Representing a vector

Back to geometric view

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= 2\left(1^{st}\text{ Basis vector}\right) + 3\left(2^{nd}\text{ basis vector}\right)$$

(or)

you get $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ by taking linear combination of the below

1. scaling the first basis vector 2 times

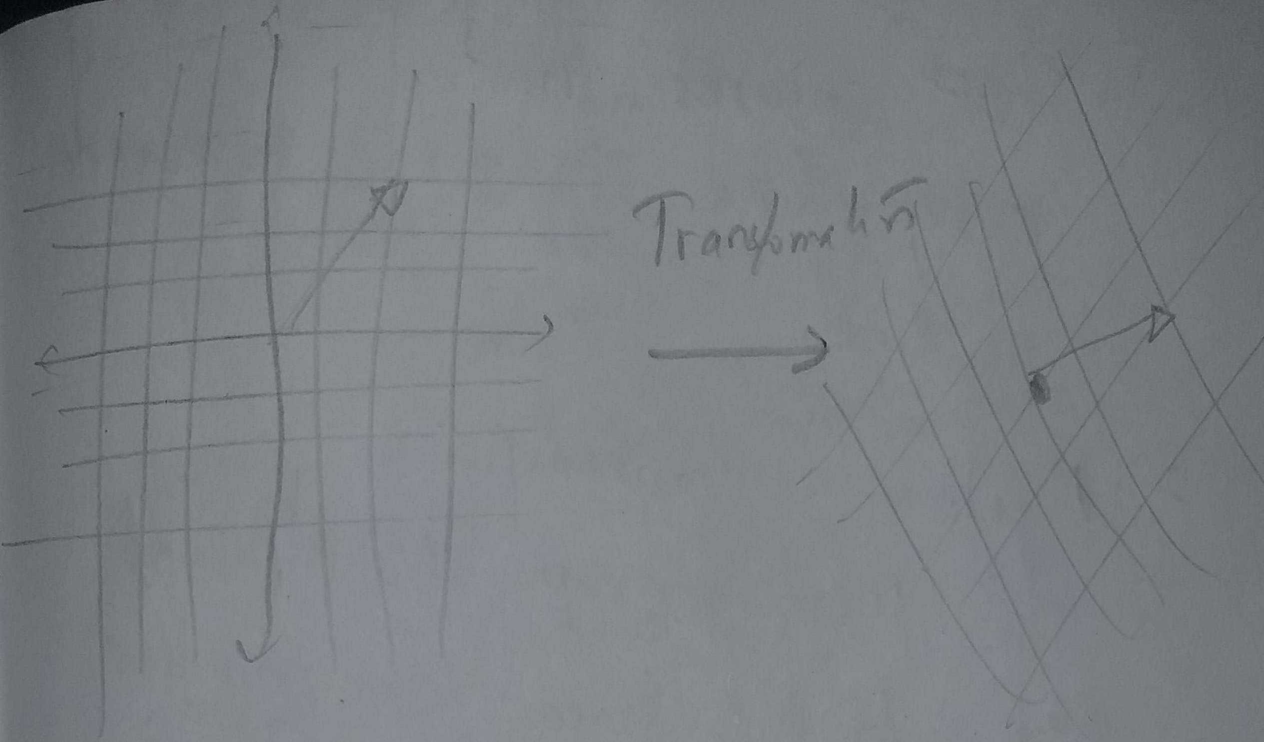
and

2. scaling the 2^{nd} basis vector 3 times

same as in 3D

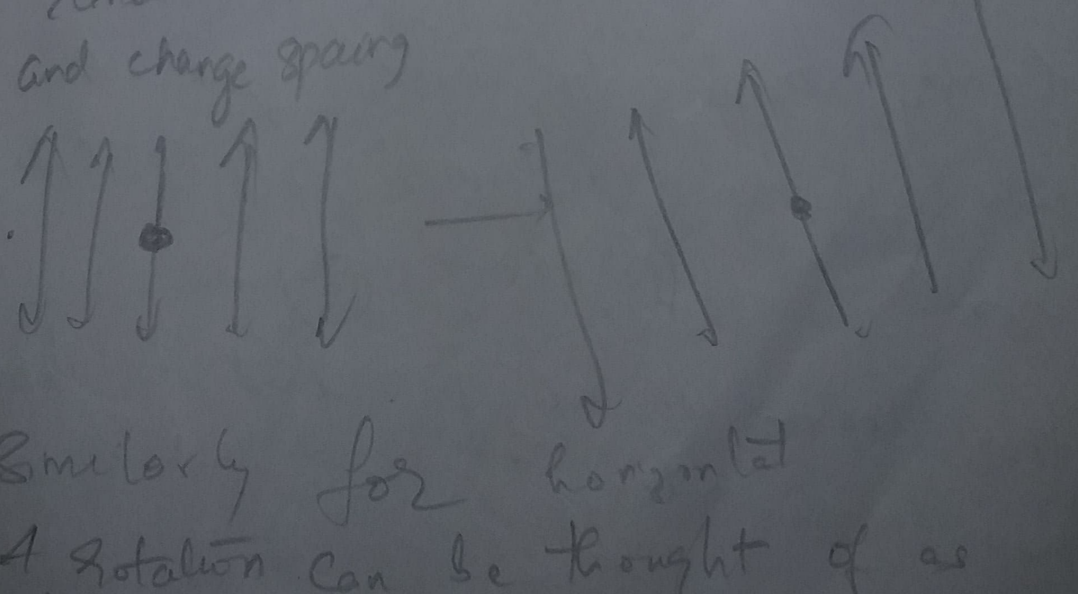
$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Now if the plane (2D-plane) is transformed. How do u represent $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$?



Here we shld mention that
we are allowing linear transformation
only

1. Geometric interpretation of linear transformation
   - The parallel lines are evenly spaced and the origin is in place
   - That means, if you take all vertical lines u can rotate them about origin and change spacing



Similarly for horizontal

A Rotation can be thought of as linear transformation

- That ensures that every line is space remains a line.

> - Algebraic interpretation
> These 2 props shld be followed

Say L is a linear transformation over 2 vector

$$L(u+v) = L(u) + L(v)$$

$$L(cu) = c \cdot L(u).$$

or simply put

$$L(cu + dv) = c \cdot L(u) + d \cdot L(v).$$

Coming back to Question, in a new linearly transformed 2D space how to represent $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

the answer is we just need to know new basis vectors.

$$\text{New}\begin{pmatrix}2\\3\end{pmatrix} = 2\begin{bmatrix}\text{New}\\ \text{1st}\\ \text{basis}\\ \text{vector}\end{bmatrix} + 3\begin{bmatrix}\text{New}\\ \text{2nd}\\ \text{basis}\\ \text{vector}\end{bmatrix}.$$

Suppose New 1st basis vector is $\begin{pmatrix}4\\5\end{pmatrix}$

that is

$$\begin{pmatrix}1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}4\\5\end{pmatrix}$$

and similarly 2nd basis vector is $\begin{pmatrix}6\\7\end{pmatrix}$

$$\begin{pmatrix}0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}6\\7\end{pmatrix}.$$

New

$$\begin{pmatrix}2\\3\end{pmatrix} = 2\begin{bmatrix}4\\5\end{bmatrix} + 3\begin{bmatrix}6\\7\end{bmatrix}$$

$\downarrow$

this can be written as

$$\begin{bmatrix}4 & 6\\5 & 7\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix} \rightarrow$$

$\boxed{\text{That implies a 2D matrix can be thought of as stacking New basis vectors side by side}}$

Similarly in a 3D linear transformation

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

New position of $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ New position of $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ New position of $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is a vector in 3D space

on applying transformation $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

X moves to a vector

$$B = AX$$

A new way to look at sys of linear eqⁿˢ

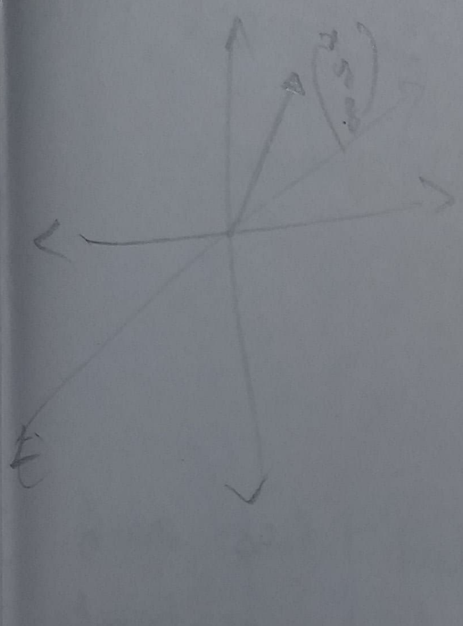
$$2x + 3y + 4z = 5$$
$$6x + 7y + 8z = 9$$
$$10x + 11y + 12z = 13$$

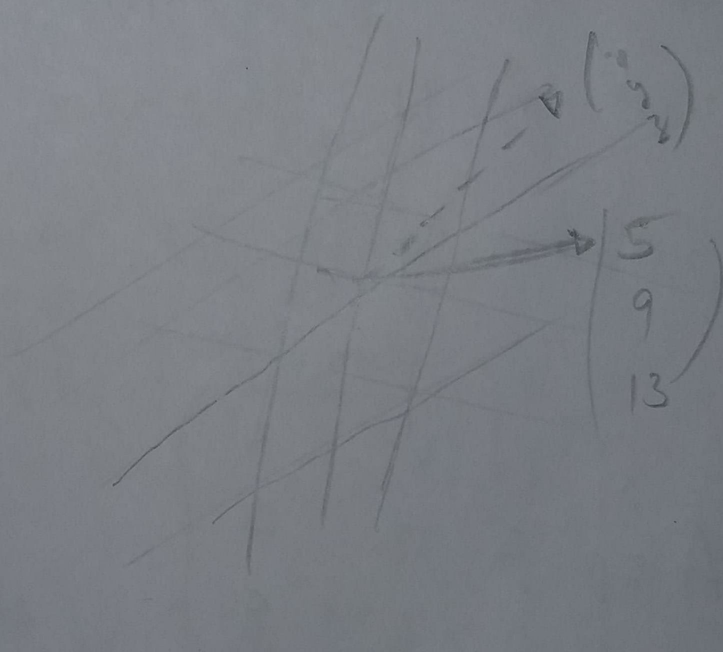
$$\begin{array}{ccc} A & X & B \\ \begin{bmatrix} 2 & 3 & 4 \\ 6 & 7 & 8 \\ 10 & 11 & 12 \end{bmatrix} & \begin{bmatrix} x \\ y \\ z \end{bmatrix} = & \begin{bmatrix} 5 \\ 9 \\ 13 \end{bmatrix} \end{array}$$

Geometrically the Quest is

On applying transformation 'A', what vector

$\begin{pmatrix}x\\y\\z\end{pmatrix}$ became $\begin{pmatrix}5\\9\\13\end{pmatrix}$





Special transformations

Some transformation are special because they reduce the dimensionality of existing space. For example, a

2D plane $\longrightarrow$ a line or a point

3D space $\longrightarrow$ a 2D plane or a line or a point.

This happens when the basis vectors are dependenent on each other, or one of the new basis vectors can be obtained from the other new basis vectors. We introduce 2 terms here Column space and Null space