# Financial Risk Management

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Credit Risk –

Estimating Default Probabilities

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THINK IN THE NEXT

# Credit Risk

- Risk of an obligor or counterparty default, which would lead to their failure to meet contractual obligations in relation to actual, contingent or potential claims.
- Examples
  - Loans: Mortgages, C&I, CRE
  - Corporate bonds, EM bonds, Muni bonds
  - Lines of Credit, Guarantees
  - Trade Credit
  - Counterparty Credit Risk
  - Credit Default Swap

# Credit Risk Measures

- Probability of Default (PD) – The likelihood that the borrower will fail to make full and timely repayment of its financial obligations
  - Usually measured per year
- Exposure At Default (EAD) – The expected amount of the debt at the time of default
- Loss Given Default (LGD) – The amount of the loss if there is a default, expressed as a percentage of the EAD
  - Equal to (1 – Recovery Rate)
- Expected Loss (EL) = PD*LGD*EAD

# Agenda

- Credit Rating and Historical Data
- Conditional vs Unconditional Default Probabilities
- Rating Methods
  – Statistical Methods
  – Expert Judgment and Scorecards
  – Validation of Ratings
- Recovery Rates
- Market Based Methods
  – CDS and bond spreads
  – Structural / Merton’s model
  – Market Implied Signals

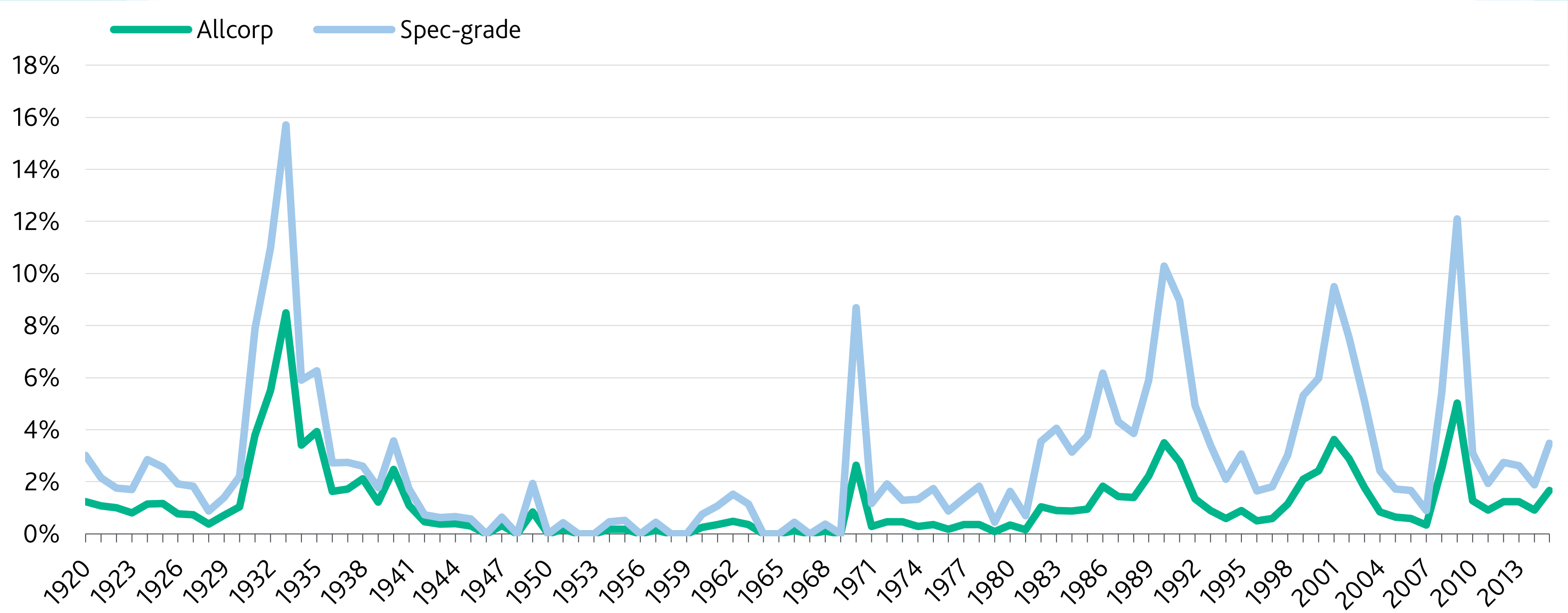
# Credit Rating

- We can allocate exposures into rating groups according to their perceived probability of default
- External ratings – performed by rating agencies (Moody's, S&P, Fitch)
- Internal ratings – performed by the bank/lender/owner of exposure

# Moody’s Rating Scale

| | Long-Term | Short-Term |
|---|---|---|
| Investment-Grade | Aaa | |
| | Aa1 | |
| | Aa2 | |
| | Aa3 | Prime-1 |
| | A1 | |
| | A2 | |
| | A3 | |
| | | Prime-2 |
| | Baa1 | |
| | Baa2 | Prime-3 |
| | Baa3 | |
| Speculative-Grade | Ba1 | |
| | Ba2 | |
| | Ba3 | |
| | B1 | |
| | B2 | |
| | B3 | |
| | Caa1 | Not Prime |
| | Caa2 | |
| | Caa3 | |
| | Ca | |
| | C | |

# Annual Corporate Default Rates




Moody's Annual Default Study, 2016

# Annual Corporate Defaults, 1920-2015

| | Aaa | Aa | A | Baa | Ba | B | Caa-C |
|---|---|---|---|---|---|---|---|
| Mean | 0.000% | 0.059% | 0.093% | 0.273% | 1.032% | 3.197% | 10.450% |
| Median | 0.000% | 0.000% | 0.000% | 0.000% | 0.561% | 2.101% | 7.699% |
| SD | 0.000% | 0.176% | 0.264% | 0.458% | 1.609% | 3.819% | 11.233% |
| Min | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% |
| Max | 0.000% | 0.855% | 1.639% | 1.990% | 11.550% | 19.444% | 50.000% |

Moody's 2016

# Default Rates by Industry - 2015

| Industry | Default Rates* | Industry | Default Rates* |
|---|---|---|---|
| metals & mining | 6.5% | fire: insurance | 0.8% |
| energy: oil & gas | 6.3% | telecommunications | 0.5% |
| consumer goods: non-durable | 4.7% | fire: real estate | 0.5% |
| environmental industries | 4.3% | high tech industries | 0.5% |
| forest products & paper | 4.2% | utilities: electric | 0.3% |
| media: diversified & production | 3.6% | automotive | 0.0% |
| services: consumer | 2.8% | capital equipment | 0.0% |
| aerospace & defense | 2.6% | chemicals, plastics, & rubber | 0.0% |
| construction & building | 2.4% | containers, packaging, & glass | 0.0% |
| hotel, gaming, & leisure | 2.3% | energy: electricity | 0.0% |
| services: business | 2.3% | fire: finance | 0.0% |
| media: advertising, printing & publishing | 2.2% | healthcare & pharmaceuticals | 0.0% |
| Retail | 2.1% | sovereign & public finance | 0.0% |
| consumer goods: durable | 2.0% | transportation: cargo | 0.0% |
| beverage, food, & tobacco | 2.0% | transportation: consumer | 0.0% |
| wholesale | 1.7% | utilities: oil & gas | 0.0% |
| Banking | 1.5% | utilities: water | 0.0% |
| media: broadcasting & subscription | 1.0% | | |



Moody's 2016

# Cumulative Average Default Rates 1983 - 2015

| Rating | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Aaa | 0.000 | 0.013 | 0.013 | 0.039 | 0.068 | 0.102 | 0.139 |
| Aa | 0.024 | 0.067 | 0.123 | 0.210 | 0.321 | 0.419 | 0.517 |
| A | 0.061 | 0.186 | 0.394 | 0.609 | 0.871 | 1.159 | 1.457 |
| Baa | 0.200 | 0.508 | 0.854 | 1.266 | 1.679 | 2.104 | 2.503 |
| Ba | 0.958 | 2.663 | 4.728 | 6.903 | 8.812 | 10.567 | 12.135 |
| B | 3.622 | 8.564 | 13.590 | 18.086 | 22.184 | 25.857 | 29.207 |
| Caa-C | 10.578 | 18.729 | 25.529 | 31.021 | 35.572 | 38.986 | 41.637 |



Moody's 2016

# Cumulative Default Chart

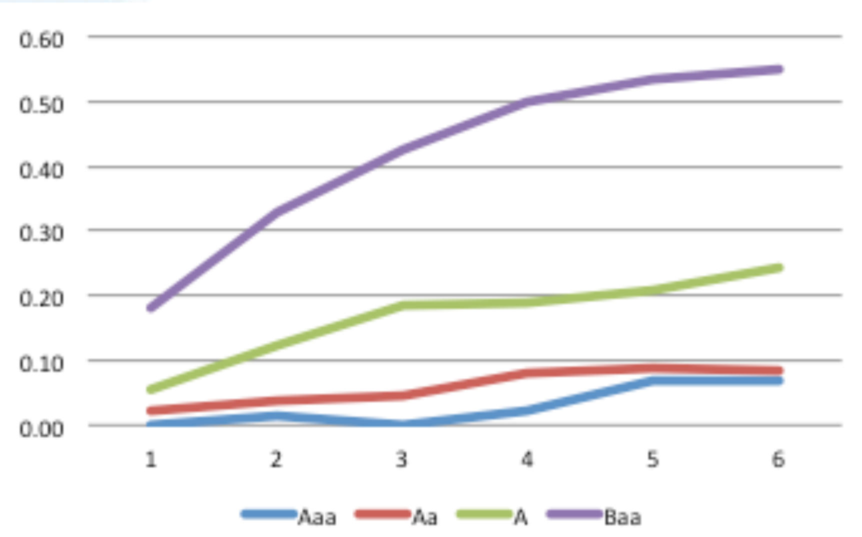
- The table shows the probability of default for companies starting with a particular credit rating
- A company with an initial credit rating of Baa has a probability of 0.200% of defaulting by the end of the first year, 0.508% by the end of the second year, and so on
- It has 2.503% chance of defaulting in 7 years, or 97.497% of not defaulting in 7 years.

# Conditional vs. Unconditional Default Probability

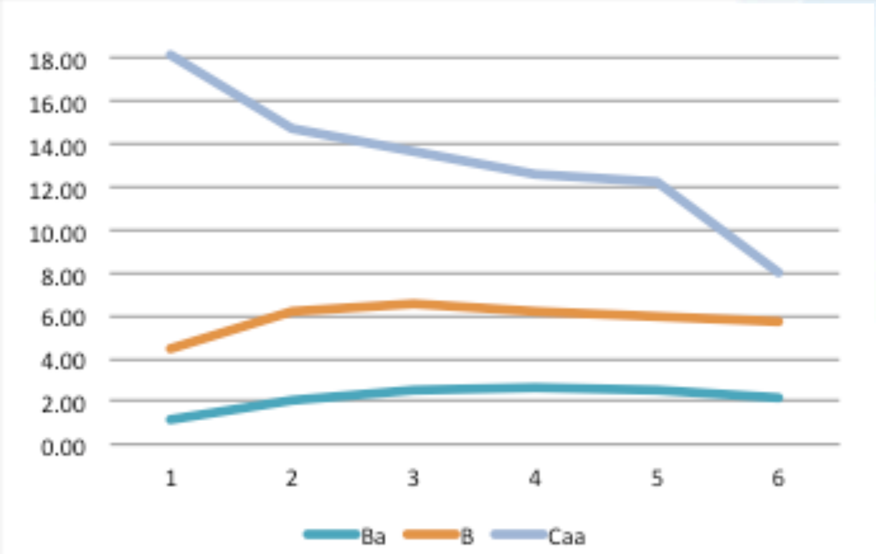
- The unconditional default probability is the probability of default as seen at time zero
  – Example: Baa in year 3: 0.854%-0.508%=0.346%
- The conditional probability of default is the probability of default over a time period conditional on no earlier default
  – Baa in year 3: 0.346%/(100%-0.508%) = 0.348%

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Cumulative PD | 0.200 | 0.508 | 0.854 | 1.266 | 1.679 | 2.104 | 2.503 |
| Unconditional PD | 0.200 | 0.308 | 0.346 | 0.412 | 0.413 | 0.425 | 0.399 |
| Conditional PD | 0.200 | 0.309 | 0.348 | 0.416 | 0.418 | 0.432 | 0.408 |

# Conditional PD in One Year in the Future




For a company that starts with a good credit rating default probabilities tend to increase with time




For a company that starts with a poor credit rating default probabilities tend to decrease with time

# Hazard rates

- Hazard rate is the conditional default probability for unit of time.
- Suppose that $\lambda(t)$ is the hazard rate at time $t$
- The probability of default between times $t$ and $t+\Delta t$ conditional on no earlier default is $\lambda(t)\Delta t$
- The probability of default by time $T$ is

$$1 - e^{-\bar{\lambda}(T) \cdot T}$$

where $\bar{\lambda}(T)$ is the average hazard rate between time zero and time $T$. (see next slide)

# Probability of Default by time T

*V(t)* – Probability of Survival till time *t*
*Q(T)* – Probability of Default by time *T*

The probability of survival till time t+Δt is the probability of survival till time t multiplied by the conditional probability of not defaulting over Δt:

$$V(t+\Delta t) = V(t) * [1 - \lambda(t)\Delta t]$$

$$\frac{V(t+\Delta t) - V(t)}{\Delta t} = -\lambda(t)V(t)$$

$$\frac{dV(t)}{dt} = -\lambda(t)V(t) \Rightarrow V(t) = e^{-\int_0^t \lambda(\tau)d\tau}$$

$$V(T) = e^{-\int_0^T \lambda(t)dt} \Rightarrow Q(T) = 1 - e^{-\int_0^T \lambda(t)dt} = 1 - e^{-\bar{\lambda}(T)\cdot T}$$

# One-Year Rating Transition Matrix (% probability, Moody's 1970-2010)

| Initial | Rating at year end | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Rating | Aaa | Aa | A | Baa | Ba | B | Caa | Ca-C | Default |
| Aaa | 90.42 | 8.92 | 0.62 | 0.01 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 |
| Aa | 1.02 | 90.12 | 8.38 | 0.38 | 0.05 | 0.02 | 0.01 | 0.00 | 0.02 |
| A | 0.06 | 2.82 | 90.88 | 5.52 | 0.51 | 0.11 | 0.03 | 0.01 | 0.06 |
| Baa | 0.05 | 0.19 | 4.79 | 89.41 | 4.35 | 0.82 | 0.18 | 0.02 | 0.19 |
| Ba | 0.01 | 0.06 | 0.41 | 6.22 | 83.43 | 7.97 | 0.59 | 0.09 | 1.22 |
| B | 0.01 | 0.04 | 0.14 | 0.38 | 5.32 | 82.19 | 6.45 | 0.74 | 4.73 |
| Caa | 0.00 | 0.02 | 0.02 | 0.16 | 0.53 | 9.41 | 68.43 | 4.67 | 16.76 |
| Ca-C | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 2.85 | 10.66 | 43.54 | 42.56 |
| Default | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 100.00 |

# Internal Rating

EAD of Advanced IRBA Credit Exposures by PD Grade (including Postbank)

| | iAAA – iAA 0.00 – 0.04 % | iA 0.04 – 0.11 % | iBBB 0.11 – 0.5 % | iBB 0.5 – 2.27 % | iB 2.27 – 10.22 % | iCCC 10.22 – 99.99 % | Default[1] |
|---|---|---|---|---|---|---|---|
| **Central Governments** | | | | | | | |
| EAD gross in € m. | 85,351 | 4,948 | 2,804 | 1,404 | 732 | 423 | – |
| EAD net in € m. | 93,599 | 6,227 | 2,533 | 583 | 207 | 50 | – |
| Average PD in % | – | 0.08 | 0.30 | 1.40 | 5.67 | 13.05 | 100.00 |
| Average LGD in % | 49.24 | 39.44 | 42.77 | 11.04 | 42.70 | 48.91 | 5.00 |
| Average RW in % | 0.49 | 23.16 | 49.88 | 25.96 | 165.01 | 215.08 | 62.50 |
| **Institutions** | | | | | | | |
| EAD gross in € m. | 15,719 | 31,913 | 13,132 | 2,706 | 2,251 | 481 | 166 |
| EAD net in € m. | 16,636 | 32,136 | 11,890 | 2,356 | 2,191 | 481 | 166 |
| Average PD in % | 0.04 | 0.07 | 0.25 | 1.08 | 3.00 | 21.77 | 100.00 |
| Average LGD in % | 31.64 | 27.03 | 19.44 | 21.83 | 4.59 | 5.51 | 13.43 |
| Average RW in % | 5.54 | 11.10 | 22.18 | 53.91 | 16.29 | 30.79 | 25.55 |
| **Corporates** | | | | | | | |
| EAD gross in € m. | 76,225 | 65,701 | 66,759 | 50,632 | 21,795 | 5,753 | 7,598 |
| EAD net in € m. | 78,535 | 64,830 | 62,096 | 45,023 | 18,351 | 4,993 | 7,361 |
| Average PD in % | 0.03 | 0.07 | 0.24 | 1.17 | 4.70 | 23.56 | 100.00 |
| Average LGD in % | 32.63 | 34.72 | 30.90 | 24.84 | 22.79 | 16.78 | 28.19 |
| Average RW in % | 9.50 | 17.86 | 31.06 | 49.72 | 79.28 | 92.15 | 24.14 |
| **Retail Exposures Secured by Real Estate Property** | | | | | | | |
| EAD gross in € m. | 2,766 | 9,976 | 45,086 | 67,241 | 12,762 | 5,432 | 2,680 |
| EAD net in € m. | 2,766 | 9,976 | 45,078 | 67,203 | 12,730 | 5,410 | 2,665 |
| Average PD in % | 0.03 | 0.08 | 0.29 | 1.05 | 4.70 | 21.24 | 100.00 |
| Average LGD in % | 12.13 | 15.18 | 10.40 | 12.21 | 9.69 | 8.85 | 17.99 |
| Average RW in % | 1.36 | 4.88 | 5.72 | 16.50 | 31.73 | 53.92 | 14.53 |

# Internal Rating Methods

- Credit Scoring –
  - Statistical methods
  - Typically used for retail credit, credit cards and increasingly for SME
  - Applied in automatic fashion
- Expert Judgment and Scorecards –
  - Combined quantitative and qualitative method
  - Typically used for larger firms, sovereigns and munis
  - Applied on a credit by credit basis

# Qualitative Response (QR) Models

- What is the probability of default in the next period, conditional on current market and firm characteristics (typically accounting ratios)?
- The models take the form of:

$$P[D_i = 1 \mid X_i = x] = F(\alpha + \beta x)$$

- Probit Model: $F(z) = N(z)$

- Logit Model: $F(z) = \dfrac{e^z}{1+e^z}$

# Fitting QR Models using MLE

- Suppose we observe N companies, with characteristic $x_i$ and an indicator whether they defaulted, $D_i$
- We can write the likelihood of one observation:

$$L_i = \left[F(\alpha + \beta x_i)\right]^{D_i} \left[1 - F(\alpha + \beta x_i)\right]^{1-D_i}$$

- The log likelihood of observing the data:

$$\log L = \sum_{i=1}^{N} D_i \log\left[F(\alpha + \beta x_i)\right] + (1 - D_i)\log\left[1 - F(\alpha + \beta x_i)\right]$$

# Example

In this example, we will analyze the data in the `CreditCard` data set in R's `AER` package. The following variables are included in the data set:

1. `card` = Was the application for a credit card accepted?
2. `reports` = Number of major derogatory reports
3. `income` = Yearly income (in USD 10,000)
4. `age` = Age in years plus 12ths of a year
5. `owner` = Does the individual own his or her home?
6. `dependents` = Number of dependents
7. `months` = Months living at current address
8. `share` = Ratio of monthly credit card expenditure to yearly income
9. `selfemp` = Is the individual self-employed?
10. `majorcards` = Number of major credit cards held
11. `active` = Number of active credit accounts
12. `expenditure` = Average monthly credit card expenditure

# Linear Discriminant Analysis

- We suppose that firm characteristics (*x*) have a multivariate Normal distribution conditional on whether the firm defaults or not.
- Assume they have the same covariance in both cases, but different means:

$$f_1(x \mid D=1) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(x-\mu_1)'\Sigma^{-1}(x-\mu_1)}$$

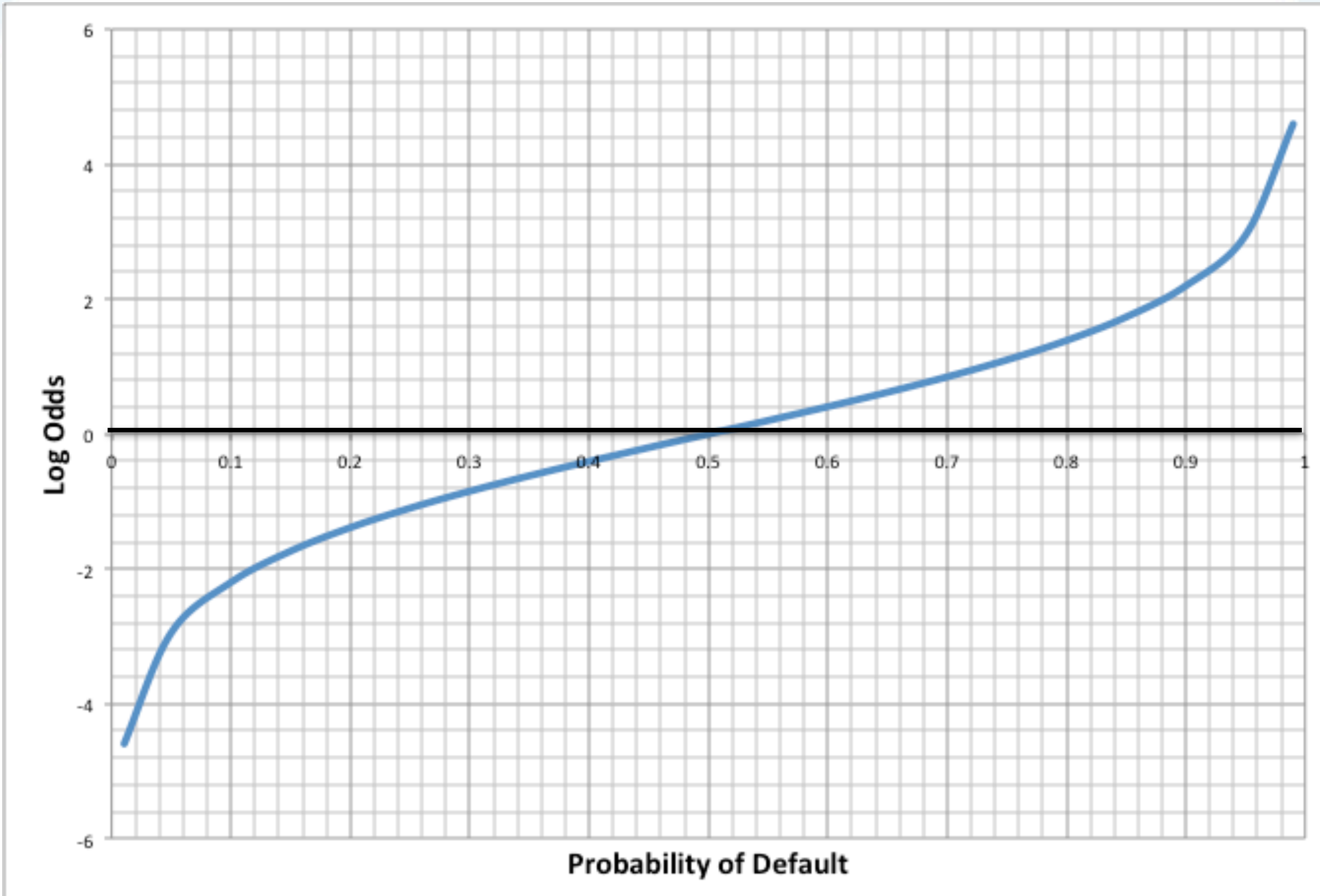
$$f_0(x \mid D=0) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(x-\mu_0)'\Sigma^{-1}(x-\mu_0)}$$

- We can estimate the mean and covariance by their sample statistics.

# Linear Discriminant Analysis (cont)

- By Bayes Rule: $$P(D=1 \mid X=x) = \frac{f_1(x) \times P(D=1)}{f_1(x) \times P(D=1) + f_0(x) \times P(D=0)}$$ $$P(D=0 \mid X=x) = \frac{f_0(x) \times P(D=0)}{f_1(x) \times P(D=1) + f_0(x) \times P(D=0)}$$
- We estimate P(D=1) by π, the proportion of defaults in the sample. P(D=0) is 1-π.
- Define log odds as: $$\log\left[\frac{P(D=1 \mid x)}{P(D=0 \mid x)}\right]$$
- It is a transformation of the probability of default

# Log Odds vs Probability



# LDA – Log Odds and Linear Rule

$$\log\left[\frac{P(D=1\mid x)}{P(D=0\mid x)}\right]=\log\left[\frac{f_1(x)\times P(D=1)}{f_0(x)\times P(D=0)}\right]=$$

$$=-\frac{1}{2}(x-\mu_1)'\Sigma^{-1}(x-\mu_1)+\frac{1}{2}(x-\mu_0)'\Sigma^{-1}(x-\mu_0)+\log\frac{\pi}{1-\pi}=$$

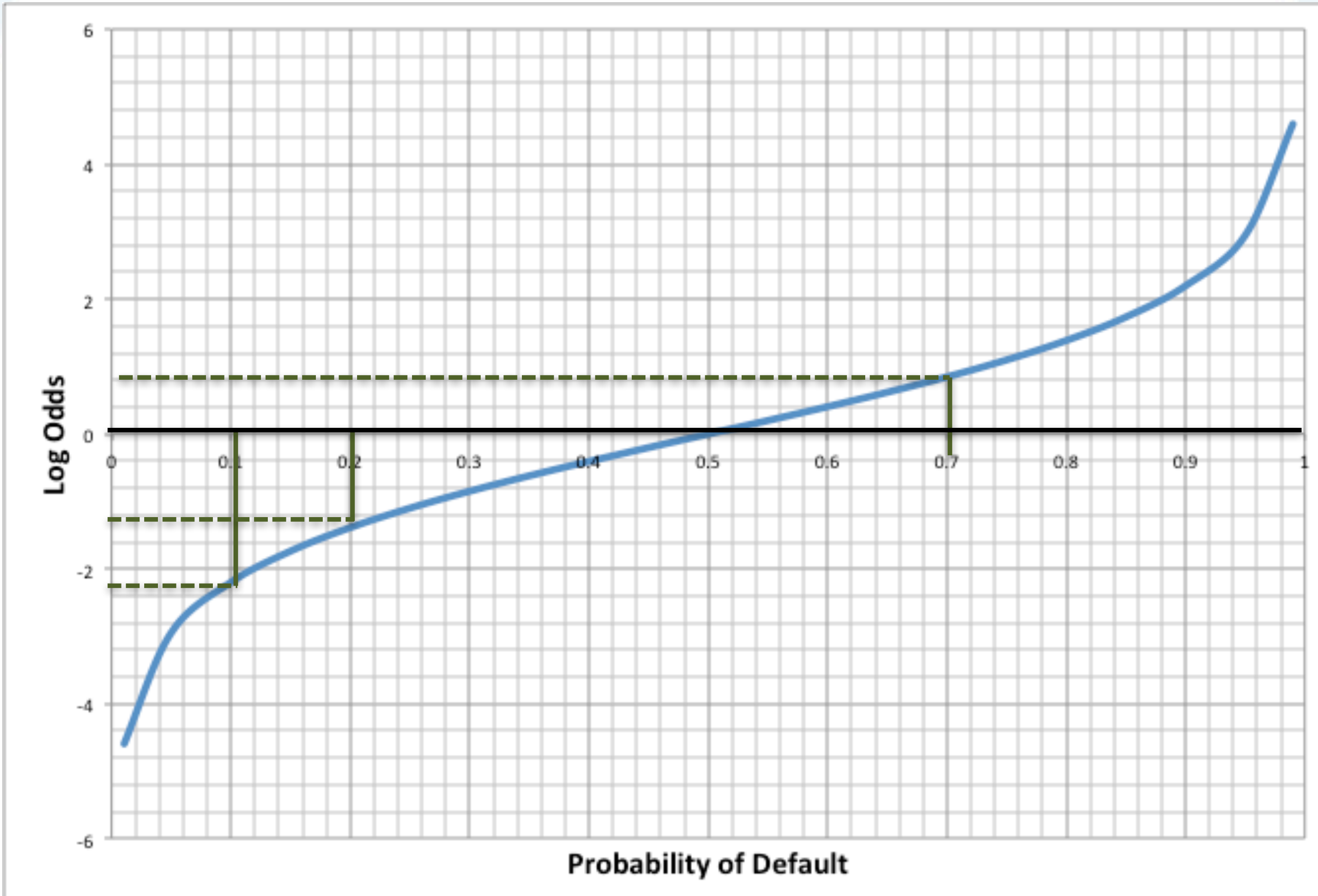
$$=(\mu_1-\mu_0)'\Sigma^{-1}x-\frac{1}{2}(\mu_1+\mu_0)'\Sigma^{-1}(\mu_1-\mu_0)+\log\frac{\pi}{1-\pi}$$

This is a linear combination of the *x*'s: $\log\text{-}odds = b_0 + b_1 \cdot x$

$$b_0=\log\frac{\pi}{1-\pi}-\frac{1}{2}(\mu_1+\mu_0)'\Sigma^{-1}(\mu_1-\mu_0)$$

$$b_1=(\mu_1-\mu_0)'\Sigma^{-1}$$

# LDA – Multiple Ratings



# Altman's Z-score
## (Manufacturing companies)

- $X_1$=Working Capital/Total Assets
- $X_2$=Retained Earnings/Total Assets
- $X_3$=EBIT/Total Assets
- $X_4$=Market Value of Equity/Book Value of Liabilities
- $X_5$=Sales/Total Assets

# What do the ratios mean?

- X1 measures liquidity
- X2 is indicative of cumulative profitability, but also age of firm and leverage
- X3 is a measure of underlying profitability
- X4 measures how much assets can drop in market value before they don’t cover liabilities
- X5 Firm’s ability to compete and generate revenues

# Conditional Means in Altman's Population

| Variable | Bankrupt Group Mean[n] | Nonbankrupt Group Mean[n] | F Ratio[n] |
|---|---|---|---|
| $X_1$ | -6.1% | 41.4% | 32.50* |
| $X_2$ | -62.6% | 35.5% | 58.86* |
| $X_3$ | -31.8% | 15.4% | 26.56* |
| $X_4$ | 40.1% | 247.7% | 33.26* |
| $X_5$ | 1.5X | 1.9X | 2.84 |

N = 33.

To estimate the LDA we use the conditional means of the characteristics conditional on bankruptcy.
There is a significant difference between the means of the variables conditional on whether there was default.

# Altman's Z-score
## (Manufacturing companies)

$$Z = 1.2X_1+1.4X_2+3.3X_3+0.6X_4+1.0X_5$$

$Z > 3.0$: default is unlikely;

$2.7 < Z < 3.0$: we should be on alert;

$1.8 < Z < 2.7$: moderate chance of default;

$Z < 1.8$: high chance of default

# Log Odds and Logit

- The log odds is linear in *x* for the LDA.
- It is also the case for Logit:

$$P(D=1|x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}} \Rightarrow P(D=0|x) = \frac{1}{1+e^{\alpha+\beta x}}$$

$$\log\frac{P(D=1|x)}{P(D=0|x)} = \log\frac{e^{\alpha+\beta x}(1+e^{\alpha+\beta x})}{1+e^{\alpha+\beta x}} = \alpha+\beta x$$

- The two models are not the same since LDA has a restriction on the conditional distribution of $x_i$ given $D_i$.

# Model Estimation

- Probit and Logit models can be estimated in R using glm:

glm(formula = card~income, family = binomial (link="probit"), data=CreditCard)

- Likelihood estimation techniques (Fisher Information and LR tests) can be used to select variables and assess goodness of fit.
- LDA implies computing sample averages and covariance matrix

# Scorecards

- Rating agencies and commercial lenders typically use scorecards that incorporate expert judgment with quantitative models
- The scorecards are particular to certain industries, types of projects, or company size
- The results are interpreted as ratings which are then calibrated to PD

## Global Packaged Goods Industry

| Broad Rating factors | Factor Weightings | Rating Sub Factor | Sub-Factor Weighting |
|---|---|---|---|
| Scale and Diversification | 44% | Total Sales (USD Billions) | 20% |
| | | Geographic Diversification | 12% |
| | | Segmental Diversification | 12% |
| Franchise Strength and Potential | 14% | Market Share | 7% |
| | | Category Assessment | 7% |
| Profitability | 7% | EBIT Margin | 7% |
| Financial Policy | 14% | Financial Policy | 14% |
| Leverage and Coverage | 21% | Debt/EBITDA | 7% |
| | | RCF/Net Debt | 7% |
| | | EBIT/Interest Expense | 7% |
| Total | 100% | Total | 100% |

## Global Pharmaceutical Industry

| Broad Rating Factors | Factor Weighting | Rating Sub-Factors | Sub-Factor Weighting |
|---|---|---|---|
| Scale | 25% | Revenue | 25% |
| Business Profile | 25% | Product and Therapeutic Diversity | 15% |
| | | Geographic Diversity | 10% |
| Patents and Pipeline | 16% | Patent exposures | 8% |
| | | Pipeline quality | 8% |
| Leverage and Cash Coverage | 24% | Debt/EBITDA | 9% |
| | | (Cash Flow from Operations)/Debt | 9% |
| | | Pharmaceutical Cash Coverage of Debt | 6% |
| Financial Policy | 10% | Financial Policy | 10% |
| **Total** | 100% | **Total** | 100% |

# Quantitative Categories

**Global Chemical Industry**

| Broad Rating Factor | Factor Weighting | Rating Sub-Factor | Sub-Factor Weighting |
|---|---|---|---|
| Scale | 20% | Revenues | 10% |
| | | PP&E (net) | 10% |
| Business Profile | 20% | Business Profile | 20% |
| Profitability | 10% | EBITDA Margin | 5% |
| | | ROA - EBIT/Avg. Assets | 5% |
| Leverage & Coverage | 30% | Debt / EBITDA | 10% |
| | | EBITDA / Interest Expense | 10% |
| | | Retained Cash Flow / Debt | 10% |
| Financial Policy | 20% | Financial Policy | 20% |
| **Total** | **100%** | **Total** | **100%** |

**Factor 1**

Scale (20%)

| Sub-factor | Sub-factor Weight | Aaa | Aa | A | Baa | Ba | B | Caa | Ca |
|---|---|---|---|---|---|---|---|---|---|
| Revenues (USD Billions) | 10% | ≥ $100 | $50 - $100 | $15 - $50 | $5 - $15 | $1.5 - $5 | $0.2 - $1.5 | $0.1 - $0.2 | < $0.1 |
| PP&E (net) (USD Billions) | 10% | ≥ $40 | $20 - $40 | $8 - $20 | $3 - $8 | $0.6 - $3 | $0.025 - $0.6 | $0.005 - $0.025 | < $0.005 |

**Factor 4**

Leverage & Coverage (30%)

| Sub-factor | Sub-factor Weight | Aaa | Aa | A | Baa | Ba | B | Caa | Ca |
|---|---|---|---|---|---|---|---|---|---|
| Debt / EBITDA | 10% | < 0.5x | 0.5x - 1.25x | 1.25x - 2x | 2x - 3x | 3x - 4x | 4x - 6x | 6x - 8x | ≥ 8x |
| EBITDA / Interest Expense | 10% | ≥ 40x | 25x - 40x | 15x - 25x | 8x - 15x | 2x - 8x | 1x - 2x | 0.5x - 1x | < 0.5x |
| Retained Cash Flow / Debt | 10% | ≥ 95% | 60% - 95% | 30% - 60% | 20% - 30% | 10% - 20% | 5% - 10% | 1% - 5% | < 1% |

# Qualitative Categories

**Factor 2**

Business Profile (20%)

| Sub-factor | Sub-factor Weight | Aaa | Aa | A | Baa |
|---|---|---|---|---|---|
| Business Profile | 20% | Expected to have highly stable cash flow generation across industry and economic cycles supported by highly diverse specialty product lines with dominant market positions, no concentration of cash flow sources, stable end markets, global leading/low cost operations and structural cost advantages. Technological leadership limits threats to competitive position and supports improving existing market positions and new market opportunities. | Expected to have very stable cash flow generation across industry and economic cycles supported by diverse specialty product lines with leading market positions, low concentration of cash flow sources, stable end markets, global low cost operations and structural cost advantages. Technological leadership results in few threats to competitive position and new market opportunities. | Expected to have stable cash flow generation across industry and economic cycles supported by multiple specialty product lines with large market positions, moderate-to-low concentration of cash flow sources, relatively stable end markets, global predominantly low cost operations and may have structural cost advantages. Technological leadership results in meaningful barriers to entry. | Expected to have moderate volatility of cash flow generation across industry cycles supported by multiple commodity or specialty product lines with significant market positions, moderate concentration of cash flow sources, cyclical end markets, cost competitive operations in more than one region, and limited structural cost advantages. Technology and operating knowhow moderates competitive threats. |

| | | Ba | B | Caa | Ca |
|---|---|---|---|---|---|
| | | Expected to have cyclical cash flow generation across industry cycles supported by two or more mostly commodity product lines with mid-sized market positions, moderately-high concentration of cash flow sources, cyclical end markets in one region, average cost operations focused on one region, little structural cost advantages. Limited differentiation based on technology and knowhow. | Expected to have highly cyclical cash flow generation, high reliance on a single commodity product line with modest market positions, high concentration of cash flow sources, cyclical end markets in one region, average-to-high cost operations with limited geographic diversity or a single plant site and no structural cost advantages. No real differentiation based on technology and knowhow. | Expected to have highly volatile cash flow generation, a single commodity product line sold to few customers for limited uses; an insignificant market position, concentrated exposure to small cyclical markets, no pricing power, and a single operating site that has an uncompetitive cost structure. Substantial structural and technological disadvantages. | Expected to have highly volatile cash flow generation, a single commodity product line sold to few customers for a single use, an insignificant market position with many large competitors, concentrated exposure to a small cyclical market and uncertain demand, no pricing power, and a single operating site that has an uncompetitive cost structure. Permanent structural and technological disadvantages. |

# Validation of PD Models

- Assessment of Discriminatory Power – The ability of the rating system to differentiate between borrowers who will default and those who won’t
  - “Does a better rating imply lower chance of default?”
- Calibration – Is the difference between estimated PD and observed default rates acceptable
  - Similar to backtesting in market risk,
  - But, with less observations to test since:
    - Annual instead of daily
    - High grade borrowers rarely default

# Confusion Matrix

Model Prediction

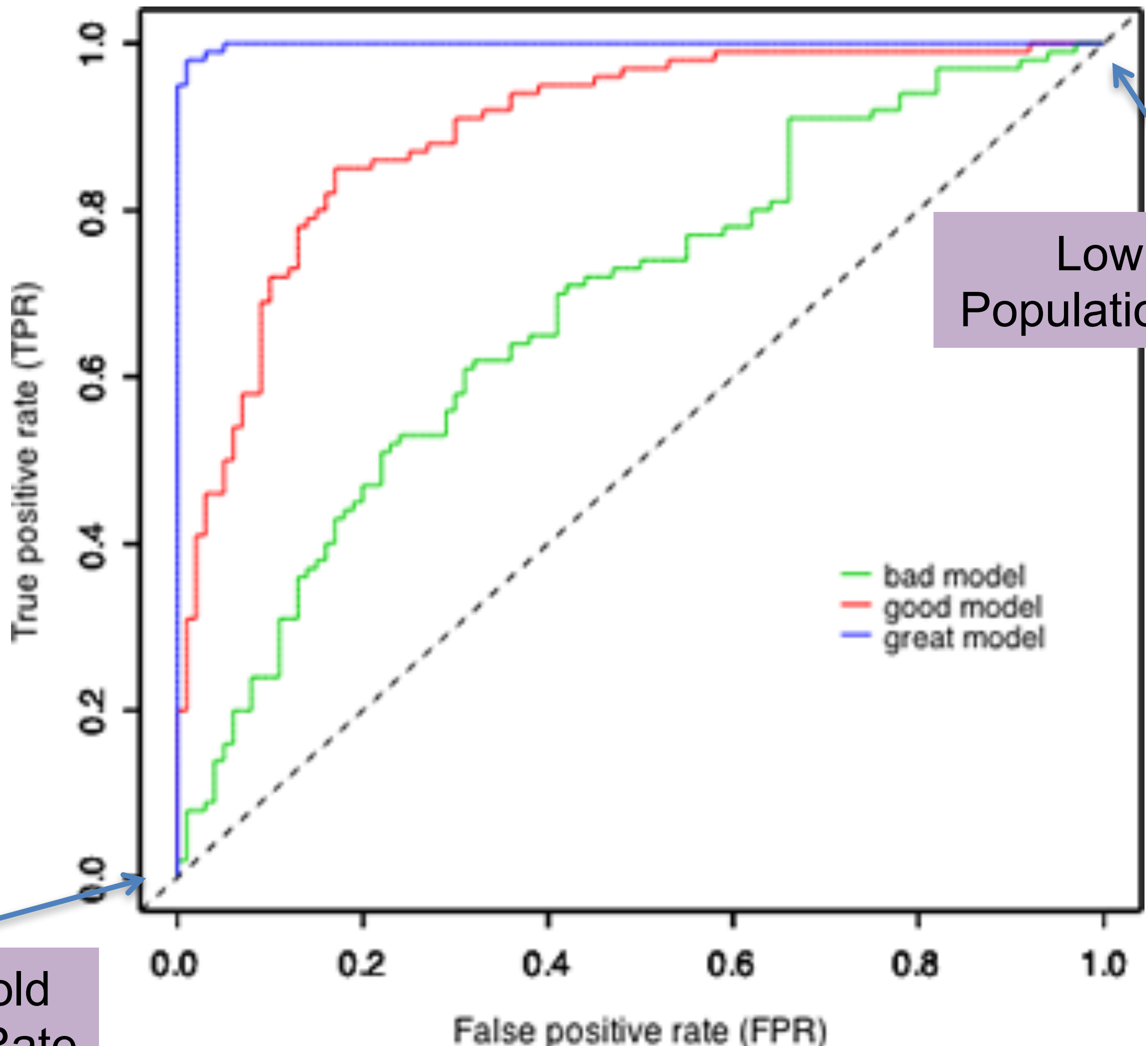
Actual Outcome

| | Good - Lend | Bad - Turn Away |
|---|---|---|
| **Good** | True Positive | False Negative |
| **Bad** | False Positive | True Negative |

True Positive Rate = TP/(TP+FN): How many of "actual goods" did we lend to?
False Positive Rate = FP/(FP+TN): How many of "actual bads" did we lend to?

As we lower the threshold score and loosen our underwriting, both rates go up.

# Receiver Operating Characteristic (ROC) Curve




Low Threshold
Population Default Rate

High Threshold
Zero Default Rate

# ROC Example

| Issuer | Rating | Default | Lend ALL | Lend Above CCC | Lend Above B | Lend Above BB |
|---|---|---|---|---|---|---|
| XYZ | CCC | 1 | FP | TN | TN | TN |
| ABC | CCC | 1 | FP | TN | TN | TN |
| … | CCC | 0 | TP | FN | FN | FN |
| | B | 1 | FP | FP | TN | TN |
| | B | 0 | TP | TP | FN | FN |
| | B | 1 | FP | FP | TN | TN |
| | B | 0 | TP | TP | FN | FN |
| | BB | 1 | FP | FP | FP | TN |
| | BB | 0 | TP | TP | TP | FN |
| | BB | 0 | TP | TP | TP | FN |
| TPR | | | 1 | 0.8 | 0.4 | 0 |
| FPR | | | 1 | 0.6 | 0.2 | 0 |

# Threshold Determination

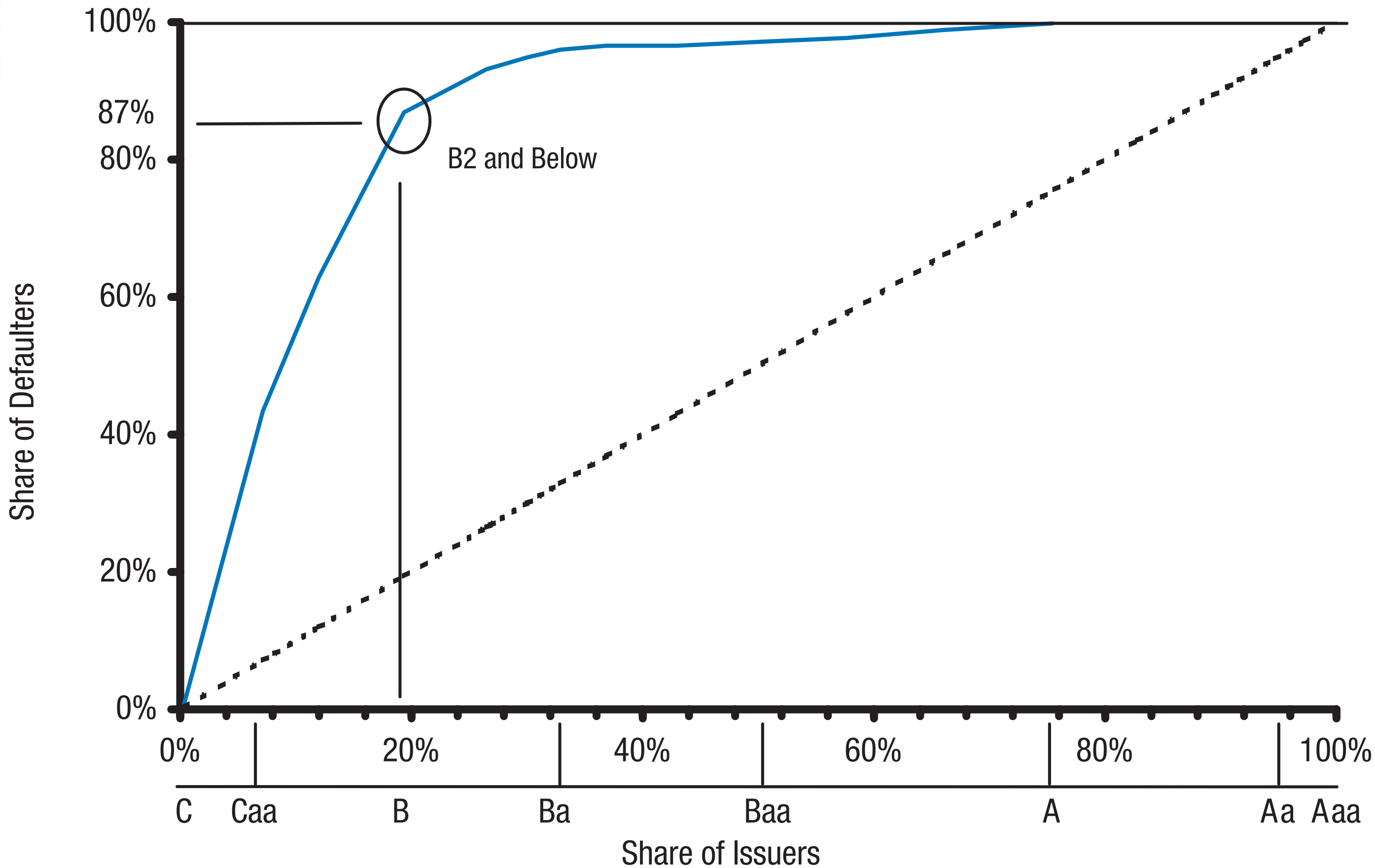
- Suppose that the life time value of a customer is LTV, the credit line is D, and we are able to recover R percent from a defaulted account.
- The expected profit for a given threshold is:

$$\text{Profit} = TP*LTV - FP*D*(1-R)$$

- We maximize profit where the slope of the ROC is:

$$D*(1-R)/LTV$$

# Cumulative Accuracy Profile (CAP)




Rank by Model score

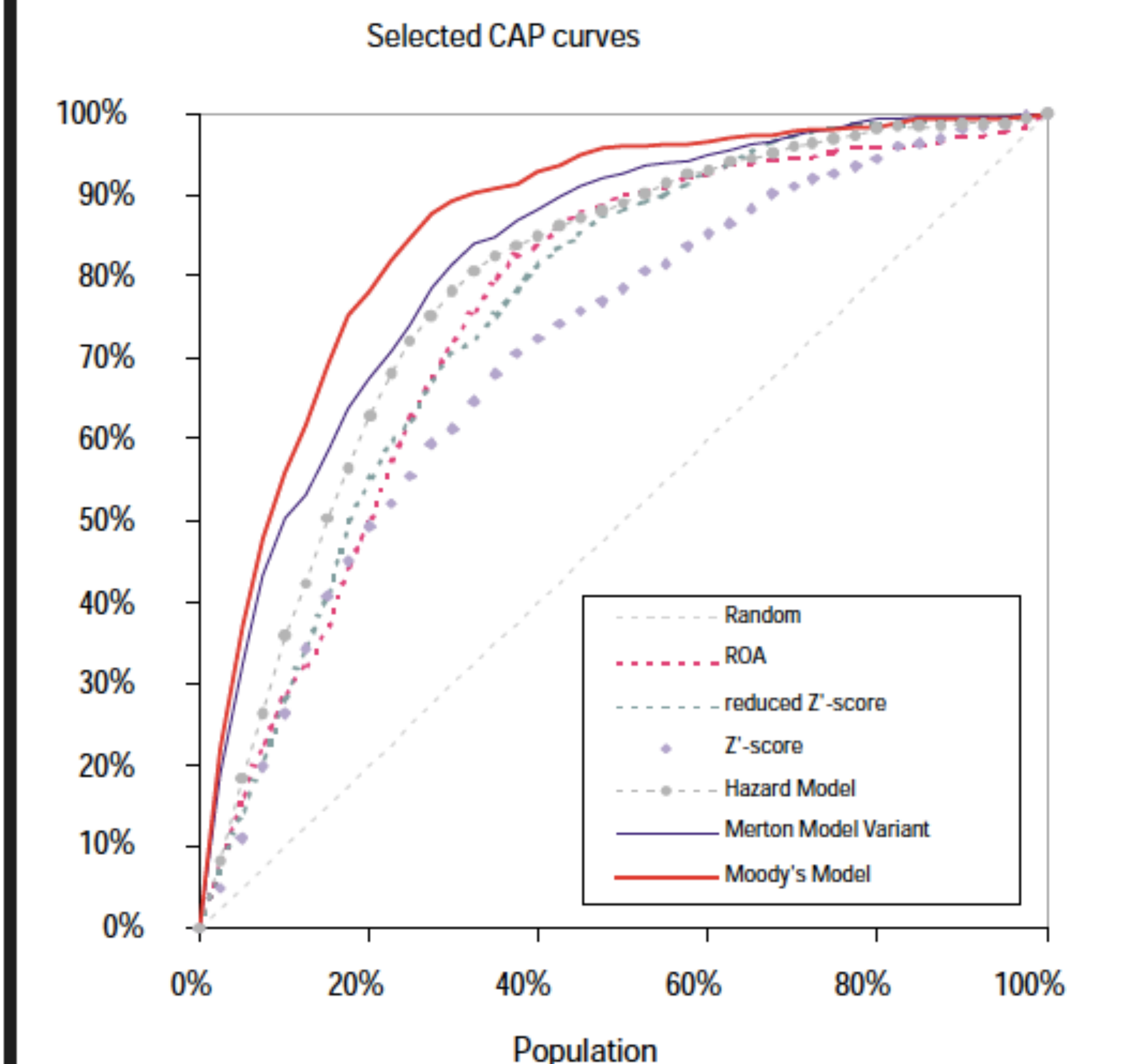
Measuring the performance of corporate bond ratings – Moody's 2003

# CAP Example

| Issuer | SORT BY<br>Rating | Default | Cum Share of Issuers | Cum Share of Defaults |
|---|---|---|---|---|
| XYZ | CCC | 1 | 10% | 20% |
| ABC | CCC | 1 | 20% | 40% |
| … | CCC | 0 | 30% | 40% |
| | B | 1 | 40% | 60% |
| | B | 0 | 50% | 60% |
| | B | 1 | 60% | 80% |
| | B | 0 | 70% | 80% |
| | BB | 1 | 80% | 100% |
| | BB | 0 | 90% | 100% |
| | BB | 0 | 100% | 100% |

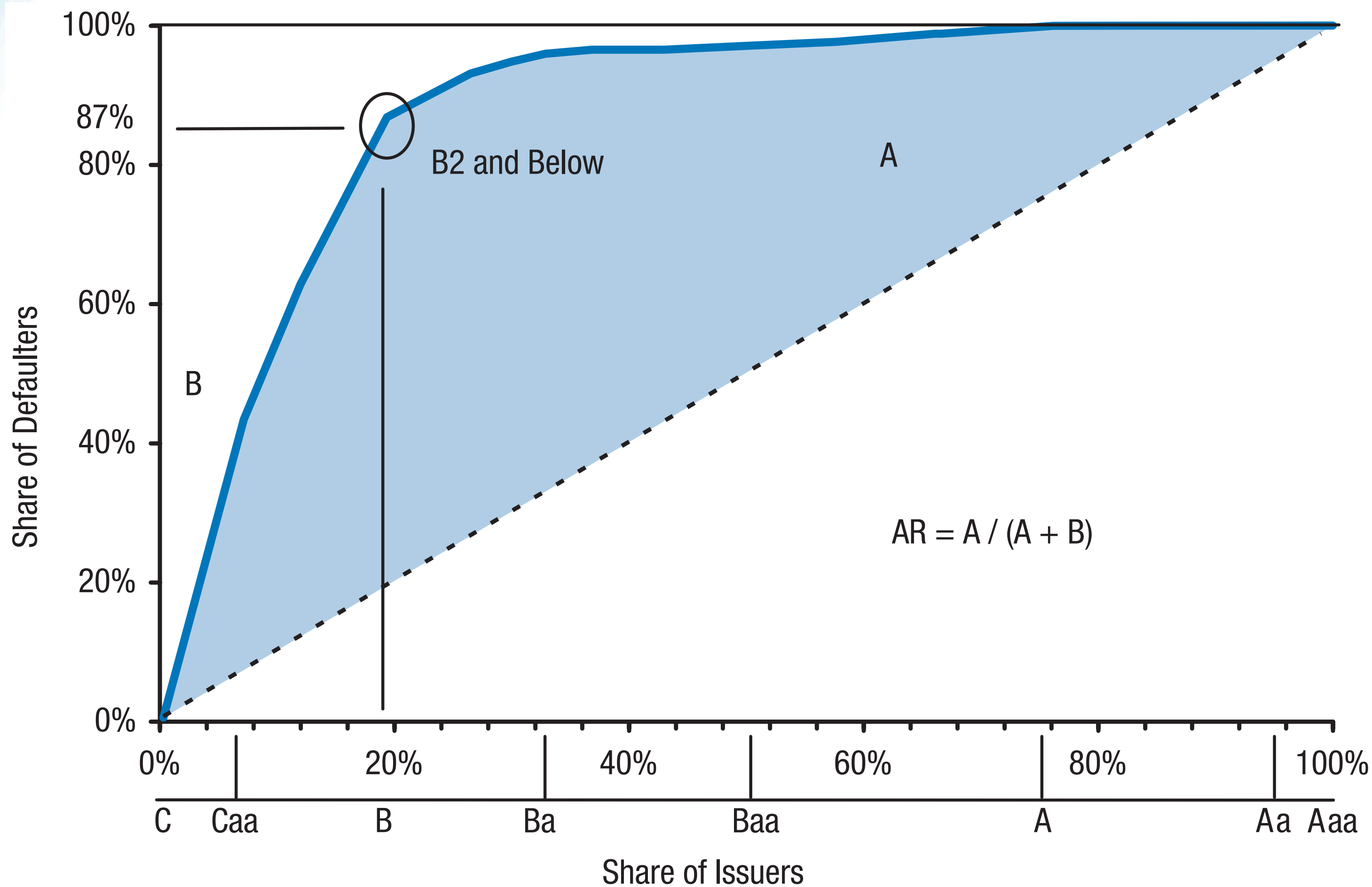
#Issuers=10 #Defaults=5

## Figure 6: CAP curves for the tested models




*Source: Moody's Risk Management Services*

# Accuracy Ratio




Measuring the performance of corporate bond ratings – Moody's 2003

# Recovery Rate

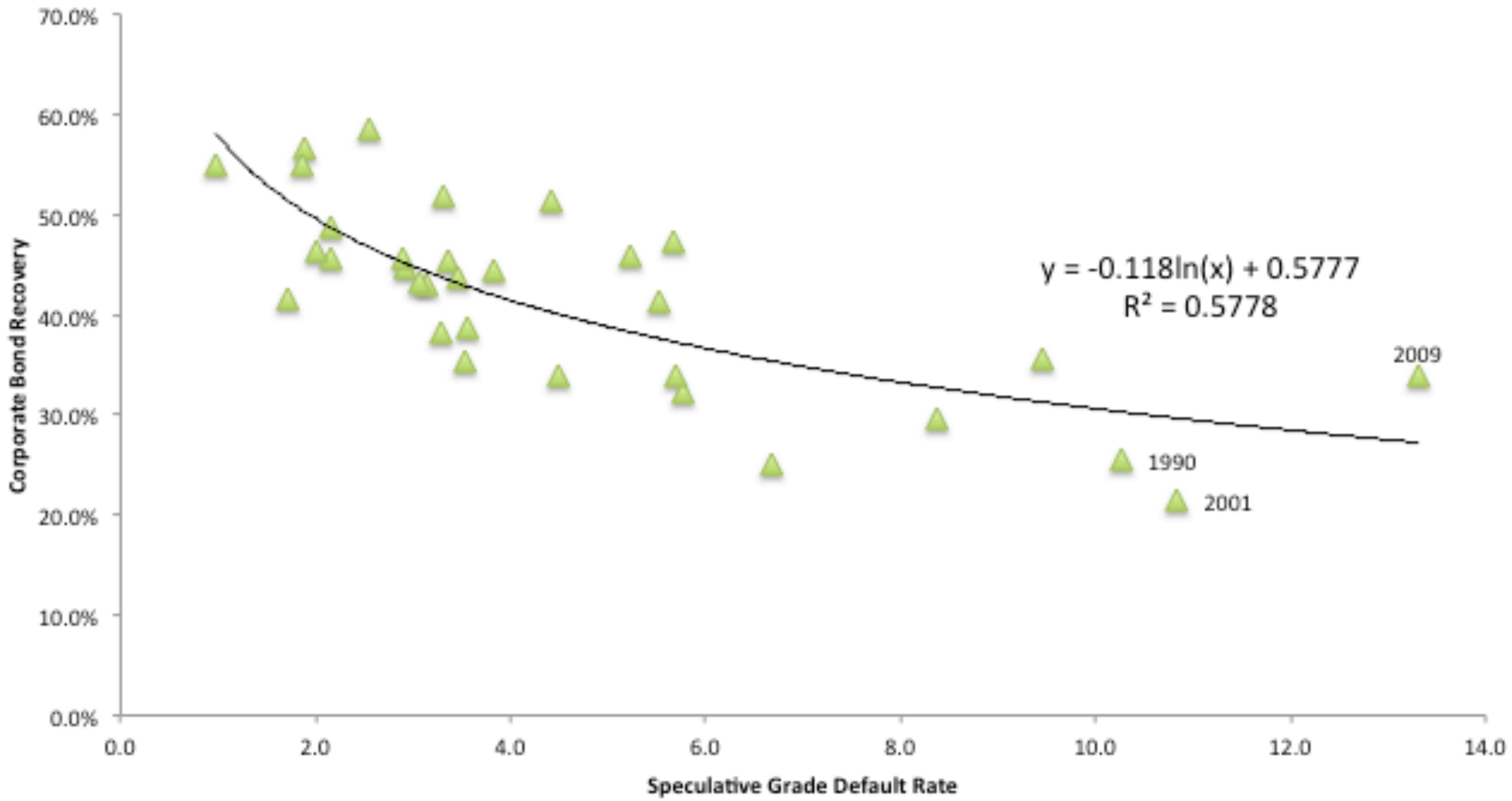
- Recovery Rate for a defaulted exposure:
  - The present discounted value at the default date of recoveries received net of costs associated with collecting on the exposure divided by the amount of the exposure at default.
- For marketable debts, we can take the traded price 30-days after default as a percentage of face value
- Loss Given Default (LGD) = 1 – Recovery Rate

# Recovery Rates Vary by Seniority and Type of Debt

| Class | Ave Rec Rate (%) |
|---|---|
| First lien bank loan | 66.6 |
| Second lien bank loan | 31.8 |
| Senior unsecured bank loan | 47.1 |
| 1st lien bond | 53.4 |
| 2nd lien bond | 49.7 |
| Senior unsecured bond | 37.6 |
| Senior subordinated bond | 31.1 |
| Subordinated bond | 31.9 |
| Junior subordinated bond | 24.2 |

Moody's: 1983 to 2015, Issuer weighted

# Recovery Rates Are Negatively Correlated with Default Rates




Moody's Annual Default Study: Corporate Default and Recovery Rates, 1920-2013

# Thanks