Financial Risk Management

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VaR II

Allocation and Aggregation of VaR

EADS, Financial Statements

A summary of the VaR position of the Group's financial instruments portfolio at 31 December 2013 and 31 December 2012 is as follows:

(In € million)	Total VaR	Equity price VaR	Currency VaR	Commodity price VaR	Interest rate VaR
31 December 2013					
FX hedges for forecast transactions or firm commitments	577	0	615	0	46
Financing liabilities, financial assets (incl. cash, cash equivalents, securities and related hedges)	156	161	16	0	19
Finance lease receivables and liabilities, foreign currency trade payables and receivables	28	0	4	0	28
Commodity contracts	13	0	1	12	0
Diversification effect	(157)	0	(18)	0	(38)
All financial instruments	**617**	**161**	**618**	**12**	**55**

VaR is used for reporting risk on total portfolio, but also on specific sub-portfolios, and risk-types.

VaR Measures for a Portfolio where an amount x_i is invested in the *i*th component of the portfolio

- Marginal VaR: $$\frac{\partial \text{VaR}}{\partial x_i}$$
- Incremental VaR: Incremental effect of the *i*th component on VaR, i.e. VaR of Portfolio including *i*th component minus VaR of the Portoflio without it.
- Component VaR: $$x_i \frac{\partial \text{VaR}}{\partial x_i}$$

Properties of Component VaR

- The total VaR is the sum of the component VaRs (Euler's theorem)

$$VaR_{Total} = VaR\left(\sum_{i=1}^{M} x_i\right) = \sum_{i=1}^{M} \frac{\partial V}{\partial x_i} x_i = \sum_{i=1}^{M} C_i$$

- The component VaR therefore provides a sensible way of allocating VaR to different activities

VaR Attribution Example

- There are two desks on a US Bank's trading floor:
  - Desk A has EUR 100B exposure
  - Desk B has GBP 75B exposure
- Current USD rates are EUR=1.11, GBP=1.53
- The daily volatility of rates are: 0.45%, 0.35% respectively, the correlation is 0.7.
- Assuming Normal daily changes, what is $VaR_{99\%}$ of the portfolio? What are the Incremental $VaR_{99\%}$ and Component $VaR_{99\%}$ of each desk?

VaR Attribution Example

	EUR	GBP	Aggregate
Position in foreign cur.	100	75	
Exchange rate (S_t)	1.11	1.53	
Dollar position (x_i)	111	114.75	
Daily volatility (σ_i)	0.45%	0.35%	
Correlation (ρ)			0.7
Variance (σ_P^2)			0.692
VaR-99%	1.162	0.934	1.935

$$\sigma_P^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2\rho$$

$$VaR_{99\%} = N^{-1}\left(0.99\right)\sigma_P$$

VaR Attribution Example

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Dollar position (x_i)	111	114.75	
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Correlation (ρ)			0.7
Variance (σ_P^2)			0.692
VaR-99%	1.162	0.934	1.935
Incremental VaR	1.000	0.773	
Marginal VaR	0.010	0.007	
Component VaR	1.091	0.844	1.935

From previous slide: $$\frac{\partial VaR}{\partial x_1} = N^{-1}(0.99) \cdot \frac{x_1\sigma_1^2 + x_2\sigma_1\sigma_2\rho}{\sigma_P}$$

Aggregating VaRs

An approximate approach that is used by many companies:

$$\text{VaR}_{\text{total}} = \sqrt{\sum_i \sum_j \text{VaR}_i \text{VaR}_j \rho_{ij}}$$

where VaR_i is the VaR for the *i*th segment, $\text{VaR}_{\text{total}}$ is the total VaR, and ρ_{ij} is the coefficient of correlation between losses from the *i*th and *j*th segments

Big question: How to determine correlation?

Aggregation under Solvency 2

Capital Requirement are based on VaR (or ES) calculations for different risks.
Various correlation matrices are then used to aggregate.

Insurance Risks:

	mortality	Longevity	disability	lapse	expenses	revision	CAT
mortality	1						
longevity	-0.25	1					
disability	0.25	0	1				
lapse	0	0.25	0	1			
expenses	0.25	0.25	0.5	0.5	1		
revision	0	0.25	0	0	0.5	1	
CAT	0.25	0	0.25	0.25	0.25	0	1

Market Risks:

	interest rate	equity	property	spread	currency
interest rate	1				
Equity	0.5/0	1			
Property	0.5/0	0.75	1		
Spread	0.5/0	0.75	0.5	1	
Currency	0.5	0.5	0.5	0.5	1

Backtesting

Model Validation

- A process whereby we check whether a model is adequate.
- Validation has garnered a lot of regulatory attention, since models are used for capital and liquidity regulation.
- Can be done with various tools
  - Backtesting
  - Stress testing
  - Independent review and oversight
- Examining for:
  - Faulty assumptions
  - Wrong parameters
  - Inaccurate modeling
  - Errors
- This process also provides ideas and directions for improvement.

Backtesting VaR

- Formal verification that actual losses are in line with those that were projected by the model.
- Compare historically the VaR forecasted for each day and the portfolio return for that day.
- We want to check that the VaR is not underestimating the risk, but we also don't want it to be too stringent.
- The testing is complicated by the fact that VaR is not a prediction for the daily return, but a statistical statement about its distribution.

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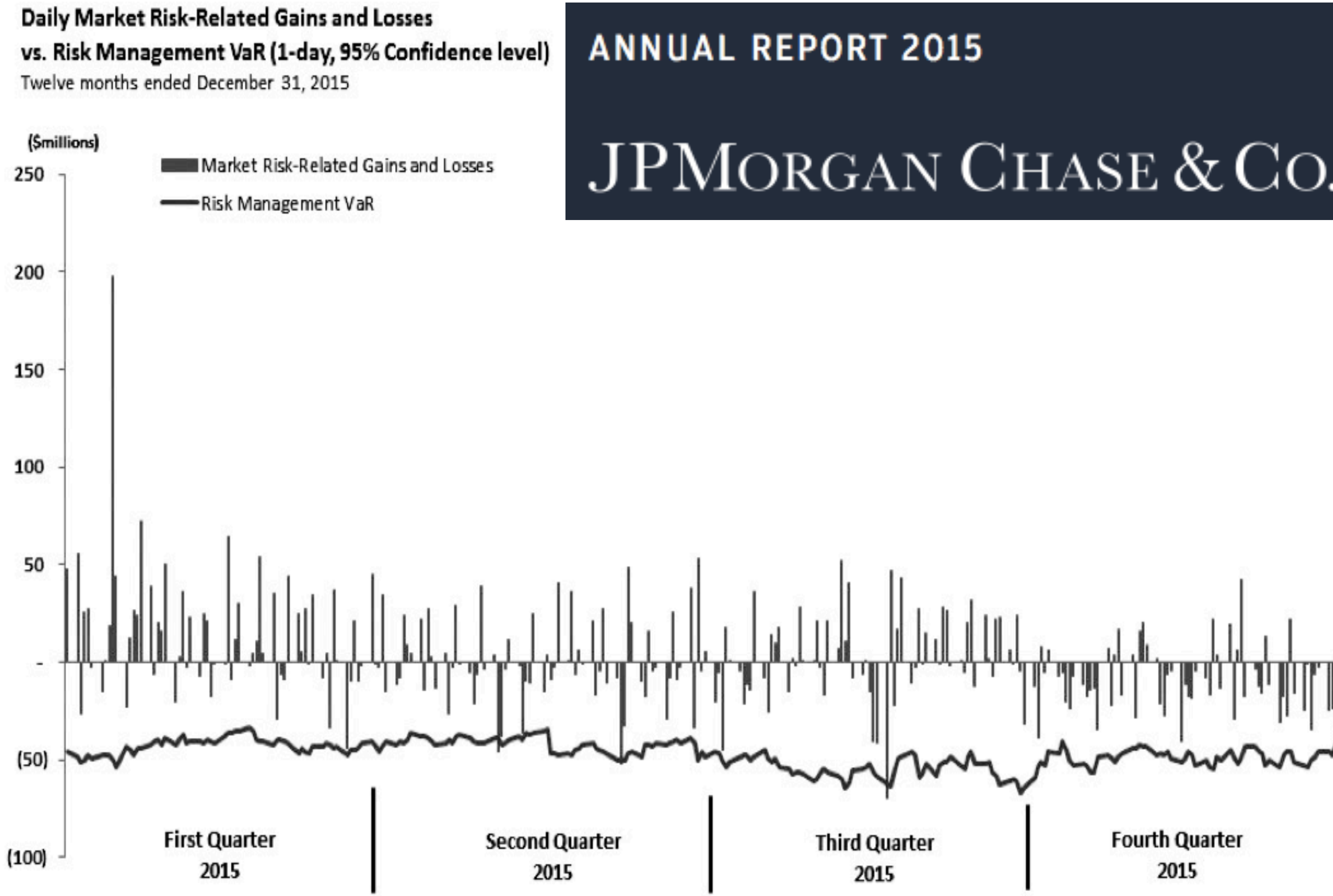


Backtesting

- We look at **exceptions**, i.e. days when loss was greater than VaR.
- If the model is valid then the number of exceptions should be in line with the confidence level.
  – For example, for a 99% 1-day VaR, we would like to have exceptions on 1% of the days
  – If exceptions occur on more than 1% of days, then we might be underestimating the risk
  – If exceptions occur on less than 1% of days, then we might be too conservative.

Example

- Suppose we back-test 1-day 99% VaR over 600 days, how many days do we expect the loss to be greater than VaR?
- Should we reject the model if we observed 9 days where losses were greater than VaR? What about 12?
- We are looking for a statistical framework that will tell us how many exceptions are too many or too few.

Actual vs. Hypothetical Returns

- Every day, VaR is computed based on the portfolio at the end of the previous day, therefore it measures the potential losses if the portfolio is "frozen" through the day.
- In fact, portfolios evolve dynamically through the day.
- We observe actual returns, which reflect intraday trades, as well as other profit items.
- Ideally, backtesting is done by comparing VaR to each of two types of returns:
  - Actual returns: Actual profit/loss that was recorded for the portfolio
  - Hypothetical returns: change in portfolio value assuming no change in portfolio composition

Type 1 vs. Type 2 Errors

	Model	

		Correct	Incorrect
Decision	**Accept**	OK	Type 2 Error
	Reject	Type 1 Error	OK

Type 1 error - What is the probability that we reject a correct model?

- If that probability is very low reject the model.
- Typical hypothesis testing.

Power of a test - What is the probability of rejecting the model when it's really incorrect?

- We need to assume a different model in order to compute this probability.

Statistical Test

To conduct a statistical test we need to determine

- H_0: The value of the parameters if the VaR model holds.
- A **level** for the test (not related to the confidence level of the VaR!!), typically 1% or 5%.
- What is the probability of observing the historical returns/exceptions given H_0. This is the **p-value**.
- If p-value lower than the level of the test then we reject the model.

H_0 Under a Simple Model

- If we assume daily results are serially independent then we have a set of Bernoulli trials.
- If the VaR model holds, then the theoretical probability of an exception is p (=1−α), e.g. for $VaR_{99\%}$: $p=0.01$.
- The probability of m or more exceptions in n days is given by the cumulative binomial distribution:

$$\sum_{k=m}^{n} \frac{n!}{k!(n-k)!} p^{k} (1-p)^{n-k} = 1 - \sum_{k=0}^{m-1} \frac{n!}{k!(n-k)!} p^{k} (1-p)^{n-k}$$

Example – Too many exceptions?

- We back-test a 1-day 99% VaR model over 600 days. There are nine days when the loss was greater than the VaR. Should we reject the model?
  - The probability of 9 or more losses is:

    p-value = 1 – BINOMDIST(8,600,0.01,TRUE) = 0.152.

    Do not reject at 5% confidence level
- What if there were 12 exceptions?
  - The probability of 12 or more losses is **0.019**.

    We reject at 5% confidence level

Example - Are we too conservative?

- Suppose a bank uses 99% daily VaR.
- During the last year there have been **no** exceptions, i.e. losses were always less than VaR.
- The CEO claims that the risk department is too conservative. Is she right?
- What is the probability of no exceptions?
  - p-value = BINOMDIST(0,250,0.01,TRUE) = 0.081
  - We cannot reject the model at 5% confidence level
- For this reason, banks like to backtest VaR at lower levels, e.g. 95%, and apply a multiplicative factor.

Binomial converges to Normal

- By the Central Limit Theorem, the Binomial goes to the Normal distribution in the limit.
- If we have T trials with a *p* probability of an exception, then the number of exceptions *(x)* follows the following:

$$E\left[x\right] = pT$$

$$Var\left[x\right] = p(1-p)T$$

$$x \sim N\left(pT, p(1-p)T\right)$$

Z-value test

This leads to a z-value test: $$z = \frac{x - pT}{\sqrt{p(1-p)T}} \sim N(0,1)$$

Consider our previous example: $$z = \frac{12 - 0.01 * 600}{\sqrt{0.01(1 - 0.01) * 600}} = 2.462$$

Computed to be a p-value of: 0.0069 → Reject the model

Regulatory VaR for Trading Portfolio

The capital required for market risk in the trading portfolio is based on 99% 10-day VaR, which is usually computed by the bank from 1-day VaR:

$$MRC_t = \max\left(VaR_t(0.01), S_t \frac{1}{60}\sum_{i=0}^{59} VaR_{t-i}(0.01)\right) + c$$

The multiplier, S_t, is computed based on N, which is the number of daily exceptions over the last 250 trading days.

$$S_t = \begin{cases} 3.0 & \text{if} \quad N \leq 4 & \text{green} \\ 3 + 0.2(N-4) & \text{if} \quad 5 \leq N \leq 9 & \text{yellow} \\ 4.0 & \text{if} \quad 10 < N & \text{red} \end{cases}$$

Type 1 and Type 2 Errors of Regulatory Test

- What is the probability of a bank with a correct model to be penalized?
  - **1 – BINOMDIST(4,250,0.01,TRUE) = 10.8%**
- What is the probability of an incorrect VaR model, with actual 97% confidence, not being penalized?
  - **BINOMDIST(4,250,0.03,TRUE) = 12.8%**
  - The power is 87.2%. Very low power.

Likelihood Ratio Test

- We can also apply Likelihood Ratio tests
- We test a VaR at confidence level: *1-p*.
- Suppose we observe *x* exceptions over *T* days.
  - MLE estimator of exception probability is $\pi = x/T$
- The (unconditional) likelihood ratio test is:

$$LR_{uc} = 2 \cdot \ln\left[\frac{\left(x/T\right)^{x}\left(1-x/T\right)^{T-x}}{p^{x}(1-p)^{T-x}}\right] \sim \chi^{2}\left(1\right)$$

- Reject at 5% if LR>3.841
- Other rejection boundaries: 6.635 at 1%, 2.706 at 10%

Bunching and conditional coverage

- Bunching occurs when exceptions are not evenly spread throughout the back testing period
  - The serial-independence assumption is invalid
- Statistical tests for bunching are based on extension of the likelihood ratio test.
- To test for independence, we need to satisfy a model for the dependence, which we are trying to reject. The most common one is a Markovian assumption, i.e. the probability of exception today depends on whether there was an exception yesterday (but not farther back than that).

Conditional Coverage – Christoffersen (1998)

	Day Before		Unconditional
Current Day	No Exception	Exception	
No exception	T_{00}	T_{10}	$T\text{-}x$
Exception	T_{01}	T_{11}	x
Total	T_0	T_1	$T=T_0+T_1$

We count the number of exceptions, and classify them according to whether or not the previous day was also an exception.

$\pi = x/T$ – unconditional probability of exception in the sample

$\pi_0 = T_{01}/T_0$ – conditional probability of exception, given no exception the day before

$\pi_1 = T_{11}/T_1$ – conditional probability of exception, given exception the day before

$$LR_{ind} = 2 \cdot \ln\left[\frac{(1-\pi_0)^{T_{00}} (\pi_0)^{T_{01}} (1-\pi_1)^{T_{10}} (\pi_1)^{T_{11}}}{(\pi)^x (1-\pi)^{T-x}}\right] \sim \chi^2(1)$$

Combining Conditional and Unconditional LR Tests

- LR_{ind} is the ratio of likelihood given the Markovian conditional model to the likelihood of the independent observations model.
- We can combine it with LR_{uc} to reject both the independence and the unconditional probability of the model:

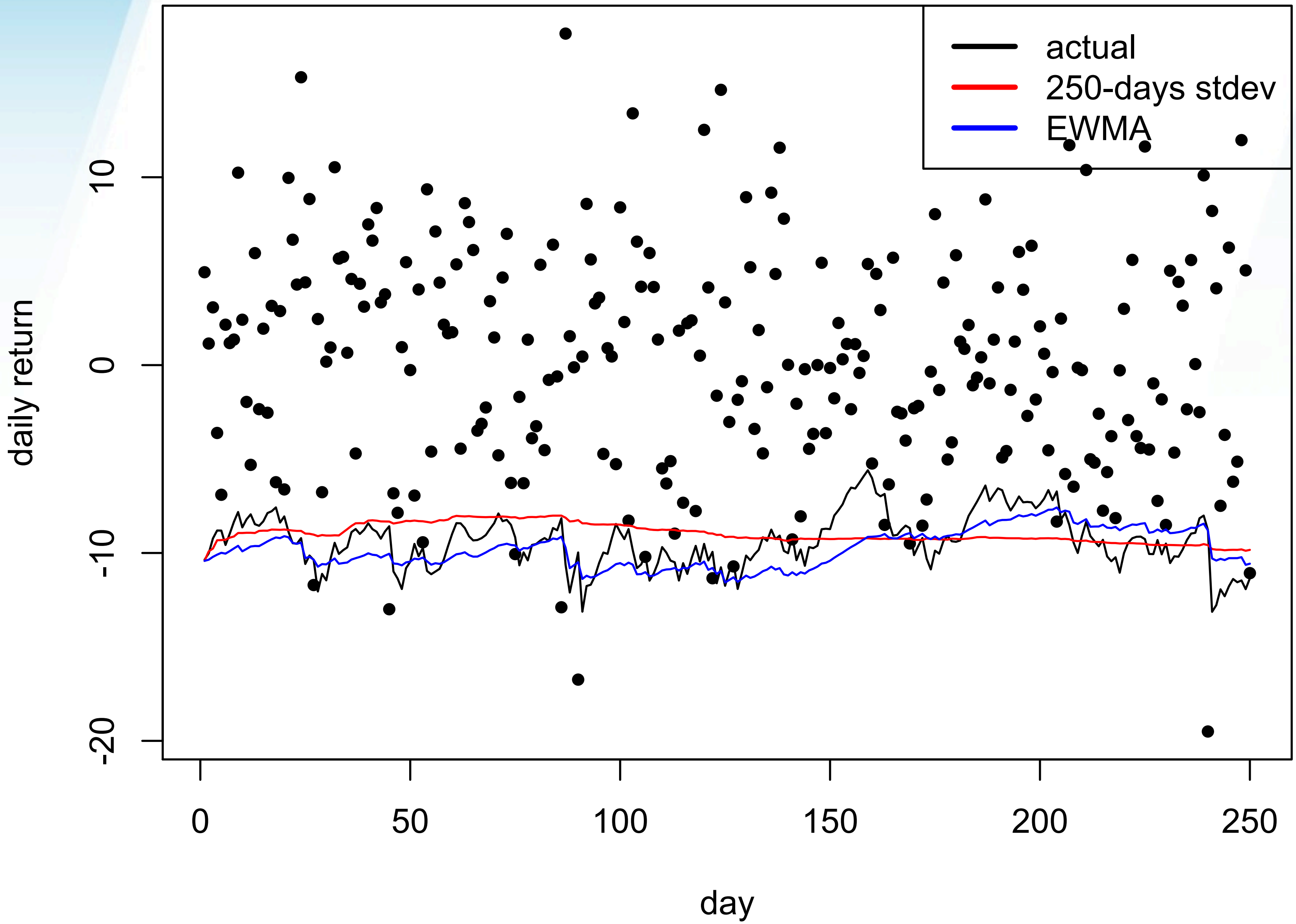
$$LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2(2)$$

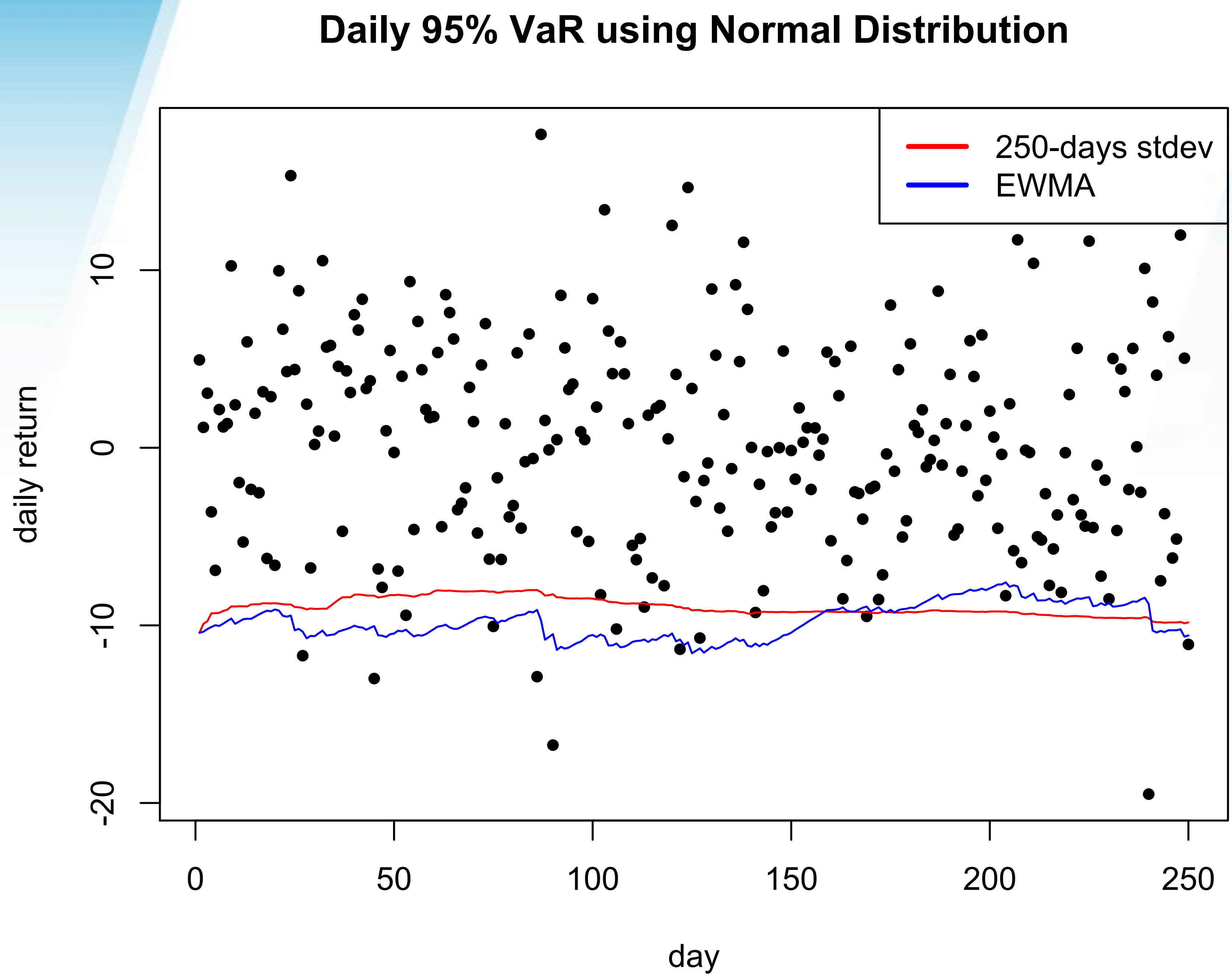
- We reject at 5% if LR > 5.991.

Homework

- Generate 250+250 daily returns, mean=0, and volatility follows EGARCH.
- Estimate daily VaR at 95% assuming: Normal, mean=0, and:
  - Volatility based on last 250 days
  - Volatility based on EWMA, λ=0.97
- Back-test using LR unconditional
- Back-test using conditional coverage.
- Compute power – the probability to reject a false model – i.e. repeat 1000 times and count how often you rejected the false models. How often does the true model get rejected?

Daily 95% VaR using Normal Distribution





Daily 95% VaR using Normal Distribution
250-days stdev
EWMA
daily return
10
0
-10
-20
0
50
100
150
200
250
day

Thanks