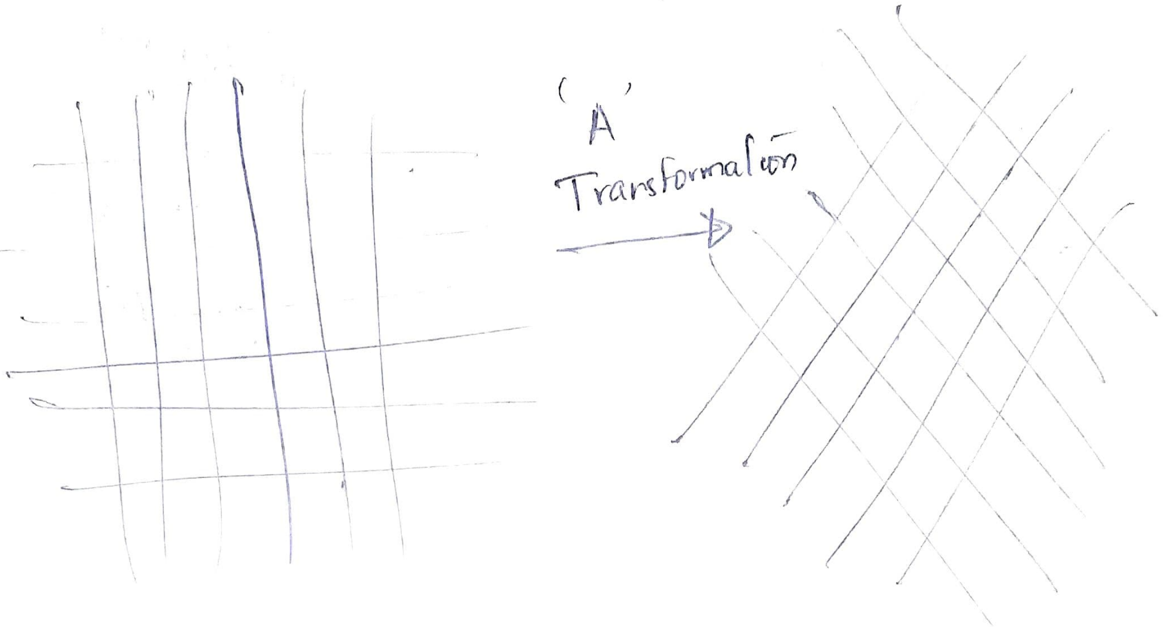
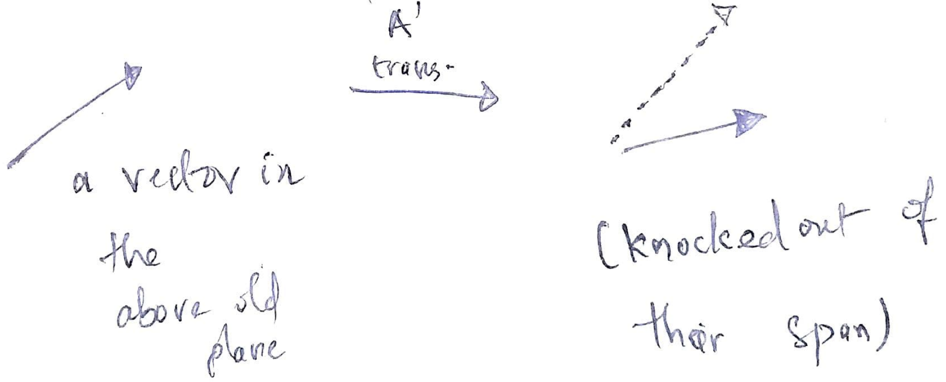
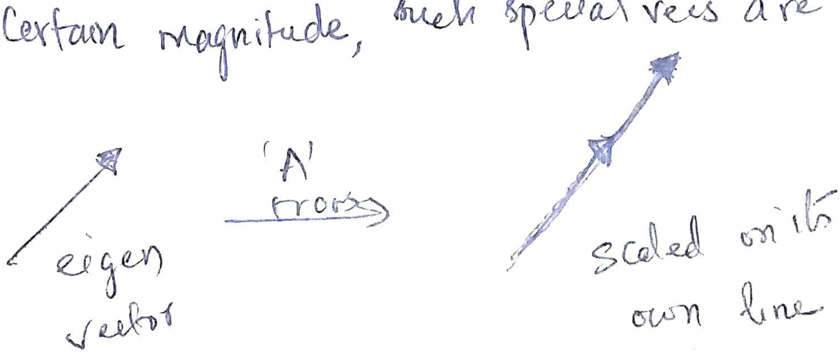
# Eigen Vectors




In the above transformation most vectors are Knocked out of their span




Some vectors are special, that they remain on their line, and are squished or scaled by a certain magnitude, such special vecs are eigen vecs



for a vector $\vec{x}$, on applying 'A' transformation

scaled or squished by '$\lambda$' quantity

$$\underbrace{A}_{\substack{\cancel{\text{vector}} \\ \text{matrix}}}\underbrace{x}_{\text{vec}} = \underbrace{\lambda}_{\text{scalar}}\underbrace{x}_{\text{vector}}$$

$$Ax = (\lambda I)\, x$$

$$(A - \lambda I)\, x = 0$$

$x$ is a non-zero vector, (a zero vector obviously stays at the same plane and hence no point in talking about it)

* if there is a transformation applied on $x$ vector, such that it becomes null vector it means it transform into a lower dimensional space, that implies

$$\det (A - \lambda I) = 0$$

the above gives $\lambda$ values, which are called eigen values, the amout scaled or squished.

Solving

$$(A-\lambda I)\cdot\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

gives the eigen vectors after putting corresponding to each eigen value.

ex

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} \text{ transformation}$$

$$|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda = 1, 1$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$x + 2y = 0$$

$$\frac{x}{-2} = \frac{y}{1} \text{ line}$$

with eigen value 1

- That means, vectors on line $x + 2y = 0$ remain on the line and their magnitude remains the same too.

- Sometimes there can be no eigen vectors, - a rotation by 90°.

- Sometimes, all vectors on plane are eigen vectors ex - $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ you are just scaling each vector twice here!

# A useful case:

Some times you are lucky enough that in a transformation, you can independent eigen vectors enough to form a basis.

That means suppose in a 2D transformation, you get 2 independent eigen vectors, you can choose them as a new basis. why?

Let's see

Suppose the eigen vectors are stacked side by side and form the change of basis matrix B. There is a transformation

A; then

$B^{-1}AB$ is always a diagonal matrix.

$B^{-1}AB$, translates 'A' transformation from our language to the equivalent transformation in new language (formed by eigen basis).

→ why is a diagonal matrix useful?

Bcoz its easy to transform it multiple times

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^{100} = \begin{bmatrix} 2^{100} & 0 \\ 0 & 3^{100} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x & 0 \\ 0 & 3y \end{bmatrix}$$

that means, suppose u have 'A' transformation you want to apply it multiple times to a vector, but matrix multiplication those many times is painful. So u find its eigen vectors, make them as a new basis - eigen basis. Transform into a new language. As its a diagonal matrix now, you can easily perform operations, then convert it back to our language. (Refer change of basis chapter).