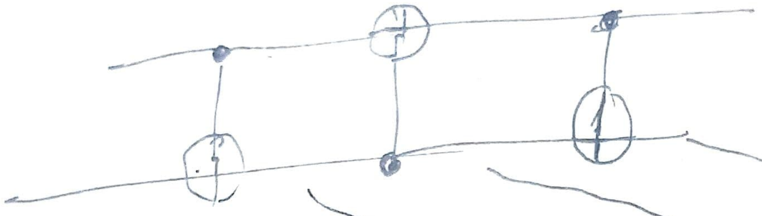
Quantum circuits

input is computational basis state (mostly $|0\rangle$).

Circuit to swap bits.

(wire need not be physical wire) (known soon!)



$|a, b\rangle \Rightarrow |a, a \oplus b\rangle$

$= |a \oplus (a \oplus b), a \oplus b\rangle$

$= |b, a \oplus b\rangle$

$= |b, (a \oplus b) \oplus b\rangle$

$= |b, a\rangle$

a XOR (a XOR b)

= a XOR $((a \vee b) \wedge (a' \vee b'))$

= $a \vee ((a \vee b) \wedge (a' \vee b'))$ → complement (a∨b)

$\wedge$

$a' \vee ((a \vee b) \wedge (a' \vee b'))$

= b

Controlled-U gate: an extension of C-NOT

C-NOT Controlled variable = 0 target as it is

Controlled = 0 target

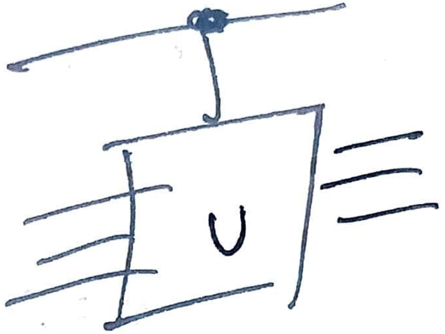
1 flips

target

C - U gate - Control 0 → as its is

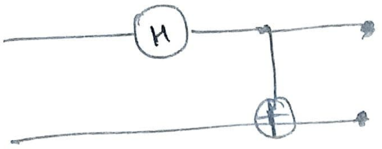
control 1. → apply U gate

Symbol



Obtaining Bell states from Circuit

6.



say $|00\rangle$ is input

H-gate transforms the first qubit

$$\frac{(|0\rangle + |1\rangle)}{\sqrt{2}}|0\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

Remember

H-gate : $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$

$= \begin{pmatrix}\frac{\alpha+\beta}{\sqrt{2}}\\ \frac{\alpha-\beta}{\sqrt{2}}\end{pmatrix}$

C. NOT gate

$$\frac{|00\rangle + |10\rangle}{\sqrt{2}} \longrightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

as CNOT flips the 2nd bit, only if 1st bit is 1

Such states are called bell states. Notations are:

$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Also called EPR States.

$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

Sending $|01\rangle$ into the above discussed gate

$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

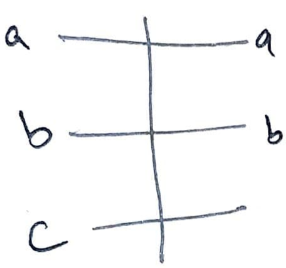
Quantum Algorithm

Simulating classical circuits

NAND gates is universal gate.

if we can simulate NAND gates, it means we can simulate other classical circuits too.

We introduce Toffoli gate (Similar to CNOT, but its called CCNOT gate)

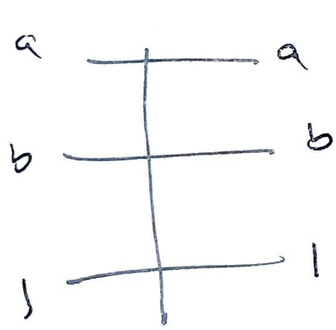


} controll bits

$c \oplus ab$.

in simple words last bit is flipped if both control bits are equal to 1

To make a ~~[illegible]~~ NAND gate using toffoli.



$\boxed{\oplus \text{ indicates XOR}}$

$1 \oplus ab = (ab)' \rightarrow$ NAND gate

$\hookrightarrow \left(\underline{(1+ab)} \cdot (0+(ab)') \right.$

$1 \cdot ((ab)')$

$\left. (ab)' \right)$

Toffoli gate can be represented by 8x8 matrix

Toffoli : 8×8 gate

000	001	011	100	110	111	010	1 1
1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	'	1	0	0	0	0	0
0	'	'	1	0	0	0	0
0	'	'	'	0	1	0	0
0	'	'	'	1	0	0	0
0	'	'	'	'	'	1	0
0	'	'	'	'	'	'	1

Quantum Parallelism

$x \to f(x)$

Classical computer : one 'x' at a time

Quantum comp. : Simultaneously multiple 'x'.

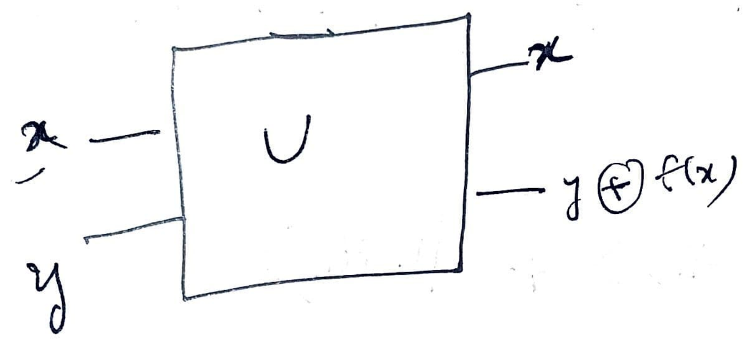
eg: $f\{0,1\} \to \{0,1\}$.

$\downarrow$

This f can be simulated using a quantum circuit as we proved.

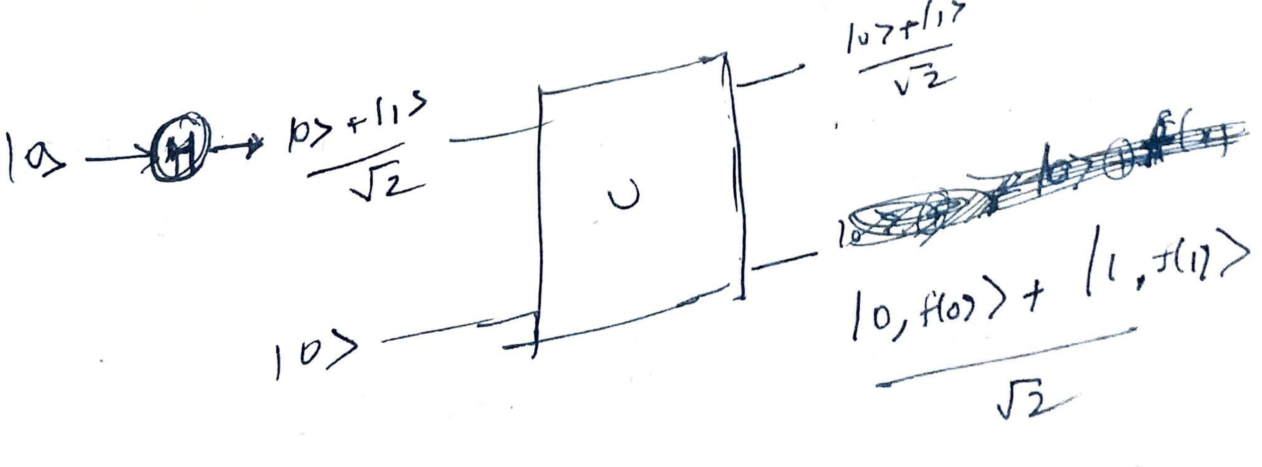
We can create another circuit with gate U_f such a way that

$$U_f|x\rangle|y\rangle \longrightarrow |x\rangle|y \oplus f(x)\rangle$$



if x is replaced with $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ [obtained by H-gate on $|0\rangle$]

y is replaced with $|0\rangle$.



$$= U\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \cdot |0\rangle\right)$$

$$= \frac{1}{\sqrt{2}} U\left(|0\rangle|0\rangle\right) + \frac{1}{\sqrt{2}} U(|1\rangle|0\rangle)$$

$$= \frac{1}{\sqrt{2}} |0\rangle|0 \oplus f(0)\rangle + \frac{1}{\sqrt{2}} (|1\rangle \cdot |0 + f(1)\rangle)$$

$$= \frac{1}{\sqrt{2}} |0\rangle|f(0)\rangle + \frac{1}{\sqrt{2}} (|1\rangle|f(1)\rangle)$$

$$= \frac{1}{\sqrt{2}} \left[|0, f(0)\rangle + |1, f(1)\rangle \right]$$

both f(0) and f(1) are calculated simultaneously in 1 go.

For 2 H-gates

$$\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = \frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2}$$

$$H \otimes H$$

for n-times, it is represented as

$$H^{\otimes n} = \frac{1}{2^{n/2}} \Sigma |x\rangle$$

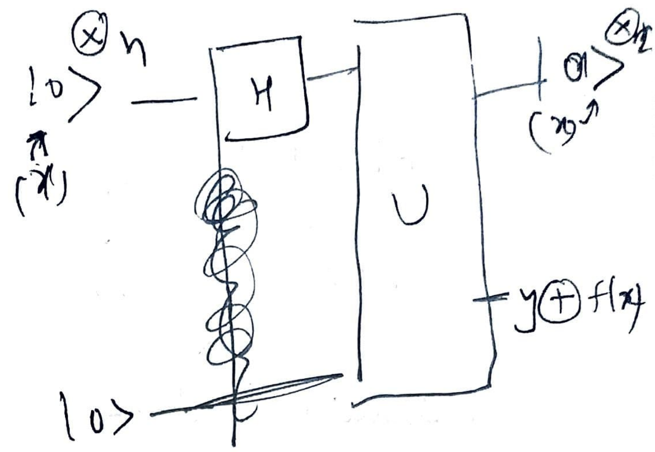
To generalise the above idea

n bit input $\longrightarrow$ 1 bit output $f(x)$

(here $n = 1$).

create $n+1$ qubit state $|0\rangle^{\otimes n} |0\rangle$

names $\begin{cases} |0\rangle \; - \\ |0\rangle \; - \\ |0\rangle \; - \end{cases}$



$|0\rangle$

$$\frac{1}{\sqrt{2^n}} \sum_x |x\rangle \, |f(x)\rangle$$

But at the end, measurement will result in only one of them - $|0, f(0)\rangle$, or $|1, f(1)\rangle$

We need ways to extract multiple values from superposition states at once.

Hence we look at Deutsche algo