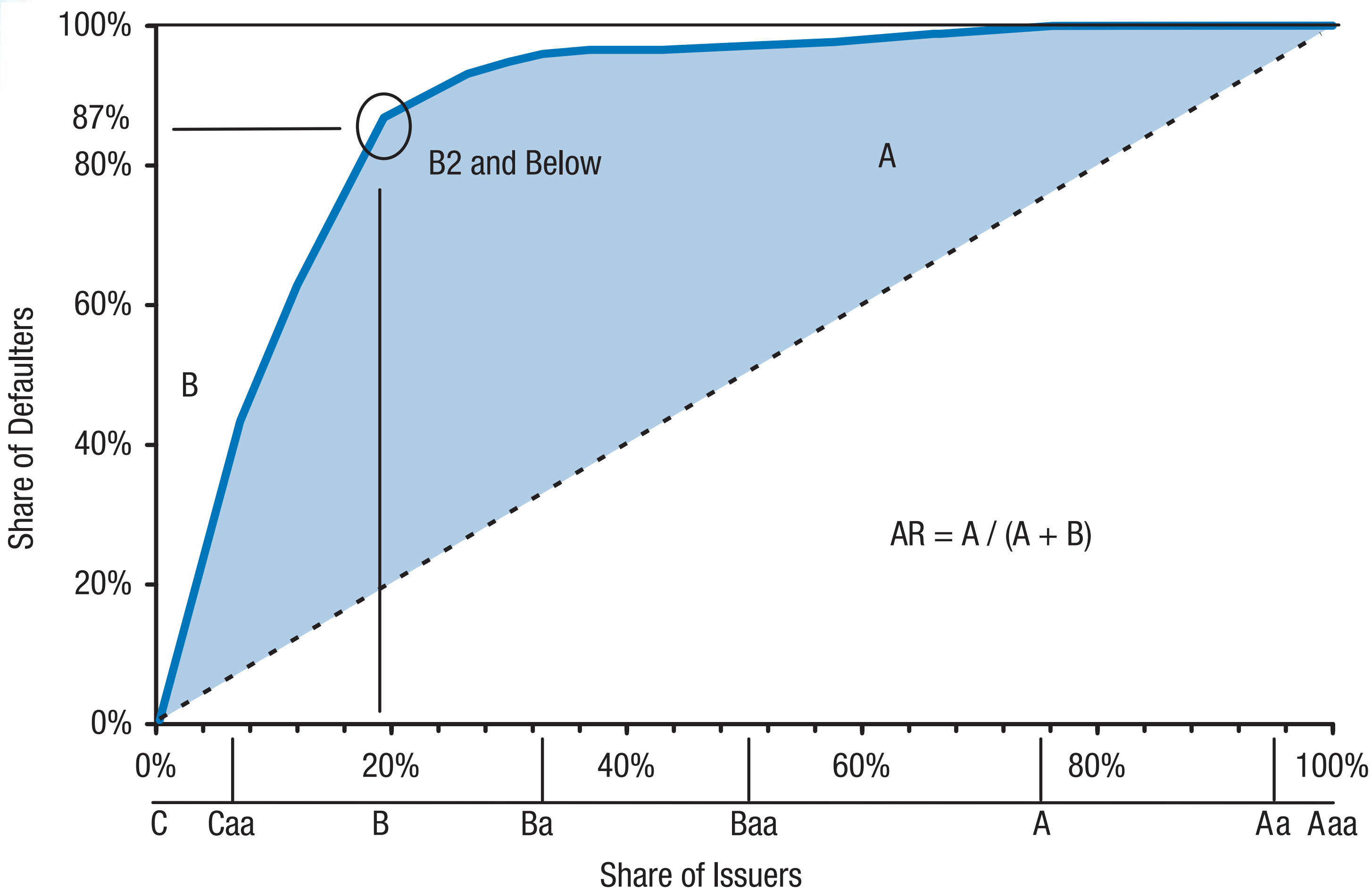
#Issuers=10 #Defaults=5

Figure 6: CAP curves for the tested models



Source: *Moody's Risk Management Services*

Accuracy Ratio



Measuring the performance of corporate bond ratings – Moody's 2003

Recovery Rate

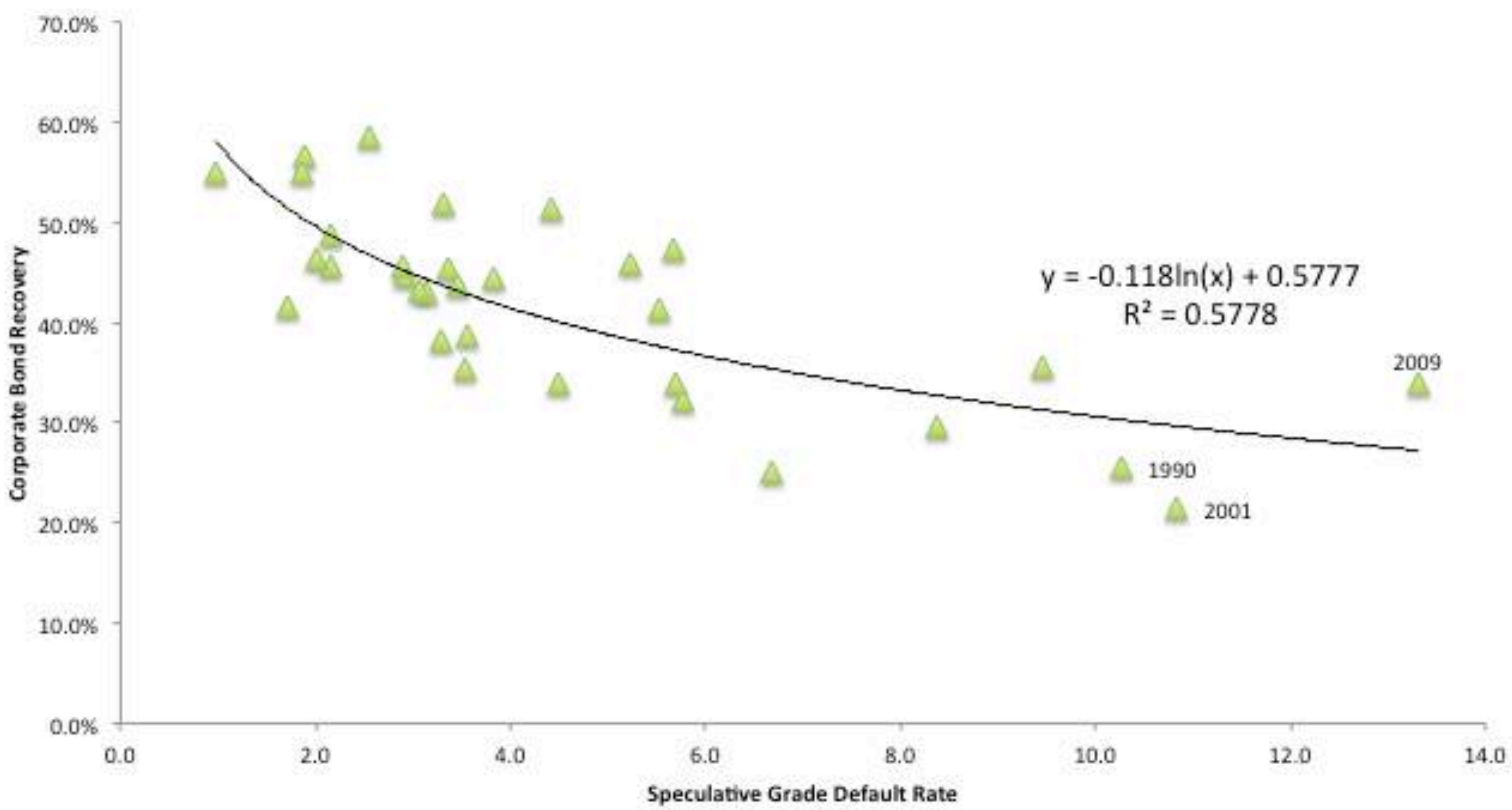
- Recovery Rate for a defaulted exposure:
  - The present discounted value at the default date of recoveries received net of costs associated with collecting on the exposure divided by the amount of the exposure at default.
- For marketable debts, we can take the traded price 30-days after default as a percentage of face value
- Loss Given Default (LGD) = 1 – Recovery Rate

Recovery Rates Vary by Seniority and Type of Debt

Class	Ave Rec Rate (%)
First lien bank loan	66.6
Second lien bank loan	31.8
Senior unsecured bank loan	47.1
1st lien bond	53.4
2nd lien bond	49.7
Senior unsecured bond	37.6
Senior subordinated bond	31.1
Subordinated bond	31.9
Junior subordinated bond	24.2

Moody's: 1983 to 2015, Issuer weighted

Recovery Rates Are Negatively Correlated with Default Rates



Moody's Annual Default Study: Corporate Default and Recovery Rates, 1920-2013

Thanks

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Credit Risk II –

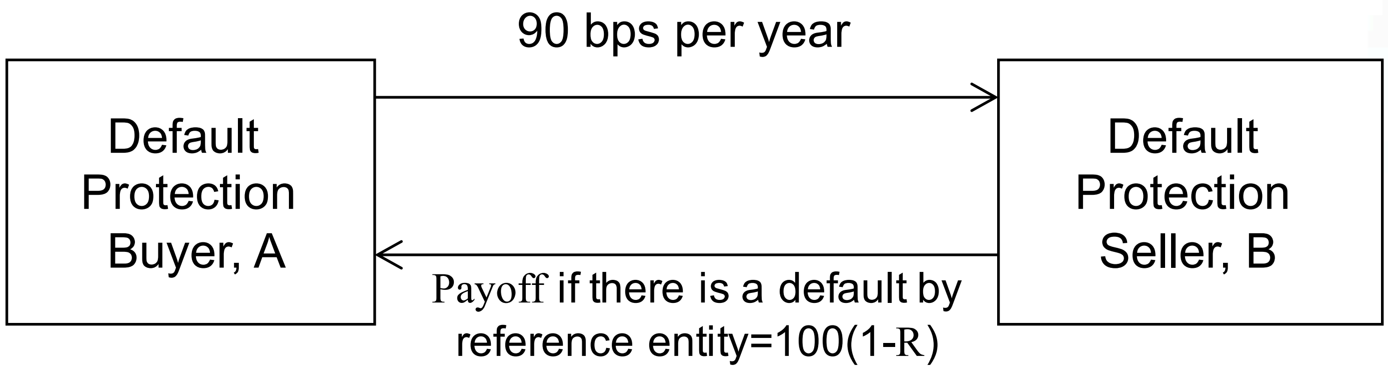
Deriving Default Probabilities From Market Prices

UCLAAnderson
THINK IN THE NEXT

Credit Default Swaps

- Buyer of the instrument acquires protection from the seller against a default by a particular issuer (the reference entity)
- Example: Buyer pays a premium of 90 bps per year for $100 million of 5-year protection against company X
- Premium is known as the *credit default spread*. It is paid for life of contract or until default
- If there is a default, the buyer has the right to sell bonds with a face value of $100 million issued by company X for $100 million (Several bonds may be deliverable)

CDS Structure



Recovery rate, R, is the ratio of the value of the bond issued by reference entity immediately after default to the face value of the bond

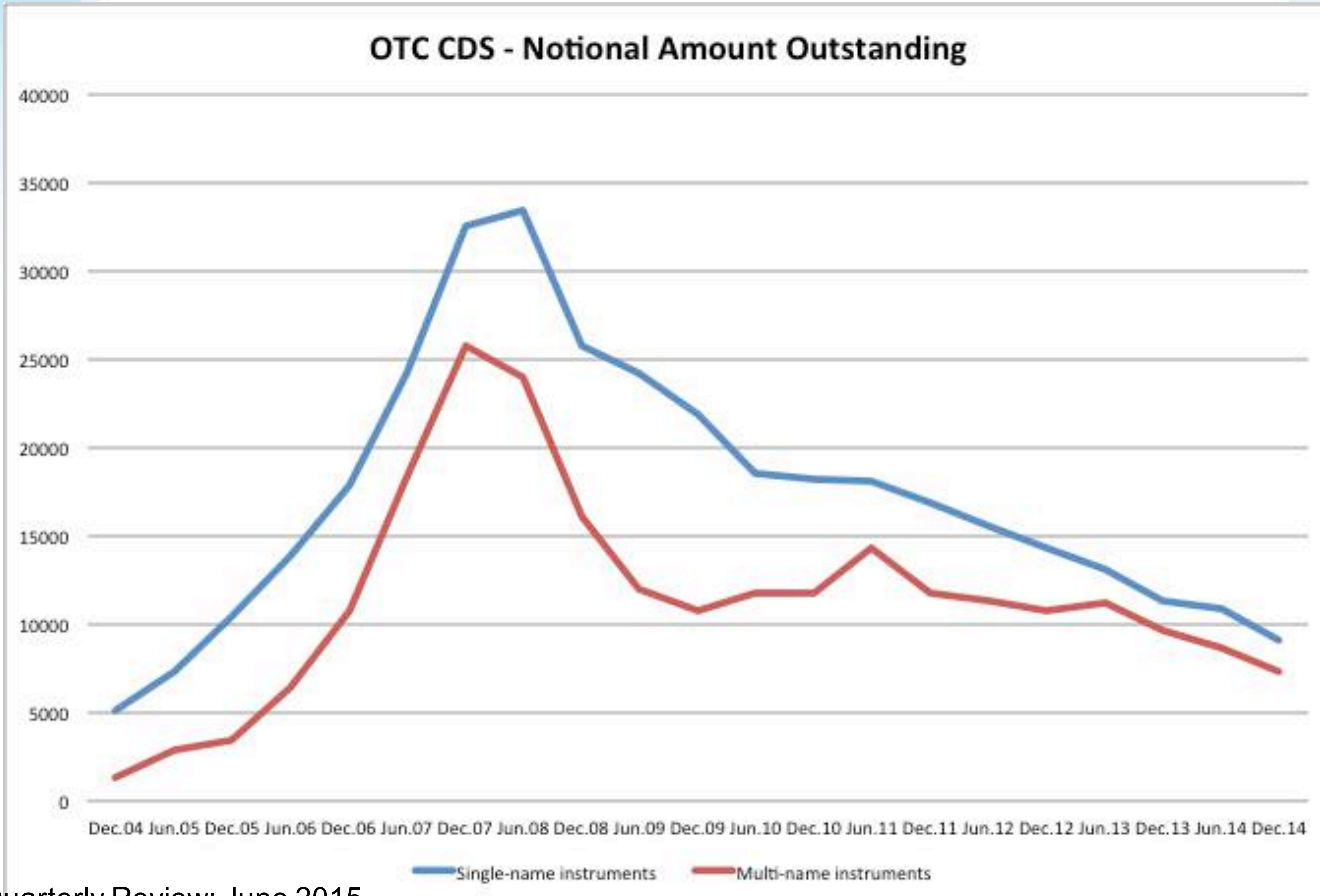
Other Details

- Payments are usually made quarterly in arrears
- In the event of default there is a final accrual payment by the buyer
- Increasingly settlement is in cash and an auction process determines cash amount
- Suppose payments are made quarterly in the example just considered. What are the cash flows if there is a default after 3 years and 1 month and recovery rate is 40%?

Attractions of the CDS Market

- Allows credit risks to be traded in the same way as market risks
- Can be used to transfer credit risks to a third party
- More liquidity in taking short position than in bond market

Declining Volumes in OTC CDS



BIS Quarterly Review: June 2015

Setting Credit Spreads

- Swaps are typically priced to have a net zero present value at time of initiation.
- The credit spread paid by the protection buyer is set so:
- PV of expected payments by the protections seller is equal PV of expected payments by the protection buyer.
- Expectations are taken in the risk neutral measure and are discounted back with the risk free rate.

Credit Spread and Hazard Rates

- One period example:
  - Suppose the risk-neutral probability of default between time t and t+1, conditional on no-default prior to time t is λ
  - The protection buyer agrees to pay s in arrears
  - The protection seller agrees to pay:
    - $1 for a bond worth R if there is a default
    - 0 if there is no default
    - The expected payment is (1-λ)*0+ λ (1-R)
  - s = λ(1-R) or λ = s/(1-R)
- For longer periods: $\bar{\lambda} = \dfrac{s(t)}{1-R}$

Credit Spread and Hazard Rate (1)

Consider a 5-year CDS, what should be the spread for a given hazard rate?
The risk-free rate is 5% and recovery on the bond is 40%
Suppose the hazard rate is 2.0%, first derive the unconditional probabilities of default:

Time (years)	Default Probability	Survival Probability
1	0.0200	0.9800
2	0.0196	0.9604
3	0.0192	0.9412
4	0.0188	0.9224
5	0.0184	0.9039

Pr (D=2) = Pr (Survival year 1) * Pr (D=2|Survival year 1)
Pr (Survival year 2) = Pr (Survival year 1)*[1- Pr (D=2|Survival year 1)]

Credit Spread and Hazard Rate (2)

Assumptions regarding payments:
If the reference entity survives a certain year, buyer pays *s* in arrears
If the reference entity defaults in a certain year, it does so mid-way through the year. At that point: seller pays *(1-R)*, buyer pays *s/2*

Present Value of protection seller's payments:

Time (years)	Probability of Default	Recovery Rate	Expected Payoff ($)	Discount Factor	PV of Expected Payoff ($)
0.5	0.0200	0.4	0.0120	0.9753	0.0117
1.5	0.0196	0.4	0.0118	0.9277	0.0109
2.5	0.0192	0.4	0.0115	0.8825	0.0102
3.5	0.0188	0.4	0.0113	0.8395	0.0095
4.5	0.0184	0.4	0.0111	0.7985	0.0088
Total					0.0511

Credit Spread and Hazard Rate (3)

Present Value of protection buyer's payments:

Time (years)	Probability of Survival	Expected Payment	Discount Factor	PV of Expected Payment
1	0.9800	0.9800*s*	0.9512	0.9322*s*
2	0.9604	0.9604*s*	0.9048	0.8690*s*
3	0.9412	0.9412*s*	0.8607	0.8101*s*
4	0.9224	0.9224*s*	0.8187	0.7552*s*
5	0.9039	0.9039*s*	0.7788	0.7040*s*
Total				4.0704*s*

Time (years)	Probability of Default	Expected Accrual Payment	Discount Factor	PV of Expected Accrual Payment
0.5	0.0200	0.0100*s*	0.9753	0.0097*s*
1.5	0.0196	0.0098*s*	0.9277	0.0091*s*
2.5	0.0192	0.0096*s*	0.8825	0.0085*s*
3.5	0.0188	0.0094*s*	0.8395	0.0079*s*
4.5	0.0184	0.0092*s*	0.7985	0.0074*s*
Total				0.0426*s*

4.0704s+0.0426s = 4.1130s

Credit Spread and Hazard Rate (4)

- $4.1130s = 0.0511$
- $s = 0.0124 = 1.24\%$
- Using the approximation:
  - $s = \lambda(1-R) = 2\%*(1-0.4) = 1.20\%$

Use of Fixed Coupons

- Increasingly CDSs and CDS indices trade like bonds
- A coupon and a recovery rate is specified
- There is an initial payments from the buyer to the seller or vice versa reflecting the difference between the currently quoted spread and the coupon

Credit Indices

- CDX.NA.IG: equally weighted portfolio of 125 Investment Grade North American companies
- CDX.NA.HY: covers 100 High Yield (Non-Investment Grade) NA borrower
- CDX.EM: covers emerging market borrowers
- iTraxx: equally weighted portfolio of 125 investment grade European companies
- Components of the index are rebalanced twice a year. Each new portfolio is given a series number, e.g. CDX.NA.IG.9

CDX.NA.IG Example

- Suppose a trader buys $80,000 protection on each of the 125 companies in the index
- A total of $10M notional
- The current spread is 203bp
- His annual spread payments are:

  0.0203*$10M = $203K
- If a company defaults, the buyer receives 80,000*(1-R), and the notional is reduced by $80K

London Whale - Background

- JP Morgan Synthetic Credit Portfolio is part of its CIO Portfolio
- Constructed as hedge to the bank's long position in credit risk created by its bond and loan portfolio
- In 2011 JPM started selling protection on IG to finance buying protection on HY
- In Nov 2011, made $400M on American Airlines' Bankruptcy
  - It was not hedging any losses in other JP Morgan portfolios.

London Whale - 2012

- In order to reduce its Risk Weighted Assets and its exposure to improving credit conditions, the bank increased selling protection on credit indexes (CDX.NA.IG.9 in particular)
- By end of March the bank was selling protection on $140B (by some estimates) in IG.9, mostly to hedge funds
- In late March, hedge funds stopped buying protection
- Eventually JP Morgan had to close out positions by buying protection at increasing prices

CDX IG CDSI S9 10Y 166.5Y as of close 5/31 CMAN Corp GP
Bid 165.5 Ask 167.5
CDX IG CDSI S9 1
Save Chart
Hide
GP - Line Chart
Page 1/4
Range 02/01/12 - 05/31/12
Upper Market Pric
Mov. Avgs
Currency USD
Period Daily
Lower None
Mov. Avg
Source CMAN
Events
HY Position Losing Money
IG.9 Position making money
Huge losses to close positions
Show Reader
Reached about $140B in sold protection on IG.9
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2012 Bloomberg Finance L.P.
SN 883704 H191-1372-0 01-Jun-12 9:15:28 EDT GMT-4:00

Approximating the loss just in April-May

JPM sold $140B protection at 120bp. What is the MtM of the position if the current spread is 170bp?

Assume defaults occur at end of year.

Derive hazard rate from current spread: Hazard rate = s/(1-R)

Recovery	40%	LGD	60%	
Current Spread	1.70%	Original Spread	1.20%	
Hazard Rate	2.83%	Notional	140	billion

t	PD	Survival	Rates	DF	PV Paying Spread if Survive	PV Paying Spread if Default	PV Spread Payer	PV Protection Seller
1	2.83%	97.17%	5%	0.9512	1.55	0.05	1.60	2.26
2	2.75%	94.41%	5%	0.9048	1.44	0.04	1.48	2.09
3	2.68%	91.74%	5%	0.8607	1.33	0.04	1.37	1.93
4	2.60%	89.14%	5%	0.8187	1.23	0.04	1.26	1.79
5	2.53%	86.61%	5%	0.7788	1.13	0.03	1.17	1.65
Total							6.87	9.73
							Difference:	**-2.86**

Whale - Report Findings

- Actually a failure of Risk Governance
- CIO Management gave conflicting and ambiguous goals, did not understand and properly monitor the portfolio position
- The Group's controls and oversight of CIO did not evolve with the increased complexity and risks of the CIO portfolio
- CIO Risk function understaffed, and some of the CIO risk personnel lacked the needed skills
- Inappropriate MTM to better prices than mid led to Income statement restatements

Whale - Limit Breaches

- The unit operated with three types of limits
  - VaR limit was breached multiple times in January leading to breach in the JP Morgan's overall VaR
    - Breaches were "fixed" by changing the VaR model
  - Non-statistical: CSBPV and CSW 10%
    - Measure the effect of spreads widening; Does not take into consideration correlations
    - Indications that portfolio was turning long on selling protection were ignored
    - Breached in March, but was downplayed as irrelevant measure
  - Stress test limits were also breached but too late

Whale - VaR Model

- The new CIO VaR model was not tested properly, and understated the risks in Q1 2012
- VaR model changed from "linear sensitivities" to "full revaluation",
  - from only considering spread changes, to considering portfolio correlations
- Limited back-testing by model validators due to "lack of historical data"
- Operational issues due to use of manual excel sheets
  - E.g. Excel mistake: divided by sum instead of average
- Made assumption regarding vendor method of calculation, which was wrong

Credit Default Swaps and Bond Yields

- Portfolio A:
  - 5-year par corporate bond that provides a yield of 6%
  - Buy protection using a 5-year CDS costing 100 bps per year
- Portfolio B:
  - long position in a riskless instrument paying 5% per year
- What are arbitrage opportunities if risk-free rate is not 5%, but 4.5%? What if it is 5.5%?
  - Buy bond to get 6% coupon and buy protection for 1%, Short risk free bond to pay 4.5% per year

Risk-free Rate

- The risk-free rate used by bond traders when quoting credit spreads is the Treasury rate
- The risk-free rate traditionally assumed in derivatives markets is the LIBOR/swap rate
- By comparing CDS spreads and bond yields it appears that in normal market conditions traders are assuming a risk-free rate 10 bp less than the LIBOR/swap rates
- In stressed market conditions the gap between the LIBOR/swap rate and the "true" risk-free rate is liable to be much higher

CDS-Bond Basis

- This is the CDS spread minus the Bond Yield Spread
- Bond yield spread to LIBOR is usually calculated as the asset swap spread
  - Asset Swaps allow to convert the bond fixed rate payments to floating (LIBOR) payment
- Tended to be positive pre-crisis
- In general, any feature that would make the insurance guaranteed by CDS more valuable will make the spread increase

Factors Affecting CDS-Bond Basis

- The bond may sell for a price different than par (prices above par decrease basis; below par increase basis)
- Counterparty default risk in a CDS (push basis in negative direction)
- Cheapest-to-deliver bond option in CDS (pushes basis in positive direction)
- Payoff on a CDS does not include accrued interest on the bond delivered (push basis in negative direction)
- Restructuring clause in CDS contract may lead to payoff when there is no default (push basis in positive direction)

Deriving PD from Bond Prices

- Consider a defaultable bond paying 100 in 1-year. The yield on the bond is 8%. A riskless bond with similar promised payment has a yield of 6%.
  – The price of the riskless bond is 100/1.06=94.34
  – The price of the risky bond is 100/1.08=92.59
  – The difference between them, 1.75, is the present value of expected loss from default

Deriving PD from Bond Prices (2)

- We have two ways of pricing the defaultable bond
  - Discount the promised cashflows with the risky yield - 100/1.08=92.59
  - Discount the expected cashflows under the risk neutral measure with the risk free rate
- Suppose the risk-neutral probability of default is Q
- The expected cashflows are

  100*(1-Q)+100*R*Q = 100[1-Q*(1-R)]
- Assume R=0.4, the two methods will give the same result if:

  100[1-Q*(1-R)]/1.06=100/1.08

  Q=[1-1.06/1.08]/(1-0.4) = 3.09%
- The present value of expected loss from default is:
  - 100*PD*(1-R)/(1+rf) = 100*3.09%*(1-0.4)/1.06 = 1.75

Applying the method for longer term bonds

- Suppose that a five year corporate bond pays a coupon of 6% per annum (semiannually). The yield is 7% with continuous compounding and the yield on a similar risk-free bond is 5% (with continuous compounding)
- The expected loss from defaults is 8.75. This can be calculated as the difference between the market price of the bond and its risk-free price (104.09 – 95.34)
- Suppose that the unconditional probability of default is Q per year and that defaults always happen half way through a year (immediately before a coupon payment).

Calculations

Time (yrs)	Def Prob	Recovery Amount	Risk-free Value	Loss	Discount Factor	PV of Exp Loss
0.5	Q	40	106.73	66.73	0.9753	$65.08Q$
1.5	Q	40	105.97	65.97	0.9277	$61.20Q$
2.5	Q	40	105.17	65.17	0.8825	$57.52Q$
3.5	Q	40	104.34	64.34	0.8395	$54.01Q$
4.5	Q	40	103.46	63.46	0.7985	$50.67Q$
Total						$288.48Q$

Calculations continued

- Note:
  - To get Risk Free Value, we need forward rates. In this case, all are assumed to be 5%
  - CDS Pays back Notional Value and recovery is assumed 40% of Par Value
- We set $288.48Q = 8.75$ to get $Q = 3.03\%$
- This analysis can be extended to allow defaults to take place more frequently
- With several bonds we can use more parameters to describe the default probability distribution (rather than Q for all maturities)

Structural Models of Default

- Structural models attempt to explain the mechanism by which default occurs.
- Merton (1974) is the prototype model
  - The firm's assets follow a stochastic process (V_t)
  - The firm has one zero coupon bond issued with face value D, maturing at time T
  - At any time the value of the firm is given by:

$$V_t = E_t + D_t$$

Value of Debt and Equity at Bond Maturity, T

- $V_T > D$: the assets are worth more than the debt. The debt holders receive D. The shareholders receive $E_T = V_T - D$
- $V_T \leq D$: the assets are worth less than the debt. The firm cannot pay its obligations.
  - The equity is worth 0.
  - Shareholders have limited liability.
  - The firm is liquidated and the bond holders receive $D_T = V_T$.

Equity and Debt as Contingent Claims

- The model regards the equity as a European call option on the assets of the firm with the strike price equal to the debt's notional value:

$$E_T = \max(V_T - D, 0)$$

- The bond equals the notional value of the bond minus a European put option with strike price equal to the debt value:

$$D_T = D - \max(D - V_T, 0)$$

Underlying Process for Assets

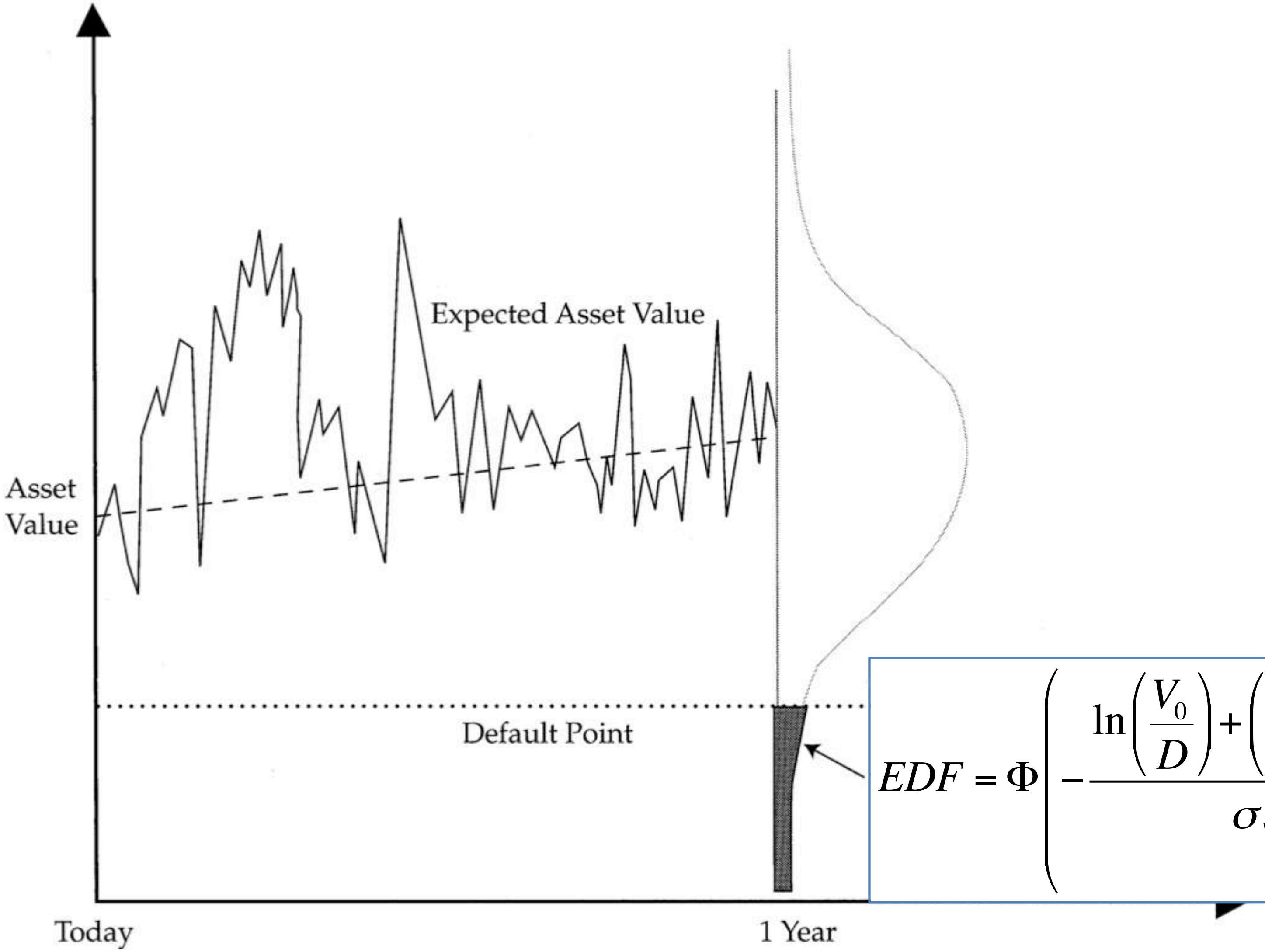
- The assets of the firm follow a Geometric Brownian Motion in the real-world probability measure: $dV_t = \mu_V V_t dt + \sigma_V V_t dW_t$
- Apply Ito's lemma and integrate from 0 to T:

$$\ln V_T \sim N\left(\ln V_0 + \left(\mu_V - \frac{1}{2}\sigma_V^2\right)T, \sigma_V^2 T\right)$$

- The probability of default is:

$$P(V_T \leq D) = P(\ln V_T \leq \ln D) = \Phi\left(\frac{\ln D - \ln V_0 - \left(\mu_V - \frac{1}{2}\sigma_V^2\right)T}{\sigma_V \sqrt{T}}\right)$$

Figure 1. Illustration: Frequency Distribution of Asset Value at Horizon and Probability of Default



$$EDF = \Phi\left(-\frac{\ln\left(\frac{V_0}{D}\right) + \left(\mu_V - \frac{1}{2}\sigma_V^2\right)T}{\sigma_V \sqrt{T}} \right)$$

V_0 = ? , μ_V = ? , σ_V = ?

Kealhofer – FAJ (2003)

Risk Neutral Probability of Default

- The risk neutral probability of default is:

$$\Phi\left(-\frac{\ln\left(\frac{V_0}{D}\right)+\left(r-\frac{1}{2}\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)=N\left(-d_2\right)$$

- RN probability of default is:
  - Decreasing with V_0
  - Increasing with D
  - Increasing in σ_V if $V_0 > D$

Risk Neutral World

- In order to find the probability of default, we have to find V_0 and σ_V.
- We use Black-Scholes to find the value of the firm's equity today , E_0, as a function of the value of its assets today, V_0, and the volatility of its assets, σ_V

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \; ; \quad d_2 = d_1 - \sigma_V \sqrt{T}$$

Applying the model

Since E is a contingent claim on V, we can use Ito's lemma to get:

$$\sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 = N(d_1) \sigma_V V_0$$

This equation together with the option pricing relationship enables V_0 and σ_V to be determined from E_0 and σ_E

Example

- A company's equity is \$3 million and the volatility of the equity is 80%
- The risk-free rate is 5%, the debt is \$10 million and time to debt maturity is 1 year
- Solving the two equations yields V_0=12.40 and σ_v=21.23%

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

$$\sigma_E E_0 = \frac{\partial E}{\partial V}\sigma_V V_0 = N(d_1)\sigma_V V_0$$

Example continued

- The Risk Neutral probability of default is $N(-d_2)$ or 12.7%
- The market value of the debt is V_0-E_0=9.40
- The risk-free present value of the debt is 9.51
- PV of expected losses = 0.11
- The expected loss at T=1 is 0.11*exp(0.05)/10= 1.2%
- (1-Recovery)*12.7% = EL = 1.2%
- The recovery rate is 91%

Distance to Default

- N(-d_2) is the probability of default.
- KMV define Distance to Default as the number of standard deviations the firm value is from the default level
- It is a measure of how unlikely is default
- It is an approximation of d_2 (T=1)

$$DD = \frac{V_0 - D}{V_0 \sigma_V} \sim \frac{\ln(V_0) - \ln(D) + (r - \sigma_V^2 / 2)}{\sigma_V} = d_2$$

The Implementation of Merton's Model to estimate real-world default probability (e.g. Moody's KMV)

- Calculate cumulative obligations to time horizon. We denote it by D
- Use Merton's model to calculate a theoretical probability of default, or Distance to Default
- Use historical data to develop a one-to-one mapping of theoretical probability into real-world probability of default.
- Assumption is that the rank ordering of probability of default given by the model is the same as that for real world probability of default

Real World vs Risk-Neutral Default Probabilities

- The default probabilities backed out of bond prices or credit default swap spreads are risk-neutral default probabilities
- The default probabilities backed out of historical data are real-world default probabilities

A Comparison

- Calculate 7-year hazard rates from the Moody's data (1970-2010). These are real world default probabilities)
- Use Merrill Lynch data (1996-2007) to estimate average 7-year default intensities from bond prices (these are risk-neutral default intensities)
- Assume a risk-free rate equal to the 7-year swap rate minus 10 basis points

Real World vs Risk Neutral Default Probabilities (7 year averages)

Rating	Historical Hazard Rate (% per annum)	Hazard Rate from bonds (% per annum)	Ratio	Difference
Aaa	0.034	0.596	17.3	0.561
Aa	0.098	0.728	7.4	0.630
A	0.233	1.145	5.8	0.912
Baa	0.416	2.126	5.1	1.709
Ba	2.140	4.671	2.2	2.531
B	5.462	8.017	1.5	2.555
Caa	12.016	18.395	1.5	6.379

Risk Premiums Earned By Bond Traders

Rating	Bond Yield Spread over Treasuries (bps)	Spread of risk-free rate used by market over Treasuries (bps)	Spread to compensate for default rate in the real world (bps)	Extra Risk Premium (bps)
Aaa	78	42	2	34
Aa	86	42	6	38
A	111	42	14	55
Baa	169	42	25	102
Ba	322	42	128	152
B	523	42	328	153
Caa	1146	42	721	383

Possible Reasons for These Results

- Corporate bonds are relatively illiquid
- Bonds do not default independently of each other. This leads to systematic risk that cannot be diversified away.
- Bond returns are highly skewed with limited upside. The non-systematic risk is difficult to diversify away and may be priced by the market
- The subjective default probabilities of bond traders may be much higher than the estimates from Moody’s historical data

Which World Should We Use?

- We should use risk-neutral estimates for valuing credit derivatives and estimating the present value of the cost of default
- We should use real world estimates for calculating credit VaR and scenario analysis

Thanks

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Credit VaR – Normal Copula Factor Model

UCLA Anderson
THINK IN THE NEXT

Credit VaR Agenda

- VaR for Independent Loans
- Simulating Dependent Loans
- Copula Factor Model
  - Closed-form solution
  - Extensions through simulation
  - Application to capital requirements

Credit VaR for Uncorrelated Loans

- A bank has 100 loans of $15 million each. The probability of default (PD) of each loan is 2.5%. In case of default there is no recovery. Loan defaults are independent of each other.
- What is the Expected Loss (EL) on the portfolio?
  - EL = 100*15*2.5% = $37.5

Credit VaR for Uncorrelated Loans (Example – Cont.)

- What is the VaR 96% of the portfolio?
  - The probability of K loans defaulting is given by the Binomial Distribution
  - P(K loans default) = BINOMDIST(K, 100,2.5%,0)
  - P(# defaults $\leq K$) = BINOMDIST(K, 100,2.5%,1)
- Excel gives us the following values:
- There is 96% that 5 loans or less will default.

Num Defaults	Cumulative Prob
0	0.08
1	0.28
2	0.54
3	0.76
4	0.89
5	**0.96**
6	0.99

Credit VaR for Uncorrelated Loans (Example – Cont.)

- VaR-96% is therefore: 5*15 = $75M, or 5% of the portfolio value.
- 5% is called Worst Case Default Rate (WCDR)
- The Unexpected Loss VaR 96% is equal to 75-37.5 = 37.5 or 2.5% of the portfolio value.

Binomial to Normal

- The Central Limit Theorem tells us that at the limit the Binomial will tend to Normal.
- Assume a portfolio of *N* equal size loans, with total value of *V*.

$$Loss = \sum_{i=1}^{N} \frac{V}{N} D_i, \qquad D_i = \begin{cases} 0 & 1-p \\ 1 & p \end{cases}$$

$$\mu = E\left[Loss\right] = \frac{V}{N} \cdot Np = Vp$$

$$\sigma^2 = Var\left[Loss\right] = \left(\frac{V}{N}\right)^2 Np\left(1-p\right) = \frac{V^2}{N} p\left(1-p\right)$$

$$\sigma = SD\left[Loss\right] = \frac{V}{\sqrt{N}} \sqrt{p\left(1-p\right)} = \mu\sqrt{\frac{1-p}{Np}}$$

Binomial to Normal - Example

- In our example:

$$\mu = Vp = 1500 \cdot 0.025 = 37.5$$

$$\sigma = \frac{V}{\sqrt{N}} \sqrt{p(1-p)} = \frac{1500}{10} \cdot \sqrt{0.025 \cdot (1-0.025)} = 23.4$$

- What is the 96 percentile?

$$Loss_{96\%} = \mu + \sigma \cdot \Phi^{-1}(0.96) = 78.5$$

- UL $VaR_{96\%}$ is 78.5 – 37.5 = 41

Binomial to Normal - Example

- What would happen if we had the same size portfolio with 1000 loans?

$$\mu = Vp = 1500 \cdot 0.025 = 37.5$$

$$\sigma = \frac{V}{\sqrt{N}}\sqrt{p(1-p)} = \frac{1500}{\sqrt{100}} \cdot \sqrt{0.025 \cdot (1-0.025)} = 7.4$$

- What is the 96 percentile?

$$Loss_{96\%} = \mu + \sigma \cdot \Phi^{-1}(0.96) = 50.5$$

- UL $\text{VaR}_{96\%}$ is 50.5 – 37.5 = 13

Limit of Independent Case

$$\mu = E\left[Loss\right] = \frac{V}{N} \cdot Np = Vp$$

$$Var\left[Loss\right] = \left(\frac{V}{N}\right)^2 Np\left(1-p\right) = \frac{V^2}{N} p\left(1-p\right)$$

$$VaR_{96\%} = \mu \cdot \left[1 + \Phi^{-1}\left(0.96\right) \cdot \sqrt{\frac{1-p}{Np}}\right]$$

- As N increases, the variance of the loss decreases, and eventually we are almost guaranteed Loss=*Vp*.
- The VaR tends to the mean, and the Unexpected Loss VaR goes to zero.

Simulating Defaults

- First, we look at one loan:
- To simulate a loss on one loan with probability of default = PD, we can sample from a uniform, $V_i \sim U[0,1]$, and count as default if $V_i < PD$
- We can alternatively sample from a Normal distribution, $U_i \sim N(0,1)$ and count as default if $U_i < N^{-1}(PD)$

Simulating VaR for Independent Loans

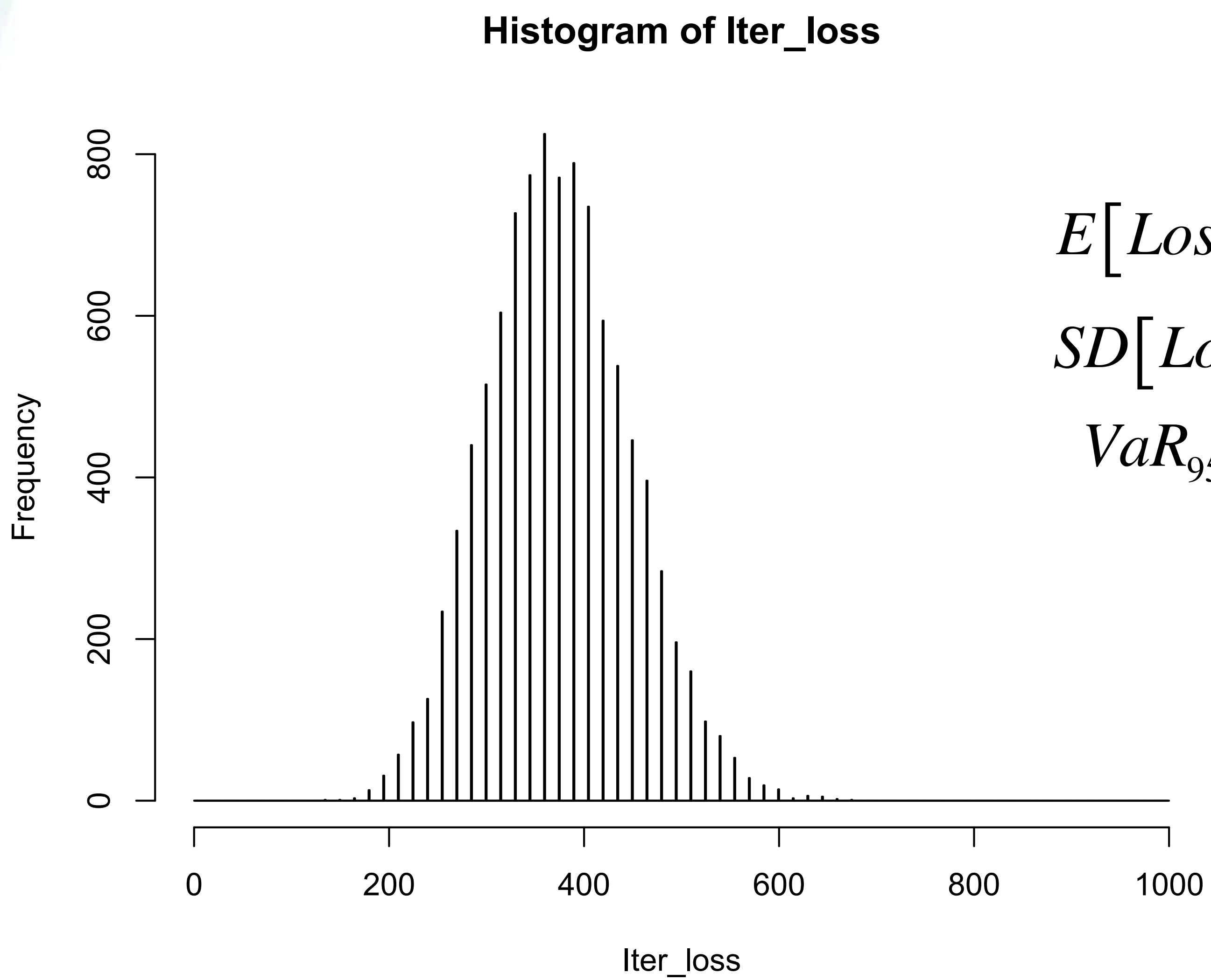
```
Num=1000 #Number of loans
Size=15 #Dollar size of each loan
PD=0.025 #PD for one loan
alpha=0.95 #VaR alpha

N=10000 #Number of iterations
Iter_loss = array (0, dim=c(N)) #Distribution of losses per iteration

for (iter in 1:N) {
        U = matrix(rnorm(Num,mean=0,sd=1), 1, Num) #Generate U_i
        Default = (U<qnorm(PD)) #Every loan, every iteration, did it default
        loan_loss = Default*Size #Total loss on each loan for this iteration (assuming LGD is
100%)
        Iter_loss[iter] = sum(loan_loss) #Total loss for this iteration
}

hist(Iter_loss) #Histogram of losses over iterations
EL = mean(Iter_loss) #Expected loss
VaR = quantile(Iter_loss, alpha) #VaR
```

Independent Loans



$$E\left[Loss\right] = 375$$

$$SD\left[Loss\right] = 74$$

$$VaR_{95\%} = 497$$

Normal Copula Factor Model

- We can generate a set of *N* correlated variables with standard normal distribution using a Factor Model.
- We generate *N+1* independent standard normal variables: the common factor F, and *N* idiosyncratic components Z_i
- Generate new variables, U_i as:

$$U_i = a_i F + \sqrt{1 - a_i^2}\, Z_i$$

- They have standard normal distributions and correlation between U_i and U_j is $a_i\, a_j$ – show this.

Normal Copula Factor Model

- We consider a case where all $a_i = \sqrt{\rho}$

$$U_i = \sqrt{\rho}F + \sqrt{1-\rho}Z_i$$

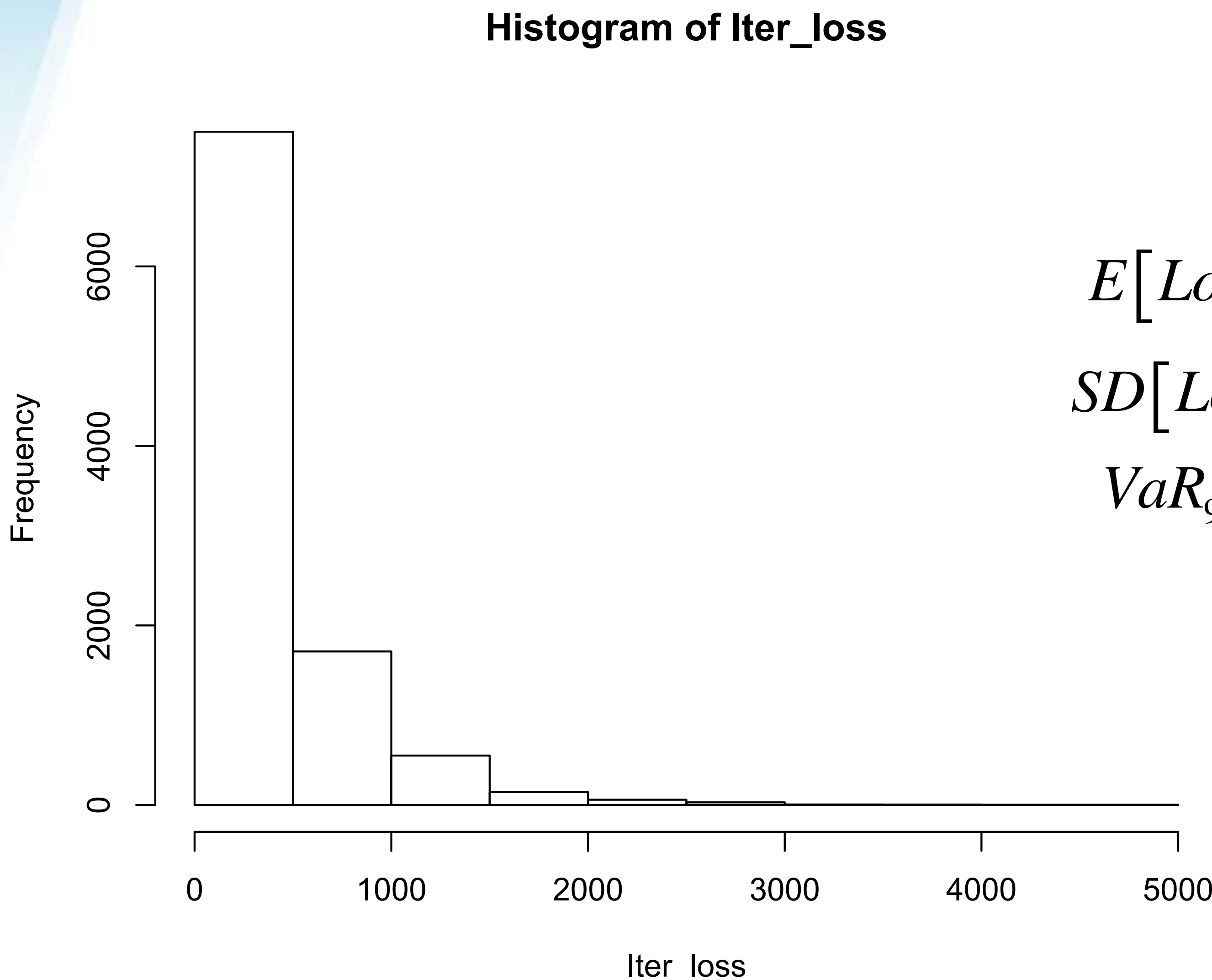
- U_i is also distributed normally: $U_i \sim N(0,1)$
- The correlation between every two latent variables (U_i) is ρ
- Count as default if

$$U_i = \sqrt{\rho}F + \sqrt{1-\rho}Z_i < N^{-1}(PD)$$

Simulating VaR for Correlated Loans

```
Num=1000 #Number of loans
Size=15 #Dollar size of each loan
PD=0.025 #PD for one loan
rho = 0.15 #Correlation between latent variables
alpha=0.95 #VaR alpha
N=10000 #Number of iterations
Iter_loss = array (0, dim=c(N)) #Distribution of losses per iteration
for (iter in 1:N) {
        F = matrix(rnorm(1),1,Num) #One F Factor value per iteration
        Z = matrix(rnorm(Num,mean=0,sd=1), 1, Num) #idiosyncratic errors for all loans
        U = sqrt(rho)*F + sqrt(1-rho)*Z #Generate the U_i for all loans
        Default = (U<qnorm(PD)) #Every loan, every iteration, did it default
        loan_loss = Default*Size #Total loss on each loan for this iteration (assuming LGD is 100%)
        Iter_loss[iter] = sum(loan_loss) #Total loss for this iteration
}
hist(Iter_loss) #Histogram of losses over iterations
EL = mean(Iter_loss) #Expected loss
VaR = quantile(Iter_loss, alpha) #VaR
```

Correlated Loans



$$E[Loss] = 375$$

$$SD[Loss] = 394$$

$$VaR_{95\%} = 1140$$

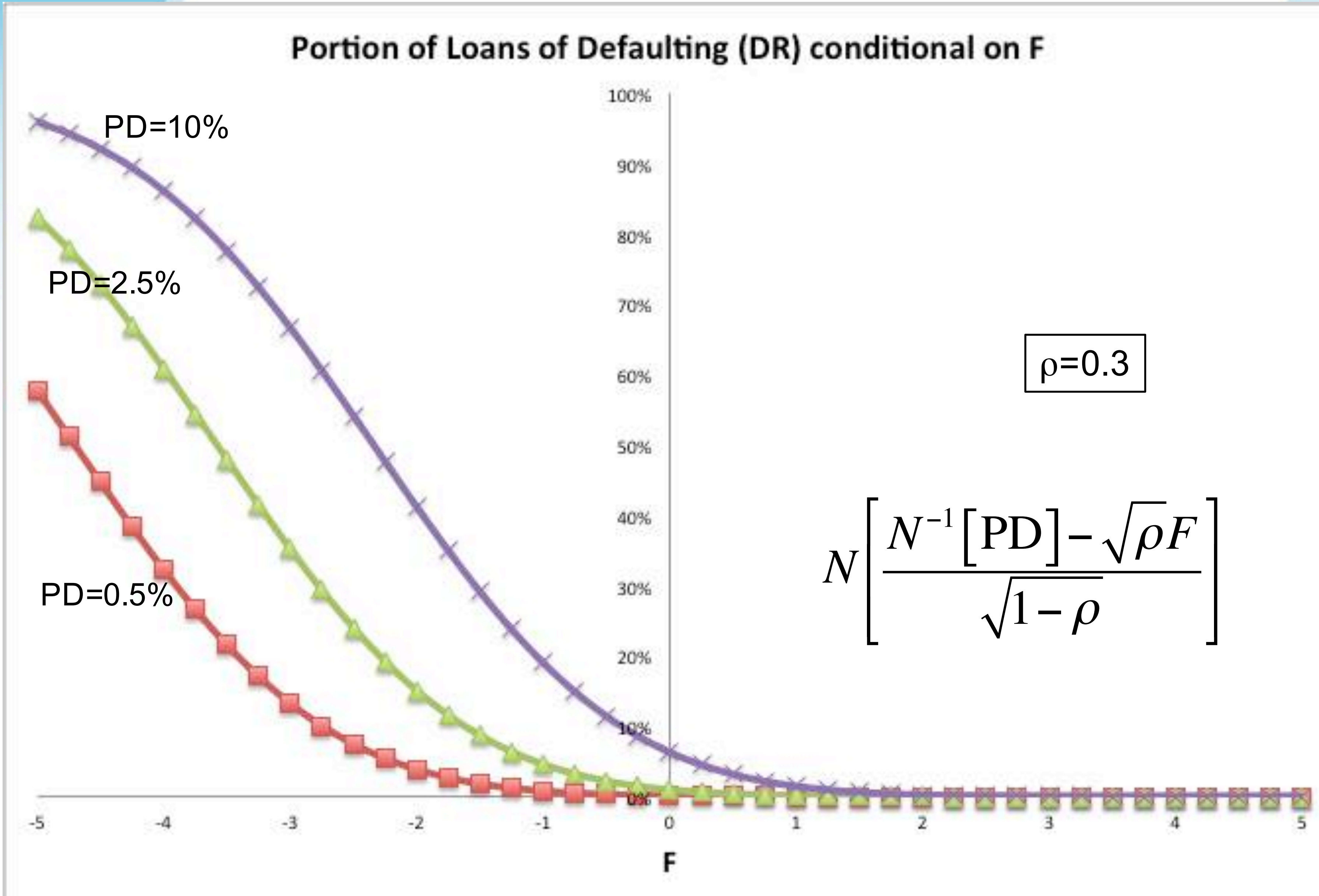
Closed Form Solution

- If we assume all loans are of same size, L, with same PD and same LGD we can reach a closed form solution
- For a given F, rewrite the condition of default as:

$$Z_i < \frac{N^{-1}(PD) - \sqrt{\rho}F}{\sqrt{1-\rho}}$$

- Given that Z_i is standard normal, the portion of loans that default conditional on F is:

$$N\left[\frac{N^{-1}\left[\mathrm{PD}\right] - \sqrt{\rho}F}{\sqrt{1-\rho}}\right]$$

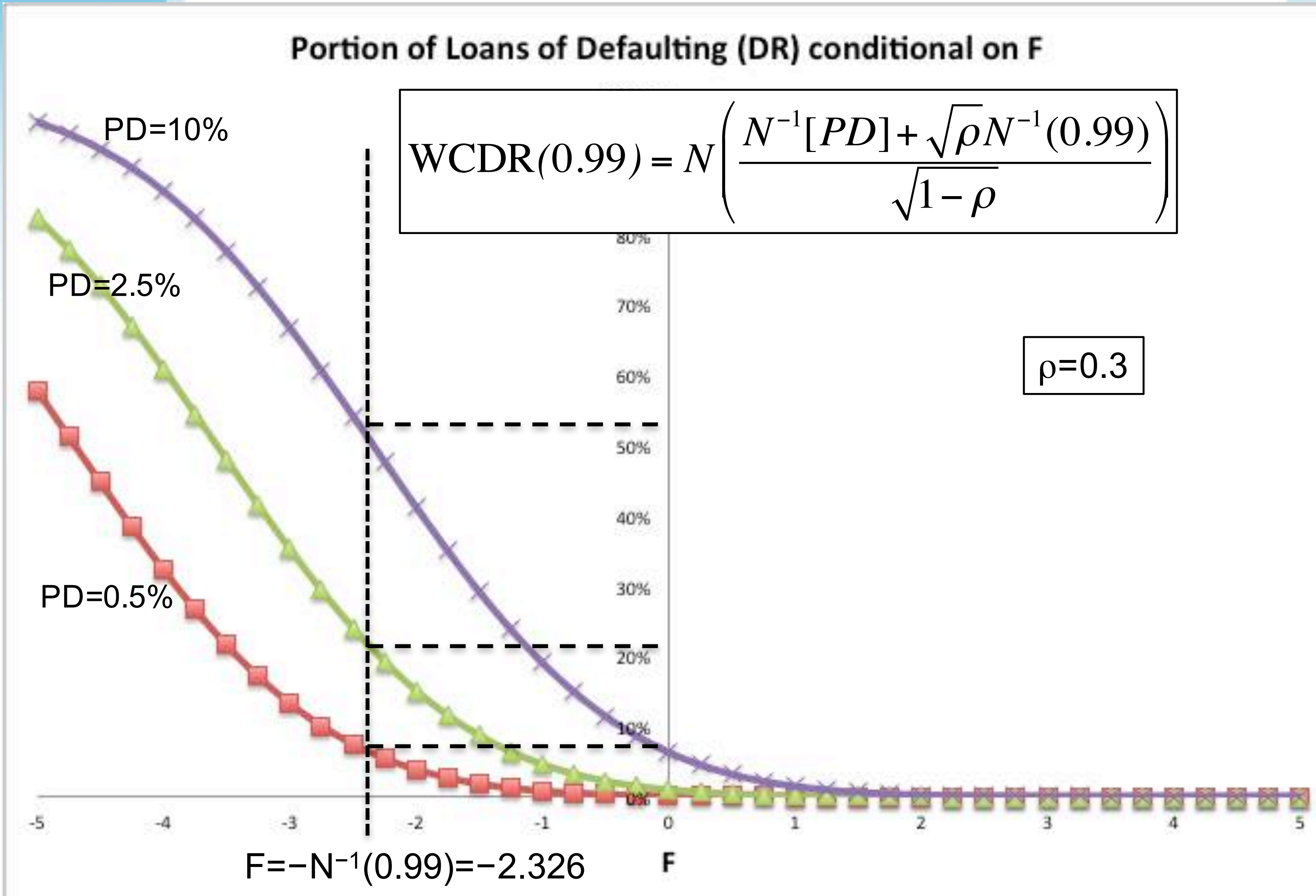


Portion of Loans of Defaulting (DR) conditional on F
PD=10%
PD=2.5%
PD=0.5%
ρ=0.3
$$N\left[\frac{N^{-1}[\mathrm{PD}]-\sqrt{\rho}F}{\sqrt{1-\rho}}\right]$$
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
-5
-4
-3
-2
-1
0
1
2
3
4
5
F

Closed Form Solution cont.

- F is the state variable (e.g. condition of the economy), when it's high the probability of default is low.
- The *X*% worst case is when *F* is $N^{-1}(1-X)=-N^{-1}(X)$
- For example, 99% worst case is when F=$N^{-1}(0.01)=-N^{-1}(0.99)$
- The worst case default rate with a confidence level of *X* is therefore:

$$\mathrm{WCDR}(X) = N\left(\frac{N^{-1}[PD]+\sqrt{\rho}N^{-1}(X)}{\sqrt{1-\rho}}\right)$$


Portion of Loans of Defaulting (DR) conditional on F
PD=10%
PD=2.5%
PD=0.5%
$$\mathrm{WCDR}(0.99) = N\left(\frac{N^{-1}[PD] + \sqrt{\rho}N^{-1}(0.99)}{\sqrt{1-\rho}}\right)$$
ρ=0.3
80%
70%
60%
50%
40%
30%
20%
10%
0%
-5
-4
-3
-2
-1
0
1
2
3
4
5
F
$F = -N^{-1}(0.99) = -2.326$

Credit VaR Formula

- The Unexpected Loss (UL) Credit VaR in Dollars is:

$$Credit\,\mathrm{VaR}(X) = L \times \mathrm{LGD} \times \left[\mathrm{WCDR}(X) - PD\right]$$

where *L* is loan principal and LGD is loss given default, *WCDR* based on previous formula.

Credit VaR Example

- A bank has a total of $100 million of loans, each exposure is small in relation to the total portfolio. The one-year probability of default (PD) for each loan is 2% and the loss given default (LGD) for each loan is 40%. The copula correlation parameter ρ is 0.1.
  - What is Worst Case Default Rate (WCDR) at 99.9%?
  - What is Unexpected Loss $\text{VaR}_{99\%}$?

Credit VaR Example

$$\text{WCDR}(0.999) = N\left(\frac{N^{-1}(0.02) + \sqrt{0.1}N^{-1}(0.999)}{\sqrt{1-0.1}}\right) = 0.128$$

$$\mathit{Credit}\,\text{VaR}(0.999) = 100 \times 0.4 \times \left[12.8\% - 2\%\right] = 4.32$$

Gordy's Result

- In a large portfolio of *M* loans where each loan is small in relation to the size of the portfolio it is approximately true that

$$Credit\mathrm{VaR}(X) = \sum_{i=1}^{M} L_i \times \mathrm{LGD}_i \times \left[\mathrm{WCDR}_i(X) - PD_i \right]$$

- Note that: loan size, probability of default and loss given default can vary between loans.

RBS Asset Protection Scheme

In 2009, Royal Bank of Scotland (RBS) was bailed out by the UK government using an Asset Protection Scheme (APS).

$325B of the Bank's assets (i.e. loans and bonds) were placed in the scheme. RBS would be liable for the first $19.5B of losses on the portfolio, and the government would be liable for the rest. Assume every asset is a small part of the portfolio.

Suppose the Probability of Default (PD) of each asset is 1% and the Loss Given Default (LGD) is 100%. The copula correlation is 0.4. What is the probability that the government will have to pay anything?

RBS Asset Protection Scheme

Call *x* the probability of losing \$19.5B or less on the portfolio. We are looking for *1-x*:

$$19.5 = L * LGD * N\left[\frac{N^{-1}(PD) + \sqrt{\rho}N^{-1}(x)}{\sqrt{1-\rho}}\right]$$

$$19.5 = 325 * 100\% * N\left[\frac{N^{-1}(1\%) + \sqrt{0.4}N^{-1}(x)}{\sqrt{0.6}}\right]$$

$$x = N\left[\frac{\sqrt{0.6}N^{-1}(0.06) - N^{-1}(0.01)}{\sqrt{0.4}}\right] = 96.2\%$$

$$1 - x = 3.8\%$$

RBS Asset Protection Scheme

What if the loan defaults were independent of each other, and there were 1000 loans?

$$Loss \sim N\left[Vp, \frac{V^2}{N} p(1-p)\right]$$

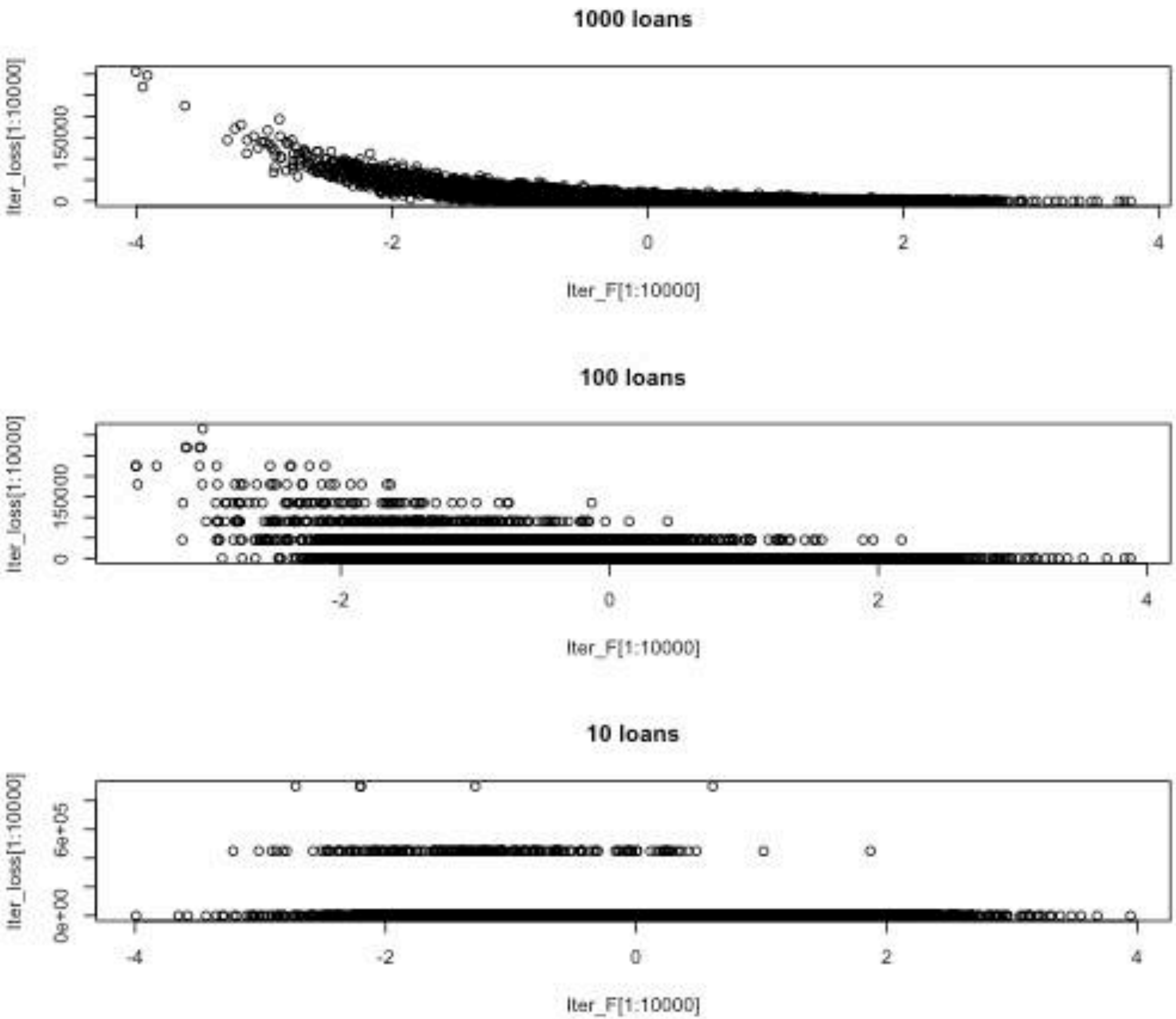
$$\mu = 3.25, \sigma^2 = 1.04$$

$$P[Loss > 19.5] = P\left[z > \frac{19.5 - 3.25}{\sqrt{1.04}}\right] = 0$$

If the loans were independent there would be almost no chance of the government paying out!

What does simulation get us?

- Using simulation we can address specific features of the portfolio
  - Size Concentration
  - Industry/Sector Concentration
- Mark to Market VaR vs. Default VaR
- LGD Simulation
- Expected Shortfall


1000 loans
Iter_loss[1:10000]
0
150000
-4
-2
0
2
4
Iter_F[1:10000]
100 loans
Iter_loss[1:10000]
0
150000
-2
0
2
4
Iter_F[1:10000]
10 loans
Iter_loss[1:10000]
0e+00
6e+05
-4
-2
0
2
4
Iter_F[1:10000]

Industry/Sector

- Use multiple factors instead of one
- For example, 2 factors will lead to every loan variable being generated as:

$$U_i = w_1 F_1 + w_2 F_2 + w_z Z_i$$

- Common choice for factors are equity indices
  - Credit Metrics uses MSCI indexes
- F_1 and F_2 are correlated Normal variables, with mean zero. Z_i are independent of Fs and other Z_i.

Sector Concentration - Example

- Estimate the correlation matrix for the factors.

Index	Volatility	Correlations		
		U.S. Chemicals	Germany Insurance	Germany Banking
U.S.: Chemicals	2.03%	1.00	0.16	0.08
Germany: Insurance	2.09%	0.16	1.00	0.34
Germany: Banking	1.25%	0.08	0.34	1.00

- Regress equity returns on the factors to get betas

$$R_{XYZ} = \alpha + \beta_1 R_{F_1} + \beta_2 R_{F_2} + \varepsilon_{XYZ}$$

- σ^2_{XYZ} is the total variance, R^2_{XYZ} out of it is due to the factors, while $1\text{-}R^2_{XYZ}$ is due to idiosyncratic risk.

Sector Concentration - Example

- Simulate factors as correlated mean-zero Normals and simulate idiosyncratic as uncorrelated

$$U_{XYZ} = \frac{\beta_1}{\sigma_{XYZ}} F_1 + \frac{\beta_2}{\sigma_{XYZ}} F_2 + \sqrt{1 - R^2_{XYZ}} \cdot Z_{XYZ}$$

- F_1, F_2 for example are indexes: *Germany:Banking* and *Germany:Insurance*
- Coefficients are set so variance due to Factors is R^2_{XYZ}. Leading to U_{XYZ} being standard normal.
- As before, count as default if $U_i < N^{-1}(PD)$

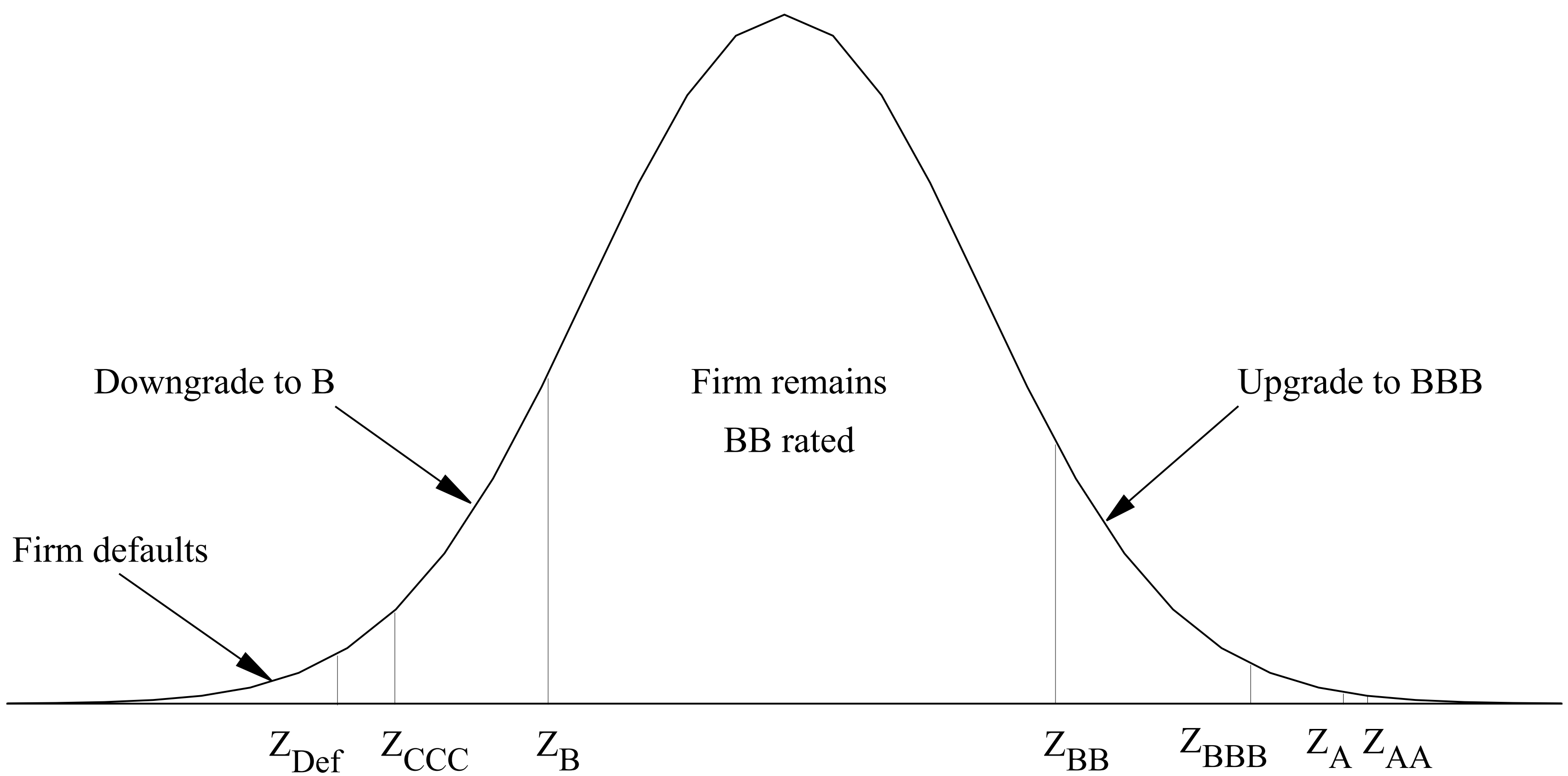
Mark-to-Market VaR

- So far, we modeled losses due to default only.
- At the end of the period some credits may be downgraded or upgraded. This will affect their price.
- Bondholders may be concerned with the MTM of the bond at the end of the period, rather than loss to default.
- In fact, loan holders too are interested in changes in the ratings, because the model is quantifying one period losses, whereas the loan maturities are longer.

One-Year Rating Transition Matrix (% probability, Moody's 1970-2010)

Initial Rating	Rating at year end Aaa	Aa	A	Baa	Ba	B	Caa	Ca-C	Default
Aaa	90.42	8.92	0.62	0.01	0.03	0.00	0.00	0.00	0.00
Aa	1.02	90.12	8.38	0.38	0.05	0.02	0.01	0.00	0.02
A	0.06	2.82	90.88	5.52	0.51	0.11	0.03	0.01	0.06
Baa	0.05	0.19	4.79	89.41	4.35	0.82	0.18	0.02	0.19
Ba	0.01	0.06	0.41	6.22	83.43	7.97	0.59	0.09	1.22
B	0.01	0.04	0.14	0.38	5.32	82.19	6.45	0.74	4.73
Caa	0.00	0.02	0.02	0.16	0.53	9.41	68.43	4.67	16.76
Ca-C	0.00	0.00	0.00	0.00	0.39	2.85	10.66	43.54	42.56
Default	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Simulating Transitions



CreditMetrics Technical Document 1997

Credit Metrics MTM VaR

- Simulate for every loan: $U_i = \sqrt{\rho} F + \sqrt{1-\rho} Z_i$
- Determine the loan's new ratings based on the standard normal cut-off points
- Re-price the loan with the credit spreads for the new rating
  - Forwards for interest rates and credit spreads have to be used
- Sum up all loan values for one possible portfolio MTM at the end of period.

MTM Example for One Bond

- Consider a 5-year, A-rated bond, paying 6% coupon.
- Simulate U_i a standard Normal, and determine the new ratings based on the transition matrix:

End of Year Rating	Probability (%)	Cum Prob (%)	lower bound	upper bound
Aaa	0.06	100	3.239	∞
Aa	2.82	99.94	1.899	3.239
A	90.88	97.12	-1.535	1.899
Baa	5.52	6.24	-2.447	-1.535
Ba	0.51	0.72	-2.863	-2.447
B	0.11	0.21	-3.090	-2.863
Caa	0.03	0.1	-3.195	-3.090
Ca-C	0.01	0.07	-3.239	-3.195
D	0.06	0.06	$-\infty$	-3.239

MTM Example Cont.

- Use the forward curves for each rating to revalue the bond:

Example one-year forward zero curves by credit rating category (%)

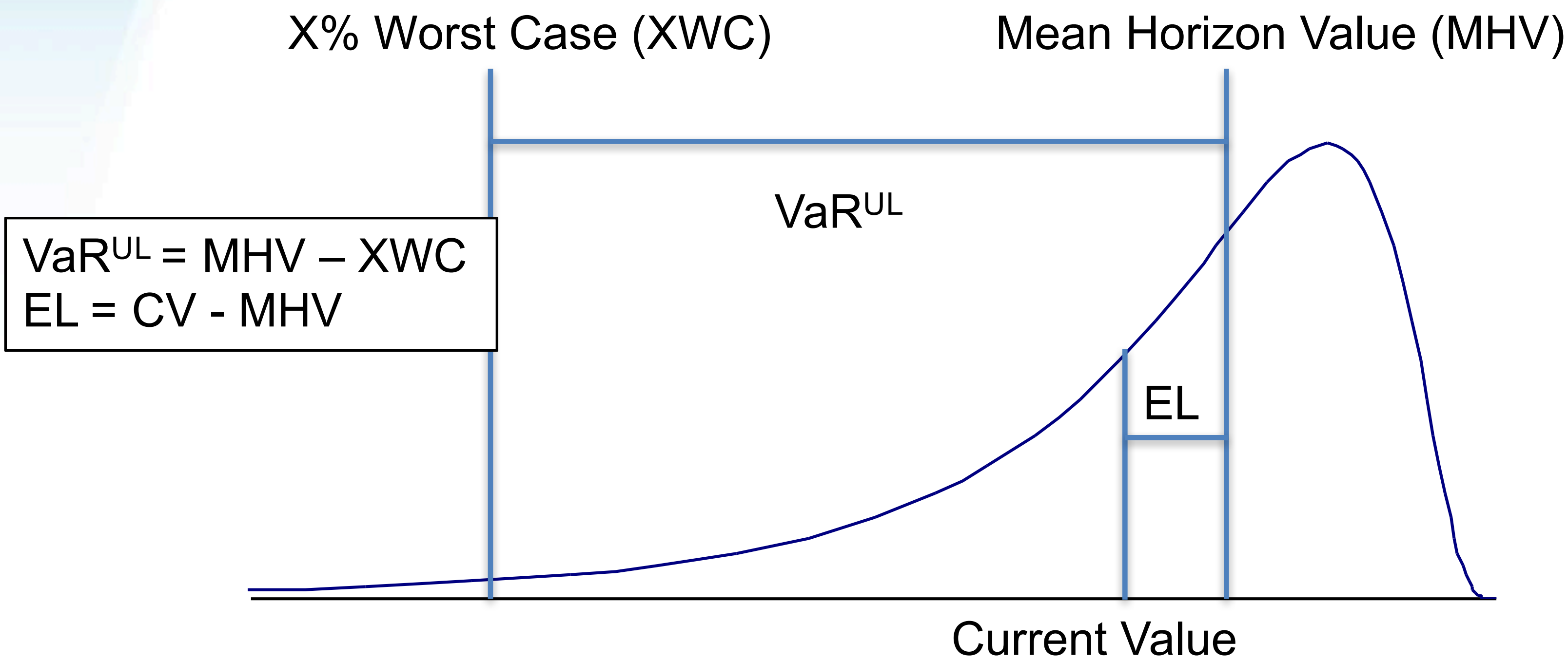
Category	Year 1	Year 2	Year 3	Year 4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

- For example, if it stays A

$$Value = 6 + \frac{6}{1+3.72\%} + \frac{6}{(1+4.32\%)^2} + \frac{6}{(1+4.93\%)^3} \frac{106}{(1+5.32\%)^4}$$

Year-end rating	Value ($)
AAA	109.37
AA	109.19
A	108.66
BBB	107.55
BB	102.02
B	98.10
CCC	83.64
Default	51.13

MTM VaR (cont)



Sum all bond values to create an end of period distribution for portfolio value.

Credit Metrics Incorporating LGD

Recovery statistics by seniority class

Par (face value) is $100.00.

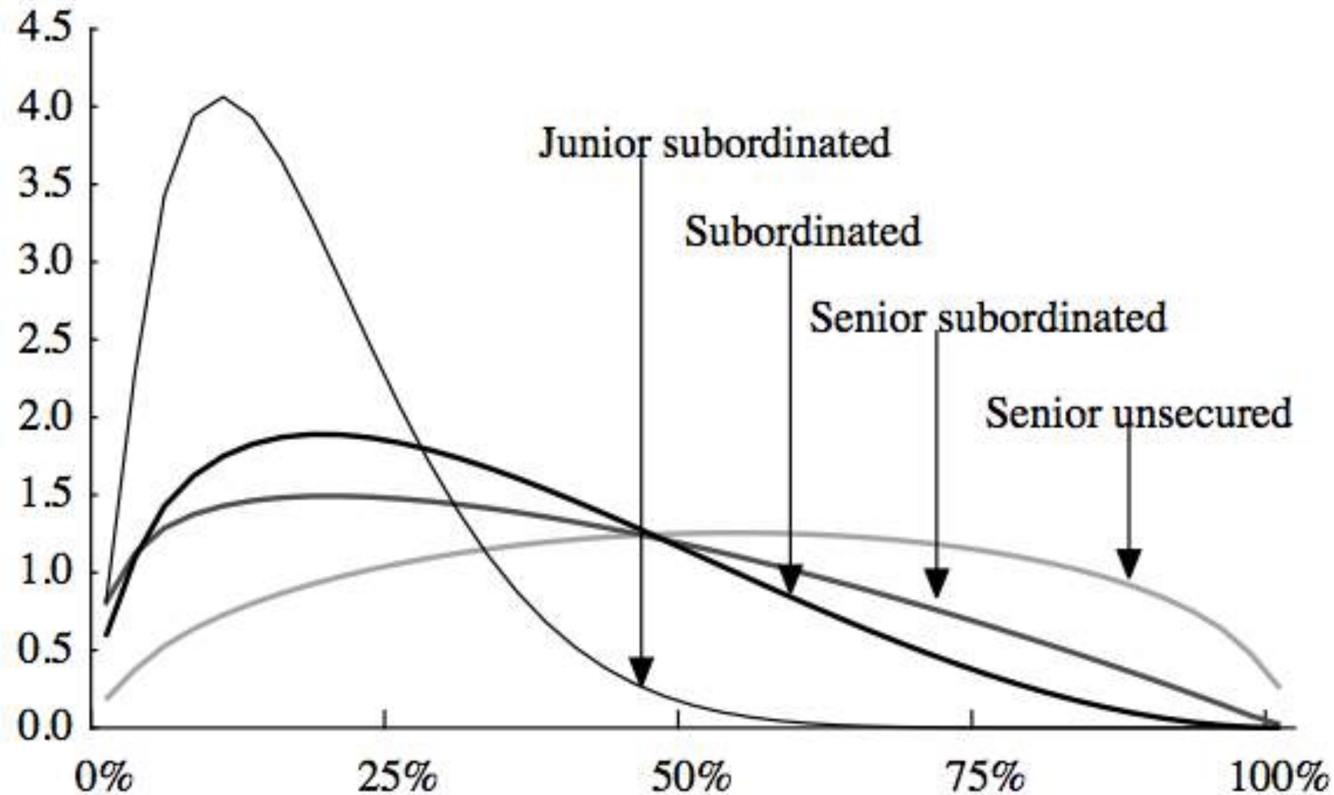
	Carty & Lieberman [96a]			Altman & Kishore [96]		
Seniority Class	**Number**	**Average**	**Std. Dev.**	**Number**	**Average**	**Std. Dev.**
Senior Secured	115	$53.80	$26.86	85	$57.89	$22.99
Senior Unsecured	278	$51.13	$25.45	221	$47.65	$26.71
Senior Subordinated	196	$38.52	$23.81	177	$34.38	$25.08
Subordinated	226	$32.74	$20.18	214	$31.34	$22.42
Junior Subordinated	9	$17.09	$10.90	—	—	—

- Recovery rates have wide variation
- Low recovery rates are correlated with high PD over time and industry

CreditMetrics Technical Document 1997

Incorporating LGD cont.

- Credit Metrics samples Recovery from Beta distribution
  - bounded between 0 and 1.
  - fitted based on empirical studies.
- More complex simulations can correlate Recovery to PDs



Basel II – Internal Rating Based Approach

Capital requirement is based on 99.9% worst case default rate using Normal copula:

$$WCDR = N\left[\frac{N^{-1}(PD) + \sqrt{\rho} \times N^{-1}(0.999)}{\sqrt{1-\rho}}\right]$$

✕– correlation between two exposures

✕- depends on PD and the type of exposure (corporate, SMB, retail, mortgage)

Capital Requirements

$$\text{Capital} = \text{EAD} \times \text{LGD} \times (\text{WCDR} - \text{PD}) \times \text{MA}$$

$$\text{where MA} = \frac{1 + (\text{M} - 2.5) \times b}{1 - 1.5 \times b}$$

M is the effective maturity and

$$b = [0.11852 - 0.05478 \times \ln(PD)]^2$$

The risk - weighted assets are 12.5 times the Capital so that Capital = 8% of RWA

Capital Requirements

- Requirements are calculated exposure by exposure and summed up
- *MA* formula was approximated by comparing a MTM simulation to Default only case
- Portfolio features like size concentration or industry concentration are NOT taken into account
  - Banks use internal Credit VaR models to approximate the additional capital required for these features
  - This process is called ICAAP

Thanks

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Banking Regulation

UCLA Anderson
THINK IN THE NEXT

Agenda

- Banking Basics
- History of Bank Failures and Regulation
- Basel Framework
- Risk Based Capital (Risk Weighted Assets)
- Internal Ratings and Internal Model Approaches
- Basel 3 and Post-Crisis Updates
- Liquidity Ratios
- Dodd Frank Act

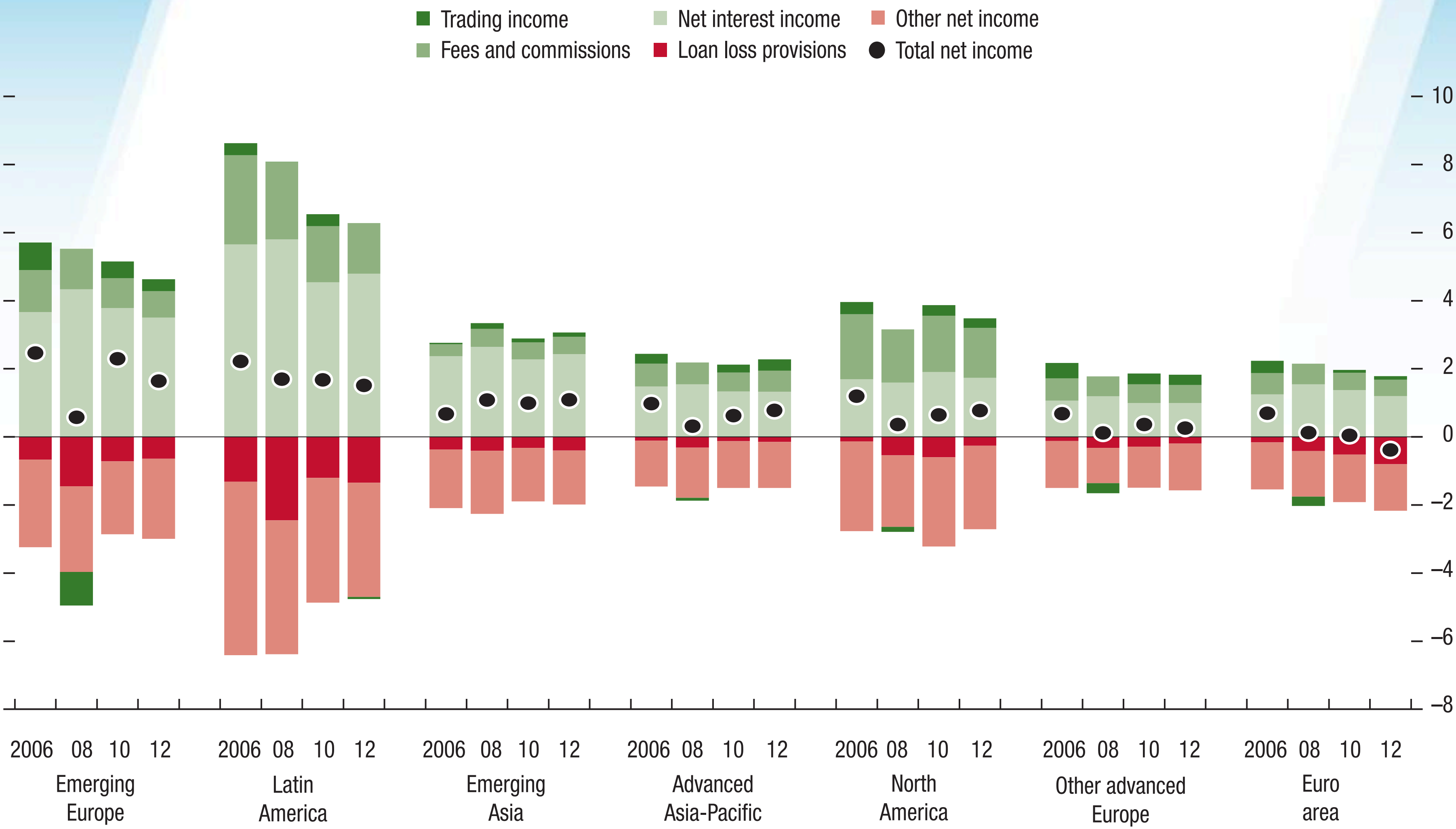
Bank Balance Sheet

Assets	Liabilities and Equity
Cash	Deposits
Securities	Long Term Debt
Loans	
Trading Assets	Trading Liabilities
	Shares
	Retained Earnings

Income Statement
+ Net Interest Income (Interest Income – Interest Expense)
+ Non-Interest Revenue (Fees, Commissions, Trading Revenue)
– Non-Interest Expenses (Compensation, Technology, Marketing)
– Provision for Loan Losses
EBT

Pre-Provision Net Revenue (PPNR)

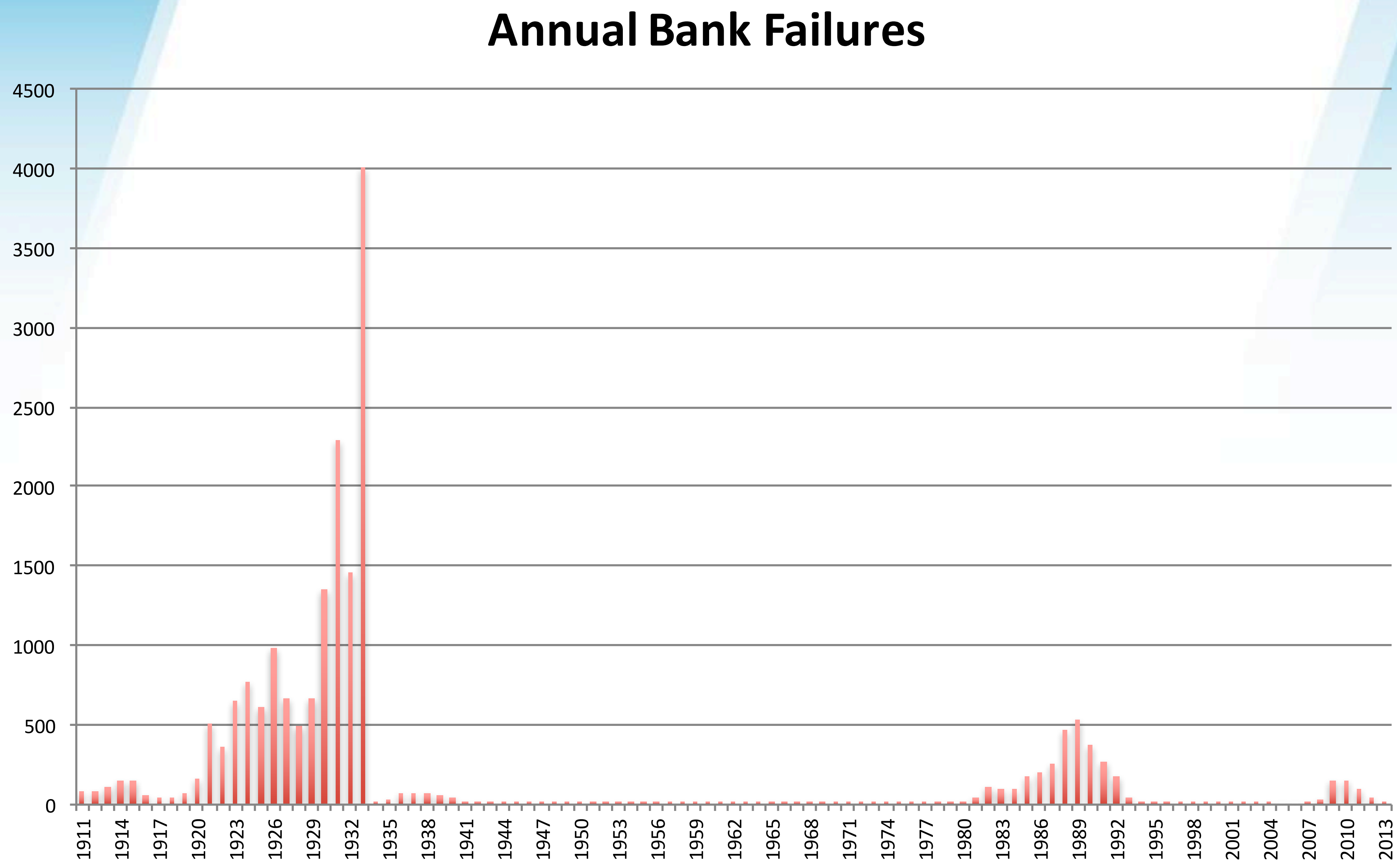
Figure 1.57. Bank Profitability Comparison
(Percent of total assets)



IMF Global Stability Report, October 2013

Bank Runs and Lender of Last Resort

- During Late 19th-early 20th Century, banking crises struck US quite often
  - characterized by runs on many small banks
  - private "clearing houses" sometimes came together to save liquidity-strained banks
- The Federal Reserve System was created in 1913 following the panic of 1907
- The original charter was to counteract at times of crisis through monetary policy and as lender of last resort



Source: "Historical Statistics of the United States: Colonial Times to 1970" and FDIC Website.

Depression Era Regulation

- Federal government increased supervisory role in 1933, leading to 50 years of no crises
- Glass-Steagall Act (1933) introduced deposit insurance, expanded supervision, and separation of commercial and investment banking
- Increased regulation and supervision were tools to counter “moral hazard”

Deposit Insurance

- Most countries have deposit insurance programs that insure depositors against losses up to a certain level
- In the US the FDIC has provided protection for depositors since 1933
- FDIC manages the Deposit Insurance Fund, which assesses fees on participating banks based on risk level and bank size.
- The amount insured was $2,500 in 1933. It has been increased several times.
- Following the recent crisis it was increased from $100,000 to $250,000 temporarily, and was made permanent with Dodd-Frank Act .
- Deposit insurance lowers bank costs of finance significantly, but creates a “moral hazard” problem.

Bank Regulation in US Since 1933

- Interstate banking and consolidation:
  - Until the 70s, regulators tried to limit interstate banking: Douglas Amendment (1956), Bank Holding Companies Act (1970). Regulations served to prevent “race to the bottom” competition, but limited monopolistic rents
  - Since the 70s interstate banking restrictions disappear until 90s, when major consolidation begins: Riegel-Neal Interstate Banking and Branching Efficiency Act (1994).
- Gramm-Leach-Bliley (1999) – Repealed Glass-Steagall

Assessment of Financial Condition

- There are various models for assessing a financial institution's condition and its viability
- Most are derived from FDIC Uniform FI Ratings System, AKA "CAMELS"
  - Capital Adequacy
  - Asset Quality
  - Management
  - Earnings
  - Liquidity
  - Sensitivity to Market Risk
- Each aspect is analyzed using current and past trends, expected performance and stress case scenarios

History of Internationally Coordinated Bank Regulation

- Pre-1988
- 1988: BIS Accord (Basel I)
- 1996: Amendment to BIS Accord
- 1999: Basel II first proposed
- 2004-2009 : Basel II implementation
- 2011: Basel 2.5
- 2013 - 2019: Basel III

Pre-Basel

- Banks were regulated using balance sheet measures such as the ratio of capital to assets
- Definitions and required ratios varied from country to country
- Enforcement of regulations varied from country to country
- Bank leverage increased in 1980s
- Off-balance sheet derivatives trading increased

1988: Basel Accord

- Basel Committee on Bank Supervision set up at the Bank International Settlements (BIS)
- Focus on regulation of leverage, or capital adequacy, as measure of risk taking
- Leverage Ratio: The assets to capital ratio must be less than 20.
- New Risk Based Capital Regulations.

Types of Capital

- **Tier 1 Capital**: common equity, non-cumulative perpetual preferred shares
- **Tier 2 Capital:** cumulative preferred stock, certain types of 99-year debentures, subordinated debt with an original life of more than 5 years

Risk-Weighted Capital in Basel I

- A risk weight is applied to each on-balance- sheet asset according to its risk. For example,
  - 0% for cash and government bonds
  - 20% for claims on OECD banks
  - 50% to residential mortgages
  - 100% to corporate loans, corporate bonds
- For each off-balance-sheet item we first calculate a credit equivalent amount and then apply a risk weight
- Cooke Ratio: Capital must be 8% of risk weighted amount. At least 50% of capital must be Tier 1.

Summing Up RWA

$$RWA = \sum_{i=1}^{N} w_i L_i + \sum_{j=1}^{M} w_j^* C_j$$

On-balance sheet items: principal times risk weight

Off-balance sheet items: credit equivalent amount times risk weight

For a derivative $C_j = \max(V_j, 0) + a_j L_j$ where V_j is value, L_j is principal and a_j is add-on factor

Credit Equivalent Amount

- The credit equivalent amount is calculated as the current replacement cost (if positive) plus an add-on factor
- The add-on amount varies from instrument to instrument (e.g. 0.5% for a 1-5 year interest rate swap; 5.0% for a 1-5 year foreign currency swap)
- Captures what might be the Exposure At Default
  - If market variables moves in a way which increases the replacement cost for derivatives
  - If certain letters of credits or lending commitments are drawn down

Add-on Factors for Derivatives (% of Principal)

Remaining Maturity (yrs)	Interest rate	Exch Rate and Gold	Equity	Precious Metals except gold	Other Commodities
<1	0.0	1.0	6.0	7.0	10.0
1 to 5	0.5	5.0	8.0	7.0	12.0
>5	1.5	7.5	10.0	6.0	15.0

Example: A $100 million swap with 3 years to maturity worth $5 million would have a credit equivalent amount of $5.5 million

The Market Risk Capital - 1996

- The capital requirement is

$$\max(\text{VaR}_{t\text{-}1}, m_c \times \text{VaR}_{\text{avg}}) + \text{SRC}$$

where m_c is a multiplicative factor chosen by regulators (at least 3), VaR is the 99% 10-day value at risk, and SRC is the specific risk charge for idiosyncratic risk related to specific companies. $\text{VaR}_{t\text{-}1}$ is the most recently calculated VaR and VaR_{avg} is the average VaR over the last 60 days

Basel II

- Implemented since 2007 at different rates across the globe
  - USA for most part skipped from Basel 1 to Basel 3
- Three pillars:
  I. New minimum capital requirements for credit and operational risk
  II. Supervisory review: guidelines for regulators how to supervise banks under their jurisdiction, and for banks how to manage risk
  III. Market discipline: significantly more disclosure regarding risk and risk management

New Capital Requirements

- Risk weights based on either external credit rating (standardized approach) or a bank's own internal credit ratings (IRB approach)
- Recognition of credit risk mitigants
- Separate capital charge for operational risk

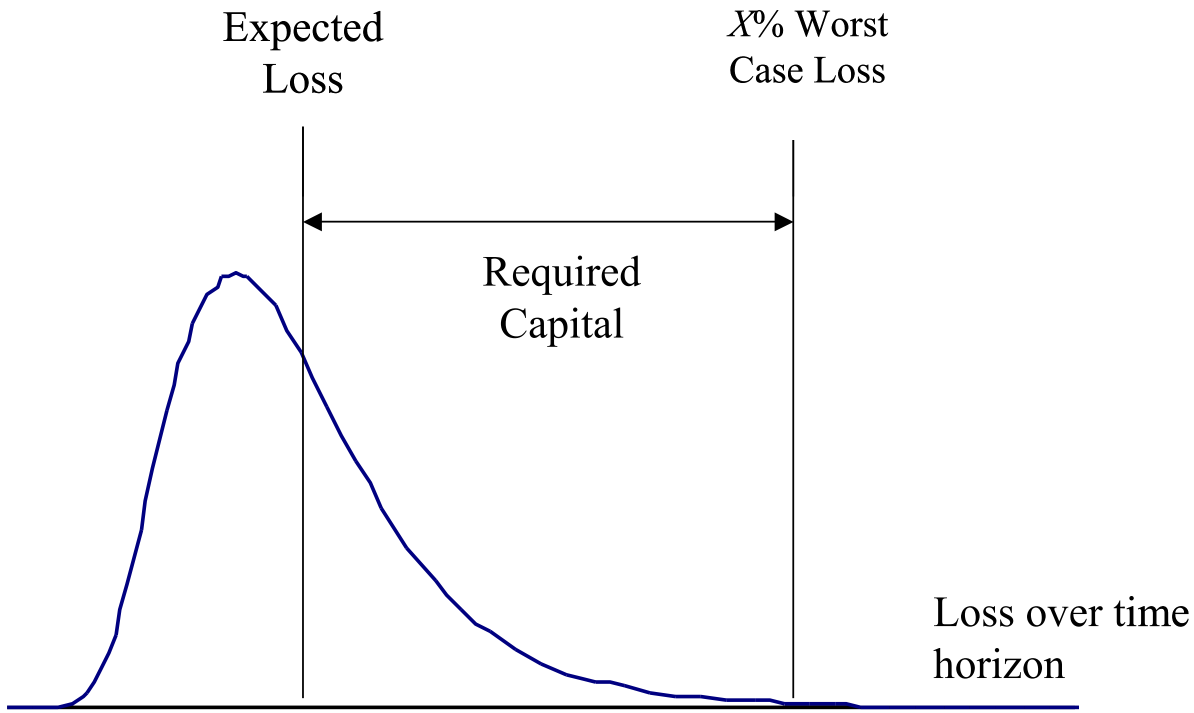
RWA - Standardized Approach

Rating	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to BB-	B+ to B-	Below B-	Unrated
Governments	0%	20%	50%	100%	100%	150%	100%
Banks	20%	50%	50%	100%	100%	150%	50%
Corporates	20%	50%	100%	100%	150%	150%	100%

Internal Ratings Based Approach (IRBA)

- Basel II provides a formula for translating PD (probability of default), LGD (loss given default), EAD (exposure at default), and M (effective maturity) into a risk weight
- Under the Advanced IRB approach banks estimate PD, LGD, EAD, and M
- Under the Foundation IRB approach banks estimate only PD and the Basel II guidelines determine the other variables for the formula

Capital Required is the Unexpected Loss at 99.9%



Key Model (Gaussian Copula)

The 99.9% worst case default rate is

$$WCDR = N\left[\frac{N^{-1}(PD)+\sqrt{\rho}\times N^{-1}(0.999)}{\sqrt{1-\rho}}\right]$$

✕– correlation between two exposures

Based on formula given by Basel, which depends on PD and the type of exposure (corporate, SMB, retail, mortgage)

Capital Requirements for a Loan under IRB

$$\text{Capital} = \text{EAD} \times \text{LGD} \times (\text{WCDR} - \text{PD}) \times \text{MA}$$

Risk Weighted Assets are 12.5*Capital. So, Capital is 8% of RWA.

MA is an adjustment for the effective maturity of the loan (M). Its role is to adjust for the fact that the model is Default-Only, and loans may deteriorate, but not default during the year. It's an approximation based on the MTM model.

$$\text{MA} = \frac{1 + (\text{M} - 2.5) \times b}{1 - 1.5 \times b}$$

$$b = [0.11852 - 0.05478 \times \ln(PD)]^2$$

Capital Requirements

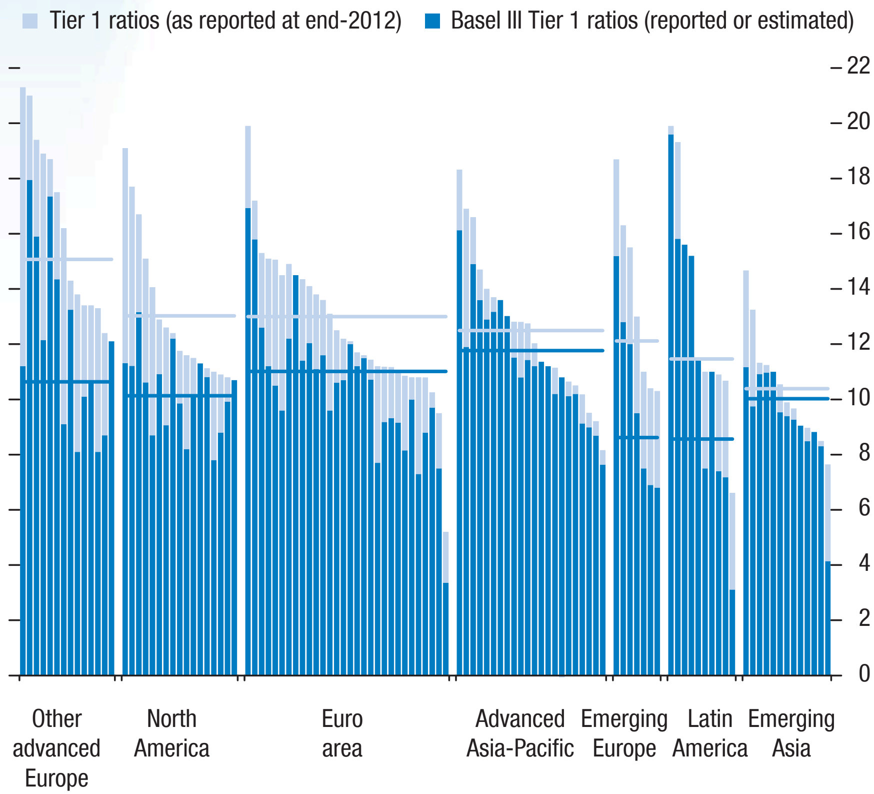
- Requirements are calculated exposure by exposure and summed up
- Portfolio features like size concentration or industry concentration are NOT taken into account
- Regulator has to approve models for PD, LGD, EAD before a bank may use them for capital calculations
  - International banks use IRB for about 60% of exposure
  - Major US banks have only recently been allowed to use such methods
  - Requires several years of running the model not for capital allocation, to perform back-testing.

Weakness of RWA measures

- Pro-cyclical requirements – in downturn loans are downgraded and may require more capital
- Vary regionally – so playing field might not be leveled.
- May be easy to manipulate in terms of classification of exposure, or through models for PD, LGD, EAD
- Ex-ante not clear which exposure are risk free (Spanish government bonds, AAA RMBS, …)
- Regulatory arbitrage between trading book and banking book
- Do not account for portfolio concentration due to large borrowers or sectors

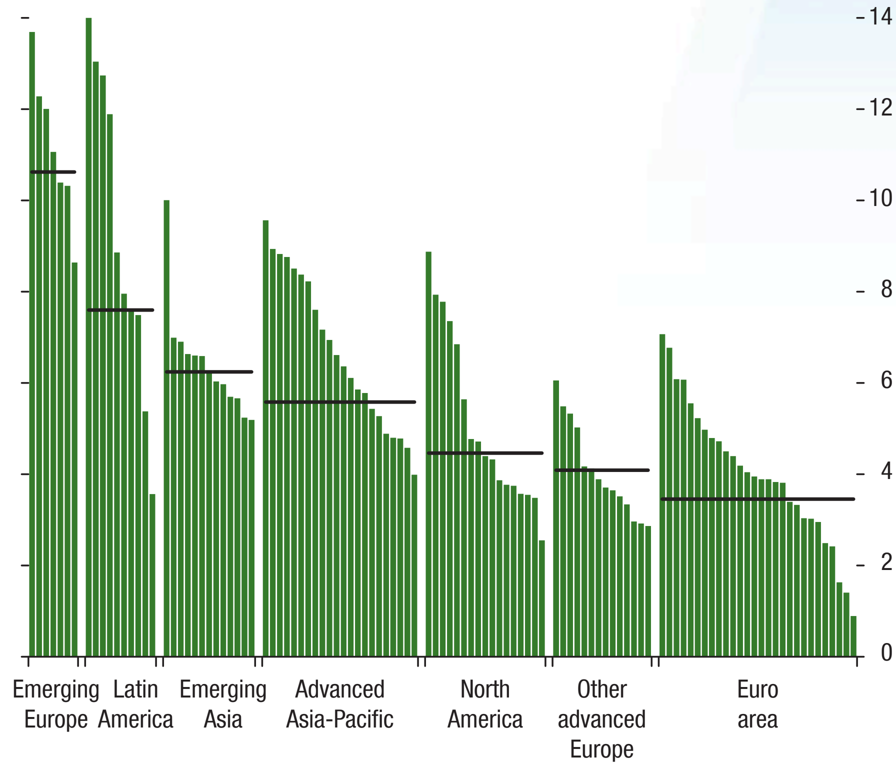
RWA vs. Asset Based measures

Figure 1.54. Large Bank Tier 1 Ratios
(Percent)



RWA in denominator

Figure 1.55. Large Bank Tangible Leverage Ratios, 2012:Q4
(Percent)



Tangible Assets in denominator

IMF Global Stability Report, October 2013

Operational Risk

Operational Risk categories considered by Basel:

1. Internal Fraud - tax evasion, intentional mismarking of positions, rogue trading
2. External Fraud- hacking damage, third-party theft and forgery
3. Employment Practices and Workplace Safety - discrimination, workers compensation, employee health and safety
4. Clients, Products, & Business Practice- antitrust, product defects fiduciary breaches
5. Damage to Physical Assets - natural disasters, terrorism
6. Business Disruption & Systems Failures - utility disruptions, software failures, hardware failures
7. Execution, Delivery, & Process Management - data entry errors, accounting errors, failed mandatory reporting

Operational Risk Capital

- Basic Indicator Approach: 15% of gross income
- Standardized Approach: different multiplicative factor for gross income arising from each business line
- Internal Measurement Approach: assess 99.9% worst case loss over one year.

Pillar 2 - Supervisory Review

- Banks perform Internal Capital Adequacy Assessment Process (ICAAP), and regulator reviews that process, SREP
- Implemented differently by different local regulators to suit local conditions
- Banks verify they have sufficient capital for stress scenarios and for risks that are not accounted for in pillar I

Pillar 3 - Market Discipline

- Banks are required to disclose
  - Scope and application of Basel framework
  - Nature of capital held
  - Regulatory capital requirements
  - Nature of institution’s risk exposures
- Reports include significant information about the risk in bank assets and about its policies and methods for managing risks

Basel 2.5 (Implementation: Dec 31, 2011)

- Stressed VaR for market risk
  - Calculated over one year period of stressed market conditions
  - Capital = $\max(VaR_{t-1}, m_c \times VaR_{avg})$
    $+\max(sVaR_{t-1}, m_s \times VaR_{avg})$
- Incremental Risk Charge
  - Ensures that products such as bonds and credit derivatives in the trading book have the same capital requirement that they would if they were in the banking book
- Comprehensive Risk Measure and additional capital for securitizations and re-securitizations
  - Designed to make sure sufficient capital is kept for instruments in the trading book that depend on credit default correlations, i.e. CDO, securitizations and re-securitizations

Basel III

- Capital Definition and Requirements
- Capital Conservation Buffer
- Countercyclical Buffer
- Leverage Ratio
- Liquidity Ratios
- Capital for CVA Risk

Capital Definition and Requirements

- Three types:
  - Common equity
  - Additional Tier 1
  - Tier 2
- Definitions tightened for each tier
- Limits
  - Common equity > 4.5% of RWA
  - Tier 1 > 6% of RWA
  - Tier 1 plus Tier 2 > 8% of RWA

Capital Conservation Buffer

- Extra 2.5% of common equity required in normal times to absorb losses in periods of stress
- If total common equity is less than 7% (=4.5%+2.5%) dividends and bonuses are restricted until the violation is remedied

Countercyclical Buffer

- Extra equity capital to allow for cyclicality of bank earnings
- Triggered by rapid growth in credit (a credit "bubble")
- Left to the discretion of national regulators
- Typically at national level, but can be limited to a subset of banks
- Can be as high as 2.5% of RWA

Leverage Ratio

- Ratio of Tier 1 capital to total exposure (not risk weighted) must be greater than 3%
- In the US a 5% minimum has been introduced, and 6% for large banks
- Denominator includes all items on balance sheet and some off-balance sheet items in Basel and in Large Bank implementation in US

Capital for CVA Risk

- CVA is the adjustment to the value of transactions with a counterparty to allow for counterparty credit risk
- Basel III requires CVA risk arising from changing credit spreads to be incorporated into market-risk VaR calculations

Basel Committee on Banking Supervision

BANK FOR INTERNATIONAL SETTLEMENTS

Basel III phase-in arrangements

(All dates are as of 1 January)

	Phases	2013	2014	2015	2016	2017	2018	2019
Capital	Leverage Ratio	Parallel run 1 Jan 2013 – 1 Jan 2017 Disclosure starts 1 Jan 2015					Migration to Pillar 1	
Capital	Minimum Common Equity Capital Ratio	3.5%	4.0%	4.5%				**4.5%**
Capital	Capital Conservation Buffer				0.625%	1.25%	1.875%	**2.5%**
Capital	Minimum common equity plus capital conservation buffer	3.5%	4.0%	4.5%	5.125%	5.75%	6.375%	**7.0%**
Capital	Phase-in of deductions from CET1*		20%	40%	60%	80%	100%	**100%**
Capital	Minimum Tier 1 Capital	4.5%	5.5%	6.0%				**6.0%**
Capital	Minimum Total Capital		8.0%					**8.0%**
Capital	Minimum Total Capital plus conservation buffer		8.0%		8.625%	9.25%	9.875%	**10.5%**
Capital	Capital instruments that no longer qualify as non-core Tier 1 capital or Tier 2 capital		Phased out over 10 year horizon beginning 2013					
Liquidity	Liquidity coverage ratio – minimum requirement			60%	70%	80%	90%	**100%**
Liquidity	Net stable funding ratio						Introduce minimum standard	

* Including amounts exceeding the limit for deferred tax assets (DTAs), mortgage servicing rights (MSRs) and financials.

– – transition periods

Globally Systemically Important Banks (G-SIBS)

- Designated by the Financial Stability Board (FSB)
- TLAC - Required to hold a minimum amount of regulatory capital (Tier 1 and Tier 2) plus long term unsecured debt of 16-20% of RWA
- Leverage Ratio – Minimum leverage ratio of 6%
- Pillar 2 – Additional capital based on qualitative firm-specific risks, recovery and resolution plans, systemic footprint, risk profile, and other factors.

Liquidity

- Top reason for banks failure is illiquidity
- Confidence-sensitive (less-stable) sources of funding include wholesale and brokered deposits, interbank deposits, and commercial paper
  - they are not based on long-term relationships, and are susceptible to adverse news and negative information
  - A bank financed by a high proportion of wholesale deposits is likely to be more vulnerable to liquidity risk than a predominately retail deposit funded bank

Liquidity

- High Quality Liquid Assets (HQLA)
  - Assets readily convertible to cash without undue loss.
  - Banks hold a portfolio of “marketable” securities that can be liquidated in a crisis.
- We would like to know are there enough liquid securities that can be sold off, if the bank’s deposits and short-term funding disappear

High Quality Liquid Assets

- Characteristics of HQLA by Basel:
  - Low Risk – Good credit ratings, but also low duration, low legal risk, low inflation and currency risk
  - Ease and certainty of valuation – Rules out structured and exotic securities
  - Low correlation with risky assets or cycle– For example, Bonds issued by Financial Institutions are more likely to be illiquid in a liquidity shortage scenario
  - Active and sizable market
  - Low volatility assets
  - Flight to quality assets

High Quality Liquid Assets

- The characteristics are translated to specific guidelines:
- Level 1 assets may be included with no haircut: cash, central bank reserves, sovereign bonds in local currency
- Level 2 assets may constitute only 40% of HQLA, with different haircuts:
  - 15% haircut for GSE and non-financial corporates rated AA or AAA
  - 25% haircut for RMBS rated AA or AAA
  - 50% haircut for non-financial corporates rated BBB or A, or stocks in a major index

Liquidity Coverage Ratio

$$\text{Liquidity Coverage Ratio} = \frac{\text{High Quality Liquid Assets}}{\text{Net Cash Outflows for 30 day period}} \geq 100\%$$

for an acute 30 - day stress period (3 notch downgrade, partial loss of deposits, loss of unsecured wholesale funding, increased haircuts on secured funding, increased collateral requirements, drawdowns on lines of credit, etc)

Net Stable Funding Ratio

We would like to know to what extent are a bank's illiquid assets (primarily loans) funded by stable core liabilities (primarily customer deposit, long-term debt and equity)

$$\text{Net Stable Funding Ratio} = \frac{\text{Amount of Stable Funding}}{\text{Required Amount of Stable Funding}} \geq 100\%$$

for a period of longer term stress. Each category of funding (capital, deposits, etc) is multiplied by an available stable funding (ASF) factor to form numerator. Each category of required funding (assets, off - balance sheet exposures is multiplied by a required stable funding factor (RSF) to form denominator

Dodd-Frank Act (USA 2010)

- New bodies to monitor systematic risk (FSOC and OFR)
- Expansion of the orderly liquidation powers of FDIC
- Volcker rule and separately capitalized affiliates for more risky business
- Exchange traded derivatives and Central clearing for OTC derivatives – SEF and CCP
- SIFIs: Higher capital requirements, living wills, stricter risk management

Dodd-Frank Act (USA 2010)

- Rating agencies: No use of external ratings in regulation, stricter oversight of agencies
- Originators of asset backed securities must keep “skin in the game”
- Federal Insurance Office
- Consumer Financial Protection Bureau

Thanks

Financial Risk Management

Spring 2016

Dr. Ehud Peleg

Stress Testing

UCLA Anderson
THINK IN THE NEXT

Stress Testing Agenda

- Stress Testing vs. Modeling
- Scenario Analysis and Multiple Variable Stress
- Top-down macro stress testing
  - Macro models
  - Satellite Models
- CCAR / DFAST
- Reverse Stress Testing

Disadvantages of VaR

- Backward looking
- Does not give insight into bad scenarios and to possible mitigation plans
- Makes restrictive assumptions about underlying variables
- Difficult to communicate to management

Individual vs. Several Variables

- Stress individual variables (Scenario Analysis)
  - Better assessment of specific risk
  - Easy to use and control as a risk management limit
  - Allows more freedom in stress
- Scenarios where several variables change
  - More realistic as variables are correlated
  - Tells a story and lends itself to a mitigation plan
  - But, limits the extremity each variable takes

Stress Testing

- Key Questions
  - How do we generate the scenarios?
  - How do we evaluate the scenarios' effect on our portfolio?
  - What do we do with the results?

Typical Stress Testing Framework

Stress Event – High level story (e.g. Subprime Crisis, Euro Crisis)

Macroeconomic Model - Links event to macroeconomic variables (e.g. GDP, interest rates, FX rates)

“Satellite” models - Link macroeconomic variables to their effects on banks’ assets (e.g. credit losses, MTM losses)

Impact on Firm/Portfolio – (e.g. earnings, capital, liquidity)

Adapted from Foglia(2009) – Stress Testing Credit Risk: A Survey of Authorities Approaches

Generating Scenarios

- Choose particular days when there were big market movements and stress all variables by the amount they moved on those days
- Form a stress testing committee of senior management and ask it to generate the scenarios
- Regulatory requirements
- Extreme shocks in Macro models

Regulatory Stress Tests

- Comprehensive Capital Analysis and Review (CCAR).
- Dodd Frank Act Stress Tests (DFAST)
- Annual tests performed by the Fed and by the Banks (BHCs) themselves
  - About 30 banks with assets greater than $50B
- Financial institutions submit capital plans that include projection of revenues, losses and balance sheet levels, as well as payout plans.
- The Fed can approve the plan, or object from quantitative or qualitative reasons.

CCAR Scenarios for 2015

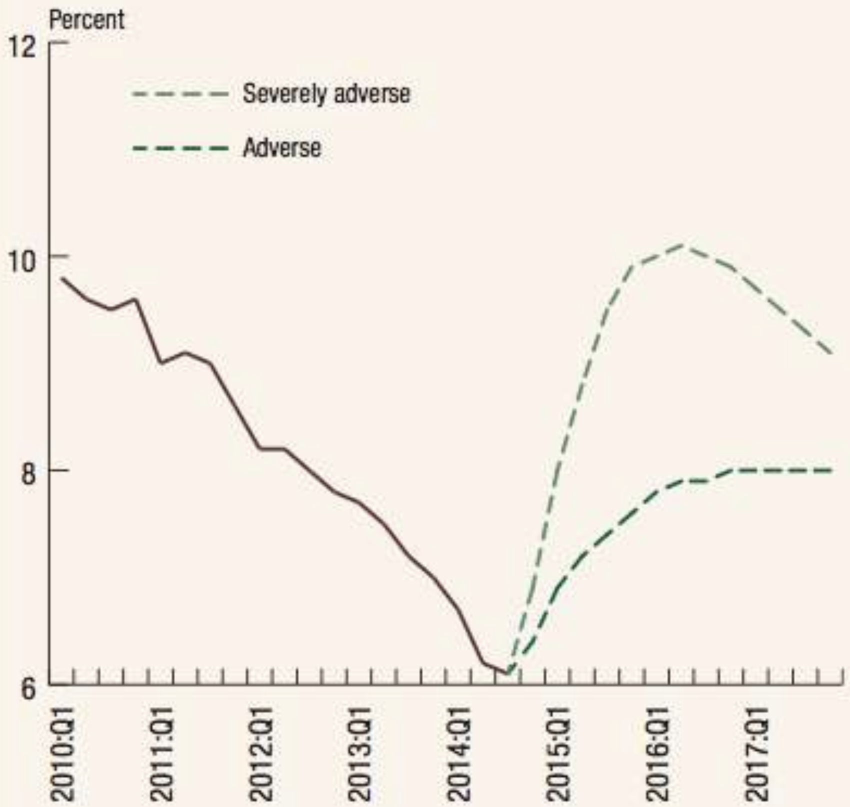
- Three scenarios: baseline, adverse, severely-adverse
- Quarterly scenarios: 2014:Q4-2017:Q4
- 28 macro variables: domestic and international
  - Economic activity: GDP, unemployment, disposable personal income, CPI
  - Asset Prices: house prices, stock market, VIX, commercial real estate prices
  - Interest Rates: 3mo, 5yr, 10yr treasury, 10-yr BBB, prime, 30yr mortgage
  - International: UK, Euro area, Asia, Japan

Severely Adverse Scenario

- Substantial weakening in economic activity across all countries
- Significant reversal of recent improvements in US housing market
- Flight-to-safety capital flows towards US cause depreciation of other currencies
- Recessions in Euro, UK and Japan, which are slow to recover
- Below-trend growth in developing Asia

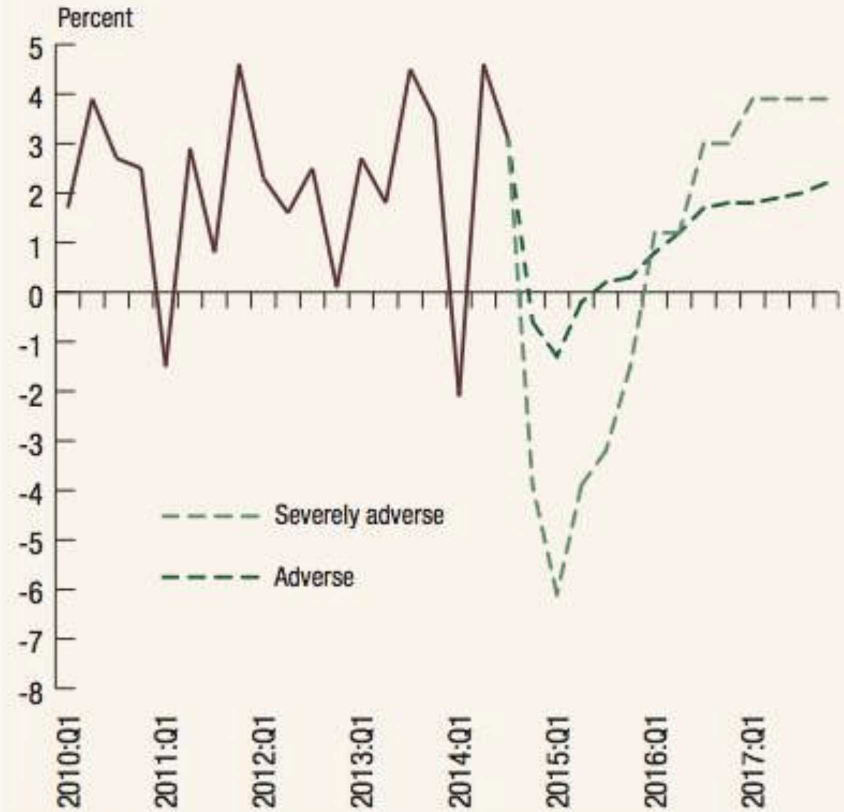
CCAR 2015 – Macro Variables in Scenarios

Figure 2. Unemployment rate in the severely adverse and adverse scenarios, 2010:Q1–2016:Q4



Source: Bureau of Labor Statistics and Federal Reserve assumptions in the supervisory scenarios.

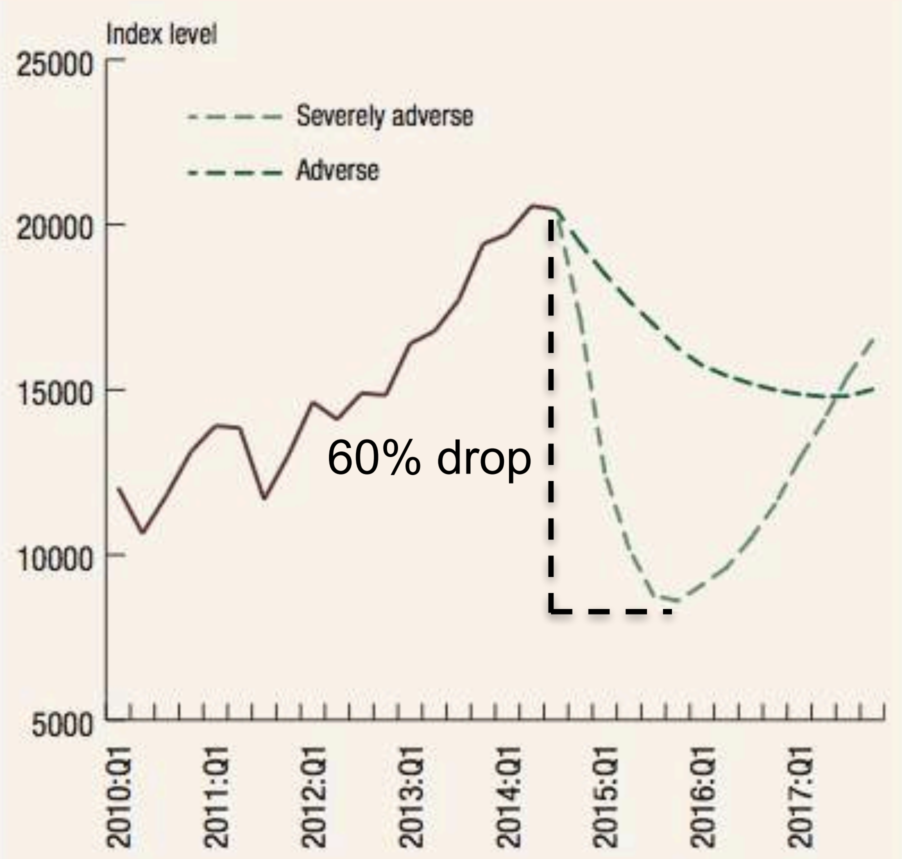
Figure 3. Real GDP growth rate in the severely adverse and adverse scenarios, 2010:Q1–2016:Q4



Source: Bureau of Economic Analysis and Federal Reserve assumptions in the supervisory scenarios.

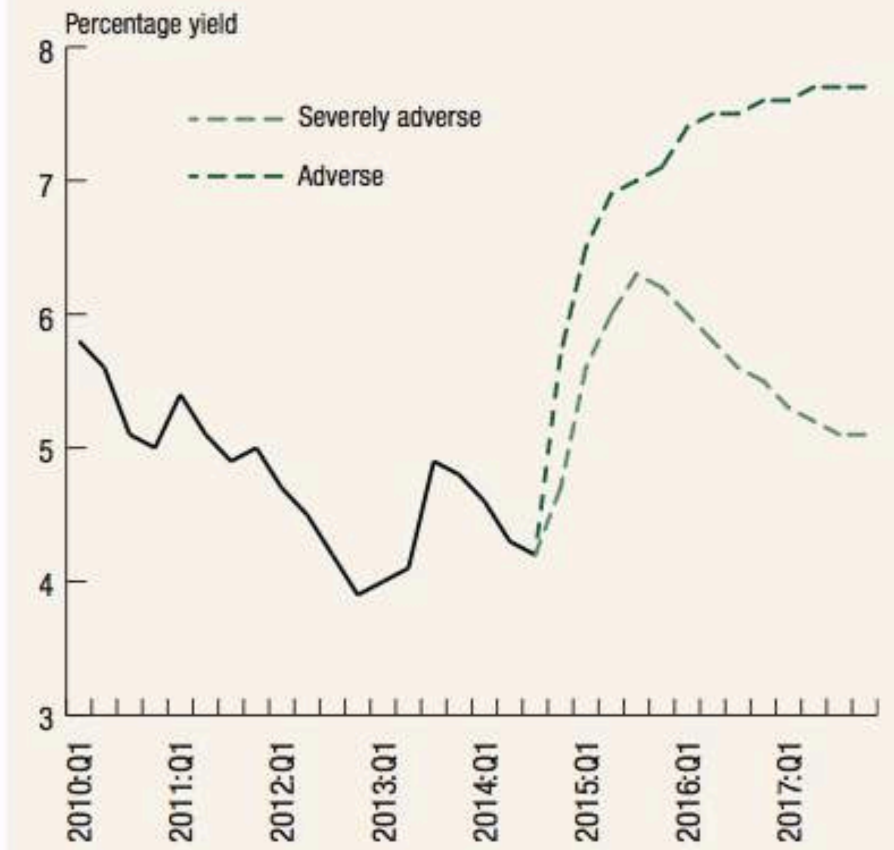
CCAR 2015 – Market Variables in Scenarios

Figure 4. Dow Jones Total Stock Market Index, end of quarter in the severely adverse and adverse scenarios, 2010:Q1–2016:Q4



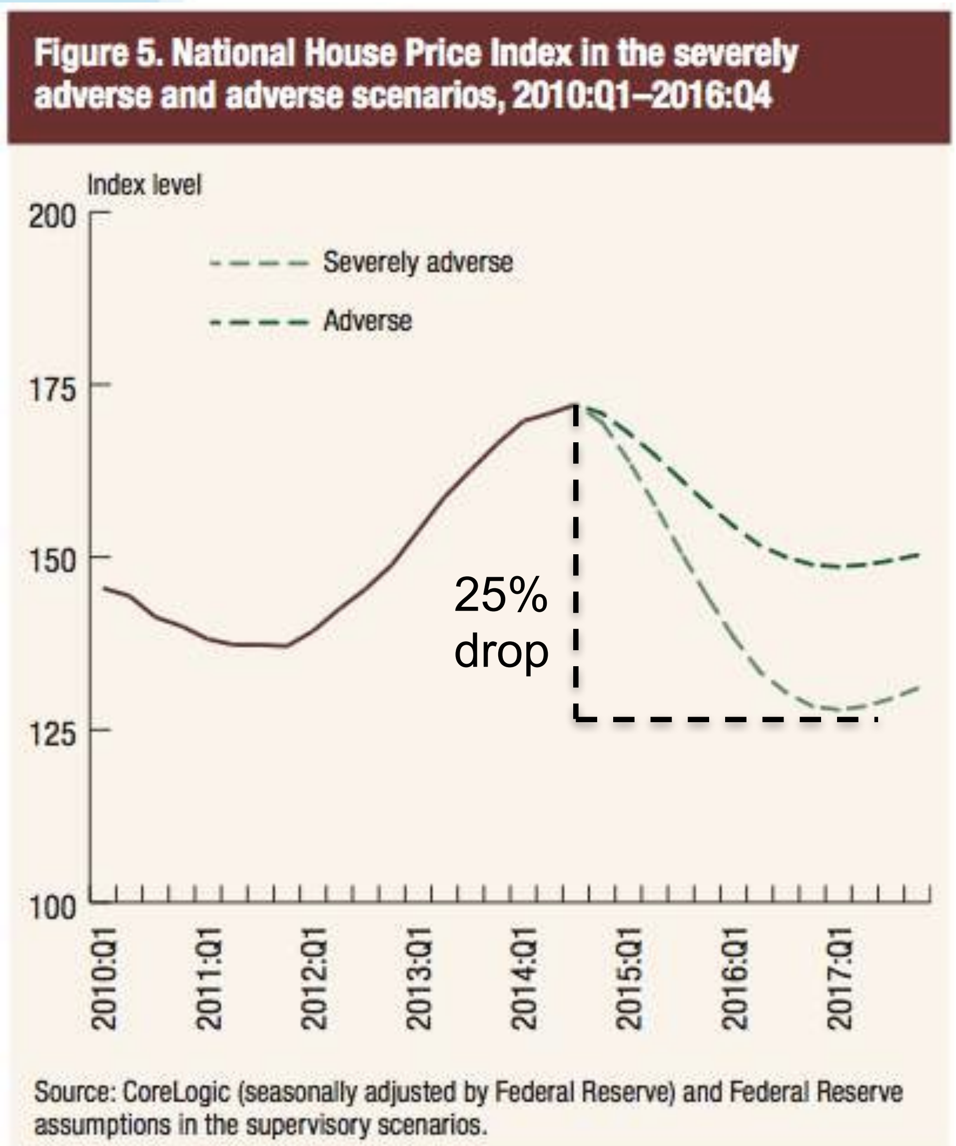
Source: Dow Jones and Federal Reserve assumptions in the supervisory scenarios.

Figure 6. U.S. BBB corporate yield, quarterly average in the severely adverse and adverse scenarios, 2010:Q1–2016:Q4



Source: Merrill Lynch (adjusted by Federal Reserve using a Nelson-Siegel smoothed yield curve model) and Federal Reserve assumptions in the supervisory scenarios.

CCAR 2015 – Market Variables in Scenarios



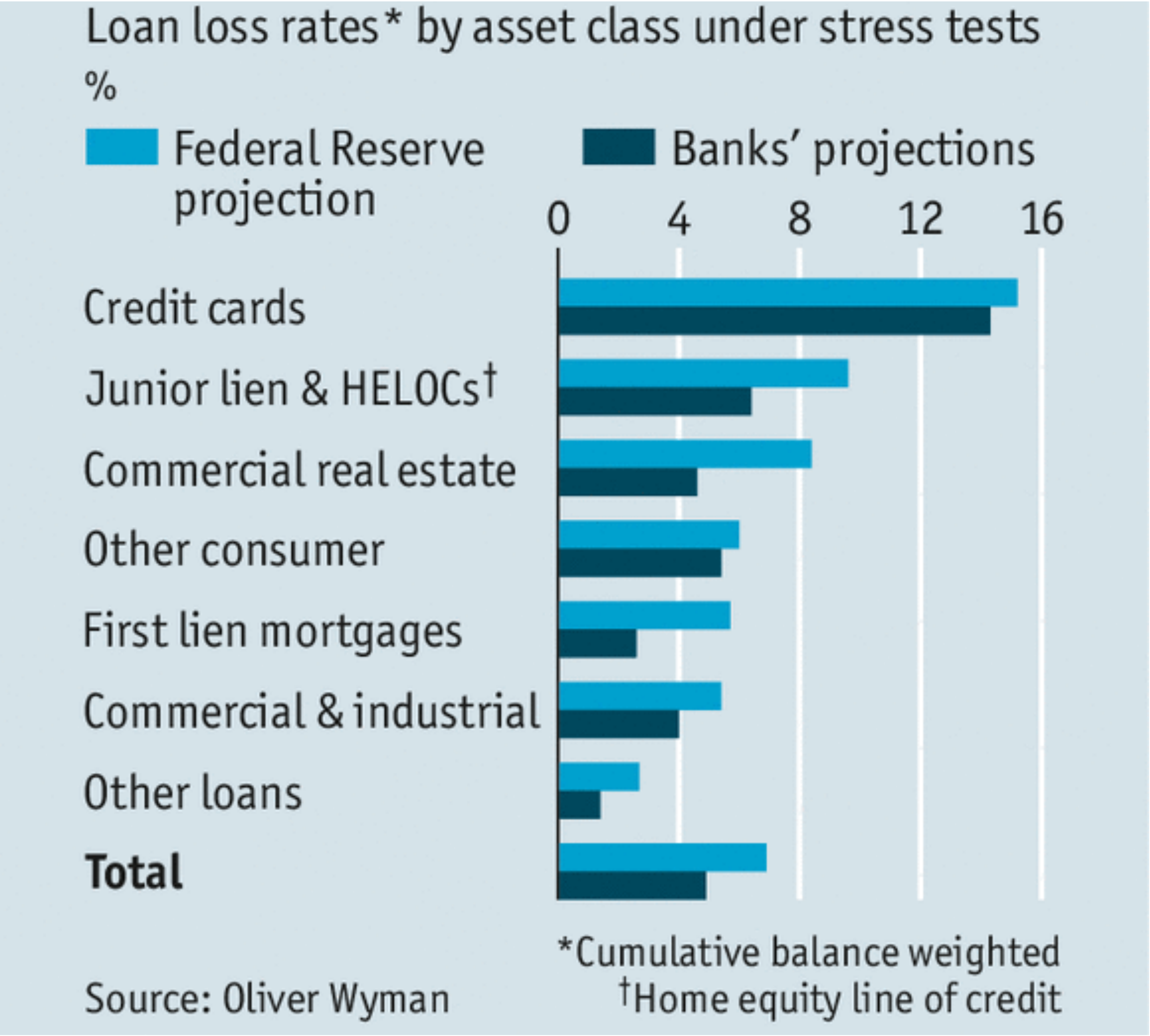
- VIX reaches 79 within one quarter of stress
- Commercial real estate prices decline by 35%
- Yield on 10-year Treasuries declines to 1%

CCAR-Additional Shocks

- 6 BHCs are subject to another severe global market shock to their trading portfolio:
  - Shift in sovereign yield curves
  - Shocks to corporate and emerging market spreads
  - A severe Euro crisis
  - Increased risk aversion across markets
  - Instantaneous as opposed to the “main” evolving scenarios
- 8 BHCs are subject to an additional Counterparty Default Scenario

Satellite Models

- How does the scenario affect the bank's results?
- Top-down approach – Macro-based regression models
  - Used by regulators. Foglia (2009) documents many variations.
  - LHS: default rate or losses; RHS: Macroeconomic factors
- Bottom-up approach – Considers effects of macro variables on specific loans/securities or well-defined portfolios
  - Commonly used by banks



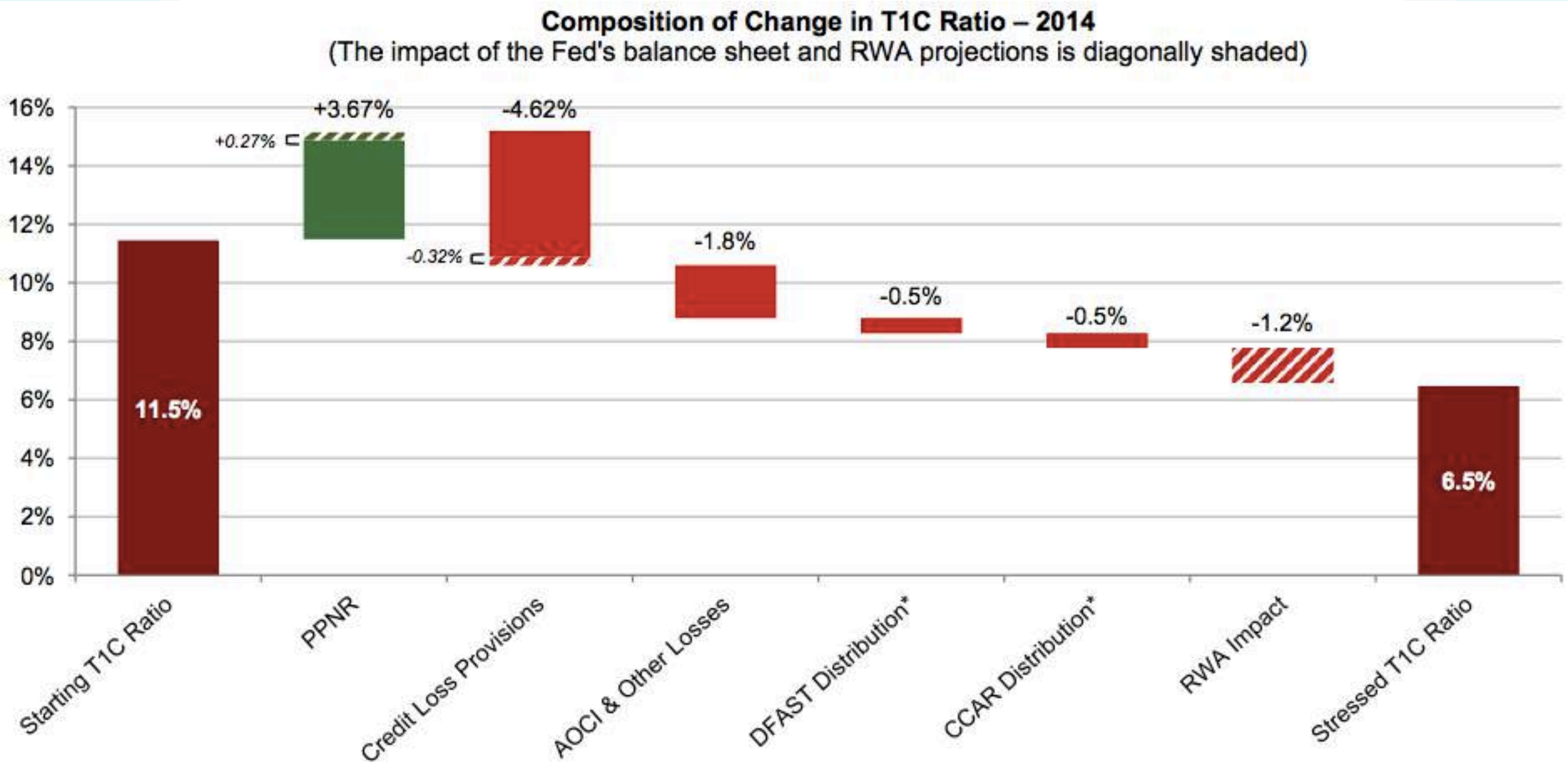
Loan loss rates* by asset class under stress tests
%
Federal Reserve projection
Banks' projections
0
4
8
12
16
Credit cards
Junior lien & HELOCs†
Commercial real estate
Other consumer
First lien mortgages
Commercial & industrial
Other loans
Total
*Cumulative balance weighted
†Home equity line of credit
Source: Oliver Wyman

Thanks

Applications for Stress Test Results

- Solvency Tests
  - Verify that institution has enough capital to cover losses
  - Under CCAR BHCs have to show that they are well-capitalized (Tier 1 common ratio above 5%) even in stress-scenarios
- Liquidity Tests
  - Verify that bank has enough liquid assets and stable funds to withstand intensive withdrawal of funds
- Decision-making – Inform return vs. risk analysis
- Risk appetite – Define limits to cap losses on a portfolio

Effect on Capital Ratios



"Stress testing: A look into the Fed's black box" (pwc, 2014)

Reverse Stress Testing

- First we assume that there is a large loss then we look for what is the story
- Typically done as a qualitative exercise
- We can also look for the worst scenarios in our VaR simulation and figure out to what macroeconomic scenario they relate
- Can be a useful input to the stress testing committee.