

# No-cloning theorem

$$\underline{|\psi\rangle = a|0\rangle + b|1\rangle} \rightarrow \boxed{U} \rightarrow a|0\rangle + b|1\rangle$$

classical

$$x \text{ OR } \begin{array}{c} \text{0} \\ \downarrow \\ \text{process.} \end{array} = x$$

Raw Material

$$\left| \begin{array}{l} 1 \text{ OR } 1 = 1 \\ 0 \text{ OR } 0 = 0 \end{array} \right.$$

you want a clone of  $x$

$$|0\rangle \rightarrow x$$

We want

$$\underline{|0\rangle} \xrightarrow{\text{become}} \underline{|\psi\rangle}$$

$$|\psi\rangle \cup (|\psi\rangle |0\rangle) \xrightarrow{\text{become}} |\psi\rangle |\psi\rangle$$

$$\text{Say } |\psi\rangle = (a|0\rangle + b|1\rangle)$$

$$|\psi\rangle |0\rangle = (a|0\rangle + b|1\rangle) \cdot |0\rangle$$

$$a|00\rangle + b|10\rangle$$

you apply CNOT to the above and get  $a|00\rangle + b|11\rangle$

expected state

$$|\psi\rangle |\psi\rangle$$

$$(a|0\rangle + b|1\rangle) \cdot (a|0\rangle + b|1\rangle)$$

$$a^2|00\rangle + b^2|11\rangle + ab|01\rangle + ab|10\rangle$$

$$a|00\rangle + b|10\rangle$$

$$= a^2|00\rangle + b^2|11\rangle + ab|01\rangle + \underline{ab|10\rangle}$$

$$\cancel{ab|01\rangle} \quad \cancel{ab|10\rangle}$$

$$a^2 = a$$

$$b^2 = b$$

$$ab = 0$$

$$\Rightarrow \frac{b = 0}{a = 1} \Rightarrow ab \neq 0$$

$$a = 1$$

$\Rightarrow$  No values of  $a, b$ .

only possible

if  $a=1, b=0$  that is it is possible to clone  $|0\rangle$  only

→ ~~but~~ main aim of cloning is to create a

Copy of unknown state.

→ If we know in advance, what already the thing is then we can create it! That's not cloning.