# Financial Risk Management

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Volatility Models

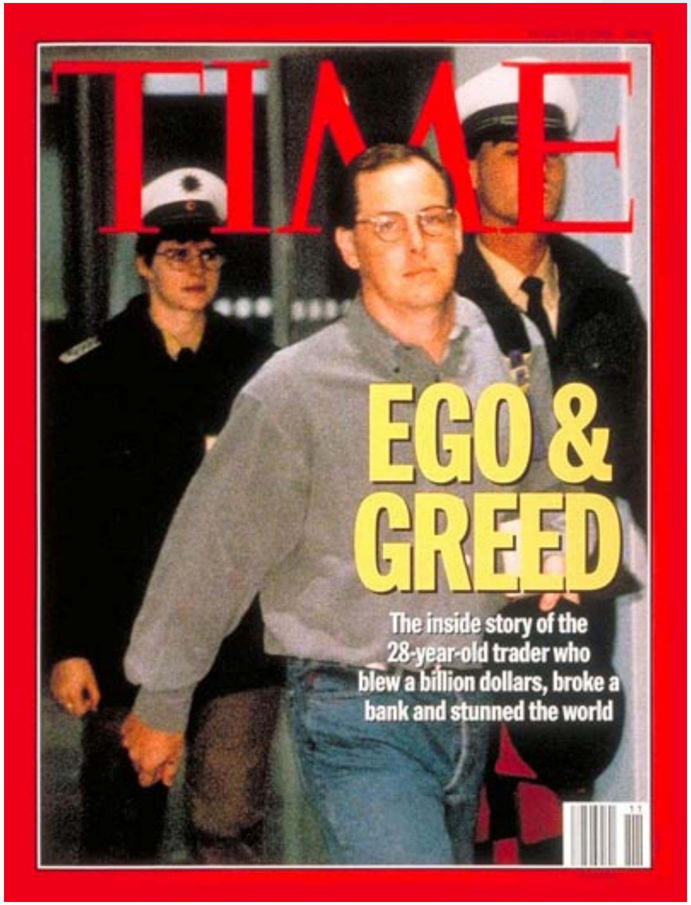
UCLA Anderson
THINK IN THE NEXT

# Agenda

- Managing Traded Market Risk
- Volatility Models
  - Exponentially Weighted Moving Average
  - GARCH (1,1)
- Forecasting Volatility
- Scenario Analysis - Volatility Shocks

# Rogue Trader

- Barings was UK's oldest merchant bank.
- Leeson managed the Singapore Trading Desk.
- In 1992, he made 10% of Barings profit.
- By 1995, He made total losses of $1.4B
- Chief trader and responsible for settling the trades.



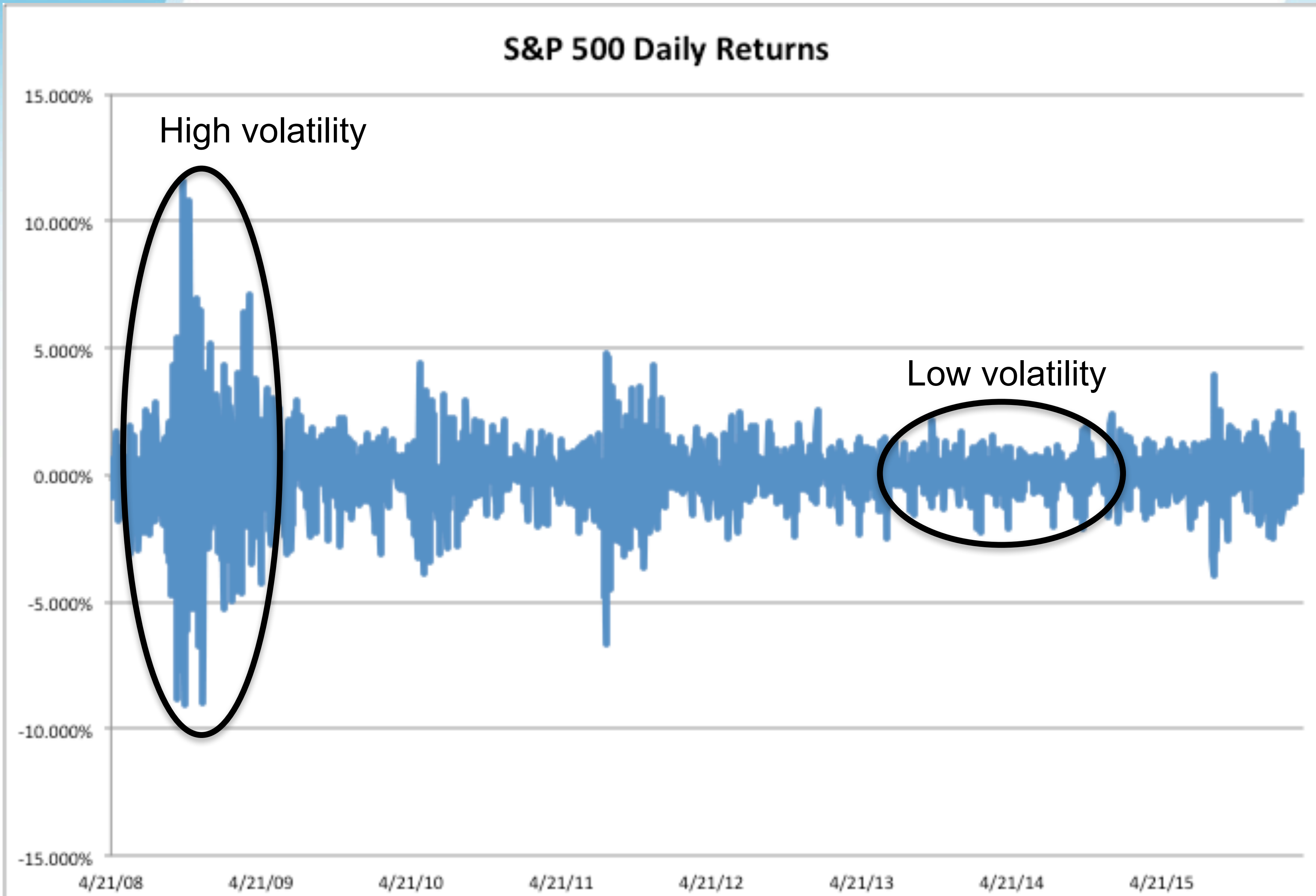
# Risk Management of Trading in Financial Institutions

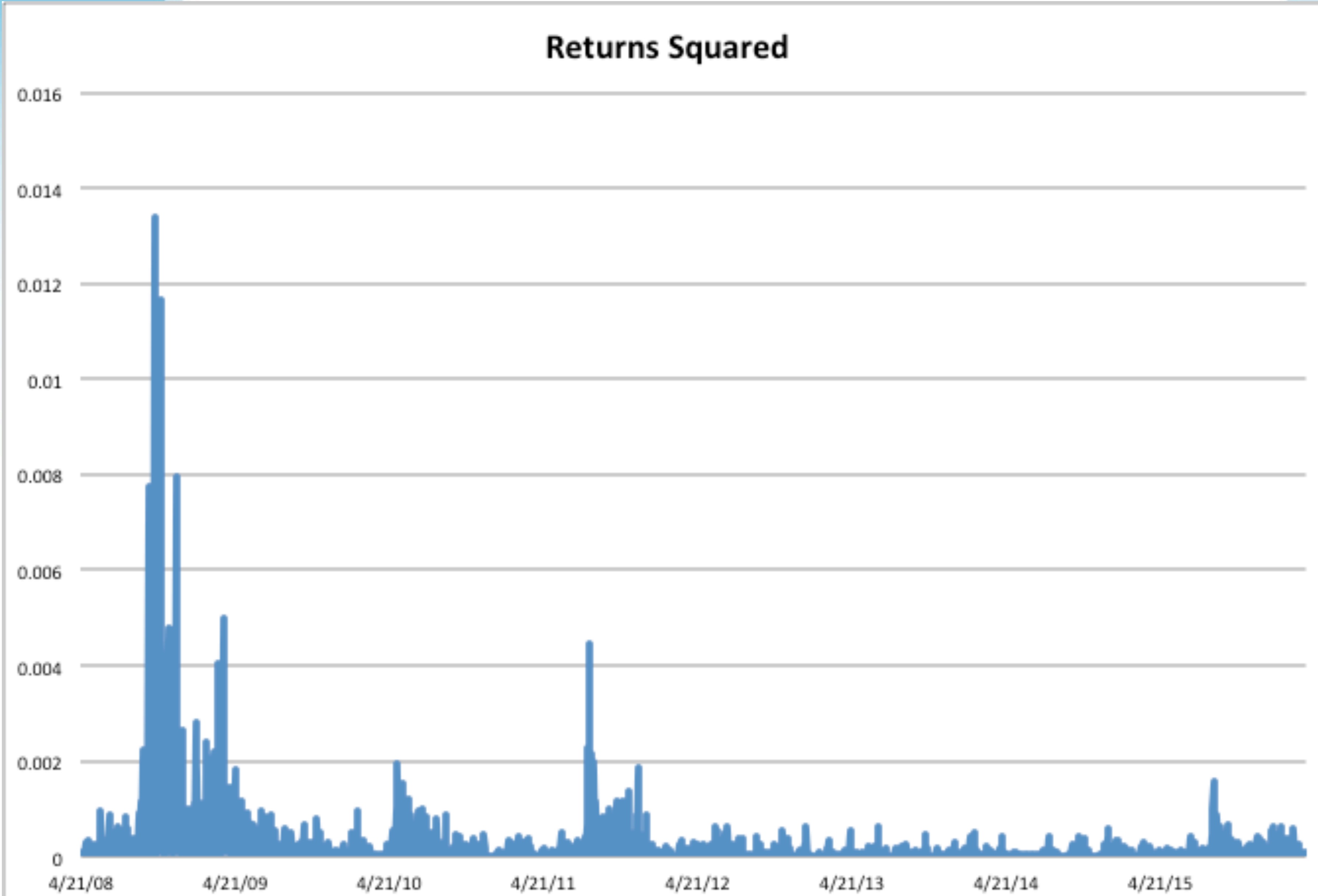
- Front Office – Take on risk as Market Makers or as Proprietary Traders
  - Take risk according to view and within risk limits
- Middle Office – Manages market and operational risk of the trading floor
  - Aggregate risk
  - Control risk limits: internal and regulatory
- Back Office – Record keeping, manage execution and control operational risks

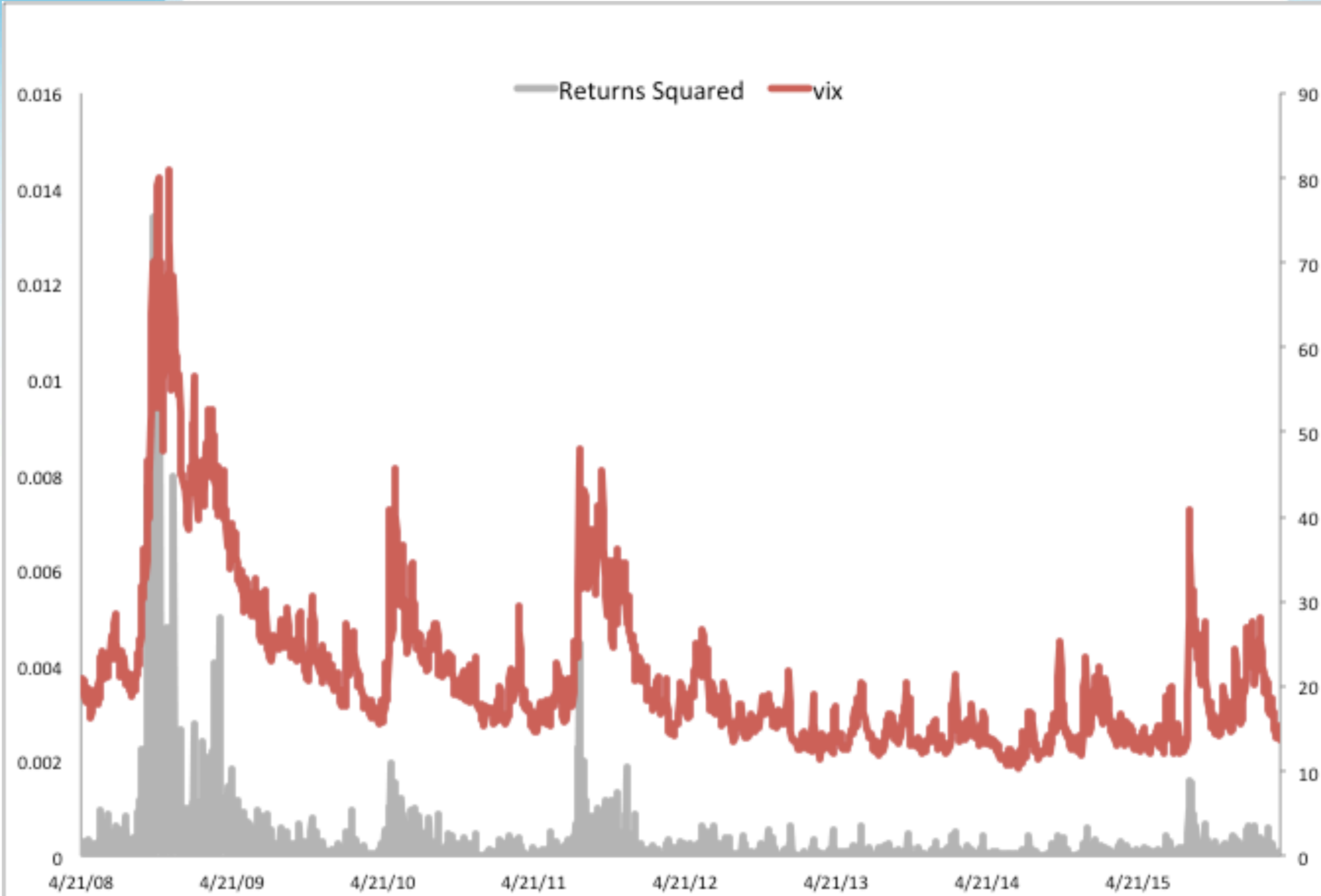
# Trading Market Risk

- Trading market risk managers typically look at daily returns.
  - Define $u_i$ as $(S_i - S_{i-1})/S_{i-1}$
  - Assume that the mean value of $u_i$ is zero
  - *Returns* and *log-returns* are very close
- We are interested in determining the expected volatility for the next day.
  - If returns are i.i.d, use m last days to estimate volatility:

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2$$


S&P 500 Daily Returns
15.000%
10.000%
5.000%
0.000%
-5.000%
-10.000%
-15.000%
4/21/08
4/21/09
4/21/10
4/21/11
4/21/12
4/21/13
4/21/14
4/21/15
High volatility
Low volatility


Returns Squared
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
4/21/08
4/21/09
4/21/10
4/21/11
4/21/12
4/21/13
4/21/14
4/21/15


Returns Squared
vix
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
90
80
70
60
50
40
30
20
10
0
4/21/08
4/21/09
4/21/10
4/21/11
4/21/12
4/21/13
4/21/14
4/21/15

# Some Stylized Facts

- Returns are not i.i.d. but show very little serial correlation.
- Squared returns, or volatilities, show high serial correlation.
- Return series is heavy tailed.
- High volatility, and extreme events, appear in clusters.

# EWMA Model

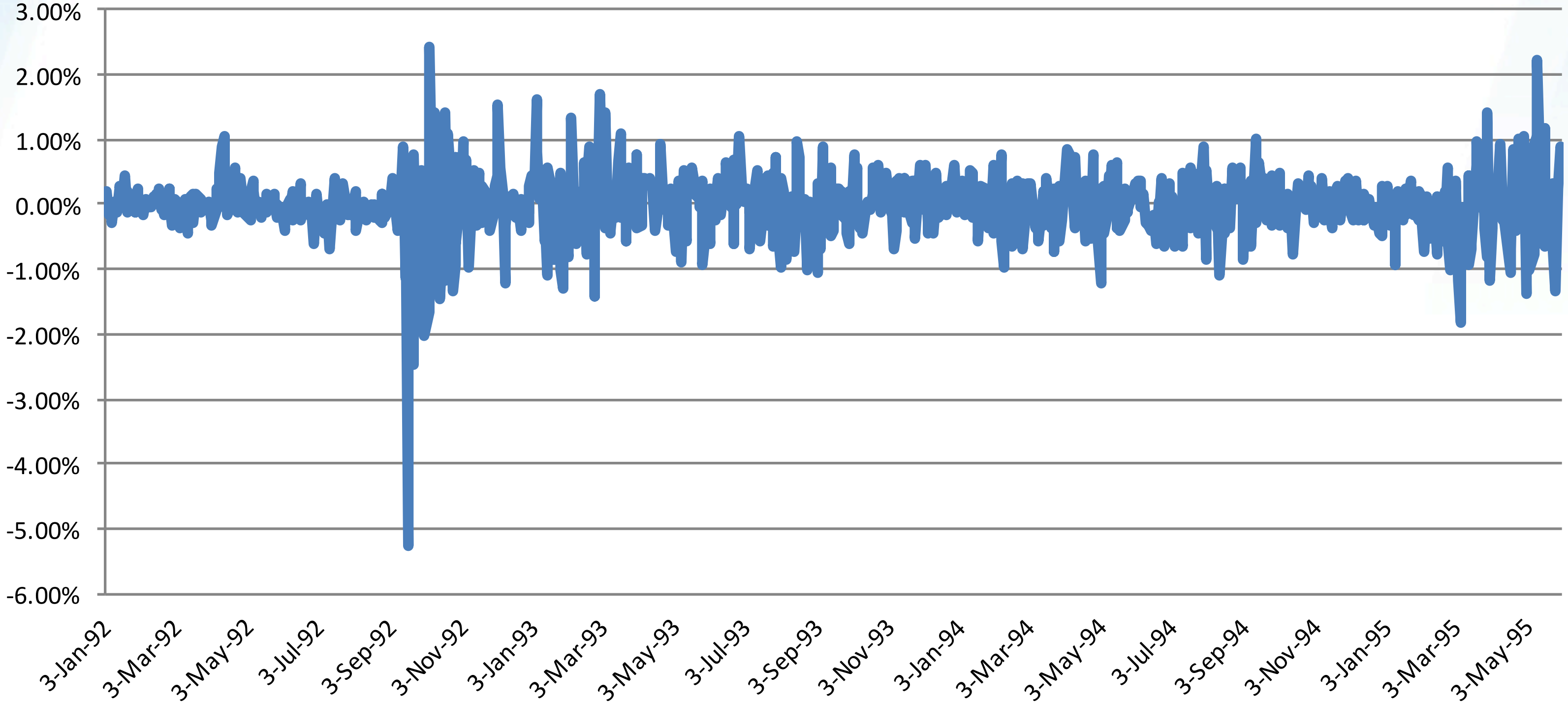
- In an exponentially weighted moving average model, the weights assigned to the $u^2$ decline exponentially as we move back through time
- This leads to $\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$
- RiskMetrics uses λ = 0.94
- Tracks volatility changes
- With large *m* this leads to a weighted average of squared returns with weights decreasing by λ.

$$\sigma_n^2 = (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}u_{n-i}^2 + \lambda^m\sigma_{n-m}^2$$

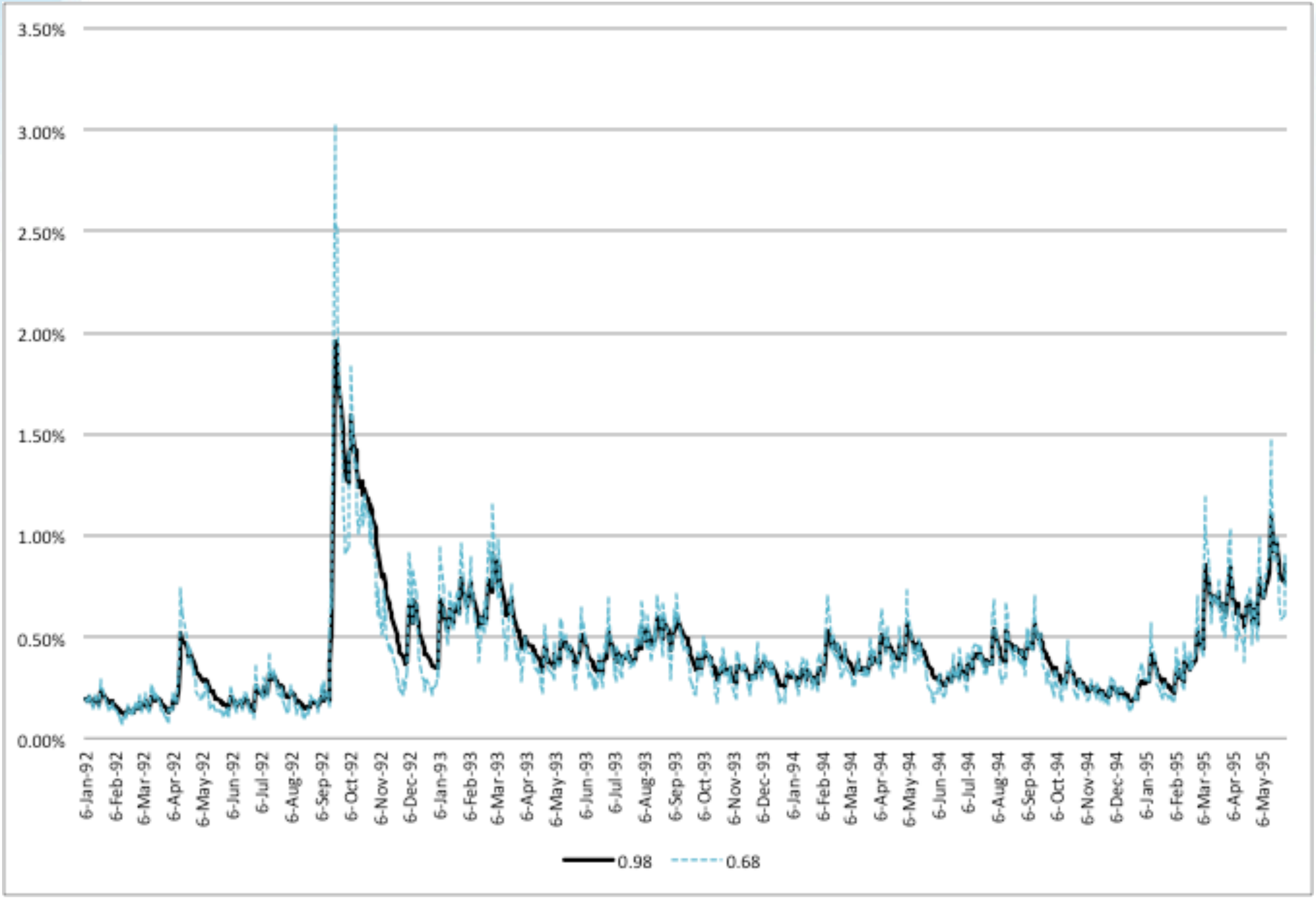
# EWMA Model (cont)

- Relatively little data needs to be stored. We need only remember the current estimate of the variance rate and the most recent observation on the market variable
- Low $\lambda$ leads to more weight on recent returns and therefore volatile estimates of volatility
- High $\lambda$ leads to less weight on recent returns and therefore slow response to changing volatility.

# Daily Returns GBP/USD



# EWMA Estimates of Daily Volatility



# GARCH (1,1)

In GARCH (1,1) we let the variance revert to a long-run average variance rate

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

Weights sum to 1: $\gamma + \alpha + \beta = 1$

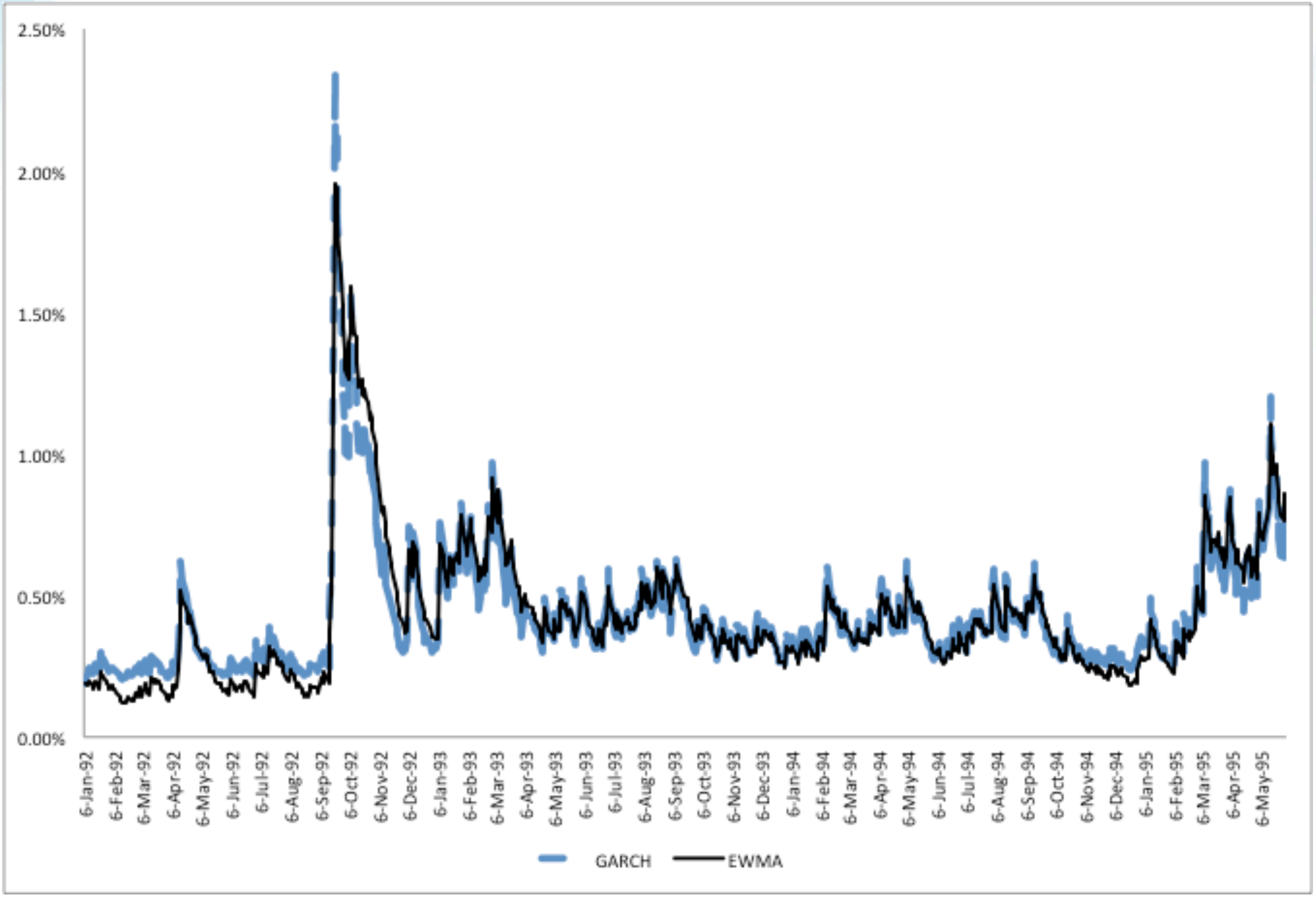
Setting $\omega = \gamma V_L$, we can write:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

And:

$$V_L = \frac{\omega}{1 - \alpha - \beta}$$

# GARCH Estimates vs EWMA



# Forecasting Future Volatility

- We now ask: What is our prediction for volatility in *t* days.
- If we assume returns are i.i.d. then: $E[\sigma_{n+t}^2] = \sigma_n^2$
- If we assume EWMA: $\sigma_{n+1}^2 = \lambda\sigma_n^2 + (1-\lambda)u_n^2$
- Since: $E[u_n^2] = \sigma_n^2$
- We get the same result:

$E[\sigma_{n+t}^2] = \sigma_n^2$ for *t=1*, and then for all *t*.

# Forecasting Future Volatility with GARCH (1,1)

Since $\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$

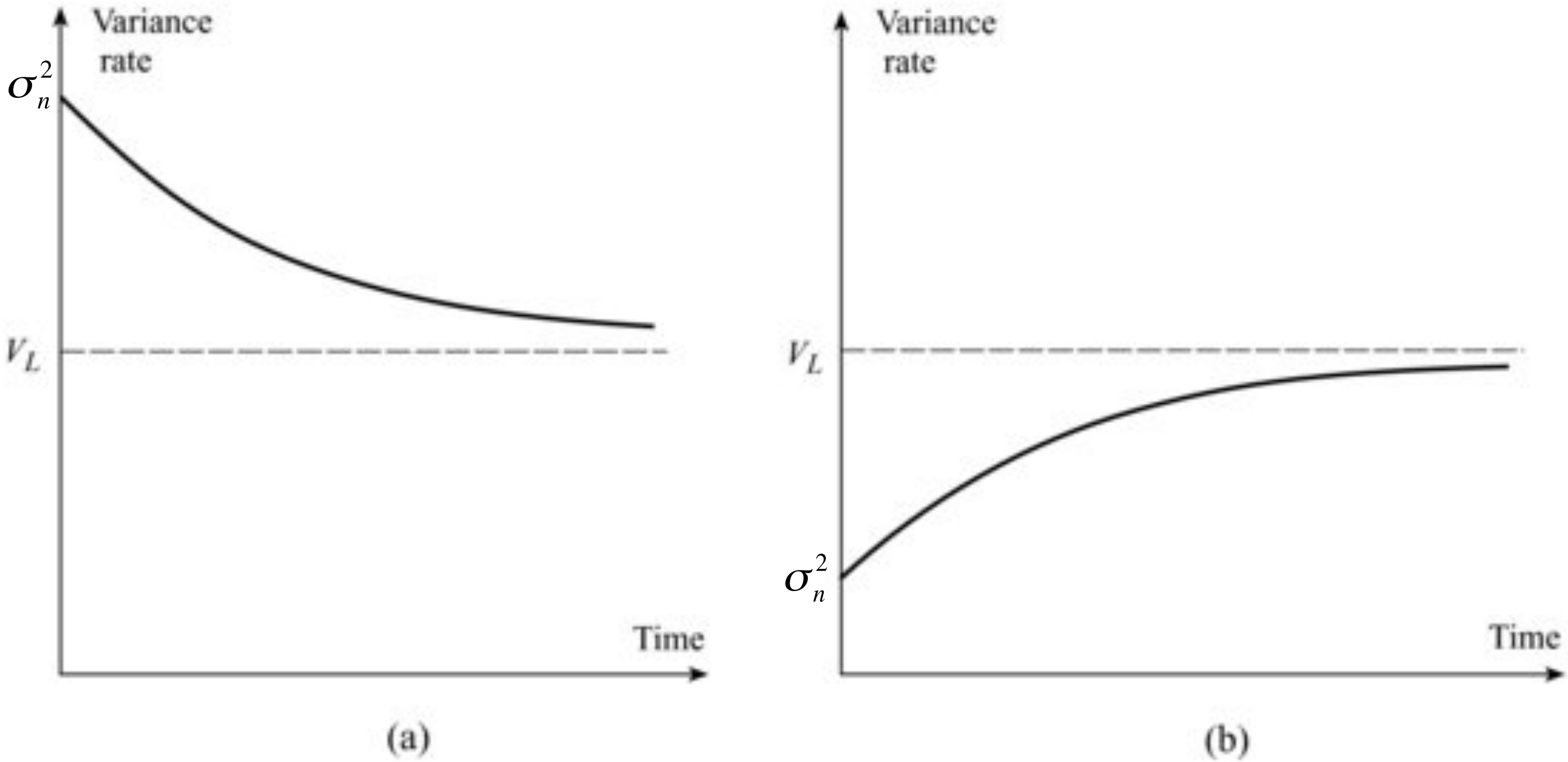
And $E[u_{n+1}^2] = \sigma_{n+1}^2$

We get by iteration that the expected future daily variance is:

$$E[\sigma_{n+t}^2] = V_L + (\alpha + \beta)^t (\sigma_n^2 - V_L)$$

If $\alpha + \beta < 1$ the daily volatility will be reverting to the long run mean

# Forecasting Future Volatility cont




$$E[\sigma_{n+t}^2] = V_L + (\alpha + \beta)^t (\sigma_n^2 - V_L)$$

# Volatility Over T Days

- We would like to know the volatility over the next T days.
- First, assume that the daily returns on the index are i.i.d. with mean zero and annualized standard deviation of 20%.
- What is the volatility of 10-day return?

$$\sqrt{\frac{10}{252}} \cdot 20\% = 3.98\%$$

# Volatility Over T Days - GARCH

- Now, suppose volatility follows GARCH(1,1) with long term annualized volatility of 15%, $\alpha=0.0603$, $\beta=0.9001$, what is the volatility of 10-day return? Current volatility estimate is 20% p.a.

# Volatility Over T Days – GARCH

- Convert the variances to daily:

$$V_L = \frac{0.15^2}{252} = 0.000089 \quad \sigma_n^2 = \frac{0.2^2}{252} = 0.000159$$

- Compute the expected future variances, using:

$$E\left[\sigma_{n+t}^2\right] = V_L + (\alpha+\beta)^t \left[\sigma_n^2 - V_L\right]$$

| **t** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
|---|---|---|---|---|---|---|---|---|---|---|
| **Variance** | 0.000159 | 0.000156 | 0.000153 | 0.000151 | 0.000148 | 0.000146 | 0.000144 | 0.000142 | 0.000140 | 0.000138 |

- Sum the variances: 0.001476
- 10-day volatility is only 3.84%

# Average Variance Rate

- Instead of summing up discretely, we can integrate over time: $\int_0^T V_L + (\alpha+\beta)^t\left(\sigma_n^2 - V_L\right)dt$
- Variance over T days is:

$$V_L \cdot T + \frac{1}{a}\left(1-e^{-aT}\right)\left(\sigma_n^2 - V_L\right) \qquad a = \ln\left(\frac{1}{\alpha+\beta}\right)$$

- The average daily variance is:

$$V_L + \frac{1}{aT}\left(1-e^{-aT}\right)\left(\sigma_n^2 - V_L\right)$$

# Annualized Average Variance Rate

- For pricing an option with T days to maturity we use the annualized average variance over the period.
- The variance per year for an option lasting $T$ days is:

$$\sigma(T)^2 = 252\left[V_L + \frac{1-e^{-aT}}{aT}\left(\sigma_n^2 - V_L\right)\right]$$

- $\sigma_n^2$ and $V_L$ are daily. $\sigma(T)$ is annualized. T in days.

- Note: $\sigma(0)^2 = 252 \cdot \sigma_n^2$

# Example – Option on S&P

- Suppose the S&P 500 is currently at 2,020. What is the value of a European option on the index, with 30 days to expiration, $K$=2,000, $r$=3%, $\sigma$=20%, $q$=3%?
  - **Black Scholes value is $65.66.**
- Now, suppose that we estimated a GARCH(1,1) process for the index and found that the long term volatility is 15%, $\alpha$=0.0603, $\beta$=0.9001. What is our estimate of the option's value?

# Example – Option on S&P (2)

- First, compute the average annualized volatility:

$$V_L = \frac{0.15^2}{252} = 0.000089 \quad \sigma_n^2 = \frac{0.2^2}{252} = 0.000159$$

$$a = \ln\frac{1}{0.0603+0.9001} = 0.0404$$

$$\sigma(T) = \sqrt{252\left\{V_L + \frac{1-e^{-aT}}{aT}\left[\sigma_n^2 - V_L\right]\right\}} = 18.07\%$$

- The Black-Scholes value is now: \$60.40.
  - \$5.26 less than with the i.i.d. assumption.

# Scenario Analysis

- Scenarios may be used to measure the sensitivity of the position value to shocks in underlying parameters.
- For example: To estimate the sensitivity of the option to volatility, we can look at the change in value position for increase/decrease of 1% in volatility.
- Regulators often require banks to consider what would happen to their portfolio if market volatility increased dramatically overnight.

# Scenario Analysis - Example

- We want to evaluate the change in value of the option from before, if volatility goes up from 20% to 21%
    1. Assuming returns are i.i.d.
    2. Using GARCH with long term volatility = 15% as before.

# Scenario Analysis - Example

1. At $\sigma$=20% the price was \$65.66.
   – Recalculating using $\sigma$=21%, price is \$68.39.
     The value has gone up by 4.15%
2. Using GARCH: average annualized volatility (aav) is 18.07%, and price is \$60.40.
   – Recalculating using $\sigma$=21%, aav is 18.71%, price is \$62.16. The value has gone up only by 2.91%

# Sensitivity to Volatility

- Differentiating the Average Annualized Volatility formula for GARCH (1,1) gives us the sensitivity of average annualized volatility to changes in $\sigma(0)$ .
- When $\sigma(0)$ changes by $\Delta\sigma(0)$, GARCH (1,1) predicts that $\sigma(T)$ changes by

$$\Delta\sigma(T) \approx \frac{1-e^{-aT}}{aT}\frac{\sigma(0)}{\sigma(T)}\Delta\sigma(0)$$

Thanks