Financial Risk Management

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Credit Risk II –

Deriving Default Probabilities From Market Prices

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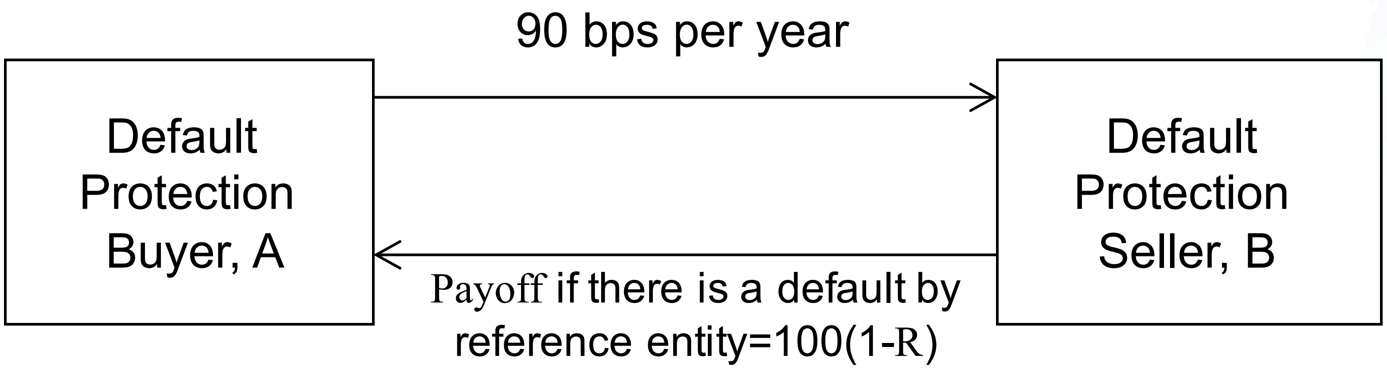
THINK IN THE NEXT

Credit Default Swaps

- Buyer of the instrument acquires protection from the seller against a default by a particular issuer (the reference entity)
- Example: Buyer pays a premium of 90 bps per year for $100 million of 5-year protection against company X
- Premium is known as the *credit default spread*. It is paid for life of contract or until default
- If there is a default, the buyer has the right to sell bonds with a face value of $100 million issued by company X for $100 million (Several bonds may be deliverable)

CDS Structure



Recovery rate, R, is the ratio of the value of the bond issued by reference entity immediately after default to the face value of the bond

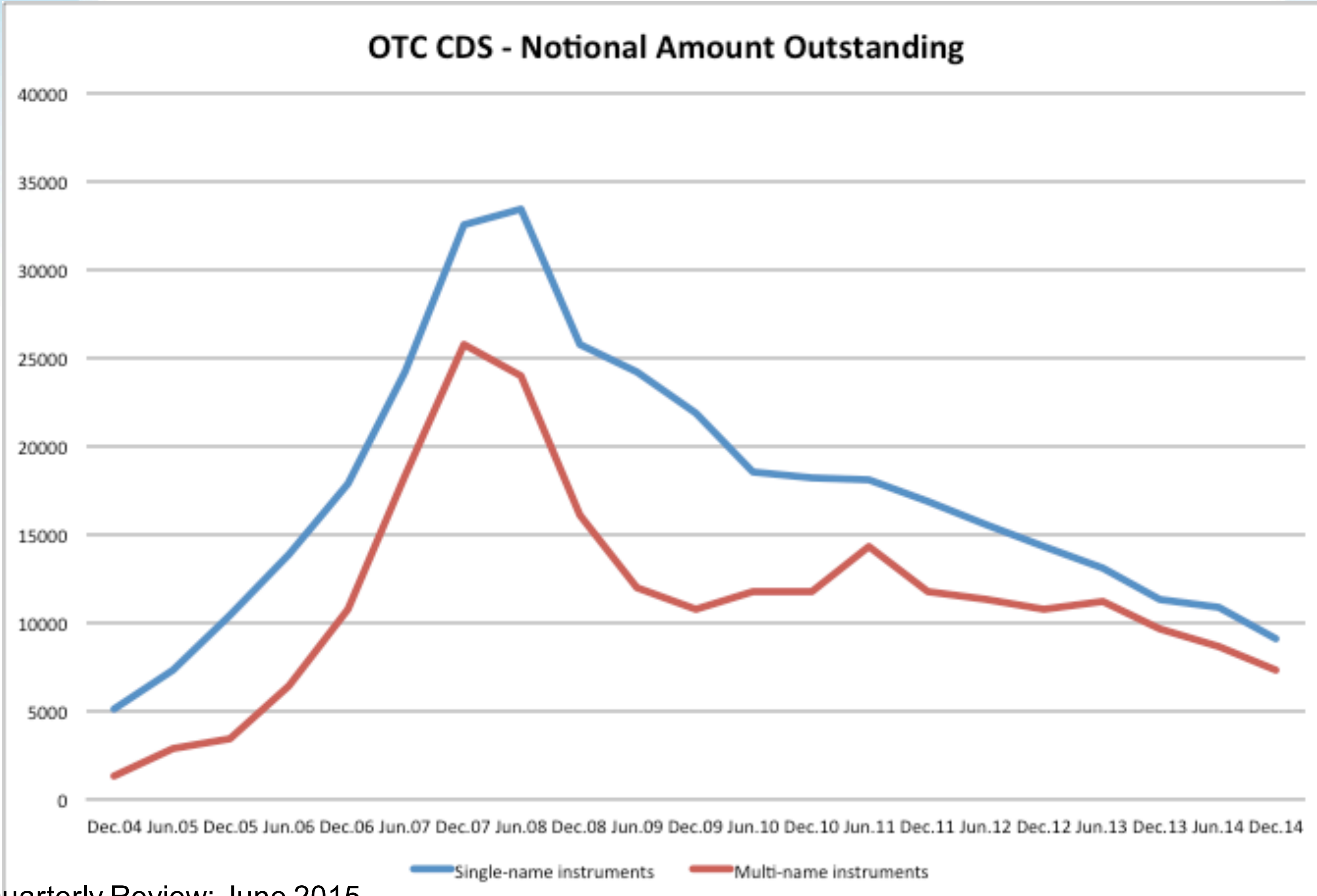
Other Details

- Payments are usually made quarterly in arrears
- In the event of default there is a final accrual payment by the buyer
- Increasingly settlement is in cash and an auction process determines cash amount
- Suppose payments are made quarterly in the example just considered. What are the cash flows if there is a default after 3 years and 1 month and recovery rate is 40%?

Attractions of the CDS Market

- Allows credit risks to be traded in the same way as market risks
- Can be used to transfer credit risks to a third party
- More liquidity in taking short position than in bond market

Declining Volumes in OTC CDS



BIS Quarterly Review: June 2015

Setting Credit Spreads

- Swaps are typically priced to have a net zero present value at time of initiation.
- The credit spread paid by the protection buyer is set so:
- PV of expected payments by the protections seller is equal PV of expected payments by the protection buyer.
- Expectations are taken in the risk neutral measure and are discounted back with the risk free rate.

Credit Spread and Hazard Rates

- One period example:
  - Suppose the risk-neutral probability of default between time t and t+1, conditional on no-default prior to time t is λ
  - The protection buyer agrees to pay s in arrears
  - The protection seller agrees to pay:
    - $1 for a bond worth R if there is a default
    - 0 if there is no default
    - The expected payment is (1-λ)*0+ λ (1-R)
  - s = λ(1-R) or λ = s/(1-R)
- For longer periods: $\bar{\lambda} = \frac{s(t)}{1-R}$

Credit Spread and Hazard Rate (1)

Consider a 5-year CDS, what should be the spread for a given hazard rate?
The risk-free rate is 5% and recovery on the bond is 40%
Suppose the hazard rate is 2.0%, first derive the unconditional probabilities of default:

Time (years)	Default Probability	Survival Probability
1	0.0200	0.9800
2	0.0196	0.9604
3	0.0192	0.9412
4	0.0188	0.9224
5	0.0184	0.9039

Pr (D=2) = Pr (Survival year 1) * Pr (D=2|Survival year 1)
Pr (Survival year 2) = Pr (Survival year 1)*[1- Pr (D=2|Survival year 1)]

Credit Spread and Hazard Rate (2)

Assumptions regarding payments:
If the reference entity survives a certain year, buyer pays *s* in arrears
If the reference entity defaults in a certain year, it does so mid-way through the year. At that point: seller pays *(1-R)*, buyer pays *s/2*

Present Value of protection seller's payments:

Time (years)	Probability of Default	Recovery Rate	Expected Payoff ($)	Discount Factor	PV of Expected Payoff ($)
0.5	0.0200	0.4	0.0120	0.9753	0.0117
1.5	0.0196	0.4	0.0118	0.9277	0.0109
2.5	0.0192	0.4	0.0115	0.8825	0.0102
3.5	0.0188	0.4	0.0113	0.8395	0.0095
4.5	0.0184	0.4	0.0111	0.7985	0.0088
Total					0.0511

Credit Spread and Hazard Rate (3)

Present Value of protection buyer's payments:

Time (years)	Probability of Survival	Expected Payment	Discount Factor	PV of Expected Payment
1	0.9800	0.9800s	0.9512	0.9322s
2	0.9604	0.9604s	0.9048	0.8690s
3	0.9412	0.9412s	0.8607	0.8101s
4	0.9224	0.9224s	0.8187	0.7552s
5	0.9039	0.9039s	0.7788	0.7040s
Total				4.0704s

Time (years)	Probability of Default	Expected Accrual Payment	Discount Factor	PV of Expected Accrual Payment
0.5	0.0200	0.0100s	0.9753	0.0097s
1.5	0.0196	0.0098s	0.9277	0.0091s
2.5	0.0192	0.0096s	0.8825	0.0085s
3.5	0.0188	0.0094s	0.8395	0.0079s
4.5	0.0184	0.0092s	0.7985	0.0074s
Total				0.0426s

$4.0704s + 0.0426s = 4.1130s$

Credit Spread and Hazard Rate (4)

- $4.1130s = 0.0511$
- $s = 0.0124 = 1.24\%$
- Using the approximation:
  - $s = \lambda(1-R) = 2\% * (1-0.4) = 1.20\%$

Use of Fixed Coupons

- Increasingly CDSs and CDS indices trade like bonds
- A coupon and a recovery rate is specified
- There is an initial payments from the buyer to the seller or vice versa reflecting the difference between the currently quoted spread and the coupon

Credit Indices

- CDX.NA.IG: equally weighted portfolio of 125 Investment Grade North American companies
- CDX.NA.HY: covers 100 High Yield (Non-Investment Grade) NA borrower
- CDX.EM: covers emerging market borrowers
- iTraxx: equally weighted portfolio of 125 investment grade European companies
- Components of the index are rebalanced twice a year. Each new portfolio is given a series number, e.g. CDX.NA.IG.9

CDX.NA.IG Example

- Suppose a trader buys \$80,000 protection on each of the 125 companies in the index
- A total of \$10M notional
- The current spread is 203bp
- His annual spread payments are:

  0.0203*\$10M = \$203K
- If a company defaults, the buyer receives 80,000*(1-R), and the notional is reduced by \$80K

London Whale - Background

- JP Morgan Synthetic Credit Portfolio is part of its CIO Portfolio
- Constructed as hedge to the bank's long position in credit risk created by its bond and loan portfolio
- In 2011 JPM started selling protection on IG to finance buying protection on HY
- In Nov 2011, made $400M on American Airlines' Bankruptcy
  - It was not hedging any losses in other JP Morgan portfolios.

London Whale - 2012

- In order to reduce its Risk Weighted Assets and its exposure to improving credit conditions, the bank increased selling protection on credit indexes (CDX.NA.IG.9 in particular)
- By end of March the bank was selling protection on $140B (by some estimates) in IG.9, mostly to hedge funds
- In late March, hedge funds stopped buying protection
- Eventually JP Morgan had to close out positions by buying protection at increasing prices


CDX IG CDSI S9 10Y 166.5Y as of close 5/31 CMAN Corp GP
Bid 165.5 Ask 167.5
CDX IG CDSI S9 1
Save Chart
Hide
GP - Line Chart
Page 1/4
Range 02/01/12 - 05/31/12
Upper Market Pric
Mov. Avgs
Currency USD
Period Daily
Lower None
Mov. Avg
Source CMAN
Events
Last Price 166.500
High on 05/31/12 166.500
Average 125.738
Low on 03/21/12
HY Position Losing Money
IG.9 Position making money
Huge losses to close positions
Show Reader
Reached about $140B in sold protection on IG.9
170
160
150
140
130
120
110
Feb 7
Feb 15
Feb 22
Feb 29
Mar 8
Mar 15
Mar 23
Mar 30
Apr 9
Apr 16
Apr 23
Apr 30
May 8
May 15
May 22
May 31
2012
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2012 Bloomberg Finance L.P.
SN 883704 H191-1372-0 01-Jun-12 9:15:28 EDT GMT-4:00

Approximating the loss just in April-May

JPM sold \$140B protection at 120bp. What is the MtM of the position if the current spread is 170bp?
Assume defaults occur at end of year.
Derive hazard rate from current spread: Hazard rate = s/(1-R)

Recovery	40%	LGD	60%
Current Spread	1.70%	Original Spread	1.20%
Hazard Rate	2.83%	Notional	140 billion

t	PD	Survival	Rates	DF	PV Paying Spread if Survive	PV Paying Spread if Default	PV Spread Payer	PV Protection Seller
1	2.83%	97.17%	5%	0.9512	1.55	0.05	1.60	2.26
2	2.75%	94.41%	5%	0.9048	1.44	0.04	1.48	2.09
3	2.68%	91.74%	5%	0.8607	1.33	0.04	1.37	1.93
4	2.60%	89.14%	5%	0.8187	1.23	0.04	1.26	1.79
5	2.53%	86.61%	5%	0.7788	1.13	0.03	1.17	1.65
Total							6.87	9.73
							Difference:	**-2.86**

Whale - Report Findings

- Actually a failure of Risk Governance
- CIO Management gave conflicting and ambiguous goals, did not understand and properly monitor the portfolio position
- The Group's controls and oversight of CIO did not evolve with the increased complexity and risks of the CIO portfolio
- CIO Risk function understaffed, and some of the CIO risk personnel lacked the needed skills
- Inappropriate MTM to better prices than mid led to Income statement restatements

Whale - Limit Breaches

- The unit operated with three types of limits
  - VaR limit was breached multiple times in January leading to breach in the JP Morgan’s overall VaR
    - Breaches were “fixed” by changing the VaR model
  - Non-statistical: CSBPV and CSW 10%
    - Measure the effect of spreads widening; Does not take into consideration correlations
    - Indications that portfolio was turning long on selling protection were ignored
    - Breached in March, but was downplayed as irrelevant measure
  - Stress test limits were also breached but too late

Whale - VaR Model

- The new CIO VaR model was not tested properly, and understated the risks in Q1 2012
- VaR model changed from “linear sensitivities” to “full revaluation”,
  - from only considering spread changes, to considering portfolio correlations
- Limited back-testing by model validators due to “lack of historical data”
- Operational issues due to use of manual excel sheets
  - E.g. Excel mistake: divided by sum instead of average
- Made assumption regarding vendor method of calculation, which was wrong

Credit Default Swaps and Bond Yields

- Portfolio A:
  - 5-year par corporate bond that provides a yield of 6%
  - Buy protection using a 5-year CDS costing 100 bps per year
- Portfolio B:
  - long position in a riskless instrument paying 5% per year
- What are arbitrage opportunities if risk-free rate is not 5%, but 4.5%? What if it is 5.5%?
  - Buy bond to get 6% coupon and buy protection for 1%, Short risk free bond to pay 4.5% per year

Risk-free Rate

- The risk-free rate used by bond traders when quoting credit spreads is the Treasury rate
- The risk-free rate traditionally assumed in derivatives markets is the LIBOR/swap rate
- By comparing CDS spreads and bond yields it appears that in normal market conditions traders are assuming a risk-free rate 10 bp less than the LIBOR/swap rates
- In stressed market conditions the gap between the LIBOR/swap rate and the "true" risk-free rate is liable to be much higher

CDS-Bond Basis

- This is the CDS spread minus the Bond Yield Spread
- Bond yield spread to LIBOR is usually calculated as the asset swap spread
  - Asset Swaps allow to convert the bond fixed rate payments to floating (LIBOR) payment
- Tended to be positive pre-crisis
- In general, any feature that would make the insurance guaranteed by CDS more valuable will make the spread increase

Factors Affecting CDS-Bond Basis

- The bond may sell for a price different than par (prices above par decrease basis; below par increase basis)
- Counterparty default risk in a CDS (push basis in negative direction)
- Cheapest-to-deliver bond option in CDS (pushes basis in positive direction)
- Payoff on a CDS does not include accrued interest on the bond delivered (push basis in negative direction)
- Restructuring clause in CDS contract may lead to payoff when there is no default (push basis in positive direction)

Deriving PD from Bond Prices

- Consider a defaultable bond paying 100 in 1-year. The yield on the bond is 8%. A riskless bond with similar promised payment has a yield of 6%.
  - The price of the riskless bond is 100/1.06=94.34
  - The price of the risky bond is 100/1.08=92.59
  - The difference between them, 1.75, is the present value of expected loss from default

Deriving PD from Bond Prices (2)

- We have two ways of pricing the defaultable bond
  - Discount the promised cashflows with the risky yield - 100/1.08=92.59
  - Discount the expected cashflows under the risk neutral measure with the risk free rate
- Suppose the risk-neutral probability of default is Q
- The expected cashflows are

  100*(1-Q)+100*R*Q = 100[1-Q*(1-R)]
- Assume R=0.4, the two methods will give the same result if:

  100[1-Q*(1-R)]/1.06=100/1.08

  Q=[1-1.06/1.08]/(1-0.4) = 3.09%
- The present value of expected loss from default is:
  - 100*PD*(1-R)/(1+rf) = 100*3.09%*(1-0.4)/1.06 = 1.75

Applying the method for longer term bonds

- Suppose that a five year corporate bond pays a coupon of 6% per annum (semiannually). The yield is 7% with continuous compounding and the yield on a similar risk-free bond is 5% (with continuous compounding)
- The expected loss from defaults is 8.75. This can be calculated as the difference between the market price of the bond and its risk-free price (104.09 – 95.34)
- Suppose that the unconditional probability of default is Q per year and that defaults always happen half way through a year (immediately before a coupon payment).

Calculations

Time (yrs)	Def Prob	Recovery Amount	Risk-free Value	Loss	Discount Factor	PV of Exp Loss
0.5	Q	40	106.73	66.73	0.9753	65.08Q
1.5	Q	40	105.97	65.97	0.9277	61.20Q
2.5	Q	40	105.17	65.17	0.8825	57.52Q
3.5	Q	40	104.34	64.34	0.8395	54.01Q
4.5	Q	40	103.46	63.46	0.7985	50.67Q
Total						288.48Q

Calculations continued

- Note:
  - To get Risk Free Value, we need forward rates. In this case, all are assumed to be 5%
  - CDS Pays back Notional Value and recovery is assumed 40% of Par Value
- We set $288.48Q = 8.75$ to get $Q = 3.03\%$
- This analysis can be extended to allow defaults to take place more frequently
- With several bonds we can use more parameters to describe the default probability distribution (rather than Q for all maturities)

Structural Models of Default

- Structural models attempt to explain the mechanism by which default occurs.
- Merton (1974) is the prototype model
  - The firm's assets follow a stochastic process (V_t)
  - The firm has one zero coupon bond issued with face value D, maturing at time T
  - At any time the value of the firm is given by:

$$V_t = E_t + D_t$$

Value of Debt and Equity at Bond Maturity, T

- $V_T > D$: the assets are worth more than the debt. The debt holders receive D. The shareholders receive $E_T = V_T - D$
- $V_T \leq D$: the assets are worth less than the debt. The firm cannot pay its obligations.
  - The equity is worth 0.
  - Shareholders have limited liability.
  - The firm is liquidated and the bond holders receive $D_T = V_T$.

Equity and Debt as Contingent Claims

- The model regards the equity as a European call option on the assets of the firm with the strike price equal to the debt's notional value:

$$E_T = \max(V_T - D, 0)$$

- The bond equals the notional value of the bond minus a European put option with strike price equal to the debt value:

$$D_T = D - \max(D - V_T, 0)$$

Underlying Process for Assets

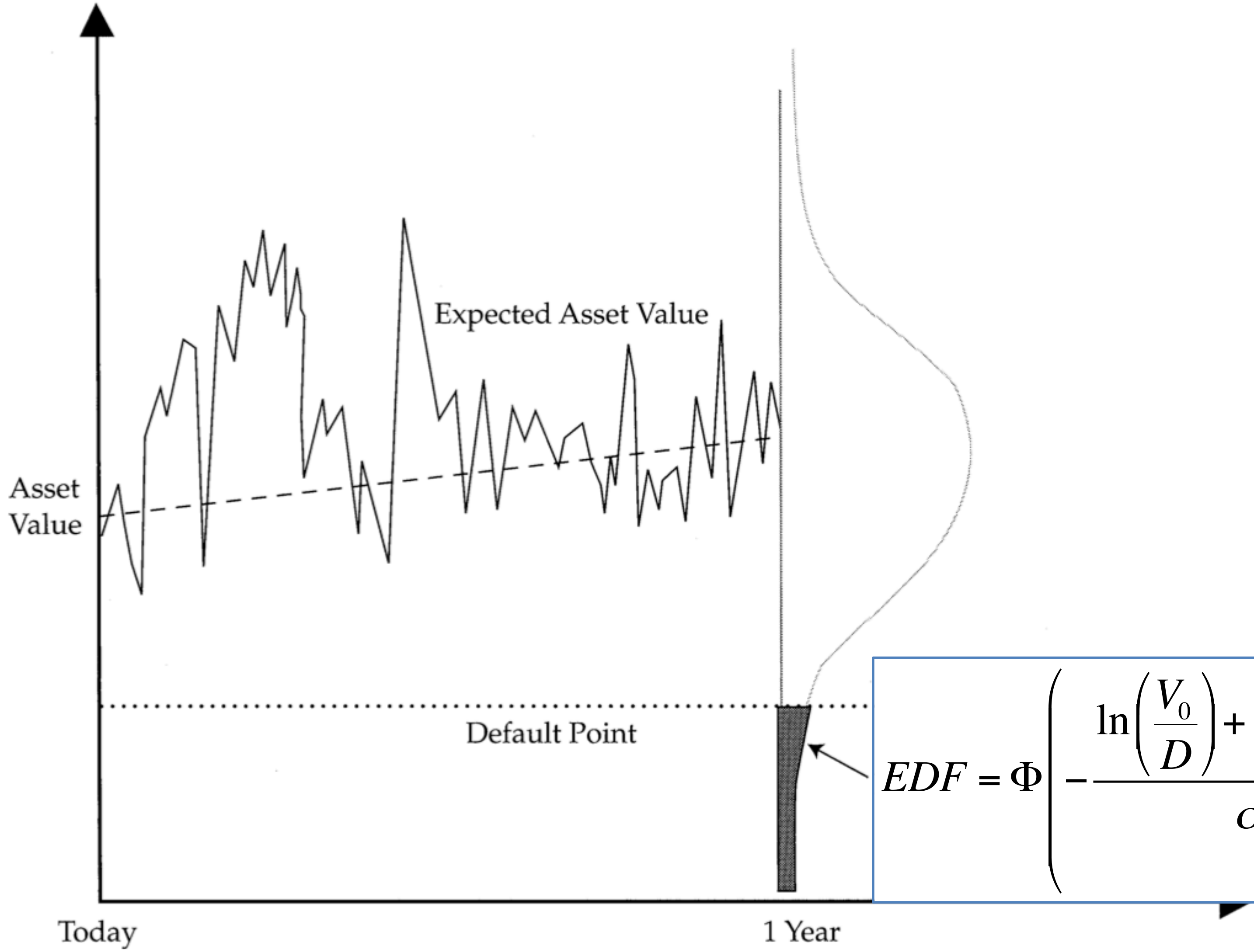
- The assets of the firm follow a Geometric Brownian Motion in the real-world probability measure: $dV_t = \mu_V V_t dt + \sigma_V V_t dW_t$
- Apply Ito's lemma and integrate from 0 to T:

$$\ln V_T \sim N\left(\ln V_0 + \left(\mu_V - \frac{1}{2}\sigma_V^2\right)T, \sigma_V^2 T\right)$$

- The probability of default is:

$$P(V_T \le D) = P(\ln V_T \le \ln D) = \Phi\left(\frac{\ln D - \ln V_0 - \left(\mu_V - \frac{1}{2}\sigma_V^2\right)T}{\sigma_V \sqrt{T}}\right)$$

Figure 1. Illustration: Frequency Distribution of Asset Value at Horizon and Probability of Default



$$EDF = \Phi\left(-\frac{\ln\left(\frac{V_0}{D}\right) + \left(\mu_V - \frac{1}{2}\sigma_V^2\right)T}{\sigma_V\sqrt{T}} \right)$$

V_0 = ? , μ_V = ? , σ_V = ?

Kealhofer – FAJ (2003)

Risk Neutral Probability of Default

- The risk neutral probability of default is:

$$\Phi\left(-\frac{\ln\left(\frac{V_0}{D}\right)+\left(r-\frac{1}{2}\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)=N\left(-d_2\right)$$

- RN probability of default is:
  - Decreasing with V_0
  - Increasing with D
  - Increasing in σ_V if $V_0 > D$

Risk Neutral World

- In order to find the probability of default, we have to find V_0 and σ_V.
- We use Black-Scholes to find the value of the firm's equity today , E_0, as a function of the value of its assets today, V_0, and the volatility of its assets, σ_V

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \; ; \quad d_2 = d_1 - \sigma_V \sqrt{T}$$

Applying the model

Since E is a contingent claim on V, we can use Ito's lemma to get:

$$\sigma_E E_0 = \frac{\partial E}{\partial V}\sigma_V V_0 = N(d_1)\sigma_V V_0$$

This equation together with the option pricing relationship enables V_0 and σ_V to be determined from E_0 and σ_E

Example

- A company's equity is \$3 million and the volatility of the equity is 80%
- The risk-free rate is 5%, the debt is \$10 million and time to debt maturity is 1 year
- Solving the two equations yields V_0=12.40 and σ_v=21.23%

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

$$\sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 = N(d_1) \sigma_V V_0$$

Example continued

- The Risk Neutral probability of default is $N(-d_2)$ or 12.7%
- The market value of the debt is V_0-E_0=9.40
- The risk-free present value of the debt is 9.51
- PV of expected losses = 0.11
- The expected loss at T=1 is 0.11*exp(0.05)/10= 1.2%
- (1-Recovery)*12.7% = EL = 1.2%
- The recovery rate is 91%

Distance to Default

- N(-d_2) is the probability of default.
- KMV define Distance to Default as the number of standard deviations the firm value is from the default level
- It is a measure of how unlikely is default
- It is an approximation of d_2 (T=1)

$$DD = \frac{V_0 - D}{V_0 \sigma_V} \sim \frac{\ln(V_0) - \ln(D) + (r - \sigma_V^2 / 2)}{\sigma_V} = d_2$$

The Implementation of Merton's Model to estimate real-world default probability (e.g. Moody's KMV)

- Calculate cumulative obligations to time horizon. We denote it by D
- Use Merton's model to calculate a theoretical probability of default, or Distance to Default
- Use historical data to develop a one-to-one mapping of theoretical probability into real-world probability of default.
- Assumption is that the rank ordering of probability of default given by the model is the same as that for real world probability of default

Real World vs Risk-Neutral Default Probabilities

- The default probabilities backed out of bond prices or credit default swap spreads are risk-neutral default probabilities
- The default probabilities backed out of historical data are real-world default probabilities

A Comparison

- Calculate 7-year hazard rates from the Moody's data (1970-2010). These are real world default probabilities)
- Use Merrill Lynch data (1996-2007) to estimate average 7-year default intensities from bond prices (these are risk-neutral default intensities)
- Assume a risk-free rate equal to the 7-year swap rate minus 10 basis points

Real World vs Risk Neutral Default Probabilities (7 year averages)

Rating	Historical Hazard Rate (% per annum)	Hazard Rate from bonds (% per annum)	Ratio	Difference
Aaa	0.034	0.596	17.3	0.561
Aa	0.098	0.728	7.4	0.630
A	0.233	1.145	5.8	0.912
Baa	0.416	2.126	5.1	1.709
Ba	2.140	4.671	2.2	2.531
B	5.462	8.017	1.5	2.555
Caa	12.016	18.395	1.5	6.379

Risk Premiums Earned By Bond Traders

Rating	Bond Yield Spread over Treasuries (bps)	Spread of risk-free rate used by market over Treasuries (bps)	Spread to compensate for default rate in the real world (bps)	Extra Risk Premium (bps)
Aaa	78	42	2	34
Aa	86	42	6	38
A	111	42	14	55
Baa	169	42	25	102
Ba	322	42	128	152
B	523	42	328	153
Caa	1146	42	721	383

Possible Reasons for These Results

- Corporate bonds are relatively illiquid
- Bonds do not default independently of each other. This leads to systematic risk that cannot be diversified away.
- Bond returns are highly skewed with limited upside. The non-systematic risk is difficult to diversify away and may be priced by the market
- The subjective default probabilities of bond traders may be much higher than the estimates from Moody's historical data

Which World Should We Use?

- We should use risk-neutral estimates for valuing credit derivatives and estimating the present value of the cost of default
- We should use real world estimates for calculating credit VaR and scenario analysis

Thanks