Complex Space

Complex matrix A, its transpose = conjugate and its transpose (represented by H)

$$\bar{Z}^T = Z^H$$

in real matrices A^T is equivalent of A^H in complex matrices.

Reason $A = \begin{bmatrix} 1 \\ i \end{bmatrix}$ $A^H = \begin{bmatrix} 1 & -i \end{bmatrix}$

if A^T was $\begin{bmatrix} 1 & i \end{bmatrix}$ or (A^H)

Sq. of len = $A^H A = 1 + i^2 = 0$, wrong

$$\begin{bmatrix} 1 & -i \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} = 1 - i^2 = 2 \checkmark$$

inner product of 2 complex vectors u & v

$u^H v$, order matters unlike scalar product in real vecs.

infact,

$$(u^H v)^H = v^H u \quad // \quad (u^H v)^H = v^H u^{H^H} = v^H u$$

(Au) with v

$$(Au)^H v = u^H A^H v$$
$$= u^H (A^H v)$$

$$(AB)^H = B^H A^H.$$

Hermitian matrices

$$S^H = S$$

diagonal elements has to be real
(bcoz they stay in same plane
and taking conjugate on real
nums have no effect)

$$A = \begin{bmatrix} 2 & 2+2i \\ 2-2i & 3 \end{bmatrix}$$

$Z \to$ is an complex vector suppose

$\boxed{Z^H A Z \text{ is real always}}$

$$[\bar{z}_1 \ \bar{z}_2] \underbrace{\begin{bmatrix} \cdot & - \\ - & - \end{bmatrix}}_{A} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$\underline{2\bar{z}_1 z + 5\bar{z}_2 z} + (2+2i)\bar{z}_1 z_2$ a num

$+$ conj of above num

$(2-2i)(z_1 \bar{z}_2)$

= real

eigen value of Hermitian matrix is real!

$$Ax = \lambda x$$

$$x^H A x = x^H(\lambda x)$$

$$= \lambda x^H x$$

$$x^H A x$$

$$\lambda = \frac{x^H A x}{x^H x}$$

$x^H A x \to$ real, proved just before

$x^H x \to$ Sq. of Length of vector always real

* eigen vectors of a Hermitian matrix are orthogonal

x, y then $y^H x = x^H y = 0$.
$\lambda_1 \neq \lambda_2$

proof

$Ax = \lambda_1 x$

y^H multiply

$y^H A x = \lambda_1 y^H x$

$Ay = \lambda_2 y$

(H)

$y^H A^H = \lambda_2 y^H$

$y^H A = \lambda_2 y^H$

x on both sides

$y^H A x = \lambda_2 y^H x$

$(\lambda_1 - \lambda_2) y^H x = 0$

$\because$ Diff eigen values $\underline{y^H x = 0}$

$S \rightarrow$ real & symmetric matrix.

$$X^{-1} S . X = \Lambda$$

X ↓ orthogonal

Λ ↘ diagonal matrix

$$X^T = X.$$

X is actually eigen vector matrix

Coming to complex context $\longrightarrow$ S'

$$Q^{-1} S Q = \Lambda$$

Q ↓ eigen vector matrix

eigen vectors are complex and orthonormal (orthogonal and unit len)

Hence called unitary

Unitary matrix

- Having orthonormal columns.

$$Q^H Q = I \Rightarrow Q^H = Q^{-1}$$

proof that eigen values of unitary matrix have absolute value = 1.

$$Q^H = Q^{-1}.$$

$$Q^H Q = I$$

$$Q^H Q z = z$$

$$z^H Q^H Q z = z^H z$$

$$(Qz)^H Qz = z^H z$$

$$\|Qz\|^2 = \|z\|^2$$

$$\|Qz\| = \|z\|$$

from eigen perspective

$$Qz = \lambda z.$$

$$|\lambda| = 1.$$

2) All eigen values have absolute value = 1