Performance Attribution: Measuring Dynamic Allocation Skill

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Abstract

Classical performance attribution methods decompose manager alpha into factor allocation and stock selection components. A manager can produce alpha through factor tilts relative to a benchmark and by stock selection within each factor. However, traditional attribution methods do not explicitly assess a manager's dynamic allocation skill in the factor domain. We propose a generalized framework for performance attribution that decomposes the allocation effect into value-added from both static and dynamic factor exposures and thus yields additional insight into sources of manager alpha. Such a decomposition can assist investors in identifying and quantifying manager skill, provide insight into a managers investment approach, style, and biases, as well as aid in benchmark selection and creation.

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I. Introduction

At the end of 2008, there were more than 8,000 mutual funds with $10 trillion in assets under management (AUM) in the United States. Of these, approximately 80 percent were actively managed.[1] In the same year, another 9,000 hedge funds actively managed $3 trillion in assets. Although some studies have cast doubt on the ability of the average active managers to consistently "beat the market", investors are nevertheless willing to entrust considerable sums of money to active management. Identifying skilled active managers is, therefore, an important goal to many investors.

Numerous evaluation methodologies have been introduced to characterize portfolio performance and identify talented active managers. The simplest of these is return-based regression analysis, such as ones based on the Fama and French (1992, 1993) three-factor model. Regression analysis can identify style tilts and estimate risk-adjusted alphas for managers with only return information. However, while regression analyses require few inputs, they also provide more limited insight into sources of managerial performance.

Holdings-based attribution can provide a more detailed analysis of manager performance, but the required input is significantly more substantial than standard regression analysis. Originally proposed by Brinson and Fachler (1985) to study manager skill in allocating to different industries, holdings-based attribution analysis has been extended to study allocation skill in other factor domains, such as value, size, momentum, and volatility. The so-called Brinson attribution analysis provides a straightforward way to decompose manager value-added into dimensions such as superior factor/sector allocation and stock selection and is an industry standard.

Classical Brinson attribution was designed to analyze manager returns over a single period under the assumption of static holdings. It has since been extended to cover multiple periods to account for changing portfolio weights over the span of analysis.[2] Commonly used multi-period attribution analyses, however, do not explicitly measure a manager's ability to dynamically allocate in the factor domain. This deficiency is important for a number of reasons.

[1] See the *2009 Investment Company Fact Book* of the Investment Company Institute (www.ici.org).
[2] For examples, see Carino (1999), Laker (2005), Menchero (2000, 2004), and Davies and Laker (2001).

For example, value stocks have historically outperformed growth stocks. A particular manager may seek to exploit this apparent value premium to generate a higher return against his benchmark by increasing portfolio weights to value stocks. We term this approach *static factor allocation*, and the resulting alpha arises from persistant style tilts towards factors with a risk premia. A different manager may possess skill in forecasting whether value stocks will outperform growth stocks in a given year and dynamically adjusts the value/growth tilt in her portfolio by increasing weights in value stocks when she believes value will do well relative to growth, and vice versa. We term this approach *dynamic factor allocation*.

Although the sources of value-added for these two managers are markedly different, traditional multi-period Brinson-type analyses do not explicitly distinguish between them. The existing methods thus provide an incomplete assessment of a portfolio manager's investment style. Our objective is to outline a methodology that decomposes the allocation effect of traditional attribution analyses into static and dynamic components in a straightforward and intuitive way to enhance the measurement of manager skill.

A number of attribution studies in the academic literature have examined a manager's ability to dynamically adjust portfolio exposures. Notably, Daniel, Grinblatt, Titman, and Wermers (1997) decomposed fund returns into "average style," "characteristic selectivity," and "characteristic timing" components relative to a characteristic-based benchmark. The attribution analysis presented in this paper is similar in spirit to the Daniel et al. approach, in that it seeks to characterize the manager's dynamic allocation skill in the factor domain. It is different, however, in that it accomplishes the analysis within the classic Brinson framework and does not require the added complexity associated with the creation of various characteristic-based benchmarks whose holdings must then be matched to each stock in the mutual fund in every period. As a consequence, the methodology presented here is significantly less complex than the Daniel et al. approach and more in line with current industry standards for performance attribution.

II. Motivation

We start with a simple example to introduce the intuition behind the attribution methodology proposed in this paper. Suppose there is a balanced fund manager who allocates his portfolio between two assets: equities (via an index fund) and cash. The equity market is expected to generate a positive return but with some variance. Cash, for simplicity, is assumed to bear zero interest and has no return variance. Suppose further that this manager attempts to time the equity market; when he believes the market will do well, he allocates away from cash and invests a fraction of the portfolio in the stock market, otherwise he invests 100 percent in cash. Over time, the arithmetic average of his returns each period can be written as

$$E\left[w_t R_t\right] = \frac{1}{T}\sum_{t=1}^{T} w_t R_t, \tag{1}$$

where w_t is the portfolio weight he places in the stock market at time t and R_t is the return on the stock market realized between t and $t+1$ (the weight in the cash account, $1-w_t$, always returns zero).

After a period of time, our manager develops a history of delivering a positive return on average and interprets this as evidence that he has been successful in timing the market. However, there are at least two explanations for his positive excess return. First, he may well have been successful in timing the market. Perhaps his return was garnered from market exposure during a limited number of days when the market did particularly well, and he managed to have low market exposure during periods of negative returns. In this case, his return is the result of a successful *dynamic factor allocation* strategy. The second possibility, however, is that, because stocks are priced to yield a positive expected return, his excess return arose simply from having equity exposure (however erratic) over time. In this case, his return is ultimately a consequence of *static factor allocation,* regardless of his time-varying equity weights.

One could attempt to distinguish between these two possibilities in a number of ways. We propose a simple and straightforward method for evaluating the dynamic skill of our manager by noting the following identity:[3]

$$E\left[w_t R_t\right] = E\left[w_t\right]E\left[R_t\right] + \operatorname{cov}\left(w_t, R_t\right). \tag{2}$$

In Equation (2), the average portfolio return for our hypothetical manager, $E\left[w_t R_t\right]$, is decomposed into two parts. We define the first term on the right-hand side of the equation, $E\left[w_t\right]E\left[R_t\right]$, as the "static allocation effect." This term captures the portion of the return gleaned from a static allocation to the equity market, Any weight on an asset that has a positive expected return can be expected to generate a positive return. In this particular example, our manager can be expected to generate a positive return simply by allocating a positive weight to the stock market on average.

We define the second term on the right-hand side of Equation (2), $\operatorname{cov}\left(w_t, R_t\right)$, as the "dynamic allocation effect." This term captures the portion of our manager's return that is attributable to his ability to time the equity market. If our investor's portfolio weight in stocks is large when market returns are high and small when market returns are low, we would observe $\operatorname{cov}\left(w_t, R_t\right) > 0$. If we observe $\operatorname{cov}\left(w_t, R_t\right) = 0$, we would conclude that the manager's performance arises from a simple positive static exposure to the equity market, and that he demonstrates no meaningful ability to tactically allocate portfolio weights. Note, if $\operatorname{cov}\left(w_t, R_t\right) < 0$, the manager may be actively destroying value. If our manager bought the market during periods of positive returns and shorted the market during periods of negative returns, his average weight in the market may have been zero, $E\left[w_t\right] = 0$, and the value he adds may be characterized as arising entirely from his dynamic allocation skill. Traditional attribution analysis is designed only to identify and measure the manager's skill in factor allocation and does not distinguish between static and dynamic components.

[3] A similar observation is made in Grinblatt and Titman (1993) in measuring manager performance without benchmarks.

Distinguishing between static and dynamic exposures may be important when analyzing manager performance for a number of reasons. Static exposure to new and innovative risk factors is valuable, but the same time, static exposure to known risk factors (in the example above, the stock market) may be easily replicable. One criticism of an alpha stream that is characterized by static factor exposures is that it can be replicated by low cost allocations to passive indexes in a buy and hold portfolio (though such replication may only be approximately possible for the limited number of factors for which passive funds exist, such as size and value). Our decomposition may help investors assess the proportion of a manager's alpha that results from static allocations to known risk factors, particularly those that may be accessed passively through style indexes. With this in mind, quantifying the dynamic component of a manager's alpha may represent an important rationale in justifying active management fees as a dynamic strategy is arguably less replicable.

Measuring the static exposure of a manager is also useful in identifying persistent manager bias relative to a benchmark and in the construction of a "normal portfolio." Normal portfolios are those which represent a manager's preferred allocation in the absence of views (see Black and Litterman (1992)) and, among other things, can be used as a benchmark when no explicit benchmark exists. In this setting, a candidate for a normal portfolio or benchmark may be one that admits little or no static allocation effect (that is, the normal portfolio should closely match the average style of the managed portfolio).

Decomposing value-added into static and dynamic components better characterizes managerial investment approach, style, and sources of value-added and is thus useful to investors.

III. Methodology

The Brinson attribution methodology is elegant, intuitive, and by any measure, an industry standard. Before we formally expand upon it to introduce our framework, a review of the original univariate (one factor dimension) Brinson approach is warranted.

Review of traditional Brinson Analysis

Over a single time period, a manager's value-added relative to a benchmark can be decomposed into "allocation" and "security selection" components as follows:

$$\begin{aligned}\sum_{i=1}^{N}\left[w_i^p R_i^p - w_i^b R_i^b\right] &= \text{Value-added} \\ &= \sum_{i=1}^{N}\left(w_i^p - w_i^b\right)\left(R_i^b \cancel{- R^b}\right) + \sum_{i=1}^{N} w_i^p\left(R_i^p - R_i^b\right) \\ &= \text{Allocation effect + Security selection effect,} \qquad (3)\end{aligned}$$

where N is the number of factor groupings (i.e., twelve industry sectors or ten price-to-book value deciles), where w_i^p, w_i^b, R_i^p, R_i^b are the weights and returns for factor group i in both the manager portfolio and the benchmark, and where R_i^b is the benchmark return. Note that the superscript p and b refer to portfolio and benchmark respectively. In the original Brinson analysis, *allocation effect* is a measure of the manager's skill at allocating across the industrial sectors. More generally, we can measure the manager's skill at allocating across value, size, momentum or other factor quintiles. *Security selection effect* is a measure of the manager's ability to overweight the higher return stocks within these groups. Note that classical Brinson analysis also includes a term for the interaction between allocation and selection effects (see Brinson, Hood, and Beebower 1995). Our selection effect here captures the sum of classical selection and interaction effects. The distinction, while important in some applications, is not critical here.[4]

Classical Brinson attribution was designed for single-period analysis with static portfolio holdings; it does not directly allow for multi-period analysis when portfolio weights are actively changed. Methodologies for multi-period attribution analysis have been developed to better account for intertemporal decision making. One of the most commonly used techniques is to repeat the standard Brinson analysis over T periods and then take a simple average; that is,

[4] The interaction effect is positive when a manager overweights sectors in which the manager has a positive stock selection ability and underweights sectors where the manager does not. The effect is often added to classical security selection to simplify the analysis (see Fabozzi and Markowitz 2002).

$$\frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}\left[w_{i,t}^{p}R_{i,t}^{p}-w_{i,t}^{b}R_{i,t}^{b}\right] = \text{Average value added}$$

$$=\frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\left(R_{i,t}^{b}-R_{t}^{b}\right)+\frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}w_{i,t}^{p}\left(R_{i,t}^{p}-R_{i,t}^{b}\right)$$

$$= \text{Avg allocation effect} + \text{Avg security selection effect.} \quad (4)$$

The arithmetic value-added is, naturally, different from geometric value-added; modified methodologies exist, which allow for geometric attribution.[5]

Weakness associated with traditional Brinson analysis

Multi-period attribution yields a more complete picture of how an active manager generates his or her alpha than single-period attribution. However, neither single-period nor multi-period Brinson analysis is able to characterize a manager's dynamic allocation or market-timing skill. To see this weakness, consider two portfolio managers who allocate between a value index and a growth index. Both managers are measured against a benchmark that is 50 percent invested in each index. The "static" manager, aware of the historical outperformance of value over growth, allocates a constant 80 percent of his portfolio weight to the value index and 20 percent to the growth index. The "dynamic" manager attempts to time the market and varies her portfolio weights on the basis of whether she believes growth will outperform in a given period.

Suppose we observe the performance of these two managers over a three-year period as summarized in Table I. The static manager behaves in a predictable way: At the beginning of each year, his portfolio weights were 80 percent value and 20 percent growth. The dynamic manager, however, places the majority of her portfolio weight in the growth index during Year 2 (when growth outperformed value). During Years 1 and 3 (when value outperformed growth), she placed the majority of her portfolio weight in the value index. The results of these portfolio weights reveal that she was apparently

[5] For convenience, the focus here will be on arithmetic attribution analysis. For representative examples of a geometric approach, see Bacon (2002) and Menchero (2000, 2001).

successful in dynamically forecasting returns and allocating portfolio weights in response.

How do the managers compare if we apply a classical multi-period Brinson analysis to their portfolios? Using the arithmetic average approach, we can determine the average allocation effect and stock selection as follows:

$$\text{3-yr ave. value-added} \quad = \frac{1}{3}\sum_{t=1}^{3}\sum_{i=1}^{2}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\left(R_{i,t}^{b}-R_{t}^{b}\right)+\frac{1}{3}\sum_{t=1}^{3}\sum_{i=1}^{2}w_{i,t}^{p}\left(R_{i,t}^{p}-R_{i,t}^{b}\right)$$
$$= \text{Avg allocation effect} + \text{Avg security selection effect.} \qquad (5)$$

In this example, by construction, the average stock selection effect is zero for both managers; the managers are allocating to indexes and make no stock selection decisions. Therefore, the only source of their value-added is from factor allocation—their ability to allocate between value and growth indexes. Here, a classical multi-period Brinson analysis says exactly the same thing for *both* managers: their value-added from factor allocation are identical at 1.5 percent. Clearly the investment strategies are markedly different for these two managers, but classical Brinson methodology fails to distinguish between them.

A new framework for capturing manager dynamic factor allocation capability

We propose a straightforward way for subdividing existing allocation effect into static and dynamic components. Similar to Equation (2), for each factor i considered, we separate the allocation effect into static and dynamic components by using the following identity:

$$E\left[\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\left(R_{i,t}^{b}-R_{t}^{b}\right)\right] = E\left[w_{i,t}^{p}-w_{i,t}^{b}\right]E\left[R_{i,t}^{b}-R_{t}^{b}\right]+\operatorname{cov}\left[\left(w_{i,t}^{p}-w_{i,t}^{b}\right),\left(R_{i,t}^{b}-R_{t}^{b}\right)\right]$$
$$= \text{Static allocation effect} + \text{Dynamic allocation effect.} \qquad (6)$$

The left hand side in Equation 6 is simply the average allocation effect from the standard attribution model, and the right hand side is the decomposition into the static and dynamic components. For a univariate analysis, where the one factor is categorized into

N groups and studied over T periods, Equation (6) can be summed for each factor in order to decompose the factor selection into dynamic and static components for the entire portfolio:

$$\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\left(R_{i,t}^{b}-R_{t}^{b}\right)=\sum_{i=1}^{N}\left[\frac{1}{T}\sum_{t=1}^{T}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\right]\left[\frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t}^{b}-R_{t}^{b}\right)\right]$$
$$+\sum_{i=1}^{N}\left\{\frac{1}{T}\sum_{t=1}^{T}\left[\left(w_{i,t}^{p}-w_{i,t}^{b}\right)-\frac{1}{T}\sum_{t=1}^{T}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\right]\left[\left(R_{i,t}^{b}-R_{t}^{b}\right)-\frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t}^{b}-R_{t}^{b}\right)\right]\right\}$$
$$=\text{Static allocation + Dynamic allocation.} \tag{7}$$

Calculation of the covariance term, although straightforward, is somewhat cumbersome and can more easily be accomplished by taking the difference between the total allocation value-added and static values-added. Summarizing, the total allocation value-added (including the decomposition into static and dynamic components) and security selection value-added for the portfolio can be computed by:[6]

$$\text{Allocation value-added } =\frac{1}{T}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(w_{i,t}^{p}-w_{i,t}^{b}\right)\left(R_{i,t}^{b}-R_{t}^{b}\right); \tag{8a}$$

$$\text{Static allocation value-added } =\sum_{i=1}^{N}\left(\frac{1}{T}\sum_{t=1}^{T}w_{i,t}^{p}-w_{i,t}^{b}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t}^{b}-R_{t}^{b}\right)\right); \tag{8b}$$

$$\text{Dynamic allocation value-added } = \text{Allocation value-added} - \text{Static value-added}; \tag{8c}$$

$$\text{Security selection value-added } =\frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}w_{i,t}^{p}\left(R_{i,t}^{p}-R_{i,t}^{b}\right). \tag{8d}$$

When we use this approach, our evaluation of the two fund managers, considered in the preceding example, is quite different. We display the decomposed allocation effect based on Equation (8) in the bottom of Table I. The static manager is precisely as we would

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[6] Note that, when summing allocation effects across all factors, $\sum_{i=1}^{N}\left(w_{i}^{p}-w_{i}^{b}\right)R^{b}=0$. Thus the portfolio level returns R_{t}^{b} can be omitted from the allocation effects in Equations (8a)-(8c) without changing the aggregate result. This eases the complexity of calculating allocation effects at the portfolio level.

expect; all 1.5 percent of the static manager's value-added is attributed to his static factor exposure. The dynamic attribution is zero for this manager. For the dynamic manager, the static value-added is roughly zero with the dynamic strategy driving the full 1.5 percent of added value.

The value-added from security selection may be similarly decomposed into static and dynamic components. Recall that the "security selection" in this paper is actually the sum of an interaction term with the classic security selection. This difference raises interpretation issues for decomposing security selection into static and dynamic components, and we do not wish to address the issues here.

Finally, we wish to emphasize that, similar to the traditional Brinson analysis, this methodology is performed strictly ex-post. For example, the expression for static value-added (Equation (8b)) requires average portfolio weights at returns, which cannot be known until the end of the period being examined. Thus, the attribution model does not reflect any real investment process but rather seeks to characterize the effect of investment decisions ex-post. However, this methodology can be used ex-ante in the development of quantitative strategies, which typically require backtesting, in order to more fully characterize historical sources of value-added. This may aid in the selection of quantitative strategies for implementation.

IV. An Application

To further illustrate this dynamic attribution framework, we apply the analysis to several large and well-known mutual funds, whose reported objectives as of June 2009 are summarized in Table II. The purpose of this exercise is not to present a comprehensive examination of all active managers using the dynamic attribution model presented in this study. Rather, our goal is (1) to illustrate the application of the proposed methodology, (2) to interpret and analyze results of the presented methodology, and (3) to demonstrate the robustness of our approach to small variations in model specification.

We examined only active managers in the large-cap and small-cap spaces. For the large-cap active managers, we required the fund to have been live in 1980; for the small-cap managers, we required the fund to have been live in 2000. Each fund also has to be ranked within the top 100 funds by AUM in its category in 2008. From the funds meeting these criteria, we selected six large-cap and three small-cap funds with the understanding that the resulting sample has significant survivorship bias. We included in our sample an index fund, the Vanguard 500 Index Fund, to illustrate the baseline case.

Information about holdings for each of these mutual funds through 2008 was obtained from the Thomson Reuters mutual fund database, which uses quarterly U.S. SEC filings. Return data are from CRSP. To benchmark the large-cap funds, we created a "Cap 1000" benchmark. The constituents of this benchmark are the largest 1,000 U.S.-listed companies (by market capitalization) at the beginning of each calendar year. The methodology produced an index that is analogous to the Russell 1000 Index. Similarly, for the small-cap funds, we create a "Cap 2000" benchmark whose constituents are the next largest U.S.-listed 2,000 companies at the beginning of each calendar year and is roughly analogous to the Russell 2000 Index. For the Vanguard 500, which seeks to mimic the S&P 500 Index, we created a "Cap 500" index as a benchmark.

To demonstrate the type of output this analysis generates, we applied our dynamic allocation attribution to each of the funds in our sample. Active managers use multiple strategies along multiple factor dimensions. For ease of illustration, however, we limited our initial analysis to three factors that are common in the investment industry and considered them separately: industry sectors, value/growth styles, and small-cap/large-cap styles. Of course, managers may chose to base their holdings on other factors, such as momentum, volatility, or market beta; the methodology demonstrated here can be easily extended to account for these additional factor dimensions.

In the case of the industry-based sectors, holdings for both the mutual fund and its benchmark were assigned to subportfolios on the basis of the Fama–French 12 industry classifications. For the value/growth style groups based on book-to-market ratios, we assigned holdings to one of the 10 book-to-market subportfolios on the basis of NYSE breakpoints at the beginning of each calendar year.[7] For the small/large cap groups based on market capitalization, we assigned holdings to one of the 10 capitalization subportfolios on the basis of NYSE breakpoints at the beginning of each calendar year.

[7] The book-to-market ratio here is the ratio of a company's book equity to its market equity. The book value of equity was calculated as the book value of stockholders' equity plus balance sheet deferred taxes and investment tax credit minus the book value of preferred stock (as of the most recent reporting date in COMPUSTAT). The market value of equity was calculated as the share price multiplied by the number of shares outstanding.

Although most of these funds invest primarily in U.S. equity, they typically also have cash holdings, some foreign stocks, and maybe even some fixed-income investments. Therefore, the value-added numbers reflect an analysis of the performance based on the funds' estimated U.S. equity holdings only.

The results of the three univariate analyses based on industry sectors, value/growth and small/large deciles are reported in Table III. The first three columns contain the name of the fund, the starting date (as recorded by the Thomson Reuters database) and the benchmark (our construction). The fourth, fifth, and sixth columns contain the total allocation effect and the decomposition into static and dynamic allocation effects. The last two columns contain the stock selection and total value-added of the fund relative to its benchmark. Quarterly data were used in the analysis, and the value-added numbers are annualized.[8]

Stock selection makes up the majority of the overall value-added in all three panels. At first glance, this result appears to be extremely positive news for the handful of active managers that we selected for this study. Consistent with the stated objectives of the funds, the managers did not pursue a strategy to systematically tilt toward growth or value stocks or particular industry sectors, although there appears to be a systematic bias for large-cap stocks, which hurt long-term performance (we will explore this effect later). For this sample, timing skill appears to be strongest in industry sector allocation; an average value-added of 96 bps came from sector timing. Dynamic allocation associated with value and growth is weak; on average, managers were able to generate 26 bps from value and growth style timing. Dynamic allocation to small-cap and large-cap stocks is actually negative on average.

The static allocations to industrial sectors or value/growth style do not appear to have contributed to value-added. The static allocation to size, however, contributed significantly to *negative* performance; this result is driven by an average overweight to large-cap stocks. Because all of the funds have significant AUM, they may be forced to shy away from the small-cap names in their benchmarks for liquidity and capacity

[8] Total value-added may be slightly different for each fund among the panels. This difference arises from the fact that we were not able to obtain industry or book-to-market information for every stock in the sample.

reasons. The outcome would be a systematic large-cap bias in these portfolios, which resulted in negative static allocation alphas over time.

As advertised in the fund descriptions, stock selection was the dominant investment performance driver. Note, however, that the results in Table III correspond to three separate analyses that examine only one factor dimension at a time. Allocation to factor dimensions that are orthogonal to the one considered will show up as stock selection, so the high stock-selection value-added may be the result of managers allocating to factors that were not considered in the univariate analysis. For example, the analysis for industry groups (Panel A) did not explicitly consider the effect from the value/growth factor tilt in the portfolio over time. To the extent that the value factor is orthogonal to industry effects, allocations to it would be absorbed by stock selection in Panel A.

One way to address this issue is to simultaneously consider multiple factor dimensions. This method can be carried out by creating mutually exclusive subportfolios that are based on multiple sorts along several risk dimensions. To illustrate this approach, we created factors that are based on the 12 Fama–French industry groups, 5 NYSE book-to-market groups, and 5 market-cap groups. This three-way sort resulted in a potential 300 subportfolios and allowed us to consider a manager who is simultaneously allocating to these three factors. Table IV presents the results of this analysis.

The allocation effect in Table IV summarizes the combined value-added from the manager's static allocation to industry sectors, value tilt, and size tilt as well as the dynamic allocation effect. In this case, the majority of value-added is still attributed to stock selection (4 percent on average), which suggests that managers are good stock pickers and/or managers are allocating to factor dimensions that we are not explicitly considering. In this limited sample, approximately 1 percent of the value-added is attributable to dynamic allocation skill.

Another feature that Table IV illustrates is the robustness of our approach. Generally, the nature of holdings-based analysis is such that the portion of value-added attributed to the allocation effect increases with the number of groups into which the portfolio is sub-divided. This effect is easy to comprehend. If we were to take the number of groups to the extreme, at which point each security was an individual group, all the value-added would be, by definition, attributed to the allocation effect. In our current application, we find that even with 300 groups, the results obtained are quite robust with stock selection still receiving the lion's share of value-added**.**

Managers of the funds in our sample, who experienced tremendous success over the long run (the selection criteria required large AUM and long track record), appear to demonstrate significant skill in stock selection and in timing industry sectors in their fund management; they also show modest skill in timing growth and value style selection. They show negative static allocation skill, however, to selecting between small-cap and large-cap stocks; although this failure may be driven by liquidity and capacity issues associated with the small-cap names in the benchmarks. What we find interesting is that value investing, which has been documented as one of the more consistent outperforming strategies, does not play a role in these funds.

In the cross-section of the managers in our sample, we do find a wide disparity in sources of value-added as well as in the split between dynamic and static allocation approaches. We refrain from drawing additional conclusions from our attribution above. The managers we have selected for our sample may either be positive outliers or truly be high alpha managers. We have not performed the proper econometrics to distinguish the two hypotheses. Our attribution is simply meant as an illustration of the methodology.

V. Conclusion

We proposed a dynamic allocation attribution methodology that retains the intuition and familiar characteristics of traditional Brinson attribution analysis. In addition to distinguishing between stock selection and factor selection, our method subdivides the allocation effect into static and dynamic components. The static component measures performance attributable to the persistant factor profile of the manager's portfolio. The dynamic component measures the performance attributable to the manager's timing ability. We argue that it is important to distinguish between static and dynamic allocation skill in the factor domain as it gives further insight into the investment approach of managers and more fully characterizes drivers of manager alpha.

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TABLE I: Static and Dynamic Manager Performance

	Year 1	Year 2	Year 3	Average
Sector				
Value return	16%	8%	3%	9.00%
Growth return	4%	10%	-2%	4.00%
Equal-Weighted Benchmark				
Value weight	50%	50%	50%	50.00%
Growth weight	50%	50%	50%	50.00%
Benchmark return	10.0%	9.0%	0.5%	6.50%
Static Manager				
Value weight	80%	80%	80%	80.00%
Growth weight	20%	20%	20%	20.00%
Static manager return	13.6%	8.4%	2.0%	8.00%
Value-added	3.6%	-0.6%	1.5%	1.50%
Static value-added				*1.50%*
Dynamic value-added				*0.00%*
Dynamic Manager				
Value weight	75%	10%	64%	49.67%
Growth weight	25%	90%	36%	50.33%
Dynamic manager return	13.0%	9.8%	1.2%	8.00%
Value-added	3.0%	0.8%	0.7%	1.50%
Static value-added				*-0.02%*
Dynamic value-added				*1.52%*

TABLE II: Fund Objectives

Mutual Fund	Investment Objective
American Fund - Fundamental Investors	Seeking to provide long-term growth of capital and income primarily through investments in common stocks.
American Fund - Growth Fund of America	Seeking to provide long-term growth of capital through a diversified portfolio of common stocks. Has the flexibility to invest wherever the best growth opportunities may be. The fund emphasizes companies that appear to offer opportunities for long-term growth and may invest in cyclical companies, turnarounds, and value situations.
Dodge & Cox Stock Fund	Seeking long-term growth of principal and income. The fund invests primarily in a broadly diversified portfolio of common stocks. In selecting investments, the fund invests in companies that, in Dodge & Cox's opinion, appear to be temporarily undervalued by the stock market but have a favorable outlook for long-term growth.
Fidelity Advisor Small Cap	Investing at least 80% of assets in securities of companies with small market capitalizations (companies with market capitalizations similar to companies in the Russell 2000 Index or the S&P SmallCap 600 Index). Investing in either growth stocks or value stocks or both. Normally invests primarily in common stocks.
Fidelity Contrafund	Investing in securities of companies whose value Fidelity Management and Research believes is not fully recognized by the public. Investing in either growth stocks or value stocks or both.
Fidelity Magellan	Investing primarily in common stocks. Investing in either growth stocks or value stocks or both.
Janus	Investing primarily in common stocks selected for their growth potential. Although the fund may invest in companies of any size, it generally invests in larger, more established companies. The portfolio manager applies a bottom-up approach in choosing investments.
T. Rowe Price Small Cap	Investing at least 80% of net assets in stocks of small companies. Stock selection may reflect either a growth or value investment approach.
Vanguard 500	Investing in stocks in the S&P 500 Index, representing 500 of the largest U.S. companies. Goal is to closely track the index's return, which is considered a gauge of overall U.S. stock returns.
Wells Fargo Advantage Small Cap	Seeking capital growth by investing primarily in the undervalued stock of small capitalization companies.

TABLE III: Univariate Manager Dynamic Allocation Skill Attribution (through December 2008

Panel A: Fama-French 12 Industry Groups

Fund	Start Date	Bench	Allocation Effect	*Static*	*Dynamic*	Stock Selection	Total
Vanguard 500	1980/09	Cap 500	0.21%	*0.06%*	*0.15%*	0.08%	0.29%
American Fund - Fundamental Investors	1980/03	Cap 1000	0.32%	*-0.25%*	*0.57%*	2.32%	2.64%
American Fund - Growth Fund of America	1980/03	Cap 1000	0.88%	*-0.33%*	*1.21%*	3.39%	4.27%
Dodge & Cox Stock Fund	1980/03	Cap 1000	0.57%	*-0.17%*	*0.74%*	2.28%	2.85%
Fidelity Contrafund	1980/12	Cap 1000	0.76%	*-0.26%*	*1.02%*	2.84%	3.61%
Fidelity Magellan	1981/06	Cap 1000	0.70%	*-0.22%*	*0.92%*	1.74%	2.44%
Janus	1980/03	Cap 1000	1.02%	*-0.17%*	*1.19%*	3.78%	4.80%
Fidelity Advisor Small Cap	1999/09	Cap 2000	0.19%	*-0.44%*	*0.63%*	5.13%	5.32%
T. Rowe Price Small Cap	1993/06	Cap 2000	0.79%	*-0.16%*	*0.95%*	1.61%	2.40%
Wells Fargo Advantage Small Cap	1998/06	Cap 2000	2.30%	*0.90%*	*1.40%*	6.53%	8.83%
Active fund average			0.84%	-0.12%	0.96%	3.29%	4.13%

Panel B: Book-to-Market Groups

Fund	Start Date	Bench	Allocation Effect	*Static*	*Dynamic*	Stock Selection	Total
Vanguard 500	1980/09	Cap 500	0.08%	*0.01%*	*0.07%*	0.21%	0.29%
American Fund - Fundamental Investors	1980/03	Cap 1000	0.44%	*0.17%*	*0.27%*	2.16%	2.60%
American Fund - Growth Fund of America	1980/03	Cap 1000	-0.09%	*-0.17%*	*0.08%*	4.38%	4.30%
Dodge & Cox Stock Fund	1980/03	Cap 1000	0.84%	*0.36%*	*0.48%*	2.00%	2.84%
Fidelity Contrafund	1980/12	Cap 1000	0.45%	*0.12%*	*0.33%*	3.23%	3.68%
Fidelity Magellan	1981/06	Cap 1000	0.22%	*-0.03%*	*0.24%*	2.21%	2.43%
Janus	1980/03	Cap 1000	0.37%	*-0.24%*	*0.62%*	4.45%	4.82%
Fidelity Advisor Small Cap	1999/09	Cap 2000	-0.63%	*-0.06%*	*-0.57%*	5.68%	5.05%
T. Rowe Price Small Cap	1993/06	Cap 2000	0.70%	*0.02%*	*0.68%*	1.61%	2.32%
Wells Fargo Advantage Small Cap	1998/06	Cap 2000	0.60%	*0.41%*	*0.19%*	7.79%	8.39%
Active fund average			0.32%	0.06%	0.26%	3.73%	4.05%

Panel C: Capitalization Groups

Fund	Start Date	Bench	Allocation Effect	*Static*	*Dynamic*	Stock Selection	Total
Vanguard 500	1980/09	Cap 500	-0.24%	*-0.25%*	*0.01%*	0.53%	0.29%
American Fund - Fundamental Investors	1980/03	Cap 1000	-0.15%	*-0.05%*	*-0.10%*	2.77%	2.61%
American Fund - Growth Fund of America	1980/03	Cap 1000	-1.17%	*-0.89%*	*-0.28%*	5.48%	4.31%
Dodge & Cox Stock Fund	1980/03	Cap 1000	0.38%	*-0.01%*	*0.39%*	2.47%	2.85%
Fidelity Contrafund	1980/12	Cap 1000	-2.32%	*-1.62%*	*-0.70%*	5.96%	3.64%
Fidelity Magellan	1981/06	Cap 1000	-1.83%	*-1.22%*	*-0.61%*	4.26%	2.43%
Janus	1980/03	Cap 1000	-0.97%	*-0.96%*	*-0.01%*	5.78%	4.82%
Fidelity Advisor Small Cap	1999/09	Cap 2000	-0.06%	*-1.07%*	*1.01%*	5.33%	5.27%
T. Rowe Price Small Cap	1993/06	Cap 2000	-2.45%	*-1.30%*	*-1.16%*	5.10%	2.65%
Wells Fargo Advantage Small Cap	1998/06	Cap 2000	-1.80%	*-1.13%*	*-0.67%*	10.60%	8.81%
Active fund average			-1.15%	-0.92%	-0.24%	5.31%	4.15%

TABLE IV: Multivariate Manager Dynamic Allocation Skill Attribution (through December 2008)

Fund	**Start Date**	**Bench**	**Allocation Effect**	***Static***	***Dynamic***	**Stock Selection**	**Total**
Vanguard 500	1980/09	Cap 500	-0.05%	*-0.06%*	*0.01%*	0.35%	0.29%
American Fund - Fundamental Investors	1980/03	Cap 1000	0.45%	*-0.15%*	*0.60%*	2.14%	2.59%
American Fund - Growth Fund of America	1980/03	Cap 1000	-0.22%	*-1.07%*	*0.85%*	4.54%	4.32%
Dodge & Cox Stock Fund	1980/03	Cap 1000	1.27%	*-0.49%*	*1.75%*	1.57%	2.84%
Fidelity Contrafund	1980/12	Cap 1000	-1.05%	*-1.94%*	*0.89%*	4.74%	3.69%
Fidelity Magellan	1981/06	Cap 1000	-0.96%	*-1.41%*	*0.45%*	3.39%	2.43%
Janus	1980/03	Cap 1000	0.20%	*-1.15%*	*1.34%*	4.64%	4.84%
Fidelity Advisor Small Cap	1999/09	Cap 2000	0.92%	*-1.34%*	*2.27%*	4.00%	4.93%
T. Rowe Price Small Cap	1993/06	Cap 2000	-1.50%	*-1.54%*	*0.04%*	3.77%	2.27%
Wells Fargo Advantage Small Cap	1998/06	Cap 2000	0.32%	*-0.25%*	*0.56%*	8.04%	8.35%
Active fund average			-0.06%	*-1.04%*	*0.97%*	4.09%	4.03%

Note: Subportfolios are composed of 12 Fama–French industry groups, 5 book-to-market groups, and 5 market-cap groups