Quantum bit

classical bits $|0\rangle$ & $|1\rangle$ | $|\ \rangle$ → dirac notation

(0) (1)

qubit → linear combination of both

$\alpha|0\rangle + \beta|1\rangle$

Superpositions

$|\alpha|^2$ → prob with qubit value $|0\rangle$

$|\beta|^2$ → prob with qubit value $|1\rangle$

$|\alpha|^2 + |\beta|^2 = 1$

qubit state $= \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ → unit vector

α, β → complex.

In Real world systems, for physical implementations things being pursued are

- Super conducting circuits
- trapped ions(?)

⋮

for our purpose, we can think of as

- e^- in ground & excited state.

geometrical interpretation

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

α & β are 2 complex nums

$\alpha = r_0 e^{i\theta_0}$ | Representing in polar form

$\beta = r_1 e^{i\theta_1}$

$$|\psi\rangle = r_0 e^{i\theta_0}|0\rangle + r_1 e^{i\theta_1}|1\rangle$$

Multiplying a qubit with a number of unit length will not change the state of qubit

So, lets multiply with $e^{-i\theta_0}$

$$e^{-i\theta_0}|\psi\rangle = r_0 |0\rangle + r_1 e^{i(\theta_1-\theta_0)}|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1 \Rightarrow r_0^2 + r_1^2 = 1$$

and we can now taken

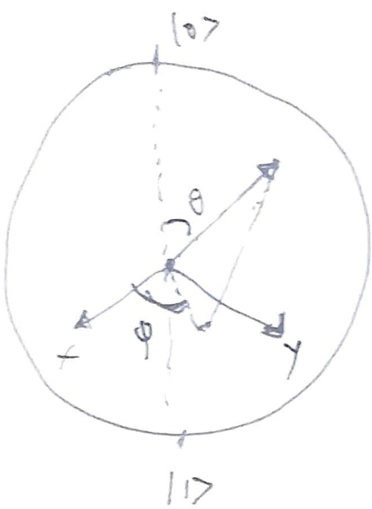
$r_0 = \cos\theta$ $r_1 = \sin\theta$

and $\theta_1 - \theta_0 = \varphi$ for conv.

$$= \cos\theta|0\rangle + \sin\theta e^{i\varphi}|1\rangle.$$

$\cos\theta \, |0\rangle + \sin\theta \cdot e^{i\varphi} \, |1\rangle$

This is represented as sphere with $|0\rangle$ and $|1\rangle$ and the top and bottom poles



$\cos\frac{\theta}{2} \, |0\rangle + \sin\frac{\theta}{2} \cdot e^{i\varphi} \, |1\rangle$

θ range is $[0, \pi)$ → (Clear from fig.)

$\theta/2$ is chosen so as to avoid repitition.

$\cos\frac{\theta}{2} : \cos 0 \to \cos\frac{\pi}{2}$

$\sin\frac{\theta}{2} : \sin 0 \to \sin\frac{\pi}{2}$

Now from sphere, one can visualise that

θ is $0°$, qubit collapses to $|0\rangle$

$\cos\frac{0°}{2} \, |0\rangle + e^{i\varphi} \sin\frac{0°}{2} \, |1\rangle = |0\rangle$

and when θ is π, qubit collapses to $|1\rangle$

$\cos\frac{\pi}{2} \, |0\rangle + e^{i\varphi} \sin\frac{\pi}{2} \, |1\rangle = e^{i\varphi} \, |1\rangle$ (as we know ~~that~~ multiplication of qubit by num of unit length doesn't change)

$= |1\rangle$

θ values tells, where the qubit's value will likely collapse, if its nearer to 0° (at top), then its $|0\rangle$

if its nearer to π (at bottom), then it will probably collapse to $|1\rangle$

- Somewhere when $\theta = \frac{\pi}{2}$

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}} e^{i\varphi} |1\rangle.$$

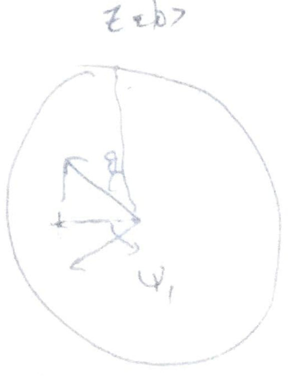
$\varphi = 0$; $\quad \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$\varphi = \pi$; $\quad \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

* Multiplication of a qubit by unit norm does not change it state, ~~it ca~~

The above pt can be realised by the sphere.

change φ
(rotate abt z axis)



In both cases, qubit is likely to collapse to $|0\rangle$

More info by a qubit :-

classical bit $|0\rangle$ and $|1\rangle$

but qubit is probabilistic in nature

it can collapse to $|0\rangle$

or

it can collapse to $|1\rangle$

it takes range of continuous

values (diff pts on sphere)

here qubit can hold more

information.

But that doesnot mean qubit

can hold ∞ values.

at the end it collapses to $|1\rangle$ or $|0\rangle$.

experimental

$q_1 \; q_2 \; q_3 \; q_4$

identical qubits

all qubits

$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

q_1 detector Result = 0

q_2 detector Result = 0

q_3 result = 1

q_4 result = 1

$\frac{1}{2}$ the times = 0

$\frac{1}{2}$ the times = 1.

Mystery

that ~~means~~ 'Nature' holds

the values $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ somewhere

if identically, there were 10^6 qubits

$\frac{10^6}{2}$ times detector will give $|0\rangle$

$\frac{10^6}{2}$ times detector will give $|1\rangle$

2 qubits system

clasical bit case → 00, 01, 10, 11

qubit case → it would be superposition of these

qubits $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

as α_{ij}^2 indicates probability

$$\sum_{\substack{i=0,1\\ j=0,1}} |\alpha_{ij}|^2 = 1.$$

But here, after measuring the first qubit,

we can eliminate terms that are not consistent

that means if the first qubit turned out to

be 0, then $|\beta_{00}|^2 + |\beta_{01}|^2 = 1$

we can eliminate α_{10}, α_{11}

$$|\psi'\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle \; ?$$

but the above shld be normalised

$$|\psi'\rangle = \frac{\alpha_{00}}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}|00\rangle + \frac{\alpha_{01}}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}|01\rangle$$

an example "partial measurement disturbs the state of the system"!