Error Gates acting on the main qubits

After applying H-gate to the first qubit,

The State of Qubit-1 = $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$

The State of Qubit-2 = $|0\rangle$

1. If no error gates act on both the Qubits, the state of the output after applying CNOT gate would be

   $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$

2. Even in the following 2 cases, the output would be the same - $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$
   a. If X-gate acts on the first Qubit, because

      $X(\frac{|0\rangle + |1\rangle}{\sqrt{2}}) = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

   b. If Z-gate acts on the second Qubit because

      $Z|0\rangle = |0\rangle$

3. But if Z-gate acts on the first Qubit, with the second Qubit remaining unchanged, then the output would be $\frac{|00\rangle - |11\rangle}{\sqrt{2}}$
4. If Z-gate acts on the first Qubit and X-gate acts on the second Qubit, then the output would be $\frac{|01\rangle - |10\rangle}{\sqrt{2}}$
5. If X-gate acts on the second Qubit, and if the first Qubit remains unchanged then the output would be $\frac{|01\rangle + |10\rangle}{\sqrt{2}}$

In cases - 1,2,3, we expect '00' with a probability of ½ and '11' with a probability of ½

In case - 4,5, we measure '01' with a probability of ½ and '10' with a probability of ½

The probability of cases - 4,5 to occur is

= Probability(Case-4) + Probability(Case-5)

= P(Z-gate on Qubit-1) x P(X-gate on Qubit-2)

+

P(No Z-gate on Qubit-1)x P(X-gate on Qubit-2)

= 1 x P(X-gate on Qubit-2)

= $\frac{p}{2}$

= 0.2

That means out of 100 times, we expect the outcomes

‘**00’: with a probability ½ and ‘11’: with a probability ½** around 80 times

‘01’ with a probability ½ and ‘11’ with a probability ½ around 20 times