When u multiply a ~~tran~~ eigenvector by its transformation, it becomes λ times eigen vector.

$$Ax = \lambda x, \quad \text{also} \quad A^2 x = \lambda (Ax)$$

$$A^2 x = \lambda^2 x$$

if the transf squishes onto a lower dimension
null space of transformation are eigen vectors,
with eigen values 0.

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0 \Rightarrow (a-\lambda)(d-\lambda) - bc = 0$$

$$+\lambda^2 - \lambda(d+a) - bc + ad = 0$$

$$\lambda^2 - \lambda(a+d) + (ad-bc) = 0$$

$\lambda \rightarrow$ eigen values

Sum of eigen values $= +(a+d) =$ Trace

product of eigen vals $= (ad-bc) =$ Det.

Triangular matrix eigen values

$$\begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix} \Rightarrow \text{upper } \Delta^{le} \text{ matrix}$$

eigen values $= 1, 0$

Reason: Trace = sum of diagonal entries = sum of eigen values

~~product~~ determinant = prod of diagonal entries = prod of eigen values

$\Rightarrow$ from above diagonal entries are eigen values if A and B share same eigen ~~vect~~ vector x, and A scales or squishes x by λ, B scales or squishes x by β

$$AB\,x = (\beta\lambda)\,x$$

<u>proof</u>

$AB\,x$	$BA\,x$
$= A(Bx)$	$= B(Ax)$
$= A\beta x$	$= B\lambda x$
$= \beta(Ax)$	$= \lambda(Bx)$
$= \underline{\underline{\beta\lambda x}}$	$= \underline{\underline{\lambda\beta x}}$

we know $\beta\lambda = \lambda\beta$ (scalars)

2) $ABx = BAx$

if $AB = BA$ then

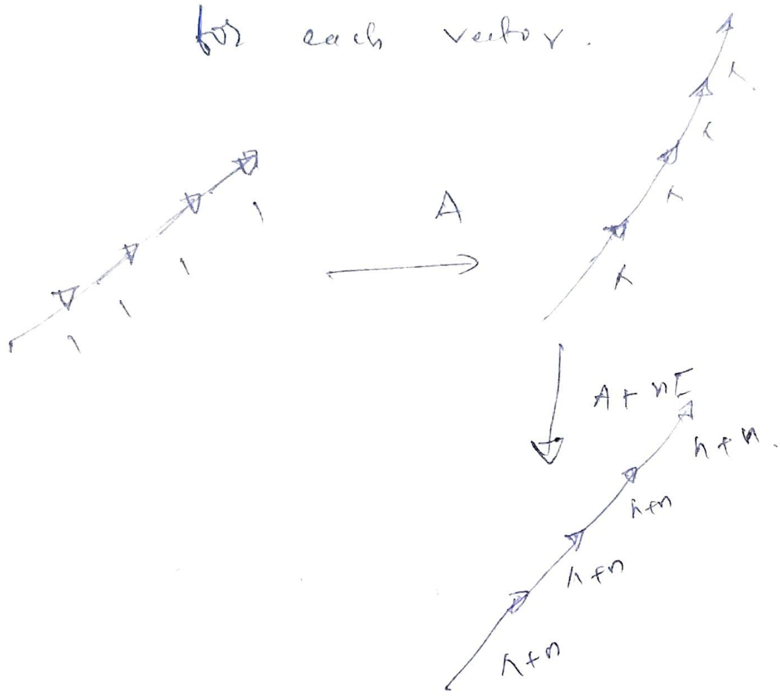
A, B share same independent eigenvectors

A has eigen values $\rightarrow \lambda_1, \lambda_2$

$A + nI$ has eigen values $\rightarrow \lambda_1 + n, \lambda_2 + n$

intuition

they are just added a bias term for each vector.



$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

Q) u is eigen vector of uv^T ($|uv^T| = 0$ $\rightarrow$ rank = 1)

$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2\times1}$ $\quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{2\times1}$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2\times1} \begin{bmatrix} v_1 & v_2 \end{bmatrix}_{1\times2} = \begin{bmatrix} u_1 v_1 & u_1 v_2 \\ u_2 v_1 & u_2 v_2 \end{bmatrix}$$

uv^T is rank 1 $\Rightarrow$ it squashes 2D plane

into a 1D line

$\Rightarrow$ det $= 0$ $\Rightarrow$ $\lambda = 0$ is a eigen value

λ_1 be other eigen value
$\lambda_2 = 0$

$\lambda_1 + \lambda_2 =$ Trace

$\lambda_1 + 0 = u_1 v_1 + u_2 v_2$

$$\boxed{\lambda_1 = u_1 v_1 + u_2 v_2}$$

challenging prob

(36) Heisenberg ineq. $\|Ax\| \|Bx\| \geq \frac{1}{2} \|x\|^2$

$AB - BA = I$

$ABx - BAx = x$

$x^T ABx - x^T BAx = x^T x$

$x^T A(Bx) - x^T B(Ax) = x^T x$

$x^T A^T (Bx) + x^T (-B)(Ax) = x^T x$

$x^T B^T$

$(Ax)^T (Bx) + (Bx)^T (Ax) = x^T x$

$|(Ax)^T Bx| \leq \|(Ax)\| \|Bx\|$

$|(Bx)^T (Ax)| \leq \|Bx\| \|Ax\|$

$\|Ax\| \|Bx\| + \|Ax\| \|Bx\| \geq x^T x$

$2\|Ax\| \|Bx\| \geq x^T x$

$|A+B| \geq |A| + |B|$ (from Minkowski's det thm)

$\downarrow$

$(\det(A+B))^{1/n} \geq (\det A)^{1/n} + (\det B)^{1/n}$

$|x^T x| = (\cancel{\text{len}}(x))^2$

$(Ax)^T (Bx) + (Bx)^T (Ax) = x^T x = (\cancel{\det})^2$

$|(Ax)^T(Bx) + (Bx)^T(Ax)| = |x^T x|$

$\underbrace{|(Ax)^T(Bx)|}_{\text{max value}} + \underbrace{|(Bx)^T(Ax)|}_{} \geq (\text{len}(x))^2$

max value is $\|Ax\|\|Bx\|$

$2\|Ax\|\|Bx\| \geq (\text{len}(x))^2$

$2\|Ax\|\|Bx\| \geq |x^T x|$

$2\|Ax\| \cdot \|Bx\| \geq |x^T||x|$

$\frac{\|Ax\|}{|x|} \cdot \frac{\|Bx\|}{|x|} \geq \frac{1}{2}$

Not wrong