Column space

$$A = \begin{pmatrix} 1 & 4 & 5 \\ 2 & 5 & 7 \\ 3 & 6 & 9 \end{pmatrix}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{New} & \text{New} & \text{New} \\ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{array}$$

How does each column contribute something new on its own?

No ×

the 3rd column is obtained by adding 1st & 2nd column

Column space is defined as

Set of all vectors that are obtained

by the linear combination of columns

that is, the column space would be

$$x\begin{bmatrix}1\\2\\3\end{bmatrix} + y\begin{bmatrix}4\\5\\6\end{bmatrix} + z\begin{bmatrix}5\\7\\9\end{bmatrix}$$

$$x\begin{bmatrix}1\\2\\3\end{bmatrix} + y\begin{pmatrix}4\\5\\6\end{pmatrix} + z\left(\begin{pmatrix}1\\2\\3\end{pmatrix} + \begin{pmatrix}4\\5\\6\end{pmatrix}\right)$$

$$\begin{pmatrix}1\\2\\3\end{pmatrix}\underbrace{(x+z)}_{X} + \underbrace{(y+z)}_{Y}\begin{pmatrix}4\\5\\6\end{pmatrix}$$

$$X\begin{pmatrix}1\\2\\3\end{pmatrix} + Y\begin{pmatrix}4\\5\\6\end{pmatrix}$$

it is clear that A transformation transforms

3D space into a 2D one.

We can say $AX = B$, has a solution if

B is in the column space of A.

Null space

In a transformation, there is a set of vectors which a transformed to null vector (origin $\begin{pmatrix}0\\0\\0\end{pmatrix}$ or $\begin{pmatrix}0\\0\end{pmatrix}$) in the case, Such set of vectors is called null space,

algebrically speaking solutions of

$$Ax = 0$$

form the null space of A.

Sometimes the set of vectors is a plane or a line. for example

$$\begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$x + 2y = 0$$

$$2x + 4y = 0$$

$\Rightarrow$ the pts lying on line

$x + 2y = 0$ or $\frac{x}{-2} = \frac{y}{1}$ line

is the null space.

The line $\frac{x}{-2} = \frac{y}{1}$ is squished to the origin.

and u can see that $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ transforms

2D plane to a line

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = x\begin{pmatrix} 1 \\ 2 \end{pmatrix} + y\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= x\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2y\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}(x + 2y)$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} x$$

Another way to interpret $Ax = B$

$$x\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} + z\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = \begin{pmatrix} 10 \\ 11 \\ 12 \end{pmatrix}$$

what values of x, y, z result in B

what linear combination of new basis result in B.