Financial Risk Management

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Interest Rate Risk

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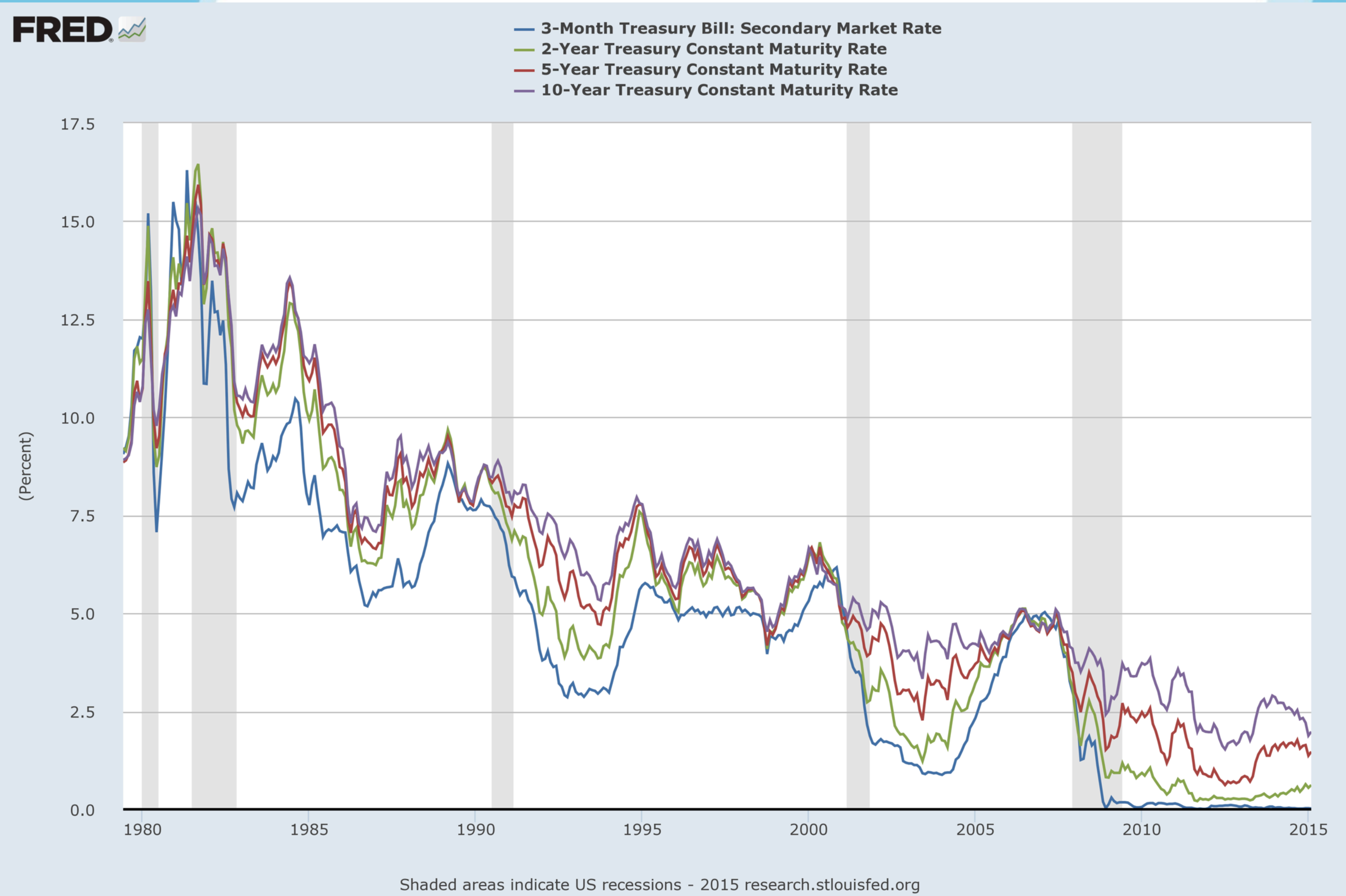
THINK IN THE NEXT

Agenda

- Interest Rate Risks
- Bond Prices and Yields
- Duration and Convexity
- Term-structure risk

Interest Rate Risks

- Portfolio Markdown due to change in levels of interest rates
  - A shift of rates at all maturities: duration risk
  - Change in relationship between maturities – yield curve moves
- Cash flow mismatch between assets and liabilities: re-pricing risk
- Change in spreads between different curves: basis risk
  - Liquidity differences
  - Credit spreads
- Interest rate related behavioral options


FRED
3-Month Treasury Bill: Secondary Market Rate
2-Year Treasury Constant Maturity Rate
5-Year Treasury Constant Maturity Rate
10-Year Treasury Constant Maturity Rate
(Percent)
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
1980
1985
1990
1995
2000
2005
2010
2015
Shaded areas indicate US recessions - 2015 research.stlouisfed.org

Orange County – Duration Risk

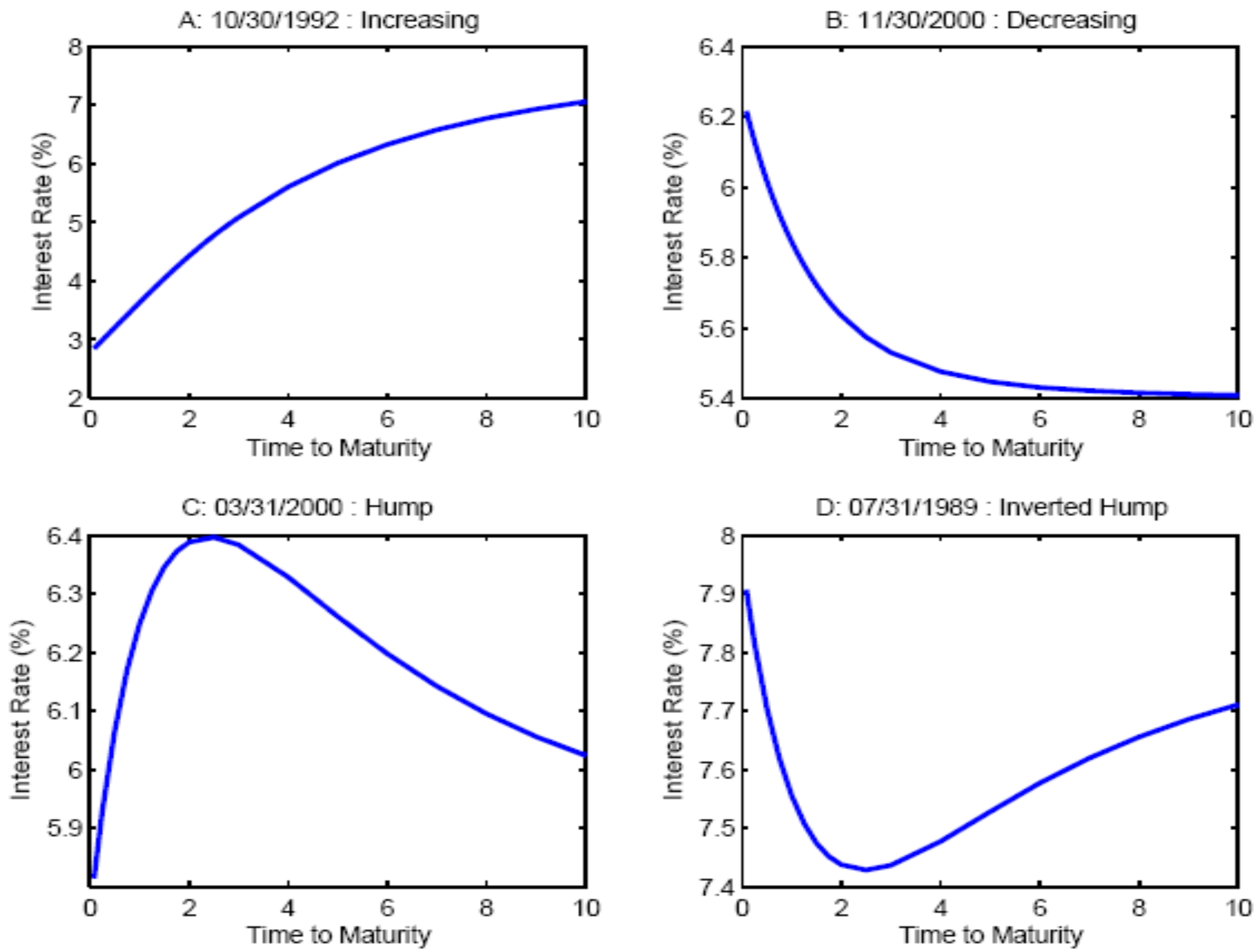
- In 1994, Orange County lost $1.6 billion when the interest rate level suddenly increased from 3% to 5.7%
- This sent the county into bankruptcy
- The county’s Treasurer, Bob Citron, had bet, through a mix of structured notes and leverage, that rates would not increase in the future
- The portfolio was too sensitive to changes in interest rates

Term Structure

- The term structure of interest rates, or spot curve, or yield curve, defines the relation between the level of interest rates and their time to maturity
- The term spread is the difference between long term interest rates (e.g. 10 year rate) and the short term interest rates (e.g. 3 month interest rate)
- The term spread depends on many variables: expected future inflation, expected growth of the economy, agents attitude towards risk, etc.
- The term structure varies over time, and may take different shapes

Term Structure of Rates

Figure 2.3 The Shapes of the Term Structure



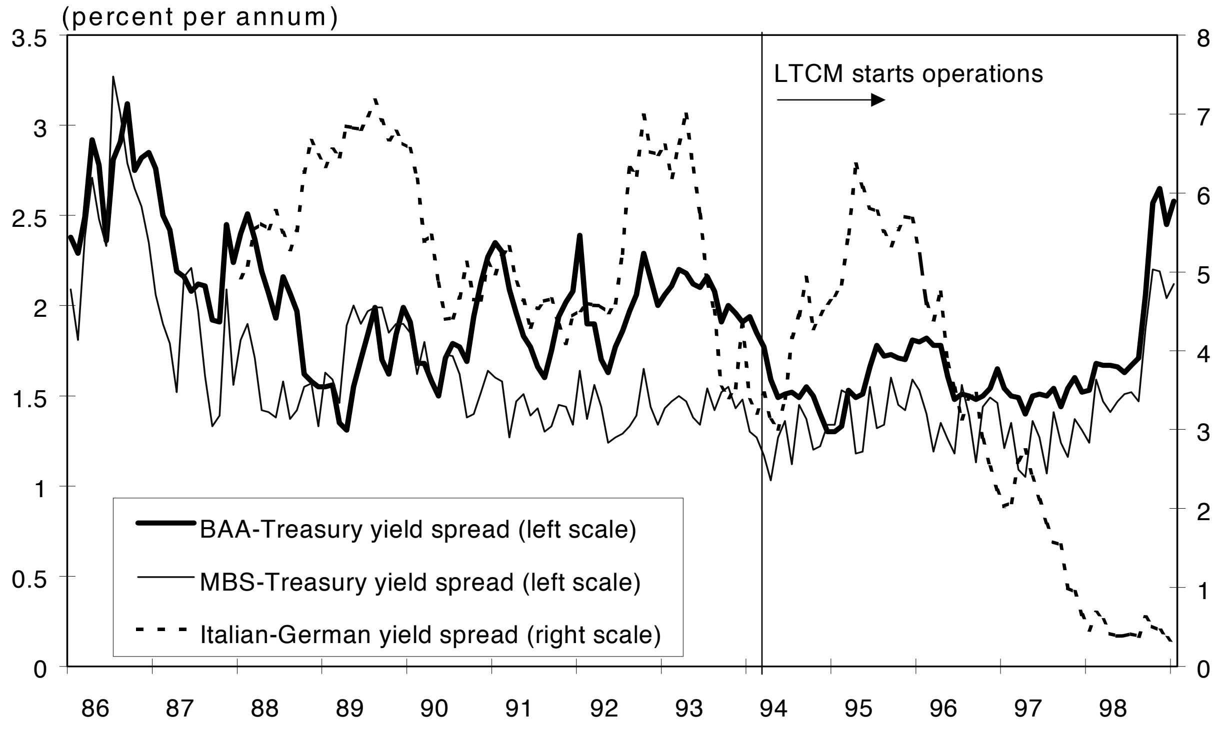
Data Source: Center for Research in Security Prices

Interest Rate Mismatch – Re-pricing Risk

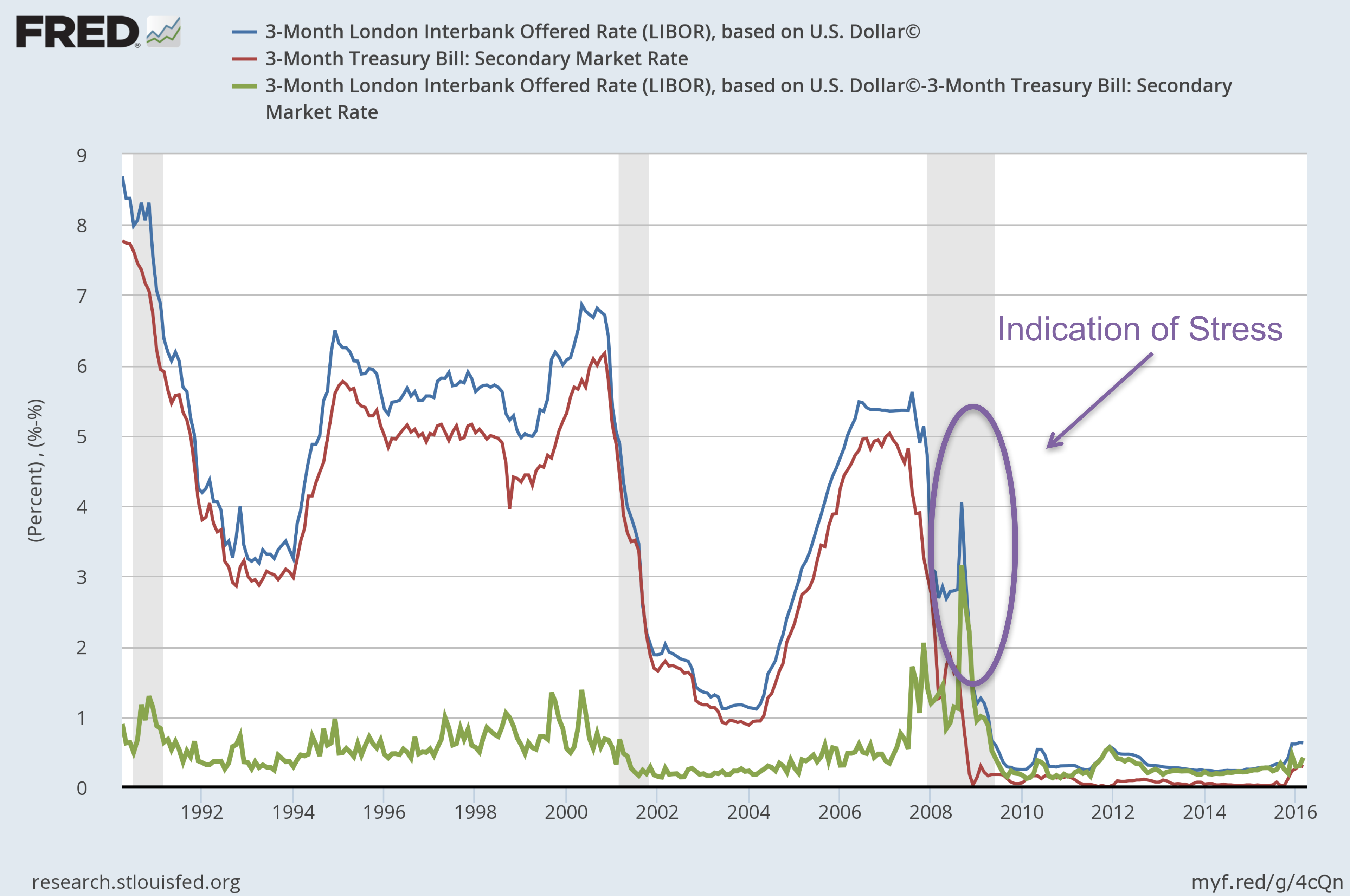
- Savings and Loan (S&L) earned revenue from the difference between long term mortgages (assets) and short term deposits (liabilities)
- Interest rates increased in late 70s and early 80s,
  - S&L received their fixed coupon from mortgages contracted in the past (when rates were low),
  - but now had to pay high interest on new deposits
- This spread sent many S&L out of business

LTCM - Basis Risk

- Long-Term Capital was trading on various, “relative value” trades
- In 1997 some spreads did not converge due to credit and liquidity issues, LTCM ran out of liquidity to fund trades



Source: “Risk Management Lessons from Long Term Capital” – P. Jorion 2000



FRED
3-Month London Interbank Offered Rate (LIBOR), based on U.S. Dollar©
3-Month Treasury Bill: Secondary Market Rate
3-Month London Interbank Offered Rate (LIBOR), based on U.S. Dollar©-3-Month Treasury Bill: Secondary Market Rate
Indication of Stress
(Percent) , (%-%)
9
8
7
6
5
4
3
2
1
0
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
2014
2016
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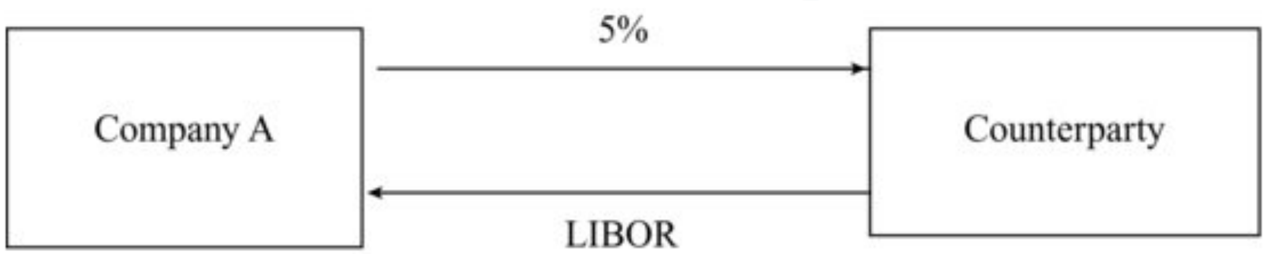
myf.red/g/4cQn

Interest Rate Swaps

- Swap – an agreement to change cash flows in the future
- Interest Rate Swap – an agreement to change cash flows indexed to a floating rate for a fixed rate.

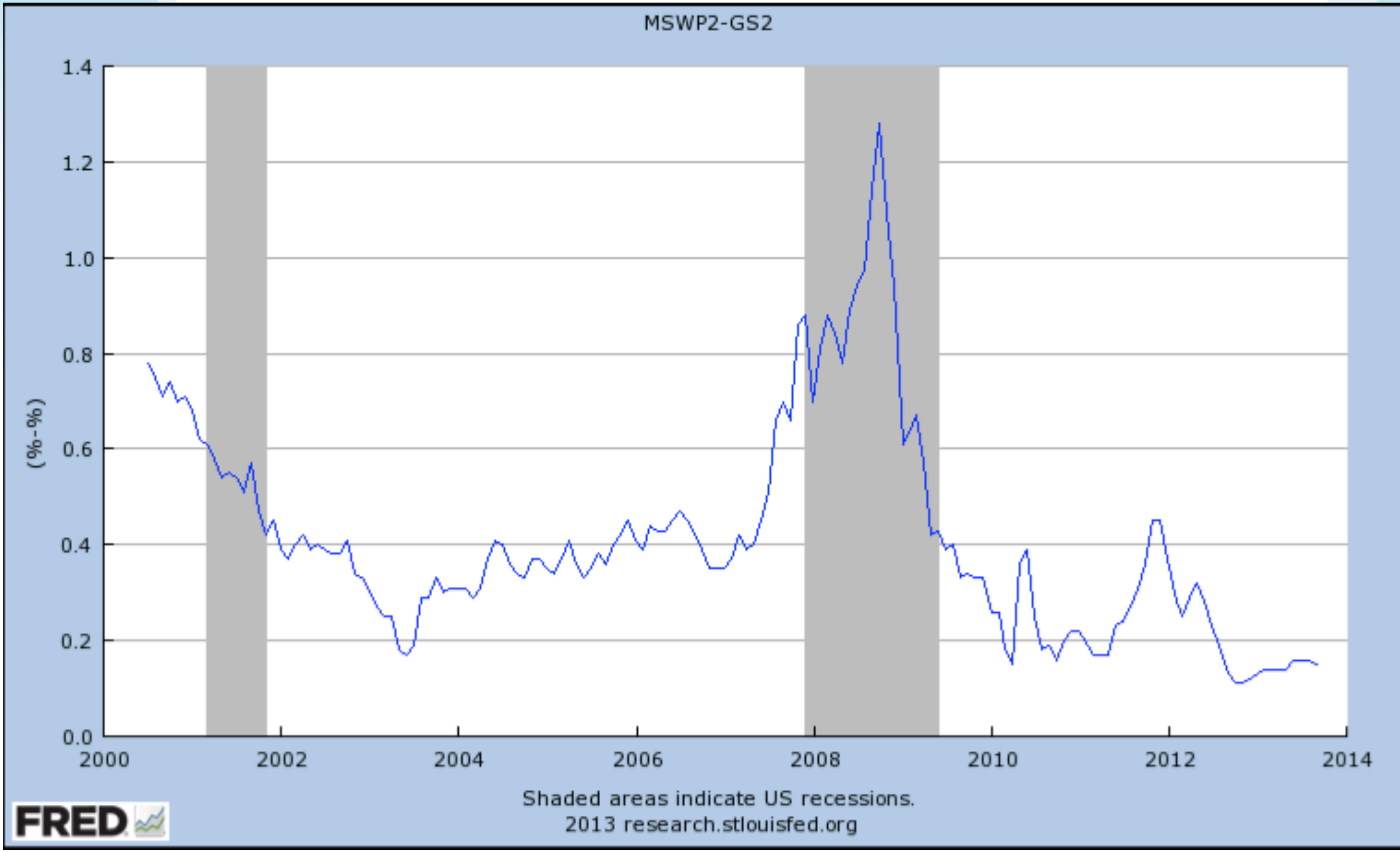
Figure 5.3 A Plain Vanilla Interest Rate Swap



Swap Rates

- Swap rates: Long term fixed rates that are swapped for LIBOR.
- LIBOR/Swap curve: Swap rates are used to extend the LIBOR curve beyond 1 year
- Serve as “risk-free” rates for pricing derivatives in banks, because they reflect AA counterparty risk

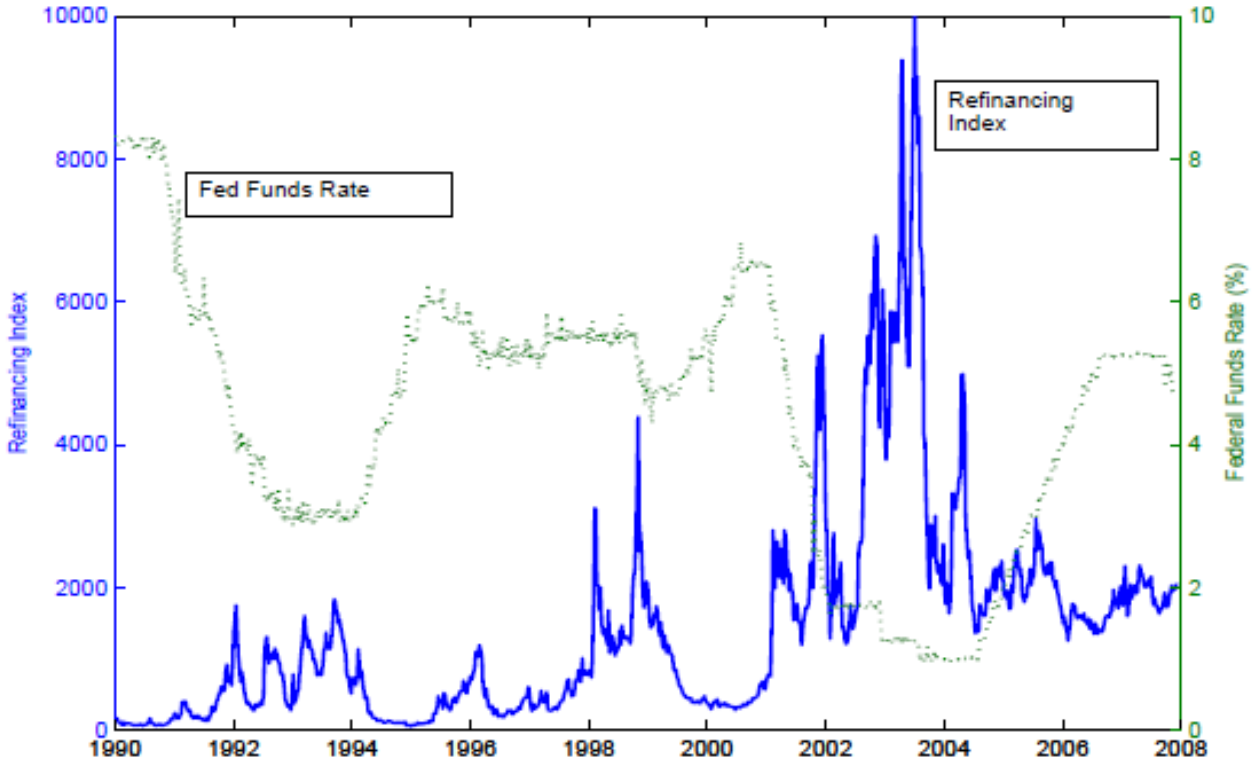
Swap Spread



MSWP2-GS2
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
(%-%)
2000
2002
2004
2006
2008
2010
2012
2014
Shaded areas indicate US recessions.
2013 research.stlouisfed.org
FRED

Prepayment of Mortgages in 2002-2003

Figure 8.4 Refinancing and the Federal Funds Rate



Source: Federal Reserve and Bloomberg.

Bond Prices

- Zero-coupon bonds pay at maturity only.
  – Their price is the value of their Notional, or Face Value, discounted by the relevant spot-rate

$$B = FVe^{-r_i t_i}$$

- Coupon bonds have periodic cash flows, including coupons and notional.
- By arguments of no-arbitrage, a coupon bond Price is equal to the appropriately discounted cash flows:

$$B = \sum_{i=1}^{n} cf_i e^{-r_i t_i} \quad \text{or} \quad B = \sum_{i=1}^{n} \frac{cf_i}{\left(1 + \frac{r_i}{m}\right)^{m \cdot t_i}}$$

Yield To Maturity

- Yield to Maturity is the single rate that will set the present value of cash flows equal to current price:

$$B = \sum_{i=1}^{n} cf_i e^{-yt_i}$$

- The compounding interval affects the yield.
- The yield can be thought of as an average of the interest rates along the different maturities.

Computing Yield - Example

- A semi-annual 10% coupon bond with 3 years to maturity is trading at 94.213, what is the continuously compounded yield on the bond?
- The bond has six more coupon payments (of \$5) and a principal payment in 3 years.
- Use Solver to find *y*:

$$5e^{-0.5y} + 5e^{-1.0y} + 5e^{-1.5y} + 5e^{-2.0y} + 5e^{-2.5y} + 105e^{-3.0y} = 94.213$$

$$y = 12\%$$

Duration

- Duration of a bond that provides cash flow cf_i at time t_i is:

$$D = \sum_{i=1}^{n} t_i \left(\frac{cf_i e^{-yt_i}}{B} \right)$$

- Since: $$\frac{\partial B}{\partial y} = \sum_{i=1}^{n} (-t_i) cf_i e^{-yt_i}$$

- An approximate relationship holds: $$\frac{\Delta B}{B} \approx -D \Delta y$$

Calculation of Duration for a 3-year bond paying a s.a. coupon 10%. Bond yield=12%.

Time (yrs)	Cash Flow ($)	PV ($)	Weight $= \frac{cf_i e^{-yt_i}}{B}$	Time × Weight
0.5	5	4.709	0.050	0.025
1.0	5	4.435	0.047	0.047
1.5	5	4.176	0.044	0.066
2.0	5	3.933	0.042	0.083
2.5	5	3.704	0.039	0.098
3.0	105	73.256	0.778	2.333
Total		94.21	1	**2.653**

Using Duration to Estimate Change in Bond Price

- What will be the change in the bond's price if the yield goes up by 10 basis points?

$$\Delta B = -B \cdot D \cdot \Delta y$$

$$\Delta B = -94.213 \cdot 2.653 \cdot 0.001$$

$$\Delta B = -0.25$$

- Verify result by repricing the bond with *y=12.1%*

Modified Duration

- When the yield y is expressed with compounding m times per year

$$\Delta B = -\frac{B \cdot D \cdot \Delta y}{1 + y/m}$$

- The expression

$$\frac{D}{1 + y/m}$$

is referred to as the "modified duration"

Properties of Duration

- Duration is a measure of average time to cash flows.
- **Duration increases with maturity.** The further out the maturity the more sensitive is the bond to yield changes.
- **Duration is lower for higher coupon bond.** The higher the coupons, the larger are the intermediate coupons relative to the last one. Thus the average time of payments gets closer to today.
- **Duration is equal to Maturity for zero coupon bonds.**

Convexity

The convexity of a bond is defined as:

$$C = \frac{1}{B}\frac{d^2B}{dy^2} = \sum_{i=1}^{n} t_i^2 \cdot \frac{cf_i e^{-yt_i}}{B}$$

which leads to

$$\frac{\Delta B}{B} \approx -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

Figure 4.2 Bond Price Approximation with Duration

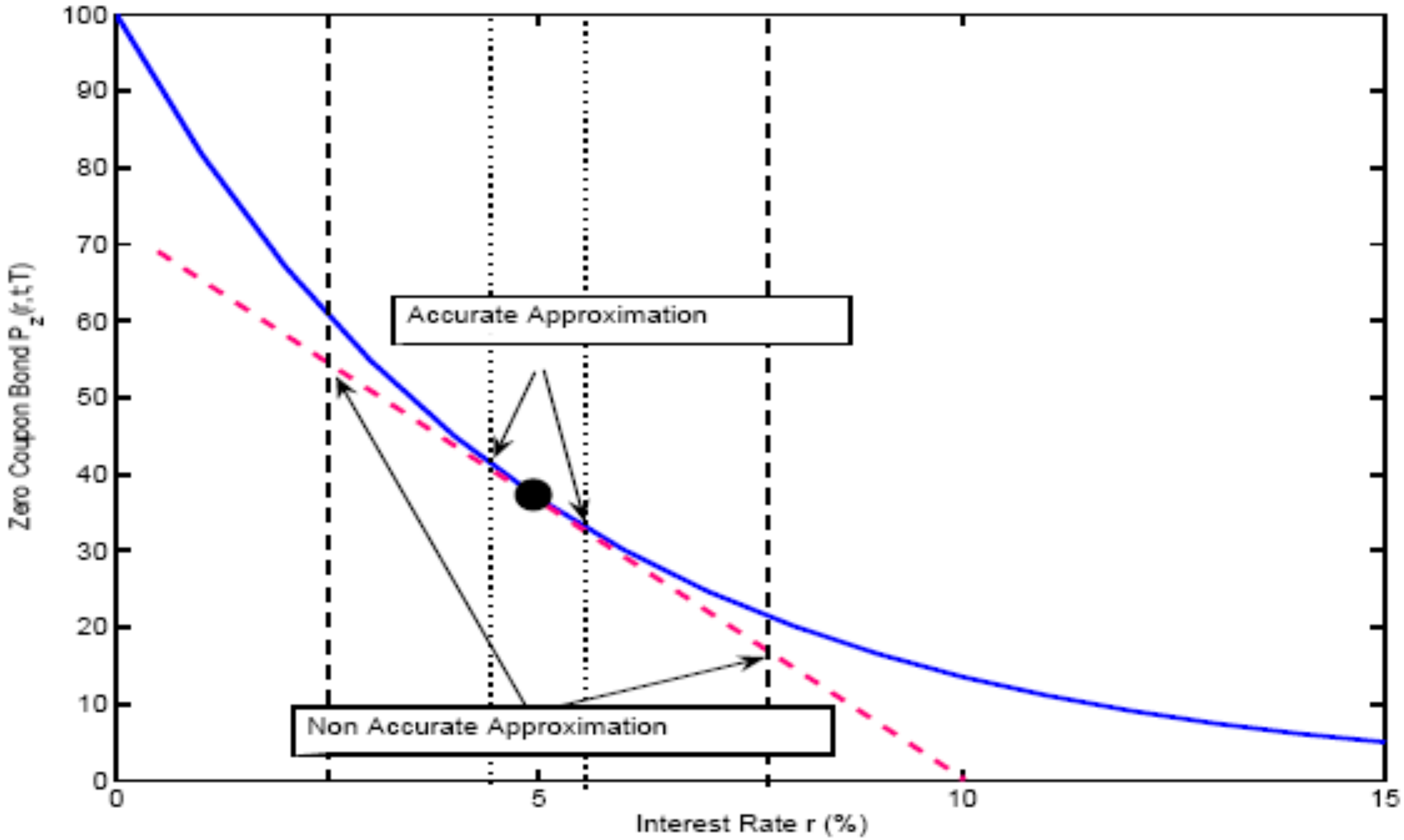
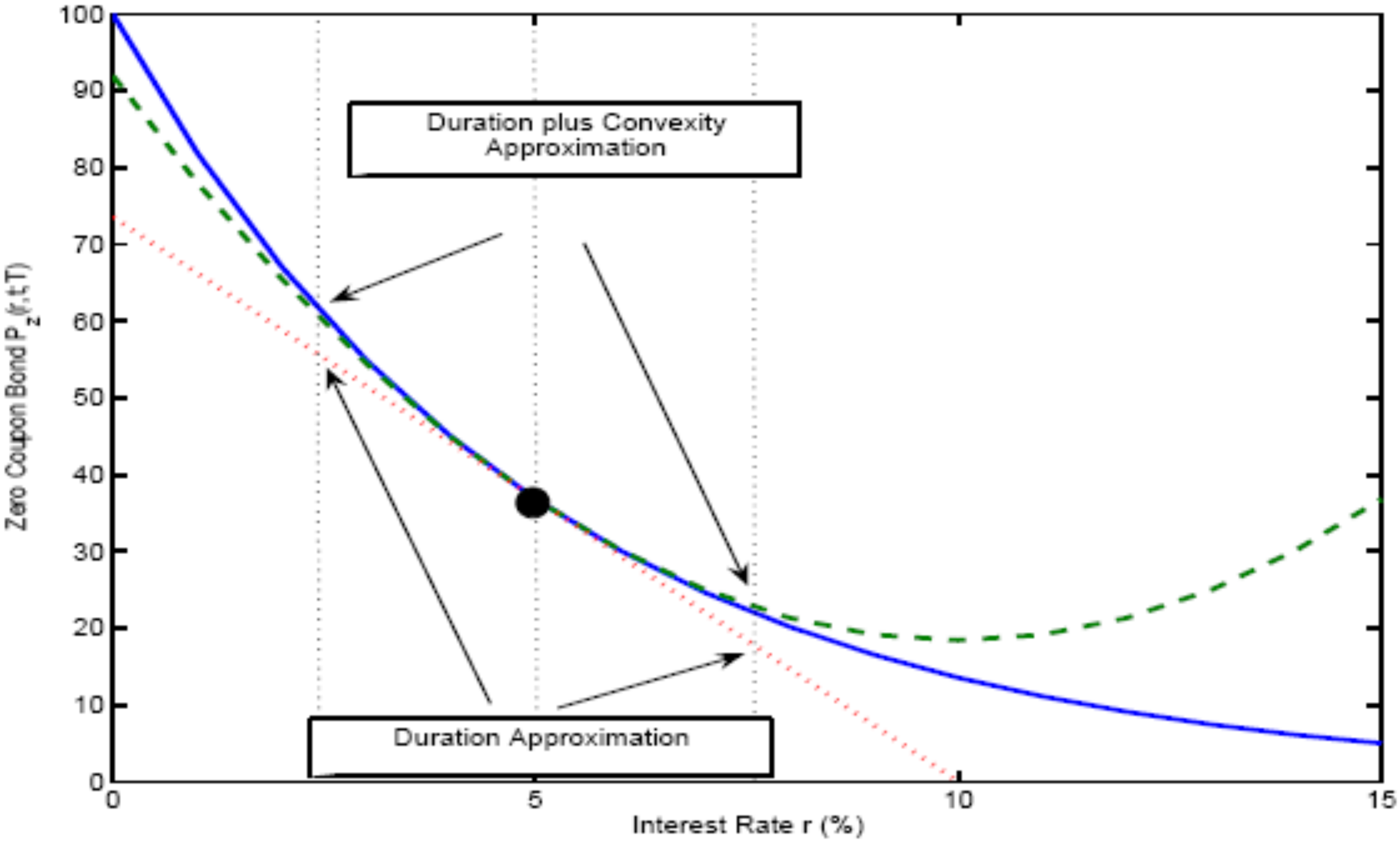


Figure 4.4 Duration plus Convexity Approximation



Portfolios

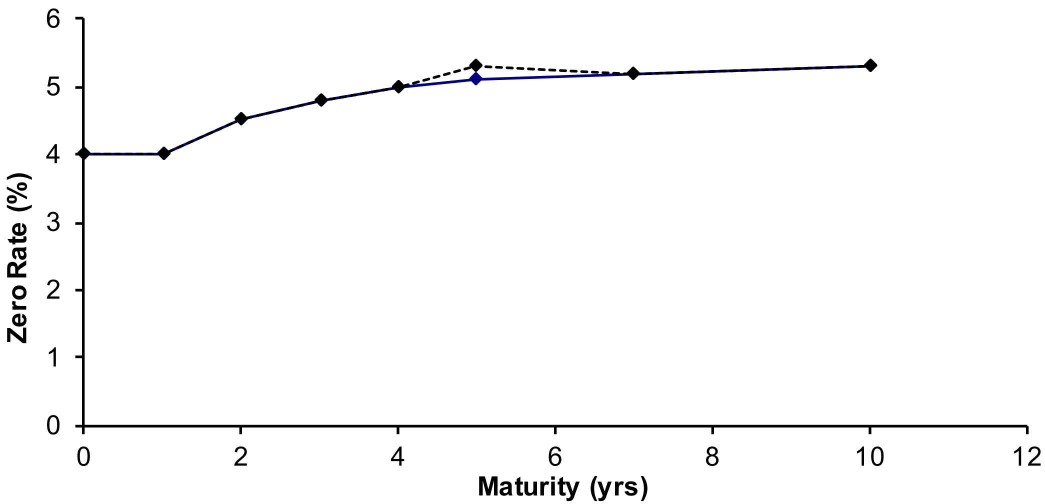
- Duration and convexity can be defined similarly for portfolios of bonds and other interest-rate dependent securities
- The duration of a portfolio is the weighted average of the durations of the components of the portfolio. Similarly for convexity.

Other Measures

- Dollar Duration: Product of the portfolio value and its duration
  - The change in dollar value of the bond for a change in yield
  - The delta of the bond with respect to the yield
- DV01 – Impact of 1 bp parallel shift in all rates
  - Dollar duration * 0.01
- Dollar Convexity: Product of convexity and value of the portfolio

Partial Duration

- Partial Duration – effect on a portfolio of a change to just one point on the zero curve
- Partial Dollar Duration – The dollar change in portfolio value due to change in one rate.



Partial Duration

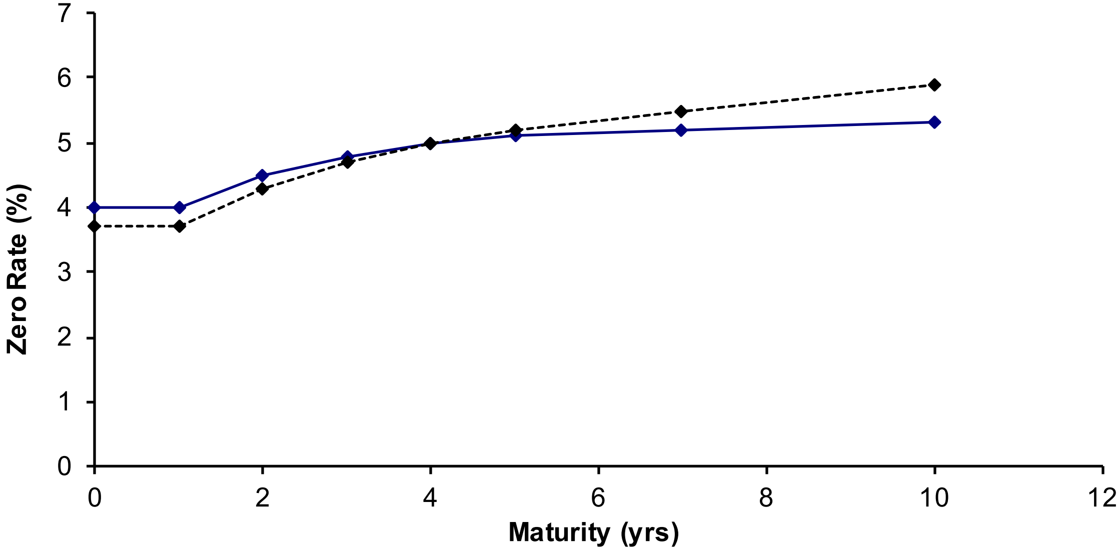
Time (yrs)	Cash Flow ($)	Rate	PV ($) of FV=100	Weight $= \frac{cf_i e^{-r_i t_i}}{B}$	Time *Weight	Time * Weight *Price
					Duration	Dollar Duration
0.5	5	12%	4.71	0.050	0.025	2.35
1	5	12%	4.43	0.047	0.047	4.43
1.5	5	12%	4.18	0.044	0.066	6.27
2	5	12%	3.93	0.042	0.083	7.87
2.5	5	12%	3.70	0.039	0.098	9.26
3	105	12%	73.26	0.778	2.333	219.77
Total			94.21	1	**2.65**	249.948

What would happen to the bond's price if 3-year rate went up by 1%?

Partial Durations and Yield Curve Changes

- Any yield curve change can be defined in terms of changes to individual points on the yield curve
- For example, a rotation can be defined by:

Maturity (yrs)	1	2	3	4	5	7	10
Shock	-3ε	-2ε	-ε	0	+ε	+3ε	+6ε



Impact of Rotation

- Suppose we have a portfolio with the following partial durations:

Maturity yrs	1	2	3	4	5	7	10	Total
Partial Duration	0.2	0.6	0.9	1.6	2.0	−2.1	−3.0	0.2

- The impact of the rotation on the proportional change in value of the portfolio:

$$-[0.2\times(-3\varepsilon)+0.6\times(-2\varepsilon)+\ldots+(-3.0)\times(+6\varepsilon)]=25.0\varepsilon$$

Portfolio Sensitivity to Rates

- An investor has the following position
  - Long FV=$1,000 of 1-year zero coupon
  - Short FV = $4,475 of 5-years zero coupon
  - Long FV=$3,000 of 10-years zero coupon
- Interest Rates are 4%, 5%, 6% continuously compounded for 1-, 5-, 10- yrs respectively
  a. Compute the change in portfolio value for 1bp increase in each of the rates.
  b. Is the portfolio sensitive to a parallel shift in interest rate?
  c. Is the portfolio sensitive to flattening of the curve, i.e. 1bp increase in 1-year and 1bp decrease in 10-yr?

Portfolio Sensitivity to Rates (cont)

Time (yrs)	Notional ($)	PV of CF ($)	Dollar Duration (DD)= PV(CF)*Time		a. Change in Value = −DD*0.0001
1	1000	960.79	960.8		-0.096
5	-4475	-3485.13	-17,425.7		1.742
10	3000	1646.43	16,464.3		-1.646

b. Parallel Shift of rates: Total change in price = -0.096+1.742-1.646= 0

c. Increase in 1-year and decrease in 10-year = -0.096+1.646 = 1.550

Portfolio will increase in value if curve flattens

Principal Components Analysis

- Daily changes in the different maturities are correlated.
- Instead of using so many rates it makes sense to use only 2-3 factors.
- Principal Component Analysis is a method to summarize daily movements using the correlation matrix between the different rates.
- The factors generated are by design independent of each other.

Table 8.7 Factor Loadings for Swap Data

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
1-year	0.216	−0.501	0.627	−0.487	0.122	0.237	0.011	−0.034
2-year	0.331	−0.429	0.129	0.354	−0.212	−0.674	−0.100	0.236
3-year	0.372	−0.267	−0.157	0.414	−0.096	0.311	0.413	−0.564
4-year	0.392	−0.110	−0.256	0.174	−0.019	0.551	−0.416	0.512
5-year	0.404	0.019	−0.355	−0.269	0.595	−0.278	−0.316	−0.327
7-year	0.394	0.194	−0.195	−0.336	0.007	−0.100	0.685	0.422
10-year	0.376	0.371	0.068	−0.305	−0.684	−0.039	−0.278	−0.279
30-year	0.305	0.554	0.575	0.398	0.331	0.022	0.007	0.032

$$\Delta y_1 = 0.216\Delta PC_1 - 0.501\Delta PC_2 + 0.627\Delta PC_3 + \ldots$$

$$\Delta y_2 = 0.331\Delta PC_1 - 0.429\Delta PC_2 + 0.129\Delta PC_3 + \ldots$$

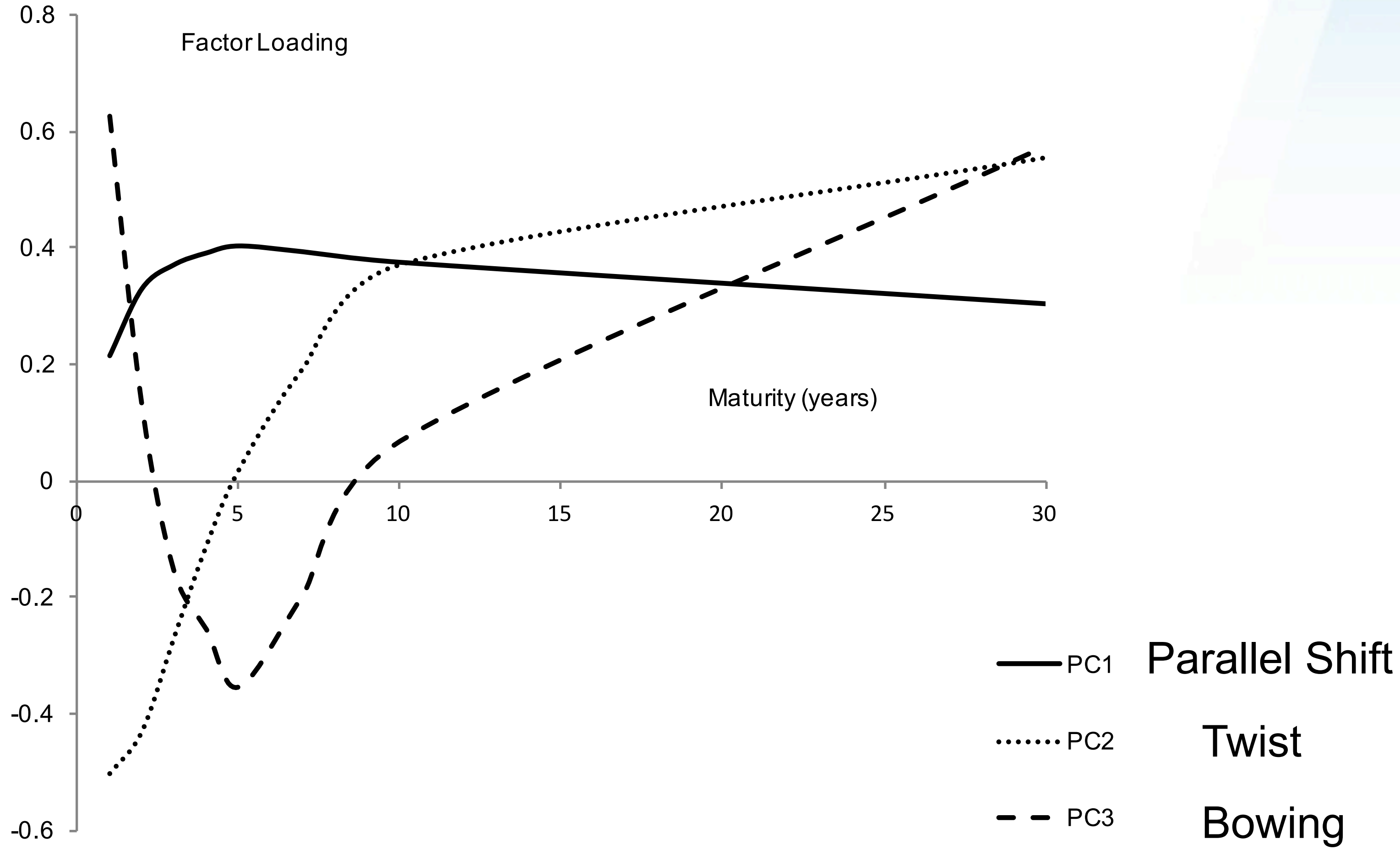
$$\vdots$$

Factor Scores and Variances

- The 8 equations imply that the daily changes in the 8 interest rates can be expressed as daily changes in the factors.
  – These are called daily factor scores.
- We can look at the standard deviation of the factor scores to see how significant is each one:

PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
17.55	4.77	2.08	1.29	0.91	0.73	0.56	0.53

The Three Factors



Three Factors Explain Most Interest Rate Moves

- The total variance is the sum of factor score variances: $17.55^2 + 4.77^2 + \cdots + 0.53^2 = 338.8$
- The first factor, parallel shift, explains 90.9% of variance: $17.55^2 / 338.8$
- The second factor, twist, explains 6.8% of variance
- The third factor, bowing, explains 1.3% of variance

Sensitivity to Changes in Yield Curve using Principal Components

Suppose a portfolio has the following sensitivities to 1-basis-point rate moves, in $ millions:

3-Year Rate	4-Year Rate	5-Year Rate	7-Year Rate	10-Year Rate
+10	+4	−8	−7	+2

How sensitive is the portfolio to each one of the factors?

PC1: $10 \times 0.372 + 4 \times 0.392 - 8 \times 0.404 - 7 \times 0.394 + 2 \times 0.376 = -0.05$

PC2: $10 \times (-0.267) + 4 \times (-0.110) - 8 \times 0.019 - 7 \times 0.194 + 2 \times 0.371 = -3.87$

To what risk is it exposed the most?

PC1: -0.05 * 17.55 = -0.88

PC2: -3.87 * 4.77 = -18.46

Thank You