MFE 237H: Quantitative Asset Management
Introduction to Active Management and Benchmarking

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Introduction

This module is designed to give an introduction to a basic framework from which the performance of an active investment manager can be measured.

Benchmarking

Active investment managers are benchmarked against passive indexes to asses the amount of value they added (or subtracted) in excess of the benchmark. Often a direct investment in the benchmark is a low-cost and easily available alternative to an active manager. Therefore, active managers must prove to potential clients that they have the ability to provide a return in excess of the benchmark large enough to justify their fee and excess trading costs.

Mathematically, the benchmark total return is written as R^b, and the return of the actively managed portfolio is R^a. The return in excess of the benchmark, or the active return, is $x^a = R^a - R^b$. We can attach a time subscript, x_t^a, to orient the returns to specific periods. Then a time series of active returns over the period $t = 1, \ldots, m$ is $x_1^a, x_2^a, \ldots, x_m^a$.

Institutional investors typically approach the choice of an active manager from the perspective of asset allocation. Or, they categorize opportunity sets of active managers according to their specified asset class. A passive benchmark is typically assigned to each asset class, and any active manager within the given class must make the case for beating the given benchmark.

For example, categories of equity investments in an institutional portfolio might be divided across U.S. Large, U.S. Small, Global ex U.S. Large, Global ex U.S. Small, Emerging Markets, and so on. In this example there are two dimensions of each class: geography and market capitalization.

The institutional investor may need to choose a manager for its allocation to U.S. Large stocks. It could specify the S&P 500 or Russell 1000 as the passive benchmark for this asset class. For the purposes of this module, we'll assume we are operating in this space by default with the benchmark S&P 500.

The most common type of actively managed fund is effectively a diversified pool of long investments. For this type of fund, a passive benchmark is typically a market-capitalization weighted index, like the S&P 500 mentioned earlier. Other types of benchmarks exist, though, such as the relative performance of the active manager amongst his/her peer group.

Why should the passive benchmark be a cap-weighted index? One of the most compelling reasons this practice persists is simply because it has been done traditionally. Arguments for using these types of benchmarks have changed through the years. Today, cap-weighted indexes are often thought of as practical alternatives: they allocate more weight to large-capitalization stocks, which tends to improve the liquidity characteristics of the benchmark. Also, as they are weighted by firms' market capitalization, cap-weighted indexes rarely need rebalancing because they adjust automatically with price fluctuations- rebalancing is usually performed once annually as a custodial issue. High liquidity, low-turnover, and low-cost access make cap-weighted indexes a practical means of accessing an asset class if one does not have a suitable active manager.

Looking more deeply into the benchmark, let's denote the weight of the ith stock w_i^b. Then, our benchmark weights can be described as a 500×1 vector of weights $w^{b\prime} = \left[w_1^b \; w_2^b \; \ldots \; w_{500}^b\right]$. Also, the weights must sum to 1.

Active Weights

An actively managed portfolio will by definition have different weights than the benchmark. Let the weights of the actively managed portfolio be written $w^{a\prime} = [w_1^a \; w_2^a \; \ldots \; w_{500}^a]$. Note the size of the active portfolio vector is always equal to the benchmark. This is done for mathematical convenience- even if only 30 of 500 stocks have nonzero weight, model the remaining 470 positions as having zero weight.

Now, suppose Apple has a weight of 2.5% in the benchmark S&P 500. If an active portfolio has a weight of only 1%, we can see the difference between the active and benchmark weights is $1\% - 2.5\% = -1.5\%$. This is also called the active weight of Apple. A positive active weight is usually associated with a bullish, or optimistic view of the stock relative to the benchmark's allocation. Conversely, a negative active weight is often representative of a bearish view, or pessimistic estimation of the asset's prospects relative to the benchmark's allocation.

More generally, an active weight is defined as:

$$\tilde{w}_i = w_i^a - w_j^b \tag{1}$$

And we can model the active weights of our entire portfolio as:

$$\tilde{w} = w^a - w^b \tag{2}$$

Which is a 500×1 vector. The sum of elements in $\tilde{w}$ is always equal to zero: there is equal weight to allocate to positive active positions and negative active positions, making the sum cancels out to zero.

Generally, the size of an active weight is proportional to the active risk the manager is taking against the benchmark, reflecting his/her level of confidence in the active view being expressed. Large active weights, either positive or negative, are the product of heightened certainty in the returns forecast.

Cash is also typically considered an active weight as the benchmark index holds none. For that reason, active managers are often heavily restricted against holding meaningful cash positions for more than a short period of time. There is considerable evidence that active managers cannot time market fluctuations anyway, and the task should therefore be in the hands of the institutional investors and their consultants.

Also, the issue of non-benchmark names comes up quite frequently. There are a myriad of reasons why a portfolio manager may end up holding a non-benchmark name: a corporate action such as a takeover may leave the portfolio manager holding shares of a company not originally in the 500 name opportunity set outlined by the benchmark. The proportion of the portfolio which can be allocated to these names are typically heavily restricted. More importantly, though, it prevents the portfolio manager from interfering with the asset allocation decisions made by the institutional investor. For example, a U.S. Large manager without a restriction of holding only the 500 largest names may be inclined to purchase smaller stocks, effectively tilting the funds return towards the U.S. Small style.

Tracking Error

As an active manager takes larger positions further away from benchmark weights, so to will the active portfolio's returns deviate from the benchmark. The variation in returns due to active weights is called active risk, or tracking error. More simply, it is the standard deviation of the returns generated by the active weights. Let Σ be the covariance matrix of the 500 names in the benchmark, then the variance of the active positions is:

$$Var(x^a) = \tilde{w}'\Sigma\tilde{w} \tag{3}$$

And the tracking error is the square root of the active variance:

$$TE^a = \sqrt{Var(x^a)} = \sqrt{\tilde{w}'\Sigma\tilde{w}} \quad (4)$$

The equation above highlights the role of covariance in a portfolio's level of active risk. Two active positions with equal weight could have a spectrum of possible effects on the level of tracking error the manager experiences given the degree of their co-movement with each other and the rest of the portfolio.

Alpha

Again, the objective of active management is to outperform the lower-cost, passive benchmark by generating higher returns. The size of the outperformance is loosely defined alpha, or α. There are many different ways to measure outperformance, such as analyzing the observed outperformance in historical data, or using a regression to control for common risk factors. All of these methods generate different estimates of α, and while the techniques and estimates are dissimilar, they are common in that they all have the objective of describing the returns achieved in excess of the benchmark.

For the purposes of this module, α is the return in excess of the benchmark, $\alpha = w^{a\prime}x^a$.

From the perspective of someone who is evaluating managers or strategies, we care about the likelihood of a manager generating alpha in the future. From practical experience, we can say a manager who has outperformed over the past year is more than likely just lucky. We can't infer that the next year will have a similar positive result, and research supports this. However, is a manager who consistently provided outperformance over the past 10 years in the same situation? Arguably not.

Which leads us to ask: is α persistent through time? Do some managers truly have skill? There is evidence suggesting that even managers who have outperformed in the last three years do not exhibit significantly larger probability of outperformance relative to someone who has not. Thus, there is some evidence historical alpha (performance) is not very indicative of future performance. Also, there is evidence showing managers who exhibit a particular style-tilt tend to exhibit some persistence in outperformance, such as value-oriented and benchmark-free (big active bets) managers. On the other hand, there is also evidence of persistent underperformance across managers with high fees.

Information Ratio

The information ratio is the ratio of a portfolio's active return to its active risk:

$$IR = \frac{\alpha}{TE} \tag{5}$$

The information ratio is a fairly intuitive measure of manager skill. It is, in effect, the signal-to-noise ratio of the active strategy. A high signal-to-noise indicates a quality source of information.

Sharpe Ratio

The Sharpe ratio is defined as the ratio of excess returns to the volatility of excess returns:

$$SR = \frac{(R - rf)}{\sigma} \tag{6}$$

Similar to the information ratio, the Sharpe ratio is the signal-to-noise ratio of the portfolio returns in excess of the risk-free rate. It measures the manager's ability to construct a portfolio which has the highest unit of return per unit of volatility risk.

The Sharpe ratio is less relevant in asset management because investors do not simply hold 100% of their wealth in equities. Investors only care about the Sharpe ratio at the total portfolio level, not at the individual manager's portfolio level.

Performance Attribution

Performance attribution seeks to answer the question: How does the manager add value? The resulting analysis gives the investor some insight into how the manager generated their performance.

One popular way of studying how a strategy performed across a certain factor, for example, economic sectors or industries, was pioneered by Gary Brinson and his associates and is thusly named a Brinson analysis. Effectively, a Brinson analysis allows us to decompose the outperformance across the levels of a factor of our choosing:

1. Let F represent the factor we have chosen with levels $F_1, \ldots, F_k$.

2. Compute the weights of the benchmark and active portfolio across the levels of the k factors:
   (a) Let w_i^b be the weight the benchmark allocates to the ith level of factor F at the beginning of period t.
   (b) Let w_i^a be the weight the active portfolio allocates to the ith level of factor F at the beginning of period t.
3. Compute the returns of the benchmark and active portfolio across the levels of the k factors over period t:
   (a) Let R_i^b be the return the benchmark of the benchmark allocation to the ith level of factor F over period t.
   (b) Let R_i^a be the return the active portfolio allocation to the ith level of factor F over the period t.
4. Next, compute the following four sums:

$$\begin{aligned} Q_1 &= \sum_{i=1}^{k} w_i^b R_i^b \\ Q_2 &= \sum_{i=1}^{k} w_i^a R_i^b \\ Q_3 &= \sum_{i=1}^{k} w_i^b R_i^a \\ Q_4 &= \sum_{i=1}^{k} w_i^a R_i^a \end{aligned} \tag{7}$$

5. Then, we can decompose α into the following components:
   (a) Allocation $= Q_2 - Q_1$
   (b) Selection $= Q_3 - Q_1$
   (c) Interaction $= Q_4 - Q_3 - Q_2 + Q_1$
   (d) Total $= Q_4 - Q_1 =$ Allocation + Selection + Interaction

Above, allocation was the active strategy's ability to allocate weight to the factors relative to the benchmark. Selection is more refined, it measures how well the strategy picked stocks within the factors relative to the benchmark. Interaction is the effect from a relationship between allocation and selection, but is more subtle to understand. Effectively, interaction answers the question: did the levels of our factor influence how the selection effect performed? Lastly, total is equivalent to α.

One of the most popular factors chosen for a Brinson analysis is sectors. If we chose sectors, we would have $k = 10$ levels of our factor, one level for each sector. Answering the same questions again in terms of sectors may help build more intuition behind a Brinson analysis:

1. Allocation answers the question: how well did the active strategy allocate to sectors relative to the benchmark?
2. Selection answers the question: Within the allocation to sectors, how well did the active strategy chose stocks relative to the benchmark?
3. Interaction answers the question: How did the sectors influence the selection effect?

A more recent contribution to the finance literature is a new model of the Brinson which corrects for one of its principal weaknesses: it only considers a single period. In reality, investment management performance plays out over many different periods, over which exposures to any given factor are likely to change. Accounting for the dynamics of exposures to the factor levels critical, but cannot be ascertained in the first method. The updated model can therefore be thought of as a "dynamic Brinson", whereas the traditional model is "static" in that it only considers the initial weights over a single period.

Consider now the situation where we have m periods. We can study the average alpha over the given time period, and therefore decompose the average value-added into dynamic and static components. The dynamic component measures the value-added from changing exposures to the factor levels, and the static component measures the value added from the proportion of the allocation to the factor levels which did not change over the measured periods.

The updated model for decomposing α has the following form:

$$\text{Total} = \text{Dynamic Allocation} + \text{Static Allocation} + \text{Selection} \tag{8}$$

Where the terms are defined below.

Allocation Added-Value:

$$\frac{1}{m} \sum_{i=1}^{k} \sum_{t=1}^{m} \left(w_{i,t}^{a} - w_{i,t}^{b} \right) \left(R_{i,t}^{b} - R_{t}^{b} \right) \tag{9}$$

Static Allocation Added-Value:

$$\sum_{i=1}^{k}\left[\frac{1}{T}\sum_{t=1}^{m}\left(w_{i,t}^{a}-w_{i,t}^{b}\right)\right]\left[\frac{1}{T}\sum_{t=1}^{m}\left(R_{i,t}^{b}-R_{t}^{b}\right)\right] \tag{10}$$

Security Selection Added-Value:

$$\frac{1}{m}\sum_{i=1}^{k}\sum_{t=1}^{m}w_{i,t}^{a}\left(R_{i,t}^{b}-R_{t}^{b}\right) \tag{11}$$

Fama-French 3-Factor Analysis

The original form of the CAPM, which links the excess returns of a stock to the excess returns of the market, began as an empirical success. Many "anomalies" regarding the returns of certain investment strategies were explained by excess exposure to the market factor, and therefore were a function of bearing excess risk, not skill. However, it was not long until new research emerged demonstrating situations where the CAPM could not sufficiently link out-sized returns to risk.

Fama and French proposed an extension of the single factor model which held significant additional explanatory power beyond the traditional CAPM. Their model added two additional factors- firms' "size", or market capitalization, and the ratio of firms' market value to their book value, or "value". The reasons for doing so were simple: firms with smaller market capitalizations tended to have greater excess returns than firms with larger market capitalizations. Similarly, firms with a high book/market, or were priced near their liquidation value, were more apt to outperform firms priced far above their book value.

Under the extension proposed by Fama and French, many of the anomalies once used as criticisms of the traditional CAPM disappeared as loadings on the two new factors. Risk, formerly argued as being one-dimensional, was now multidimensional.

The Fama-French 3-factor model describes a linear relationship between the excess returns of x_t and the excess returns of the market (mkt), size (SMB), and value (HML) factors:

$$x_t = \alpha + \beta_{mkt}x_{mkt,t} + \beta_{SMB,t}x_{SMB,t} + \beta_{HML}x_{HML,t} + \epsilon_t \tag{12}$$

While x_{mkt} are simply the excess returns of the market capitalization weighted market, the excess returns of SMB and HML are structurally different. SMB, for example, is generated from a special portfolio designed to measure the premium associated with investing in small stocks. This portfolio is built by dividing the market into two cohorts: large stocks and small stocks. The division is made at the median market capitalization of the market; thus, both halves have equal market value. The large and small cohorts are then partitioned into three value portfolios: firms with high book/market are value, firms with low book/market are growth, and firms between are neutral.[1]

The average return of the three portfolios in the small partition is calculated, and same is done for the large. The return of the large portfolio is then subtracted from the small portfolio. The extra work building value portfolios was done as a means of suppressing correlation of the SMB factor with HML.

Remember, the large and small cohorts have equal market value (approximately), so, theoretically we could short the stocks in the large portfolio and use the proceeds to (almost exactly) purchase the small portfolio. Thus, this is sometimes described as a self-financing portfolio, or zero-investment portfolio.[2] The takeaway should be that we have a long/short portfolio which is designed to specifically measure the risk premium associated with the cross-sectional characteristics we are interested in.[3]

One criticism of the Fama-French model is that the argument for using size and value is not necessarily obvious. The reasoning behind using the cap-weighted market in the original CAPM was elegant, and grounded in economic theory. Despite this criticism, the Fama-French model is still wildly popular due to its empirical power. Thus, one may dispute the economic justification of the SMB and HML factors, but most simply accept it for its empirical power.

The computations behind the Fama-French model can be done in MS Excel quite rapidly. However, in a computer we can do OLS regression rather quickly with only a few steps of matrix algebra. Let y be our dependent variable, or our time series of excess returns, and let X be our design matrix. The first column of X is simply 1's, which will be used to model the intercept α. The second, third, and fourth columns are the time series of the market,

[1] The specific construction methodology can be seen here:`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/six_portfolios.html`

[2] In practice, this is neither easy or cheap.

[3] There are many other ways to build portfolios designed to access specific risk premiums. Sophisticated techniques designed to efficiently sample risk premiums are of great interest to practitioners.

size, and value factors, respectively:

$$X = \begin{bmatrix} 1 & x_{1,mkt} & x_{1,SMB} & x_{1,HML} \\ 1 & x_{2,mkt} & x_{2,SMB} & x_{2,HML} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{t,mkt} & x_{t,SMB} & x_{t,HML} \end{bmatrix} \tag{13}$$

And our data y is:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix} \tag{14}$$

Then, we can estimate the coefficients $\hat{\beta} = \begin{bmatrix} \hat{\alpha} & \hat{\beta}_{mkt} & \hat{\beta}_{SMB} & \hat{\beta}_{HML} \end{bmatrix}$ by the following equation:

$$\hat{\beta} = (X'X)^{-1} X'y \tag{15}$$

And, the variance of the estimated coefficients is:

$$Var(\hat{\beta}) = \sigma^2 (X'X)^{-1} \tag{16}$$

Of course, these can be used to compute the standard errors for testing hypotheses about the estimated coefficients.

Carhart 4-Factor Model

While the Fama-French 3-factor model has replaced the traditional single-factor model as the status quo, another subsequent innovation by Carhart attracted considerable attention. In the same way Fama and French extended the original CAPM, Carhart adds a price momentum factor to the Fama-French 3-factor model. This additional factor aims to measure the premium associated with momentum strategies: stocks whose price has risen recently are more likely to have positive future returns. Similarly, stocks whose price has fallen recently tend to produce a subsequent negative return:

$$x_t = \alpha + \beta_{mkt} x_{mkt,t} + \beta_{SMB,t} x_{SMB,t} + \beta_{HML} x_{HML,t} + \beta_{MOM} x_{MOM,t} + \epsilon_t \tag{17}$$

Debate still surrounds the use of momentum factors in asset pricing. Empirically, the effect does exist in the data. However, there are some considerations when porting this strategy to reality: momentum tends to be more pronounced in smaller stocks with greater volatility. Some argue that after transaction costs and market impact, there does not exist a momentum premium worth pursing.

Exercises

1. Consider a manager who uses equal weighting as his sole strategy; he invests in the 500 largest stocks by market capitalization and equal weight the stocks. He rebalances the portfolio at a monthly frequency.
   (a) Using data from 1960, construct the monthly portfolio weights and compute the monthly portfolio returns.
2. Please create a nicely formatted table showing:
   (a) Since inception portfolio (total) return, volatility and SR alongside the benchmark S&P500 index (total) return, volatility and SR.
   (b) In addition, please report the alpha, TE and IR.
3. Please use the standard Brinson analysis to examine the contribution from industry bets and from stock picking.
4. Please use the dynamic attribution method to examine the static and dynamic contribution from industry bets and also report the stock picking contribution.
5. Please perform the Fama-French 3-factor analysis and report the estimated $\hat{\alpha}$ and $\hat{\beta}$ coefficients.
6. Please perform the Carhart 4-factor analysis and report the estimated $\hat{\alpha}$ and $\hat{\beta}$ coefficients.

Data for the last two problems are available at Ken French's data library: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

The standard Fama-French factors are labeled as such; to perform the Carhart analysis please append the time series labeled "Momentum Factor" to the "Fama/French" data.