Financial Risk Management

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Market VaR II

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THINK IN THE NEXT

MONTE CARLO SIMULATION

Monte Carlo Simulation

To calculate VaR using MC simulation we

- Value portfolio today
- Sample once from the multivariate distributions of the Δx_i
- Use the Δx_i to determine market variables at end of one day
- Revalue the portfolio at the end of day

Monte Carlo Simulation continued

- Calculate ΔP, the change in portfolio value.
- Repeat many times to build up a probability distribution for ΔP
- VaR is the appropriate percentile of the distribution
- For example, with 1,000 trials the 1 percentile is the 10th worst case.

Speeding up Calculations with the Partial Simulation Approach

- In most cases, the positions in the portfolio are too complicated to be fully valued on every iteration
- On every iteration, we use the approximate delta/gamma relationship between ΔP and the Δx_i to calculate the change in value of the portfolio
  - Greeks for the positions are usually already calculated.

Market Risk VaR: Historical Simulation Approach

Historical Simulation

- Collect data on the daily movements in all market variables.
- The first simulation trial assumes that the percentage changes in all market variables are as on the first day
- The second simulation trial assumes that the percentage changes in all market variables are as on the second day
- and so on

Historical Simulation continued

- Suppose we use n days of historical data with today being day n
- Let v_i be the value of a variable on day i
- There are n-1 simulation trials
- The ith trial assumes that the value of the market variable tomorrow (i.e., on day n+1) is: $v_n \frac{v_i}{v_{i-1}}$
- Compute all the values of the market variables for day n+1 for each trial.
- Compute changes in the value of the portfolio.

Example: Portfolio on Sept 25, 2008 (Table 14.1, page 304)

Index	Amount Invested ($000s)	Current Price of Index ($)
DJIA	4,000	11,022.06
FTSE 100	3,000	9,599.90
CAC 40	1,000	6,200.40
Nikkei 225	2,000	112.82
Total	10,000	

U.S. Dollar Value of Stock Indices

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	11,219.38	11,131.84	6,373.89	131.77
1	Aug 8, 2006	11,173.59	11,096.28	6,378.16	134.38
2	Aug 9, 2006	11,076.18	11,185.35	6,474.04	135.94
3	Aug 10, 2006	11,124.37	11,016.71	6,357.49	135.44
….	……	……	…..	……	……
499	Sep 24, 2008	10,825.17	9,438.58	6,033.93	114.26
500	Sep 25, 2008	11,022.06	9,599.90	6,200.40	112.82

501 days → 500 Returns

Scenario #1

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	11,219.38	11,131.84	6,373.89	131.77
1	Aug 8, 2006	11,173.59	11,096.28	6,378.16	134.38
….	……	……	…..	……	……
500	Sep 25, 2008	11,022.06	9,599.90	6,200.40	112.82

Recalculate index levels:

11,022.06*11,173.59 / 11,219.38 = **10,977.08**	9,599.90*11,096.28 / 11,131.84 = **9,569.23**	6,200.40*6,378.16 / 6,373.89 = **6,204.55**	112.82*134.38 / 131.77 = **115.05**

Revalue positions:

4000 * 10,977.08/11,022.06 = **3983.67**	3000 * 9,569.23/9,599.90 = **2990.42**	1000 * 6,204.55/6,200.40 = **1000.67**	2000* 115.05/112.82 = **2039.61**

Portfolio Value = 10,014.334

500 1-day Scenarios

Scenario Number	DJIA	FTSE	CAC	Nikkei	Portfolio Value	Loss
1	10,977.08	9,569.23	6,204.55	115.05	10,014.334	-14.334
2	10,925.97	9,676.96	6,293.60	114.13	10,027.481	-27,481
3	11,070.01	9,455.16	6,088.77	112.40	9,946.736	53,264
….	……	…..	…..	…..	…..	…..
499	10,831.43	9,383.49	6,051.94	113.85	9,857.465	142.535
500	11,222.53	9,763.97	6,371.45	111.40	10,126.439	-126.439

Recall: Current portfolio value: 10,000

Highest Losses

Scenario Number	Loss ($000s)
494	477.841
339	345.435
349	282.204
329	277.041
487	253.385
227	217.974
131	205.256

One-day 99% VaR is the 5th worst loss out of the 500 scenarios=**$253,385**

Using Delta-Gamma Approximation

- To avoid revaluing a complete portfolio 500 times a delta/gamma approximation is sometimes used
- When a derivative depends on only one underlying variable, S

$$\Delta P \approx \delta \Delta S + \frac{1}{2}\gamma(\Delta S)^2$$

Weighting Observations

- Let weights assigned to observations decline exponentially as we go back in time.

$$\frac{\lambda^{n-i}(1-\lambda)}{1-\lambda^{n}}$$

- Rank observations from worst to best
- Starting at worst observation sum weights until the required quantile is reached

VaR Using Weighted Observations λ=0.995

Scenario Number *(i)*	Loss ($000s)	Weight $\frac{\lambda^{n-i}(1-\lambda)}{1-\lambda^n}$	Cumulative Weight
494	477.841	0.00528	0.00528
339	345.435	0.00243	0.00771
349	282.204	0.00255	0.01027
329	277.041	0.00231	0.01258
487	253.385	0.00510	0.01768
227	217.974	0.00139	0.01906
131	205.256	0.00086	0.01992

One-day 99% VaR=**$282,204**

The lower the λ the more weight will be put on recent observations.

Filtered Historical Simulation / Volatility Updating

- Use a volatility updating scheme, like EWMA or GARCH(1,1) to estimate daily volatilities
- Adjust the percentage change observed on day i for a market variable for the differences between volatility on day i and current volatility
- Value of market variable under ith scenario becomes

$$v_n \frac{v_{i-1} + (v_i - v_{i-1})\sigma_{n+1} / \sigma_i}{v_{i-1}} = v_n \left[1 + \frac{u_i}{\sigma_i} \sigma_{n+1} \right]$$

Volatilities (% per Day) Estimated for Next Day, Using EWMA

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	1.11	1.42	1.40	1.38
1	Aug 8, 2006	1.08	1.38	1.36	1.43
2	Aug 9, 2006	1.07	1.35	1.36	1.41
3	Aug 10, 2006	1.04	1.36	1.39	1.37
….	……	……	……	……	……
499	Sep 24, 2008	2.21	3.28	3.11	1.61
500	Sep 25, 2008	2.19	3.21	3.09	1.59

Scenario #1 with Vol. Update

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	11,219.38	11,131.84	6,373.89	131.77
1	Aug 8, 2006	11,173.59	11,096.28	6,378.16	134.38
….	……	……	…..	……	……
500	Sep 25, 2008	11,022.06	9,599.90	6,200.40	112.82

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	1.11	1.42	1.40	1.38
….	……	……	……	……	……
500	Sep 25, 2008	2.19	3.21	3.09	1.59

DJIA Level for Sep 26, 2008 on scenario #1:

$$11{,}022.06 \times \left[1 + \left[{}^{11{,}173.59}\!\big/_{11{,}219.38} - 1 \right] \frac{2.19}{1.11} \right]$$

Volatility Adjusted Losses

Scenario Number	Loss ($000s)
131	1,082.969
494	715.512
227	687.720
98	661.221
329	602.968
339	546.540
74	492.764

McKinsey Working Papers on Risk, Number 32

Managing market risk: Today and tomorrow

May 2012

Market-risk practices at 18 financial institutions, 2011, %

Simulation approach	Valuation approach[1]: Sensitivities	Hybrid	Full revaluation
Historical simulation		35	40
Hybrid	5		5
Monte Carlo	15		

1 Banks are deemed to use the sensitivities approach if they use it exclusively, hybrid if they use it at least 30 percent of the time, and full revaluation if less than 30 percent.

Source: McKinsey Market Risk Survey and Benchmarking 2011

Exhibit 2 Most banks use equal weighting and look back for one year.

VAR[1] historical-simulation practices at 18 financial institutions, %

	Year				
	1	2	3	4	5
Equal weighting	40	20	5	10	5
Time weighting		10	5		

1 Value at risk.
Note: Numbers may not add up to 100 due to rounding.
Source: McKinsey Market Risk Survey and Benchmarking 2011

Stressed VaR

- Basel requires banks to compute Stressed VaR in conjunction with regular VaR
- Historical simulation based on 250-day period of stressed market conditions
- The one-year chosen should reflect the bank's portfolio, in the sense of worst year for its exposure

ATTRIBUTION IN SIMULATION BASED VAR

Top Losses out of 1000 simulations

Rank	Loss on A	Loss on B	Loss on C	Total Loss
1	1626	976	734	**3337**
2	2360	440	152	**2952**
3	1792	622	524	**2937**
4	2071	361	19	**2451**
5	1005	336	491	**1832**
6	1170	373	231	**1774**
7	1293	354	66	**1713**
8	1411	38	174	**1623**
9	94	529	969	**1592**
10	1337	61	86	**1484**
11	275	293	831	**1399**
12	186	514	582	**1281**
13	571	253	194	**1018**
14	718	27	225	**970**
15	278	296	258	**832**

$VaR_{99\%}$

Allocating Simulation Based VaR

- Suppose we want to allocate the VaR, i.e. understand how each position is contributing to the total
- If we look at the 10^{th} scenario, which is the VaR 99%, Position A is contributing 90% of the Loss, equal to $1337
- But this is not stable, if we look at the 9^{th} scenario, it is contributing 6%, at $94

Allocating Based on Component Volatility

- One method is to recall our result for Component VaR when VaR only depends on the volatility (e.g. Normal distribution)
- Then allocate the VaR we computed in proportion to the allocation of the volatility.

Allocating Based on Component Volatility

Write the dollar value of the portfolio as: $X = x_i + \sum_{j \neq i} x_j$

Changes in each position are: $\Delta x_i = x_i \cdot R_i, Var\left(R_i\right) = \sigma_i^2$

Changes in portfolio value: $\Delta X = x_i \cdot R_i + \sum_{j \neq i} x_j \cdot R_j$

The variance of portfolio value is:

$$\sigma_P^2 = x_i^2 \sigma_i^2 + 2 \sum_{j \neq i} x_i x_j \sigma_i \sigma_j \rho_{ij} + Var\left(\sum_{j \neq i} \Delta x_j \right)$$

Marginal Volatility is therefore: $\frac{\partial \sigma_P}{\partial x_i} = \frac{2 x_i \sigma_i^2 + 2 \sum_{j \neq i} x_j \sigma_i \sigma_j \rho_{ij}}{2\sqrt{\sigma_P^2}}$

$$Component\,vol = x_i \frac{\partial \sigma_P}{\partial x_i} = \frac{\operatorname{cov}\left(\Delta x_i, \Delta X\right)}{\sigma_P}$$

Allocating Based on Component Volatility

- Estimate with sample standard deviation and covariance of simulation results:

$$CoVol_i = x_i \frac{\partial \sigma_P}{\partial x_i} = \frac{1}{\sigma_P} \times \frac{1}{N-1} \sum_{n=1}^{N} (S_i^n - \bar{S}_i)(P^n - \bar{P})$$

σ_P SD of Portfolio Values

S_i^n Value of position i in simulation n

P^n Value of portfolio in simulation n

- Allocate VaR proportionally:

$$CoVaR_i = \frac{VaR}{\sigma_P} \cdot CoVol_i$$

Allocating Based on Expected Shortfall

- We can average the simulations around the VaR to get better results
  - But how many simulations?
- A method preferred by risk managers is to look at the Expected Shortfall:
  - Consider the average losses on each position in the simulations where total loss is at least VaR
  - It is more stable and better reflects the position's contribution to the tail
  - We're guaranteed that the sum of these conditional tails will give us the conditional tail:

$$ES_{99\%} = E\left[L \mid L \geq VaR_{99\%}\right] = E\left[L_1 + L_2 + L_3 \mid L \geq VaR_{99\%}\right] =$$

$$= E\left[L_1 \mid L \geq VaR_{99\%}\right] + E\left[L_2 \mid L \geq VaR_{99\%}\right] + E\left[L_3 \mid L \geq VaR_{99\%}\right]$$

Allocating Based on Expected Shortfall

Rank	Loss on A	Loss on B	Loss on C	Total Loss
1	1626	976	734	**3337**
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3	1792	622	524	**2937**
4	2071	361	19	**2451**
5	1005	336	491	**1832**
6	1170	373	231	**1774**
7	1293	354	66	**1713**
8	1411	38	174	**1623**
9	94	529	969	**1592**
10	1337	61	86	**1484**
Expected Shortfall	1416	409	345	**2170**
Percentage	65%	19%	16%	
Allocated VaR	969	280	236	**1484**

CONFIDENCE INTERVAL FOR VAR

Bootstrapping for Confidence Intervals

- Suppose we have 500 days of data.
- Sample with replacement from this set to obtain 1000 sets of 500 days of data
- Calculate VaR for each set, by finding the required percentile
- Rank the 1,000 VaRs: The 25th largest and 975th largest form a 95% confidence interval

Parametric Method for Confidence Interval

- We would like to find the Confidence Interval for VaR 99%, that was computed based on *n* daily losses:

1. Estimate a loss distribution (e.g. Normal mean zero) that fits the data
2. x is the qth quantile of the distribution
3. f(x) is the p.d.f. at that point
4. The standard error is: $$\frac{1}{f(x)}\sqrt{\frac{q(1-q)}{n}}$$

Confidence Interval - Example

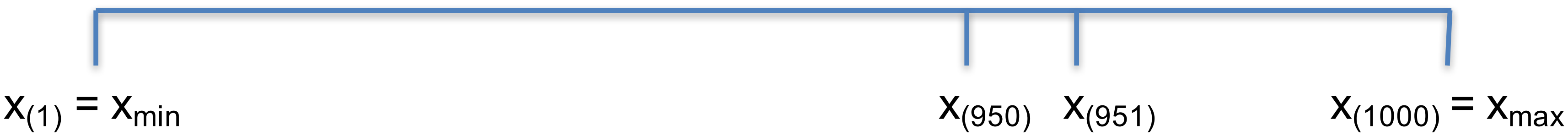
1. We used 500 daily portfolio losses to estimate \$25 as the 99th percentile
2. We find that $N(0,10^2)$ fits the data
3. $2.33*10=23.3$ is the 99th percentile given the normal assumption
4. Using Normal pdf, $f(23.3)=0.0027$
5. The standard error is: $\frac{1}{0.0027}\sqrt{\frac{0.01\times 0.99}{500}}=1.67$

6. 95% C.I. for VaR-99% is:

$$25 \pm 1.96 \times 1.67 = 25 \pm 3.3$$

CI Using Binomial - Example

- Suppose we create a sample of 1000 x_i, and estimate $VaR_{95\%}$ as $x_{(950)}$.
- Consider a sorted view of the sample:



- What is the probability of observing a sample where true $VaR_{95\%}$ is between $x_{(950)}$ and $x_{(951)}$?
- Same as the probability of 950 draws being less than $VaR_{95\%}$.

CI Using Binomial - Example

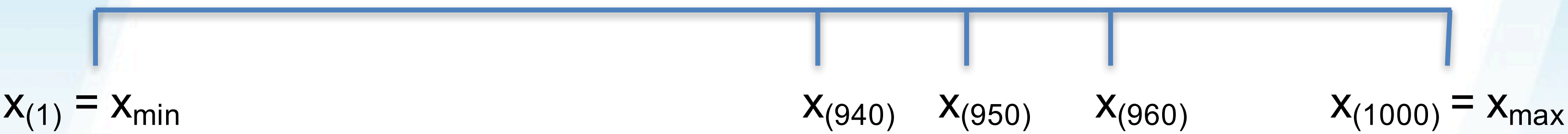
- The probability of exactly 950 draws being less than $VaR_{95\%}$ is binomial, N=1000, p=0.95, x=950:

$$\binom{1000}{950}(0.95)^{950}(0.05)^{50} = 0.06$$

- It is actually not that high!
- What is the probability of observing a sample where $VaR_{95\%}$ is between $x_{(940)}$ and $x_{(941)}$?
- Same as the probability of 940 draws being less than $VaR_{95\%}$.

$$\binom{1000}{940}(0.95)^{940}(0.05)^{60} = 0.02$$

CI Using Binomial - Example



- To find a confidence interval for $VaR_{95\%}$ add up the probability of observing samples where $VaR_{95\%}$ is in consecutive sub-intervals [$x_{(940)}$ to $x_{(941)}$], [$x_{(941)}$ to $x_{(942)}$] etc…
- Each probability is a binomial similar to the previous one. [$x_{(940)}$ to $x_{(960)}$] is a 85% confidence interval. $$\sum_{i=940}^{959} \binom{1000}{i} (0.95)^i (0.05)^{n-i} = 0.85$$

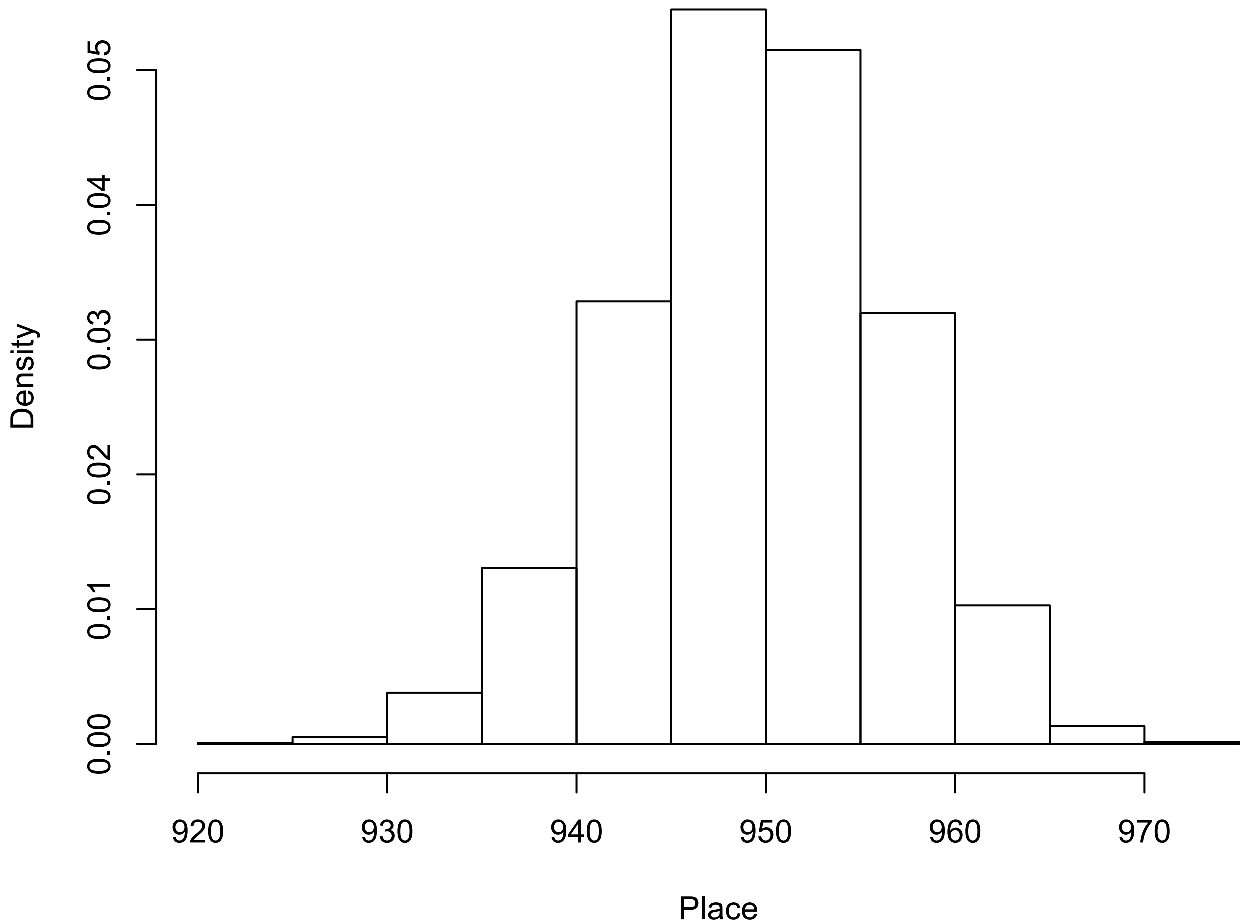
CI Using Binomial - Example

- Expanding the interval to between $x_{(935)}$ and $x_{(965)}$ gives:

$$\sum_{i=935}^{964} \binom{1000}{i} (0.95)^i (0.05)^{n-i} = 0.97$$

- For every sample size, we can find the interval that will give us the required probability.

Location of $N^{-1}(0.95) = 1.645$ in 10,000 samples of 1,000 $x_i \sim N(0,1)$



Thanks