Change of basis

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ Means Addition of 2 times the 1^{st} basis vector and 3 times the 2^{nd} basis vector

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are most common basis vectors

But sometimes a change of perspective helps solving few problems easier,
hence we might want to change the basis

Suppose Jennifer is using $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ as her basis, then $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ vector would mean a very different vector in ~~our~~ her world. [$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ are the basis vectors of Jennifer as described in our lang]

(I)

Suppose Jennifer gives a vector

say $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

What would $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ mean to us?

Let's break it

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ means 2 times 1^{st} basis vector + 3 times 2^{nd} basis vector

1^{st} basis vector of Jennifer in our Lang is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

2^{nd} basis vector of Jennifer in our Lang is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \Rightarrow 2\begin{bmatrix} 1 \\ +1 \end{bmatrix} + 3\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$= \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ in our language

is $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ in Jennifer's Lang

The prev operation is similar to

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$\downarrow$

Change of basis matrix ~~too~~

So to convert a vector from alien Language to our Language multiply by change of basis matrix.

II) Suppose we want reverse case.

There is a ~~matrix~~ vector in our language,

we want it in Jennifer's language.

Suppose we want to tell $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$ in Jennifer's lang

$\begin{bmatrix} 5 \\ -1 \end{bmatrix}$ = x times Jennifer's 1st basis + y times Jennifer's Second basis

$$\begin{bmatrix} 5 \\ -1 \end{bmatrix} = x \begin{bmatrix} 1 \\ 1 \end{bmatrix} + y \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$\Rightarrow \quad x = 2, \; y = 3$

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is ~~what~~ $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$ in our lang.

in Jennifer's lang

Prev operation was similar to

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

If we want convert a vector from our Language to alien language multiply it by change of basis matrix

III There is a matrix transformation in our Language & How to convey it to Jennifer, to Convey the same transformation

→ Jennifer has a vector $\begin{bmatrix} x \\ y \end{bmatrix}$

→ Lets Convert the vector to our Language

⇒ Change of Basis Matrix $\cdot \begin{bmatrix} x \\ y \end{bmatrix}$ | Let J is change of basis matrix

⇒ Jx is Vector

→ Now Lets apply transforation 'A'

AJx

→ Lets convert it back to Jennifer's lang

$J^{-1}AJx$

→ If u have a vector in her lang → x and u want to know what happens to the vector, when we apply the transformation We know in our language, then multiply it by $J^{-1}AJ$

In other words

A transformation in our langage

is

$J^{-1}AJ$ transformation in Jennifer's language

Similarly

A transformation in her language

is

JAJ^{-1} transformation in our langage

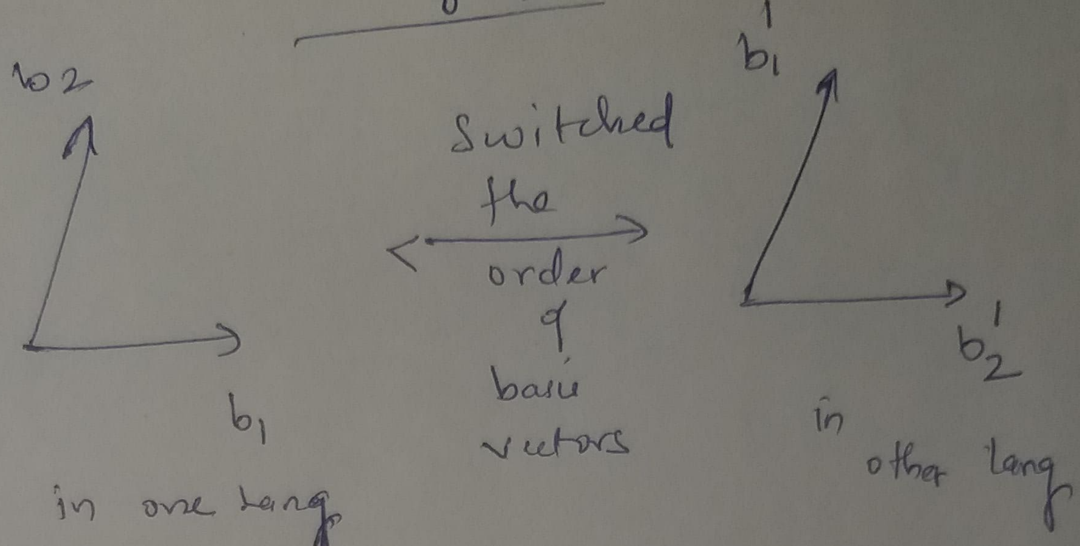
(applying the similar reasoning of prev page)

- $\vec{x}$ in our Lang
- convert to her Lang $J^{-1}x$
- Apply transformation $AJ^{-1}x$
- Convert it back to our Language $JAJ^{-1}x$

nice Use case of JJ explained in eigen vectors.

Looking glass Qs

1



what is change of basis matrix?

in other lang

$b_1' =$ 0 times 1st basis + 1 time 2nd basis

$= 0\, b_1 + 1\, b_2$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$b_2' =$ 1 times 1st basis + 0 times 2nd basis

$= 1\, b_1 + 0\, b_2$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

- stack side by side $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$