+1 → balanced

Deutsch Algo

Aim: ~~To find~~ We have a func

f : Domain = $\{0,1\}$

Range : $\{0,1\}$

f can be

Constant OR Balanced

(i) $f(0) = f(1) = 0$

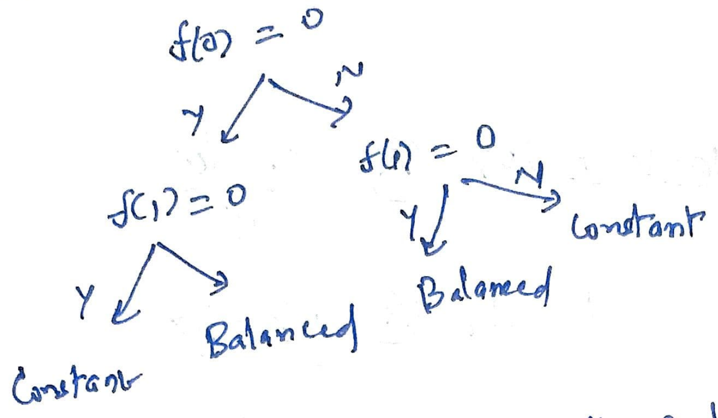
(ii) $f(0) = f(1) = 1$

(i) $f(0) = 1;\ f(1) = 0$

(ii) $f(0) = 0,\ f(1) = 1$

We wish to devise an algo, which tells whether f is constant or balanced.

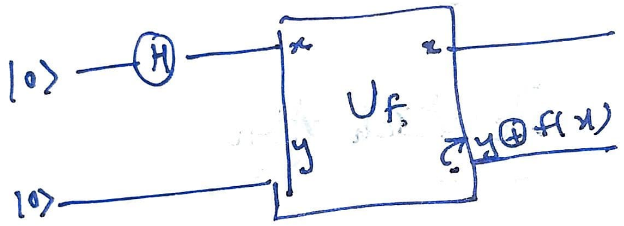
Classical Algo:



We need to calculate both f(0) and f(1).

Quantum Algo:

1st attempt



$$U_f\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}},\ |0\rangle\right)$$

$$\frac{1}{\sqrt{2}}\, U_f\left(|\underset{x}{0}, \underset{y}{0}\rangle + |\underset{x}{1}, \underset{y}{0}\rangle\right)$$

$$\frac{1}{\sqrt{2}}\ |\overset{x}{0}, \overset{y \oplus f(x)}{0 \oplus f(0)}\rangle + |\overset{x}{1}, 0 \oplus f(1)\rangle$$

$$\frac{1}{\sqrt{2}}\ |0, f(0)\rangle + |1, f(1)\rangle$$

~~Balanced~~ Constant

(i) $\frac{|0,0\rangle + |1,0\rangle}{\sqrt{2}}$

(ii) $\frac{|0,1\rangle + |1,1\rangle}{\sqrt{2}}$

Balanced

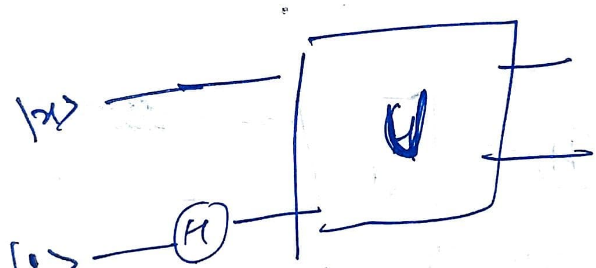
(i) $\frac{|0,1\rangle + |1,0\rangle}{\sqrt{2}}$

(ii) $\frac{|0,0\rangle + |1,1\rangle}{\sqrt{2}}$

Still need to measure both qubits

Can't find any pattern

2nd attempt



$$U\left(|x\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$U\left(\frac{|x,0\rangle - |x,1\rangle}{\sqrt{2}}\right)$$

$$\frac{|x, 0 \oplus f(x)\rangle - |x, 1 \oplus f(x)\rangle}{\sqrt{2}}$$

$f(x) = 0$ $\qquad$ $f(x) = 1$

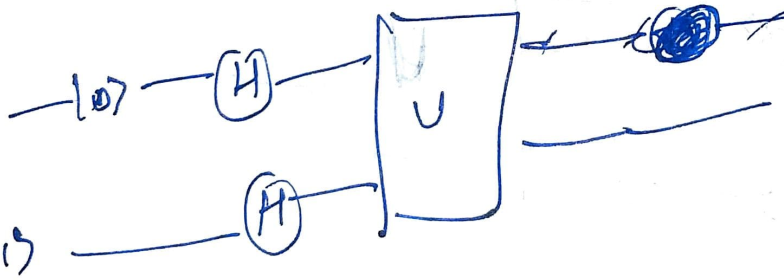
$\frac{1}{\sqrt{2}} |x,0\rangle - |x,1\rangle$

$\frac{1}{\sqrt{2}} |x,1\rangle - |x,0\rangle$

$= \frac{-1}{\sqrt{2}} \left(|x,0\rangle - |x,1\rangle \right)$

$$(-1)^{f(x)} \frac{|x,0\rangle - |x,1\rangle}{\sqrt{2}} = (-1)^{f(x)} |x\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

let x be $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$ $\left(|0\rangle - H - \right)$



$$\frac{(-1)^{f(0)} |0\rangle + (-1)^{f(1)} |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

f is constant

(i) $+\frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

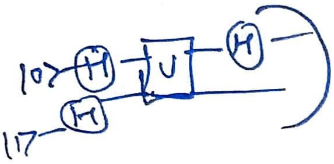
(ii) $-\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

f is balanced

(i) $-\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

(ii) $+\frac{|0\rangle - |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

<= if H is applied to top qubit



$H \times H = I$

$H(H|0\rangle) = |0\rangle$

$H(H|1\rangle) = |1\rangle$

f is constant

(i) $|0\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

(ii) $-|0\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

f is balanced

(i) $-|1\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

(ii) $|1\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

if top qubit = 0 => constant func

if top qubit = 1 => balanced func

an elementary example to demonstrate how speed can be improved.