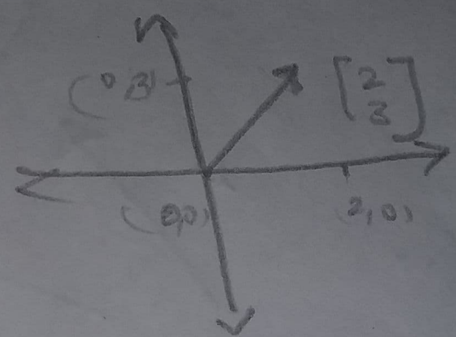
Linear Algebra

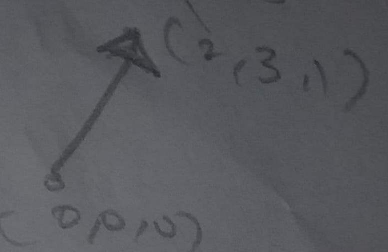
What is a vector?

Speaking geometrically, the ordered list of numbers representing an arrow rooted from origin to a point in space.

eg: $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$



Similary $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$



Well in general, a vector can be much more, but the above gives a simple place to start.

Vector Space :-

Set of elements form a vector space if any Linear combination of those 2 lie in the vector space.

For ex, you have a space, you pick 2 vectors say u and v

Suppose c and d are two scalars (Rea
num

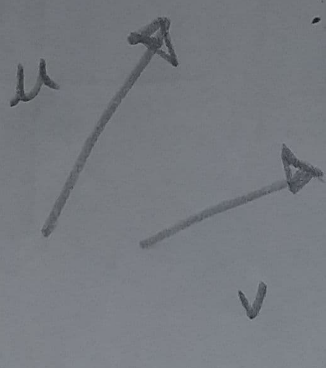
Linear Combination of u and v, would b

cu + dv also lies in the vector space. Note that c and d can be zero too.

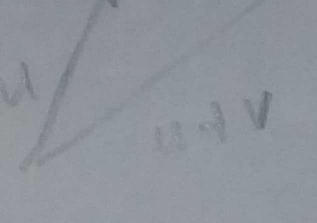
Coming back to geometric point of view.

u → vector

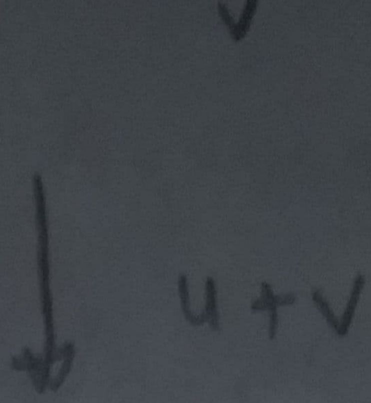
v → vector

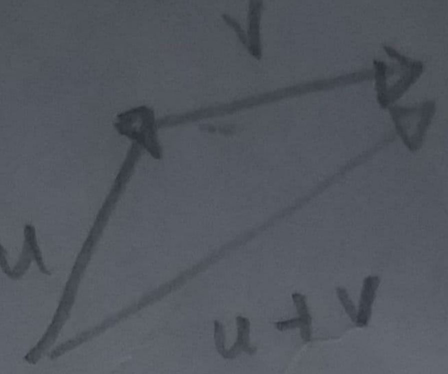


$\downarrow$ u+v



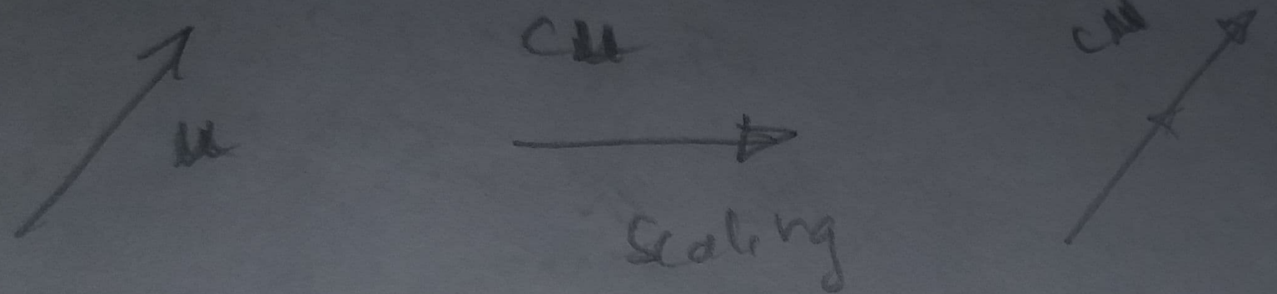
(v is shift
parallel)



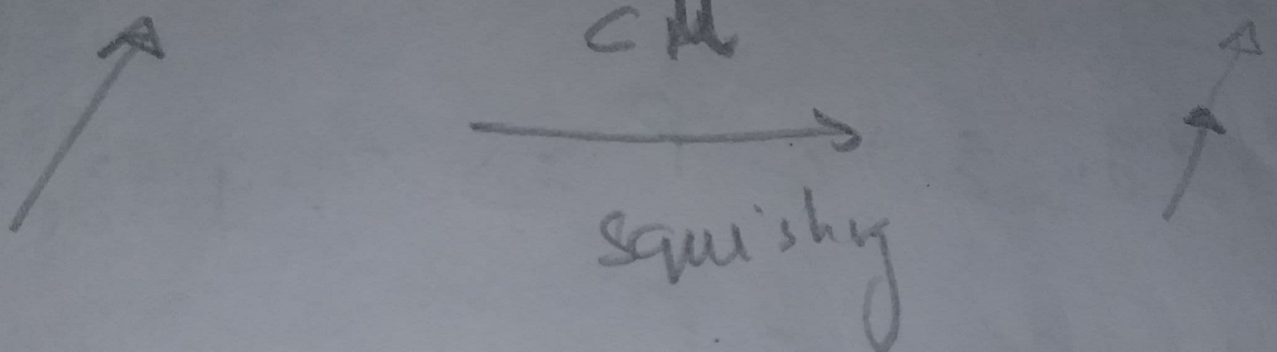


(v i[illegible]
paral[illegible]

(it can be thought of as walking along 2 vectors—
walk from (0,0) to tip of u, then from tip of u to tip of v,
the resultant is the pt that you could have reached directly from (0,0



vector has been scaled

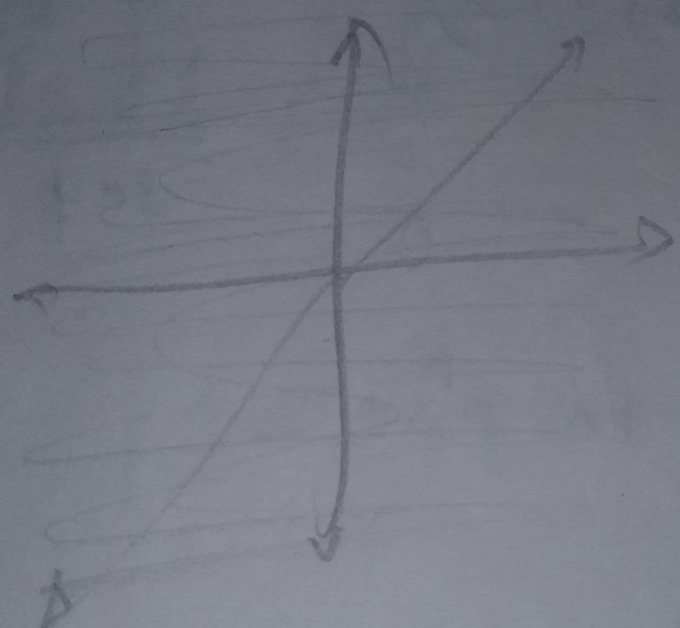


vector can be squished as well ($c < 1$)

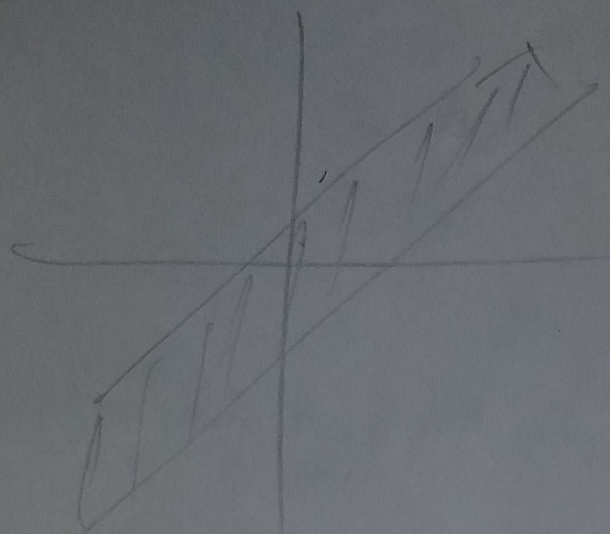
Subspace

Subset of vector space, that form a vector space itself.

example consider R^3 as vector space



All of R^3 space is is a vector space, u take any linear Combination, the result will lie in R^3



A plane passing thru origin would be a subspace of R^3.

For example $P: x + y + z = 0$

proof

$u : (x_1, y_1, z_1) \in P \Rightarrow x_1 + y_1 + z_1 = 0$ (1)

$v : (x_2, y_2, z_2) \in P \Rightarrow x_2 + y_2 + z_2 = 0$ (2)

linear combination of u, v

$cu + dv$

$(cx_1, cy_1, cz_1) + (dx_2, dy_2, dz_2) = (cx_1 + dx_2, cy_1 + dy_2, cz_1 + dz_2)$ say W

$(1) \times c + (2) \times d = 0$

$\Rightarrow$ W also lies on plane

And another example could be a line passing thru origin.

Notes: Subspace S_1, Subspace S_2

1) $S_1 \cup S_2$ → is not a subspace

(eg take a pt on line thru origin & another on plane thru origin try adding them up)

2) $S_1 \cap S_2$ → is a subspace

(since the pt belongs to both the subspaces, the resultant linear combination would also be a subspace by definition.