Error Gates also act on Ancilliary Qubit

Here we are only considering the outcomes of measurement as per expectation values, hence the phase is NOT being considered here.

1. Z-gates acting on first main Qubit affect only its phase(which we are not considering in this case)
2. Z-gate has no effect on the ancillary Qubits of the first main Qubit because $Z|0\rangle$ = $|0\rangle$
3. Z-gate has no effect on second main Qubit - $|0\rangle$ and it's ancillary Qubits - $|00\rangle$ because $Z|0\rangle =$ $|0\rangle$
4. Also, X-gate has no effect on when it acts on the first main-Qubit because

$$X\left(\frac{|00\rangle+|11\rangle}{\sqrt{2}}\right) = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$$

Hence we expect that the output will be different only in these cases

1. If both the ancillary Qubits of first main Qubit are affected by X-gate
2. Among the 2nd main Qubit along with its ancillary bits, majority of them are affected by X-gate

Case-1:

The state of the sub-system consisting of First main Qubit along with its ancillary bits $= \frac{|000\rangle+|100\rangle}{\sqrt{2}}$

Encoding the Qubits by applying CNOT results in $\frac{|000\rangle+|111\rangle}{\sqrt{2}}$

Flipping both the ancillary bits we get $\frac{|011\rangle+|100\rangle}{\sqrt{2}}$

Bitflip code

Applying CNOT gate $\frac{|011\rangle+|111\rangle}{\sqrt{2}}$

Applying Toffoli Gate $\frac{|111\rangle+|011\rangle}{\sqrt{2}}$

We still will get $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ when the first main Qubit is measured

Case-2:

State of subsystem consisting of Second main Qubit along with its ancillary bits $= |000\rangle$

Encoding the Qubits by applying CNOT results in $|000\rangle$

Flipping the majority of the Qubits - Say the first and last bits are flipped
$= |101\rangle$

Bitflip Code

Applying CNOT gate results $|110\rangle$
Applying Toffoli gate results $|110\rangle$
We get $|1\rangle$ when the second main Qubit is measured, Hence the output would be
$\frac{|01\rangle + |11\rangle}{\sqrt{2}}$ which on further applying CNOT gate would result in
$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$

Hence our output in case of error will not be $\frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$ in cases when the majority of Qubits among second main Qubit along with its ancillary bits are flipped.

Therefore, we mostly expect the output where '00' with a probability ½ and '11' with a probability ½. In rest of the cases, we get '01' with a probability ½ and '10' with a probability ½. Probability of this happening would be
= P(Any 2 out of 3 Qubits are flipped) + P(All the 3 Qubits are flipped)
$= 3 \times (\frac{p}{2})^2 \times (1-\frac{p}{2}) \quad + \quad (\frac{p}{2})^3$
Putting p = 0.4 we get 0.104
When repeated 1000 times, we can expect
'00' with probability ½ and '11' with a probability ½ 896 times
'01' with probability ½ and '10' with a probability ½ 104 times