

Secondary

MATHEMATICS

Students' Book Two

Third Edition

Question

A commemorative stone is sculptured in a shape of a frustum with the diameters of the top and bottom faces being 28 cm and 49 cm respectively. If the vertical distance between the faces is 45 cm, find the volume of the stone.

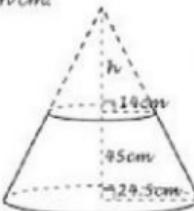
Solution

The two cones in the figure are similar.

Let the height of the smaller cone be h cm.

$$\begin{aligned}\frac{14}{24.5} &= \frac{h}{h+45} \\ 24.5h &= 14(h+45) \\ 24.5h - 14h &= 630 \\ h &= \frac{630}{10.5} \\ &= 60 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Volume of } &\text{volume of } - \text{ volume of} \\ \text{the stone} &\text{larger cone} - \text{smaller cone} \\ &= \frac{1}{3} \times \frac{22}{7} \times 24.5^2 \times 105 - \frac{1}{3} \times \frac{22}{7} \times 14^2 \times 60\end{aligned}$$



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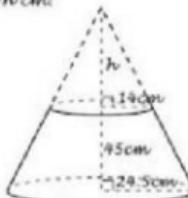
A commemorative stone is sculptured in a shape of a frustum with the diameters of the top and bottom faces being 28 cm and 49 cm respectively. If the vertical distance between the faces is 45 cm, find the volume of the stone.

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The two cones in the figure are similar.

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$$\begin{aligned}\text{Volume of stone} &= \text{volume of larger cone} - \text{volume of smaller cone} \\ &= \frac{1}{3} \times \frac{22}{7} \times 24.5^2 \times 105 - \frac{1}{3} \times \frac{22}{7} \times 14^2 \times 60\end{aligned}$$



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Secondary

MATHEMATICS

Compiled by Lec Rakdfha :0770112040

Students' Book Two

KENYA LITERATURE



BUREAU

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Prologue

The new mathematics syllabus for the Kenya Certificate of Secondary Education was developed in accordance with the objectives of the secondary cycle of the 8-4-4 system of education. Secondary Mathematics Student's Book Two has been written to match the objectives as spelt out in the syllabus.

The main thrust of the book is on meeting mathematical needs of the

crosssection of learners found in our secondary schools. This has been ably done by the team of writers that consists of mathematics educators with a vast experience in the teaching of the subject at different levels.

I am grateful to all those who have participated at various levels in the writing and revision of this title.

THE MANAGING DIRECTOR

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INTRODUCTION

This is the second students' title of the KLB Secondary Mathematics series and is intended for use at form two level. Although the book is written mainly for the 8-4-4 secondary school mathematics syllabus, it will prove useful to students pursuing similar courses both within and outside Kenya.

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In this book, mathematics is presented in a simple and precise manner.

Elaborate examples aimed at illustrating specific mathematical ideas are offered as a precursor to wide-ranging exercises. This ensures that the varying abilities and interests of students are adequately catered for. The mixed exercises further give the students an opportunity to consolidate the mathematical ideas met, so that an allround positive result is attained.

It is expected that the series will provide a strong foundation in mathematics as per the new syllabus.

Chapter 1

CUBES AND CUBE ROOTS

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1.1: Cubes

$$2^3 = 2 \times 2 \times 2$$

$$3^3 = 3 \times 3 \times 3$$

$$4^3 = 4 \times 4 \times 4$$

The cube of a number is the number multiplied by itself three times.

In general, $a^3 = a \times a \times a$.

Example 1

Find the value of 5^3 .

Solution

$$5^3 = 5 \times 5 \times 5$$

$$= 25 \times 5$$

$$= 125$$

Example 2

Find the cube of 1.4.

Solution

$$(1.4)^3 = 1.4 \times 1.4 \times 1.4$$

$$= 1.96 \times 1.4$$

$$= 2.744$$

Exercise 1.1

1. Find the cube of each of the following:
 - (a) 6
 - (b) 1.6
 - (c) 3.2
2. Find the value of $(2.17)^3$.
3. Find the value of each of the following: (a) $(0.4)^3$

(b) $(0.5)^3$

(c) $(1.1)^3$

(d) $(0.03)^3$

4. Find the volume of a cube of size 33 cm.

5. Find the value of each of the following: (a) $(-6)^3$

(b) $(-2.4)^3$

(c) $\left(\frac{-3}{4}\right)^3$

1.2: Use of Tables to find Cubes

Cubes of numbers can be read directly from the cube table. The technique of reading cubes from the cube table is similar to that used for the table of squares and square roots.

Table 1.1 is an extract from the table of cubes. Table 1.1

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1'0	1-000	1-030	1-061	1-093	1-125	1-158	1-191	1-225	1-260	1-295	3	7	10	13	16	20	23	26	30
1'1	1-331	1-368	1-405	1-443	1-482	1-521	1-561	1-602	1-643	1-685	4	8	12	16	20	24	28	31	35
1'2	1-728	1-772	1-816	1-861	1-907	1-953	2-000	2-048	2-097	2-147	5	9	14	19	23	28	33	37	41
1'3	2-197	2-248	2-300	2-353	2-406	2-460	2-515	2-571	2-628	2-686	5	11	16	22	27	33	38	43	49
1'4	2-744	2-803	2-863	2-924	2-986	3-049	3-112	3-177	3-242	3-308	6	13	19	25	31	38	44	50	56
1'5	3-375	3-443	3-512	3-582	3-652	3-724	3-795	3-870	3-946	4-020	7	14	21	29	36	43	50	57	64
1'6	4-096	4-173	4-252	4-331	4-411	4-492	4-574	4-657	4-742	4-822	8	16	24	32	41	49	57	65	73
1'7	4-913	5-000	5-088	5-178	5-268	5-359	5-452	5-545	5-640	5-735	9	18	27	37	46	55	64	73	82
1'8	5-831	5-930	6-039	6-128	6-230	6-334	6-435	6-539	6-645	6-751	10	20	31	41	51	61	71	81	91
1'9	6-859	6-968	7-078	7-189	7-301	7-415	7-530	7-645	7-762	7-881	11	23	34	45	57	68	79	91	102
2'0	8-900	8-121	8-242	8-365	8-490	8-615	8-742	8-870	8-999	9-120	13	25	38	50	63	75	88	100	113
2'1	9-261	9-394	9-528	9-664	9-800	9-938	10-078	10-218	10-350	10-503	14	28	41	55	69	83	97	110	124
2'2	10-749	10-994	11-090	11-199	11-349	11-543	11-697	11-852	12-009	12-155	15	20	45	60	76	91	106	121	136
2'3	12-167	12-326	12-487	12-649	12-813	12-978	13-144	13-313	13-481	13-652	16	33	49	66	81	99	115	132	148
2'4	13-398	13-998	14-172	14-349	14-517	14-706	14-887	15-069	15-253	15-438	18	36	54	72	90	108	126	143	161
2'5	15-625	15-813	16-003	16-194	16-387	16-581	16-777	16-975	17-174	17-374	19	39	58	78	97	117	136	155	175
2'6	17-576	17-780	17-985	18-191	18-400	18-610	18-821	19-034	19-250	19-465	21	42	63	84	105	127	147	168	189
2'7	19-683	19-903	20-124	20-346	20-571	20-797	21-025	21-254	21-485	21-716	23	45	68	90	113	136	158	181	203
2'8	21-951	22-188	22-426	22-665	22-906	23-149	23-394	23-640	23-888	24-138	24	49	73	97	121	146	170	194	216
2'9	24-389	24-643	24-897	25-154	25-412	25-672	25-934	26-198	26-464	26-731	26	52	78	104	130	156	182	208	234
3'0	27-000	27-271	27-544	27-818	28-094	28-373	28-653	28-934	29-218	29-504	28	56	83	111	139	167	195	223	250
3'1	29-791	30-080	30-371	30-664	30-959	31-256	31-554	31-853	32-157	32-462	30	59	89	119	148	178	208	237	267
3'2	32-768	33-076	33-386	33-698	34-012	34-328	34-646	34-966	35-288	35-611	32	63	95	126	158	190	221	253	284
3'3	35-937	36-165	36-394	36-926	37-260	37-595	37-933	38-273	38-614	38-938	34	67	101	134	163	191	215	235	269
3'4	39-304	39-632	40-002	40-354	40-708	41-064	41-422	41-782	42-144	42-509	39	71	107	142	178	214	249	285	320
3'5	42-87	43-24	43-61	43-99	44-36	44-74	45-12	45-50	45-88	46-27	4	8	11	15	19	23	26	30	34
3'6	46-66	47-05	47-44	47-83	48-23	48-63	49-03	49-43	49-84	50-24	5	8	12	16	20	24	28	32	36
3'7	50-65	51-06	51-48	51-90	52-31	52-73	53-16	53-58	54-01	54-44	4	8	13	17	21	25	29	34	38
3'8	54-87	55-31	55-74	56-18	56-62	57-07	57-51	57-96	58-41	58-86	4	9	13	18	22	27	31	35	40
3'9	59-32	59-78	60-24	60-70	61-16	61-63	62-10	62-57	63-04	63-52	5	9	14	19	23	28	33	37	42
4'0	64-00	64-48	64-96	65-45	65-94	66-43	66-93	67-42	67-91	68-42	5	10	15	20	25	29	34	39	44
4'1	68-92	69-43	69-93	70-44	70-96	71-47	71-99	72-51	73-03	73-56	5	10	15	21	26	31	35	41	46
4'2	74-09	74-62	75-15	75-69	76-23	76-77	77-31	77-85	78-40	78-95	5	11	16	22	27	32	38	43	49
4'3	79-51	80-06	80-62	81-18	81-75	82-31	82-88	83-45	84-03	84-60	6	11	17	23	28	34	40	45	51
4'4	85-18	85-77	86-35	86-94	87-53	88-12	88-72	89-31	89-92	90-52	6	12	18	24	30	36	41	47	53
4'5	91-12	91-73	92-35	92-96	93-58	94-20	94-82	95-46	96-07	96-70	6	12	19	25	31	37	43	50	56
4'6	97-34	97-97	98-61	99-25	99-90	100-54	101-19	101-85	102-50	103-16	6	13	19	26	32	39	45	52	58
4'7	103-82	104-49	105-15	105-82	106-50	107-17	107-85	108-53	109-23	109-90	7	14	20	27	34	41	47	54	61
4'8	110-59	111-28	111-98	112-68	113-38	114-08	114-79	115-50	116-21	116-93	7	14	21	28	35	42	49	56	63
4'9	117-65	118-37	119-10	119-82	120-55	121-29	122-02	122-76	123-51	124-25	7	15	22	29	37	44	51	59	66
5'0	125-00	125-75	126-51	127-26	128-02	128-79	129-55	130-32	131-10	131-87	8	15	23	31	38	46	53	61	69
5'1	131-65	133-43	134-22	135-01	135-80	136-59	137-39	138-19	138-99	139-80	8	16	24	32	40	48	56	64	71
5'2	140-61	141-42	142-24	143-06	143-89	144-70	145-53	146-36	147-20	148-04	8	17	25	33	41	50	58	66	74
5'3	148-88	149-72	150-57	151-42	152-57	153-13	153-99	154-85	155-72	156-59	9	17	26	34	43	51	60	69	77
5'4	157-46	158-34	159-22	160-10	160-99	161-88	162-77	163-67	164-57	165-47	9	18	27	36	44	53	62	71	80

Example 3

Use cube table to find the cube of each of the following:

- (a) 1.8
- (b) 2.12
- (c) 3.254
- (d) 0.76

Solution

- (a) $(1.8)^3 = 5.832$
- (b) $(2.12)^3 = 9.528$

$$(c) (3.254)^3 = 34.454$$

$$= 34.45 \text{ (4 s.f.)}$$

$$(d) (0.76)^3 = (7.6 \times 10^{-1})^3$$

$$= \left(\frac{7.6}{10}\right)^3$$

$$= \frac{438.9}{1\ 000}$$

$$= 0.4389$$

Exercise 1.2

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1. Use cube tables to find the cube of each of the following:
 - (a) 8.3
 - (b) 1.01
 - (c) 2.504
 - (d) 0.87
 - (e) 15.45
2. Use cube tables to find:
 - (a) $(4.06)^3$
 - (b) $(6.312)^3$

$$(c) (0.0912)^3$$

$$(d) (381.7)^3$$

$$(e) (2.1534)^3$$

$$(f) (5.3679)^3$$

3. A cubic building block measures 21 cm. Find its volume.
4. A cubic water tank has sides of length 2.143 m. Find the capacity of the tank in litres.

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1.3: Cube Roots using the Factor Method

$$\begin{aligned}2 \times 2 \times 2 &= 2^3 \\&= 8\end{aligned}$$

The cube root of 8 is 2, written as;

$$\sqrt[3]{8} = 2$$

Similarly, $27 = 3 \times 3 \times 3$

$$\therefore \sqrt[3]{27} = 3$$

In general, the cube root of a number is the number that when multiplied by itself three times gives the cube root.

Example 4

Evaluate: $\sqrt[3]{216}$

Solution

$$\begin{aligned}\sqrt[3]{216} &= \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{3 \times 3 \times 3} \\ &= 2 \times 3 \\ &= 6\end{aligned}$$

Example 5

Evaluate: $\sqrt[3]{\frac{125}{343}}$

Solution

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$$\begin{aligned}\sqrt[3]{\frac{125}{343}} &= \frac{\sqrt[3]{125}}{\sqrt[3]{343}} \\&= \frac{\sqrt[3]{5 \times 5 \times 5}}{\sqrt[3]{7 \times 7 \times 7}} \\&= \frac{5}{7}\end{aligned}$$

Example 6

Find $\sqrt[3]{0.064}$

Solution

$$\begin{aligned}
 \sqrt[3]{0.064} &= \sqrt[3]{\frac{64}{1000}} \\
 &= \frac{\sqrt[3]{64}}{\sqrt[3]{1000}} \\
 &= \frac{\sqrt[3]{4 \times 4 \times 4}}{\sqrt[3]{10 \times 10 \times 10}} \\
 &= \frac{4}{10} \\
 &= 0.4
 \end{aligned}$$

Example 7

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Find: $\sqrt[3]{a^6b^3}$

Solution

$$\begin{aligned}\sqrt[3]{a^6b^3} &= \sqrt[3]{a^6} \times \sqrt[3]{b^3} \\&= \sqrt[3]{a \times a \times a \times a \times a \times a} \times \sqrt[3]{b \times b \times b} \\&= \sqrt[3]{a \times a \times a} \times \sqrt[3]{a \times a \times a} \times \sqrt[3]{b \times b \times b} \\&= a \times a \times b \\&= a^2b\end{aligned}$$

Example 8

The volume of a cube is $1\ 000\text{ cm}^3$. What is the length of the cube?

Solution

Volume of the cube, $V = l^3$

$$l^3 = 1\,000$$

$$l = \sqrt[3]{1\,000}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$$

$$= \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{5 \times 5 \times 5}$$

$$= 2 \times 5$$

$$= 10$$

∴ The length of the cube is 10 cm.

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Exercise 1.3

1. Evaluate:

(a)

$$\sqrt[3]{64} \quad (b)$$

$$\sqrt[3]{125}$$

$$\sqrt[3]{3\ 375}$$

(c)

$$\sqrt[3]{\frac{27}{64}} \quad (\text{d})$$

$$\sqrt[3]{0.512} \quad (\text{e})$$

$$\sqrt[3]{2 \frac{93}{125}} \quad (\text{f})$$

2. Evaluate:

(a)

$$\sqrt[3]{a^3b^3} \quad (b)$$

$$\sqrt[3]{x^6y^6}$$

$$\sqrt[3]{w^6y^3}$$

$$\sqrt[3]{27x^3y^9}$$

(c)

(d)

3. Find:

$$(a) \sqrt[3]{\frac{0.064 \times 125}{343}}$$

$$(b) \sqrt[3]{(x - y)(x + y)}, \text{ when } x = 76 \text{ and } y = 49$$

4. The volume of a sphere is $\frac{4}{3}\pi r^3$,
where r is its radius. Find the radius
when the volume is 38.808 cm^3 .

(Take $\pi = \frac{22}{7}$)

5. The volume of the material used to
make a cube is 1728 cm^3 . Find the
length of the cube.

6. The volume of water in a measuring cylinder is 200 cm^3 . When a cube is immersed into the water, the cylinder reading is 543 cm^3 . Find:
- (a) the volume of the cube.
 - (b) the length of the side of the cube.
7. A metallic cuboid measuring 16 cm by 8 cm by 4 cm was melted. The material was then used to make a

cube. What was the length of the cube?

Chapter 2

RECIPROCALS

2.1: Reciprocals of Numbers by
Division

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If the reciprocal of a number a is b , this implies that $a \times b = 1$. This means that $b = \frac{1}{a}$

For example, the reciprocal of 2 is $\frac{1}{2}$, i.e., $2 \times \frac{1}{2} = 1$.

The reciprocal of 4 is
is $\frac{1}{20}$

$\frac{1}{4}$ and that of 20

The bigger the number, the smaller the value at its reciprocal.

Example 1

Find the reciprocal of 3.6.

$$\begin{aligned}\frac{1}{3.6} &= \frac{1}{3.6} \times \frac{10}{10} \\&= \frac{10}{36} \\&= \frac{5}{18}\end{aligned}$$
$$\begin{array}{r} 0.277\dots \\ 18 \overline{)50} \\ - 36 \\ \hline 140 \\ - 126 \\ \hline 14 \end{array}$$

Therefore, the reciprocal of 3.6 is 0.27

Exercise 2.1

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1. Find the reciprocal of each of the following integers:
 - (a) 3
 - (b) 49
 - (c) 49
 - (d) 50
2. Find the reciprocal of each of the following decimal numbers using division: (a) 2.5

- (b) 3.8
- (c) 4.2
- (d) 8.6
- (e) 9.4

2.2: Reciprocals of Numbers from Tables

Reciprocals can be found using reciprocal tables. The method of

reading reciprocal tables is similar to that of squares and square roots.

Example 2

Find the reciprocal of 3.665 using reciprocal tables.

Solution

Using reciprocal tables, the reciprocal of 3.665 is $0.2732 - 0.0004 = 0.2728$

Example 3

Find the reciprocal of 45.8.

Solution

45. is first written in standard form as
 4.58×10^1 .

$$\begin{aligned}\text{then, } \frac{1}{45.8} &= \frac{1}{4.58 \times 10^1} \\ &= \frac{1}{10} \times \frac{1}{4.58} \\ &= \frac{1}{10} \times 0.2183 \\ &= 0.02183\end{aligned}$$

The reciprocal of 45.8 from the tables is
0.02183.

Example 4

Using mathematical tables, evaluate

$$\frac{3}{0.0236}$$

Solution

$$\begin{aligned}\frac{3}{0.0236} &= \frac{3}{2.36 \times 10^{-2}} \\&= 3 \times \frac{1}{2.36} \times \frac{1}{10^{-2}} \\&= 3 \times 0.4237 \times 10^2 \\&= 127.1\end{aligned}$$

Exercise 2.2

1. Find the reciprocal of each of the following numbers using reciprocal tables:
 - (a) 56.2
 - (b) 102.4
 - (c) 0.234
 - (d) 0.0458
 - (e) 4368
 - (f) 0.002978

2. Use reciprocal tables to work out each of the following:

(a) $\frac{1}{0.0125} + \frac{1}{12.5}$

(b) $\frac{3}{0.364}$

(c) $\frac{17}{0.051} + \frac{3}{0.0027}$

(d) $\frac{4}{0.375} - \frac{5}{37.5}$

3. Use mathematical tables to evaluate each of the following:

(a)

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$$\frac{1}{\sqrt{0.2468}} \quad (b)$$

$$\frac{1}{(2.437)^2}$$

$$\frac{1000}{92.56}$$

$$100 \times \frac{1}{0.5789}$$

$$\frac{7}{\sqrt[3]{6.859}}$$

(c)

(d)

(e)

$$\left(\frac{4}{0.2976}\right)^3$$

(f)

$$\frac{1}{29.43} + \sqrt{2.358}$$

(g)

$$3.045^2 + \frac{1}{\sqrt{49.24}}$$

(h)

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

(i) If $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, find f, given that $u = 0.5$ and $v = 0.8$.

Chapter 3

INDICES AND LOGARITHMS

3.1: Indices

The power to which a number is raised is called its index (plural indices).

2^5 is the shortest form of $2 \times 2 \times 2 \times 2 \times 2$.

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5 s called the power or index to which
2 is raised and 2 is the base.

If n is a positive integer, a^n means $a \times a \times a \dots n$ times. For example; $a^3 = a \times a \times a$

$$2^3 = 2 \times 2 \times 2$$

$$= 8$$

$$3^4 = 3 \times 3 \times 3 \times 3$$

$$= 81$$

$$4^4 = 4 \times 4 \times 4 \times 4$$

$$= 256$$

3.2: Laws of Indices

Consider the following:

$$(i) a^5 \times a^2 = (a \times a \times a \times a \times a) \times (a \times a)$$

$$= a^7$$

$$= a^{(5+2)} \quad [\text{Sum of the two}$$

powers]

$$(ii) \quad a^5 + a^2 = \frac{a \times a \times a \times a \times a}{a \times a} \\ = a^3$$

[Difference
of the

$$= a^{(5-2)}$$

two powers]

$$(a^5)^2 = a^5 \times a^5 \\ = a^{10} \\ = a^{(5 \times 2)}$$

(iii) [Product
of the two

powers]

The three examples illustrate the following laws of indices;

(i) $a_m \times a_n = a_{(m+n)}$

$$(ii) \quad a_m \div a_n = a_{(m-n)}$$

$$(iii) \quad (a^m)^n = a^{mn}$$

Note

For these laws to hold, the base must be the same (common base).

The Zero Index

Consider the expression $a^n \div a^n$.

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$$a^n \div a^n = \frac{a^{n1}}{a^n 1} = 1$$

For example, $a^3 \div a^3 = \frac{a^{31}}{a^3 1} = 1$

Also, using the second law of indices;

$$a^3 \div a^3 = a^{(3 - 3)} = a^0$$

This implies that $a^0 = 1$.

Example 1 Evaluate:

(a) $2^6 \times 2^4$

$$(b) \ 3^{10} \div 3^4$$

$$(c) \ (5^2)^3$$

Solution

$$\begin{aligned}(a) \ 2^6 \times 2^4 &= 2^{(6+4)} \\&= 2^{10} \\&= 1\ 024\end{aligned}$$

$$\begin{aligned}(b) \quad 3^{10} \div 3^4 &= 3^{(10-4)} \\&= 3^6 \\&= 729\end{aligned}$$

$$\begin{aligned}(c) \quad (5^2)^3 &= 5^{(2 \times 3)} \\&= 5^6 \\&= 15\,625\end{aligned}$$

Example 2 Solve:

$$(a) \quad (2^x)^3 = 2^{12}$$

$$(b) \quad (4^3)^x = 64$$

Solution

(a)

$$(2^x)^3 = 2^{12} \quad (\text{b})$$

$$2^{3x} = 2^{12}$$

$$\therefore 3x = 12$$

$$x = 4$$

$$(4^3)^x = 64$$

$$4^{3x} = 4^3$$

Negative Indices

$$\therefore 3x = 3$$

$$x = 1$$

Consider the expression

$$a^{-n} \times a^n.$$

Using first the law of indices,

$$a^{-n} \times a^n = a^{(-n+n)} \\ = a^0$$

Since $a^{-n} \times a^n = a^0$,

then $a^{-n} = \frac{a^0}{a^n}$

But $a^0 = 1$

Therefore, $a^{-n} = \frac{1}{a^n}$

$$2^3 + 2^4 = \frac{2^3}{2^4}$$

Consider

$$\begin{aligned} &= \frac{2^1 \times 2^1 \times 2^1}{2 \times 2 \times 2 \times 2} \\ &= \frac{1}{2^1} \end{aligned}$$

Using second law of indices;

$$\begin{aligned} 2^3 + 2^4 &= 2^{(3-4)} \\ &= 2^{-1} \end{aligned}$$

Therefore, $2^{-1} = \frac{1}{2^1}$

$$a^3 + a^5 = a^{(3-5)}$$

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Similarly,

$$\begin{aligned} &= a^{-2} \\ &= \frac{1}{a^2} \end{aligned}$$

Example 3

Evaluate:

(a) $2^3 \times 2^{-3}$

(b) $\left(\frac{2}{3}\right)^{-2}$

Solution

$$2^3 \times 2^{-3} = 2^{(3 + -3)}$$

$$= 2^{(3 - 3)}$$

(a)

$$= 2^0$$

$$= 1$$

$$\begin{aligned}(b) \quad \left(\frac{2}{3}\right)^{-2} &= \left(\frac{1}{\frac{2}{3}}\right)^2 \\ &= \frac{1}{\left(\frac{2}{3}\right)^2} \\ &= \frac{9}{4} \\ &= 2\frac{1}{4} \\ \left(\frac{a}{b}\right)^{-n} &= \left(\frac{b}{a}\right)^n\end{aligned}$$

In general,

Fractional Indices

Fractional indices are of the form, $\frac{p}{q}$, where p and q are integers and q is not equal to zero. The fraction $\frac{p}{q}$ should be in its simplest form.

If $a^2 = b$, then a is the square root of b, written as; \sqrt{b}

If $a^3 = b$, then a is the third root (or cube root) of b, written as $\sqrt[3]{b}$.

If $a^4 = b$, then a is the fourth root of b,

written as $\sqrt[4]{b}$.

n

written as $\sqrt[n]{b}$

For example; $\sqrt{16} = 4$

$$\sqrt[3]{8} = 2$$

$$\sqrt[4]{16} = 2$$

G Consider the number $4^{\frac{1}{2}}$ generally, if $a = b$, a is the n^{th} root of b,

$$4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^{\left(\frac{1}{2} + \frac{1}{2}\right)}$$
$$= 4^1$$
$$= 4$$

But $4 = 2 \times 2$

Therefore, $4^{\frac{1}{2}} = 2$

$$= \sqrt{4}$$

Similarly, $4^{\frac{1}{3}} \times 4^{\frac{1}{3}} \times 4^{\frac{1}{3}} = 4^{\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right)}$

$$= 4^{\left(\frac{3}{3}\right)} = 4^1$$
$$= 4$$

Therefore, $4^{\frac{1}{3}} = \sqrt[3]{4}$

$$\begin{aligned}
 \text{Also, } 4^{\frac{1}{4}} \times 4^{\frac{1}{4}} \times 4^{\frac{1}{4}} \times 4^{\frac{1}{4}} &= 4^{(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4})} \\
 &= 4^{(\frac{4}{4})} \\
 &= 4^1 \\
 &= 4
 \end{aligned}$$

Therefore, $4^{\frac{1}{4}} = \sqrt[4]{4}$

In general, $a^{\frac{1}{n}} = \sqrt[n]{a}$

$$\begin{aligned}
 \text{Therefore, } (a^{\frac{1}{n}})^m &= a^{\frac{m}{n}} \\
 &= (\sqrt[n]{a})^m
 \end{aligned}$$

Example 4

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Evaluate:

(a) $27^{\frac{1}{3}}$

(b) $16^{\frac{3}{4}}$

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$$c \quad 4^{-\frac{1}{2}}$$

Solution

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$$(a) \quad 27^{\frac{1}{3}} = \sqrt[3]{27}$$
$$= 3$$

$$(b) \quad 16^{\frac{3}{4}} = (\sqrt[4]{16})^3$$
$$= 2^3$$
$$= 8$$

$$(c) \quad 4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}}$$
$$= \frac{1}{\sqrt{4}}$$
$$= \frac{1}{2}$$

Exercise 3.1

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1. Find the numerical value of:
- (a) $9^{\frac{1}{2}}$
 - (b) $25^{-\frac{1}{2}}$
 - (c) $36^{-\frac{1}{2}}$
 - (d) 35769°

$$81^{\frac{1}{2}} \quad (\text{e})$$

$$36^{\frac{3}{2}} \quad (\text{f})$$

$$8^{-\frac{5}{3}} \quad (\text{g})$$

$$27^{-\frac{1}{3}} \quad (\text{h})$$

$$16^{\frac{5}{2}} \quad (\text{i})$$

$$100^{\frac{9}{2}} \quad (\text{j})$$

$$64^{\frac{4}{3}}$$

(k)

|

()

$$(m)_{256}^{\frac{3}{4}}$$

$$729^{\frac{1}{3}}$$

$$(49^2)^{-\frac{1}{2}}$$

$$(10^3)^{\frac{5}{3}}$$

$$(3^{-2})^3$$

$$(216^2)^{\frac{1}{3}}$$

Compil $243^{\frac{1}{3}}$ by Lec *Rakdfha*

:07701 $4096^{-\frac{3}{4}}$

$$64^{-\frac{1}{6}}$$

(n)

(p)

(q)

(r)

(s)

(t)

(u)

()

2. Simplify each of the following expressions. You may leave your answer in index form:

(a) $3^3 \times 3^4$

(b) $5^2 \times 5^{-3}$

(c) $2^3 \times 2^5 \times 2^{-4}$

(d) $3^{\frac{3}{2}} \times 3^{-\frac{3}{4}} \times 3^{\frac{1}{4}}$

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$$(e) \ 6^2 \times 7^{-4} \times (8^{-2})^2 \times 6^3 \times 7^2 \times 8^4$$

$$(f) \ 20^{-3} \times 25^2 \times 20^3 \times 25^{-4}$$

$$(g) \ 7^{-3} \times 8^4 \times 7^2 \times 8^{-3}$$

$$27^{\frac{1}{3}} \times 9^{\frac{1}{2}} \quad (h)$$

$$512^{\frac{1}{3}} \times 8^{\frac{1}{3}} \div 2^6 \quad (i)$$

()

$$j \quad \left(\frac{3}{4}\right)^2 \times \left(\frac{3}{4}\right)^3$$

$$(k) \quad \left(\frac{6}{7}\right)^{-2} \times \left(\frac{7}{6}\right)^{-2}$$

$$(l) \quad \frac{2^{-2} \times (2^2)^{-6}}{2^{-4} \times 2^{-5}}$$

$$(m) \quad \frac{20^{-2} \times 20^{-2} \times 20^{-2}}{20}$$

$$(n) \quad 2^{-6} \times 5^{\frac{1}{2}} \times 2^7 \div 2^{-4}$$

3 Simplify: by Lec Rakdfha
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$$(a) x^2 \times x^4$$

$$y^3 \times y^6$$

$$(c) n^{13} \times n^{-7}$$

$$(d) a^9 \times a^3$$

$$(e) a^{-2} \times 3a^4$$

$$(f) (2n)^3 \times (2n)^5$$

$$x^{16} \div n^{12}$$

$$a^{15} \div a^{14}$$

$$(g)$$

$$(h)$$

()

(i) $a^2 \times b^2 \times a^4$

(j) $b^2 \times x^2 \times b^{11}$

(k) $b^3 \times n^2 \times b^\circ \times n^3$

(l) $a^5 \times c^2 \times a^3 \times b^3 \times a^4$

(m) $a^{-2} \times b^2 \times a^{-8} \times b^{-4}$

(n) $r^3 \times r^2t^2 \times r^{-3}t^{-2}$

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$$(p) \quad x^2y^2 \times x^2y^5 \times x^{-3}y^{-9}$$

$$(q) \quad (xyt)^{11} \div (xyt)^2$$

$$r = \frac{a^n \times a^m}{a^{-n} \times a^{-m}}$$

$$(s) \frac{a^x \times a^{2x}}{a^x \times a^x \times a^x}$$

$$(t) \frac{x^2y^3 \times x^4y^2}{x^3y^4}$$

$$(u) \frac{a^4b^3 \times a^2c^4 \times b^{-2}c^3}{a^{-3}b^3c^3}$$

4. Simplify:

$$(a) \frac{25y^9k^{-7}}{81y^7k^9}$$

$$(b) \frac{12a^5b^3}{4a^3b}$$

Com(p)I $\frac{3^{-4}a^4b^6}{3a^5b^8}$, *Lec* *Rakdfha*
:0770112040

$$(d) \frac{a^{-2} \times b^{-3} \times c^{-2}}{a^{-2a} \times b^{-2a} \times c^{-2a}}$$

$$(e) \frac{(a^{-n})^m \times a^{-2m}}{a^{mn} \times a^{2m}}$$

$$(f) \frac{rs^2t^3 \times r^4s^5t^{-6} \times r^7s^8t^{-9}}{r^{10}s^{11}t^{-12} \times r^{13}s^{-14}t^{15}}$$

$$(g) \frac{(9a^2)^{\frac{1}{3}}}{6^4} \times \frac{(4b^2)^{\frac{1}{3}}}{5a}$$

$$(h) \frac{(16a^2)^{\frac{1}{2}} \times (36a^4)^{-\frac{1}{2}}}{2a^{\frac{1}{2}} \times 5a^{\frac{3}{2}} \times 8a^{\frac{9}{4}}}$$

$$(i) \sqrt[3]{\frac{27x^3y^9}{x^6y^3}}$$

$$(j) \frac{\sqrt[5]{(32y^{10}k^{15}2^{10})}}{\sqrt[4]{(16y^{12}k^{16}2^4)}}$$

()

5. Solve each of the following equations:

$$a (3^{2x})^3 = 3^4 \times 3^8$$

$$(b) (7^5)^x = (7^4)^x \div 7^2$$

$$(c) (3^{2x})^4 = 81$$

$$(d) 4^{5x} \div (2^{3x})^2 = 256$$

$$(e) 9^{2x} = 729$$

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$$(f) \quad 2^{8x} = 512$$

$$(g) \quad (7^4)^{2x} = (7^4)^3$$

$$(h) \quad (5^3)^a \times (5^8)^a = 5^{72}$$

$$(i) \quad 9^{4x} \div 3^{2x} = 2 \ 187$$

$$(j) \quad (2^{2c})^3 = (5^4)^c \times 625$$

$$(k) \quad 2^{\frac{x}{4}} = 8$$

$$(l) \quad 3^{(2x-5)} = 2 \ 187$$

() m $3^{4x} \div 3^{-7} = 3^{15}$

n $2 \times 3^x = 162$

(p) $\frac{64^{2y} \div 16^y}{128^y \times 4^{2y}}$

6. Write in the form of a^n .

(a) Where $a = 2$

(i) 2

(ii) 4

(iii) 2

(iv) 1

$$\frac{1}{2}$$

$$\frac{1}{4}$$

$$\frac{1}{4}$$

(v)

(vi)

(vii)

b Where a = 3

(i) 27

(ii) 9

()

(iii) 3

(iv) 1

$$\frac{1}{3}$$

$$\frac{1}{9}$$

$$\frac{1}{27}$$

(v)

(vi)

(vii)

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(c) Where $a = 10$

(i) 1 000

(ii) 100

(iii) 10

(iv) 1

(v)

()

$$\frac{1}{10} \quad (\text{vi})$$

$$\frac{1}{100} \quad (\text{vii})$$

$\frac{1}{1\,000}$ Logarithms

Consider the following:

$$3^0 = 1$$

$$3^1 = 3$$

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

The indices 0, 1, 2, 3, 4 ... are the logarithms of the corresponding

numbers to base 3.

For example; logarithm of 9 to base 3 is 2.

logarithm of 81 to base 3 is 4.

These are written in short form
as; $\log_3 9 = 2$ $\log_3 81 = 4$

Generally, the expression $a^m = n$ is written as $\log_a n = m$. $a^m = n$ is the index notation while $\log_a n = m$ is the logarithmic notation.

Example 5

Write in logarithmic form:

(a) $2^5 = 32$

(b) $9^{\frac{1}{2}} = 3$

(c) $b^n = m$

Solution

(a) If $2^5 = 32$, then $\log_2 32 = 5$

(b) If $9^{\frac{1}{2}} = 3$, then $\log_9 3 = \frac{1}{2}$

(c) If $b^n = m$, then $\log_b m = n$

Example 6

Write in index form:

(a) $\log_{10} 1000 = 3$

(b) $\log_3 81 = 4$

(c) $\log_b 1 = 0$

Solution

(a) $10^3 = 1000$

(b) $3^4 = 81$

(c) $b^0 = 1$

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Exercise 3.2

1. Copy and complete the table below:

Index form	Logarithmic form
(i) $2^3 = 8$	$\log_2 8 = 3$
(ii) $2^4 = 16$	
(iii)	$\log_2 128 = 7$
(iv) $4^2 = 16$	
(v) $5^2 = 25$	

(iv)	$\log_5 625 = 4$
(vii) $10^1 = 10$	
(viii) $10^2 = 100$	

2. Write in logarithmic form:

(a) $3^2 = 9$

(b) $2^4 = 16$

(c) $3^3 = 27$

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$$(d) 2^5 = 32$$

$$(e) 3^4 = 81$$

$$(f) 5^3 = 125$$

$$(g) 10^0 = 1$$

$$(h) 2^{10} = 1\,024$$

$$(i) a^n = b$$

3. Write each of the following in index form:

$$(a) \log_2 8 = 3$$

$$(b) \log_4 16 = 2$$

$$(c) \log_5 125 = 3$$

$$(d) \log_{10} 8 = x$$

$$(e) \log_b a = c$$

$$(f) \log_3 27 = 3$$

$$(g) \log_6 216 = 3$$

$$(h) \log_x 40 = y$$

$$(i) \log_4 6 = y \quad (j) \log_y x = 2 \quad (k) \log_{10} 10$$

$$000 = 4$$

$$(1) \log_2 16 = 4$$

Standard Form

Consider the following:

$$12 = 1.2 \times 10^1$$

$$120 = 1.2 \times 10^2$$

$$1200 = 1.2 \times 10^3$$

$$\begin{aligned}0.12 &= 1.2 \times \frac{1}{10} \\&= 1.2 \times 10^{-1}\end{aligned}$$

$$\begin{aligned}0.286 &= 2.86 \times \frac{1}{10} \\&= 2.86 \times 10^{-1}\end{aligned}$$

$$\begin{aligned}0.0074 &= 7.4 \times \frac{1}{1\,000} \\&= 7.4 \times 10^{-3}\end{aligned}$$

A number is said to be in standard form when it is written in the form $A \times 10^n$ where $1 \leq A < 10$ and n is an integer.

Write each of the following numbers in standard form:

(i) 26

(ii) 357

(iii) 4 068

(iv) 15 000 000

(v) 0.031

(vi) 0.00215

(vii) 0.005012

(viii) 0.000000152

(ix) 100 000

(x) $\frac{46}{100\ 000}$

3.3: Powers of 10 and Common Logarithms

1, 10, 100 and 1000 can be expressed as powers of 10 as follows;

$$1 = 10^0$$

$$10 = 10^1$$

$$100 = 10^2$$

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$$1000 = 10^3$$

The indices 0, 1, 2 and 3 are called the logarithms to base 10 of 1, 10, 100 and 1000 respectively. When the base is 10, the powers are referred to as common logarithms. What is the common logarithm of 10 000, 100 000 and 1 000 000?

In this book, logarithms to base 10 are discussed. The logarithm of a number, say 100, to base 10 is $\log_{10}100 = 2$. This is written as $\log 100 = 2$.

Some numbers cannot be expressed as exact powers of 10, for example, 70. Note that 70 lies between 10 or 10^1 and 100 or 10^2 . Therefore, its logarithm to base 10 is a number between 1 and 2. The logarithms of such numbers to base 10 are obtained from the table of logarithms.

Below is an extract from the table of logarithms.

[Logarithms to base 10](#)

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x	o	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	
ADD																				
10	-0000	0043	0086	0128	0170	0212	0212	0253	0294	0334	0374	4	8	13	17	21	25	29	34	38
11	-0414	0453	0492	0531	0569	0607	0607	0645	0682	0719	0755	4	8	12	16	20	24	28	32	36
12	-0792	0828	0864	0899	0934	0969	0969	1004	1038	1072	1106	4	7	11	15	19	22	26	30	33
13	-1139	1173	1206	1239	1271	1303	1303	1335	1367	1399	1430	3	7	10	13	16	20	23	26	30
14	-1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27	
15	-1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25	
16	-2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23	
17	-2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22	
18	-2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	10	12	14	17	19	22	
19	-2788	2810	2833	2856	2878	2906	2923	2945	2967	2989	2	4	7	9	11	13	15	18	20	
20	-3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19	
21	-3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18	
22	-3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	11	13	15	17	
23	-3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	5	7	9	11	13	14	16	
24	-3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	13	14	16	
25	-3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15	
26	-4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	6	8	10	11	13	14	
27	-4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	10	11	13	14	
28	-4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14	
29	-4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13	
30	-4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	8	10	11	13	
31	-4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	13	
32	-5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	10	12	
33	-5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12	
34	-5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	12	
35	-5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	8	10	11	
36	-5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11	
37	-5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	4	5	6	7	8	10	11	
38	-5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	4	6	7	8	9	10	
39	-5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	6	7	8	9	10	
40	-6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	7	8	9	10	
41	-6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9	
42	-6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9	
43	-6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9	
44	-6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9	
45	-6532	6542	6551	6561	6571	6580	6592	6599	6609	6618	1	2	3	4	5	6	7	8	9	
46	-6628	6637	6646	6656	6665	6675	6684	6693	6703	6712	1	2	3	4	5	5	6	7	8	
47	-6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8	
48	-6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8	
49	-6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8	
50	-6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	4	4	5	6	7	8	
51	-7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	2	3	4	5	6	6	7	

The technique of reading logarithm table is similar to that applied for tables such as squares and square roots. For numbers between 1 and 10 the logarithms can be read directly from the table as in Example 7.

Example 7

Find the logarithm of each of the following numbers from the table of logarithm:

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- (a) 2.56
- (b) 3.752

Solution

(a) $\log 1 = 0$ and $\log 10 = 1$. Therefore,
 $\log 2.56$ lies between $\log 0$ and 1.
From the table $\log 2.56 = 0.4082$.

This means $10^{0.4082} = 2.56$

(b) $\log 3.75 = 0.5740$

Difference of 2 is 0.0002

$$\begin{aligned}\therefore \log 3.752 &= 0.5740 + 0.0002 \\ &= 0.5742\end{aligned}$$

Therefore, $10^{0.5742} = 3.752$

For numbers greater than 10, the steps are as follows:

- (i) The number is expressed in standard form $A \times 10^n$. Then n will be the whole part of the logarithm.
- (ii) The logarithm of A is read from the table which is the decimal part of the logarithm.

(iii) The sum of the whole part and decimal part is the logarithm of the number.

Example 8

Find the logarithm of the following numbers using the table of logarithm:

- (a) 28.5
- (b) 379
- (c) 5280000

Solution

(a) $28.5 = 2.85 \times 10^1$ in
standard form \log
 $2.85 = 0.4548$, i.e.,
 $10^{0.4548} =$

$$\begin{array}{r} 2.85 \quad \log \\ 10^1 = 1 \end{array}$$

$$\begin{aligned} 28.5 &= 2.85 \times 10^1 = 10^{0.4548} \times 10^1 = \\ &10_{1.4548} \end{aligned}$$

Therefore, $\log 28.5 = 1.4548$

$$\begin{aligned}(b) \quad 379 &= 3.79 \times 10^2 \log \\ 10^2 &= 2 \log 3.79 = \\ 0.5786 &\quad \text{Therefore,} \\ \log 3.79 &= 2 + 0.5786 \\ &= 2.5786\end{aligned}$$

$$\begin{aligned}(c) \quad 5280000 &= 5.28 \times 10^6 \\ \log 10^6 &= 6 \log 5.28 = \\ 0.7226 &\quad \text{Therefore, } \log 5280000 = 6 + \\ &0.7226 \\ &= 6.7226\end{aligned}$$

The whole number part of the logarithm is called the characteristic and the decimal part is called the mantissa.

State the characteristic and the mantissa in the examples above.

Exercise 3.3

1. Use tables of logarithm to express each of the following numbers in the form 10^x . (a) 8.2

(b) 1.47

(c) 4.73

(d) 7.25

(e) 9.83

(f) 5.672

(g) 8.137

(h) 3.142

(i) 2.718

(j) 3.333

(k) 12.3

(l) 59.7 (m) 82.9

(n) 72

(p) 96.1

(q) 431.5

(r) 7 924

(s) 1 025

(t) 1 913

(u) 4 937

- (v) 273.7
- (w) 475 000
- (x) 3 910 000
- (y) 958 312

3.4: Logarithms of Positive Numbers less than 1

Consider the logarithm to base 10 of 0.034.

Logarithm of 0.034 cannot be read directly from the table of logarithms,

therefore, to obtain its logarithm, the steps are as follows:

- (i) 0.034 is expressed in standard form, i.e., 3.4×10^{-2} .
- (ii) The logarithm of 3.4 is 0.5315.
- (iii) The sum of 0.5315 and -2 is the log of 0.034, i.e., $\log 0.034 = -2 + 0.5315$ -2 + 0.5315 is written as $\bar{2}.5315$ and read as bar two point

five, three, one, five. The negative sign is written above 2 to emphasise that only the characteristic is negative.

Example 9

Find the logarithm of:

- (a) 0.57
- (b) 0.00063

Solution

$$(a) 0.57 = 5.7 \times 10^{-1}$$

$$\log 10^{-1} = -1 \log$$

$$5.7 = 0.7559$$

$$\therefore \log 0.57 = -1 + 0.7559$$

$$= \bar{1}.7559$$

Therefore, $\log 0.57 = \bar{1}.7559$

$$(b) 0.0063 = 6.3 \times$$

$$10^{-3} \log 10^{-3} = -3$$

$$\log 6.3 = 0.7993$$

Therefore, \log

0.0063

$$= -3 + 0.7993$$

$$= \overline{3}.7993$$

Example 10

Evaluate $\overline{2}.49 + \overline{3}.53$

Solution

$$\overline{2}.49 = -2 + 0.49$$

$$\overline{3}.53 = -3 + 0.53$$

$$\therefore \overline{2}.49 + \overline{3}.53 = -2 + 0.49$$

$$\begin{array}{r} -3 + 0.53 \\ \hline -5 + 1.02 \end{array} = -5 + 1 + 0.02$$

$$= -4 + 0.02$$

$$= \overline{4}.02$$

Antilogarithms

If the logarithm of a number is known, the table of antilogarithms is used to find the number. Finding the antilogarithm of a number is the reverse of finding the logarithm of a number.

Consider finding the number whose logarithm is 0.6107, i.e., $10^{0.6107}$. The number lies between 1 and 10.

In the table of antilogarithms, the antilogarithm of $0.6107 = 4.074$

The difference corresponding under 7 is 0.007.

Therefore, the antilogarithm of $0.6107 = 4.074 + 0.007$

=

4.081

Find the numbers whose logarithms are 0.5490, 2.6107 and 3.5026.

Example 11

Find the number whose logarithm is:

2.3031

4.5441

(a)

(b)

Solution

(a) Let the number be x

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$$\therefore \log x = 2.3031$$

$$x = 10^{2.3031}$$

$$= 10^{(-2 + 0.3031)}$$

$$= 10^{-2} \times 10^{0.3031}$$

$$= 10^{-2} \times 2.01$$

$$= \frac{1}{100} \times 2.01$$

$$= \frac{2.01}{100}$$

$$= 0.0201$$

(b) Let the number be x

$$\therefore \log x = 4.5441$$

$$x = 10^{4.5441}$$

$$= 10^{(-4 + 0.5441)}$$

$$= 10^{-4} \times 10^{0.5441}$$

$$= 10^{-4} \times 3.500$$

$$= \frac{1}{10\,000} \times 3.500$$

$$= \frac{3.500}{10\,000}$$

$$= 0.00035$$

Exercise 3.4

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e

1. Find the logarithms to base 10 of:
 - (a) 0.149
 - (b) 0.2843
 - (c) 0.3520
 - (d) 0.4286
 - (e) 0.0694
 - (f) 0.0485
 - (g) 0.0239
 - (h) 0.000376
 - (i) 0.0000784

(j) 0.0000523

2. Find the antilogarithms of the following logarithms:

(a)

e

1.1080 (b)

1.2480

2.3927

2.5403

(c)

(d)

()

(f)

3.6503 (g)

4.8938 (h)

3.9750

2.5658

5.4533

3.6821

(i)

(j)

e

3. Calculate:

$$2.37 + 1.20$$

$$5.63 + 2.57$$

$$2.34 + 4.30$$

$$3.79 - 2.24$$

$$7.12 - 5.54$$

(a)

(b)

(c) (d)

(e)

(f) $5.27 - 2.73$

3.5: Application of Logarithms

Multiplication and Division from the Laws of Indices

$$10^m \times 10^n = 10^{(m+n)} \text{ and}$$

$$10^m \div 10^n = 10^{(m-n)}$$

e

The above relationships can be used to perform multiplication and division.

Example 12

Use tables of logarithm to evaluate 357×47.9

Solution

$$357 = 10^{2.5527}$$

$$47.9 = 10^{1.6803}$$

$$\begin{aligned} \text{Therefore, } 357 \times 47.9 &= 10^{2.5527} \times 10^{1.6803} \\ &= 10^{(2.5527 + 1.6803)} \end{aligned}$$

$$= 10^{4.2330}$$

$$= 10^4 \times 10^{0.2330}$$

$$= 10^4 \times 1.71 \text{ (antilog}$$

of 0.2330)

$$= 17100$$

Note: The actual value of 357×47.9 by long multiplication is 17100.3. The difference is small and can be neglected.

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e

Example 13

Use logarithms to evaluate $864 \div 136$

Solution

$$\begin{aligned}864 \div 136 &= 10^{2.9365 - 102.1335} \\&= 10^{(2.9365 - 2.1335)} \\&= 10^{0.8030} \\&= 10^0 \times 6.353 \text{ (antilog of } \\&\quad 0.8030)\end{aligned}$$

Therefore, $864 \div 136 = 6.353$

Examples 12 and 13 can be arranged as follows:

<u>Number</u>	<u>Standard form</u>	<u>Logarithm</u>
357	357×10^2	2.5527
47.9	4.79×10^1	<u>1.6803</u> +
17100	1.71×10^4	4.2330

<u>Number</u>	<u>Standard form</u>	<u>Logarithm</u>
864	8.64×10^2	2.9365
136	1.36×10^2	<u>2.1335</u> -
6.353	6.353×10^0	0.8030

Example 14

Use logarithms to evaluate:

(a) $\frac{456 \times 398}{271}$

(b) 3.14^2

(c) 8.36^3

Solution

(a)
$$\frac{456 \times 398}{271}$$

No.	Log
456	2.6590
398	2.5999 +
	5.2589
271	2.4330 -
$10^2 \times 6.697$	2.8259

Therefore, $\frac{456 \times 398}{271} = 6.697 \times 10^2 = 669.7$

(b) $3.14^2 = 3.14 \times 3.14$

No.	Log
3.14^2	0.4969×2
$10^0 \times 9.858$ (antilog of 0.9938)	0.9938

$$\begin{aligned}\text{Therefore, } 3.14^2 &= 9.858 \times 10^0 \\ &= 9.858\end{aligned}$$

(c) 8.36^3

No.	Log
8.36^3	0.9222×3
$10^2 \times 5.842$ (antilog of 0.7666)	2.7666

$$\begin{aligned}\text{Therefore, } 8.36^3 &= 5.842 \times 10^2 \\ &= 584.2\end{aligned}$$

Example 15 Evaluate

$$\frac{415.2 \times 0.0761}{135} \text{ Solution}$$

Number	Logarithm
415.2	2.6182
0.0761	2.8814 +
	1.4996
135	2.1303 -
$10^{-1} \times 2.341$ (antilog of 0.3693)	1.3693

$$\begin{aligned}\text{Therefore, } \frac{415.2 \times 0.0761}{135} &= 2.341 \times 10^{-1} \\ &= 0.2341\end{aligned}$$

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Exercise 3.5

Use logarithms and antilogarithms to evaluate:

1. (a) 251×367

(b) 4192×3078

(c) 21.47×362.1

(d) 7.32×199

(e) $26.1 \times 91.2 \times 45.7$

(f) $33.2 \times 172 \times 44.32$

(g) $1527 \times 3196 \times 4157$

(h) $7.312 \times 49.45 \times 157.2$

(i) $16.31 \times 152.1 \times 3290$

(j) $100 \times 245 \times 175 \times 396$

2. (a) $1500 \div 750$

(b) $2145 \div 560$

(c) $412 \div 241$

(d) $882 \div 144$

$$(e) \ 3612 \div 452$$

$$(f) \ 1111 \div 222$$

$$(g) \ 3.142 \div 2.718$$

$$(h) \ 43.1 \div 3.17$$

$$(i) \ 9250 \div 4312$$

$$(j) \ 6380 \div 2\ 137$$

3. (a) $\frac{291 \times 681}{372}$

(b) $\frac{634 \times 436 \times 688}{784}$

(c) $\frac{3\ 041 \times 3\ 211}{2\ 112}$

(d) $\frac{788 \times 576}{675}$

(e) $\frac{294 + 578}{368 - 275}$

(f) $\frac{1\ 024 \times 6\ 561}{4\ 096 \times 729}$

Compill $\frac{839 + 672}{762 + 393}$ *Rakdfha :0770112040*

$$(h) \frac{3\ 267 - 37}{2\ 236 + 89}$$

$$(i) \frac{29^2 \times 33^3}{64^2}$$

$$(j) \frac{18^2 \times 391^4}{15^3 \times 56^4}$$

$$4. (a) \frac{876.9}{61.2 \times 3.85}$$

$$(b) \frac{518.3}{29.5 \times 714}$$

$$(c) \sqrt[3]{\frac{1.794 \times 0.038}{12.43}}$$

$$(d) \frac{3.74 \times 7.82}{5.4}$$

$$(e) \frac{399.6}{15.2 \times 4.83 \times 1.98}$$

$$(f) \frac{734.4}{25.1 \times 6.34 \times 3.94}$$

$$\frac{94.7 \times 16.45}{12.5 \times 8.93}$$

$$\frac{(4.48)^3}{(3.16)^2}$$

$$\frac{(171.5)^3}{56.3 \times 26.98}$$

$$\frac{(79.36)^3}{8.2 \times (9.2)^2}$$

(g)

(h)

(i)

(j)

5. Use logarithms to evaluate:

(a) 0.9063×3.387

(b) 0.5060×0.05707

(c) $36.65 \times 0.4163 \times 0.007022$

(d) $26.68 \div 255.4$

(e) $3.404 \div 628$

(f) $\frac{0.0075 \times 0.8181}{0.00509}$

(g) $\frac{77.9}{0.988 \times 9100}$

(h) $\frac{2.89 \times 5.27}{62.25 \times 1.908}$

(i) $\frac{34.53 \times 361.6}{343.7 \times 615.8}$

(j) $\frac{91.3 \div 18.26}{75.4 \div 12.09}$

Roots

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The tables of logarithms can be used to find the roots of numbers. Consider the expressions:

$$\sqrt{892}$$

$$\sqrt[3]{407.6}$$

(i)

(ii)

$$(iii) \sqrt{0.945}$$

$$(iv) \sqrt[3]{0.0618}$$

The solutions are as follows:

$$(i) \sqrt{892} = 892^{\frac{1}{2}} = (10^{2.9504})^{\frac{1}{2}} = 10^{1.4752}$$

$$\text{Therefore, } \sqrt{892} = 10^1 \times 2.986 \text{ (antilog of 0.4752)} \\ = 29.86$$

The working can be arranged as follows:

No.	Log
$\sqrt{892}$	$2.9504 \times \frac{1}{2}$
$10^1 \times 2.986$ (antilog of 1.4152)	1.4752

$$\text{Therefore, } \sqrt{892} = 29.86$$

$$(ii) \sqrt[3]{407.6}$$

No.	Log
$\sqrt[3]{407.6}$	$2.6103 \times \frac{1}{3}$
$10^0 \times 7.415$ (antilog of 0.8701)	0.8701

$$\text{Therefore, } \sqrt[3]{407.6} = 7.415$$

$$(iii) \sqrt{0.945} = (9.45 \times 10^{-1})^{\frac{1}{2}} = \left(10^{1.9754} \times \frac{1}{2}\right)$$

In order to divide 1.9754 by 2, the logarithm is rewritten such that the

negative characteristic is divisible by 2
as explained below.

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$$\bar{1.9754} = -1 + 0.9754$$

$= -2 + 1.9754$ (-1 is added to the characteristic and 1 added to the mantissa to make the characteristic divisible by 2).

$$\begin{aligned}\text{Therefore, } \frac{1}{2}(\bar{1.9754}) &= \frac{1}{2}(-2 + 1.9754) \\ &= -1 + 0.9877 \\ &= \bar{1.9877}\end{aligned}$$

$$\begin{aligned}\text{Therefore, } \sqrt{0.945} &= 9.720 \times 10^{-1} \\ &= 0.9720\end{aligned}$$

(iv) $\sqrt[3]{0.0618}$

Number	Logarithm
$\sqrt[3]{0.0618}$	$2.7810 \times \frac{1}{3}$
3.954×10^{-1}	$= (\bar{3} + 1.7910) \times \frac{1}{3}$ $= \bar{1.5970}$

$$\text{Therefore, } \sqrt[3]{0.0618} = 0.3954$$

Exercise 3.6

1. Divide each of the following logarithms by: (i) 2 (ii) 3 (iii) 4:
- (a) 0.8938
 - (b) 1.8624

$\bar{2.9754}$ (c)

$\bar{1.7076}$ (d)

$\bar{1.8538}$ (e)

$\bar{2.3502}$

$\bar{2.1644}$

$\bar{3.4928}$

$\bar{3.6946}$

$\bar{4.7938}$

(f)

(g)

(h)

(i) (j)

2. Use logarithms to find (i) the square root and (ii) the cube root of:

(a) 478

(b) 2 461

(c) 3 572

(d) 4 683

(e) 0.0346

(f) 0.00457

(g) 0.0067

(h) 0.00072

(i) 78.039

(j) 361.472

3. Use tables to evaluate:

(a) $\sqrt{\frac{3.45 \times 16.7}{31.5}}$

(b) $\sqrt{\left(\frac{3.142 \times 2.718}{6.49 \times 81.2}\right)}$

(c) $\frac{41.56 \times 52.3}{\sqrt{42.88}}$

(d) $\sqrt[3]{\left(\frac{1.794 \times 0.038}{12.43}\right)}$

(e) $\sqrt[3]{\left(\frac{39.51 \times 725}{0.758}\right)}$

(f) $\sqrt[3]{\left(\frac{4\ 862 \times 725}{6\ 437 \times 1\ 024}\right)}$

(g) $\sqrt[4]{\left(\frac{6\ 978 \times 25.1}{132.7}\right)}$

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Chapter 4

GRADIENT AND EQUATIONS OF STRAIGHT LINES

4.1: Gradient

A technician is to climb to the top of a wall 3 metres high using ladder AB or CB as shown below.

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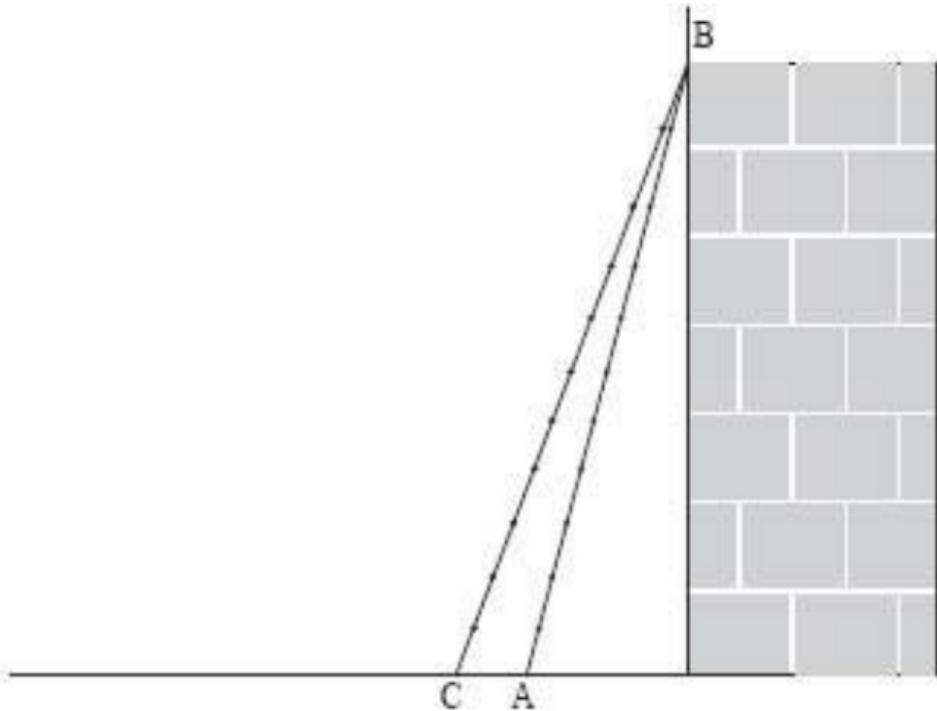


Fig. 4.1

The distance from the feet of the ladders to the wall are 2 m and 2.5 m respectively. If the technician uses

ladder AB, he will have to move through a horizontal distance of 2 m. For every one metre moved horizontally, the corresponding vertical displacement is $1\frac{1}{2}$ or $\frac{3}{2}$ metres.

The ratio $\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{3}{2} = 1.5$

For ladder CB, the corresponding ratio
 $= \frac{3}{2.5} = 1.2$

Notice that the ratio is higher for the steeper ladder.

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The ratio $\frac{\text{vertical distance}}{\text{horizontal distance}}$, which measures steepness or slope, is known as gradient.

Gradients of straight lines are determined in the same way. For example, in figure 4.2, OP is a straight line through the origin. P(4, 3) is a point on the line. The gradient of OP is;

$$\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{PQ}{OQ} = \frac{3}{4}$$

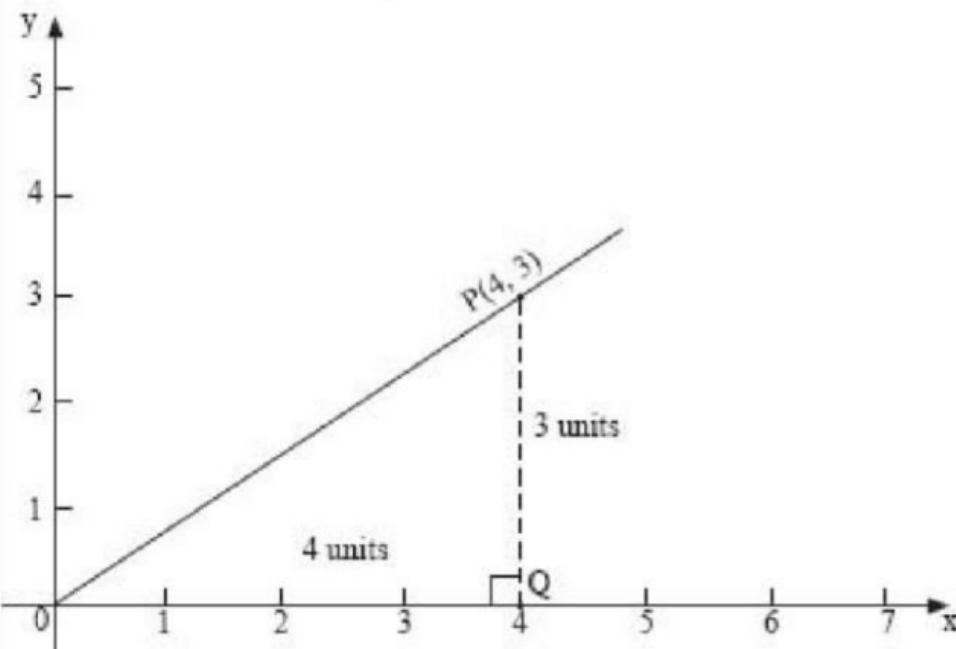


Fig. 4.2
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Example 1

Find the gradient of line AB in figure 4.3.

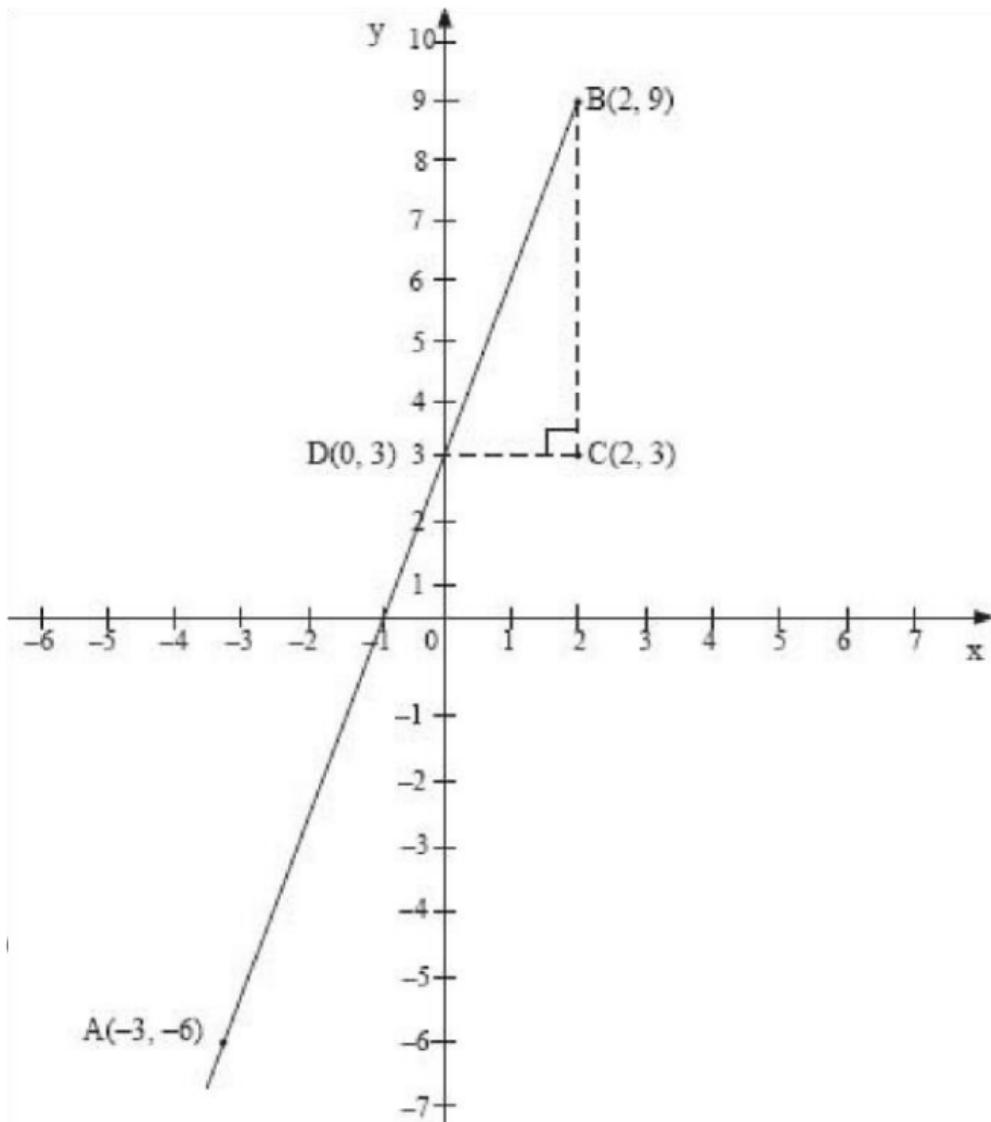


Fig. 4.3 Solution

Gradient of AB $\frac{\text{vertical distance}}{\text{horizontal distance}} =$

Using the co-ordinates of A $= \frac{9 - 6}{2 - 3}$ and B, $= \frac{15}{5}$

The gradient is $= 3$
 $=$

Determine the gradient of a line using any other convenient point on the line.

Figure 4.4 shows a line through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$.

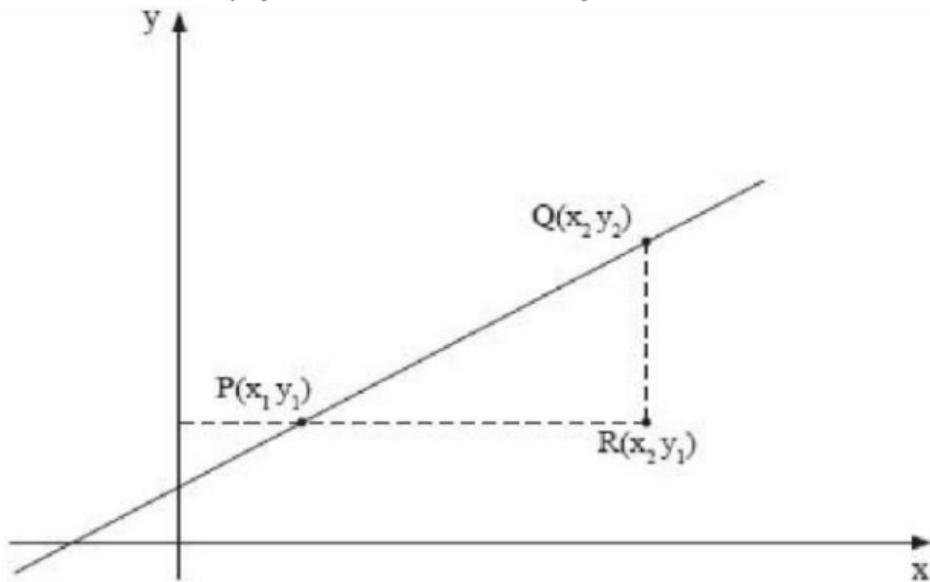


Fig. 4.4

The gradient of line PQ is given by;

$$\frac{\text{change in y co-ordinates}}{\text{corresponding change in x co-ordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 2

Find the gradient of the line PQ in figure 4.5.

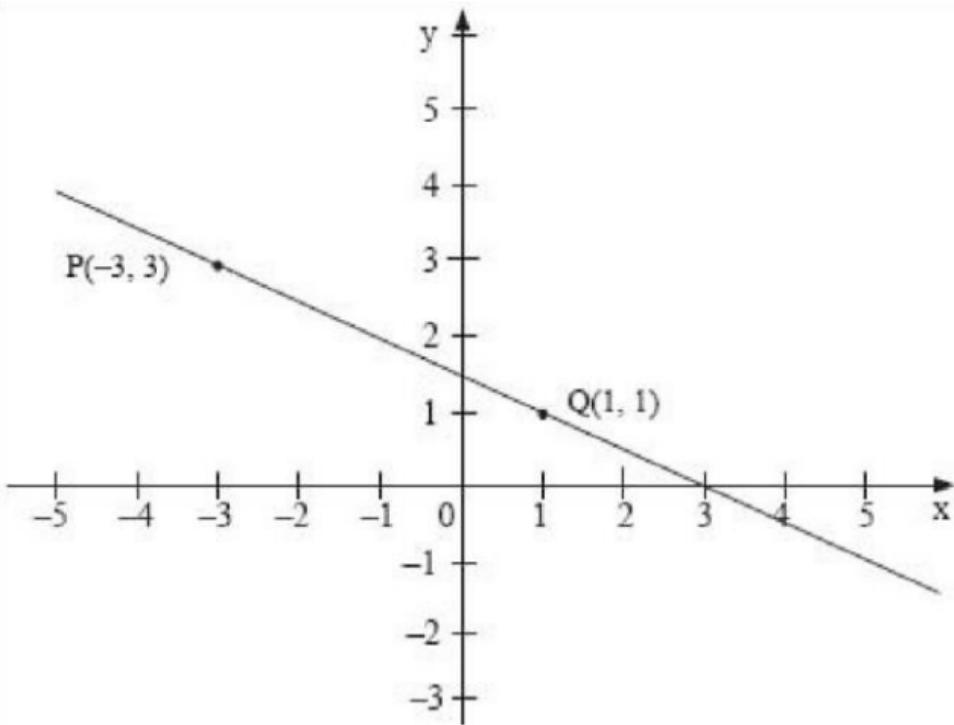


Fig. 4.5

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Solution

Gradient of PQ =

$$\frac{\text{change in y co-ordinates}}{\text{corresponding change in x co-ordinates}}$$

Change in y co-ordinates = 1 - 3

$$= -2$$

Change in x co-ordinates = 1 - (-3)

$$= 4$$

$$\begin{aligned}\text{Gradient} &= \frac{2}{4} \\ &= -\frac{1}{2}\end{aligned}$$

Work out the same gradient by subtracting the co-ordinates of Q from those of P.

Note:

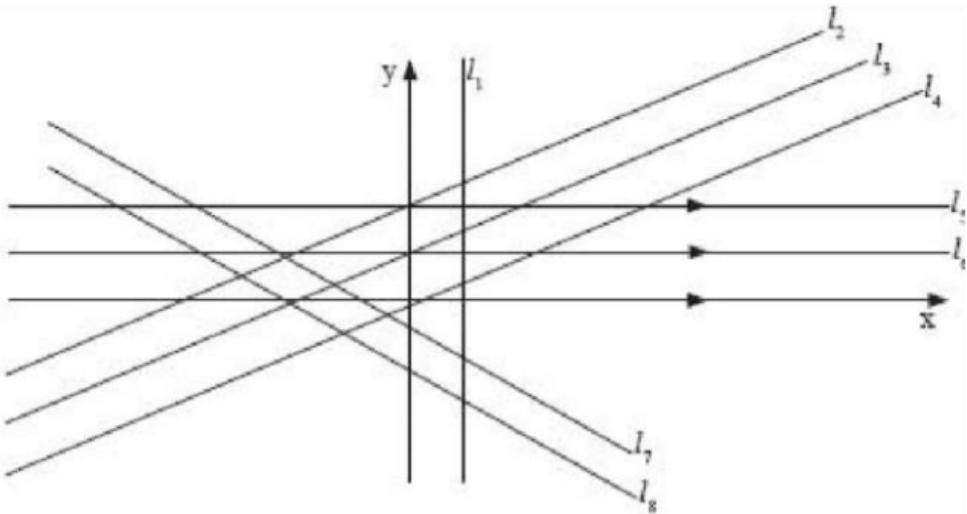
- (i) If an increase in the x co-ordinate also causes a corresponding increase in the y co-ordinate, the gradient is positive.
- (ii) If an increase in the x co-ordinate also causes a decrease in the value

of the y co-ordinate, the gradient is negative.

- (iii) If for an increase in the x coordinate, there is no change in the value of the y co-ordinate, the gradient is zero.
- (iv) For a vertical line, the gradient is not defined because the horizontal change is zero. Determination of the gradient would entail division by zero which is not defined.

Exercise 4.1

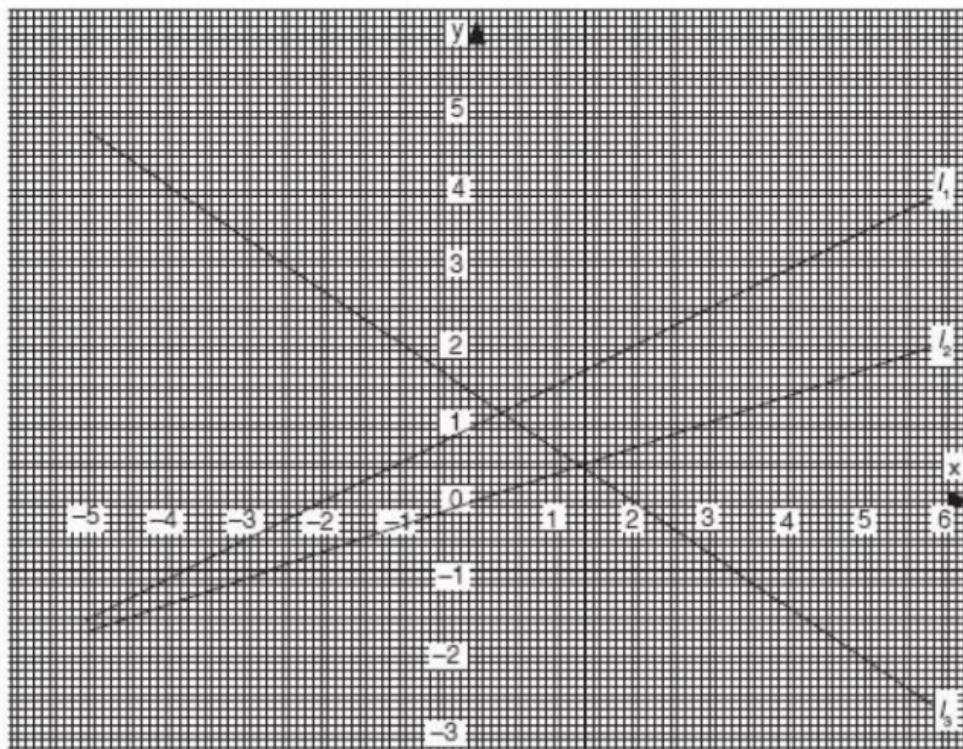
1. In the figure below, name the lines whose gradient is:
 - (a) positive
 - (b) negative
 - (c) zero
 - (d) undefined



2. For each of the following pairs of points, find the gradients of the lines passing through them:
- (a) A(2, 3), B(5, 6)
 - (b) C(5, 10), D(12, 20)
 - (c) E(-5, 6), F(2, 1)

- (d) G(4, 5), H(6, 5)
- (e) I(8, 0), J(12, -6)
- (f) K(5, -2), L(6, 2)
- (g) M(6, 3), N(-6, + 2)
- (h) P(2, -5), Q(2, 3)
- (i) R($\frac{1}{4}$, $\frac{1}{3}$), S($\frac{1}{3}$, $\frac{1}{4}$)
- (j) T(0.5, 0.3), U(-0.2, -0.7)

3. Find the gradients of the lines marked l_1 l_2 and l_3 in the figure below.



4.2: Equation of a Line

(a) Given two Points

Example 3

Find the equation of the line through the points A(1, 3) and B(2, 8).

Solution

The gradient of the required line is

$$\frac{8-3}{2-1} = 5$$

Take any point P(x, y) on the line.

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Using the points P and A, the gradient is

$$\frac{y-3}{x-1}$$

Therefore, $\frac{y-3}{x-1} = 5$

This simplifies to $y = 5x - 2$.

Find the equation of the line using the points P and B.

(b) Given the Gradient and One Point on the Line

Example 4

Determine the equation of a line with gradient 3, passing through the point (1, 5).

Solution

Let the line pass through a general point (x, y) . The gradient of the line is

$$\frac{y-5}{x-1} = 3.$$

This equation simplifies to $y = 3x + 2$.

4.3: The Linear Equation $y = mx + c$

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We can express linear equations in the form $y = mx + c$. For example, $4x + y = \frac{4}{3}x - \frac{8}{3}$ or $3y = -8$ is equivalent to $\frac{4}{3}$ and $c = -\frac{8}{3}$.

In this case, $m = .$

The constants m and c are significant in the geometry of a straight line. Consider the three lines l_1 , l_2 and l_3 whose equations are $2y = x + 6$, $3y = 4x - 5$ and $2y = 7 - 5x$ respectively. Their graphs are given in figure 4.6.

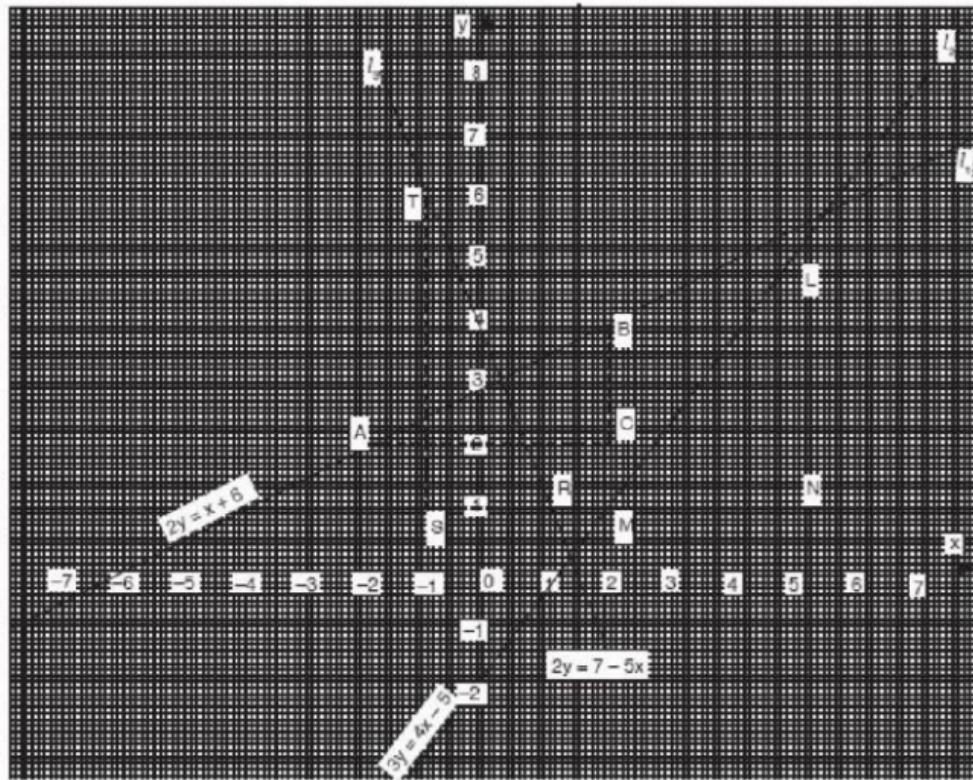


Fig. 4.6

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The gradient of l_1 is $\frac{BC}{AC} = \frac{2}{4} = \frac{1}{2}$

$$\frac{LN}{MN} = \frac{4}{3}$$

The gradient of l_2 is $\frac{TS}{SR} = \frac{5}{2}$

The gradient of l_3 is

Re-write each of the above equations in the form $y = mx + c$, where m and c are constants and compare the value of m with that of the gradient. These are shown in the table below.

<i>Equation of line</i>	<i>The form $y = mx + c$</i>	<i>m</i>	<i>c</i>	<i>Gradient</i>
l_1 $2y = x + 6$	$y = \frac{1}{2}x + 3$	$\frac{1}{2}$	3	$\frac{1}{2}$
l_2 $3y = 4x - 5$	$y = \frac{4}{3}x - \frac{5}{3}$	$\frac{4}{3}$	$-\frac{5}{3}$	$\frac{4}{3}$
l_3 $2y = 7 - 5x$	$y = -\frac{5}{2}x + \frac{7}{2}$	$-\frac{5}{2}$	$\frac{7}{2}$	$-\frac{5}{2}$

In each case, the gradient equals the corresponding value of m. If the equation of a straight line is in the form $y = mx + c$, the gradient is equal to m.

Example 5

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Find the gradient of the line whose equation is $3y - 6x + 7 = 0$

Solution

Write the equation in the form of $y = mx + c$.

$$3y = 6x - 7 \quad y = 2x - \frac{7}{3}m = 2$$

and so the gradient is 2.

The y-intercept

Consider the following equations of straight lines: (i) $y = 3x + 2$

$$(ii) y = -5x + 3 \quad (iii) y$$

$$= \frac{5}{2}x - 4$$

For each of the lines, find y when x = 0.

$$(i) \quad y = 3(0) + 2$$

$$= 2$$

$$(ii) \quad y = -5(0) + 3$$

$$= 3$$

$$(iii) \quad y = \frac{5}{2}(0) - 4$$

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$$= -4$$

2, 3 and – 4 are the y-intercepts.

The x-intercept

Consider a line whose equation is $y = 3x + 6$, when $y = 0$.

$$3x + 6 = 0$$

$$3x = -6$$

$$x = -2$$

The line crosses the x-axis at the point where $x = -2$, thus -2 is the x-intercept.

Find the x-intercepts for the lines whose equations are:

(i) $2y - 3x = 12$

(ii) $\frac{1}{2}x + \frac{1}{3}y = 6$

Example 6

Find the equation of a line whose x-intercept is -3 and y-intercept is 6 .

Solution

Points $(-3, 0)$ and $(0, 6)$ lie on the line.

Therefore, the gradient of the line

$$= \frac{6 - 0}{0 - (-3)}$$

$$= \frac{6}{3}$$

$$= 2$$

In $y = mx + c$, m is the gradient of the line and c is the y -intercept. In the above case, $m = 2$ and $c = 6$.

Therefore, the equation of the line is $y = 2x + 6$.

Exercise 4.2

1. For each of the following straight lines, determine the gradient and

the y-intercept without drawing the line.

- (a) $3y = 7x$
- (b) $2y = 6x + 1$
- (c) $7 - 2x = 4y$
- (d) $3y = 7$
- (e) $2y - 3x + 4 = 0$
- (f) $3(2x - 1) = 5y$
- (g) $y + 3x + 7 = 0$

$$(h) 5x - 3y + 6 = 0$$

$$(i) \frac{3}{2}y - 15 = \frac{2}{3}x$$

$$(j) 2(x + y) = 4$$

$$(k) \frac{1}{3}x + \frac{2}{5}y + \frac{1}{6} = 0$$

$$(l) -10(x + 3) = 0.5y$$

$$(m) ax + by + c = 0$$

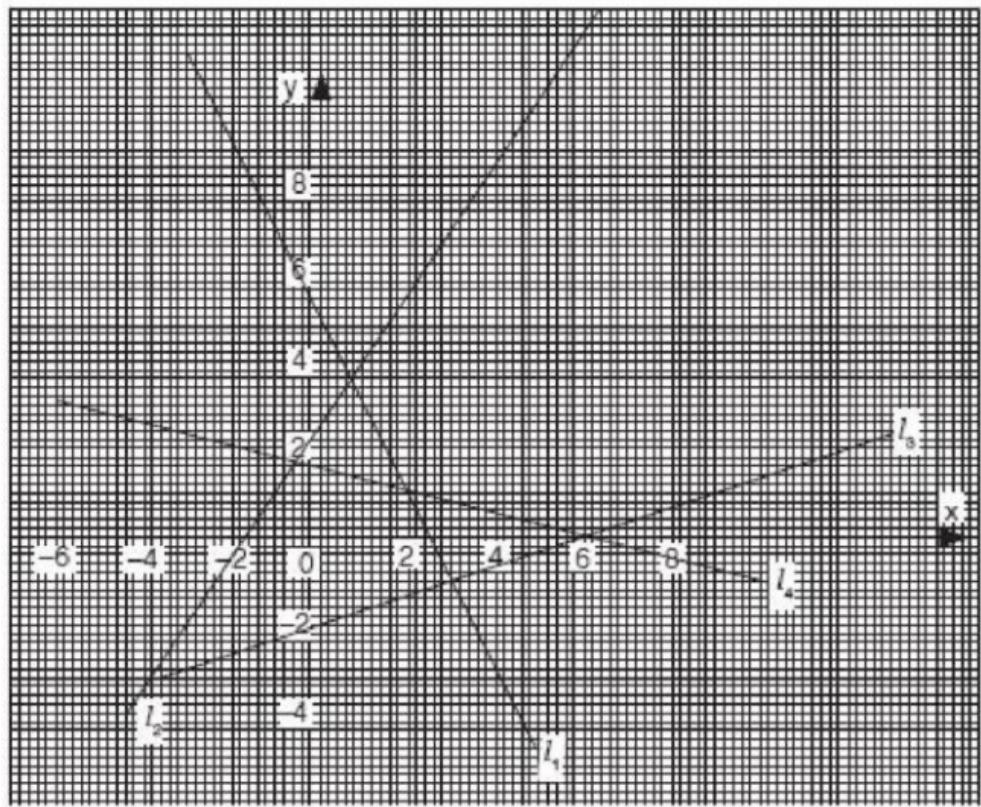
2. Find the equations of the lines with the given gradients and passing through the given points:

- a) 4; (2, 5)
- (b) $\frac{3}{4}$; (-1, 3)
- (c) -2; (7, 2)
- (d) $-\frac{1}{3}$; (6, 2)
- (e) 0; (-3, -5)
- (f) $-\frac{3}{2}$; (0, 7)
- (g) m;(1, 2)

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(h) $m;(a, b)$

3. Write the equations of the lines l_1 , l_2 , l_3 , l_4 shown in the figure below in the form $y = mx + c$.



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4. Find the equation of the line passing through the given points:

(a) $(0, 0)$ and $(1, 3)$

(b) $(0, -4)$ and $(1, 2)$

(c) $(0, 4)$ and $(-1, -2)$

(d) $(1, 0)$ and $(3, -3)$

(e) $(3, 7)$ and $(5, 7)$

(f) $(-1, 7)$ and $(3, 3)$

(g) $(11, 1)$ and $(14, 4)$

(h) $(5, -2.5)$ and $(3.5, -2)$

(j) (a, b) and (c, d)

(k) (x_1, y_1) and (x_2, y_2)

5. Find the co-ordinates of the point where each of the following lines cuts the x-axis: (a) $y = 7x - 3$

(b) $y = -(3x + 2)$

(c) $y = \frac{1}{3}x + 4$

(

d) $y = 0.5 - 0.8x$

(e) $y = mx + c$

(f) $ax + by + c = 0 \ (a \neq 0)$

6. The x and y-intercepts of a line are given below. Determine the equation of the line in each case:

x-intercept y-intercept

(a) -2 -2

(b) -3 4

(c) 5 -1

(d) 3 4

(e) a b ($a \neq 0, b \neq 0$)

in

7. The equation of the base of an isosceles triangle ABC is $y = -2$ and the equation of one of its sides is $y + 2x = 4$. If the co-ordinates of A are $(-1, 6)$, find the coordinates of B and C. Hence, find the equation of the remaining side.

4.4: The Graph of a Straight Line

Example 7

Draw the graph of a line passing through $(0, -4)$ with a gradient of 2.

(in
Solution

The equation of the line is;

$$\frac{y+4}{x} = 2$$

$$y+4 = 2x$$

$$y = 2x - 4$$

The x-intercept is 2, and the y-intercept is – 4. The line cuts the x-axis at (2, 0), and the y-axis at (0, – 4).

The two points can then be plotted on cartesian plane and joined as shown in figure 4.7.

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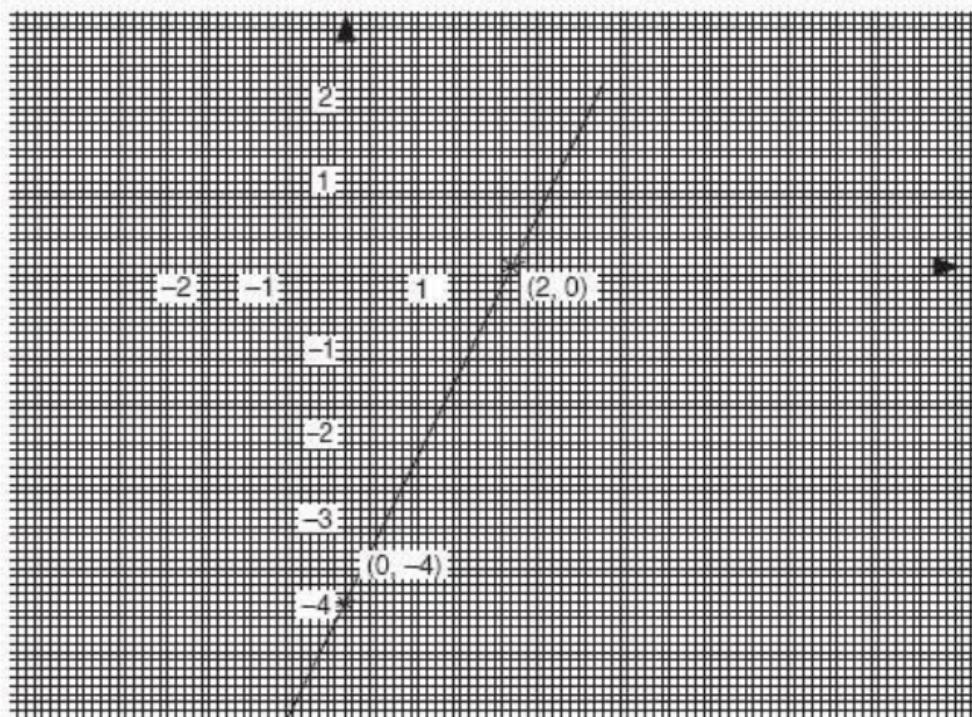


Fig. 4.7

Exercise 4.3

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1. Draw the lines passing through
the

in

given points and having the given gradients:

(a) $(0, 3); 3$

(b) $(0, 2); 5$

(c) $(4, 3); 2$

2. Draw the graphs of the following lines using the x and y-intercepts:

(a) $y = \frac{1}{2}x + 3$

(b) $3y - 4x = 5$

(c) $y + 2x - 3 = 0$

$$(d) \quad y = -2x + 2$$

4.5: Perpendicular Lines

In figure 4.8, line AB is perpendicular to AC and EF is perpendicular to DE.

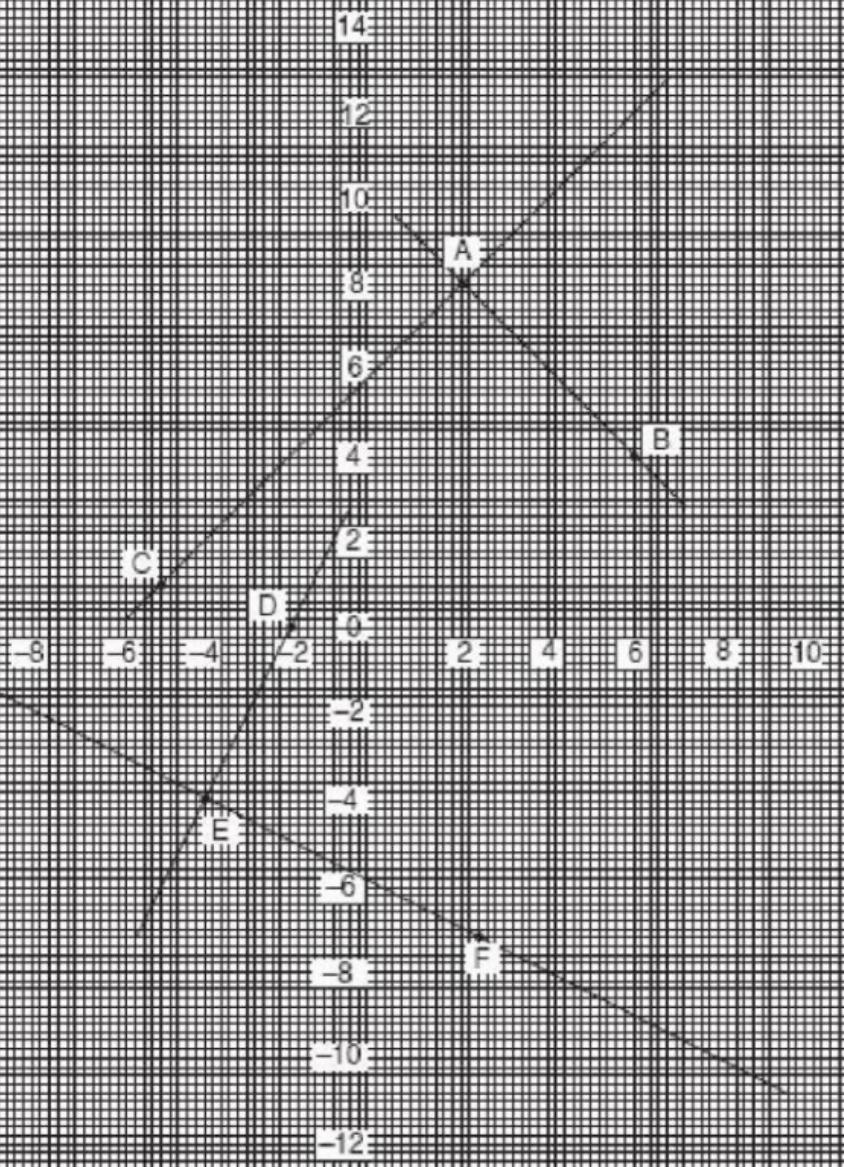


Fig. 4.8

From figure 4.11, find the gradients of AB and AC and the product of the two gradients.

Repeat the same for DE and EF.

Note that the product in both cases is -1 .

If the product of gradients of two lines is equal to -1 , then the two lines are perpendicular to each other.

Example 8

Which of the following lines is perpendicular to the line $y = 4x + 2$? (a) $l_1: y = \frac{1}{2} - \frac{1}{4}x$

(b) $l_2: y + 4x = 6$

Solution

$l_1: y = \frac{1}{2} - \frac{1}{4}x$ in the form $y = mx + c$ is $y = -\frac{1}{4}x + \frac{1}{2}$

(a) The gradient is $-\frac{1}{4}$

The gradient of $y = 4x + 2$ is 4.

The product of the two gradients is

$$-\frac{1}{4} \times 4 = -1$$

Therefore, the two lines are perpendicular to each other.

(b) $l_2: y + 4x = 6$ in the form $y = mx + c$ is

$$y = -4x + 6$$

The gradient is -4

The product of the gradient of l_2 and the line $y = 4x + 2$ is;

$$-4 \times 4 = -16$$

Therefore, the two lines are not perpendicular to each other.

Exercise 4.4

1. Without drawing, determine which of the following pairs of lines are perpendicular:
 - (a)

$$y = 2x + 7 \quad (b)$$

$$y = -\frac{1}{2}x + 3$$

$$3y = x + 3$$

$$y = -3x - 2$$

$$y = 2x + 7$$

$$y = -2x + 5$$

$$y = 5x + 1$$

$$5y = 10 - x$$

$$y = \frac{2}{3}x - 1$$

$$y = \frac{3}{2}x - 4$$

(c)

(d)

(e)

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$$y = \frac{2}{3}x + 4 \quad (f)$$

$$2y + 3x = 8$$

$$2y - 7x = 4 \quad (g)$$

$$y = \frac{2}{7}x - \frac{1}{2}$$

$$y = -\frac{3}{4}x - 2 \quad (h)$$

$$y = -\frac{4}{3}x + 5$$

2. Determine the equations of the lines perpendicular to the given lines and passing through the given points:

- (a) $y - 5x + 3 = 0$; $(3, 2)$
 - (b) $y = 8 - 7x$; $(-3, -4)$
 - (c) $y + 3x + 5 = 0$; $(0.25, -0.75)$
 - (d) $y + x = 17$; $(-4, 2)$
 - (e) $y = 17$; $(-2, -1)$
 - (f) $y = mx + c$; (a, b) .
3. A triangle has vertices $A(2, 5)$, $B(1, -2)$ and $C(-5, 1)$. Determine:
- (a) the equation of the line BC .

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- (b) the equation of the perpendicular line from A to BC.
4. ABCD is a rhombus. Three of its vertices are A(1, 2), B(4, 6) and C(4, 11). Find the equations of its diagonals and the co-ordinates of vertex D.
5. The point D(-2, 5) is one of the vertices of a square ABCD. The equations of the lines AB and AC are $y = \frac{1}{3}x - 1$ and $y = 2x - 1$ respectively. Determine the equations of lines DA, DB and DC.

Hence, find the co-ordinates of the remaining three vertices.

6. ABCD is a rectangle with the centre at the origin. A is the point (5, 0). Points B and C lie on the line $2y = x + 5$. Determine the co-ordinates of the other vertices.

4.6: Parallel Lines

In figure 4.9, line l_1 , is parallel to l_2 and line l_3 is parallel to l_4 . Find the gradients

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of I_1 , and I_2 . Likewise, find the gradients of I_3 and I_4 .

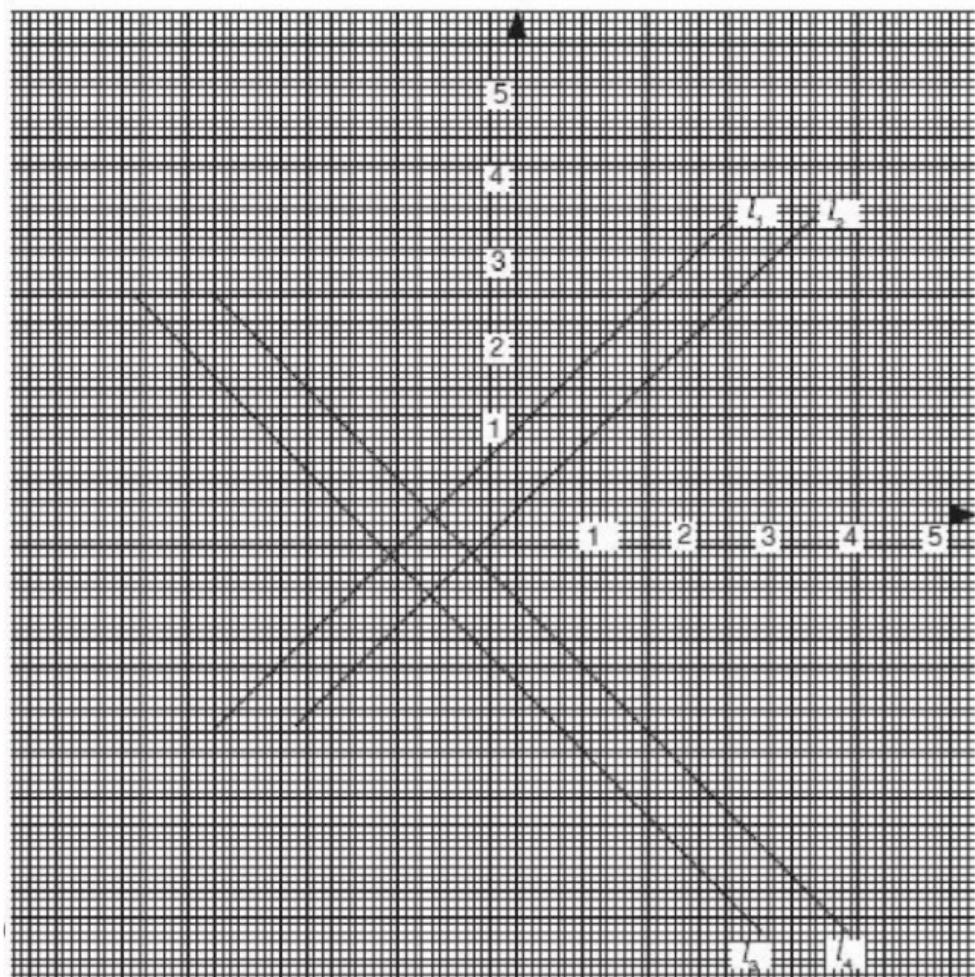


Fig. 4.9

Note that in each case, the gradients are equal. Parallel lines have equal gradients.

Exercise 4.5

1. Determine which of the following pairs of straight lines are parallel.
Do not draw the lines:

(a) $y = \frac{1}{2}x + 7$
 $y = \frac{1}{2}x - 20$

(b) $\frac{3}{2}x + \frac{2}{3}y + \frac{3}{2} = 0$

$$(c) 2x + 3y - 8 = 0$$

$$21 - 4x - 6y = 0$$

$$(d) 5x + 6y - 4 = 0$$

$$y = \frac{6}{5}x + \frac{2}{3}$$

$$(e) 2y - 7x - 8 = 0$$

$$4y + 17 = 14x$$

$$(f) y = -\frac{4}{5}x + 13 \text{ and the line through } (-1, -3) \text{ and } (-3.5, -11)$$

- (g) $6(x - 3y) + 7 = 0$ and the line through the origin having a gradient -3 .
- (h) A line through $(0, 7)$ and $(1, 11)$ and another line through $(1, 11.8)$ and $(2, 14.6)$.
- (i) A line through $(1, 4)$ and $(-1, -1)$ and another line through $(99, 103)$ and $(51, 7)$.
2. In each of the following, find the equation of the line through the given point and parallel to the given line:

- (a) $(0, 0)$; $y = -\frac{2}{7}x + 1$
- (b) $(3.5, 0)$; $x + y = 10$
- (c) $(5, 2)$; $5y - 2x - 115 = 0$
- (d) $(-3, 5)$; $7y = 3x$
- (e) $(-\frac{7}{3}, \frac{3}{4})$; $2(y - 2x) = 1.1$
- (f) $(0, -3)$; $2x + y = -3$
- (g) $(3\frac{1}{7}, -1\frac{1}{7})$; $15(1 - x) = 22y$
- (h) $(-3, 4)$; $x = 101$

(i) $(2, 3); y = 0$

3. The equations of two sides of a parallelogram are $y = -2$ and $y = x - 2$. If one of the vertices of the parallelogram has co-ordinates $(14, 5)$, find the equations of the other two sides and the remaining vertices.
4. In a rectangle ABCD, the equation of the line AB is $y = x + 6$. The x coordinate of A is -3 . The line AD is parallel to the line $y + 3x = 7$. If C

has the coordinates (2, 6),
determine:

- (a) the equation of lines AD, BC and CD.
- (b) the co-ordinates of A, B and D.
- (c) the equations of the diagonals.

Chapter 5

REFLECTION AND CONGRUENCE

5.1: Symmetry

Figure 5.1 shows a rectangle ABCD. X and Y are the midpoints of AD and BC respectively, while P and Q are the midpoints of AB and DC respectively.

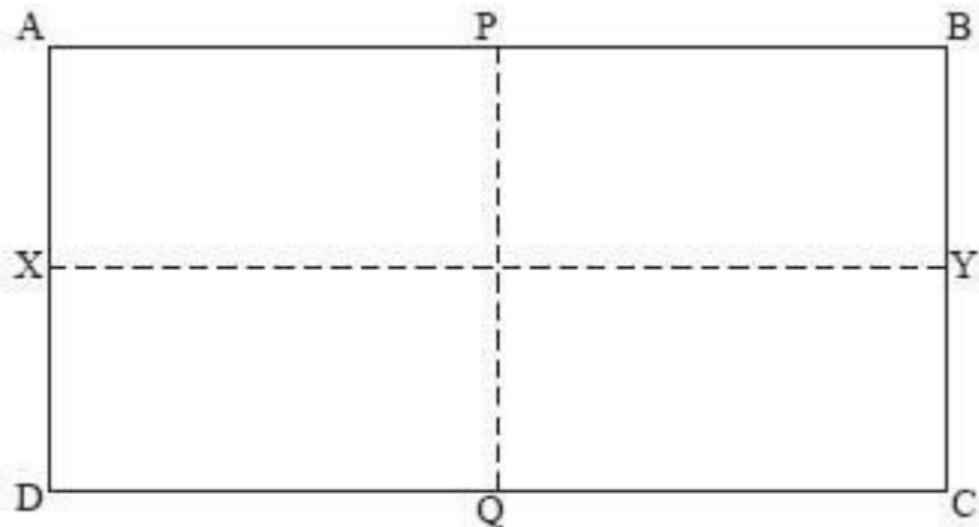


Fig. 5.1

Trace the figure and fold the tracing along the line XY. Note that ABYX fits exactly onto DCYX. Similarly, fold the

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tracing along PQ. Note that DAPQ fits exactly onto CBPQ.

The lines of folds such as XY and PQ which divide a plane figure into two identical parts are called lines of symmetry.

Fold the tracing along line AC and BD. Note that ABC does not fit exactly onto ADC. Similarly, BCD does not fit exactly onto BAD. The diagonals of a rectangle are not lines of symmetry. Therefore, a rectangle has only two lines of symmetry.

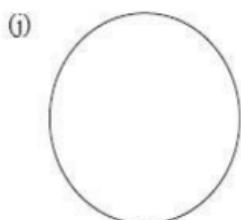
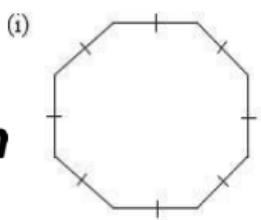
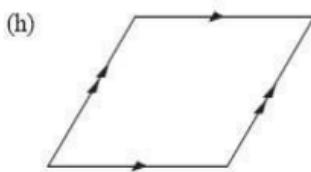
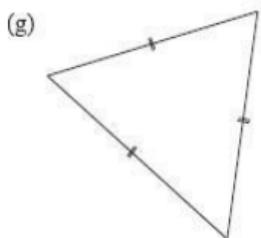
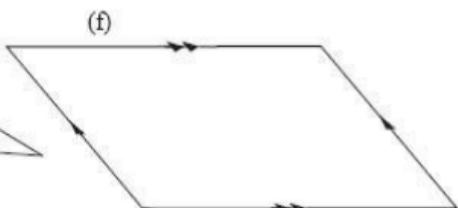
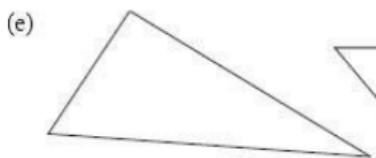
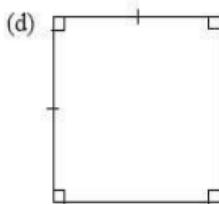
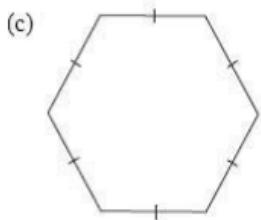
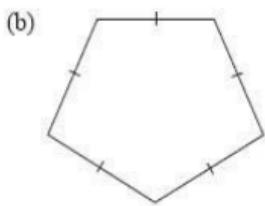
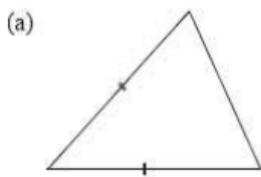
- (a) How many lines of symmetry do the following shapes have?
- (i) a square
 - (ii) a kite
 - (iii) a circle
- (b) Name other shapes which have lines of symmetry.

If an orange is cut into two identical

parts, the cut surface is called the plane of symmetry. How many planes of symmetry does an orange have? Give other examples in nature that have planes of symmetry.

Exercise 5.1

1. How many lines of symmetry does each of the following shapes have?

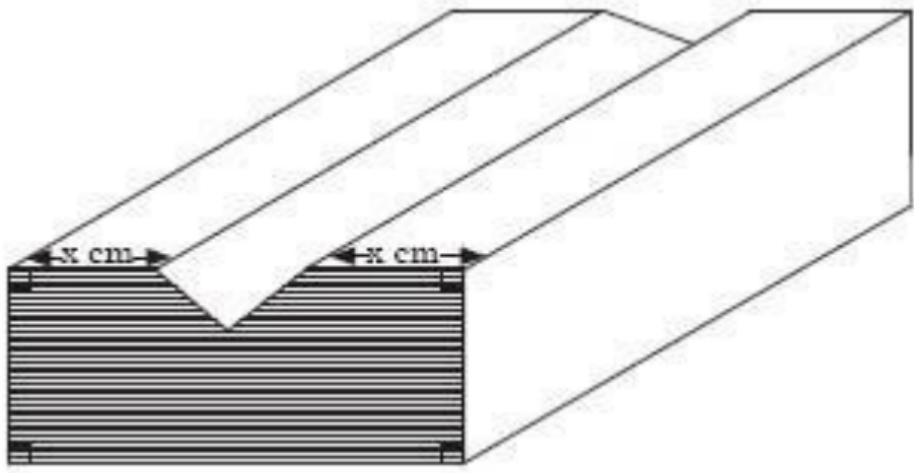


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2. Which letters of the alphabet have lines of symmetry?
3. State the numbers of planes of symmetry the following solids have:
 - (a) A square-based pyramid.
 - (b) A spherical ball.
 - (c) A cuboid.
 - (d) A cylindrical piece of wood.
 - (e) A cone.
 - (f) A prism whose cross-section is:

- (i) an isosceles triangle.
 - (ii) an equilateral triangle.
 - (iii) a scalene triangle.
4. Name some parts of your body that are mirror images of each other.
5. How many planes of symmetry has the solid in the figure below.



5.2: Reflection

Consider figure 5.2:

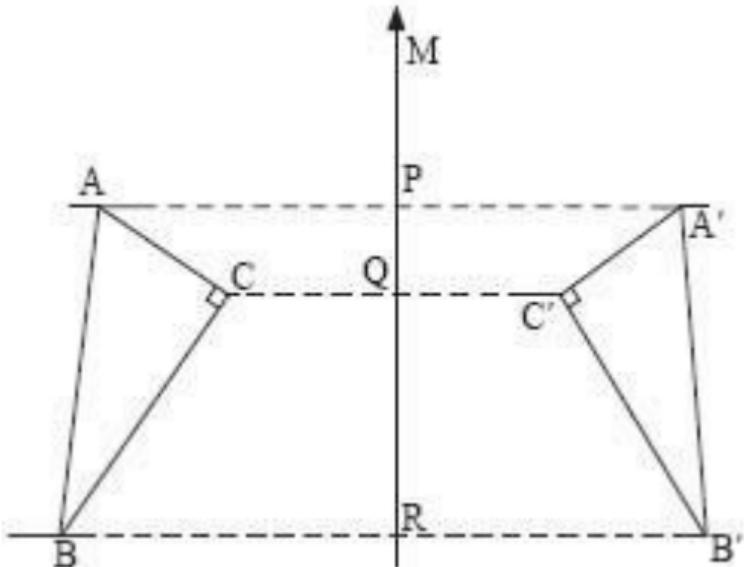


Fig. 5.2

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In the figure, triangle $A'B'C'$ is the image of triangle ABC under a reflection in the line M .

A is mapped onto A'

B is mapped onto B'

C is mapped onto C'

Measure AP and PA' , BR and RB' and CQ and QC' Measure also the angles between the mirror line and lines AA' , BB' and CC' .

Note that:

- (i) a point on the object and a corresponding point on the image are equidistant from the mirror line.
- (ii) the line joining a point and its image is perpendicular to the mirror line.
- (iii) the object and its image have the same shape and size.

Example 1

Draw the image of triangle ABC under a reflection in the mirror line M.

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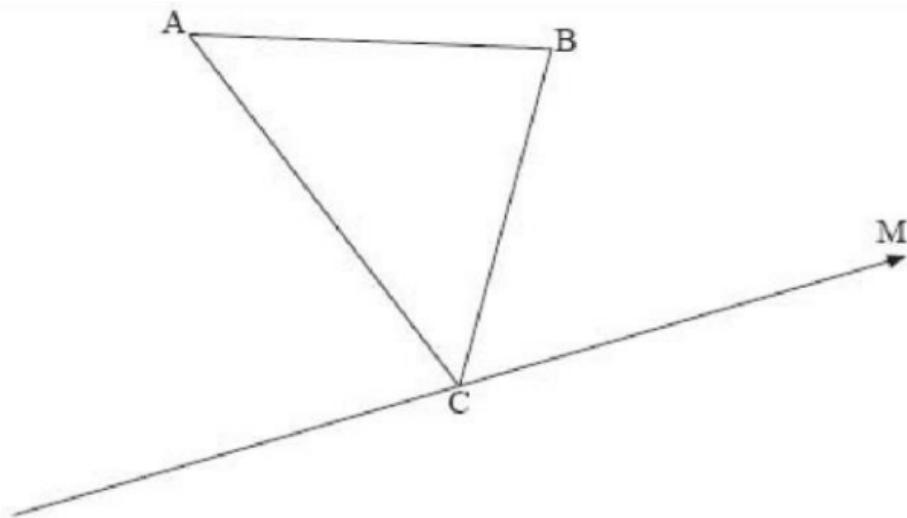


Fig. 5.3

Solution

To obtain the image A, drop a perpendicular from A to the mirror line and produce it. Mark off A', the image of A' equidistant from the mirror line as A.

Similarly, obtain B' , the image of B . Since C is on the mirror line, it is mapped onto itself. Join A' to B' , B' to C' and C' to A' to obtain triangle $A'B'C'$, the image of triangle ABC as in figure 5.4.

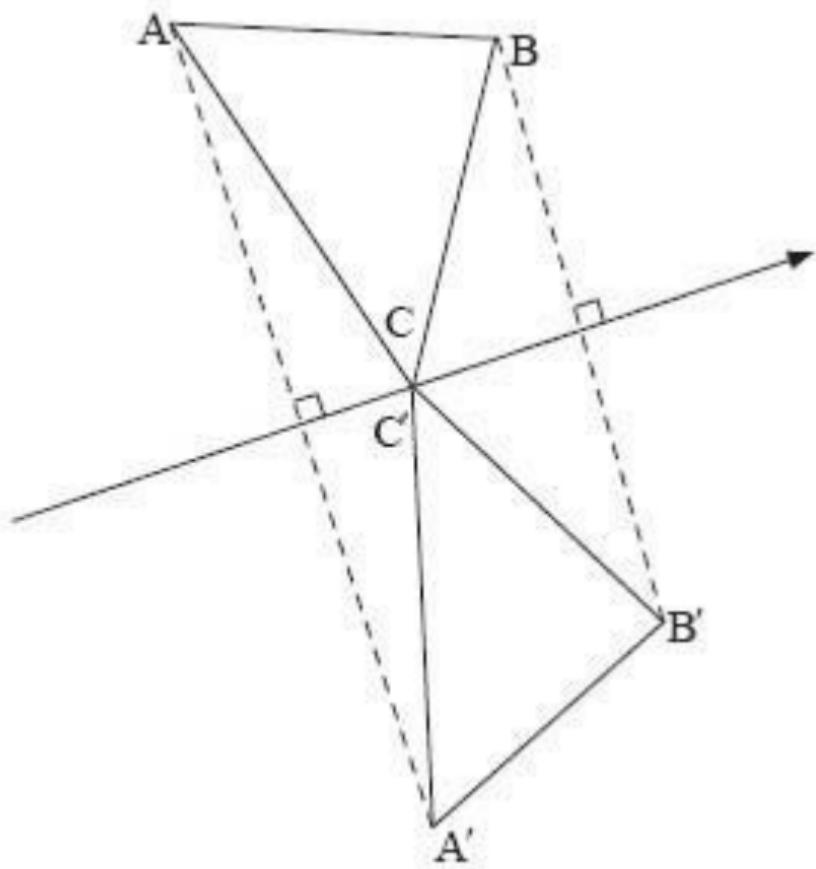


Fig. 5.4

Measure the angles between the mirror line and the lines:

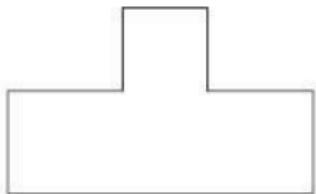
(a) $B'C$ and BC

(b) $A'C$ and AC

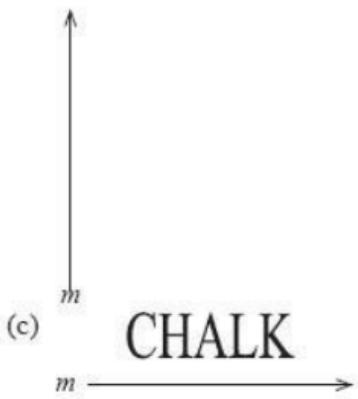
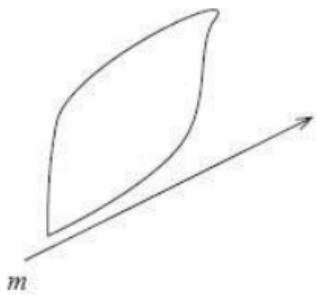
Exercise 5.2

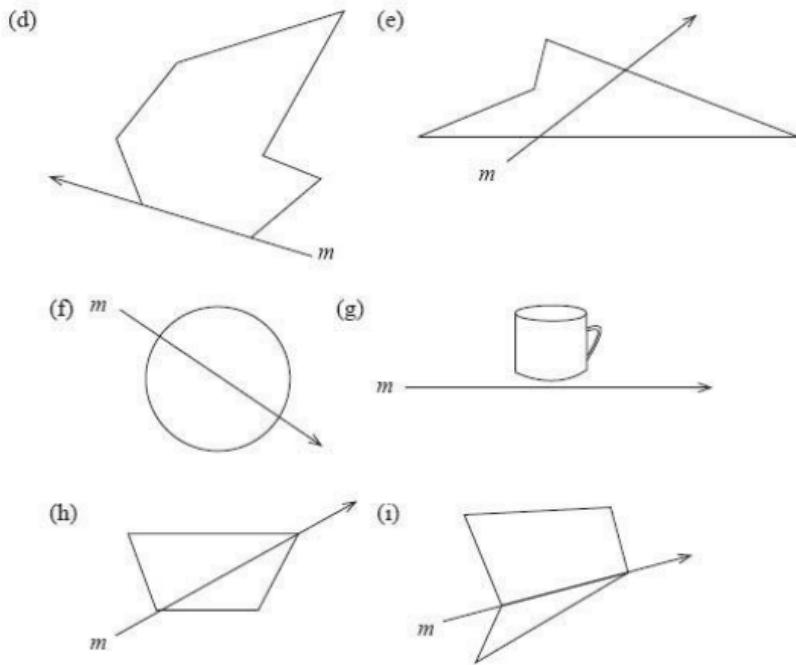
1. In each of the diagrams in the figure below, m is the mirror line. Make a tracing of each and draw their images under a reflection in the mirror line.

(a)



(b)





2. An object line AB, 4 cm long, makes an angle of 35° with the mirror line

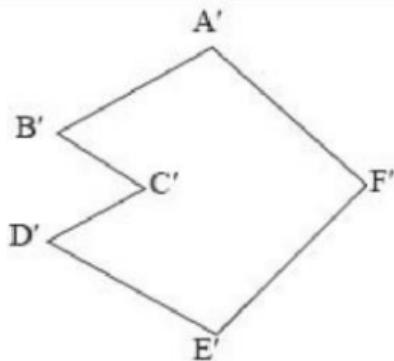
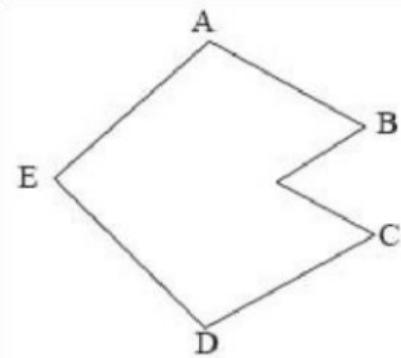
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at point B. Draw the image line and find the angle between A'B' and the mirror line.

3. The angle between an image line and its object line is 80° . Draw a diagram to show the position of the

i
mirror line.

4. The figure below shows an object and its image after reflection. Trace them and draw the mirror line.



5. The vertices of a quadrilateral ABCD are A(4.5, 1.5), B(7.5, 3), C(5, 3.5) and D(4.5, 2.5). Draw on the same

i

axes the quadrilateral and its image after a reflection in:

- (a) x-axis
- (b) line $y = x$.
- (c) y-axis
- (d) line $y = -x$.

6. If $A(2, -7)$, $B(2, -2)$ and $C(7, -2)$ are the vertices of a triangle, find the image of the triangle under a reflection in the line:

- (a) $y = 2.5$
- (b) $y = -2.5$

i

(c) $x = 3.5$

(d) $x = -3.5$

7. A(-5, -2), B(-2, -5) and C(-12, -2) are vertices of a triangle. Find the image of the triangle when it is reflected in:

(a) y-axis followed by a reflection in the line $y = -x$.

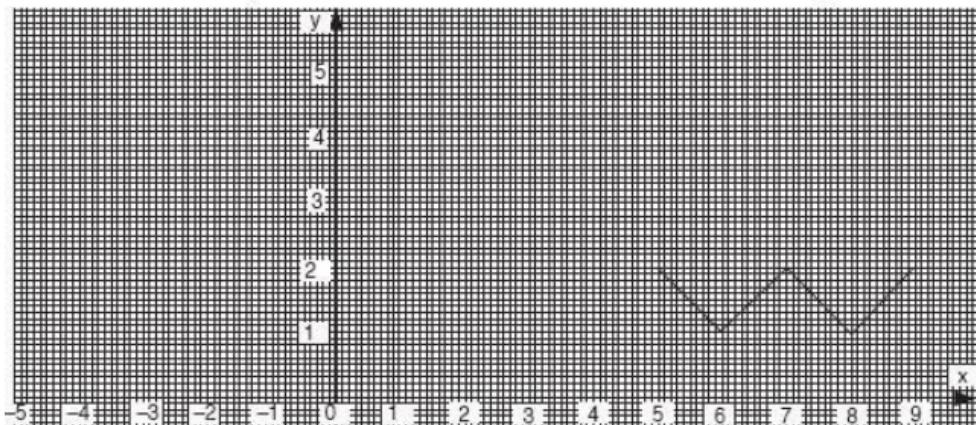
(b) x-axis followed by a reflection in the line $y = x$.

i

8. The vertices of a polygon ABCDEF are A(- 4, 6), B(-3, 2), C(-7, 1), D(-7, 4), E(-8. 5) and F(-7, 6). Find the final image of the polygon under the reflection in line:
- (a) $y = x$, followed by a reflection in $y = -x$.
- (b) $y = x$, followed by a reflection in y axis, and finally followed by one in the line $y = -x$.
9. A circle with centre A(7, 4) and radius 3 units is reflected in the line $x = 4.5$, followed by a reflection in the y -axis. Draw the final image.

i

10. In the figure below, the letter W is first reflected in the line $y = x$ followed by a reflection in the yaxis. Determine the co-ordinates of its final image.



i

11. A letter T has vertices at (3, 8), (4, 8), (4, 6) and (5, 8). It is reflected in the x-axis. Draw the image of T.
12. A letter N has vertices at (2, 6), (3,

6), (3, 8) and (4, 8). It is reflected in the y-axis followed by a reflection in the line $y = 2$. Determine the coordinates of the final image.

13. The co-ordinates of the vertices of the letter V are (8, 4), (9, 2), and (10, 4). If the letter is reflected in the line $x = -2$ followed by reflection in the line $y = x$, determine the coordinates of the:
- (a) first image.

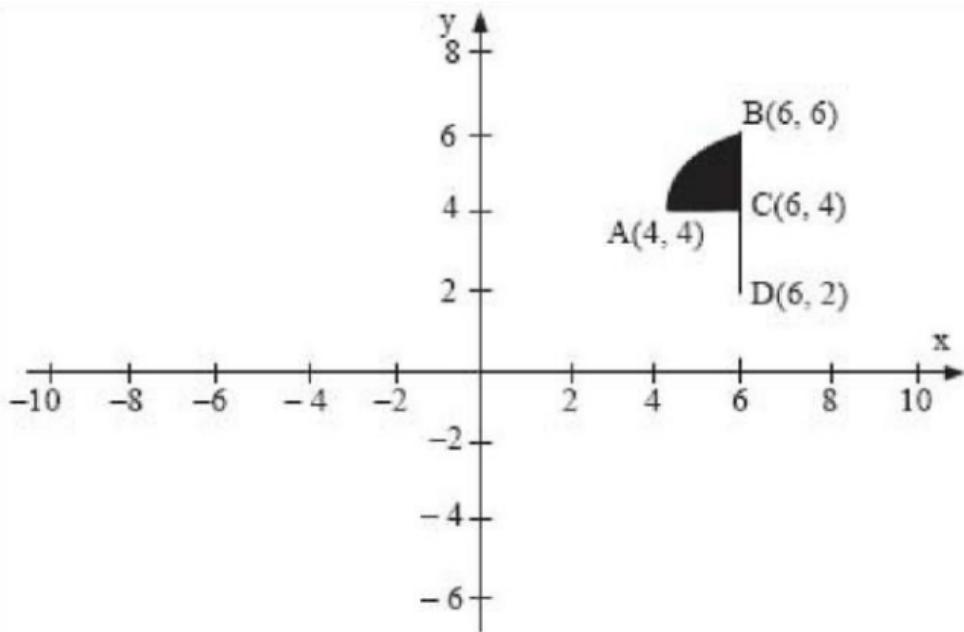
(b) final image.

14. The figure below shows a flag made from a quadrant of a circle ABC and fixed to a pole BCD. The diagram is reflected in the line $y = 8$, followed by a reflection in the line $y = x$.

Draw the image of the flag under the:

(a) first reflection.

(b) final reflection.

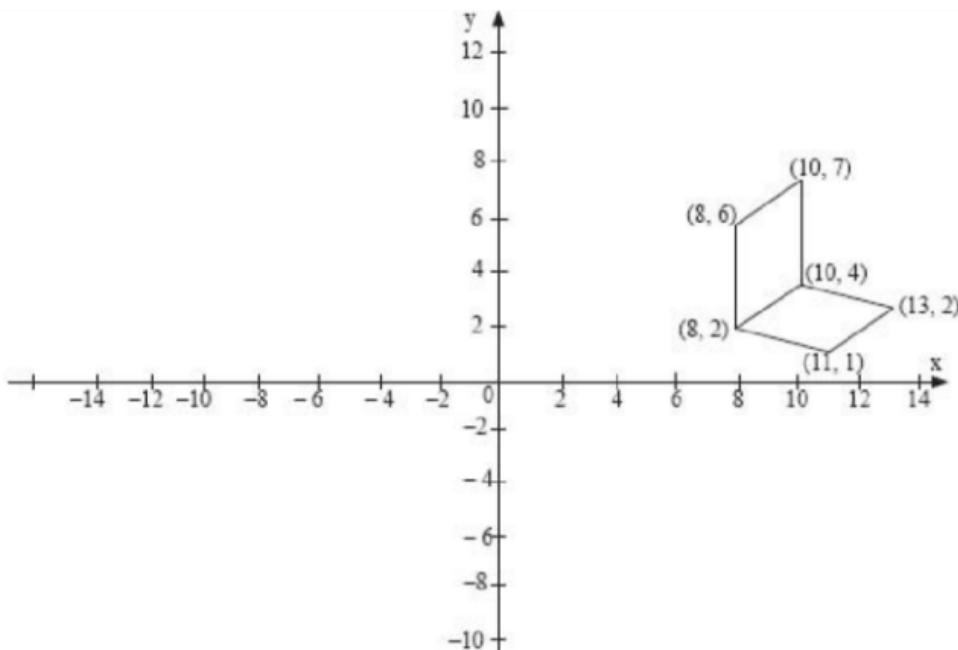


15. In the figure below the object is reflected in the line $x = 6$ followed by a reflection in the line $y = -x$.

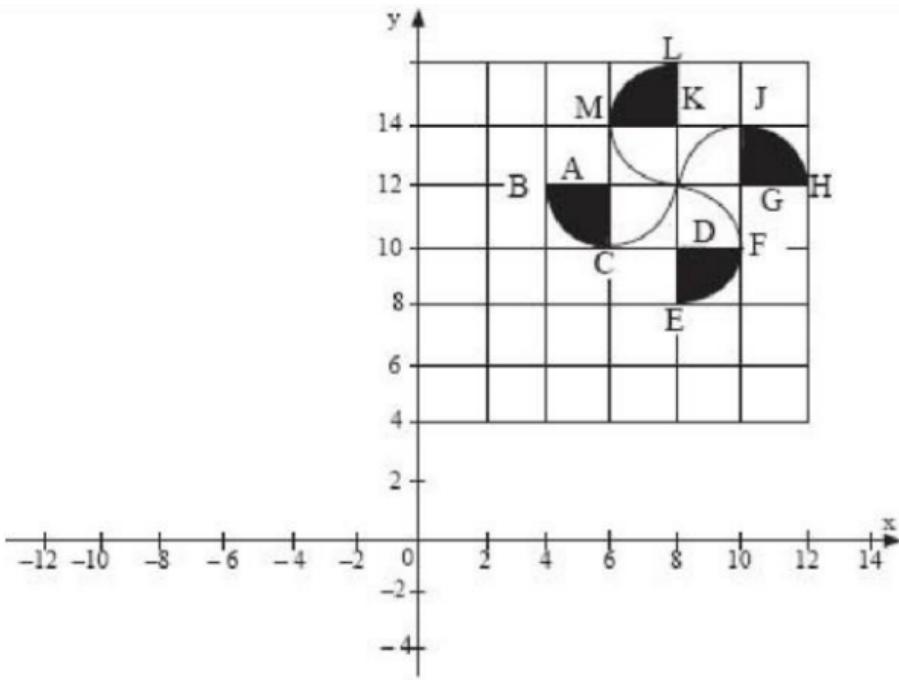
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Find the co-ordinates of the image under the:

- (a) first reflection.
- (b) two reflections.

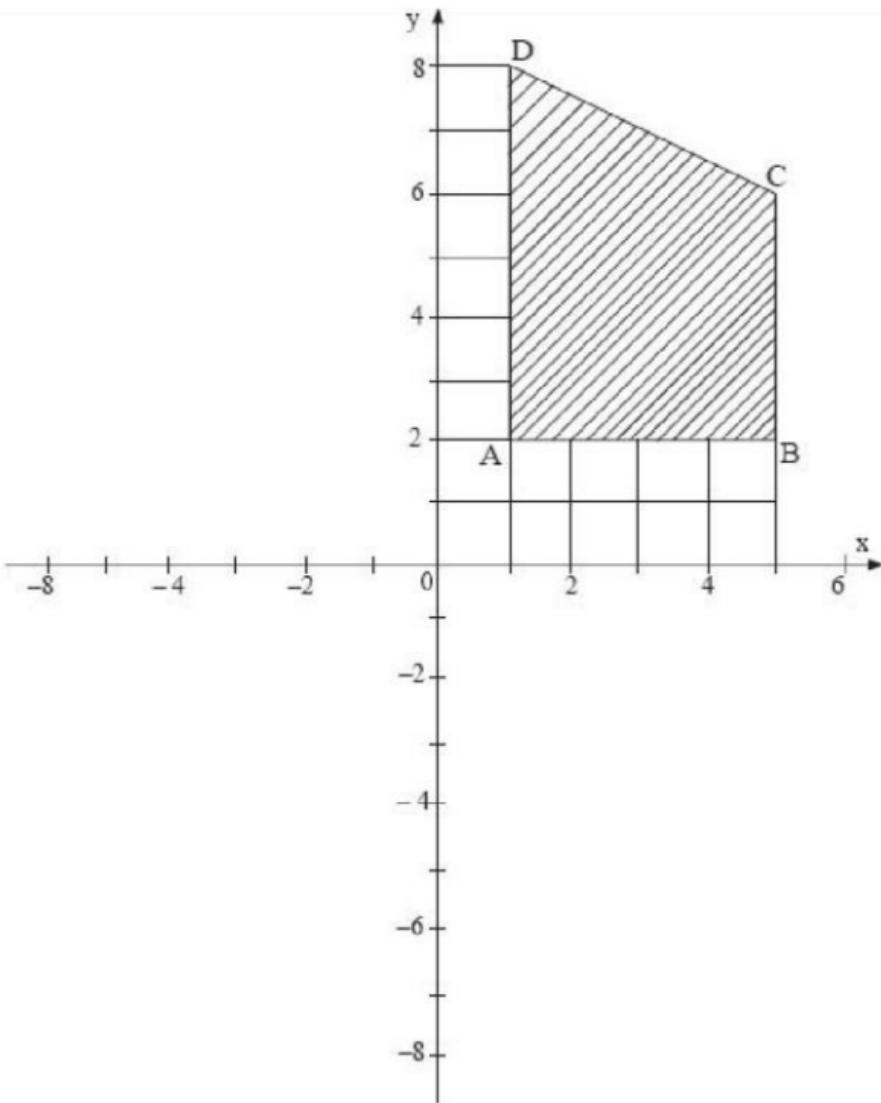


16. Draw the pattern shown in the figure below on a graph paper and show its image after a reflection in the line $y = x$:

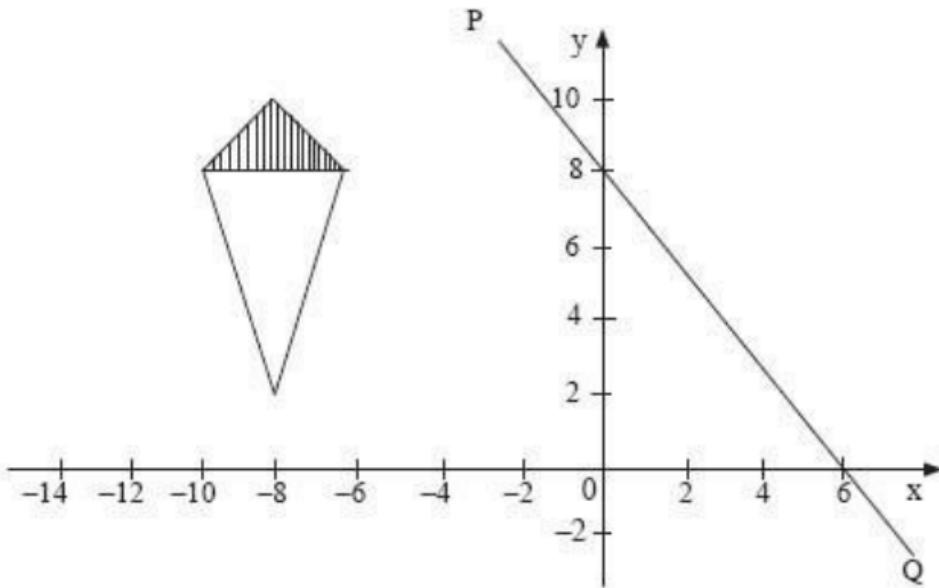


17. The object shown below is reflected in the line $x = -4$, followed by a reflection in the line $y = x$. Find the position of the image under the:

- (a) first reflection.
- (b) two reflections.

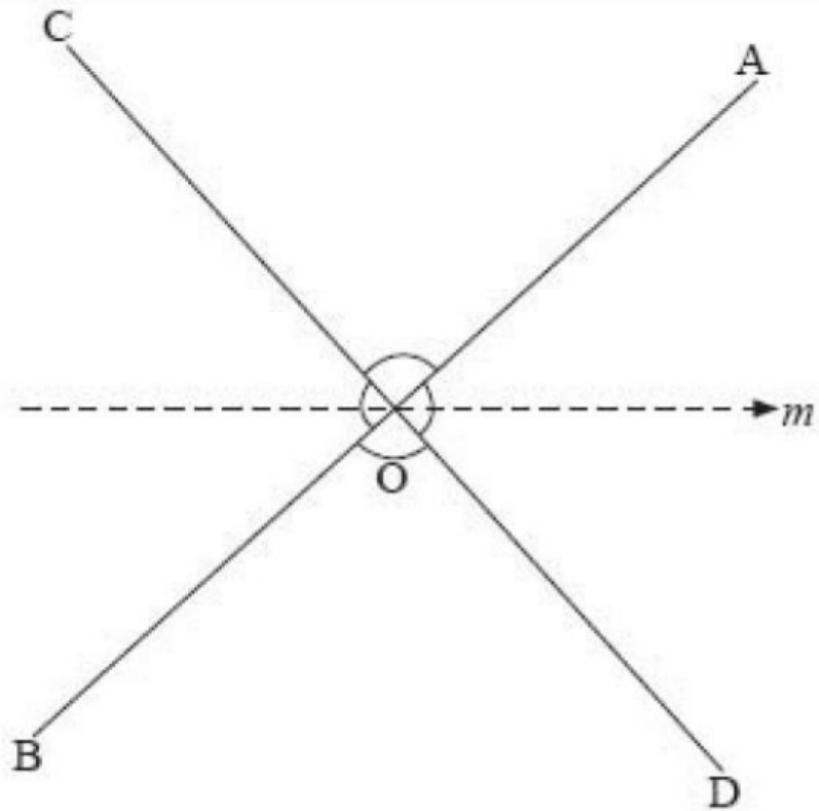


18. Draw the image of the object shown in the figure below under a reflection in the line PQ followed by a reflection in the line $y = x$:



5.3: Some Geometrical Deductions Using Reflection

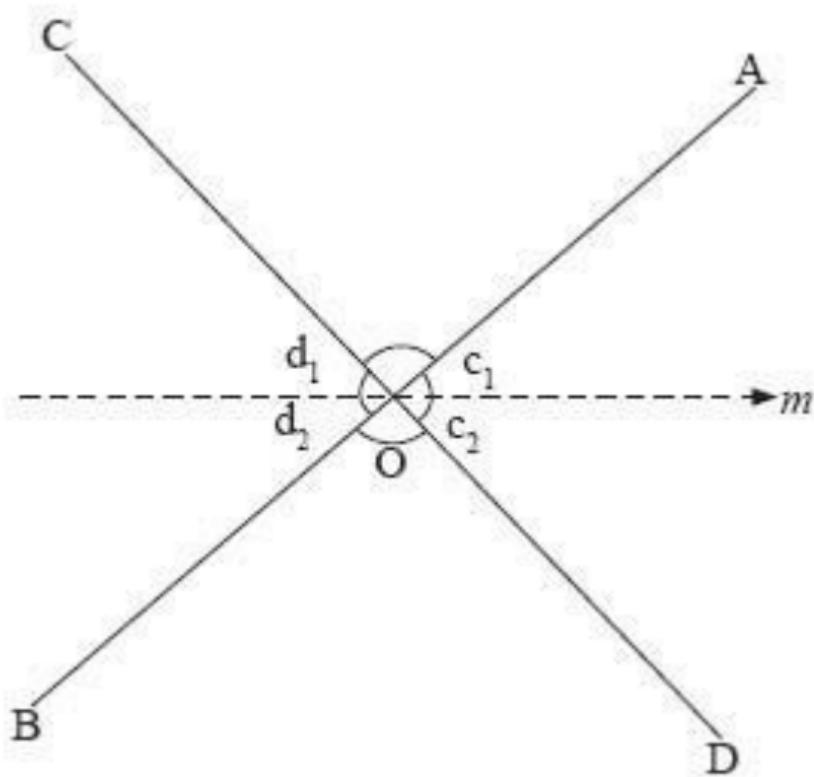
- (a) Vertically opposite angles are equal
- The lines AB and CD intersect at O as shown in figure 5.5:



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Fig. 5.5

The broken line showing the mirror line m bisects angle AOD and angle COB as shown in figure 5.6. Show that angle BOC is equal to angle AOD and angle BOD is equal to angle AOC .



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Fig. 5.6

Let $\angle BOC = d_1 + d_2$ and

$\angle AOD = c_1 + c_2$ (reflection in
m)

$c_1 + \angle COA + d_1 = 180^\circ$ (angles
on a straight line). $c_2 + \angle BOD +$
 $d_2 = 180^\circ$ (angles on a straight
line).

But $c_1 = c_2$ and $d_1 = d_2$

Therefore, $\angle BOD = \angle COA$

Therefore, vertically opposite angles are equal.

$$\angle COA + c_1 + c_2 = d_2 + d_1 + \angle COA$$

$$\therefore c_1 + c_2 = d_1 + d_2, \text{ but } c_1 + c_2 = \angle AOD \text{ and } d_1 + d_2 = \angle COB$$

Therefore, $\angle AOD = \angle COB$

Therefore, if two straight lines

intersect, the vertically opposite angles are equal.

- (b) Base angles of an isosceles triangle are equal

Figure 5.7 shows a right-angled triangle ABC and its image \overline{ABC} under a reflection in AB.

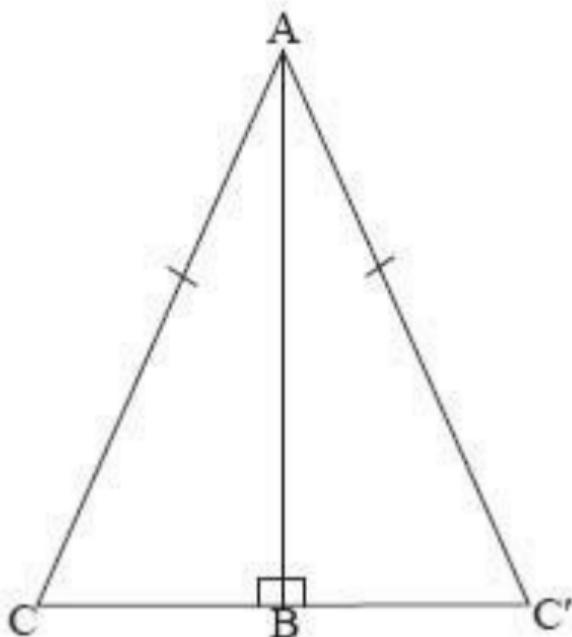


Fig. 5.7

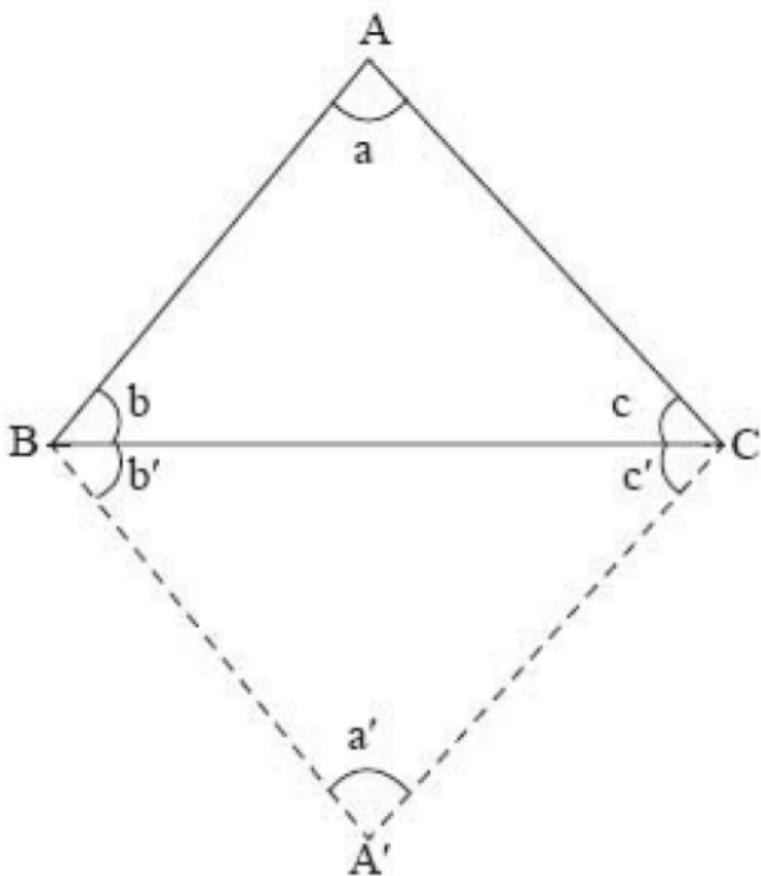
By reflection, $AC = AC'$, $BC =$

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BC' and $\angle C = \angle C'$

Triangle CAC' is therefore isosceles. Hence, by reflection, the base angles of an isosceles triangle are equal.

(c) Sum of interior angles of a triangle is 180°



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Fig. 5.8

Figure 5.8 shows two triangles, ABC and BCA'. BC is the mirror line. By reflection, $\angle a = \angle a'$, $\angle b = \angle b'$ and $\angle c = \angle c'$.
But $\angle a + \angle b + \angle c + \angle a' + \angle b'$ and $\angle c' = 360^\circ$ (sum of interior angles of a quadrilateral)

Thus, $2a + 2b + 2c = 360^\circ$

$$2(a + b + c) = 360^\circ$$

Therefore, $a + b + c = 180^\circ$

Angle sum of a triangle is 180°

(d) The mirror line as a perpendicular bisector

Consider the right angled triangle LMN.

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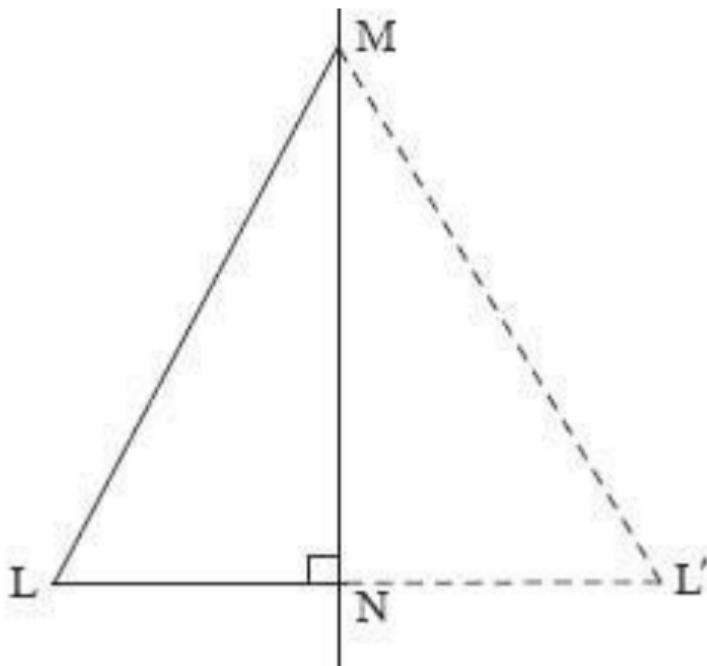


Fig. 5.9

If MN is a mirror line, then L' is the image of L under reflection. $LN = NL'$ and $LM = L'M$. Therefore MN bisects line LL' .

Exercise 5.3

1. Line $y = 2$ intersects line $x = 0$. Find the equations of two possible mirror lines under which the lines will be images of each other.
2. Find the equation of the bisector of the line joining each of the following pairs of points:
 - (a) A(2, 8) and B(2, 10)

- (b) C(2, 8) and D(-2, 8)
 - (c) E(-2, -3) and F(-4, -3)
 - (d) G(2, -4) and H(3, -4)
 - (e) I(5, 0) and J(0, 5)
3. Draw triangle ABC, A(-2, -3), B(-4, -4), and C(-4, -2).
- (a) Name the type of triangle formed.
 - (b) Write the equation of the mirror line if B and C are images of each other.

4. Four of the vertices of an irregular octagon, which is symmetrical about line $y = x$, are $A(1, -\frac{1}{2})$, $B(4, -3)$, $C(4, 0)$ and $D(3, 0)$. Find the coordinates of the remaining vertices.

The octagon is reflected in the line $y = 0$. Find the equation of line of symmetry of the image.

5. Plot points $A(0, 5)$, $B(3, 9)$, $C(3, 4)$ and $D(0, 0)$.

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- (a) Name the quadrilateral ABCD.
- (b) Write down the equations of the two lines of symmetry.
6. Plot points A(4, 1) and B(2, 5).
- (a) Calculate the distance AB.
- (b) Reflect line AB along the line $y = -x$.
7. Plot points P(-3, 7), Q(-5, 3), R(-3, 1) and S(-1, 3).
- (a) Name the quadrilateral PQRS.

- (b) Find the equation of the line of symmetry.
- (c) Reflect figure PQRS in the line $y = x$ and find the co-ordinates of its image.
- (d) Find the equation of the line of symmetry of the image.
8. Given point L(2, 1) and L'(6, 1), the image of L under a reflection, find the equation of the mirror line.

9. Draw triangle UVX with vertices $U(1, 4)$, $V(5, 4)$ and $X(3, 1)$. A point T is such that TX is the angle bisector of $\angle UXV$. Locate one possible position of T and find the equation of TX .

5.4: Congruence

In figure 5.10, ΔABC is reflected in the mirror line M to obtain $\Delta A'B'C'$.

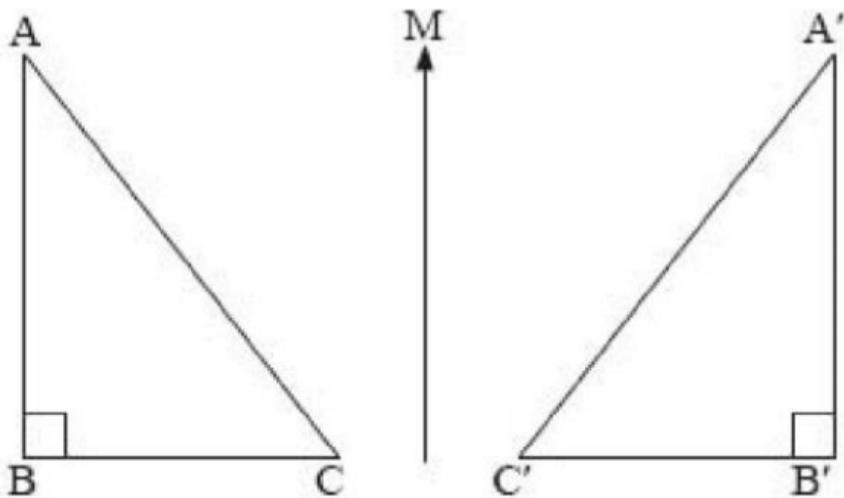


Fig. 5.10

Since $\Delta A'B'C'$ is a reflection of ΔABC , the two triangles:

- (a) have the same shape.

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(b) have the same size.

In general, figures which have the same shape and same size are said to be congruent.

The symbol for congruency is \equiv . Thus,
 $\Delta ABC \equiv \Delta A'B'C'$

Figure 5.11 shows two successive reflections. ΔABC is reflected in line M_1 to get $\Delta A'B'C'$. $\Delta A'B'C'$ is reflected in M_2 to get $\Delta A''B''C''$. A tracing of ΔABC fits exactly on $\Delta A''B''C''$ directly

(without lateral inversion). ΔABC and $\Delta A''B''C''$ are said to be directly congruent.

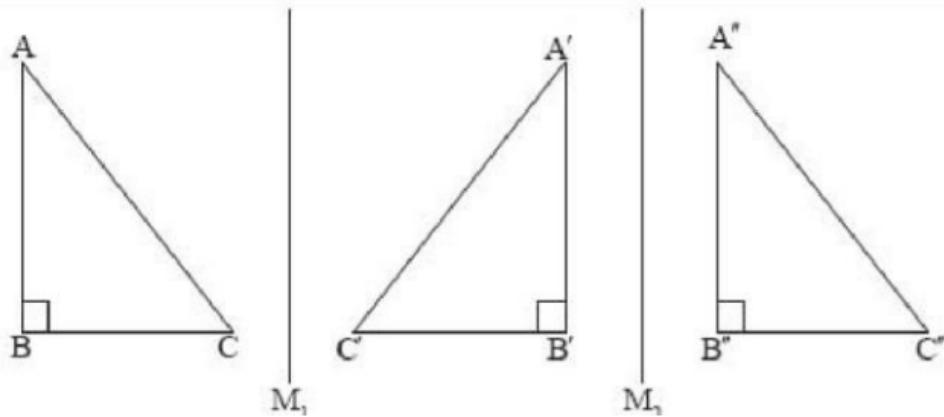


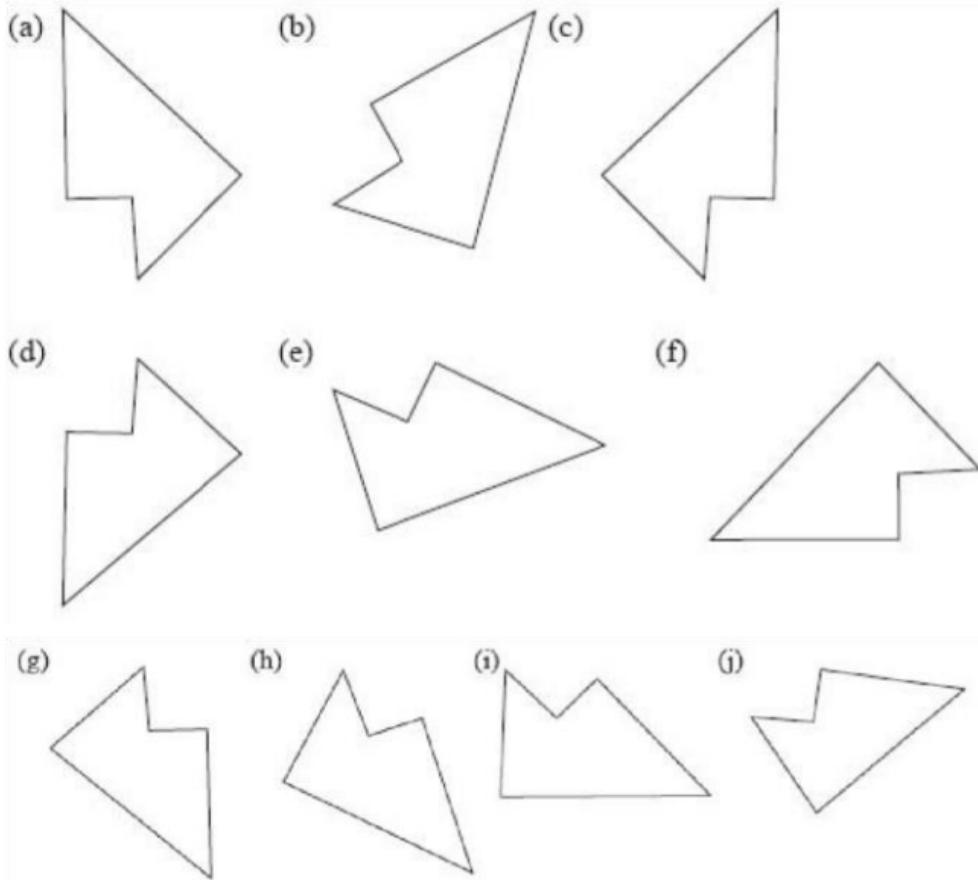
Fig. 5.11

But a tracing of ΔABC cannot fit directly on $\Delta A'B'C'$ unless it is turned over. This congruence is called opposite congruence. Similarly, $\Delta A'B'C'$ and $\Delta A''B''C''$ are oppositely congruent.

Therefore: (i) triangles ABC and $A''B''C''$ are directly congruent.

(ii) triangles ABC and $A'B'C$ are oppositely congruent.

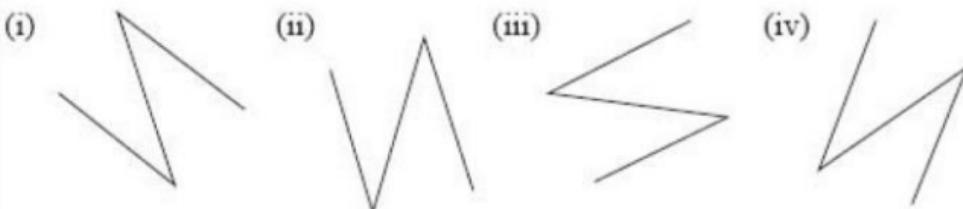
Which of the following shapes in figure 5.12 are directly congruent? Which ones are oppositely congruent?



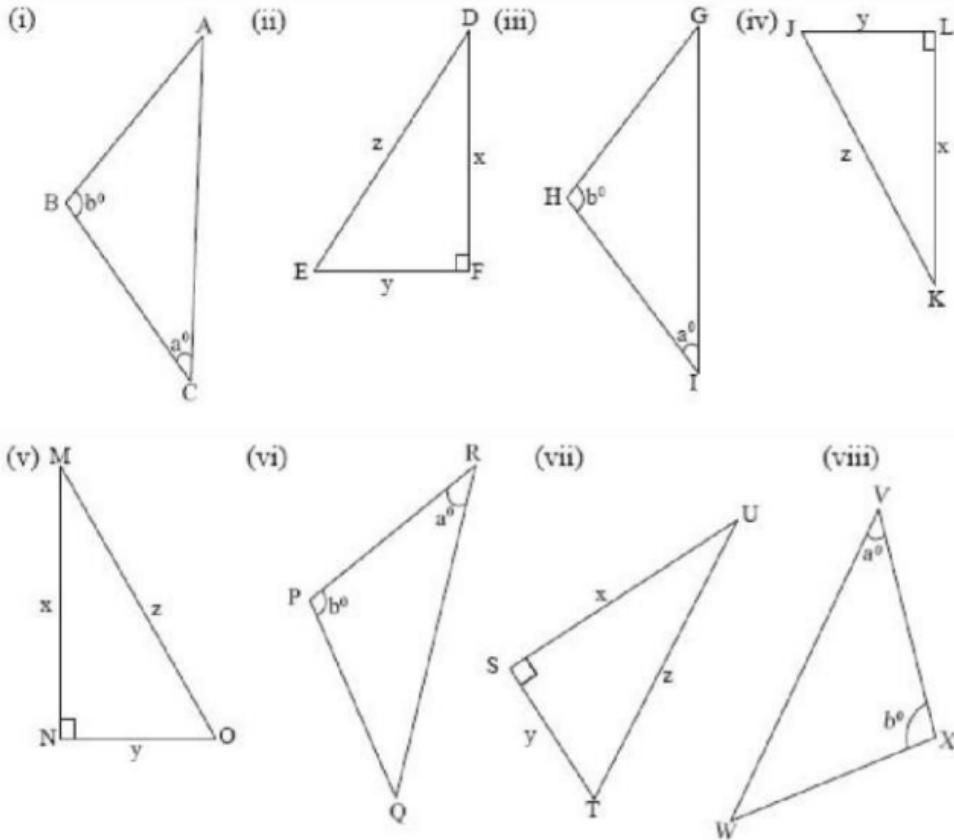
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Fig. 5.12

Exercise 5.4

1. Which of the following figures are:
 - (a) directly congruent?
 - (b) oppositely congruent?



2. In the figure below, write all pairs of triangles that are:
 - (a) directly congruent.
 - (b) oppositely congruent.



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directly and oppositely congruent to its image after a reflection?

5.5: Congruent Triangles

Triangles which are congruent are named in such a way that the letters are written in the correct order so as to indicate the corresponding sides or the corresponding angles.

If $\triangle ABC$ is congruent to $\triangle PQR$, AB corresponds to PQ, BC to QR and CA to RP. Similarly, if $\angle ABC$ corresponds to

$\angle PQR$, $\angle BCA$ corresponds to $\angle QRP$ and $\angle CAB$ corresponds to $\angle RPQ$.

Two or more triangles are congruent if they satisfy any of the following conditions:

- (i) Three sides of one triangle are equal to the three corresponding sides of the other triangle (SSS).
- (ii) Two sides and the included angle of one triangle are equal to the

corresponding two sides and the included angle of the other triangle (SAS).

- (iii) Two angles and a side of one triangle are equal to the corresponding two angles and a side of the other triangle (AAS or ASA).
- (iv) A right angle, a hypotenuse and another side of one triangle are equal to the corresponding right-angle, a hypotenuse and another side of the other triangle (RHS).

The following examples illustrate the four conditions:

(i)

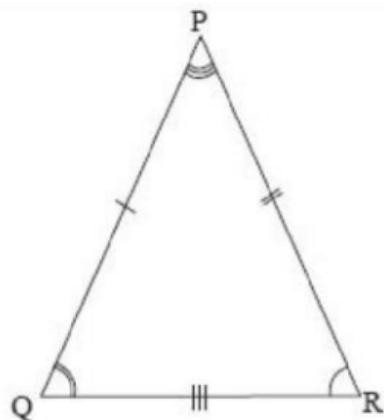
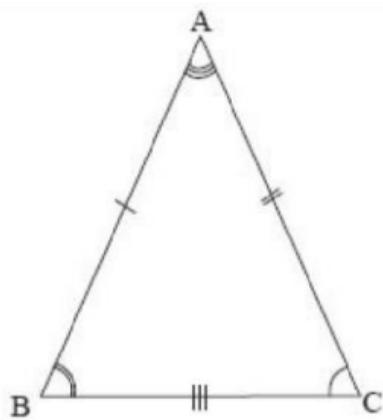


Fig. 5.13

In figure 5.13, $\triangle ABC$ and $\triangle PQR$ are congruent. All the three corresponding

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sides are equal (SSS). $AB = PQ$, $BC = QR$ and $CA = RP$.

The corresponding angles are equal since the corresponding sides are equal, e.g., $\angle ABC = \angle PQR$. Name other angles that are equal.

(ii)

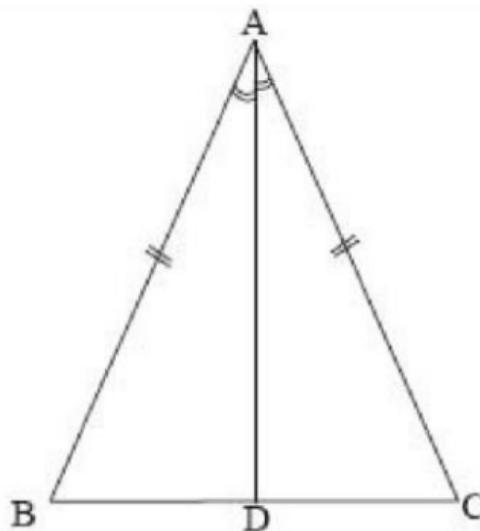


Fig. 5.14

Given the isosceles triangle ABC (figure 5.14), AD is a bisector of $\angle BAC$ and ΔABD is congruent to ΔACD , because line AD is common. $\angle BAD = \angle DAC$ (given) and line AB = AC (given). Two sides and an included angle are equal (SAS).

(iii)

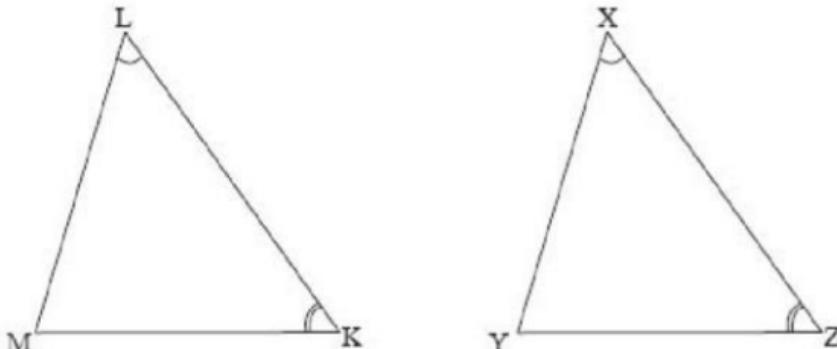


Fig. 5.15

Figure 5.15 shows two triangles in which $LK = XZ$, $\angle MLK = \angle YXZ$ and $\angle LKM = \angle XZY$. This implies that $\angle LMK = \angle XYZ$, $LM = XY$ and $MK = YZ$. The two triangles are, therefore, congruent (ASA).

At times, the given sides may not be between the given angles as shown in figure 5.16. In the figure, $\angle FEG = \angle SRT$, $\angle GFE = \angle TSR$ and $FG = ST$.

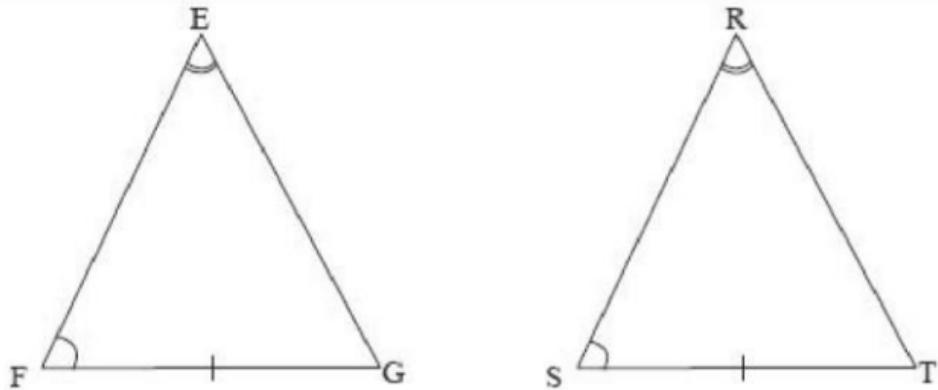


Fig. 5.16

Since two pairs of angles are equal, the third pair must also be equal. The two triangles are therefore congruent, provided the equal sides correspond (AAS or SAA).

(iv)

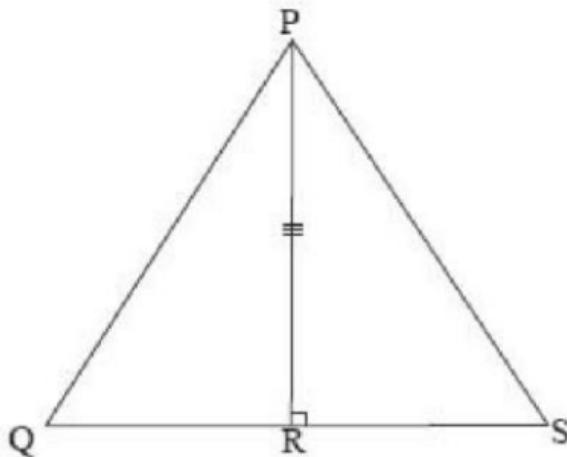


Fig. 5.17

Given the isosceles triangle PQS in figure 5.17, $PQ = PS$ and line PR is perpendicular to the line QS. Triangles PQR and PSR are congruent. This is because:

- PR is a common line.
- $\angle PRQ = \angle PRS = 90^\circ$
- $PQ = PS$ (given).

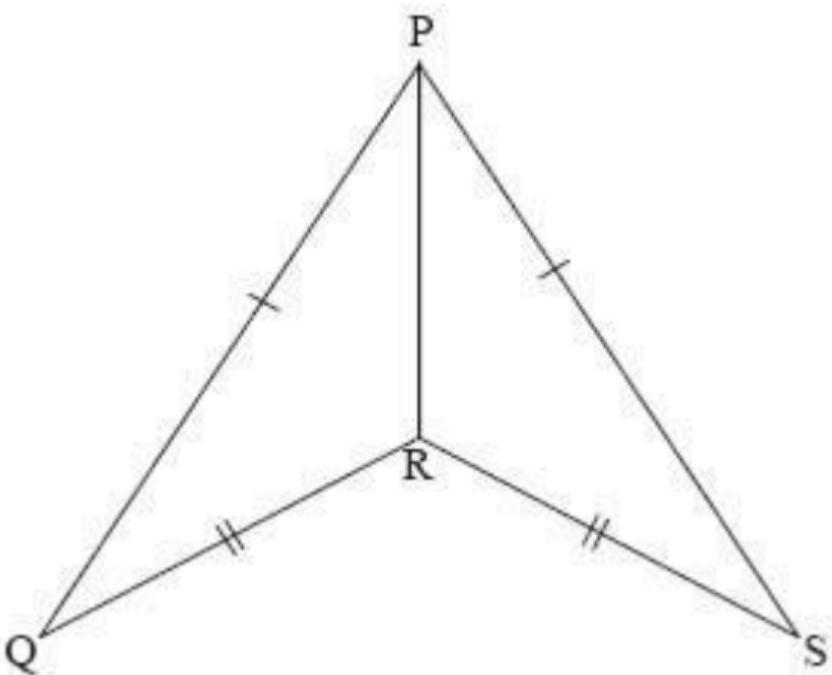
Therefore, there is a right angle, a hypotenuse and another side equal

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(RHS). Note that in this case, the SAS test of congruence is not necessary.

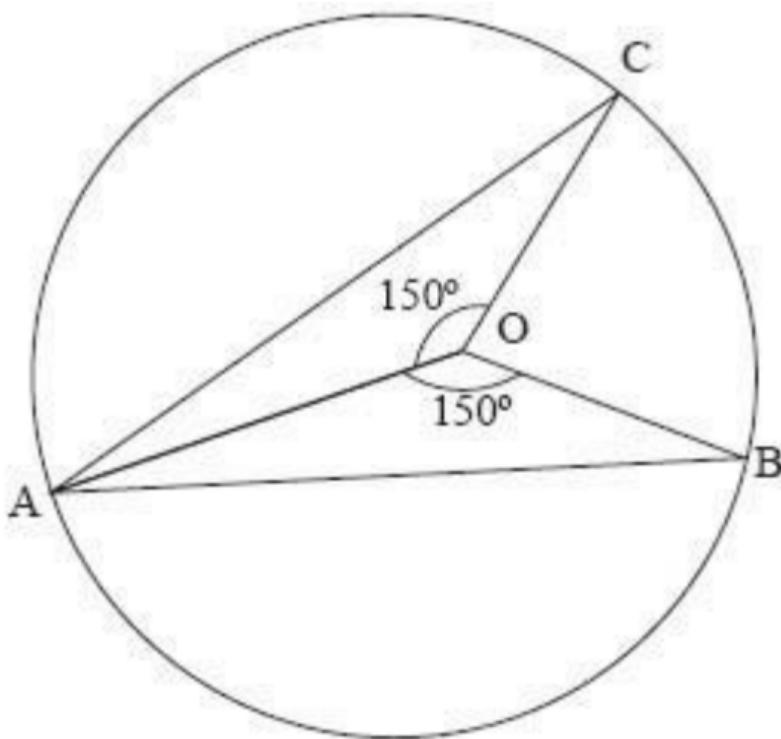
Exercise 5.5

1. In the figure below, $PQ = PS$ and $QR = RS$. Show that ΔPQR are congruent.

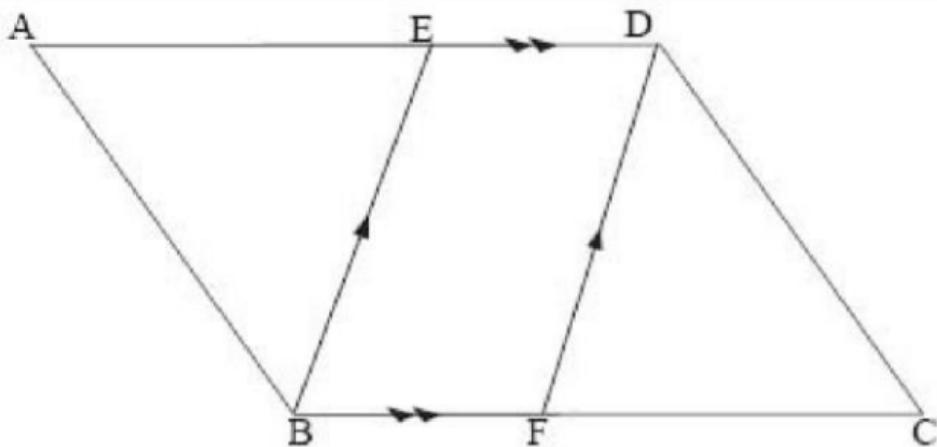


2. In the figure below, points A, B and C are on the circumference of the

circle with centre O. Show that $\triangle AOB$ and $\triangle AOC$ are congruent.

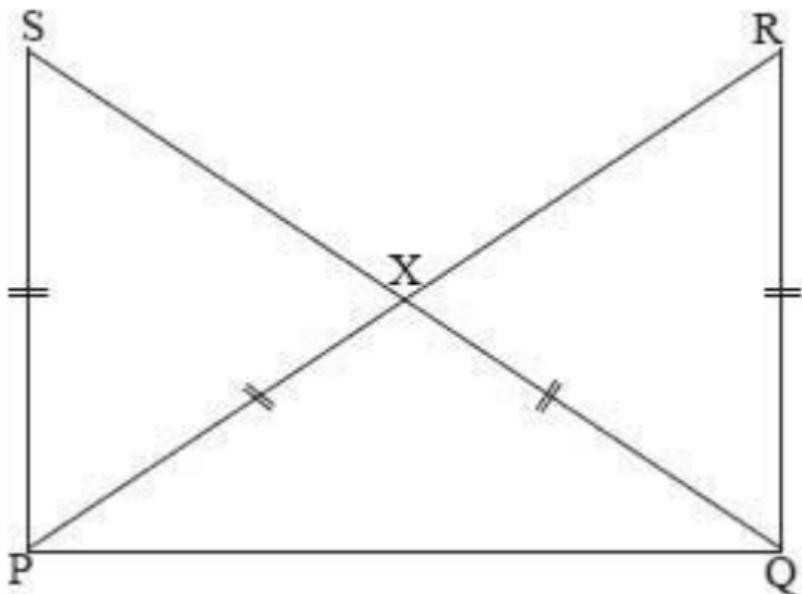


3. In the figure below, ABCD is a parallelogram. Line BE is parallel to line FD.
- Show that ΔABE and ΔCDF are congruent.



4. Two cords AB and AC of a circle are equal in length and the bisector of angle BAC meets the arc BC at D . Show that:
- $BD = DC$
 - AD is perpendicular to BC .

5. In the figure below, $PS = QR = PX = XQ$. Show that ΔPQS and ΔPQR are congruent.



6. In a triangle ABC , $AB = AC$. H and K are points on AB and AC respectively, such that HK is parallel to BC . Show that $\triangle ABK$ and $\triangle ACH$ are congruent.

7. Two lines CB and ED meet at A when produced. A perpendicular line to AC through B meets a perpendicular line to AE through D at F. If $DF = BF$, show that $\Delta BAF = \Delta DAF$.
8. In ΔXYZ , Q is a point on side YZ. The triangle is reflected in XZ to form the image $XY'Z$ with Q' the image of Q. Show that $\Delta Q'YZ$ is congruent to ΔQYZ .

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Chapter 6

ROTATION

6.1: Introduction

In figure 6.1, triangle $A'B'C'$ is the image of triangle ABC under a transformation R .

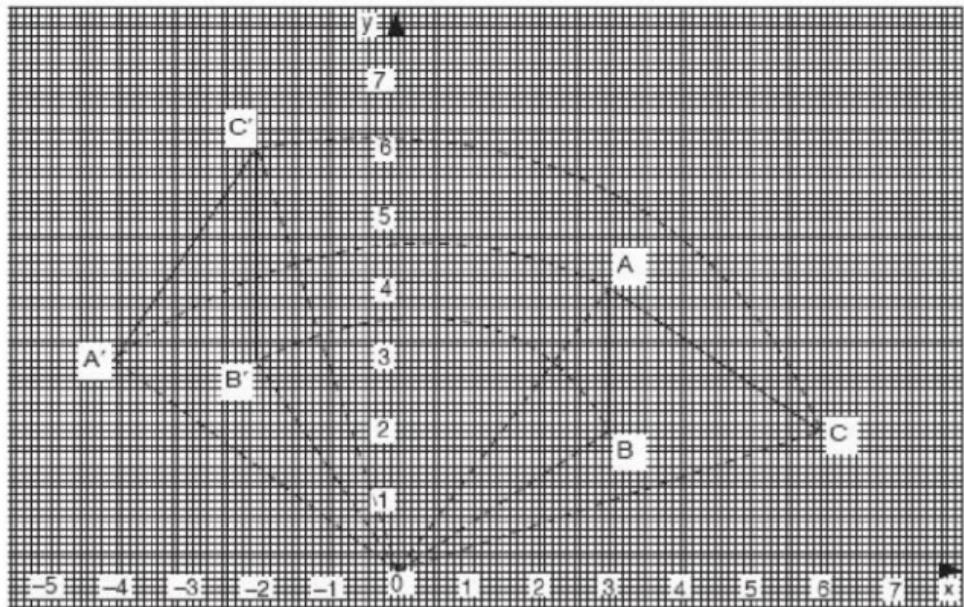


Fig. 6.1

To obtain A' , the image of A , rotate OA about O through an angle of 90°

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anticlockwise. Similarly, to obtain B' , the image of B , rotate OB about O through an angle of 90° anticlockwise. The image of the point C is obtained in a similar way. Such a transformation is called rotation. In this example, point O is called the centre of rotation and angle 90° is called the angle of rotation.

Note:

A rotation in the anticlockwise direction is taken to be positive, whereas a rotation in the clockwise direction is taken to be negative. For example, a

rotation of 90° anticlockwise is $+90^\circ$ whereas the rotation of 90° clockwise is taken to be negative (-90°).

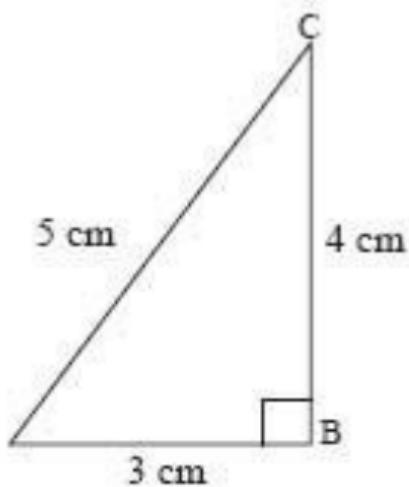
In general, for a rotation to be completely defined, the centre and the angle of rotation must be stated.

Example 1

Figure 6.2 shows a triangle ABC in which $AB = 3 \text{ cm}$, $BC = 4 \text{ cm}$ and $AC = 5 \text{ cm}$. The triangle is rotated through $+60^\circ$ about a

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point X. Copy the figure and draw $\Delta A'B'C'$, the image of ΔABC .



• X

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Fig. 6.2
Solution

To get $\Delta A'B'C'$, proceed as follows:

- (i) Draw angle $AXL = 60^\circ$ as in figure 6.3. To obtain A' on XL , measure $XA' = XA$.

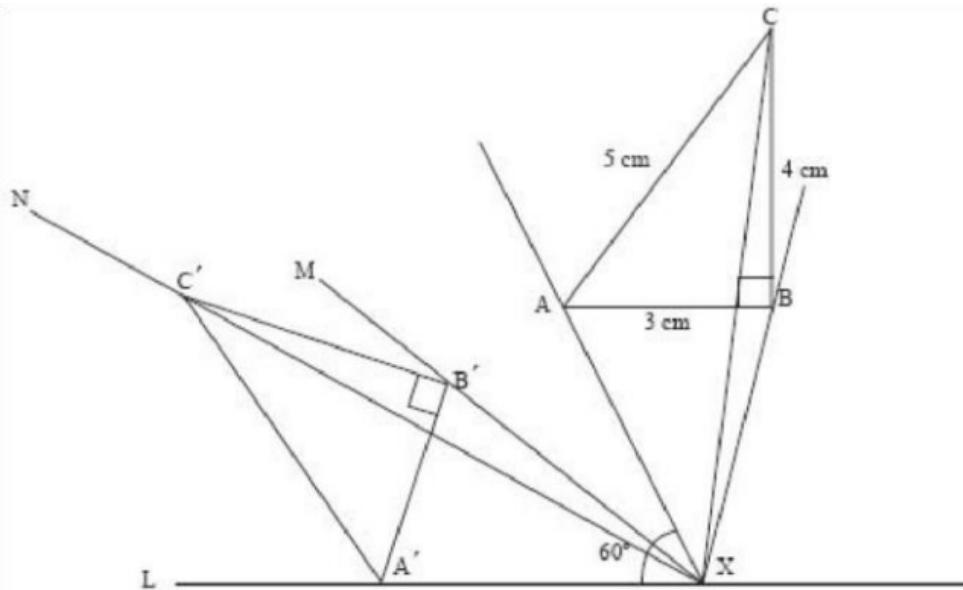


Fig. 6.3

- (ii) Draw angle $BXM = 60^\circ$. To obtain B' on XM , measure $XB' = XB$.

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- (iii) Point C' is similarly obtained by measuring $XC' = XC$, see the figure.
- (iv) Join $A'B'$, $B'C'$ and $A'C'$ to obtain $\Delta A'B'C'$.

With the same centre X, find the images of the ΔABC in figure 6.3 after a rotation of:

(i) -60°

(b) $+45^\circ$

(c) -120°

(d) 360°

From figure 6.1 to 6.3, note that:

- (i) each point and its image are equidistant from the centre of rotation.
- (ii) the object figure and its image are directly congruent.
- (iii) the centre and the angle of rotation completely defines the rotation.

We can use the letter R to denote a rotation about a given point through a given angle. For example, if R represents

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a rotation, centre origin through $+90^\circ$, then the image of $P(5, -3)$ under R [$R(P)$] is $(3, 5)$.

6.2: Centre and Angle of Rotation

In figure 6.4, the square $A'B'C'D'$ is the image of the square $ABCD$ after a rotation.

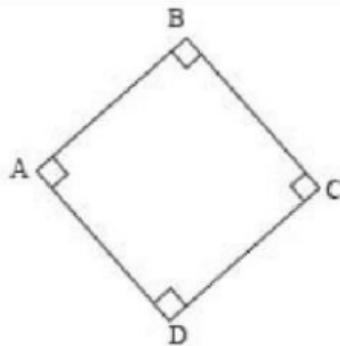
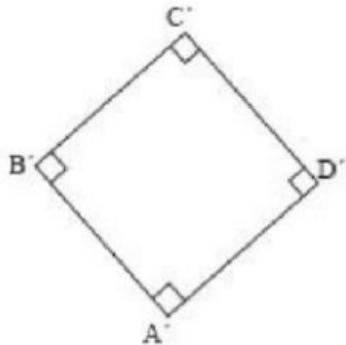


Fig. 6.4

To determine the centre and angle of rotation:

- (a) Trace the figure. Join A to A' and construct a perpendicular bisector to AA'.
- (b) Join D to D' and construct the perpendicular bisector to DD'. The point of intersection of the two perpendicular bisectors is the

centre of rotation. Note that any two of the perpendicular bisectors to AA', BB', CC' and DD' are sufficient to locate the centre of rotation.

- (c) Join O to A and A'. Measure $\angle AOA'$. In this case, $\angle AOA' = 90^\circ$. This is the angle of rotation.

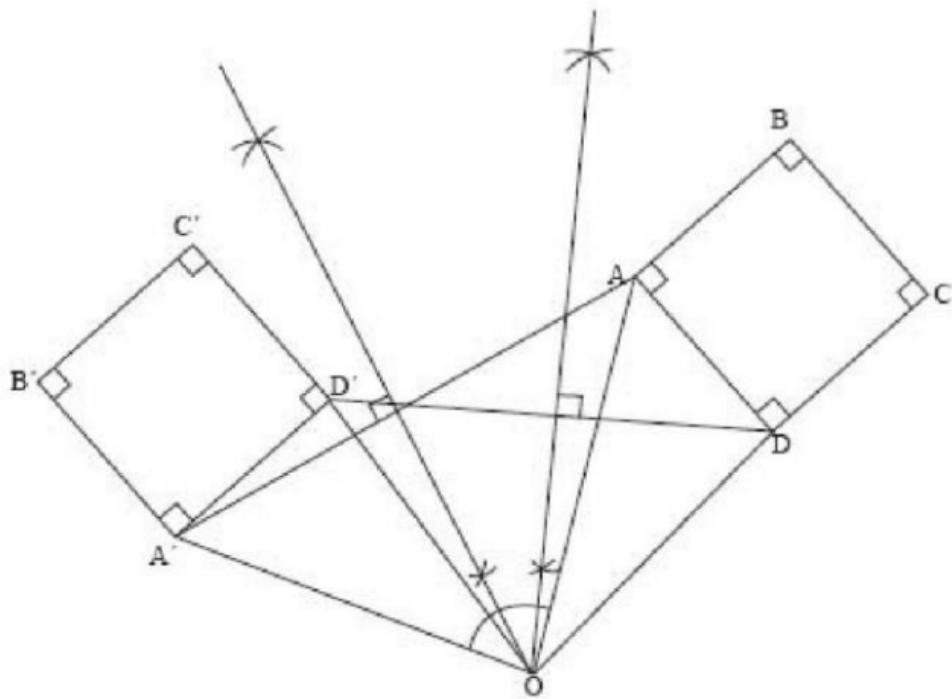


Fig. 6.5

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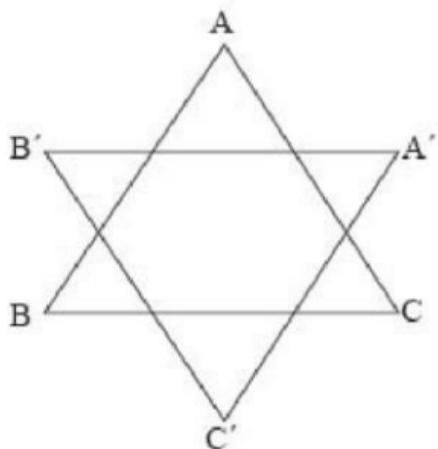
Note

$$\angle AOA' = \angle BOB' = \angle COC' = \angle DOD'$$

A'B'C'D' is the image of ABCD under a rotation of 90° in anticlockwise direction ($+90^\circ$).

Each pair of diagrams in figure 6.6 represents an object and its image after a rotation. Copy each and find the centre and angle of rotation.

(a)



(b)

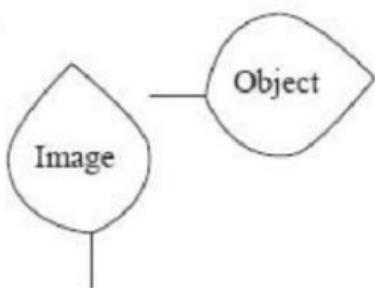
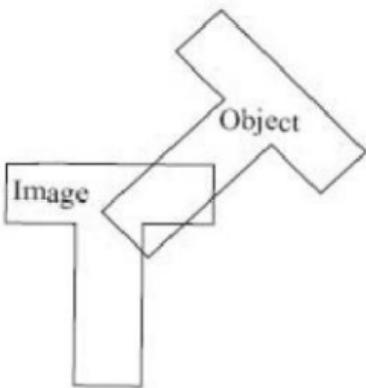


Fig 6.6 (a) and b

(c)



(d)

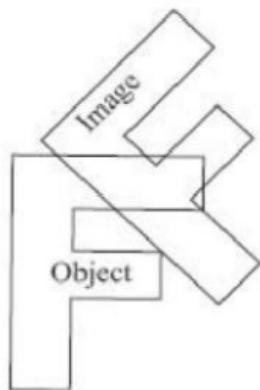


Fig. 6.6 c and d

6.3: Rotation in the Cartesian Plane

Rotation about the Origin through
 $+90^\circ$, $+180^\circ$, $+360^\circ$

Figure 6.7 shows a point P and its images after rotations about $(0, 0)$ with different angles of rotation.

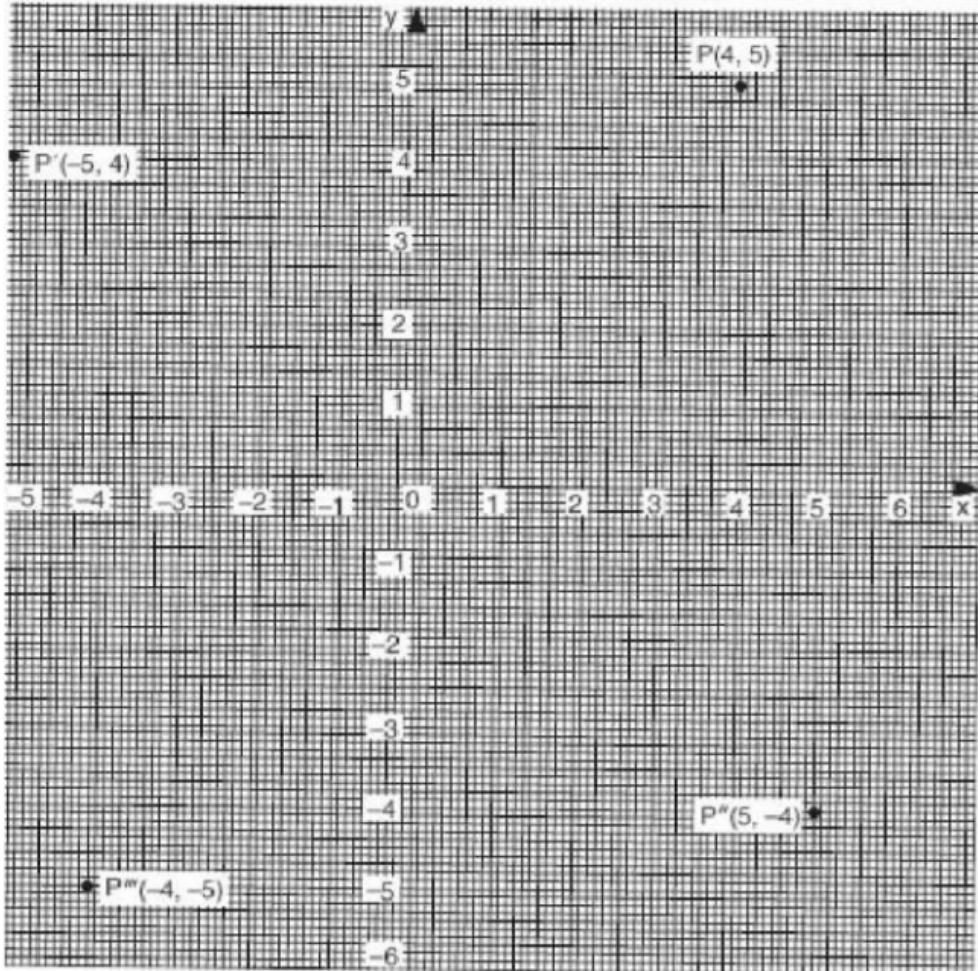


Fig 6 7

Table 6.1 below shows the images of P after a rotation about the origin through the indicated angles.

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Table 1

<i>Object point</i>	P(4, 5)	P(4, 5)	P(4, 5)	P(4, 5)	P(4, 5)	P(4, 5)	P(4, 5)
<i>Angle of rotation</i>	+90°	-90°	+180°	-180°	+360°	-360°	0°
<i>Image point</i>	P'(-5, 4)	P''(5, -4)	P'''(-4, -5)	P''''(-4, -5)	P(4, 5)	P(4, 5)	P(4, 5)

Note:

- (i) A rotation through $\pm 180^\circ$ about (0, 0) gives the same image.
- (ii) A rotation through $\pm 360^\circ$ or 0° about (0, 0) does not change the position of the object.

Repeat this for Q(3, 2), R(-1, 4), S(-2, 6), T(3, -1) and (a, b). The following table

|

shows the image of the point (a, b) through the indicated angle of rotation about the origin.

Table 6.2

Angle of rotation	0°	$+90^\circ$	-90°	$+180^\circ$	360°	-360°
Image of (a, b)	(a, b)	$(-b, a)$	$(b, -a)$	$(-a, -b)$	(a, b)	(a, b)

Rotation of a point (a, b) through 180° , centre (x, y)

Consider the points $(2, 3)$ and $(7, 5)$. Taking $(1, 4)$ as the centre, rotate the two points through 180° .

Note:

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- (i) $(2, 3)$ is mapped onto $(0, 5)$. To obtain $(0, 5)$ without a graph, consider the point $(2, 3)$ and the centre of rotation $(1, 4)$;
- $$(0, 5) = (2 \times 1 - 2, 2 \times 4 - 3).$$
- (ii) $(7, 5)$ is mapped onto $(-5, 3)$, which is obtained as follows:

$$(-5, 3) = (2 \times 1 - 7, 2 \times 4 - 5).$$

Following the same pattern, the image of a point (a, b) rotated about $(1, 4)$ through 180° is $(2 \times 1 - a, 2 \times 4 - b)$.

In general, a point (a, b) rotated through 180° about (x, y) is mapped onto the point $(2x - a, 2y - b)$.

Write down the images of the following points under rotation through 180° about the stated centres of rotation:

- (i) $(3, -4)$ about $(2, 1)$
- (ii) $(4, 5)$ about $(3, 8)$
- (iii) $(-5, 6)$ about $(4, 4)$
- (iv) $(4, -7)$ about $(-3, 2)$
- (v) $(-2, -3)$ about $(5, -3)$
- (vi) $(-6, -8)$ about $(-2, -5)$
- (vii) $(7, 1)$ about $(-3, -4)$

(viii) $(9, 2)$ about $(-1, 3)$

(ix) $(-3, 6)$ about $(7, 0)$

(x) $(0, 5)$ about $(0, 2)$

Exercise 6.1

1. A triangle whose vertices are $A'(-1.5, -2.5)$, $B'(-1.5, -1.5)$ and $C'(-3.5, -1.5)$ is an image of the triangle whose vertices are $A(1.5, 2.5)$, $B(1.5, 1.5)$ and $C(3.5, 1.5)$ under a rotation. Find:
 - (a) the centre and the angle of rotation.

- (b) the image of points $(0, 3)$, $(2, 2)$ and $(0, 0)$ under the same rotation.
2. Describe the rotation which maps the square whose vertices are $A(5, 3)$, $B(6.5, 3)$, $C(6.5, 1.5)$ and $D(5, 1.5)$ onto the square whose vertices are $A'(8, -3)$, $B'(6.5, -3)$, $C'(6.5, -1.5)$ and $D'(8, -1.5)$. Find the image of the point $(2, 0)$ under the same rotation.

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3. Find the co-ordinates of the vertices of the image of a parallelogram whose vertices are $A(3, 5)$, $B(7, 5)$, $C(5, 0)$ and $D(1, 0)$ when rotated about the origin through:

- (a) -90°
- (b) 180° .
4. A point $P(-3, 2)$ maps onto $P'(1, -2)$ under a rotation R centre $(1, 2)$.

Find:

- (a) the angle of rotation.
- (b) $R(P')$.

5. The vertices of a trapezium are $A(-3, 2)$, $B(-2, 2)$, $C(-1.5, 1.5)$ and $D(-3.5, 1.5)$. The trapezium is rotated about $(0.5, -4)$ through an angle of -60° . Find the co-ordinates of the vertices of its image. Find the image of $(-1.5, 3.5)$ under the same rotation.
6. Give the co-ordinates of the image of each of the following points when rotated through 180° about $(3, 1)$:

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(a) $(-2, 0)$

(b) $(4, 6)$

(c) $(5, -3)$

(d) $(-6, -6)$

7. The transformation R represents a negative quarter turn about the point $(-1, 0)$. The vertices of the image of a triangle ABC under R are $A'(3, 1)$, $B'(0, 5)$ and $C'(0, 1)$. Find the vertices of ΔABC .
8. The square whose vertices are $O(0, 0)$, $P(3, 0)$, $Q(3, 3)$ and $R(0, 3)$ is

rotated through $+90^\circ$ to give an image whose vertices are $O'(3, -1)$, $P'(3, 2)$, $Q'(0, 2)$ and $R'(0, -1)$.

- (a) Find the centre of rotation.
- (b) If the square $O'P'Q'R'$ is reflected in the line $x = 0$, find the coordinates of $O''P''Q''$ and R'' .

9. Describe the rotation which maps the right-angled triangle with vertices at $L(-4, -5)$, $M(-2, -5)$ and $N(-4, -3)$ onto the triangle with

t

vertices at $L'(-6, -3)$, $M'(-6, -5)$ and $N'(-4, -3)$. Find the image of the point $(3, -4)$ under the same rotation.

6.4: Rotational Symmetry of Plane Figures

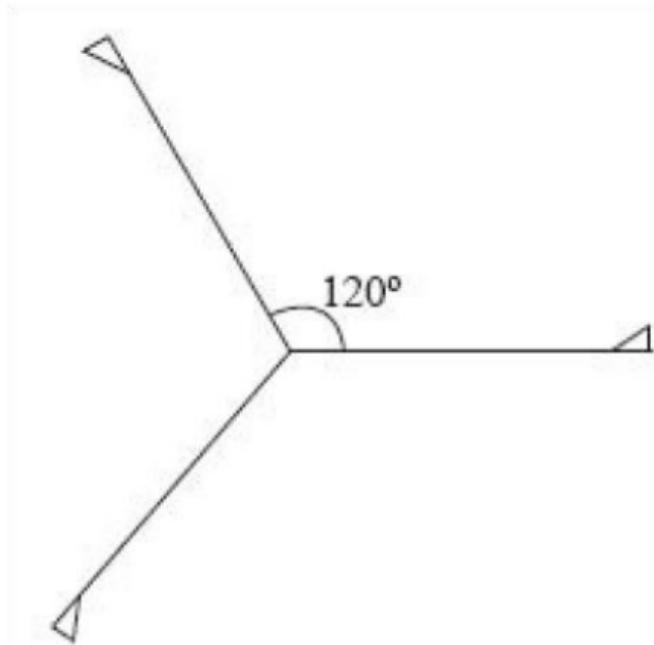


Fig. 6.8

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Trace figure 6.8 above. Make a copy of the tracing in your exercise book. Place the tracing on top of the copy and stick a pin through their centres so that the tracing can rotate. Rotate the tracing until it fits on top of the copy again. How many times can the figure fit onto itself in one complete turn?

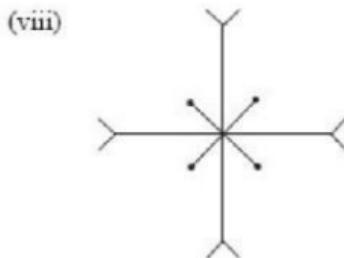
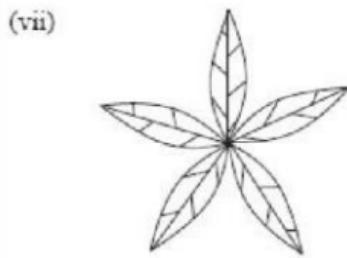
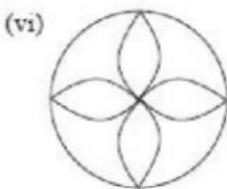
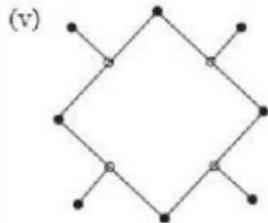
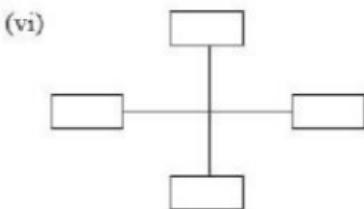
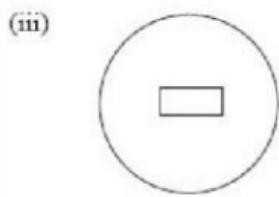
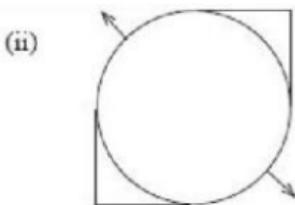
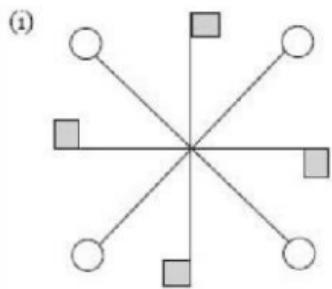
The number of times he figure fits onto itself in one complete turn is called the order of rotational symmetry. Figure 6.8 has a rotational symmetry of order 3.

Note:

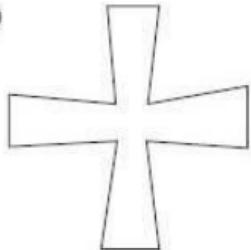
The order of rotational symmetry of a figure = $\frac{360^\circ}{\text{angle between identical parts of the figure}}$

In figure 6.8, the angle between identical parts is 120° .

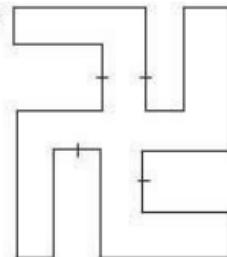
State the order of rotational symmetry of each of the following plane figures:



(ix)



(x)



(xi)

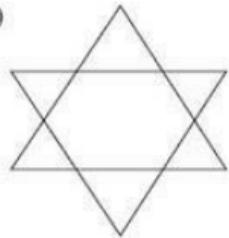


Fig. 6.9

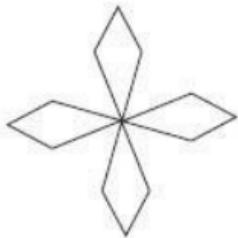
Exercise 6.2

1. State the order of rotational symmetry of each of the following letters of the alphabet: A, B, H, I, N, O, S, X.

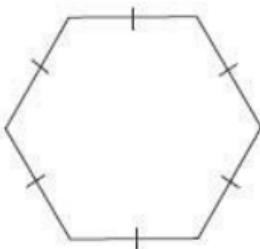
2. A figure has a rotational symmetry of order 2 about $(-2, 2.5)$. Two of its vertices are $(-2.5, 4)$ and $(1.5, 4)$. Find the other vertices and draw the figure.
3. Trace and copy each of the following figures. Fill the order of rotational symmetry for each of the figures in

table 6.3.

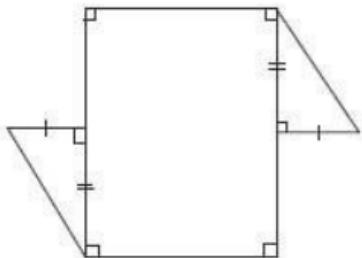
(a)



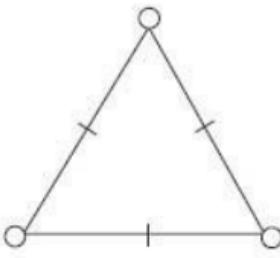
(b)



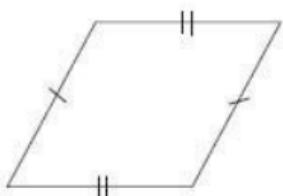
(c)



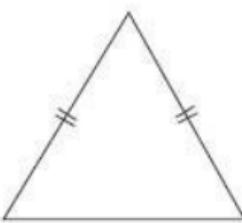
(d)



(e)



(f)



Con

140

(g)



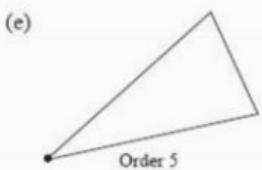
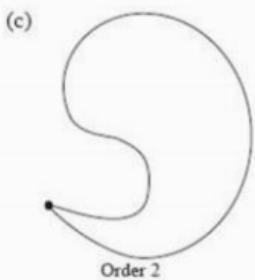
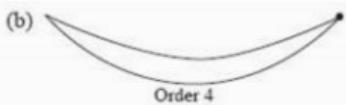
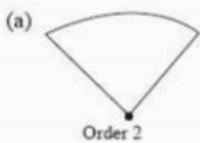
(h)



Table 6.3

<i>Diagram</i>	a	b	c	d	e	f	g	h
<i>Order of rotational symmetry</i>								

4. Each of the following shapes is part of a symmetrical figure. For each, the point and order of rotational symmetry is indicated. Copy and complete the figures:



6.5: Rotational Symmetry of Solids

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Figure 6.10 shows a model of a pyramid with a square base. A straight stiff wire AB runs vertically through the vertex V and the centre C of the base:

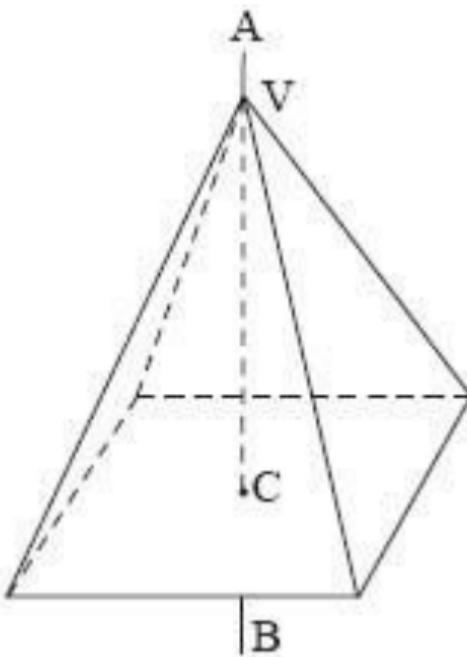


Fig. 6.10

If the pyramid is rotated about the stiff wire, it will fit onto itself four times in one complete turn. The pyramid, therefore, has a rotational symmetry of order 4 about its axis VC.

The line about which a solid has a rotational symmetry is called an axis of symmetry.

Other examples of Rotational Symmetry

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A triangular prism whose cross-section is an equilateral triangle has an axis of symmetry passing through the centre of triangular faces, as shown in figure 6.11. The prism has a rotational symmetry of order 3 about the axis.

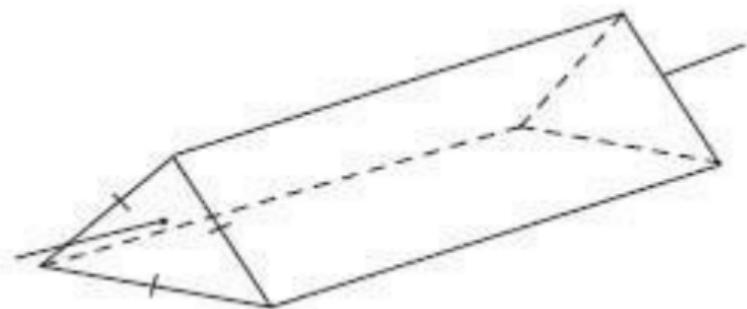


Fig. 6.11

The prism also has three other axes of symmetry each of order 2 such as the one shown in figure 6.12.

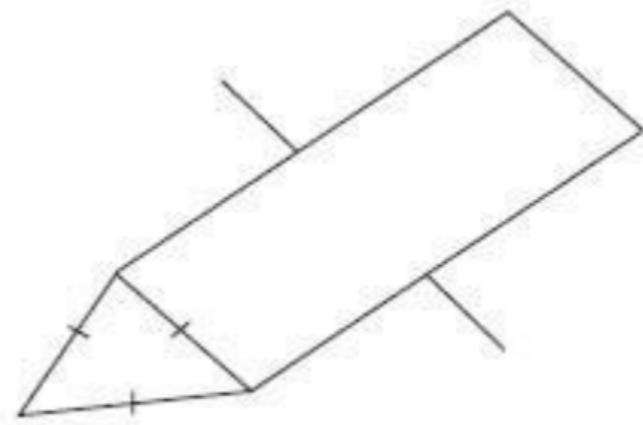


Fig. 6.12

A cone has one axis of rotational symmetry as shown in figure 6.13.

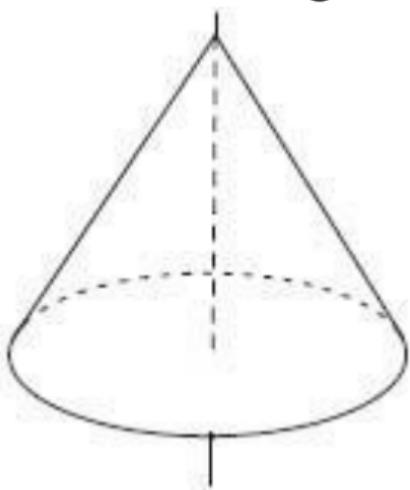


Fig. 6.13

The order of rotational symmetry is infinite.

A regular tetrahedron has an axis of symmetry passing through one vertex

and the centre of the opposite face as shown in figure 6.14(a).

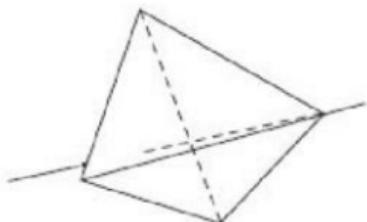
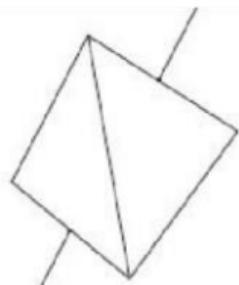


Fig. 6.14 (a)



(b)

- (i) State the number of such axes of rotational symmetry of a regular tetrahedron.

- (ii) Find the order of rotational symmetry about each of these axes.

A regular tetrahedron also has an axis of symmetry passing through the midpoints of two opposite edges as shown in figure 6.14 (b).

- (iii) State the number of such axes of symmetry.

- (iv) Find the order of rotational symmetry about each of these axes.

6.6: Rotation and Congruence

In figure 6.15, triangle ABC is mapped onto triangle A'B'C' after a rotation through -60° and centre X.

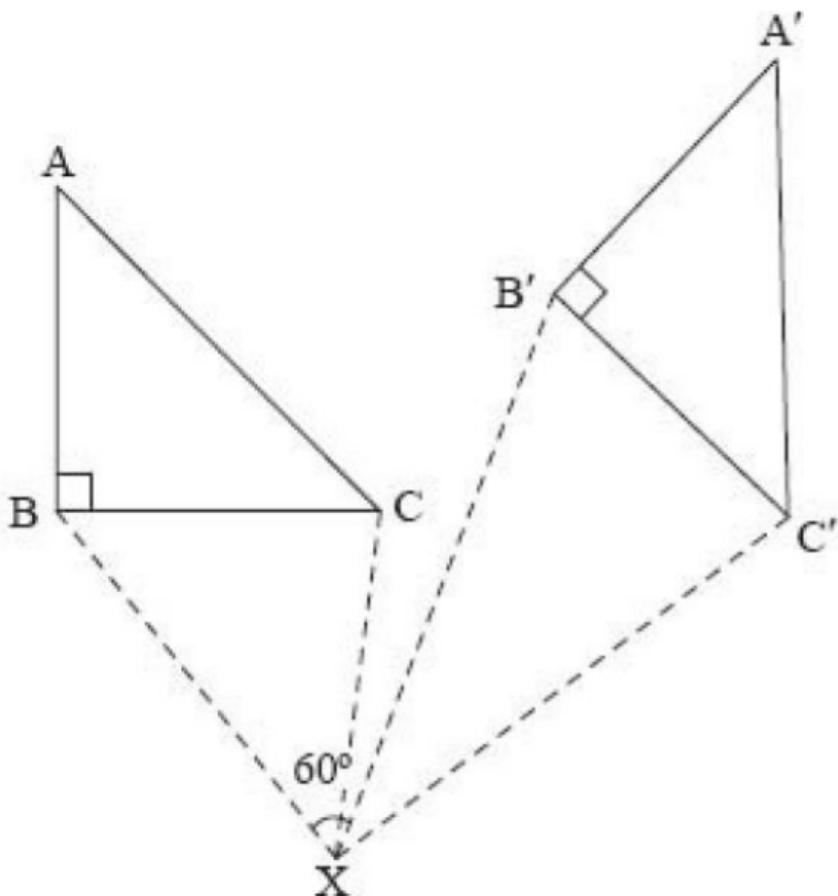


Fig. 6.15 Note:

The two triangles have:

- (i) the same shape.
- (ii) the same size.

The two triangles are therefore congruent. State the type of congruence.

In general, a rotation preserves lengths, angles and area, and the object and its image are directly congruent.

Exercise 6.3

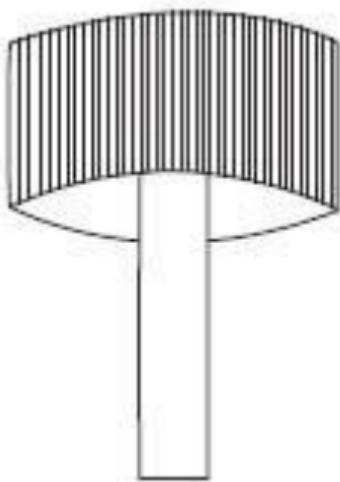
Compiled by Lec Rakdfha :0770112040

1. Find:

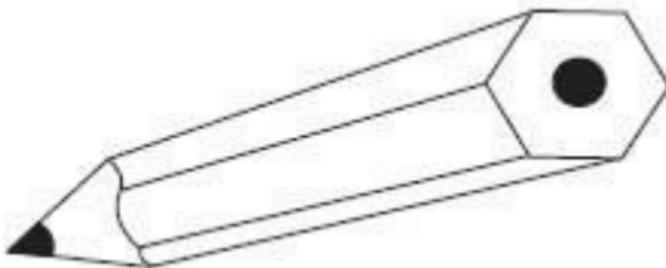
- (i) the number of axes of symmetry.
- (ii) the order of rotational symmetry about each of the axes in the following:
 - (a) Sunflower



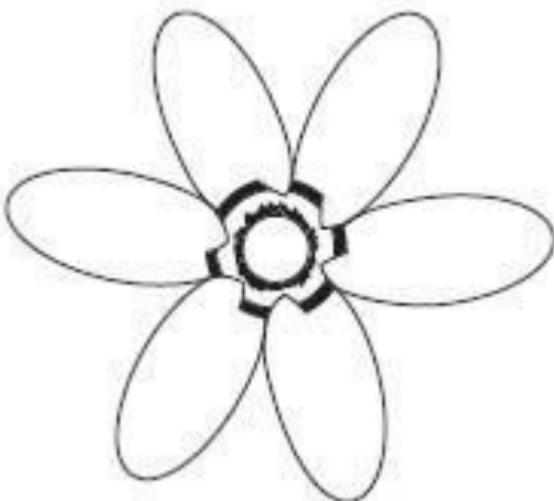
(b) A water tower



(c) A hexagonal pencil



(d) A car fan



2. Identify the axes of symmetry and state the order of rotational

symmetry about each of the axes of
the following solids:

- (a) cuboid
- (b) cylinder
- (c) hexagonal prism (pencil)
- (d) cube
- (e) a regular octahedron.

Chapter 7

SIMILARITY AND ENLARGEMENT

7.1: Similar Figures

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Figure 7.1 shows two different sizes of a plan of a cross-section of a house.

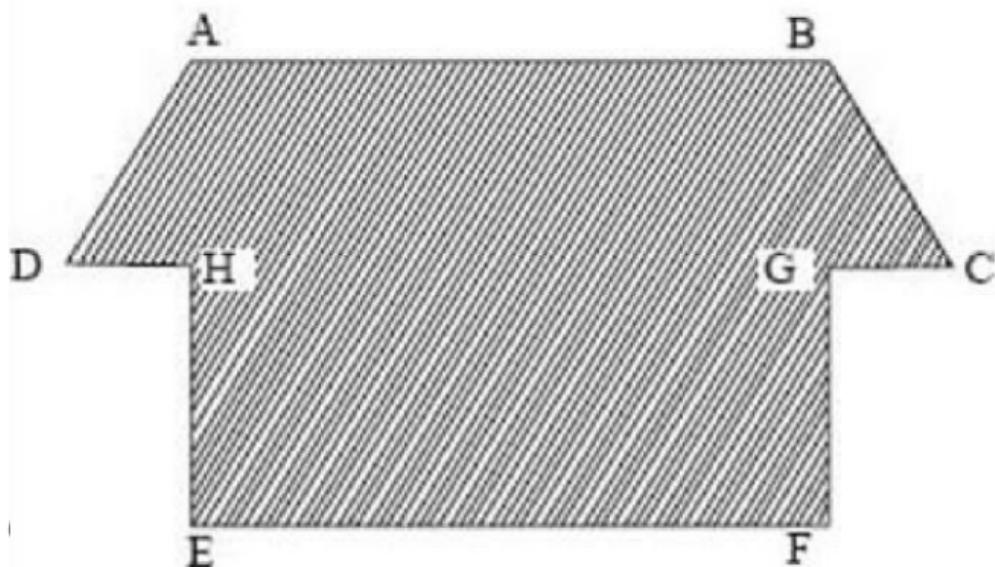


Fig. 7.1

Measure the corresponding sides of the two cross-sections. Copy and complete the following on the ratio of corresponding sides:

$$\frac{AD}{A'D'} = 2 \quad \frac{\dots\dots}{EF'} = 2$$

$$\frac{AB}{A'B'} = \dots\dots \quad \frac{GF}{\dots\dots} = 2$$

$$\frac{DC}{\dots\dots} = 2 \quad \frac{DH}{D'H'} = \dots\dots$$

Measure all the corresponding angles.

Notice that the ratio of corresponding sides is 2 and the corresponding angles

are equal. The two figures are said to be similar.

In general, two or more figures are similar if:

- (i) The ratio of the corresponding sides is constant.
- (ii) The corresponding angles are equal.

Example 1

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In figure 7.2, the triangles PQR and WXY are similar. Calculate the lengths of PR and XY.

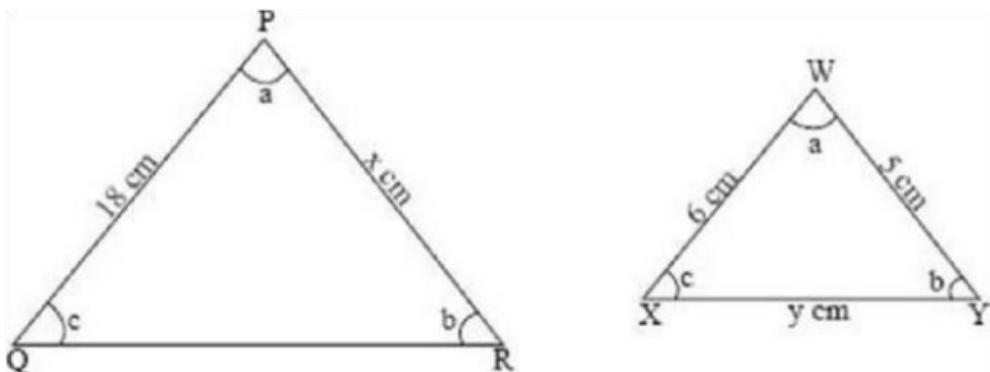


Fig. 7.2

Solution

PQ corresponds to WX (each of them is opposite angle b)

QR corresponds to XY and PR
corresponds to WY

Therefore, $\frac{PQ}{WX} = \frac{QR}{XY} = \frac{PR}{WY}$

That is, $\frac{18}{6} = \frac{24}{y} = \frac{x}{5}$

Thus, $\frac{18}{6} = \frac{24}{y}$

$$18y = 24 \times 6$$

$$y = \frac{24 \times 6}{18}$$

$$y = 8$$

Therefore, XY = 8 cm.

Also, $\frac{18}{6} = \frac{x}{5}$

$$6x = 18 \times 5$$

$$x = \frac{18 \times 5}{6}$$

$$x = 15$$

Therefore, PR = 15 cm.

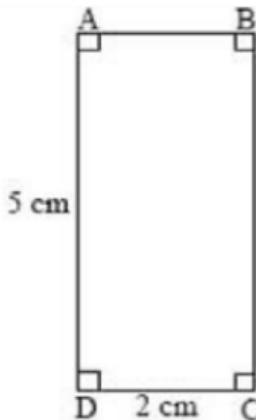
Note:

The corresponding sides of similar triangles are those opposite equal angles.

Example 2

Figure 7.3 (a) and (b) shows two rectangles ABCD and EFGH whose measurements are as shown. Determine whether or not the two are similar.

(a)



(b)

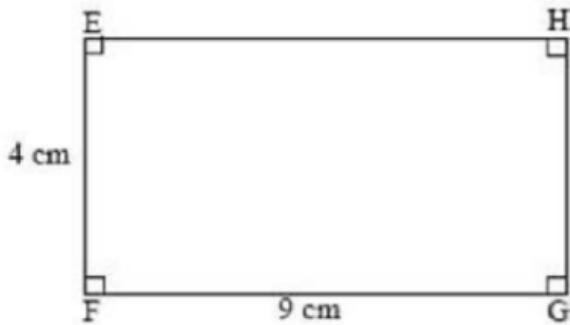


Fig. 7.3

Solution

The size of each angle is 90°

The ratio of the corresponding sides are;

$$\frac{AD}{EH} = \frac{5}{9}$$

$$\frac{DC}{HG} = \frac{2}{4} = \frac{1}{2}$$

The ratio of their corresponding sides is not constant. Therefore, the two rectangles are not similar.

Notice that although the corresponding angles of the two

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rectangles are equal, the rectangles are not similar because the ratios of their corresponding sides are not equal.

Note also that two figures could have the ratio of corresponding sides equal but fail to be similar if the corresponding angles are not equal. For example, a square of side 4 cm and a rhombus of sides 5 cm are not similar.

- (i) Construct two triangles with corresponding angles equal to 70° , 50° and 60° , with sides of different

lengths. Find the ratio of the corresponding sides. Are the two triangles similar?

- (ii) Construct two triangles whose sides measure 2 cm, 3 cm, 4 cm and 4 cm, 6 cm, 8 cm respectively. Measure the corresponding angles. Are the two triangles similar?

You should notice that two triangles are similar if either all their corresponding

angles are equal or the ratio of their corresponding sides is constant.

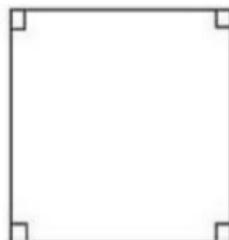
Exercise 7.1

1. Trace the following figures and determine those that are similar:

(a)



(b)



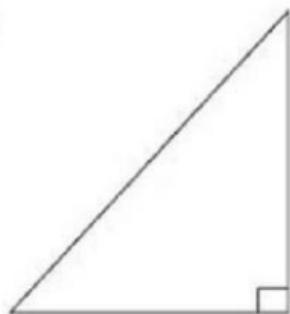
(c)



(d)



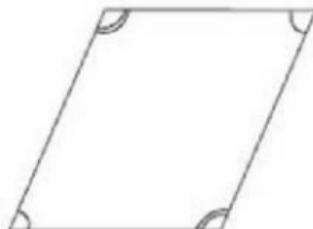
(e)



(f)

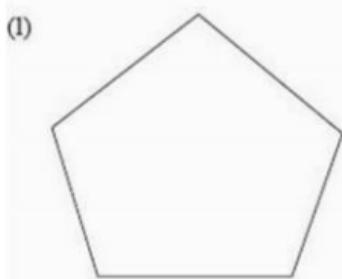
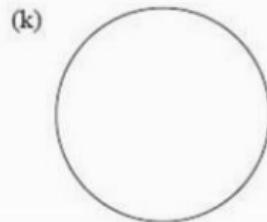
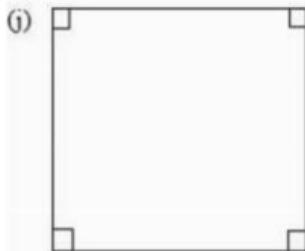
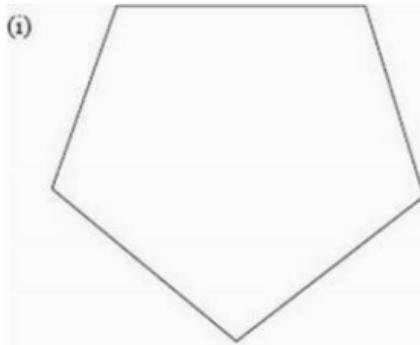


(g)

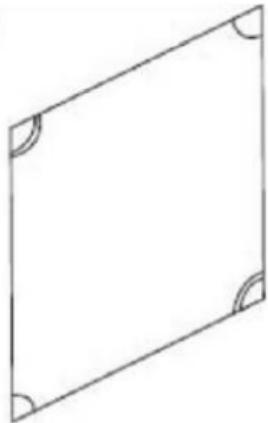


(h)

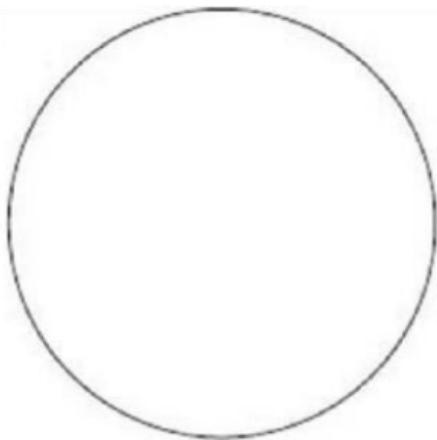




(p)



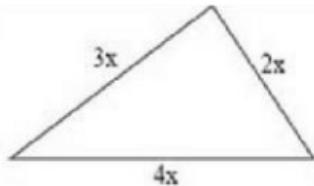
(q)



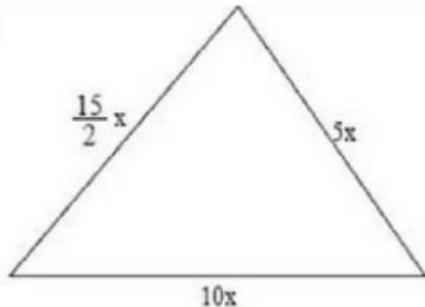
In questions 2 to 4, identify the similar triangles.

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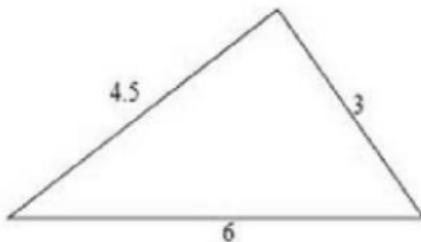
2. (a)



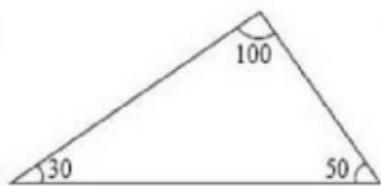
(b)



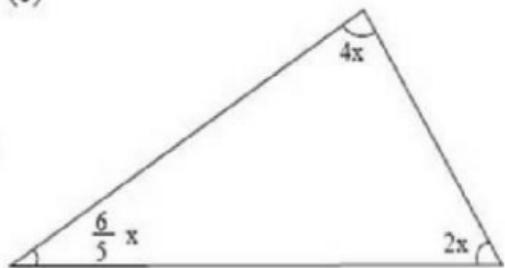
(c)



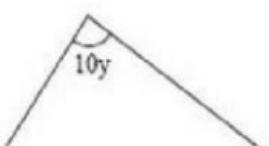
3. (a)



(b)

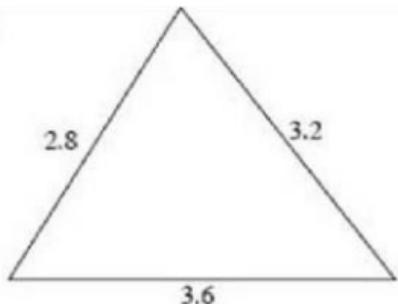


(c)

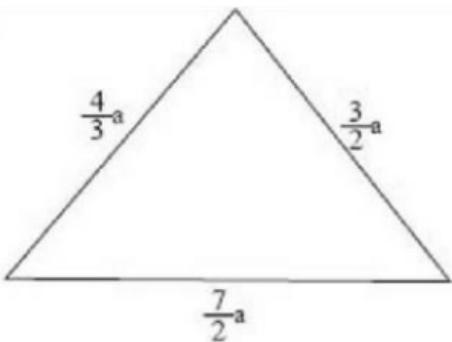


4.

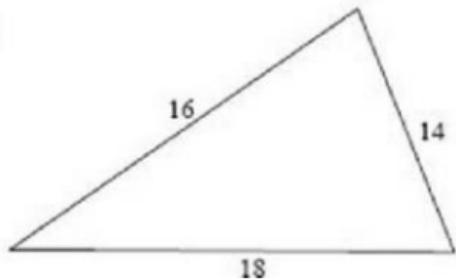
(a)



(b)

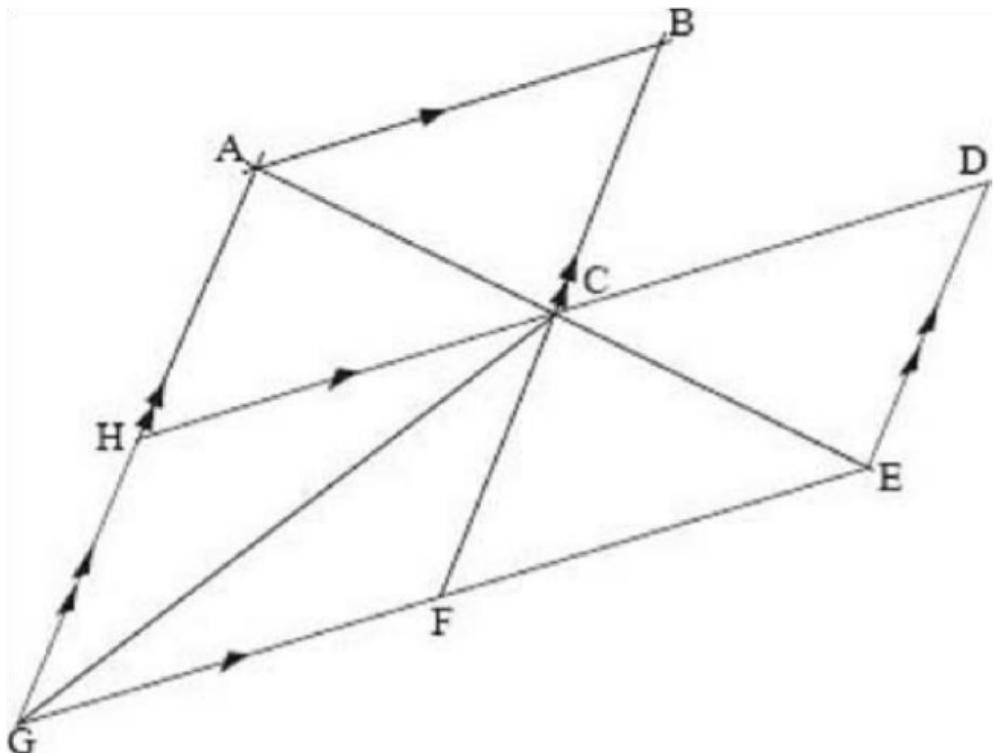


(c)



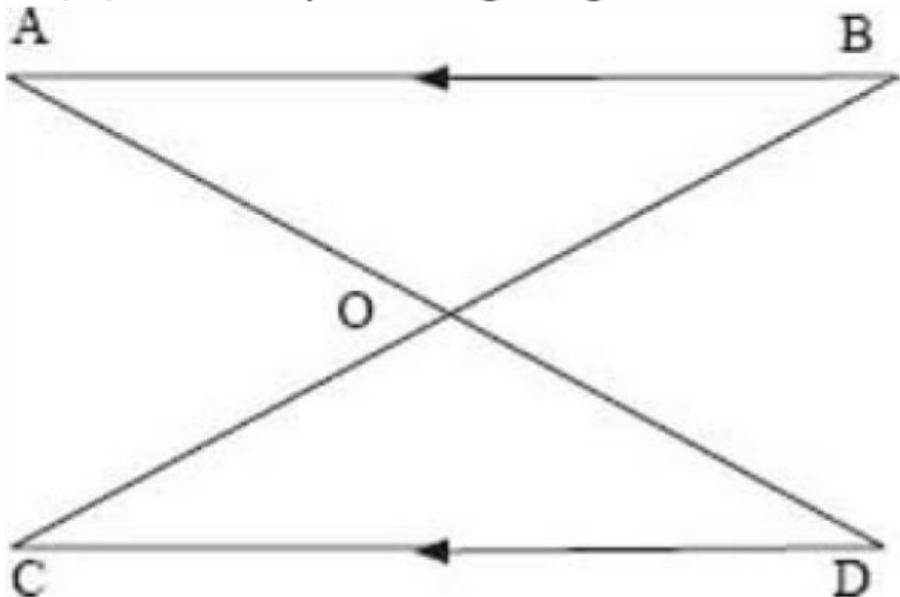
5. In the figure below, identify all the geometric figures which are similar:

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6. In the figure below, name three pairs of:
- (a) Corresponding sides.

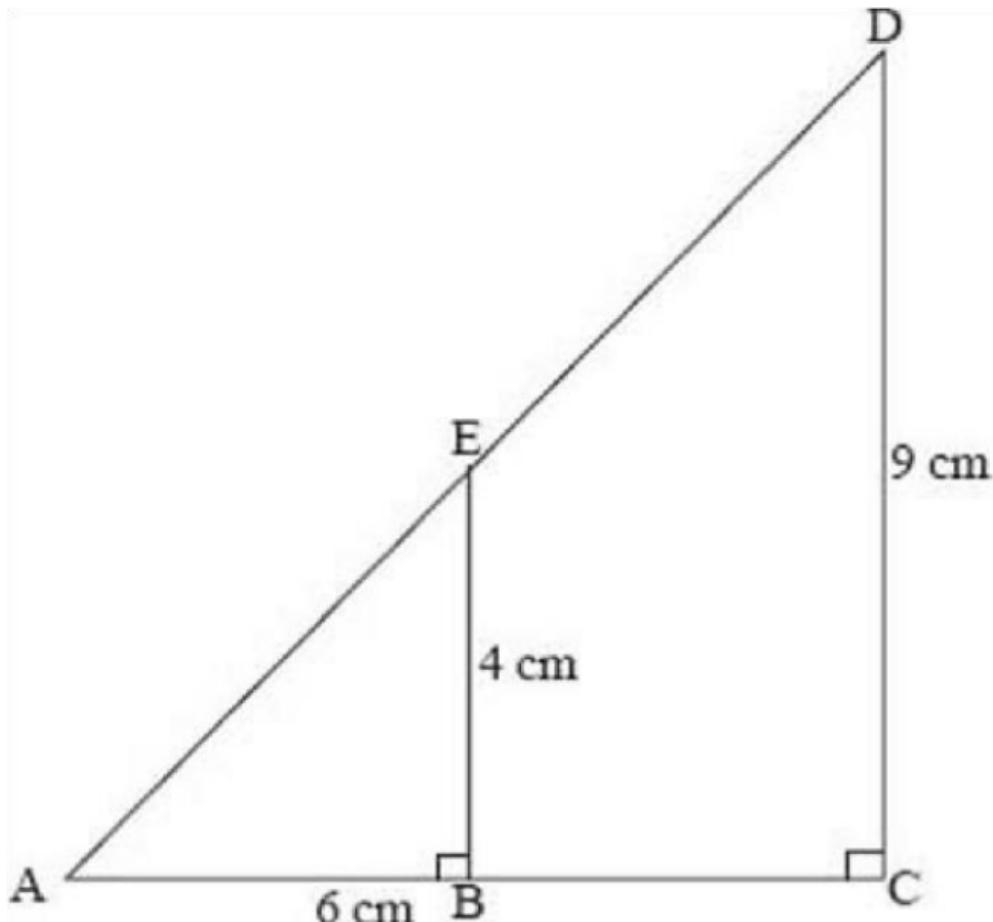
(b) Corresponding angles.



What can you say about triangles
AOB and COD?

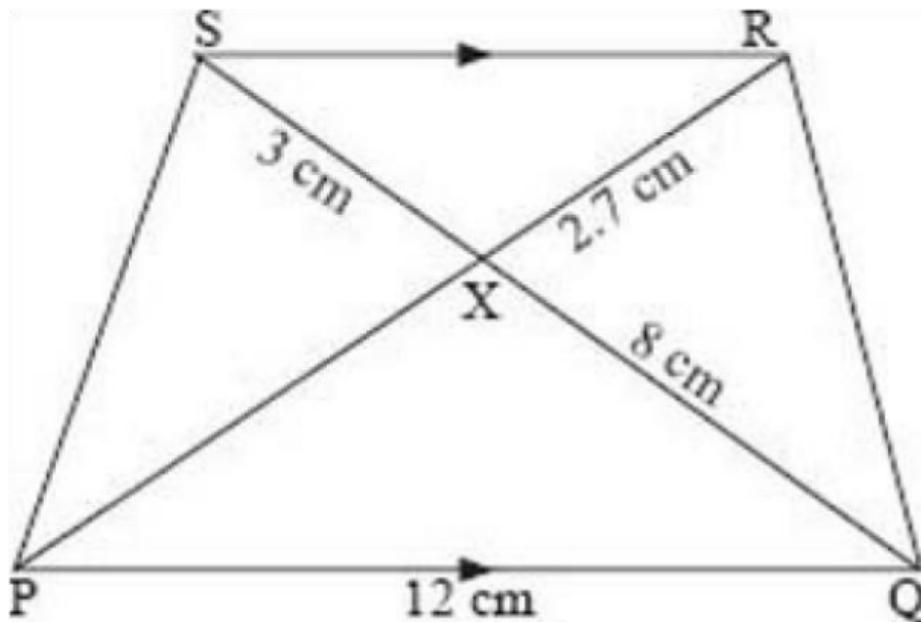
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7. In the figure below, triangle ABE is similar to triangle ACD. Calculate the length of BC.



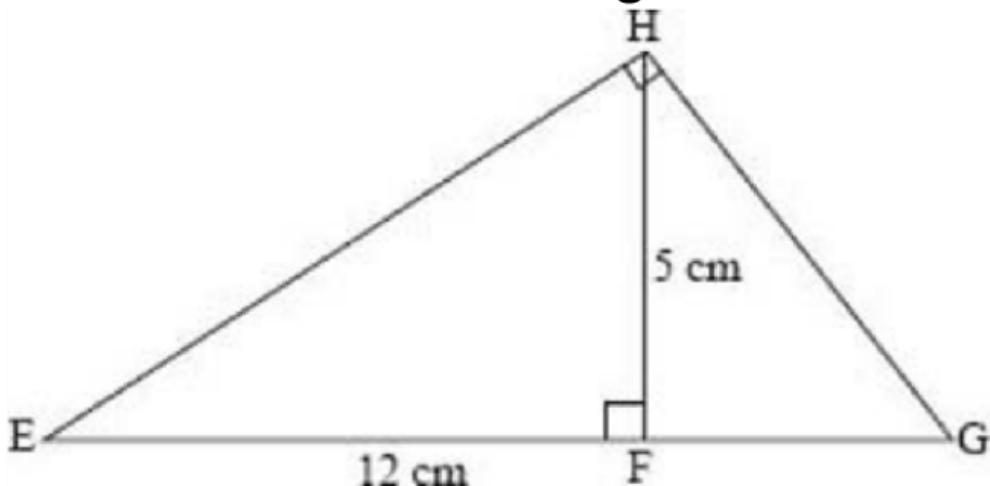
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8. In the figure below, SR is parallel to PQ. Calculate the lengths of SR and PX.



9. In the figure below, $\angle EHG =$

$\angle EFH = 90^\circ$, $HF = 5 \text{ cm}$ and $EF = 12 \text{ cm}$. Calculate the length HG and FG .

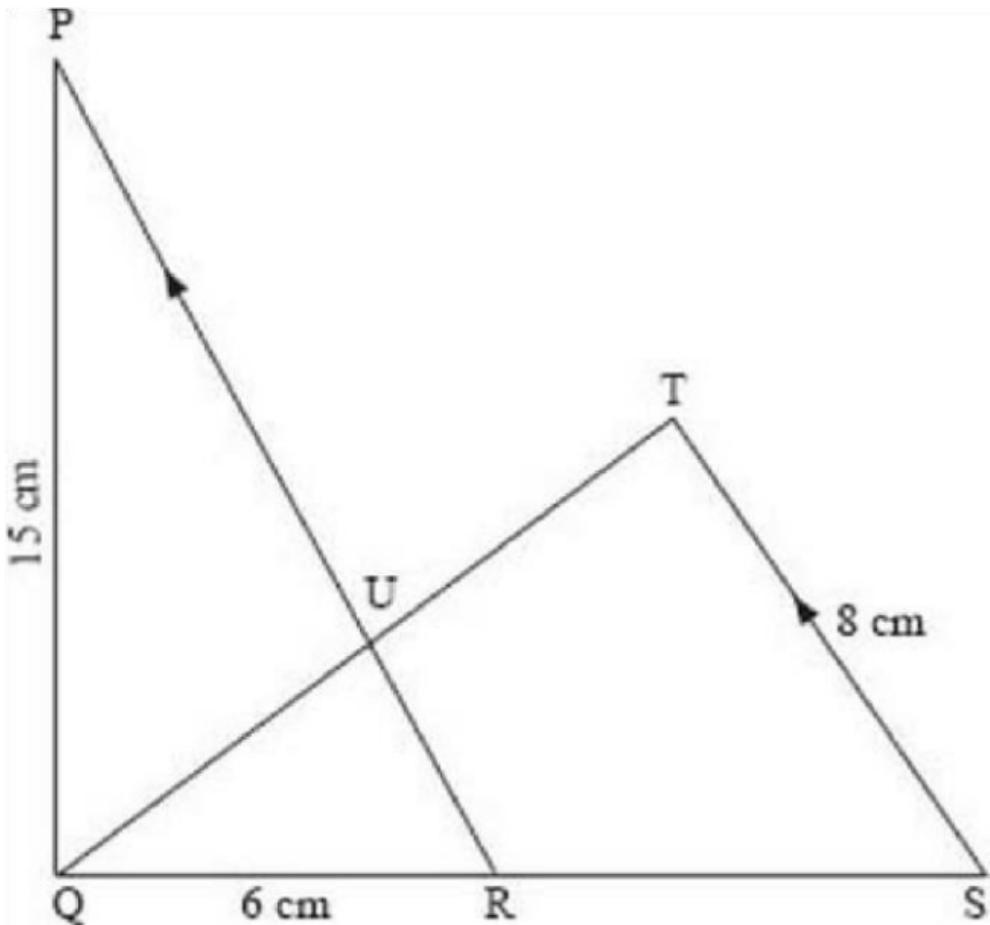


10. A triangle KLM is similar to a triangle PQR. $KL = 5 \text{ cm}$, $LM = 4 \text{ cm}$ and

$\angle KLM = 46^\circ$. Construct triangle PQR such that QR = 6 cm.

11. ABCD and PQRS are similar rectangles. AB = 3 cm and BC = 7 cm. If PQ = 5 cm, construct rectangle PQRS.
12. WXYZ is a rectangle in which WX = 8 cm and XY = 6 cm. F is a point on YZ such that YF = 6 cm. Construct a rectangle YEFG similar to WXYZ such that YF is the diagonal.

13. In the figure below, $\angle TQS = \angle QPR$, $PR//TS$, $PQ = 15$ cm, $QR = 6$ cm and $ST = 8$ cm. Calculate QT.



7.2: Enlargement

In figure 7.4. PQR is a triangle in which $PQ = 2$ cm. $QR = 1.5$ cm and $PR = 2.5$ cm. O is a point such that $OP = 4.1$ cm, $OQ = 2.5$ cm and $OR = 3.8$ cm.

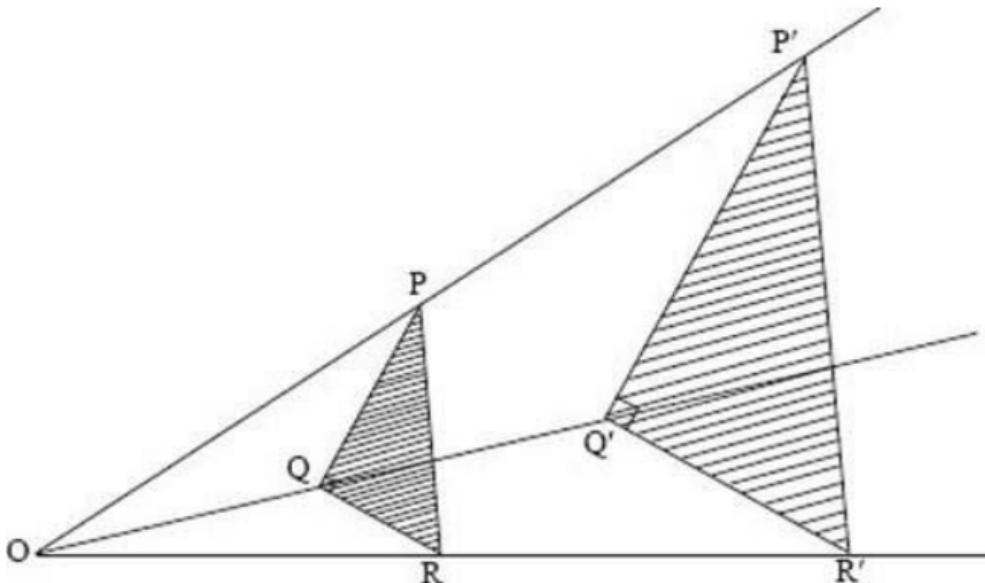


Fig 7.4

Triangle $P'Q'R'$ has been obtained from $\triangle PQR$ as follows:

OP , OQ and OR are produced such that $OP' = 8.2$ cm, $OQ' = 5$ cm and $OR' = 7.6$ cm.

This process of obtaining $\Delta P'Q'R'$ from ΔPQR is called enlargement. Triangle PQR is said to be the object and triangle $P'Q'R'$, its image under enlargement. The point O is called the centre of enlargement.

Note:

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$$\frac{OP'}{OP} = \frac{OQ'}{OQ} = \frac{OR'}{OR} = 2$$

$$\frac{P'Q'}{PQ} = \frac{P'R'}{PR} = \frac{Q'R'}{QR} = 2$$

(i)

(ii)

The ratio 2 is the scale factor of this enlargement. Under enlargement, the object and its image are similar. The scale factor is the linear scale factor of the enlargement.

Example 3

n

Construct any triangle ABC. Take a point O outside the triangle. With O as the centre of enlargement and scale factor of 3, construct the image of ABC under the enlargement.

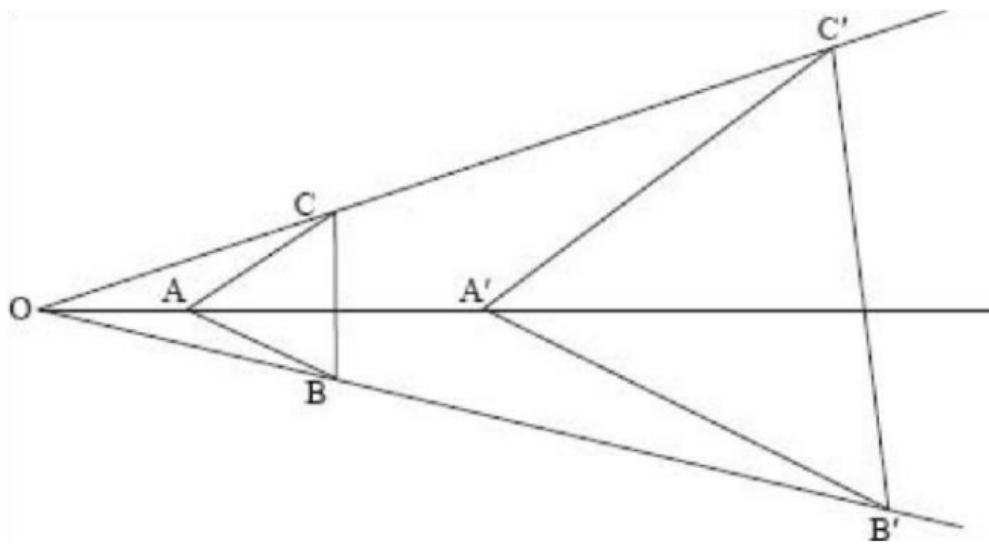


Fig. 7.5

By measurement;

O

OA = 1.5 cm, OB = 3 cm and OC = 2.9 cm.
To get A', the image of A, proceed as follows: OA' = 1.5 cm

$$\frac{OA'}{OA} = 3 \text{ (3 is the scale factor)}$$

$$OA' = 1.5 \text{ cm} \times 3 \\ = 4.5 \text{ cm}$$

Produce OA and measure 4.5 cm from O to get A'. Similarly;

$$\frac{OB'}{OB}$$

$$OB' = 3 \times 3 = 9 \text{ cm}$$

$$\frac{OC'}{OC}$$

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p

$$= 3$$

$$OC' = 2.9 \times 3$$

$$= 8.7 \text{ cm}$$

Produce OB and OC to obtain B' and C' respectively.

Note:

Lines joining object points to their corresponding image points meet at the centre of enlargement. This fact is used to locate the centre of enlargement if given an object and its image.

Example 4

q

Triangle A'B'C' is the image of triangle ABC under an enlargement.

- (a) Locate the centre of the enlargement.
- (b) Find the scale factor of the enlargement.

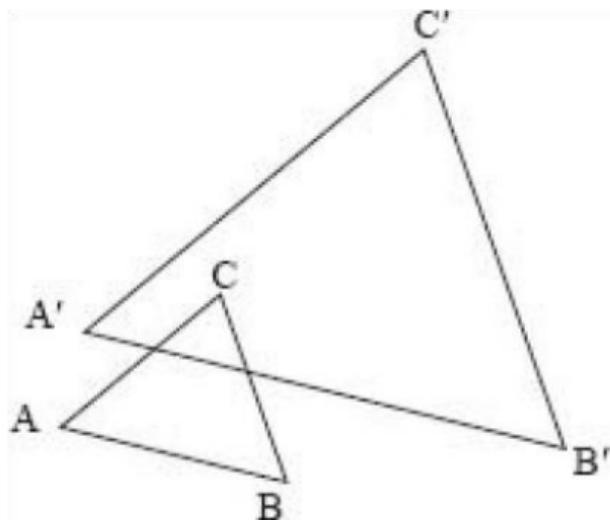


Fig. 7.6

Solution

(a) To locate the centre of enlargement, proceed as follows:

Join A to A', B to B' and C to C' and produce them. The point O

where the lines meet is the centre of enlargement.

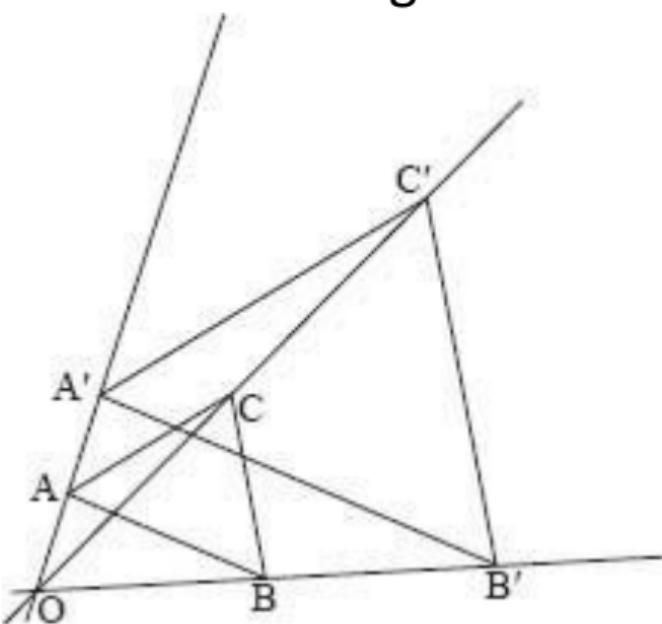


Fig 7.7

(b) By measurement;

$$\frac{OA'}{OA} = 2$$

$$\frac{OB'}{OB} = 2$$

$$\frac{OC'}{OC} = 2$$

Therefore, the scale factor of the enlargement is 2.

Example 5

In figure 7.8, $\Delta P'Q'R'$ is the image ΔPQR under an enlargement, centre O.

(a) If $OQ = 6 \text{ cm}$ and $QQ' = 4 \text{ cm}$, find the scale factor of the enlargement.

(b) If $PQ = 4 \text{ cm}$, calculate the length of $P'Q'$.

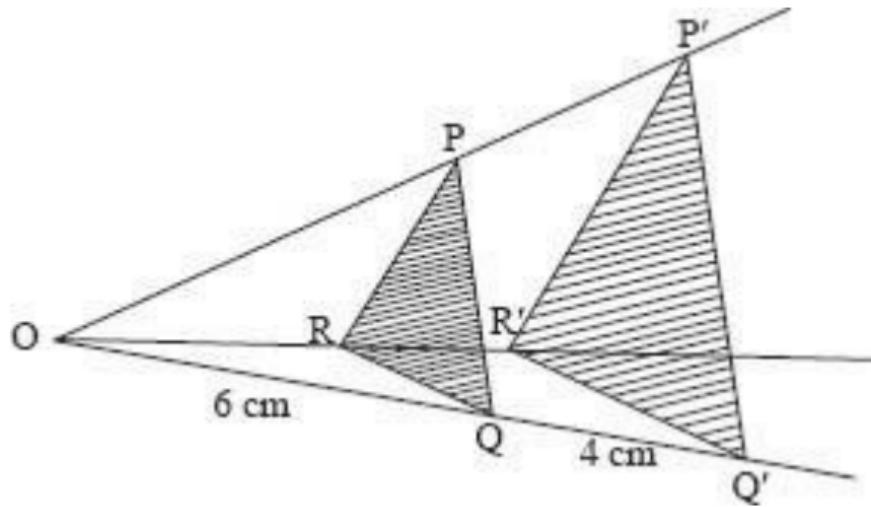


Fig. 7.8

Solution

(a) Linear scale factor is

$$\frac{OQ'}{OQ} = \frac{(6+4)}{6} = \frac{10}{6} = \frac{5}{3}$$

(b) Linear scale factor = $\frac{P'Q'}{PQ}$

But $PQ = 4 \text{ cm}$

Therefore, $\frac{P'Q'}{4} = \frac{5}{3}$

$$P'Q' = 6\frac{2}{3} \text{ cm}$$

Example 6

In figure 7.9, rectangle $A'B'C'D'$ is the image of rectangle $ABCD$ under an enlargement with centre at O. $OA = 12$

cm, $OA' = 4$ cm, $AB = 6$ cm and $A'D' = 3$ cm.

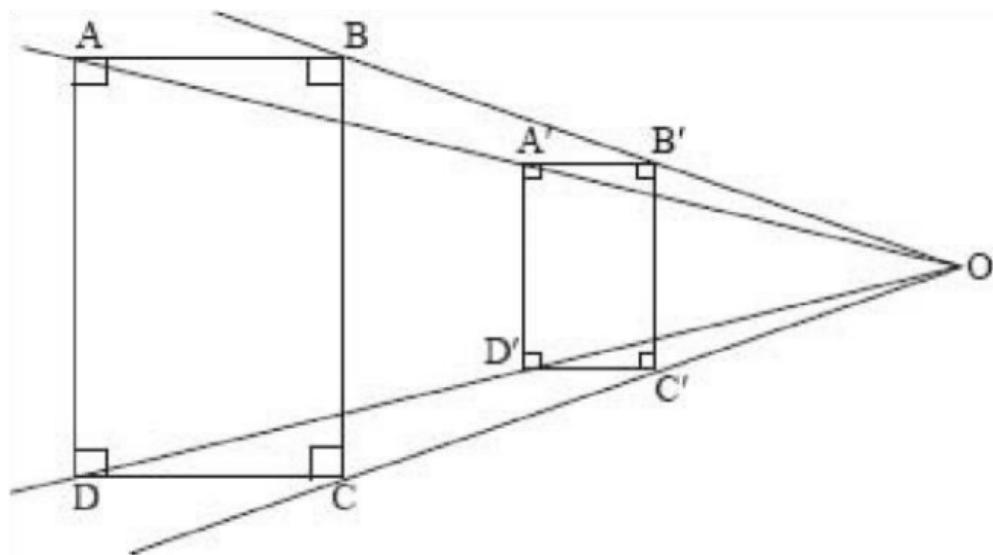


Fig. 7.9

Calculate:

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- (a) the linear scale factor.
- (b) the length of A'B'
- (c) the length of BC.

Solution

(a) The linear scale factor is given by;

$$\frac{OA'}{OA} = \frac{4}{12} = \frac{1}{3}$$

(b) $\frac{A'B'}{AB} = \frac{1}{3}$

Therefore, $A'B' = 6 \times \frac{1}{3} = 2 \text{ cm}$

(c) $\frac{B'C'}{BC} = \frac{1}{3}$

Therefore, $BC = 3B'C'$

$$= 3 \times 3$$

$$= 9 \text{ cm}$$

Note that when the scale factor is a proper fraction, as in this case, the image is smaller than the object.

Figure 7.10 shows a rhombus of side 3 cm. Trace the figure. Using O as the centre of enlargement and linear scale

factor 2, draw the image of the rhombus.

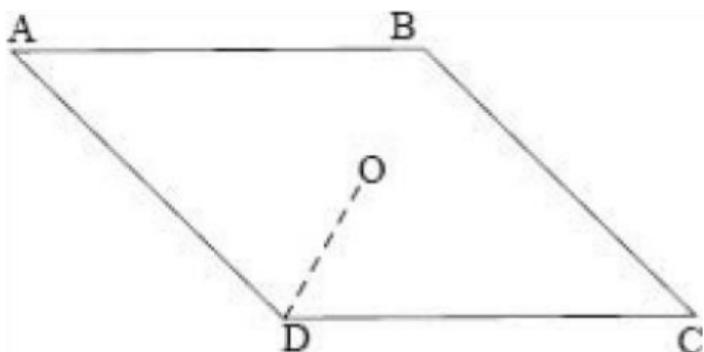
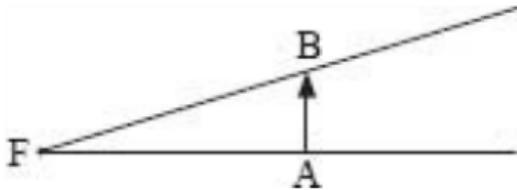


Fig 7.10

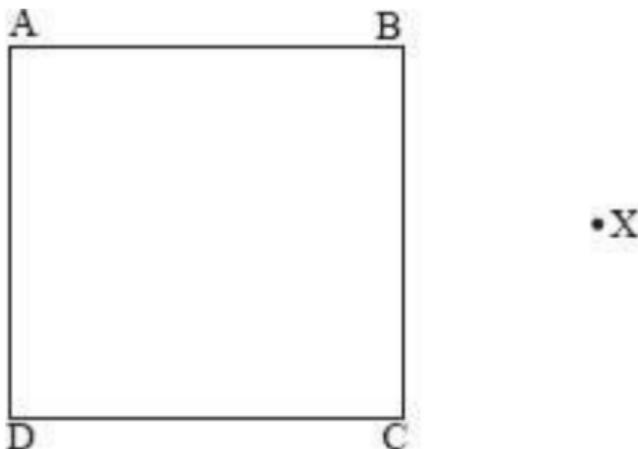
Exercise 7.2

In questions 1 to 4, copy the figures using tracing paper

1. In the figure below, locate the image of AB under enlargement with centre F and scale factor of:
- (a) 2
 - (b) 3
 - (c) $\frac{1}{2}$



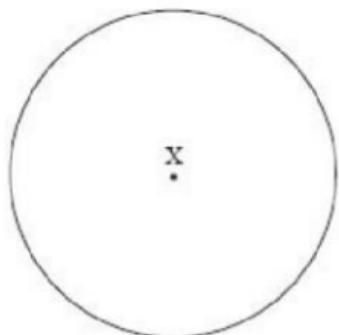
2. Draw the image of the square ABCD in the figure below under an enlargement scale factor 3 and centre X.



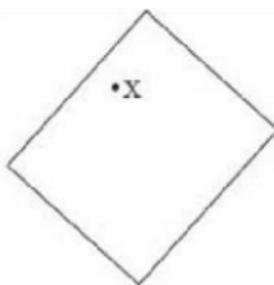
3. Find the image of (a), (b) and (c) in the figure 5 below, given the

centres of enlargement marked X
and a scale factor of 1.5:

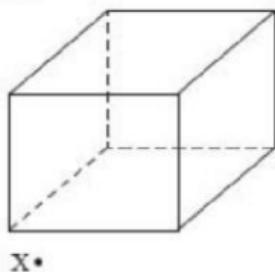
(a)



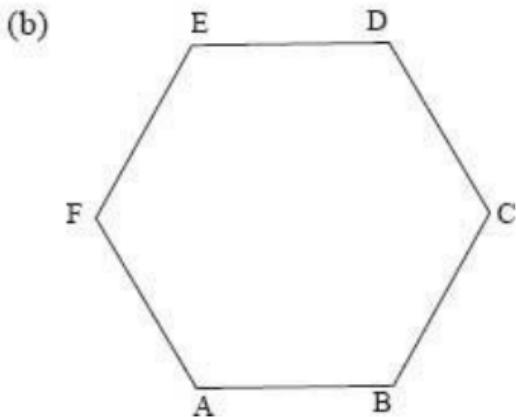
(b)



(c)



4. Using B as centre of enlargement
and a scale factor of $\frac{1}{2}$, find the
images of (a) and (b) in the figure
below.



5. The vertices of an object and its image after an enlargement are
A(0,
0), B (1, 1), C (2, 0), D(1, -1) and A
'(3, 3), B'(5, 5), C'(7, 3), D'(5, 1)
respectively. Find the centre and
scale factor of the enlargement.

6. Given that P (3, 4), Q(4, 4), R(6, 4), S (7, 1) and T (5, 0) are the vertices of an object, find the vertices of the image after an enlargement with the centre at (0, 0) and scale factor.
- (a) $\frac{1}{2}$
- (b) 3

Negative Scale Factor

In figure 7.11, $P'Q'R'$ and $P''Q''R''$ are the images of PQR under an enlargement, centre O .

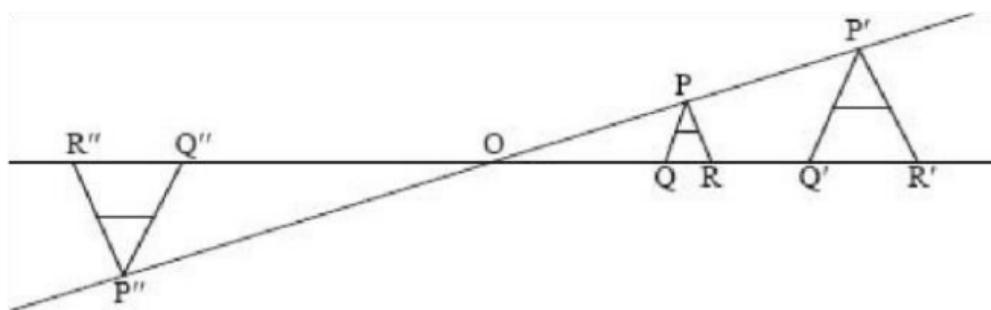


Fig. 7.11

$P'Q'R'$ is the image of PQR under an enlargement centre O , scale factor 2. How can we describe $P''Q''R''$, the image of PQR under an enlargement with O as the centre?

Note:

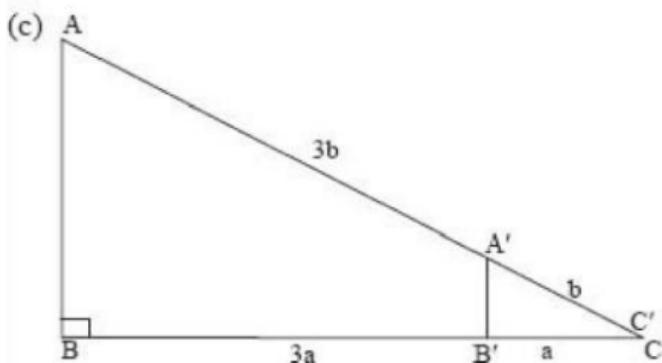
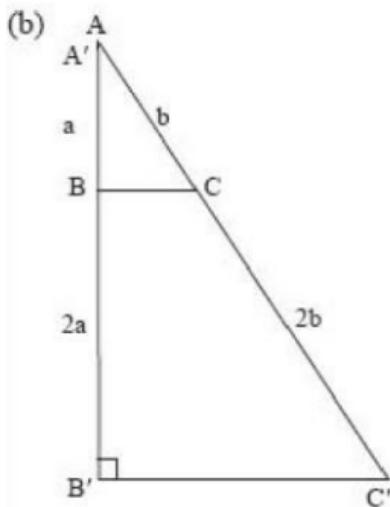
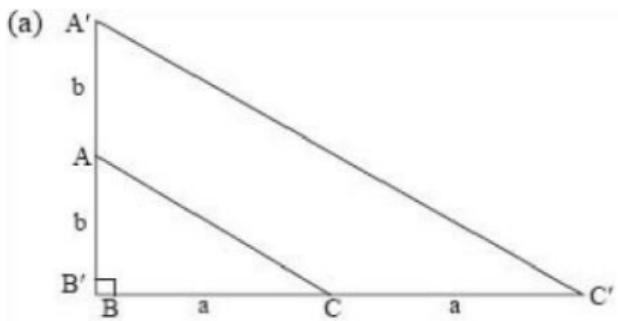
- (i) PQR and its image $P''Q''R''$ are on opposite sides of the centre of enlargement.
- (ii) The image is inverted with a scale factor 2.
When this happens, the scale factor is said to be negative. In this case, the scale factor is -2 .

To locate the image of the object under an enlargement with a negative scale factor, the same procedure as the positive scale factor is followed. However, the object and the image fall on opposite sides of the centre of enlargement.

Exercise 7.3

1. Points A (4, 2), B (9, 2), C(7, -2) and D (2, -2) are the vertices of a parallelogram. Taking the origin as the centre of enlargement, find the image when the scale factor is:

- (a) -1
 - (b) -2
 - (c) $-\frac{1}{4}$
2. State the centre of enlargement and the linear scale factor in each of the following figures if $\Delta A'B'C'$ is the image of ΔABC :

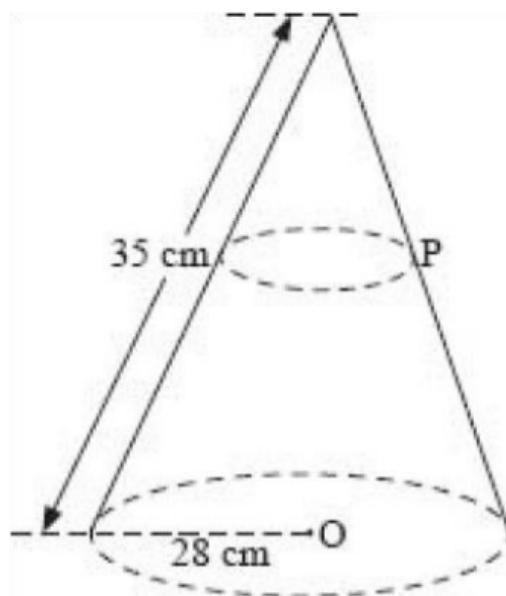


3. In ΔABC , $AB = 3 \text{ cm}$, $BC = 4 \text{ cm}$ and $AC = 5 \text{ cm}$. The sides AB , BC and AC are enlarged by a linear scale factor of 2.5. Find the length of the new triangle.
4. If a road 8 km long is represented by 4 cm on a map, find the scale of the map.

5. A rectangle measures 3 cm by 5 cm. Find the corresponding measurements of the image of the rectangle after an enlargement scale factor of:
- (a) -0.75
 - (b) -1.5
 - (c) -3
6. The radius of a circle is 3.5 cm. Find the circumference of its image after an enlargement scale factor of 1.75.

7. A triangular field PQR is such that $PQ = 20$ m, $PR = 30$ m and $QR = 40$ m. The field is drawn to scale such that the side PQ is 3 cm long. Find the linear scale factor and the lengths of QR and PR .
8. The figure below shows a cone of base radius 28 cm and the slant side of length 35 cm. At a point P, 14 cm vertically below the vertex, the cone is cut across to form a

smaller one. Calculate the base radius of the smaller cone.



7.3: Area Scale Factor

In figure 7.12, $A'B'C'D'$ is the image of rectangle $ABCD$ after an enlargement whose scale factor is 2.

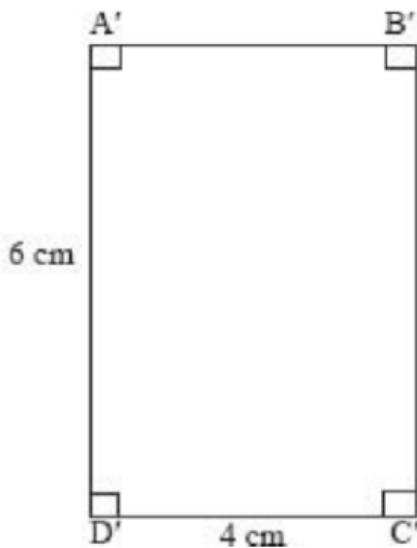
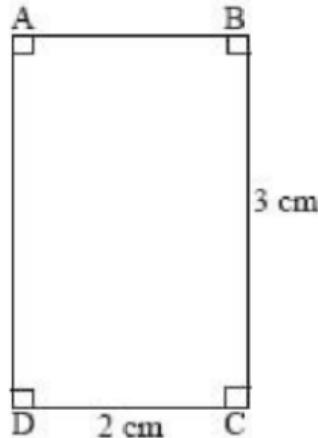


Fig. 7.12

Area of ABCD is $(2 \times 3) \text{ cm}^2 = 6 \text{ cm}^2$

Area of A'B'C'D' is $(4 \times 6) \text{ cm}^2 = 24 \text{ cm}^2$

$$\frac{\text{Area of A'B'C'D'}}{\text{Area of ABCD}} = \frac{24 \text{ cm}^2}{6 \text{ cm}^2} \\ = 4$$

The ratio of the area of the image to the area of the object is called area scale factor.

A right angled triangle of base 3 cm and height 4 cm is given an enlargement with scale factors:

(a) 3

(b) $\frac{1}{2}$

(c) -2

In each case, find:

- (i) the corresponding dimensions of the three images and hence their areas.
- (ii) The area scale factors

Notice that area scale factor is the square of linear scale factor.

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Example 7

A triangle whose area is 12 cm^2 is given an enlargement with a linear scale factor 3.

Find the area of the image.

Solution

Linear scale factor is 3.

Area scale factor is $3^2 = 9$

$$\frac{\text{Area of image}}{12} = 9$$

Therefore, area of image is $12 \times 9 = 108 \text{ cm}^2$

Example 8

A map of a certain town is drawn to a scale of 1: 50 000. On the map, the railway quarters cover an area of 10 cm². Find the area of the railway quarters in hectares.

Solution

The linear scale factor is $\frac{50\ 000}{1} = 50\ 000$

The area scale factor is $50\ 000^2$.

Therefore, 10 cm² on the map would

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represent $50\ 000^2 \times 10\ \text{cm}^2$ on the ground. In hectares, this would be:

$$\frac{50\ 000 \times 50\ 000 \times 10}{100 \times 100 \times 10\ 000} = 250\ \text{ha}$$

Example 9

The ratio of the area of two circles is $\frac{16}{9}$.

- .
- (a) Find the ratio of their radii.
- (b) If the larger one has a radius of 20 cm, find the radius of the smaller one.

Solution

(a) Area scale factor = $\frac{16}{9}$

Therefore, linear scale factor is

$$\sqrt{\left(\frac{16}{9}\right)} = \frac{4}{3}$$

Hence, the ratio of the radius of the smaller to the radius of the larger is 3: 4.

(b) If the radius of the larger circle is 20 cm, then the radius of the smaller circle is:

$$20 \div \frac{4}{3} = 20 \times \frac{3}{4}$$
$$= 15 \text{ cm}$$

Exercise 7.4

1. A floor in a plan of a house measures 20 cm by 13 cm. Find the area of the actual floor if the linear scale factor is 50.
2. The corresponding lengths of two similar photographs are 12 cm and 30 cm. The area of the larger photograph is 750 cm^2 . Find the:
 - (a) area scale factor.

- (b) area of the smaller photograph.
3. The floor of a room measuring 15 m by 9 m requires 3 375 square tiles. Find the measurements of each tile. If the tiles available were twice as long, determine the number of tiles that would be required.
4. The ratio of the area of two similar rooms is $\frac{4}{25}$.

- (a) Find the area of the bigger room if the area of the smaller room is 8 m^2 .
- (b) Find the ratio of their lengths.
- (c) If the length of the larger room is 10 m, find the length of the smaller one.
5. The corresponding sides of two similar regular pentagons are 3 cm and 7 cm respectively.
- (a) Find the ratio of their areas.

- (b) Calculate the area of the larger if the area of the smaller is 36 cm^2 .
6. Mwangi has two balloons, one yellow and the other green. The radius of the yellow balloon is 7 cm while that of the green balloon is 21 cm. Find the ratio of their surface areas.

7.4: Volume Scale Factor

Figure 7.13 represents two similar cuboids, the big one measuring $2 \text{ cm} \times 3 \text{ cm} \times 4 \text{ cm}$.

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$cm \times 5\text{ cm}$ and the smaller one $1\text{ cm} \times 1.5\text{ cm} \times 2.5\text{ cm}$. The linear scale factor is 2. The ratio of the volume of the large cuboid to the volume of the smaller cuboid is;

$$\frac{(2 \times 3 \times 5) \text{ cm}^3}{(1 \times 1.5 \times 2.5) \text{ cm}^3}$$

$$= 2^3 \text{ (2 is the linear scale factor).}$$

This ratio is the volume scale factor.

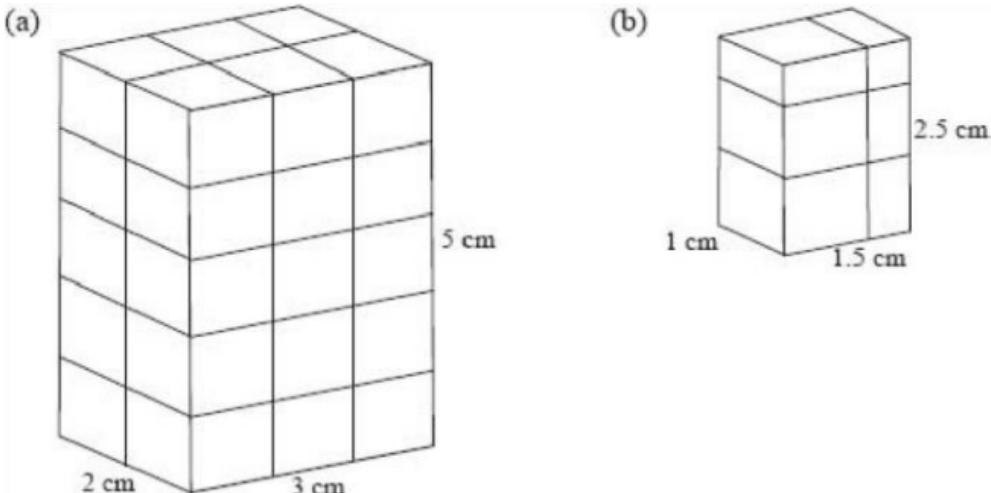


Fig. 7.13 Note:

When the linear scale factor is 2, the volume scale factor is 2^3 .

Volume scale factor = (linear scale factor)³

Example 10

The base radii of two similar cones are 6 cm and 8 cm. If the volume of the smaller cone is 324 cm^3 , find the volume of the larger one.

Solution

The ratio of their radii is $\frac{8 \text{ cm}}{6 \text{ cm}} = \frac{4}{3}$.

Linear scale factor is

Therefore, the volume scale factor is

$$\left(\frac{4}{3}\right)^3 = \frac{64}{27}$$

Thus, volume of the larger cone =

$$\frac{64}{27} \times 324$$

$$= 768 \text{ cm}^3$$

Example 11

A model of a swimming pool is shown in figure 7.14. Find the capacity in litres of

the actual swimming pool if its length is 50 metres.

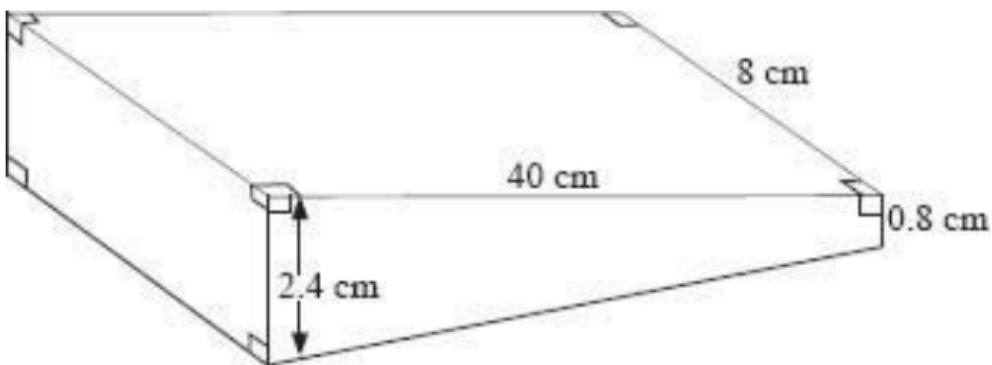


Fig. 7.14

Volume = cross sectional area \times length

$$= \frac{1}{2} (2.4 + 0.8) \times 40 \\ = 512 \text{ cm}^3$$

Linear scale
= $\frac{\text{actual length of pool in cm}}{\text{length of the model}}$
= $\frac{50 \times 100}{40}$
= 125 factor

Volume scale factor is given by;

$$125^3 = 1\ 953\ 125$$

Actual capacity of the pool; $\frac{1\ 953\ 125 \times 512}{1\ 000} = 1\ 000\ 000$ litres

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Exercise 7.5

1. The volume scale factor of two similar cylinders is 27. Find the:
 - (a) linear scale factor.
 - (b) area scale factor.
2. The corresponding sides of two similar blocks of wood measure 15 cm and 3 cm. Find the ratio of their volumes.
3. A spherical balloon 5 cm in diameter is inflated so that its diameter is 10

cm. By what factor has the volume increased?

r

4. The corresponding lengths of two similar iron bars are 5 cm and 15 cm.
 - (a) Find the ratio of their masses.
 - (b) If the smaller iron bar has a mass of 12 kg, determine the mass of the large bar.
5. A spherical solid lid of diameter 12.0 cm weighs 6.4 kg. Calculate the mass of a similar solid of diameter 10.0 cm.
6. The heights of two similar pails are 12 cm and 8 cm. The larger pail can hold 2 litres. Find the capacity of the smaller pail.

r

7. The mass of fat in two similar tins are 2 000 g and 500 g respectively. If the area of the base of the smaller tin is 100 cm^2 , find the area of the base of the larger tin.
8. A soft drink manufacturing company makes a giant model bottle 2 m long as the factory symbol. If the real bottle from the factory is 20 cm long and has a volume of 300 cm^3 , find the volume of the model in cm^3 .

r

9. The shortest side of a triangle is 12 cm and the area of the triangle is 8 cm^2 . A similar triangle has an area of 18 cm^2 . Calculate the shortest side of this triangle.
10. The volumes of two similar cans are 96 cm^3 and $1\,500 \text{ cm}^3$. Find the ratio of:
- thei heights.
 - the area of their curved surfaces.
11. The radius of a soap bubble increases by 4%. Calculate the percentage increase in its:

r

(i) surface area.

(ii) volume (to three significant figures).

12. The corresponding sides of two similar figures measure 16 cm and 24 cm respectively. Calculate the ratio of:

(a) their perimeters.

(b) their areas.

13. In figure 7.31, VAB, VCD and VEF

are similar cones. Their radii are 14 cm, 10.5 cm and 7 cm respectively. If the height of VAB is 42 cm, calculate:

- (a) the heights of the cones VCD and VEF.
- (b) (i) the ratio of the curved surface areas of cones VCD and VEF.
(ii) the ratio of their volumes.

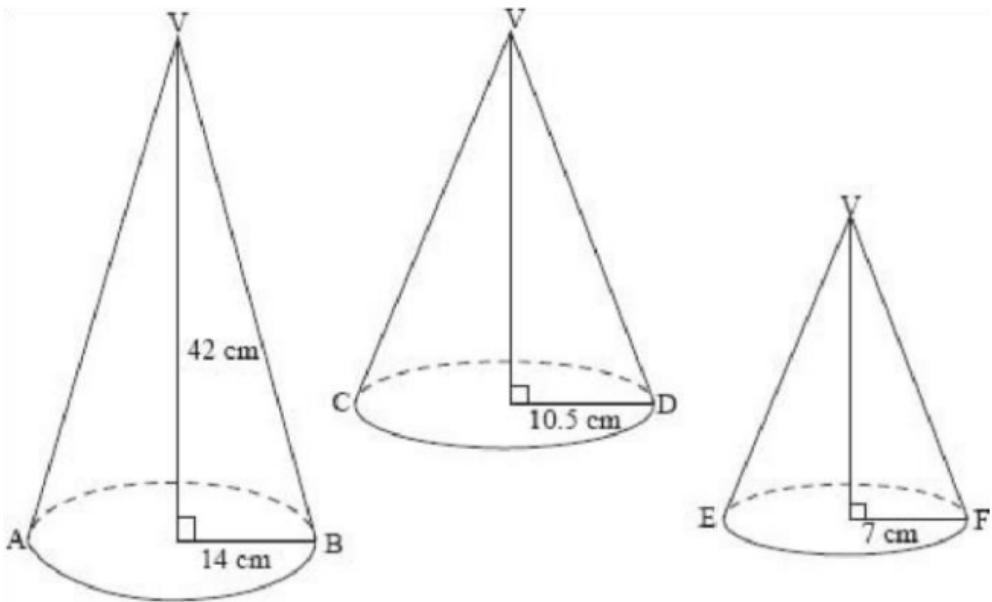
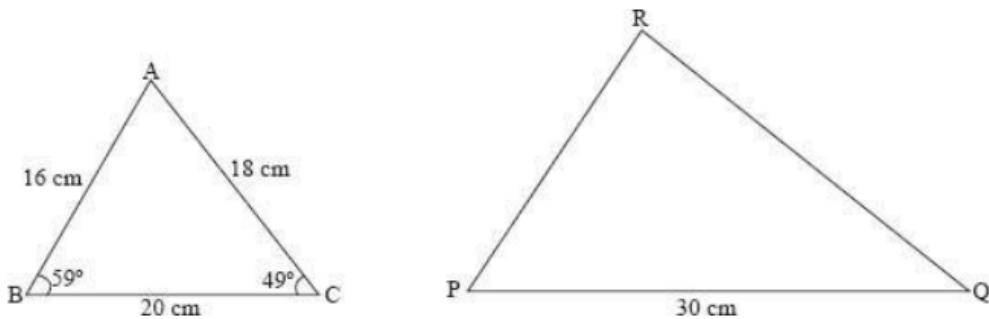


Fig. 7.31 Mixed Exercise 1

1. Evaluate: $\sqrt{\left(\frac{0.64 \times (1.69)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}} \times 38.44} \right)}$

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2. Given that the triangles ABC and PQR in the figure below are similar, find:
- (a) the size of $\angle QPR$.
- (b) the lengths of RQ and PR.

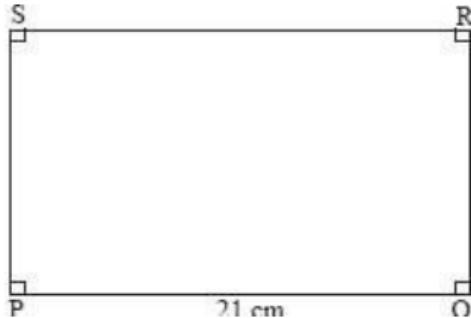
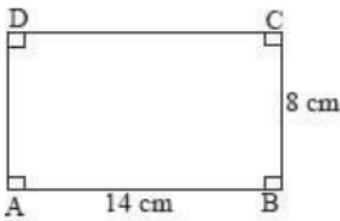


3. Evaluate $\frac{a^b - b^a}{b^b \times a^a}$, if $a = 2$ and $b = -2$.
4. Given that $y = ax^n$, find the value of y

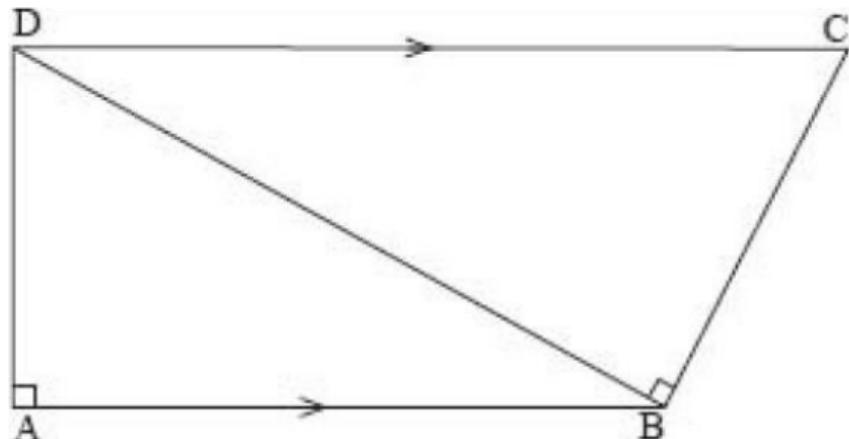
when $a = \frac{5}{3}$, $x = 2$ and $n = -4$.

5. A straight line l_1 has a gradient $-\frac{1}{2}$ and passes through the point $P(-1, -3)$. Another straight line l_2 passes through the points $Q(1, -3)$ and $R(4, 5)$. Find:
- (a) the equation of l_1 (b) the gradient of l_2 .
- (c) the equation of l_2 .

- (d) the co-ordinates of the point of intersection of l_1 and l_2 .
- (e) the equation of a line through R parallel to l_1 .
- (f) The equation of a line passing through a point S (0, 5) and perpendicular to l_2 .
6. In the figure below, rectangles ABCD and PQRS are similar. Find the area of PQRS.



7. In the figure below, $AB \parallel DC$, $\angle DAB = \angle DBC = 90^\circ$, $CD = 7\text{ cm}$ and $BC = 4\text{ cm}$. Calculate the length of AB correct to four significant figures.



8. On a map with a scale of 1: 50 000, a coffee plantation covers an area of 20 cm^2 . Find the area of the plantation in hectares.
9. Determine the equation of the straight line with:
- (a) gradient $\frac{4}{3}$ and y-intercept -2.

(b) gradient – $\frac{1}{4}$. and x-intercept

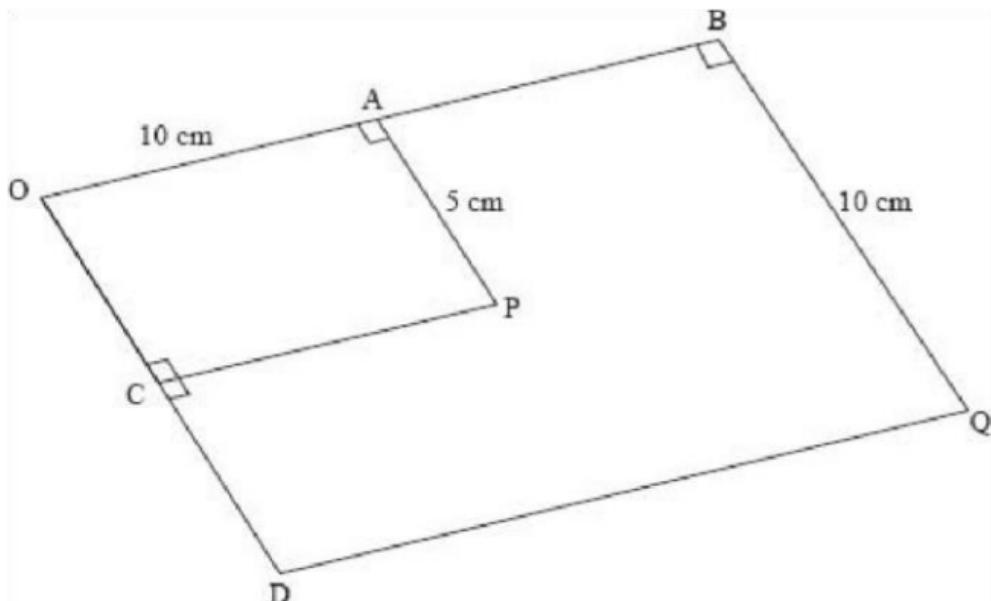
In each case, give your answer in the form $ax + by + c = 0$, where a, b and (c) are integers.

10. In the figure $OA = BQ = 10 \text{ cm}$. $AP = 5 \text{ cm}$. Given that OPQ is a straight line, calculate:

(a) OB

(b) AB

(c) PQ



11. The gradient of a straight line l_1 passing through the points P (3, 4) and Q (a, b) is $-\frac{3}{2}$. A line l_2 is

perpendicular to l_1 and passes through the points Q and R (2, -1). Determine the values of a and b.

12. Evaluate $\{(3.2)^{\frac{1}{2}} - 4\}^{-2}$, giving your answer in standard form.
13. The scale of a map is 1:125 000. Find the actual distance in kilometres represented by 16.8 cm on the map.
14. A triangle with the vertices P (3, 0), Q (5, 1) and R(4, 4) is enlarged. If the

centre of enlargement is $(1, 0)$, find the co-ordinates of the image of the triangle when the scale factor is:

(a) -2

(b) $\frac{1}{2}$

15. A line L is perpendicular to $y = 3x$. If L passes through point $(0, 4)$, find:

(a) the equation of L.

(b) the point Q where L intersects the line $y = 3x$.

16. If $A = \sqrt[3]{\frac{t(W-B)}{r^2}}$ find A if $t = 0.0034$,

$$W = 4.634, B = 2.342 \text{ and } r = 3.006.$$

17. The width of a rectangle is 10 cm and its area is 120 cm^2 . Calculate the width of a similar rectangle whose area is 480 cm^2 .
18. Three vertices of a rectangle ABCD are A (3, -4), B(7, -4) and C(3, -2). The images of A and B under an enlargement are A'(7, 6) and B'(3, 6). Find:

(a) the scale factor and centre of enlargement.

(b) the image of C.

19. Two perpendicular lines intersect at $(3, 9)$. If one of them passes through $(2, 9\frac{1}{3})$, find the equation of each line.

20. The volume of a sphere is given by $V = \frac{4\pi r^3}{3}$. Find the value of r if $V = 311$ and $\pi = 3.142$.

21. The vertices of a triangle are $P(4, -3)$, $Q(2, 1)$ and $R(-8, 2)$. Find the gradients of PQ , QR and RP .
Arrange the gradients in increasing order.
22. (a) A square whose vertices are $P(1, 1)$, $Q(2, 1)$, $R(2, 2)$ and $S(1, 2)$ is given an enlargement with centre at $(0, 0)$. Find the images of the vertices if the scale factors are:

(i) 1

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(ii) -1

(iii) 3

(iv) -3

$\frac{1}{2}$ (v)

$\frac{1}{2}$ (vi) -

- (b) If the image of the vertices of the same square after an enlargement are P'(1, 1), Q'(5, 1), R(5, 5) and S'(1, 5), find:
- (i) the centre.

(ii) the scale factor of the enlargement.

23. The vertices of a quadrilateral are A (5, 1), B(6, 3), C(4, 4) and D(2, 3). It is reflected in the line $x = -4$ followed by a reflection of the line $y = 0$. Find the co-ordinates of the final image.
24. Given the co-ordinates of A and B as (4, 2) and (8, 2) respectively, find the equation of the perpendicular bisector of AB.

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25. Find the co-ordinates of the vertices of the image of a kite whose vertices are $P(0, 8)$, $Q(3, 3)$ $R(0, 1)$, and $S(-$

e

3, 3) when rotated about the origin through:

(a) -90°

(b) 180°

26. Plot the points A (4, -1) B(5, -3), C(4, -4) and D(3, -3) on graph paper. Join all the points to form a polygon ABCD.

(a) Name of the polygon formed.

(b) Write down the equation of the line of symmetry of the polygon.

(c) Find the angle between AC and BD.

e

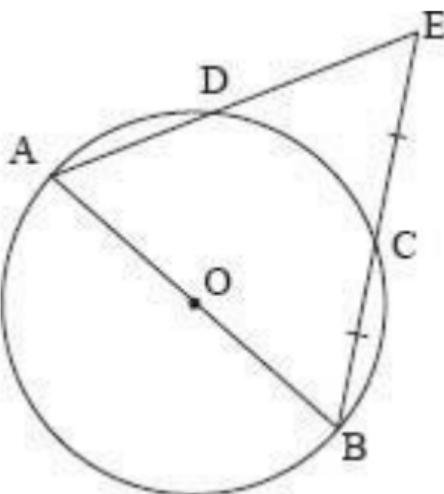
27. An isosceles triangle ABC is rightangled at B. Points E and D on AC are such that $AE = ED = DC$. P is opposite B on the other side of AC such that $BE = PE$. Show that:
- (a) triangles BEP and BDP are congruent.
 - (b) triangles AEP and CDP are congruent.
28. A triangle whose vertices are P (2, 2), Q(4, 2) and R(4, 4) is mapped onto a triangle whose vertices are P '(4, -2), Q'(2, -2) and R'(2, -4) under a rotation. Find:

e

- (a) the centre and angle of rotation.
- (b) the images of points $(0, 4)$ and $(-1, 2)$ under the same rotation.

29. In the figure below, O is the centre of the circle. Chords AD and BC are produced to meet at E:

e



If $BC = CE$:

- show that $AB = AE$.
- show that triangles CDE and ABE are similar.

30. A figure has a rotational symmetry of order 4 about the point $(6, -6)$. Two of its vertices are $(8, -4)$ and $(8, -10)$.

e

– 8). Find the co-ordinates of the other

vertices and draw the figure.

31. AOC and BOD are diameters of a circle ABCD, centre O. Show that:
- (a) the Δ s ACD and BDC are congruent.
 - (b) $\angle ACD = \angle BDC$.
32. The ratio of the radii of two spheres is 2:3. Calculate the volume of the first sphere if the volume of the second is 20 cm^3 .
33. Use mathematical tables to solve

$$0.1468^3 + \frac{1}{27.38^3}.$$

34. Evaluate without using tables:

$$\sqrt[3]{\frac{0.729 \times 409.6}{0.1728}}$$

35. A prism has a regular decagonal cross-section, find the:

- (a) number of axes of symmetry the solid has.
- (b) order of its rotational symmetry.

(c) number of planes of symmetry
of the prism.

Chapter 8

THE PYTHAGORAS THEOREM

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Consider the right-angled triangle ACB below.

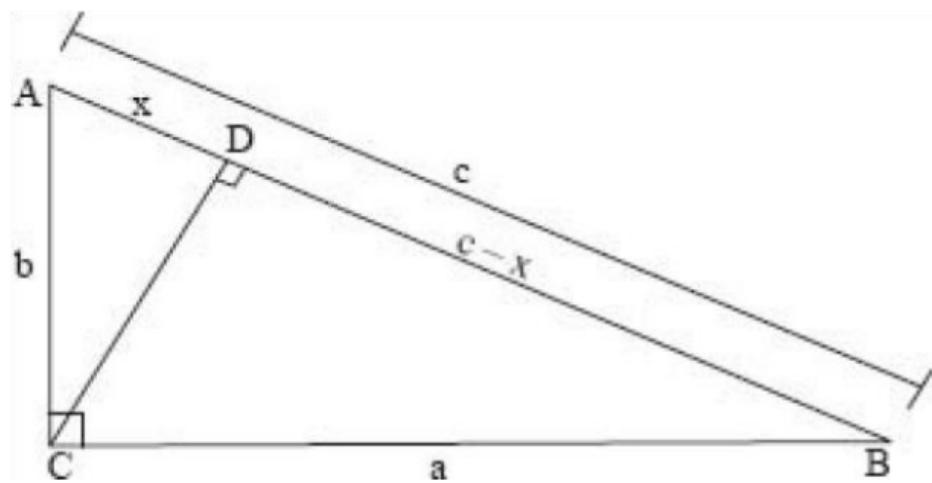
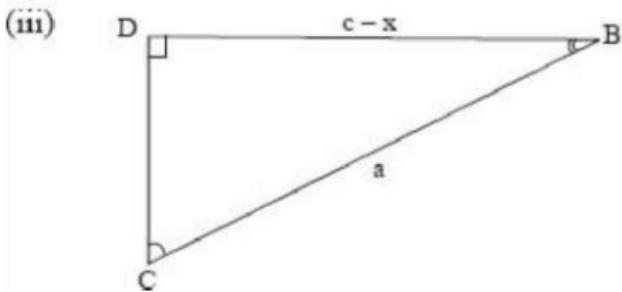
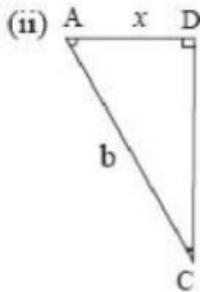
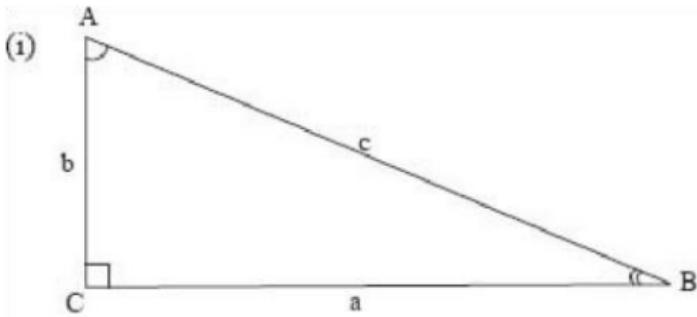


Fig. 8.1

From ΔACB , ΔABC , ΔACD and ΔBCD are similar as shown below.



Using triangles (i) and (iii)

$$\frac{BA}{BC} = \frac{BC}{BD}$$

$$\frac{c}{a} = \frac{a}{c-x}$$

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Using triangles (i) and (ii)

$$\frac{AB}{AC} = \frac{AC}{AD}$$

$$\frac{c}{b} = \frac{b}{x}$$

$$b^2 = cx \dots \dots \dots \text{(ii)}$$

Adding equation (i) and (ii)

$$a^2 + b^2 = c(c - x) + cx$$

$$a^2 + b^2 = c^2 - cx + cx$$

This is the Pythagoras's theorem, which states that for a right-angled triangle, the square on the hypotenuse is equal

to the sum of the squares on the two shorter sides.

Example 1

In a right-angled triangle, the two shorter sides are 6 cm and 8 cm. Find the length of the hypotenuse.

Solution

Let the hypotenuse be H

Using the Pythagoras's theorem

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$$H^2 = 6^2 + 8^2$$

$$H^2 = 36 + 64$$

$$= 100$$

$$H = 10 \text{ cm.}$$

Example 2

A ladder 20 metres long leans against a building and reaches a point on the building that is 14 metres above the ground. How far from the bottom of the building is the foot of the ladder?

Solution

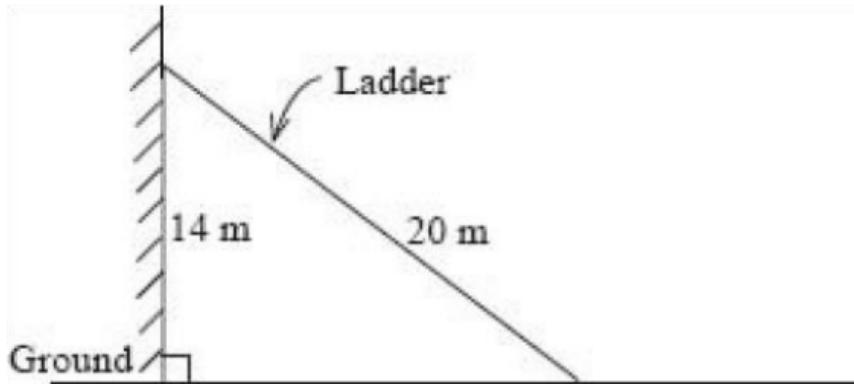


Fig. 8.2

Let the distance between the foot of the ladder and the building be x . Using the Pythagoras's

$$\text{theorem } x^2 + 14^2 =$$

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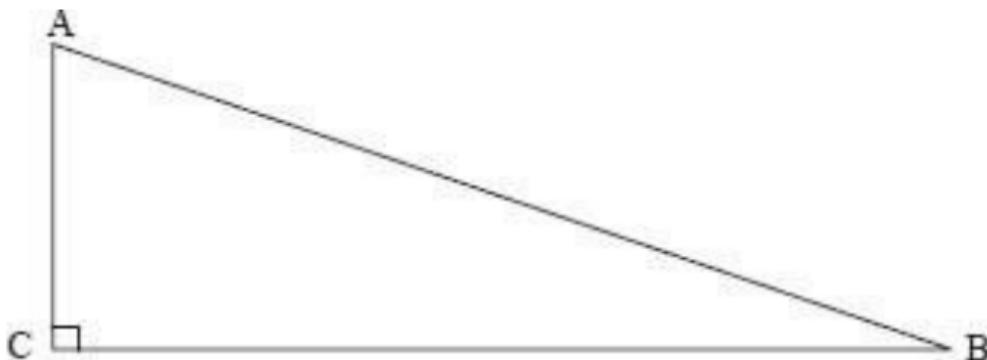
$$20^2 x^2 + 196 = 400 x^2$$

$$= 400 - 196 = 204$$

$$\therefore x = \sqrt{204}$$

Exercise 8.1

1. The figure shows a triangle ABC which is right-angled at C.



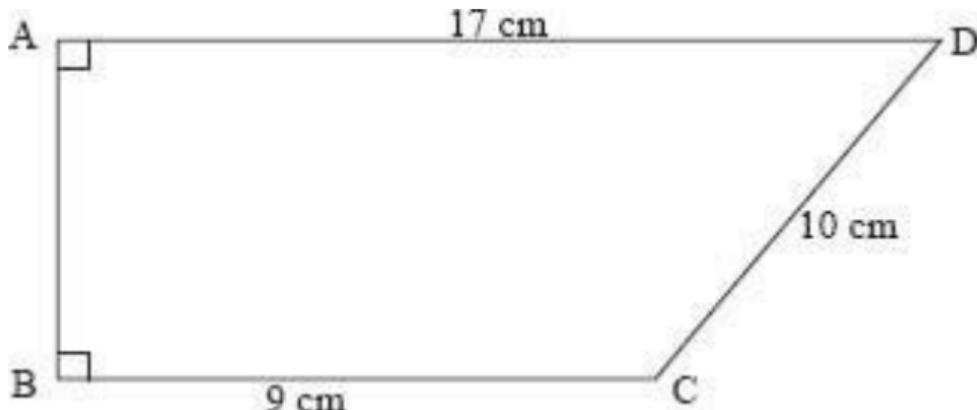
Find:

- (a) AB when AC = 5 cm and BC = 12 cm.
- (b) AC when AB = 20 cm and BC = 16 cm.
- (c) BC when AC = 10 cm and AB = 26 cm.
- (d) AB when AC = 5 m and BC = 6 m.

2. Calculate the length of the diagonal of a rectangle whose sides are 6 cm and 8 cm long.
3. Find the length of the diagonal of a square of side 4 cm.
4. Find the height of an isosceles triangle if the equal sides are each 26 cm and the base is 48 cm long.
5. Find the length of a side of a square whose diagonal is 6 cm.
- 6.

- (a) Determine the length of a side of a rhombus whose diagonals are 18 cm and 24 cm long.
- (b) The shorter diagonal of a rhombus of side 25 cm is 30 cm. Find the length of the other diagonal.
7. Find the height of an equilateral triangle of side 10 cm.

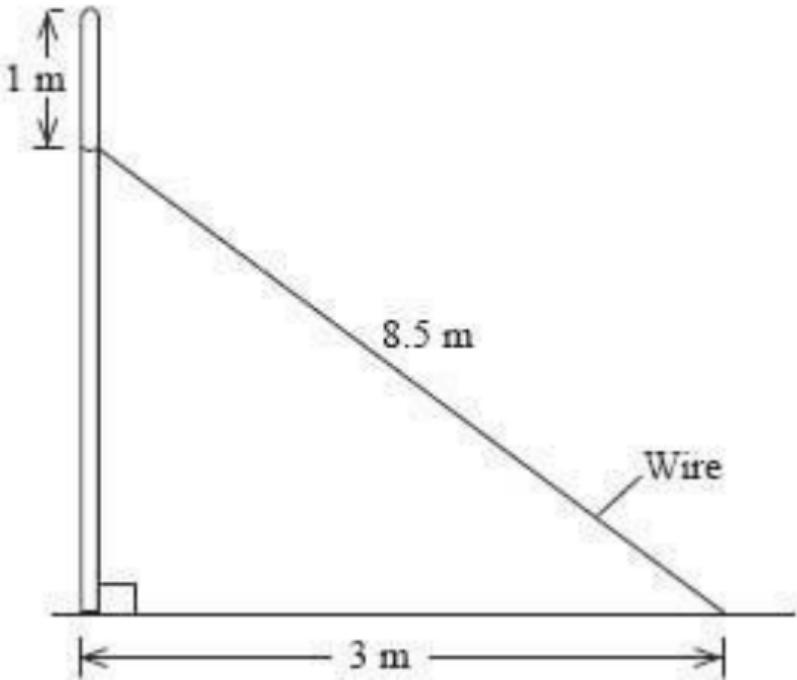
8. The figure below shows a trapezium ABCD in which side AB is perpendicular to both AD and BC. Side AD = 17 cm, DC = 10 cm and CB = 9 cm. Calculate the length AB.



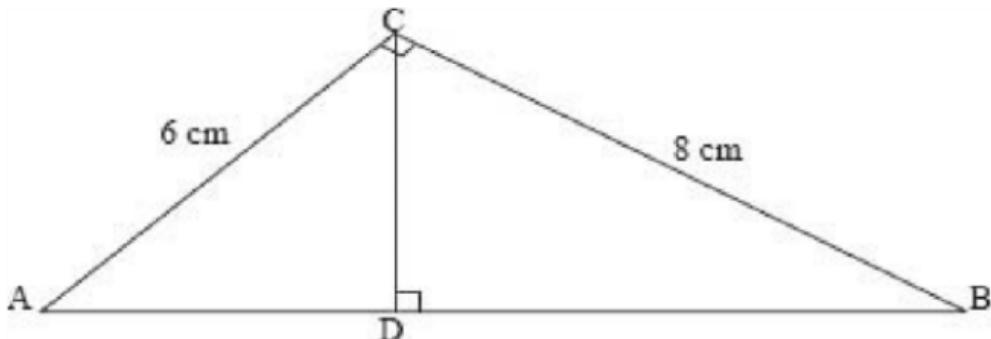
9. An electricity pole is supported to stand vertically by a tight wire as

shown below. Find the height of the pole.

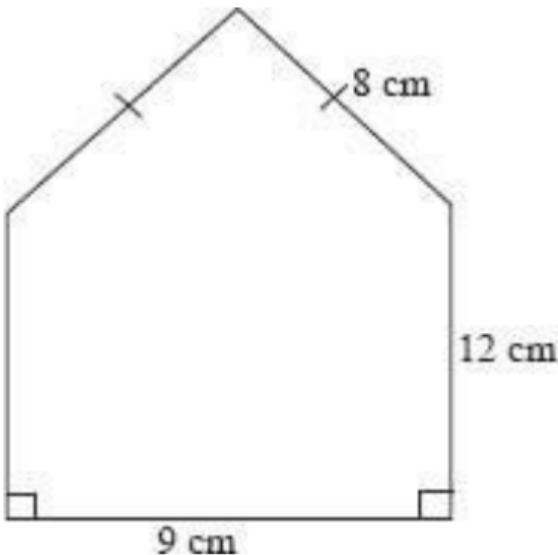
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10. The figure below shows a triangle ABC which is right-angled at C. $CB = 8 \text{ cm}$ and $AC = 6 \text{ cm}$. Find CD.



11. The figure below represents the cross section of a building. The building is 9 metres wide and 12 metres high. If the slant edge measures 8 metres, calculate the height from the floor to its roof.



12. A ship leaves port Q and moves straight due West to port R covering a distance of 320 km. Port R is 180 km to the North of port S. Find the shortest distance in km between

port S and Q. (Give the answer to 4 s.f).

Chapter 9

TRIGONOMETRIC RATIOS

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9.1: Tangent of an Acute Angle

The angle of elevation to the top of a flag post 9 m high as viewed by an observer 10 m away from the post can be found by scale drawing as shown in figure 9.1. In the figure, the angle of elevation is marked x .

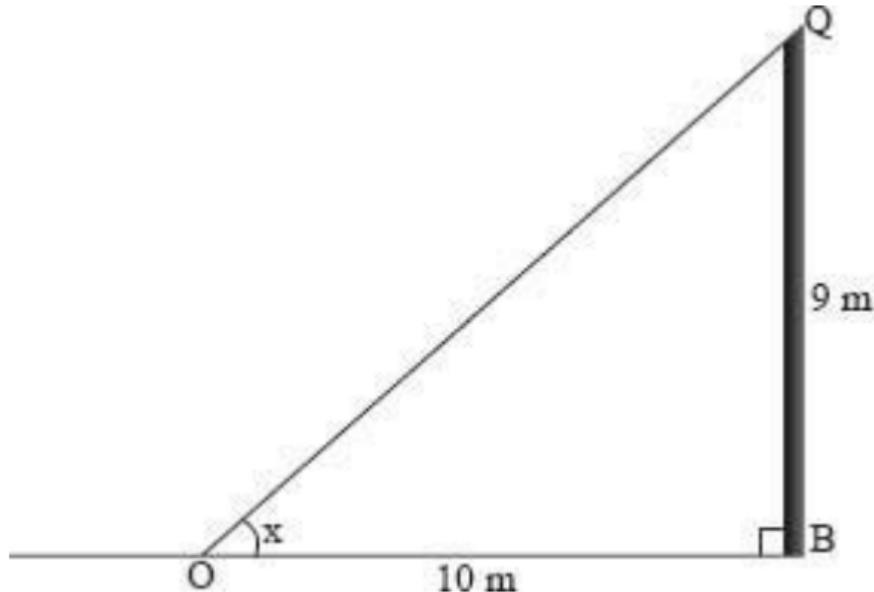


Fig. 9.1

In figure 9.2, OB is produced to V and OQ to T. A and C are two points on OV and

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P and R are points on OT such that AP and CR are parallel to BQ.

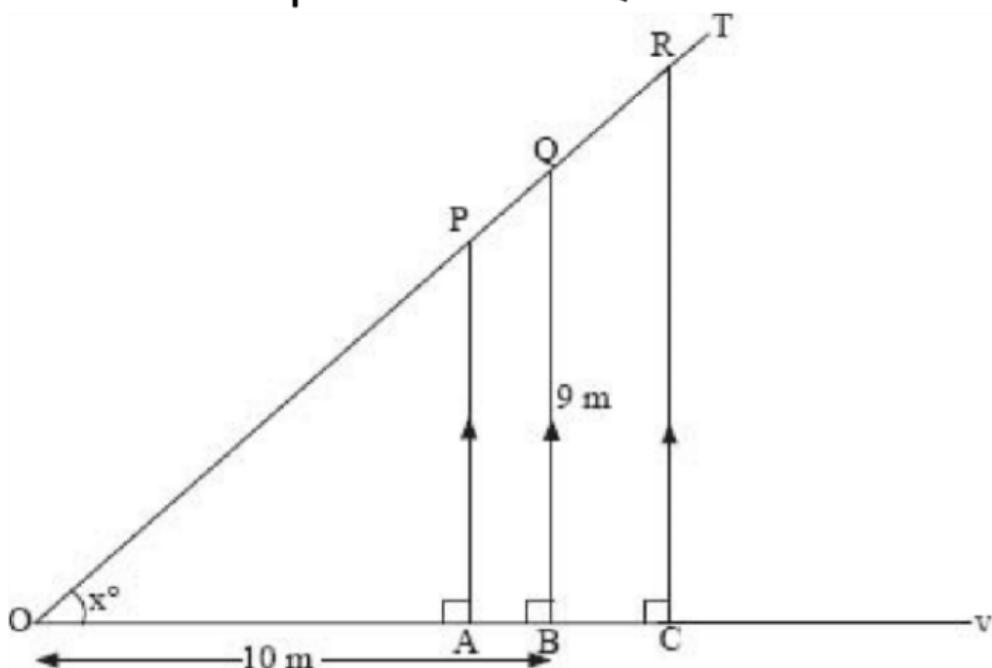


Fig. 9.2

Triangles OPA , OQB and ORC are similar.
Therefore,

$$\frac{PA}{OA} = \frac{QB}{OB} = \frac{RC}{OC} = \frac{9}{10} = 0.9.$$

By drawing any lines parallel to BQ, the ratio $\frac{\text{vertical distance}}{\text{horizontal distance}}$ will be the same for each triangle, that is, 0.9.

This constant ratio, is $\frac{\text{vertical distance}}{\text{horizontal distance}}$ called the tangent of angle TOV.

Therefore, tangent of $x^\circ = 0.9$.

Note:

Tangent is abbreviated as 'tan'

Therefore, tangent of $x^\circ = 0.9$ is written as $\tan x^\circ = 0.9$

The tangent of an angle depends on the size of the angle only. Figure 9.3 shows a right-angled triangle ABC, in which $\angle CAB = \theta^\circ$.

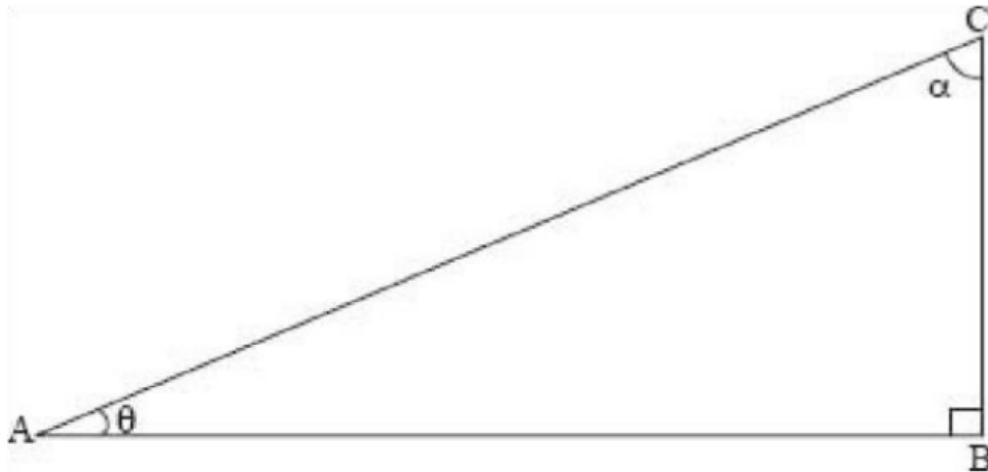


Fig. 9.3

The vertical side CB is opposite angle θ .
The horizontal side AB is adjacent to angle θ .

In this case, $\tan \theta^\circ = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{CB}{AB}$

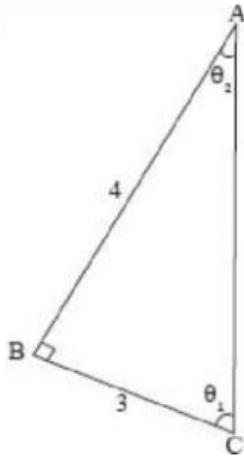
Similarly, express $\tan \alpha^\circ$ in terms of the lengths of the sides of the triangle.

Exercise 9.1

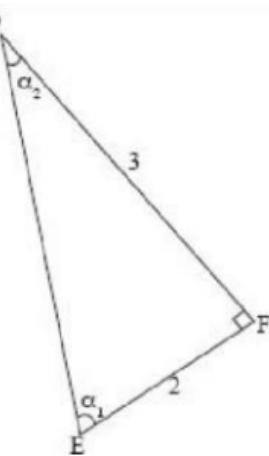
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1. Identify opposite and adjacent sides of the marked acute angles in the figure below.

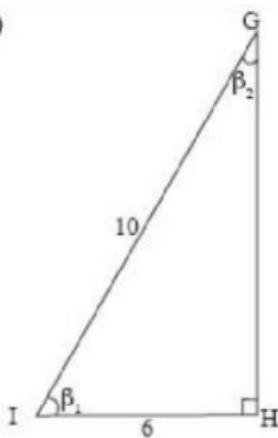
(a)



(b)



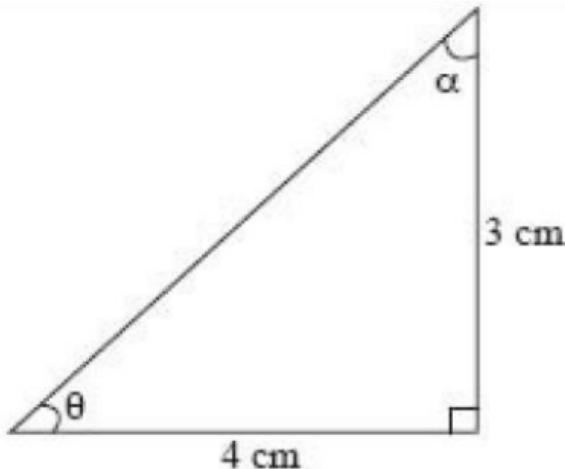
(c)



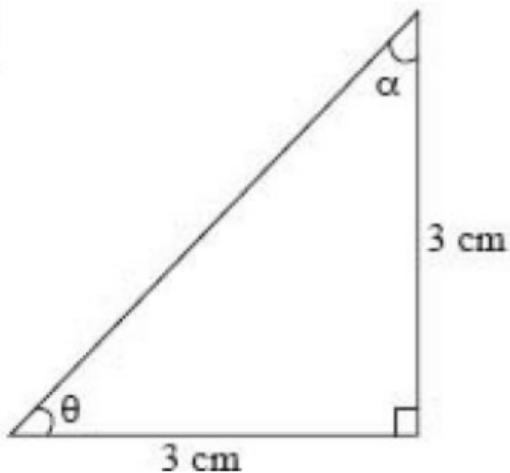
2. Find the tangents of the marked acute angles in question 1 above.
3. Draw the following triangles accurately. Using your diagram:

- (a) measure the indicated angles in degrees.
- (b) find the tangents of the indicated angles using the given measurements.

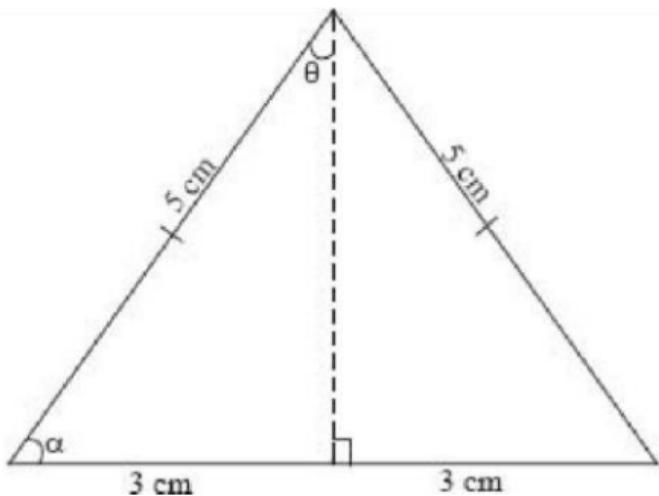
(i)



(ii)



(iii)



9.2: Table of Tangents

Figure 9.4 shows a right-angled triangle ABC in which $AB = 10 \text{ cm}$ and $\angle CAB = 35^\circ$.

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By scale drawing, it is found that $BC = 7$ cm and therefore $\tan 35^\circ = 0.7$.

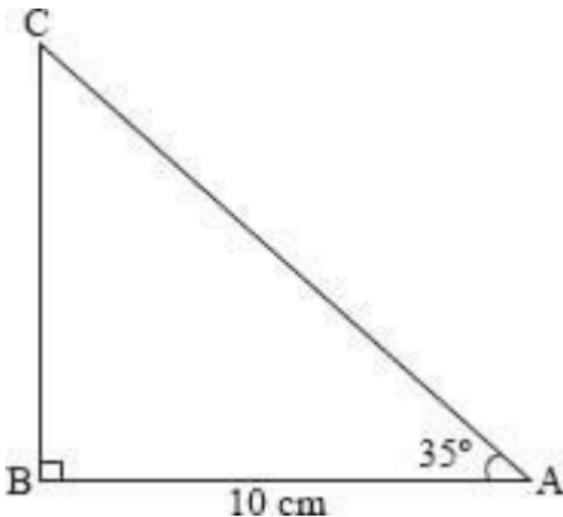


Fig. 9.4

Scale drawing is one method of obtaining the tangent of an angle. Special tables have been prepared and

can be used to obtain tangents of acute angles (see tables of natural tangents in your mathematical tables). The technique of reading tables of tangents is similar to that of reading tables of logarithms or square roots.

- (i) In the tables of tangents, the angles are expressed in decimals and degrees or in degrees and minutes.

(ii) One degree is equal to $60'$ (60 minutes). Thus, $30' = 0.50^\circ$, $54' = 0.9^\circ$ and $6' = 0.1^\circ$.

Note:

From the table, the values of tangents increase abruptly as the angles approach 90° .

Example 1

Find the tangent of each of the following angles from the table:

(a) 42°

(b) 42.75°

(c) $42^\circ 47'$

Solution

From the tables:

(a) $\tan 42^\circ = 0.9004$

(b) Using decimal tables, $\tan 42.7 = 0.9228$. From the difference column under 5, read 0.0016.

Therefore, $\tan 42.75^\circ = 0.9228 +$

0.0016

= 0.9244

$$(c) \left(\frac{47}{60} \right)^{\circ} = 0.78^{\circ}$$

Therefore, $\tan 42^{\circ} 47' = \tan 42.78^{\circ}$

= 0.9228 +

0.0027

= 0.9255 Example 2

In $\triangle PQR$, $\angle PRQ = 90^{\circ}$, $\angle PQR = 36^{\circ}$ and $RQ = 4$ cm. Calculate PR.

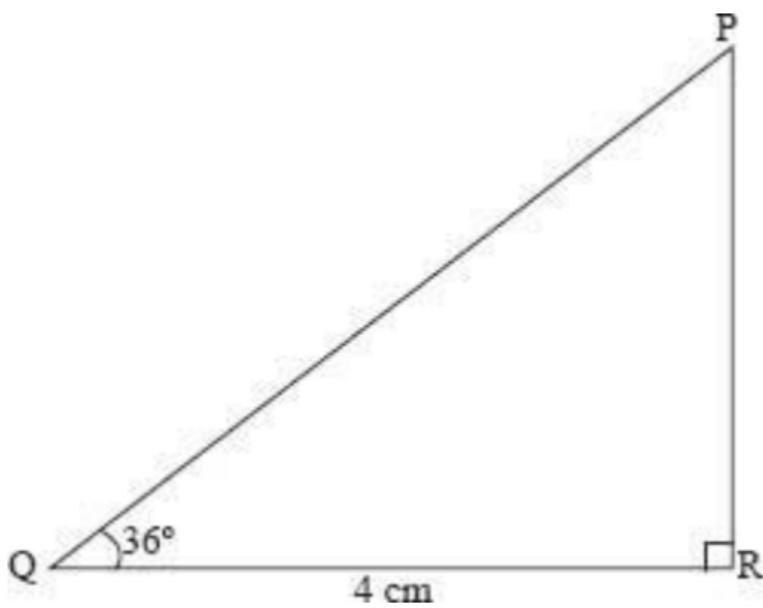


Fig. 9.5

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Solution Tan

$$36^\circ = \frac{\text{opp}}{\text{adj}} 4$$
$$= \frac{PR}{4}$$

$$\tan 36^\circ = PR$$

$$\text{Therefore, } PR = 4 \times 0.7265$$

$$= 2.9060 \text{ cm}$$

Example 3

In figure 9.6, $AB = 5 \text{ cm}$, $BC = 12 \text{ cm}$ and $\angle ABC = 90^\circ$. Find x .

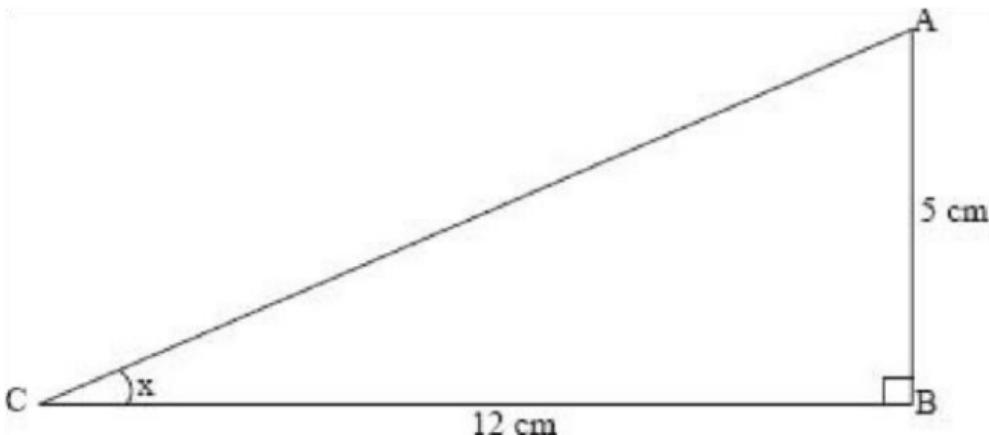


Fig. 9.6

Solution $\tan x =$

$$\frac{AB}{CB} = \frac{5}{12} = 0.4167$$

Note
that 0.4167 cannot be

read directly from the tables of tangents.

Therefore, look for a number nearest to 0.4167 from the tables. In this case, the nearest number is 0.4163. The angle whose tangent is 0.4163 is 22.6° .

The difference between 0.4167 and 0.4163 is 4. From the difference column in the tangent tables, the nearest number to 4 is 3 which gives a difference of 0.02.

Adding 0.02 to 22.6 gives 22.62°

Therefore, the angle whose tangent is 0.4167 is 22.62°

Thus, $x = 22.62^\circ$.

Example 4

In figure 9.7, ABCD is a rhombus whose diagonals BD and AC intersect at O. If $BD = 10 \text{ cm}$ and $AC = 6 \text{ cm}$, find the angles of the rhombus:

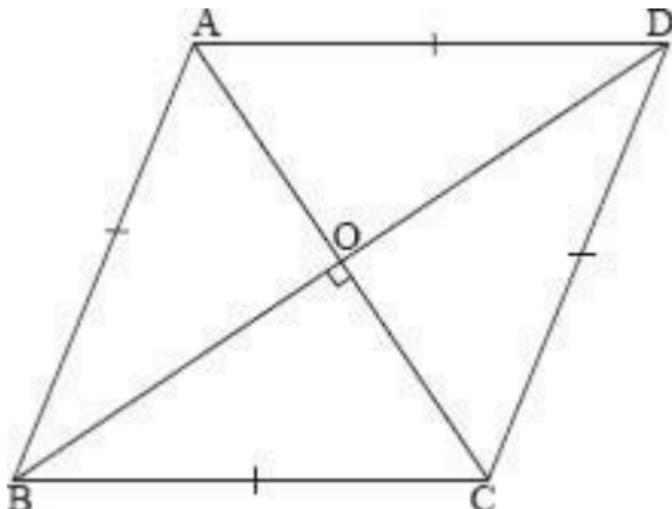


Fig. 9.7

Solution

Consider ΔBOC in figure 9.8 from figure 9.7.

$$\tan \theta = \frac{5}{3} = 1.6667$$

$$\text{Therefore, } \theta = 59.04^\circ$$

$$\tan \alpha = \frac{3}{5} \quad \alpha = 30.96^\circ$$

Therefore, in figure 9.7;

$$\angle BCD = 59.04^\circ \times 2$$

$$= 118.08^\circ$$

$$\angle CBA = 30.96^\circ$$

$$= 61.92^\circ$$

$$\angle BCD = \angle BAD$$

$$= 118.08^\circ$$

$$\angle CBA = \angle ADC$$

$$= 61.92^\circ$$

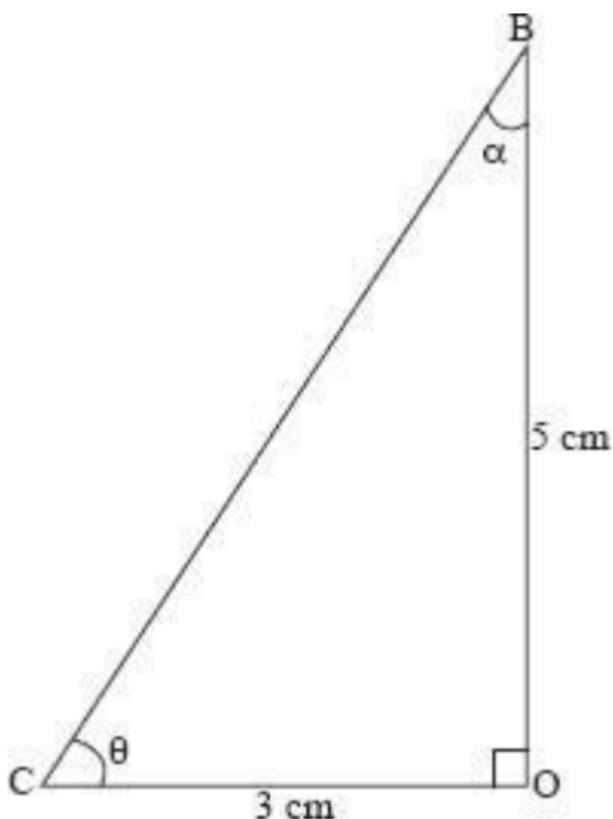


Fig. 9.8

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Exercise 9.2

1. Express each of the following in degrees and minutes:
 - (a) 15.3°

()

()

(b) 25.75° c

$30\frac{1}{2}^\circ$

$34\frac{3}{4}^\circ$ (d)

2. Read from tables the tangent of:

(a) 70.53°

(b) 18.73°

(c) $55^\circ 53'$

(d) $63^\circ 12'$

()

()

(e) $81^\circ 08'$

(f) $10^\circ 30'$

(g) $75^\circ 58'$

(h) $89^\circ 54'$

(i) $29^\circ 34'$

(j) $87^\circ 50'$ k $78^\circ 08'$

(l) $48^\circ 42'$

(m) 84°

(n) $43^\circ 51'$

(p) 57.17°

()

()

3. Find from the tables the angle whose tangent is:

(a) 0.3317

(b) 0.6255

(c) 1.6391

(d) 0.4444

(e) 0.0122

(f) 0.8799 g 0.1867 (h) 0.5903

(i) 5.1006

()

()

(j) 1.0000

(k) 0.2839

(l) 2.0011

(m) 3.6703

(n) 0.7400

(p) 40.92

4. A boy on top of a vertical wall 13.5 m high throws a ball down and notices that the ball hits a stone on the ground 18 m away from the foot

of the wall. Calculate the angle of depression of the stone from the top of the wall.

5. A ladder leans against a wall so that its foot is 2.5 m away from the foot of the wall and its top is 4 m up the wall. Calculate the angle it makes with the ground.
6. A pedestrian notices that a tower is at a horizontal distance of 30 m away from him and that the angle of elevation of the top of the tower

i

from where he is, is 35° . Find the height of the tower.

7. A tree casts a shadow 20 m long. Find the height of the tree if the angle of elevation of the top of the tree from the top of the shadow is 31° .
8. In a right-angled triangle, the shorter sides are 4.5 cm and 9.2 cm long. Find the sizes of its acute angles.
9. One of the diagonals of a rhombus is

i

28 cm long and one of its angles is 70° . Calculate the length of the second diagonal and hence the side of the rhombus (two possible answers).

10. From a window 25 m above a street, the angle of elevation of the top of a wall on the opposite side is 15° . If the angle of depression of the base of the wall from the window is 35° , find:

i

- (i) the width of the street.
- (ii) the height of the wall on the opposite side.

11. An aircraft flying into an airport calls the control tower and says it is 500 m above the tower. If its horizontal distance from the tower is 8 km, calculate its angle of elevation from the top of the tower.

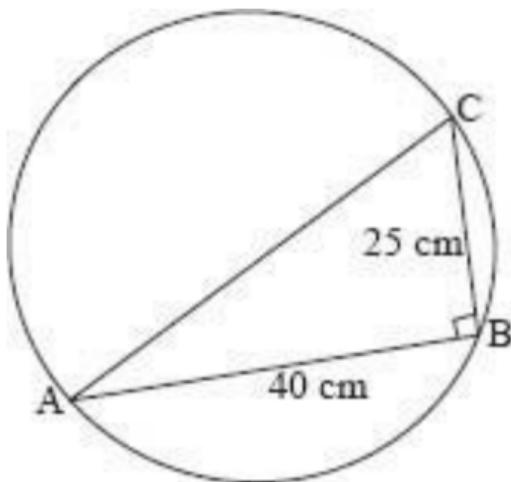
12. Two buses X and Y are moving towards each other on a straight East-West main road. At a

i

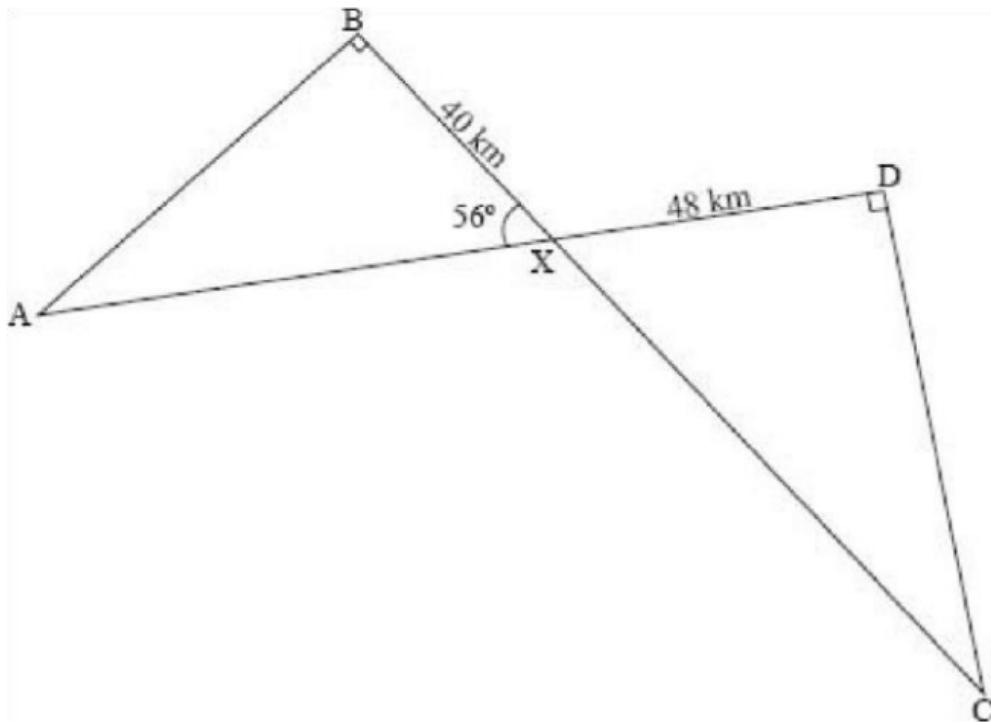
particular instant, another bus Z on another road is 50 km South of the main road and the bearings of X and Y from Z are 330° and 360° respectively. Find the distance between X and Y.

13. ABC is a right-angled triangle inscribed in a circle ABC with AC as the diameter. See the figure below. If BC = 25 cm and AB = 40 cm, calculate angles BAC and ACB.

i

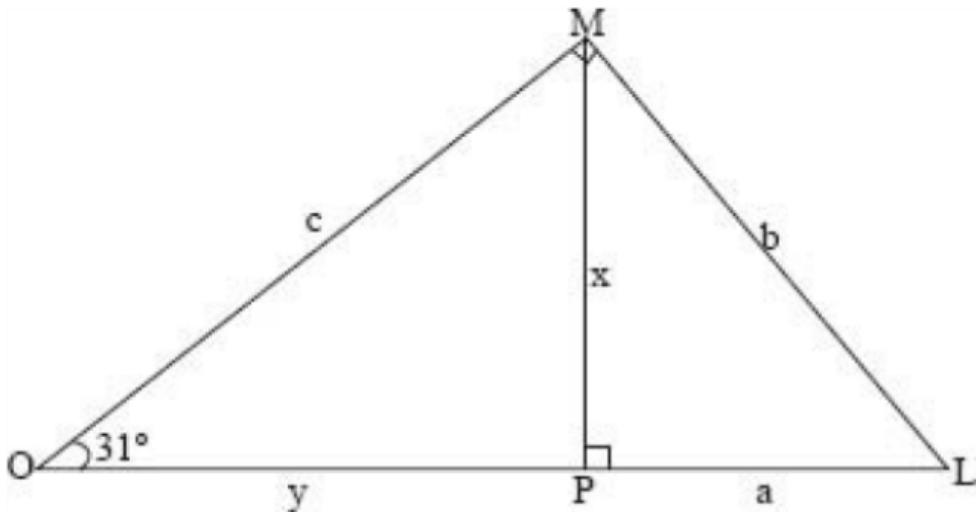


14. The figures below shows five towns A, B, C, D and X connected by straight roads. $BX = 40 \text{ km}$ and $XD = 48 \text{ km}$. If $\angle AXB = 56^\circ$, calculate the distances AB and BC.

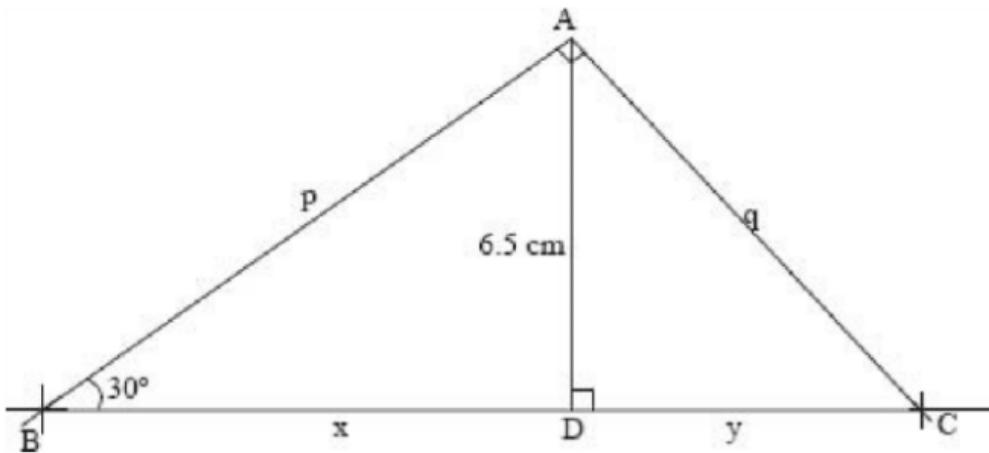


15. Calculate b , x , c and y in centimetres if $a = 24$ cm.

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16. The figure below shows a rightangled triangle ABC. AD is perpendicular to BC. If $\angle ABC = 30^\circ$ and $AD = 6.5 \text{ cm}$, find the sides marked x, y, p and q.



9.3: Sine and Cosine of an Acute Angle

In figure 9.9, $\angle A$, $\angle B$ and $\angle C$ are perpendicular to OY and $\angle XOY = \theta$.

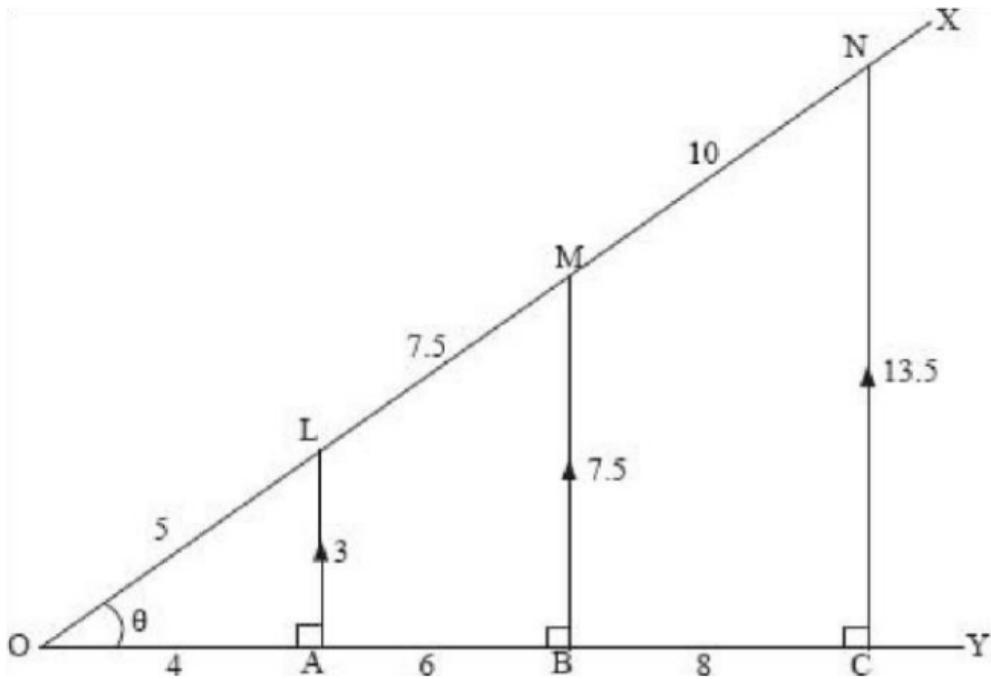


Fig. 9.9

(a) Copy the following and fill in the blank spaces:

(i)

$$\frac{AL}{OL} = \frac{3}{5} \quad (ii)$$

$$\frac{BM}{OM} = \text{---} \quad (iii)$$

$$\frac{CN}{ON} = \text{---} \quad \text{Note:}$$

$$\frac{AL}{OL} = \frac{BM}{OM} = \frac{CN}{ON} = \frac{3}{5}$$

This constant value is obtained by taking the ratio of the side opposite the angle 8 to the hypotenuse side in each case. This ratio is called the sine of angle θ , abbreviated as $\sin \theta$.

(b) Copy the following and fill in the blank spaces:

$$(i) \frac{OA}{OL} = \underline{\hspace{2cm}}$$

$$(ii) \frac{OB}{OM} = \underline{\hspace{2cm}}$$

$$(iii) \frac{OC}{ON} = \underline{\hspace{2cm}}$$

Note that: $\frac{OA}{OL} = \frac{OB}{OM} = \frac{OC}{ON} = \frac{4}{5}$

This constant value is obtained by taking the ratio of the side adjacent to angle 6 to the hypotenuse in each case. This constant ratio is called the cosine of angle θ , abbreviated as $\cos \theta$.

In general, given a right-angled triangle ABC as shown in figure 9.10 below:

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$= \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$\cos \theta =$$

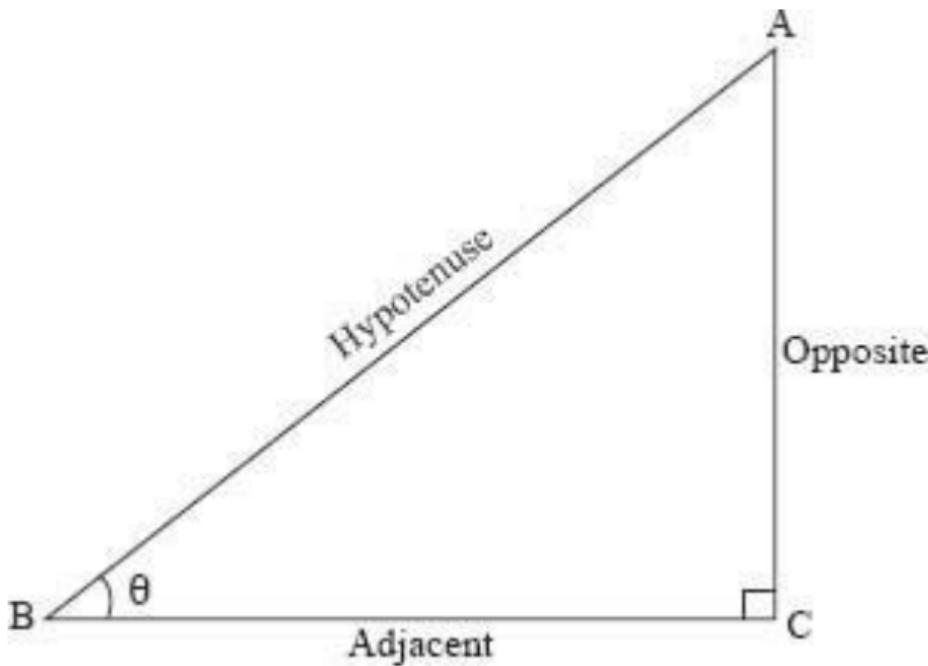


Fig. 9.10

Example 5

In figure 9.11, $AB = 5 \text{ cm}$, $CB = 12 \text{ cm}$ and $\angle ABC = 90^\circ$. Calculate:

(a) $\sin x$.

(b) $\cos x$.

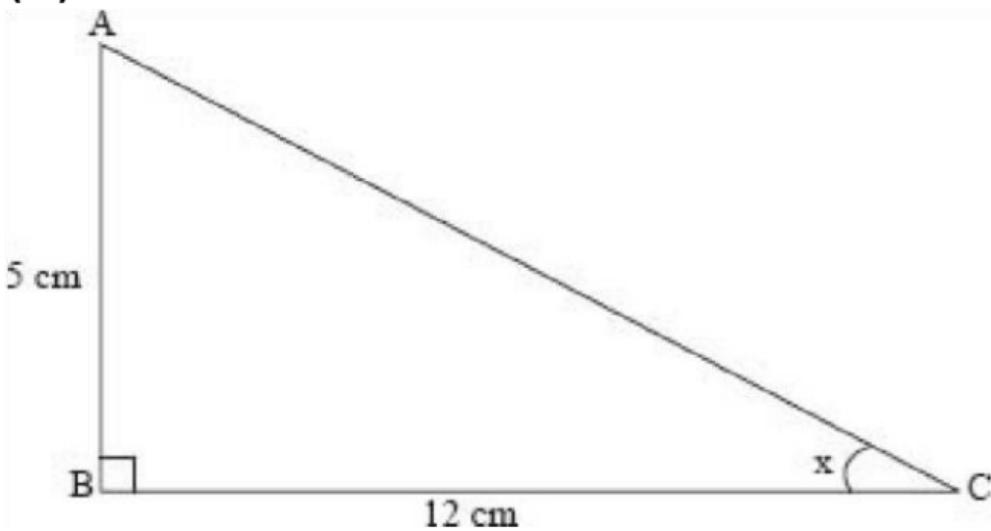


Fig. 9.11

Solution

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$$(a) \sin x = \frac{\text{opp}}{\text{hyp}} = \frac{AB}{AC} = \frac{5}{AC}$$

$$\text{But } AC^2 = 12^2 + 5^2$$

$$= 144 + 25$$

$$= 169$$

Therefore, $AC = 13 \text{ cm}$

$$\text{Thus, } \sin x = \frac{5}{13}$$

$$= 0.3846$$

$$\begin{aligned}(b) \cos x &= \frac{\text{adj}}{\text{hyp}} \\&= \frac{CB}{AC} \\&= \frac{12}{13} \\&= 0.9231\end{aligned}$$

Example 6

Find each of the following by scale drawing:

(a) (i) $\sin 30^\circ$

(ii) $\cos 30^\circ$

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(b) The angle whose:

- (i) sine is 0.7
- (ii) cosine is 0.9

Solution

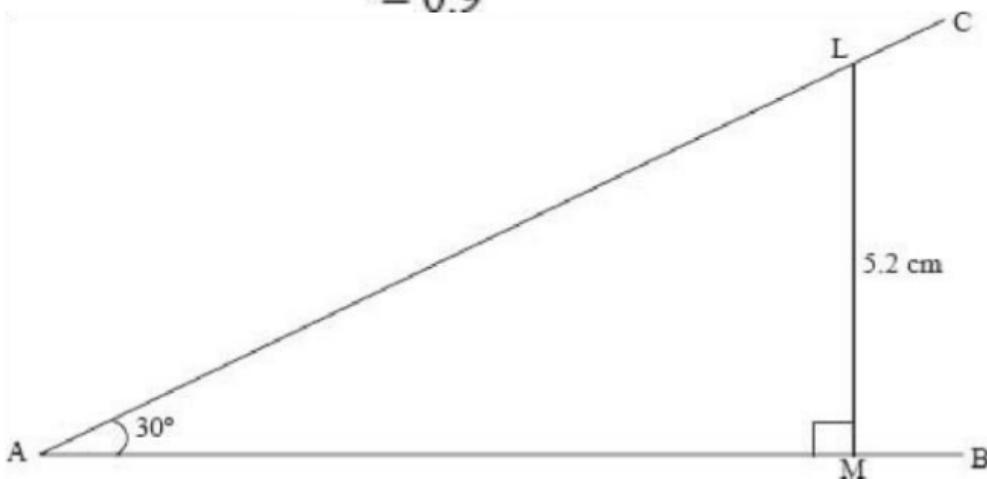
- (a) • Draw a line AB of any length.
- Measure $\angle CAB = 30^\circ$ (see figure 9.12)
- From a point L on AC, draw a perpendicular to AB to meet AB at M.

In this case, $LM = 5.2$ cm and $AL =$

10.4 cm.

$$\frac{LM}{AL} = \frac{5.2 \text{ cm}}{10.4 \text{ cm}} \quad \sin 30^\circ = \\ = 0.5$$

$$\cos 30^\circ \quad \frac{AM}{AL} = \frac{9.0}{10.4} \\ = 0.9 \quad =$$



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Fig. 9.12

Using the same method as in (a) find:

(i) $\sin 65^\circ$

(ii) $\cos 65^\circ$

(b) (i) $0.7 = \frac{7}{10}$

- Draw a line XY of any length.
- Construct a line LM, 7 cm long and perpendicular to XY at M.
- With centre L and a radius of 10 cm, draw an arc to cut XY at N.

- By measuring, $\angle MNL = 44.5^\circ$.

Therefore, $\sin 44.5^\circ = 0.7$

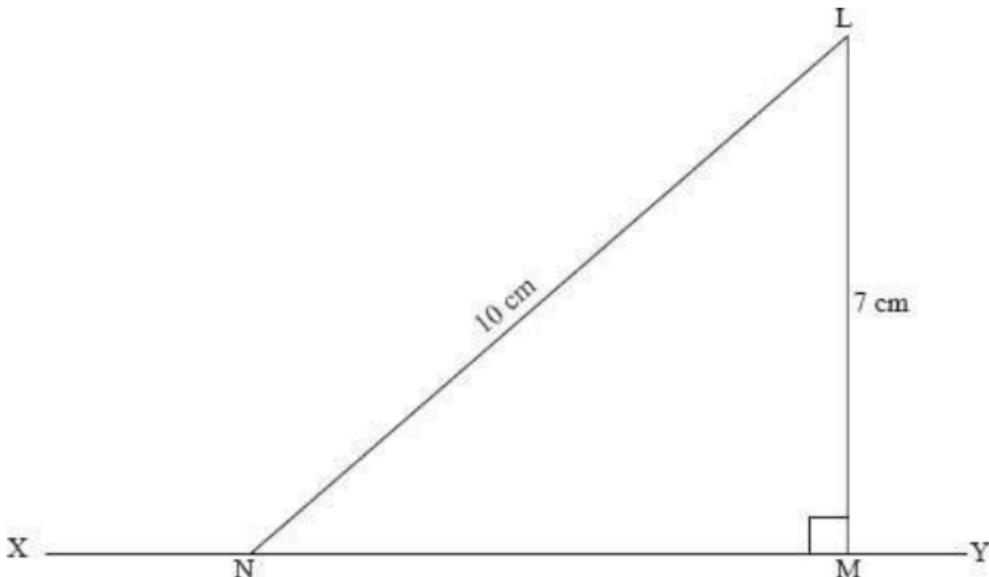


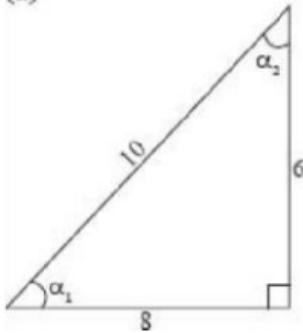
Fig. 9.13

Follow the same procedure to find the angle whose cosine is 0.9.

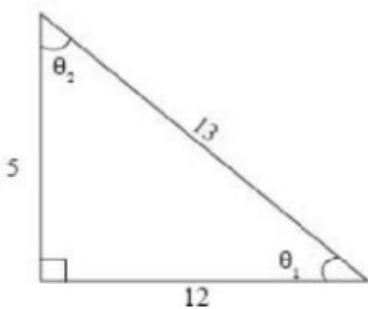
Exercise 9.3

- Find the cosine and sine of each in the following marked angles. (Units are in centimetres)

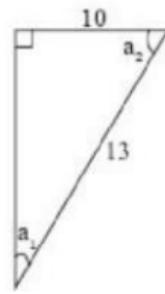
(a)



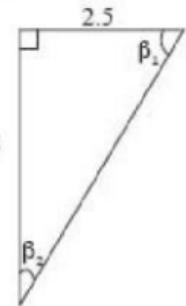
(b)



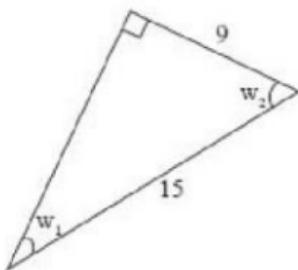
(c)



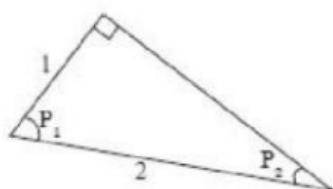
(d)



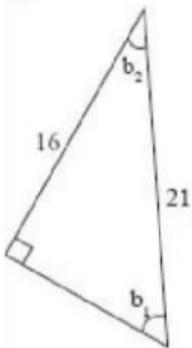
(e)



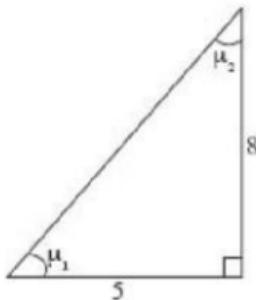
(f)



(g)



(h)



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2. (a) Find by scale drawing the angle whose sine is:

(i) $\frac{7}{12}$

(ii) 0.5

(iii) 0.30

(iv) 0.6

(v) 0.7

$\frac{3}{5}$ (vi)

$\frac{4}{7}$ (vii)

$\frac{2}{5}$ (viii)

$\frac{5}{6}$

(ix)

(x) 0.42

(b) Find by scale drawing the angle whose cosine is:

(i) $\frac{1}{3}$

(ii) $\frac{7}{9}$

(iii) $\frac{1}{5}$

(iv) 0.6

$\frac{5}{13}$

(iv)

$$\frac{4}{5}$$

$$\frac{10}{17}$$

(vi)

(vii)

(viii) 0.72

(ix) 0.24

(x) 0.56

3. (a) If $\sin \theta = \frac{3}{5}$, find:

(i) $\cos \theta$

(ii) $\tan \theta$

- (b) If $\sin \theta = \frac{1}{2}$ find $\tan \theta$.
- (c) If $\tan \theta = \frac{1}{\sqrt{3}}$ find $\cos \theta$.
- (d) If $\cos \theta = \frac{1}{\sqrt{2}}$ find $\sin \theta$.

9.4: Tables of Sines and Cosines

The sine and cosine tables are read and used in the same way as the tangent

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tables. As the angles increase from 0° to 90° :

- (i) the values of their sines increase from 0 to 1
- (ii) the values of their cosines decrease from 1 to 0.

Therefore, the values in the difference column of cosine tables have to be subtracted and those in the difference columns of the sine tables have to be added.

Example 7

Read the sine and cosine values of the following angles from the tables.

- (a) 47.3°
- (b) 69.55°

Solution

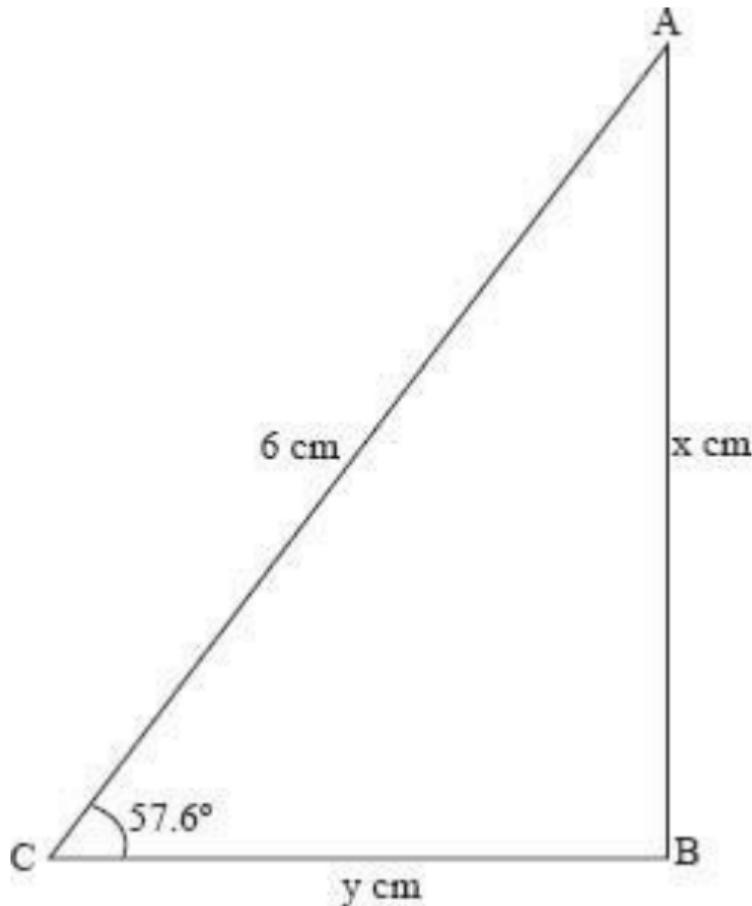
(a) $\sin 47.3^\circ = 0.7349$ $\cos 47.3^\circ = 0.6782$

$$(b) \sin 69.55^\circ = 0.9370$$

$$\cos 69.55^\circ = 0.3494$$

Example 8

Find x and y in figure 9.14.



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Fig. 9.14

Solution sin

$$57.6^\circ = \frac{x}{6}$$

$$6 \sin 57.6^\circ = x \text{ Therefore, } x = 6 \times 0.8443$$

$$= 5.0658$$

$$= 5.066(\text{to 4 s.f}) \cos$$

$$57.6^\circ = \frac{y}{6}$$

$$6 \cos 57.6^\circ = y$$

$$\text{Therefore, } y = 6 \times 0.5358$$

$$= 3.2148$$

$$= 3.215 \text{ (to 4 s.f)}$$

Exercise 9.4

1. Find from tables the angle whose sine is:
 - (a) 0.3367
 - (b) 0.5871
 - (c) 0.0523

()

- (d) 0.8500
 - (e) 0.1822
 - (f) 0.9834
 - (g) 0.5012
 - (h) 0.2518
2. Find from the tables the angle whose cosine is:
- (a) 0.1643
 - (b) 0.7196
 - (c) 0.9970
 - (d) 0.8660

()

(e) 0.4009

(f) 0.9481

(g) 0

(h) 0.7371

3. Read from the tables the sine of:

(a) 31.46°

(b) $77^\circ 34'$

(c) $52^\circ 9'$

(d) $66^\circ 31'$

()

(e) 6.76°

(f) 40.13°

(g) $26^\circ 47'$

(h) 13.07°

4. Read from the tables the cosine of:

(a) $79^\circ 42'$

(b) 24.23°

(c) $5^\circ 37'$

(d) 60°

(e) $88^\circ 59'$

(f) 55.97°

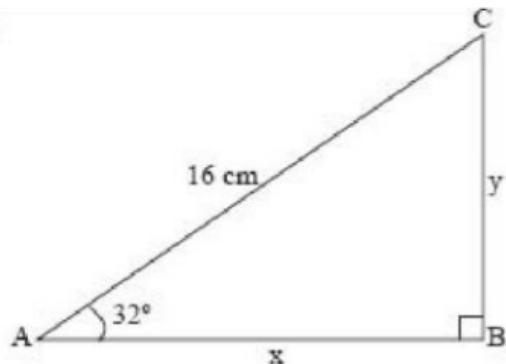
()

(g) 33.33°

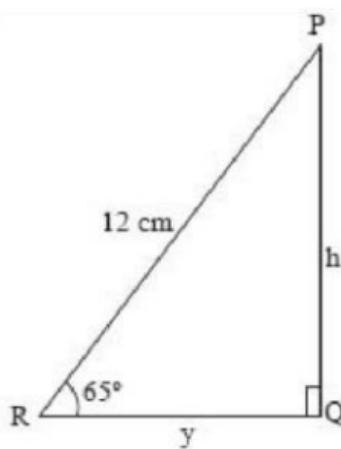
(h) $17^\circ 52'$

5. Find the length of the unknown side
in each of the following figures:

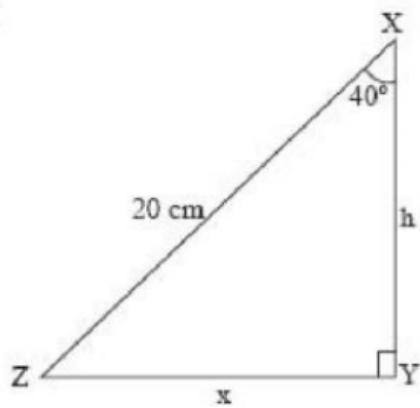
(a)



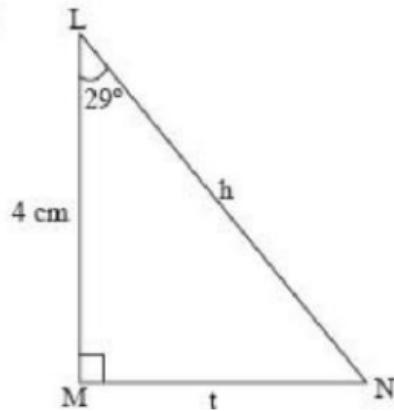
(b)



(c)

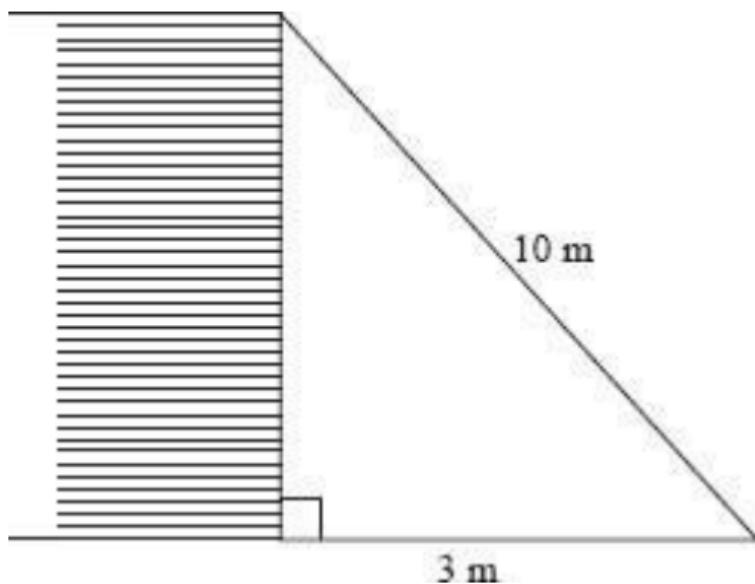


(d)



6. PQRS is a rhombus of side 7 cm and $\angle PQR$ is 65° . Calculate the lengths of PR and QS.
7. A ladder 10 m long leans against a wall as shown in the figure below. If its foot is 3 m from the wall, calculate:
- (a) the vertical distance from the top of the ladder to the ground.

- (b) the angle the ladder makes with the ground.

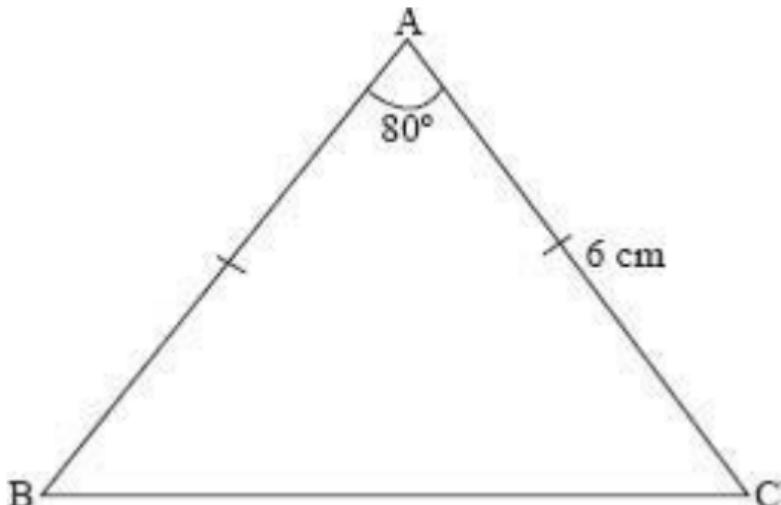


8. From a point P on the ground, 20 m away from the foot of a building, the

angle of elevation of the top of the building is 25° . Find:

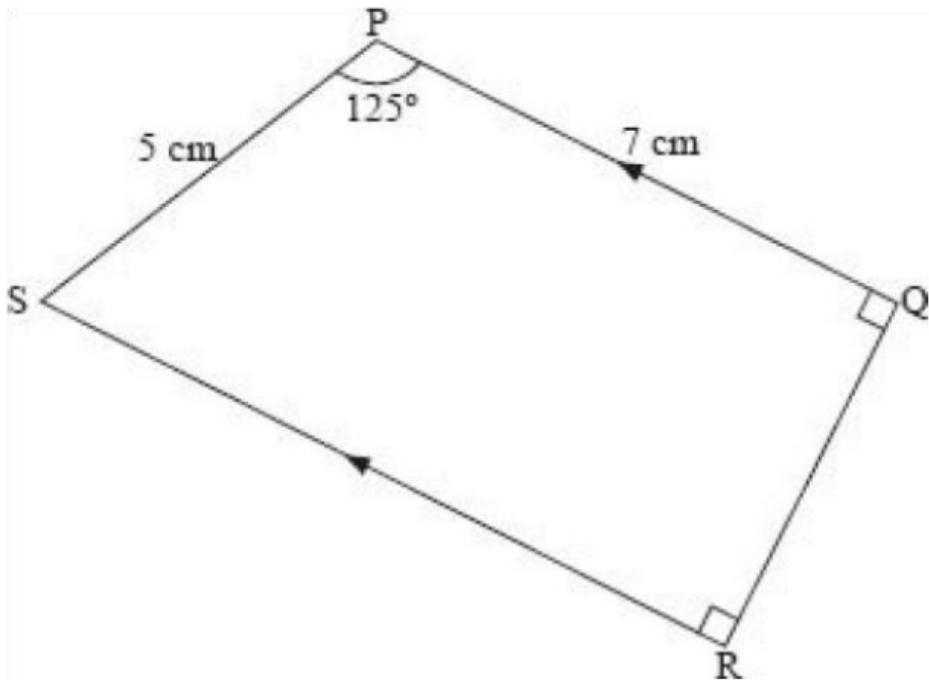
- (a) the height of the building.
- (b) the shortest distance to the top of the building from point P.

9. The figure below shows an isosceles triangle in which $AB = AC = 6 \text{ cm}$. Angle BAC is 80° . Calculate the length of BC .

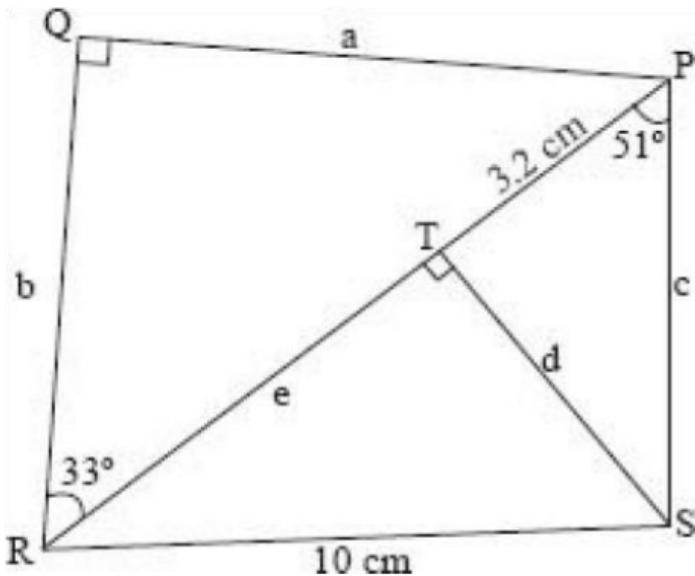


10. The angle of depression of a car from the top of a building 8 m high is 44° . Find the distance of the car from the foot of the building.
11. The figure below shows a trapezium PQRS in which $PQ = 7 \text{ cm}$, $PS = 5 \text{ cm}$, $\angle PQR = \angle QRS = 90^\circ$ and $\angle SPQ =$

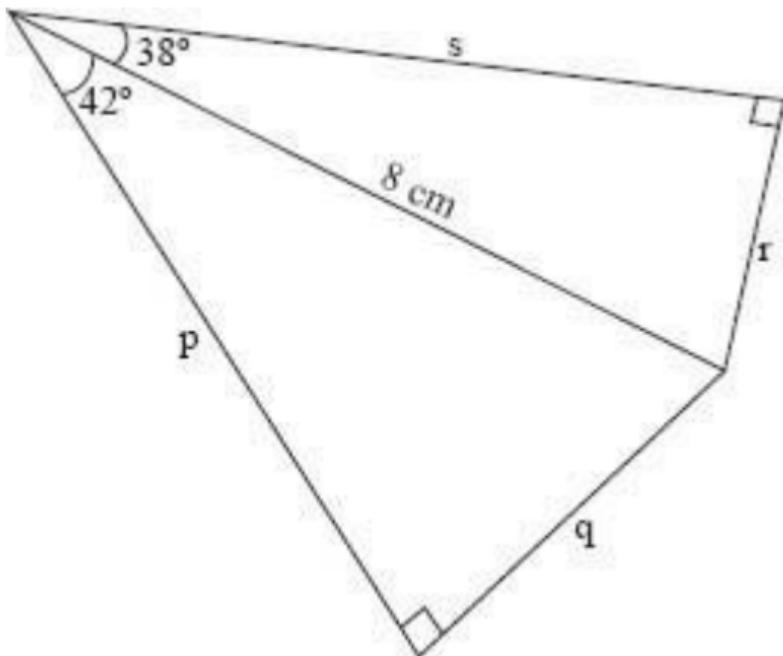
125° . Calculate the lengths QR and RS.



12. In the figure below, $PT = 3.2 \text{ cm}$, $RS = 10 \text{ cm}$, $\angle SPT = 51^\circ$ and $\angle PRQ = 33^\circ$. Calculate lengths of the sides marked a, b, c, d and e.

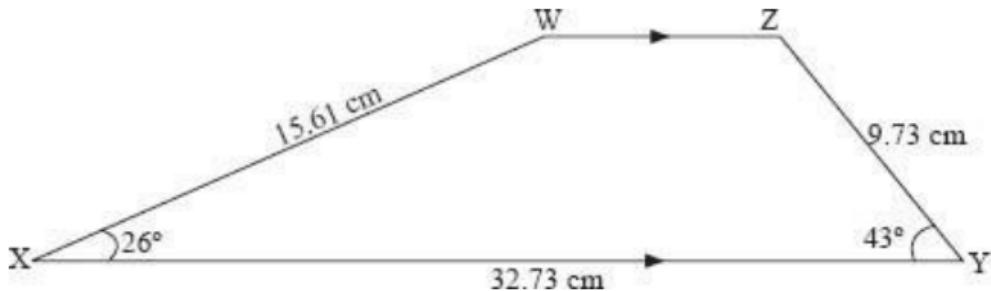


13. Calculate the values of p, q, r and s.



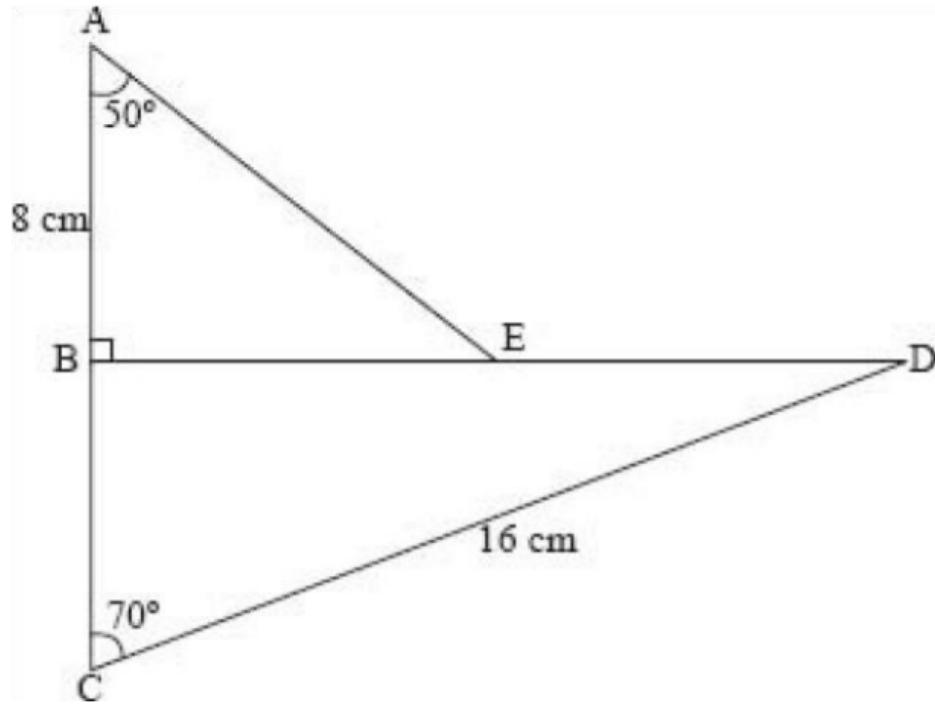
14. The diagonals of a rectangle PQRS are 13 cm long. If $\angle SQR = 27^\circ$, find the dimensions of the rectangle.
15. In the figure below, WXYZ is a trapezium with WZ parallel to XY.

$WX = 15.61 \text{ cm}$, $XY = 32.73 \text{ cm}$, $ZY = 9.73 \text{ cm}$, $\angle WXY = 26^\circ$ and $\angle ZYX = 43^\circ$. Calculate the length of WZ .



16. In the figure below, $AB = 8 \text{ cm}$, CD

$= 16 \text{ cm}$, $\angle ACD = 70^\circ$ and $\angle BAE = 50^\circ$. Calculate ED.



17. The angle at the vertex of a pair of dividers is 54° . The tips of the arms

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are 12 cm apart. If the dividers are held upright, calculate:

(a) the angle between the horizontal and the arms.

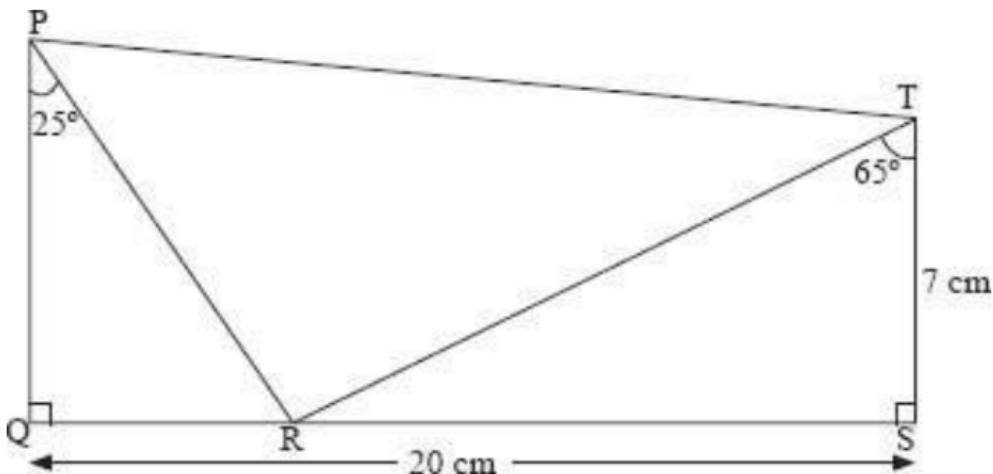
(b) the length of each arm.

(c) the height of the vertex above the horizontal.

18. A regular octagon of side 6 cm is inscribed in a circle. Calculate the diameter of the circle.

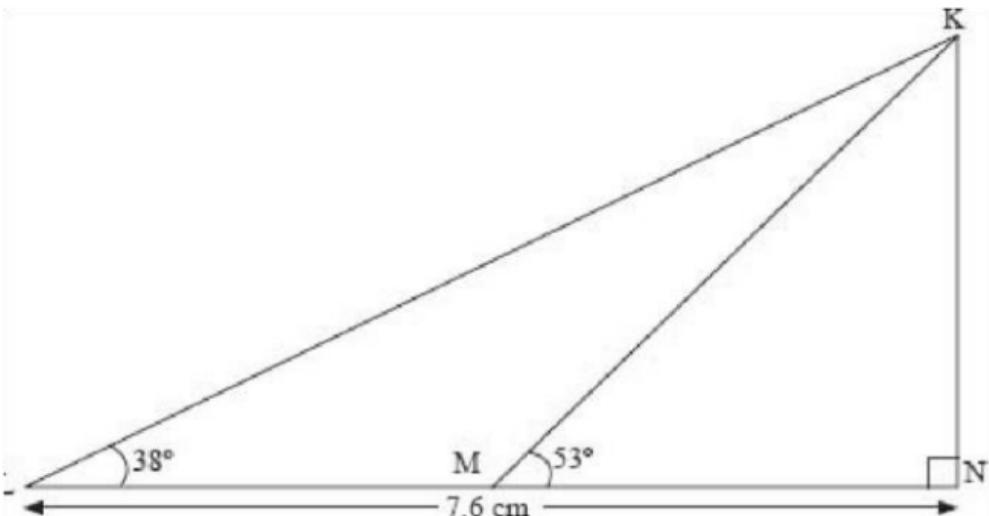
19. In the figure below, $QS = 20$ cm, $TS = 7$ cm, $\angle RTS = 65^\circ$ and $\angle QPR =$

25° . Find the length of PT.



20. In the figure below, $LN = 7.6 \text{ cm}$, $\angle KLM = 38^\circ$, $\angle KMN = 53^\circ$ and $\angle KNL = 90^\circ$. Calculate KM and KL.

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9.5: Sines and Cosines of Complementary Angles

Read from the tables, the values of the following pairs of angles.

- (i) $\sin 60^\circ, \cos 30^\circ$

(ii) $\cos 20^\circ, \sin 70^\circ$

(iii) $\sin 30^\circ, \cos 60^\circ$

Note that: $\sin 60^\circ = \cos 30^\circ$

$\cos 20^\circ = \sin 70^\circ$ $\sin 30^\circ$

= $\cos 60^\circ$

For any two complementary angles x and y , $\sin x = \cos y$ and $\cos x = \sin y$.

Example 9

Find acute angles α and β if:

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$$(a) \sin \alpha = \cos 33^\circ.$$

$$(b) \cos \beta = \sin 3\beta$$

Solution

$$(a) \sin \alpha = \cos 33^\circ$$

$$\text{Therefore, } \alpha + 33^\circ = 90^\circ$$

$$\alpha = 90^\circ - 33^\circ$$

$$= 57^\circ$$

$$(b) \cos \beta = \sin 3\beta$$

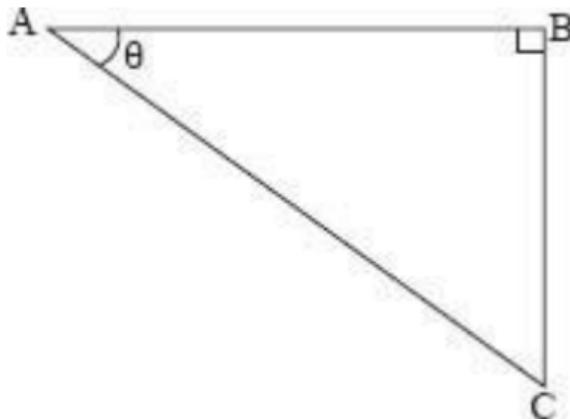
$$\text{Therefore, } \beta + 3\beta = 90^\circ$$

$$4\beta = 90^\circ$$

$$\beta = 22\frac{1}{2}^\circ$$

Exercise 9.5

1. If A and B are complementary angles and $\sin A = \frac{4}{5}$, find $\cos B$.
2. In the figure below, AB = 16 cm and AC = 20 cm. Find:
 - (a) $\sin \theta$.
 - (b) $\cos \theta$.



3. X and B are complementary angles.
If $\sin B = 0.9975$, find X.

4. A and B are complementary angles.
If $A = \frac{1}{2} B$, find:
 - (a) $\sin A$.
 - (b) $\cos A$.

5. Find the acute angle x , given that $\cos x^\circ = \sin 2x^\circ$.

9.6: Trigonometric Ratios of Special Angles (30° , 45° and 60°)

The trigonometrical ratios of 30° , 45° and 60° can be deduced by the use of an isosceles right-angled triangle and an equilateral triangle as follows.

Tangent, Cosine and Sine of 45°

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Figure 9.15 shows a right-angled isosceles triangle PQR. The dimensions are as shown.

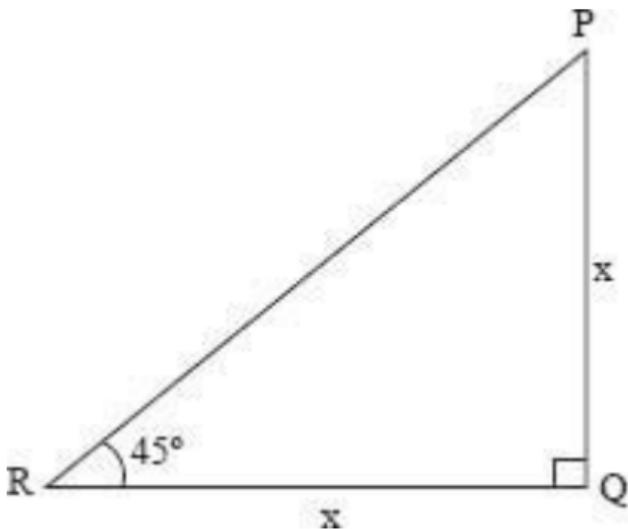


Fig. 9.15

- (i) Find the length of PR in terms of x.

(ii) Find $\cos 45^\circ$, $\sin 45^\circ$ and $\tan 45^\circ$ Note
that:

$$(i) \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$(ii) \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$(iii) \tan 45^\circ = 1$$

Tangent, Cosines and Sine of 30° and
 60°

Figure 9.16 shows an equilateral triangle ABC. AN is the perpendicular bisector of BC. The dimensions are shown.

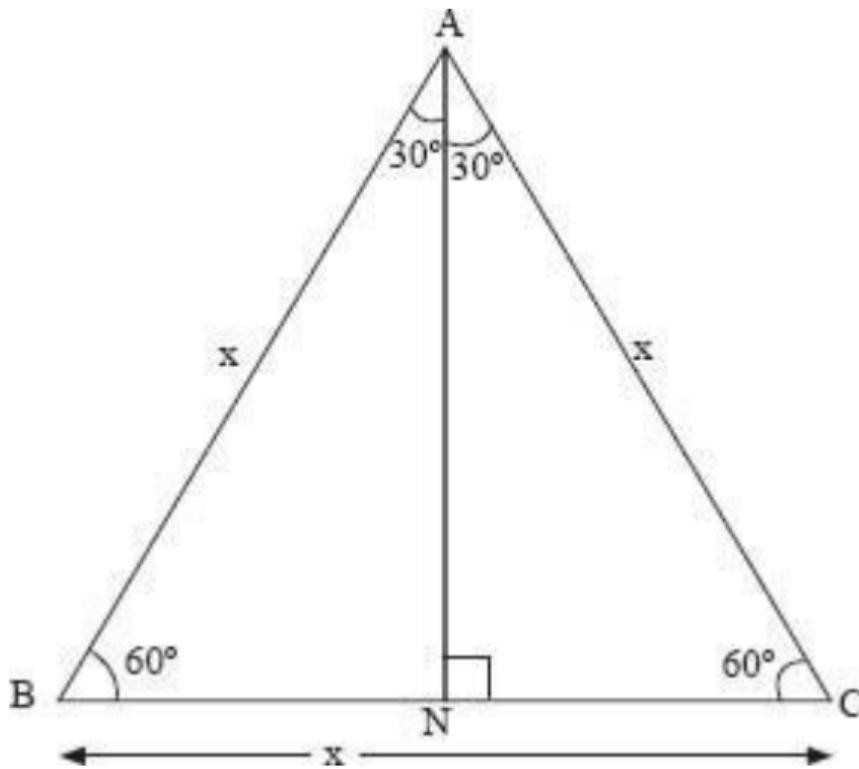


Fig. 9.16

- (i) Find an expression for the length of AN in terms of x.
- (ii) Find $\sin 30^\circ$, $\cos 30^\circ$ and $\tan 30^\circ$
- (iii) Find $\sin 60^\circ$, $\cos 60^\circ$ and $\tan 60^\circ$

Note that:

$$(i) AN = \frac{\sqrt{3}}{2}x$$

$$(ii) \sin 30^\circ = \frac{1}{2}$$

$$30^\circ = \frac{1}{\sqrt{3}}, \cos 30^\circ = \frac{\sqrt{3}}{2} \text{ and } \tan$$

$$(iii) \sin 60^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ = \frac{1}{2} \text{ and } \tan 60^\circ = \sqrt{3}$$

Exercise 9.6

1. Simplify the following without using tables.

Note: $\sqrt{a} \times \sqrt{a} = a$

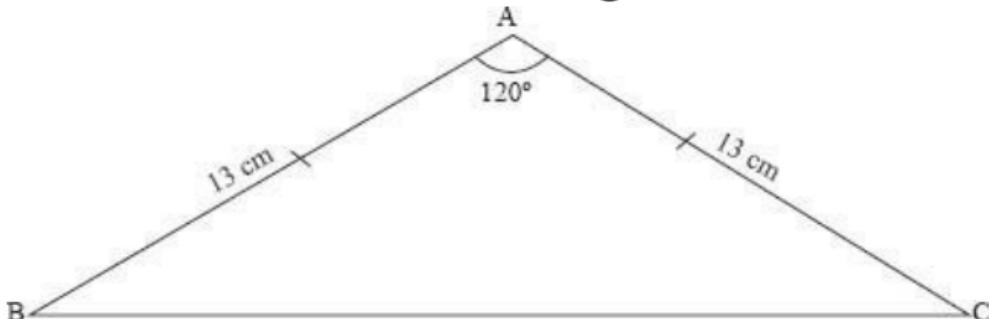
$$(a) \sin 30^\circ \cos 30^\circ$$

- (b) $4 \cos 45^\circ \sin 60^\circ$
 - (c) $3 \cos 30^\circ + \cos 60^\circ$
 - (d) $\tan 45^\circ + \cos 45^\circ \sin 45^\circ$
 - (e) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$
 - (f) $\cos^2 60^\circ + \sin^2 60^\circ$ ($\sin \theta \times \sin \theta$ is written as $\sin^2 \theta$)
2. An isosceles triangle is such that AB = AC = 8 cm. If the perpendicular distance from A to BC is 6 cm, find:

- (a) the length of BC.
 - (b) $\angle BAC$.
3. The angle made by the arms of an upright pair of dividers and the horizontal is 30° . The vertical distance from the horizontal to the vertex is 10 cm. Find without using tables:
- (a) the horizontal distance between the tips of the arms.
 - (b) the length of the arms.

4. The angle at the vertex of a cone is 90° . If the slant height is $3\sqrt{2}\text{cm}$, find without using tables:
- (a) the diameter of the cone.
 - (b) the height of the cone.
5. Find the height of an equilateral triangle of side $x \text{ cm}$. Use the triangle to show that $\sin^2 60^\circ + \cos^2 60^\circ = 1$ (without using tables).

6. In the figure below, ABC is an isosceles triangle. $\angle BAC = 120^\circ$ and $AB = AC = 13 \text{ cm}$. Calculate BC and the area of the triangle.



9.7: Logarithms of Tangents, Sines and Cosines

In this section, the use of logarithms in computations involving trigonometry is considered.

For example, to evaluate $234 \sin 36^\circ$, use the following method: From tables, $\sin 36^\circ = 0.5878$.

Therefore, $234 \sin 36^\circ = 234 \times 0.5878$

No.	Log
234	2.3692
0.5878	1.7692 +
1.375×10^2	2.1384

Therefore, $234 \sin 36^\circ = 137.5$

Alternatively, $\log \sin 36^\circ$ can be read directly from the tables of logarithms of sines.

No.	Log
234	2.3692
$\sin 36^\circ$	1.7692 +
1.375×10^2	2.1384

Therefore, $234 \sin 36^\circ = 137.5$

Similarly, values of $\log (\cos x)$ and $\log (\tan x)$ can be read from their respective tables.

Example 10

Evaluate $\frac{69.6 \cos 42^\circ}{\sin 64^\circ}$

Solution

No.	Log
69.6	1.8426
$\cos 42^\circ$	1.8711 +
$\sin 64^\circ$	1.7137 1.9537 -
5.75×10^1	1.7600

Therefore, $\frac{69.6 \cos 42^\circ}{\sin 64^\circ} = 57.54$

Example 11

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Three towns P, Q and R are such that Q is 150 km from P on a bearing of 043° , see figure 9.17. The bearing of R from P is 133° and the bearing of R from Q is 160° . Calculate the distance of R from P, Q from R and the bearing of P from R.

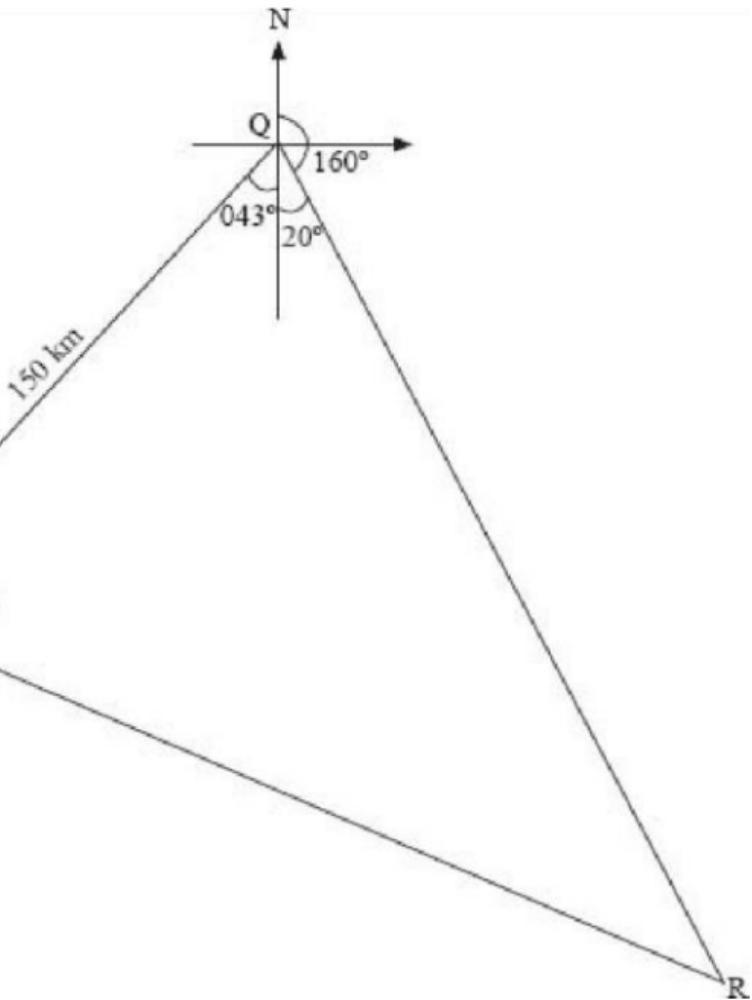


Fig. 9.17

Solution

In the figure, ΔPQR is right-angled at P
and $\angle PQR = 63^\circ$

$$\text{Hence, } \tan 63^\circ = \frac{150^\circ}{QR}$$

$$150 \tan 63^\circ = PR$$

No.	Log
150	2.1761
$\tan 63^\circ$	0.2928 +
2.944×10^2	2.4689

Therefore, $PR = 294.4$ km

In the right-angled ΔPQR , QR is the hypotenuse; $\cos 63^\circ =$

$$QR \frac{150^\circ}{QR} =$$
$$\frac{150^\circ}{QR}$$

No.	Log
150	2.1761
$\cos 63^\circ$	1.6571 –
3.305×10^2	2.5192

Therefore, $QR = 330.5$ km

The bearing of P from R;

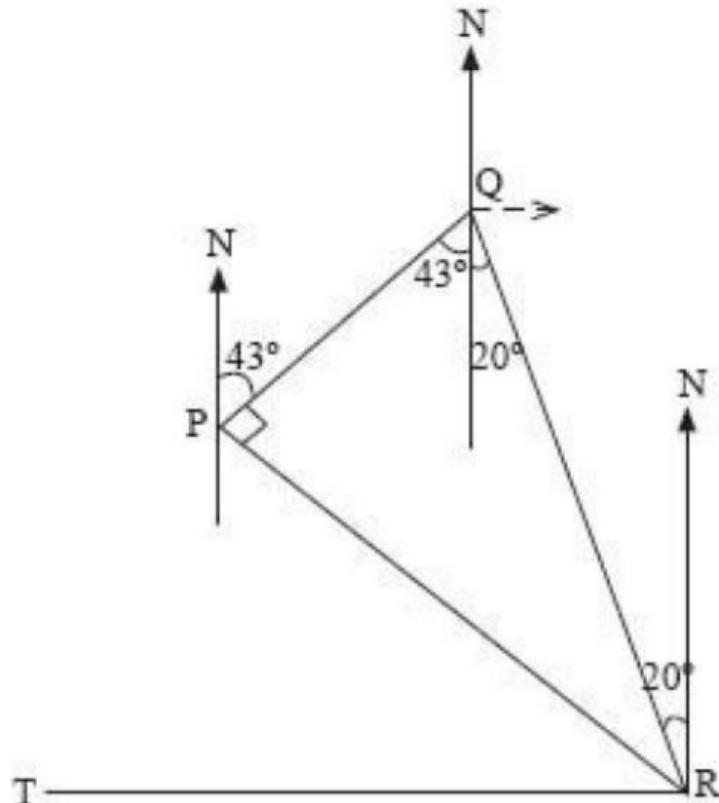


Fig. 9.18

From figure 9.18

$$\begin{aligned}\angle PRQ &= 180^\circ - (90 + 63)^\circ \\ &= 27^\circ\end{aligned}$$

$$\begin{aligned}\angle PRT &= 90^\circ - (20 + 27)^\circ \\ &= 43^\circ\end{aligned}$$

Therefore, the bearing of P from R =
 $270^\circ + 43^\circ = 313^\circ$

Exercise 9.7

1. Evaluate:

(a) $8.52 \tan 42.2^\circ$

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(b) $7.9 \sin 79^\circ$

$$\frac{69}{\cos 63.6^\circ}$$

$$\frac{7 \cos 50.2^\circ}{9.5 \sin 60^\circ}$$

(c)

(d)

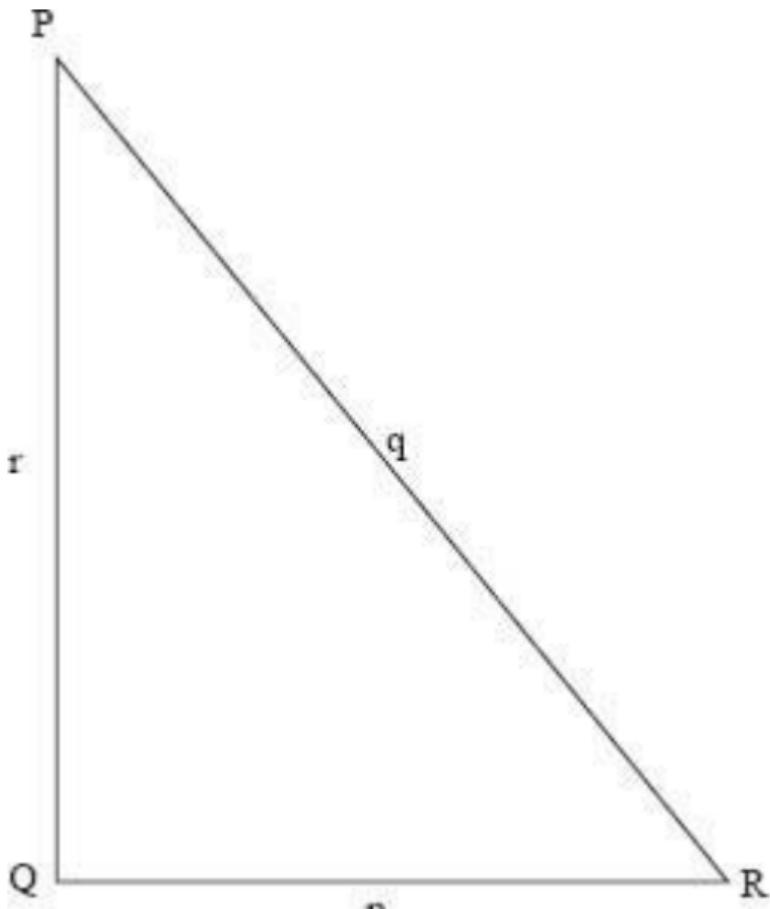
2. Use the figure below to find the unknown angles and sides in each case:

(a) $\angle PRQ = 46^\circ$, $q = 7.83$

(b) $p = 13.6$, $q = 17.2$

(c) $\angle QPR = 56^\circ 10'$, $p = 4.53$

(d) $\angle QPR = 47^\circ 35'$, $r = 3.47$

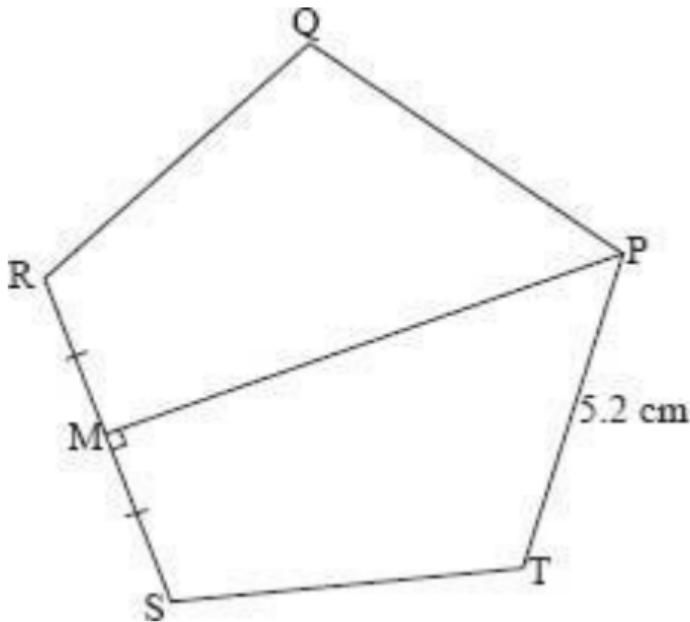


3. In a triangle PQR, $\angle PQR = \angle PRQ = 58^\circ$ and QR = 5.2 cm. Calculate

the length of PQ.

4. In a triangle PQR, $QR = 5.2$ cm and $PQ = PR = 8.2$ cm. Calculate:
 - (a) $\angle PQR$ and $\angle QPR$
 - (b) the area of ΔPQR
5. In triangle XYZ, $XY = XZ = 4.1$ cm, XN is the height of the triangle, which is 3.2 cm long. Calculate:
 - (a) the base angles of the triangle.

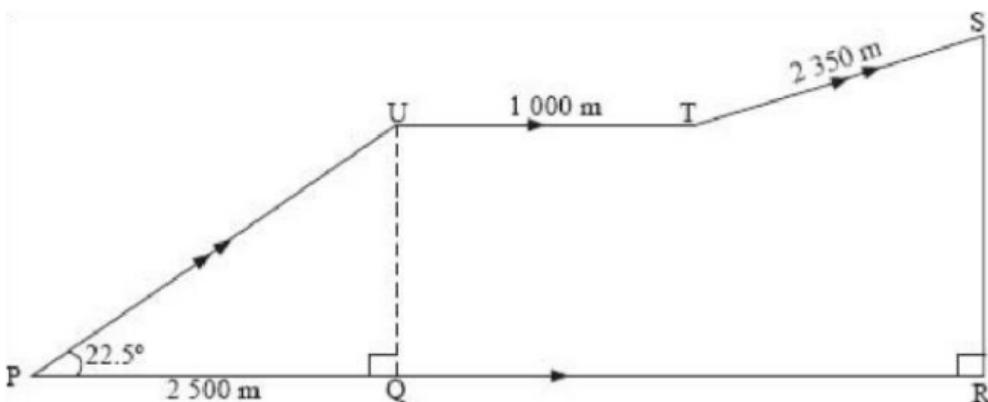
- (b) the size of the vertex angle.
 - (c) area of triangle XYZ.
6. Calculate the area of a parallelogram PQRS in which $PS = 3.1$ cm, $PQ = 7.2$ cm and $\angle SPQ = 73^\circ$.
7. The figure below shows a regular pentagon PQRST of side 5.2 cm. Find the length of PM if $\angle PMS = 90^\circ$ and M bisects RS.



8. In the figure below, $PQ = 2\ 500\text{ m}$, $UT = 1\ 000\text{ m}$ and $TS = 2\ 350\text{ m}$. PQR is a straight line parallel to UT and PU is

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parallel to TS. Calculate PR to the nearest kilometre.



9. A ship moves 4.5 km on a bearing of 330° . It then changes course to a bearing of 270° and moves 11 km, then changes its course again to a bearing of 315° for 8 km. Find, to the nearest km, how far north and west the ship is from the starting point.

Chapter 10

AREA OF A TRIANGLE

10.1: Area of a Triangle, given Two

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Sides and an Included Angle

The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where b is the base and h the perpendicular height. At times, situations arise which make it difficult to apply this formula directly. Consider $\triangle ABC$ in fig. 10.1.

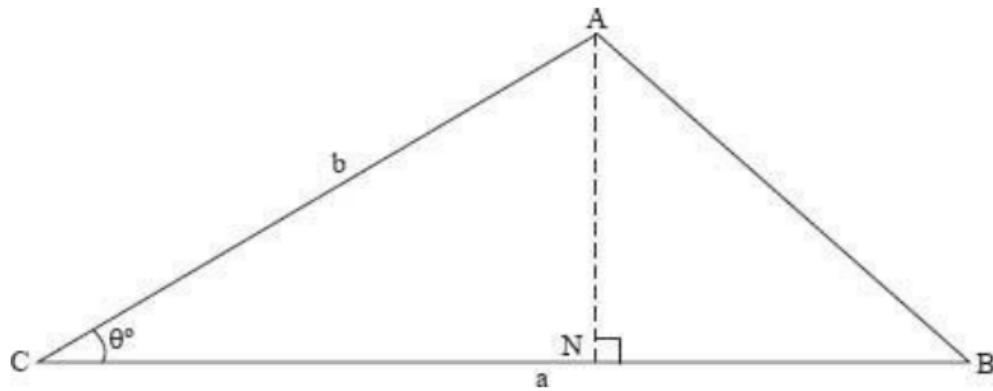


Fig. 10.1

From A, drop a perpendicular to meet BC at N. AN is the perpendicular height of triangle ABC.

$$\begin{aligned}\text{In triangle ACN } \sin \theta &= \frac{AN}{AC} \\ &= \frac{AN}{b}\end{aligned}$$

$$AN = b \sin \theta$$

$$\text{Area of } \Delta ABC = \frac{1}{2} CB \cdot AN$$

$$= \frac{1}{2} ab \sin \theta$$

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In general, given a triangle ABC with two sides and an included angle θ , the area is given by, area $A = \frac{1}{2} ab \sin \theta$.

Example 1

Figure 10.2 shows a triangle ABC in which $AB = 6 \text{ cm}$, $BC = 7 \text{ cm}$ and $\angle ABC = 50^\circ$. Find the area of the triangle.

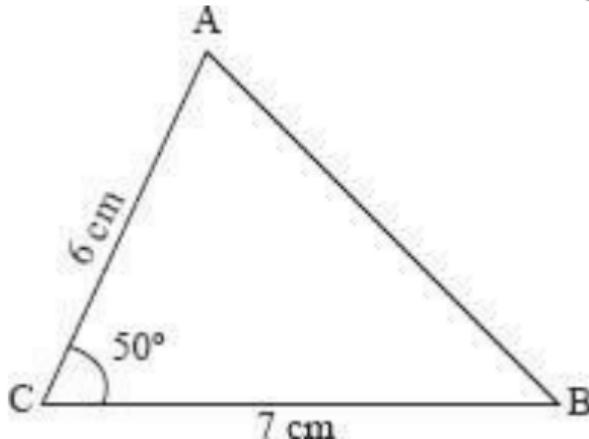


Fig. 10.2

Solution

Using the formula $A = \frac{1}{2}ab\sin \theta$, $a = 7 \text{ cm}$,
 $b = 6 \text{ cm}$ and $\theta = 50^\circ$.

$$\begin{aligned}\text{Therefore, area} &= \frac{1}{2} \times 6 \times 7 \sin 50^\circ \\&= 3 \times 7 \times 0.7660 \\&= 16.09 \text{ cm}^2\end{aligned}$$

10.2: Area of Triangle, given the

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Three Sides

When only the three sides of a triangle are given the formula;

$$\sqrt{s(s-a)(s-b)(s-c)} \quad A = \text{(Hero's formula),}$$

where $s = \frac{1}{2}$

triangle of the
 $= \frac{1}{2} (a + b + c)$ perimeter of
the

a, b and c are the lengths of the sides of the triangle.

Example 2

Find the area of a triangle ABC in which AB = 5 cm, BC = 6 cm and AC = 7 cm.

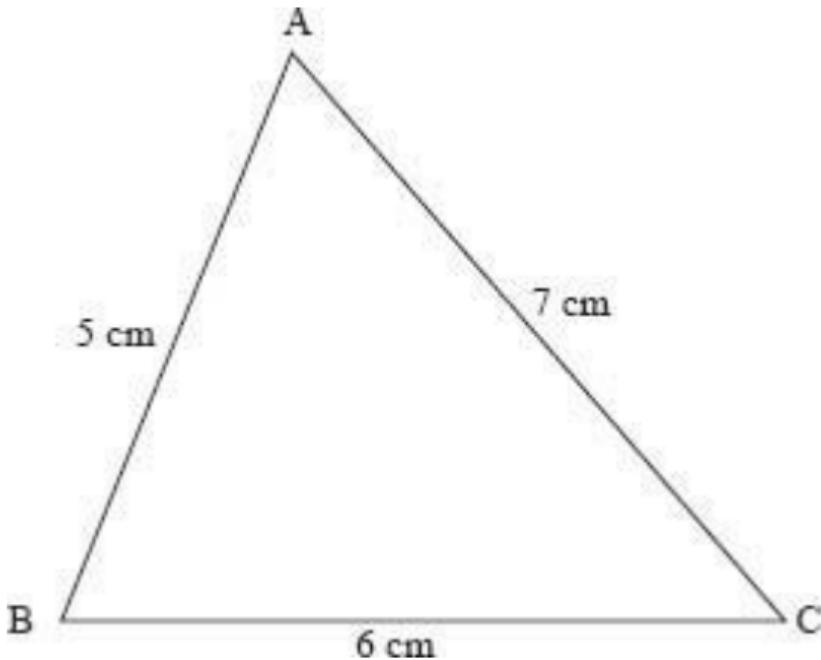


Fig. 10.4

In the figure, $a = 6$, $b = 7$ and $c = 5$.

Therefore; $s = \frac{1}{2}$

$$(6 + 7 + 5)$$

$$= 9$$

$$\text{and } A = \sqrt{9(9-6)(9-7)(9-5)} = \\ \sqrt{9 \times 3 \times 2 \times 4}$$

=

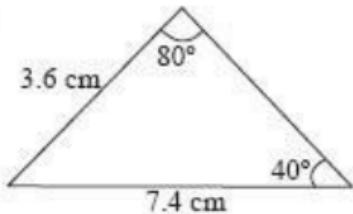
$$= \sqrt{216}$$

$$= 14.70 \text{ cm}^2$$

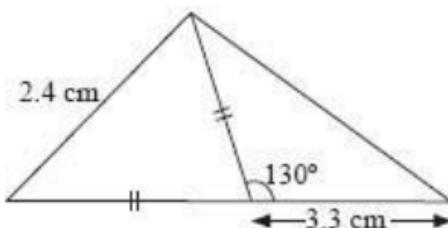
Exercise 10.1

- Find the area of each of the following figures:

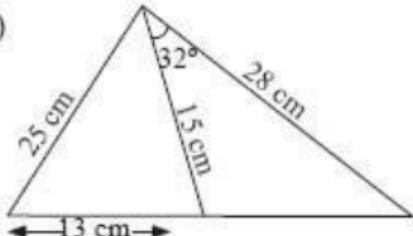
(a)



(b)

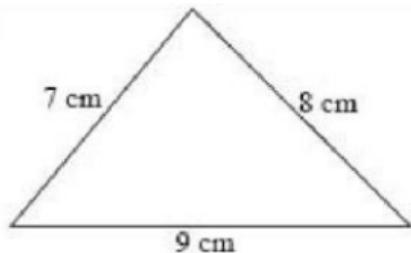


(c)

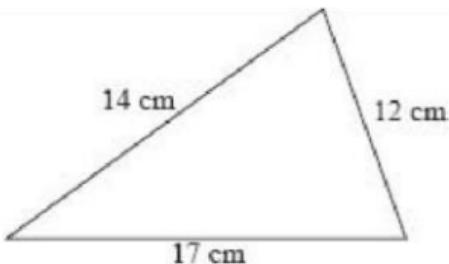


2. Find the area of each of the triangles below:

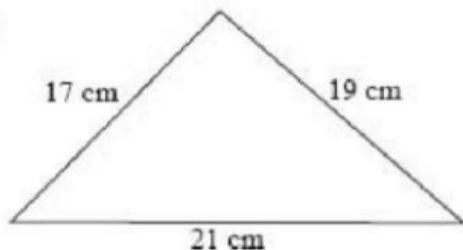
(a)



(b)



(c)

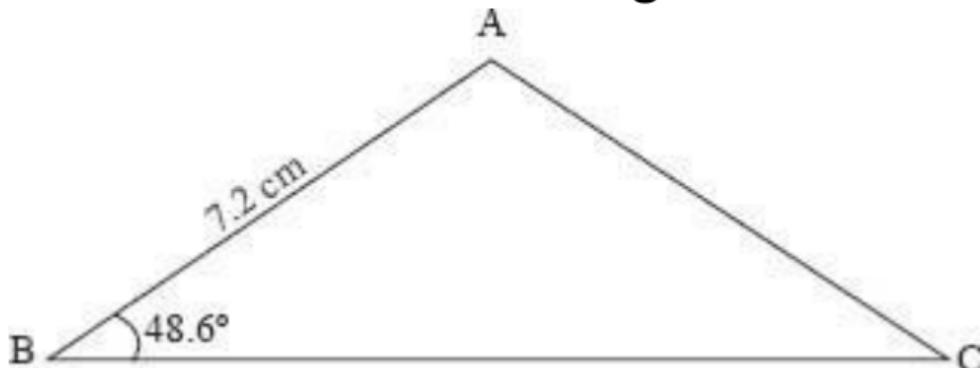


3. A triangle has sides 10 cm, 7 cm and 9 cm. Find:

- (a) its area.

(b) the sizes of its angles.

4. The area of triangle ABC below is 28.1 cm^2 . Find the length BC.



5. In a triangle XYZ, $XY = 7 \text{ cm}$, $YZ = 24 \text{ cm}$ and $XZ = 25 \text{ cm}$. Find the length of the perpendicular from vertex Y to side XZ.

6. The perimeter of a triangle is 22 cm. If one of the sides is 9 cm, find the other sides if the area of the triangle is 20.976 cm^2 .
7. A traditional stool has a triangular top which measures 27 cm, 35 cm and 42 cm. Calculate the area of the top.
8. A triangular flower garden measures 10 m, 15 m and 24 m. Find the area of the garden.

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Chapter 11

AREA OF

QUADRILATERALS

11. 1: Quadrilaterals

A quadrilateral is a four-sided figure.
Some examples of quadrilaterals are a

rectangle, square, rhombus,
parallelogram, trapezium and a kite.

Example 1

Find the area of a rectangle whose length is 12 cm and width 7 cm.

Solution

$$A = l \times b$$

$$= 12 \times 7 \text{ cm}^2$$

$$= 84 \text{ cm}^2$$

11.2: Area of Parallelogram

A parallelogram is a quadrilateral whose opposite sides are equal and parallel. The area of a parallelogram is given by;

$A = bh$, where b is the base and h the perpendicular height of the parallelogram. For example, the area of a parallelogram whose base is 8 cm and height 6 cm is; $A = bh$

$$= 8 \times 6 \text{ cm}^2$$

$$= 48 \text{ cm}^2$$

Example 2

ABCD is a parallelogram of sides 5 cm and 10 cm. If $\angle ABC$ is 70° , find the area of the parallelogram.

Solution

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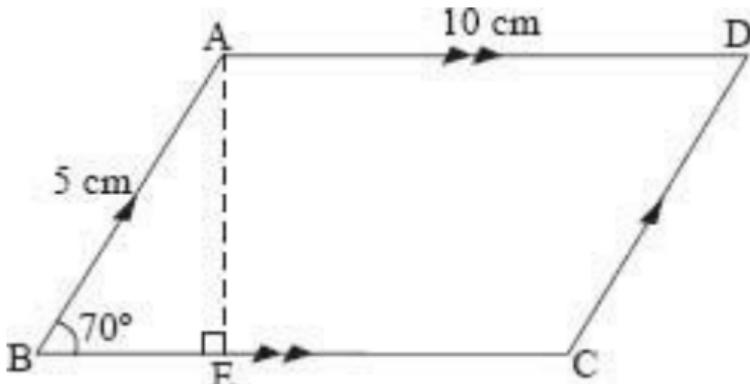


Fig. 11.1

From A, drop a perpendicular to meet BC at E. Considering $\triangle ABE$;

$$AE = 5 \sin 70^\circ$$

$$\text{Area of } ABCD = BC \times AE$$

$$= 10 \times 5 \sin 70^\circ$$

$$= 50 \sin 70^\circ$$

$$= 50 \times 0.9397$$

$$= 46.99 \text{ cm}^2$$

11.3: Area of a Rhombus

A rhombus is a parallelogram with all sides equal and the diagonals bisect at 90° .

Example 3

Find the area of rhombus PQRS in figure

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11.2 below, given that $PR = 4 \text{ cm}$, $QS = 7 \text{ cm}$, $PQ = QR = RS = SP$ and $QR // PS$.

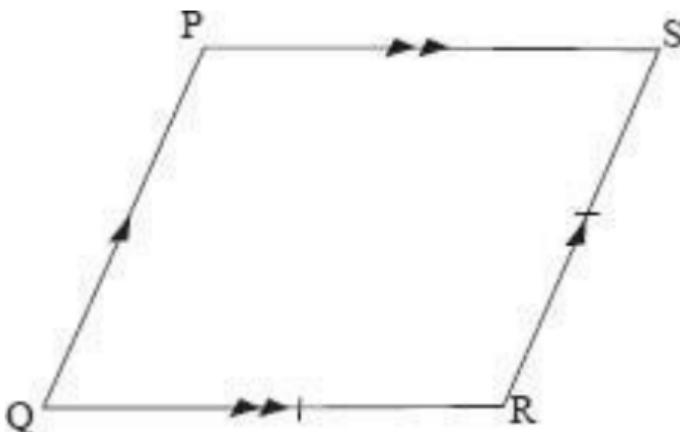


Fig. 11.2

Solution

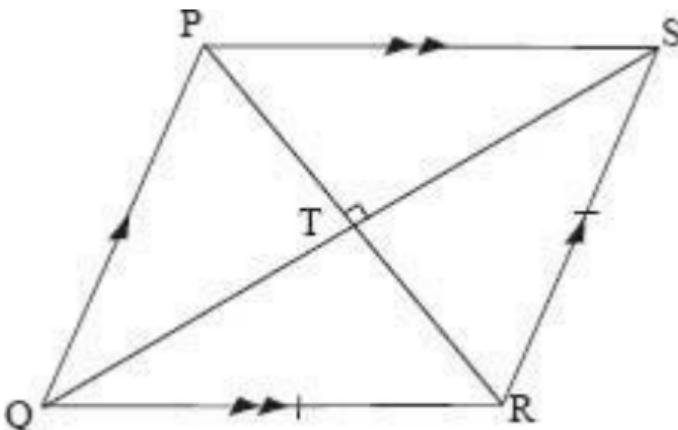


Fig 11.3

Figure 11.3 is a rhombus. The diagonals of a rhombus bisect at 90°.

$$QT = \frac{1}{2} \times 7 \text{ cm} = 3.5\text{cm}^2$$

$$\begin{aligned}\text{Area of } \triangle PQR &= \frac{1}{2} \times 4 \text{ cm} = 3.5 \text{ cm}^2 \\ &= 7 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of PQRS} &= 2 \times \text{area of } \triangle PQR \\ &= 2 \times 7 \text{ cm}^2 \\ &= 14 \text{ cm}^2\end{aligned}$$

For a rhombus whose diagonals are a and b, the area is given by $\frac{1}{2}ab$.

11.4: Area of a Trapezium

A trapezium is a quadrilateral with two of its opposite sides being parallel.

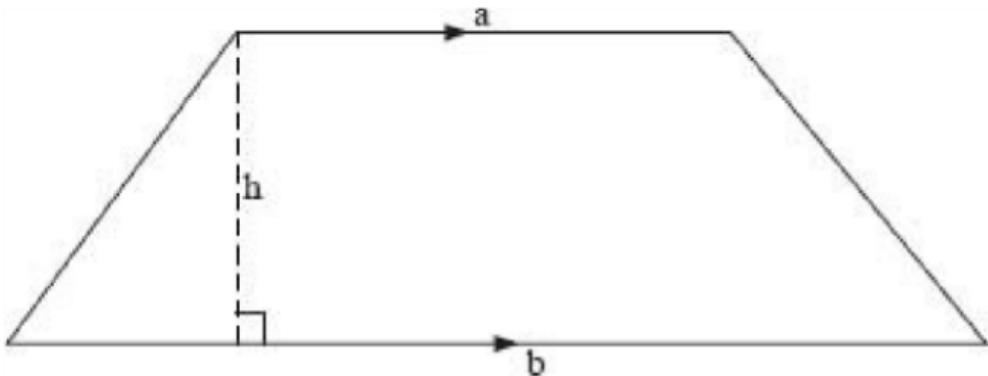


Fig. 11.4

The area of a trapezium is given by the formula;

$$A = \left(\frac{a+b}{2} \right) h, \text{ where } a \text{ and } b \text{ are the two}$$

parallel sides and h the perpendicular distance between them.

Example 4

Figure 11.5 is a trapezium in which $PS \parallel QR$, $PS = 15 \text{ cm}$, $QR = 20 \text{ cm}$, $RS = 8 \text{ cm}$ and $\angle QRS = 35^\circ$. Calculate the area of the trapezium.

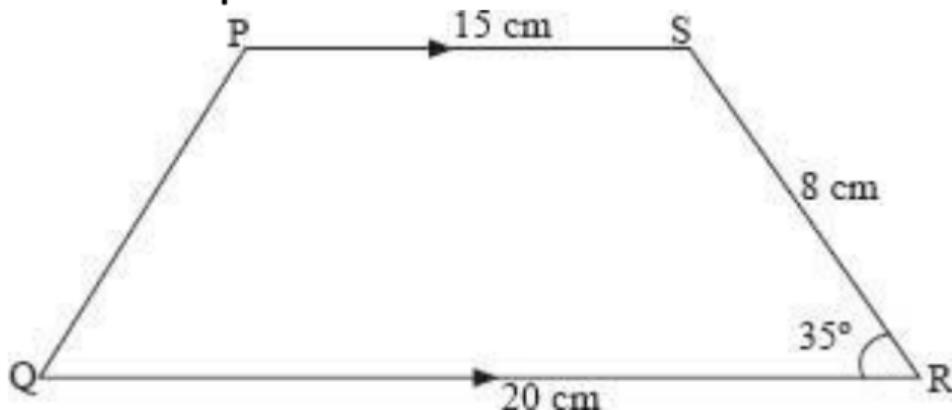


Fig. 11.5

Solution

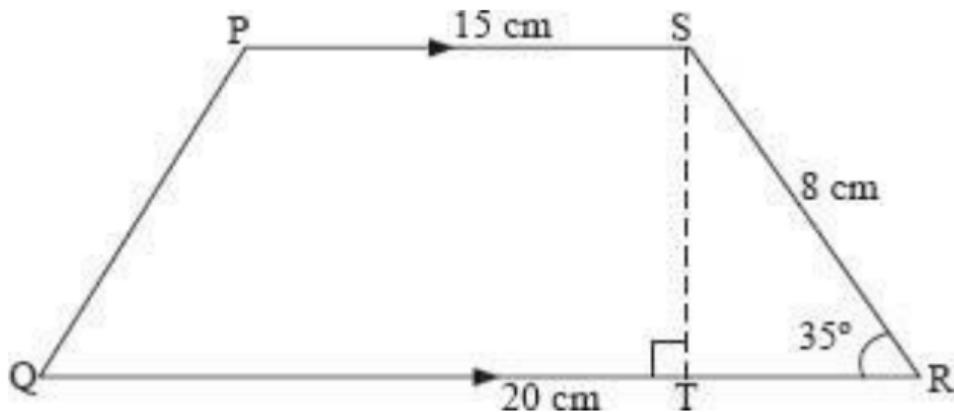


Fig. 11.6

From S, drop a perpendicular to meet

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QR at T. ST is the height of the trapezium. Area of PQRS

$$= \left(\frac{PS + QR}{2} \right) \times ST$$

$$= \left(\frac{15 + 20}{2} \right) \times 8 \sin 35^\circ$$

$$= 140 \sin 35^\circ$$

$$= 80.30 \text{ cm}^2$$

11.5: Area of a Kite

Figure 11.7 represents a kite.

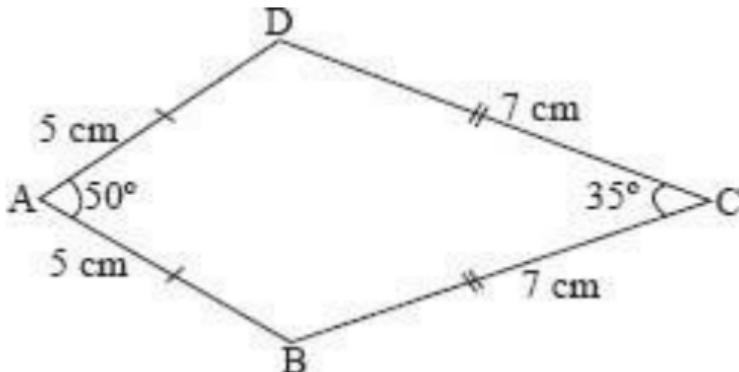


Fig. 11.7

Example 5

Find the area of the kite in figure 11.7 above.

Solution

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Join D to B.

$$\begin{aligned}\text{Area of } \triangle ABD &= \frac{1}{2} \times 5 \times 5 \sin 50^\circ \\ &= 12.5 \sin 50^\circ \\ &= 9.576\end{aligned}$$

$$\begin{array}{lclcl}\text{Area of } \triangle ABD & & & & \\ 9.576 \text{ cm}^2 & = & \frac{1}{2} \times 7 \times 7 \sin 35^\circ & & \\ \text{Area of } \triangle BCD & & = 24.5 \sin 35^\circ & & \\ & & = 14.05 & & \end{array} =$$

$$\text{Area of } \triangle BCD = 14.05 \text{ cm}^2$$

$$\text{Area of } \triangle BCD = \text{area of } \triangle ABD + \text{area of } \triangle BCD$$

$$= 9.576 + 14.05$$

$$= 23.63 \text{ cm}^2$$

11.6: Area of a Regular Polygon

Any regular polygon can be divided into isosceles triangles by joining the vertices to the centre. The number of isosceles triangles formed is equal to the number of sides of the polygon.

Example 6

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Figure 11.8 represents a regular octagon ABCDEFGH with O as its centre. If OA is 6 cm, find its area.

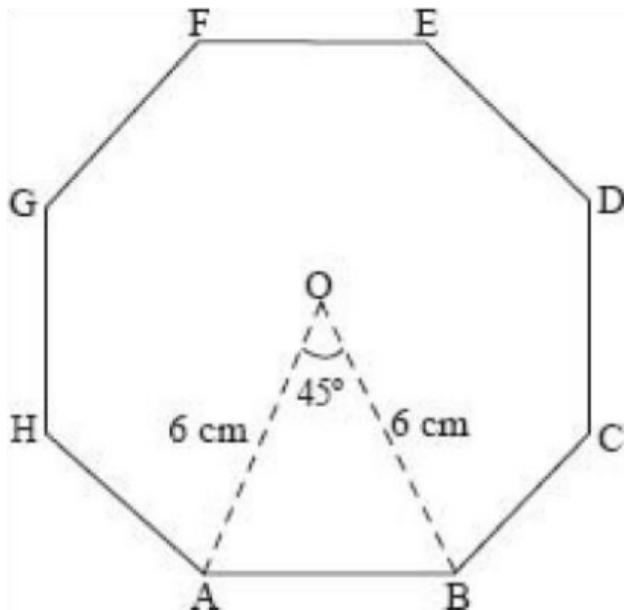


Fig. 11.8

Solution

Area of triangle AOB

$$= \frac{1}{2} \times 6 \times 6 \sin 45^\circ$$

$$= \frac{1}{2} \times 36 \times 0.7071 \text{ cm}^2$$

There are 8 triangles since an octagon has 8 sides;

Therefore, total area

$$= 8 \times (\frac{1}{2} \times 36 \times 0.7071)$$

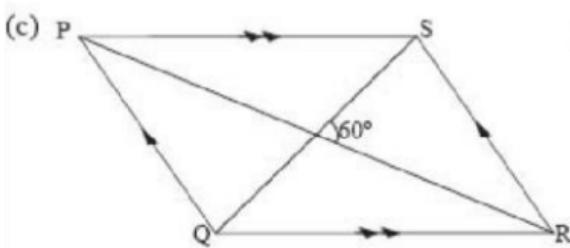
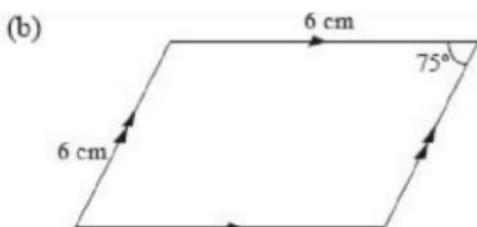
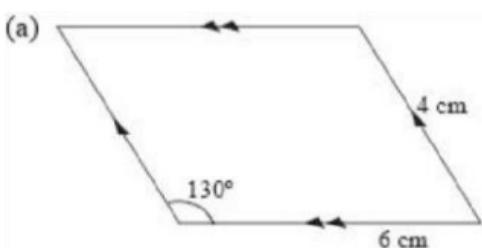
$$= 8 \times 18 \times 0.7071$$

$$= 101.8 \text{ cm}^2$$

Exercise 11.1

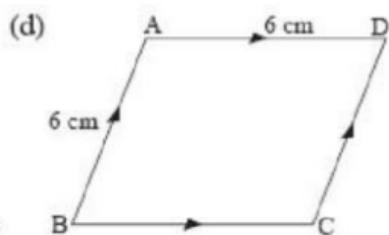
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1. Find the area of each of the following figures 11.9: (a) – (d)



$$PR = 7.8 \text{ cm}$$

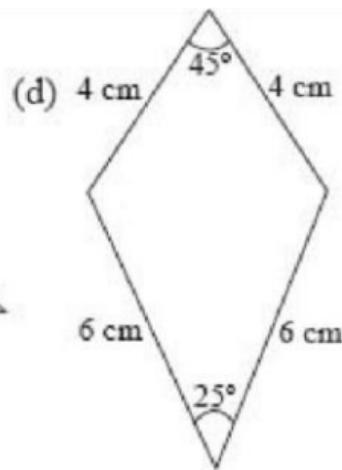
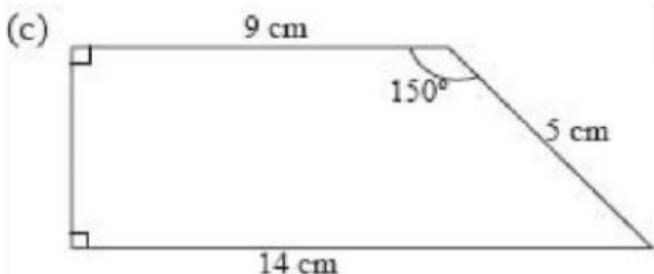
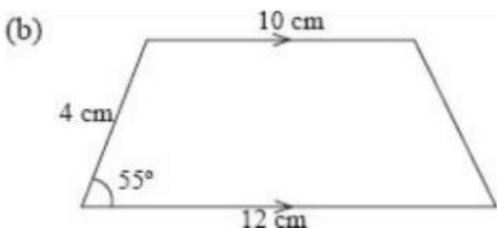
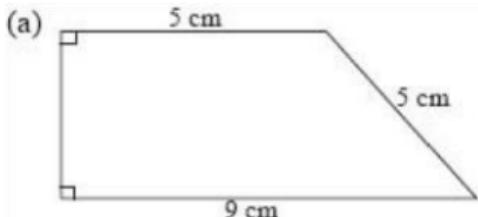
$$QS = 5 \text{ cm}$$



$$AC = 3.8 \text{ cm}$$

$$BD = 6 \text{ cm}$$

2. Calculate the area of each of the figure below 11.10: (a) – (d)



3. Find the area of a regular hexagon of

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side 4.8 cm.

4. Find the area of a rhombus whose diagonals are 9.6 cm and 6.0 cm long.
5. A pentagon PQRST is such that $PQ = 2$ cm, $QR = 3$ cm, $RS = 1.5$ cm and $PT = 4$ cm. If $\angle TPQ = \angle QRS = 90^\circ$ and $\angle TQS = 36^\circ$, find the area of the pentagon.
6. Find the length of the sides of a regular heptagon of area 168 cm^2 .

7. The area of a parallelogram is 121 cm^2 and its sides are 11 cm and 16 cm long. Find the sizes of all the angles of the parallelogram.
8. One of the angles of a rhombus is 120° and its sides are 15 cm long. Find the area of the rhombus.
9. ABCD is a parallelogram of area 120 cm^2 . Its base is 10 cm and

$\angle ABC = 30^\circ$. Find the length of the other side of the parallelogram.

10. Find the value of a in figure 11.11, if its area is 128 cm^2 .

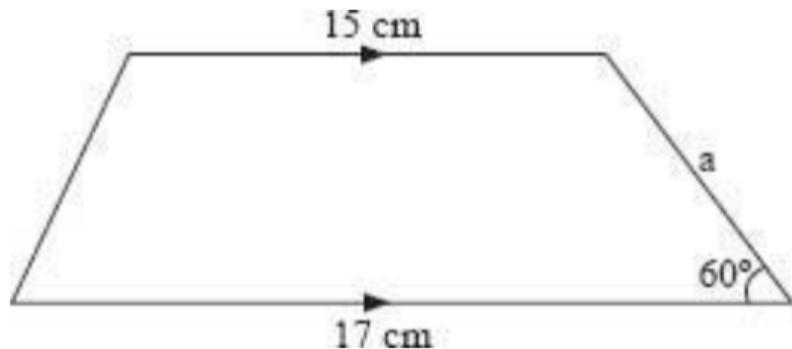


Fig 11.11

Chapter 12

AREA OF A PART OF A CIRCLE

12.1: Sector

A sector is a region bounded by two radii and an arc. A minor sector is one whose area is less than a half of the area of the circle while a major sector is one whose

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area is greater than a half of the area of the circle, as in figure 12.1.

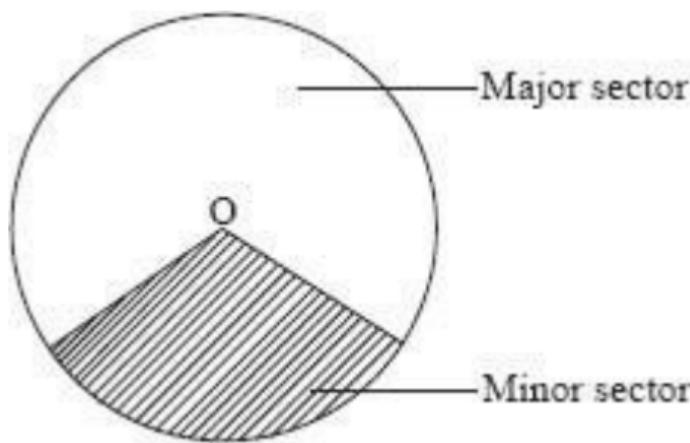


Fig. 12.1

The Area of a Sector

Consider the area of the shaded region in figure 12.2.

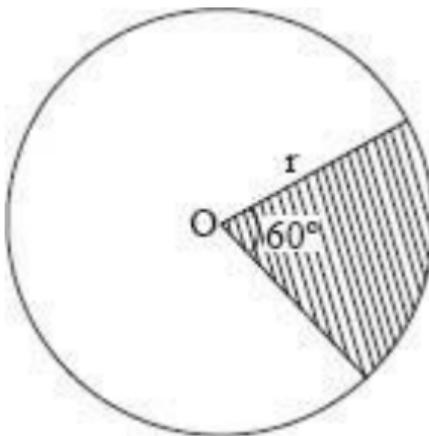


Fig. 12.2

The area of the whole circle is πr^2 . The whole circle subtends 360° at the centre. Therefore, 360° corresponds to πr^2 .

1° corresponds to $\frac{1}{360} \times \pi r^2$
to $\frac{60}{360} \times \pi r^2$

60° corresponds to

In general, the area of a sector whose arc subtends an angle θ at the centre of the circle is given by; $A = \frac{\theta}{360} \times \pi r^2$.

Example 1

Find the area of a sector of a circle of radius 3 cm if the angle subtended at the centre is 140° . (Take $\pi = \frac{22}{7}$)

Solution

Area A of a sector is given by;

$$A = \frac{\theta}{360} \times \pi r^2$$

$$\begin{aligned}\text{Therefore, area} &= \frac{140}{360} \times \frac{22}{7} \times 3^2 \\ &= 11 \text{ cm}^2\end{aligned}$$

Example 2

The area of a sector of a circle is 38.5 cm^2 . Find the radius of the circle if the angle subtended at the centre is 90° .
(Take $\pi = \frac{22}{7}$).

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Solution

From $A = \frac{\theta}{360} \pi r^2$, then

$$38.5 = \frac{90}{360} \times \frac{22}{7} \times r^2$$

$$r^2 = \frac{38.5 \times 360 \times 7}{90 \times 22}$$

$$r^2 = 49$$

$$r^2 = 7 \text{ cm}$$

Example 3

The area of a sector of a circle radius 63 cm is 4158 cm^2 . Calculate the angle subtended at the centre of the circle.

(Take $\pi = \frac{22}{7}$)

Solution

$$4158 = \frac{\theta}{360} \times \frac{22}{7} \times 63 \times 63$$

$$\begin{aligned}\theta &= \frac{4158 \times 360 \times 7}{22 \times 63 \times 63} \\ &= 120^\circ\end{aligned}$$

Exercise 12.1

(Take $\pi = \frac{22}{7}$)

1. A sector of a circle of radius r is subtended at the centre by an angle

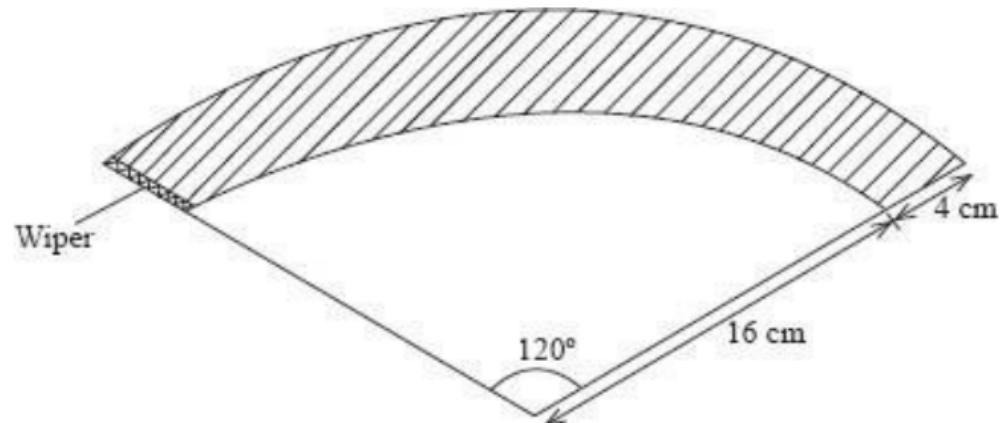
of θ . Calculate the area of the sector if:

- (a) $r = 1.4 \text{ cm}$, $\theta = 30^\circ$
- (b) $r = 2.1 \text{ cm}$, $\theta = 45^\circ$
- (c) $r = 8 \text{ cm}$, $\theta = 33^\circ$
- (d) $r = 8.4 \text{ cm}$, $\theta = 60^\circ$
- (e) $r = 9.1 \text{ cm}$, $\theta = 24^\circ$
- (f) $r = 4 \text{ cm}$, $\theta = 259^\circ$
- (g) $r = 10 \text{ cm}$, $\theta = 301^\circ$

2. A floodlight can spread its illumination over an angle of 50° to

a distance of 49 cm. Calculate the area that is lit by the floodlight.

3. The shaded region in the figure below shows the area swept out on a flat windscreen by a wiper. Calculate the area of this region.



4. The two arms of a pair of dividers are spread so that the angle between them is 45° . Find the area of the sector formed if the length of an arm is 8.4 cm.
5. A goat is tethered at the corner of a fenced rectangular grazing field. If

the length of the rope is 21 m, what is its grazing area?

12.2: Area of a Segment of a Circle

A segment is the region of a circle bounded by a chord and an arc. In figure 12.3, the shaded region is a segment of the circle with centre O and radius r. $AB = b$ cm, $OM = h$ cm and $\angle AOB = \theta^\circ$.

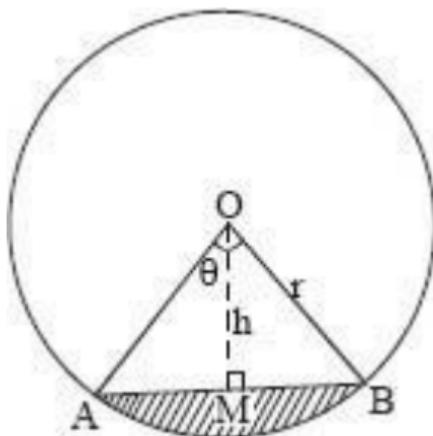


Fig. 12.3

The area of the segment = area of minor sector OAB – area of triangle OAB

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} b h$$

Example 4

Figure 12.4 is a circle with centre O and radius 5 cm. If ON = 3 cm, AB = 8 cm and $\angle AOB = 106.3^\circ$, find the area of the shaded region:

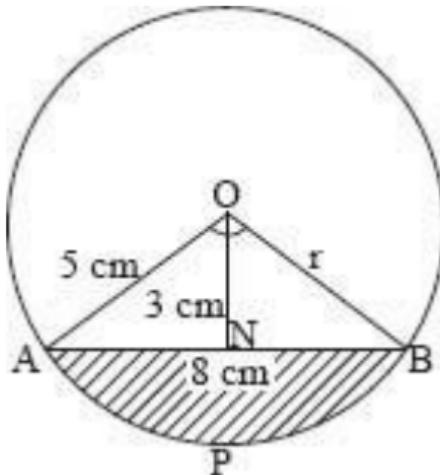


Fig. 12.4

Solution

Area of segment = area of sector OAPB
– area of triangle OAB

$$= \left[\frac{106.3}{360} \times 3.142 \times 5^2 \right] - \frac{1}{2} \times 8 \times 3$$

$$= \frac{106.3}{360} \times 25 \times 3.142 - 12$$

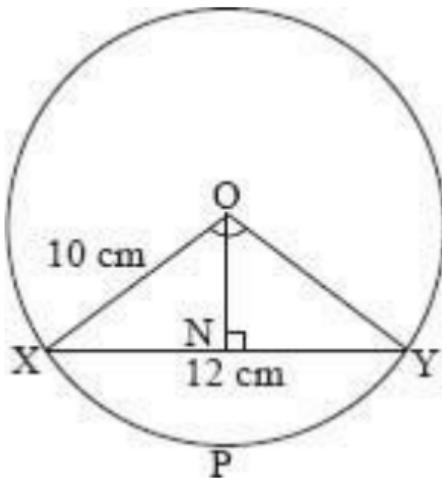
$$= 23.19 - 12$$

$$= 11.19 \text{ cm}^2$$

Exercise 12.2

(Take $\pi = 3.142$)

1. A chord XY of length 12 cm is drawn in a circle with centre O and radius 10 cm, as in the figure below.

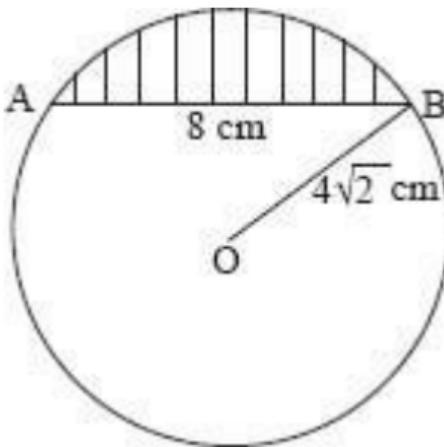


Calculate:

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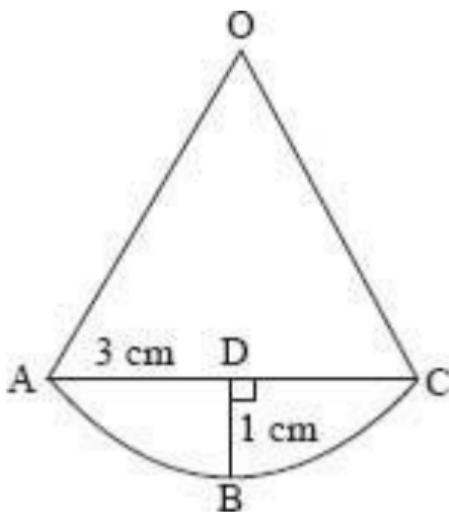
- (a) the distance ON.
- (b) the area of the sector OXPY.
- (c) the area of triangle OXY
- (d) the area of the minor segment.
- (e) the area of the major segment.
2. A chord XY subtends an angle of 120° at the centre of a circle of radius 13 cm. Calculate the area of the minor segment.
3. The figure below shows a circle with centre O and radius $4\sqrt{2}$ cm. If

the length of the chord AB is 8 cm,
show that the shaded area is $(8\pi - 16)$ cm².



4. In the figure below, ADC is a chord of a circle with centre O passing

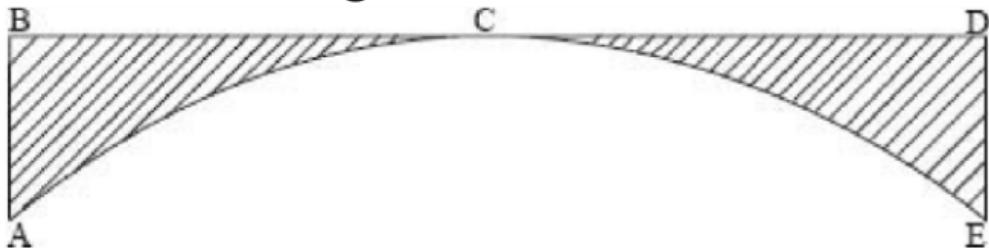
through A, B and C. BD is a perpendicular bisector of AC. AD = 3 cm and BD = 1 cm.



Calculate:

- the radius OA of the circle.
- the area of the sector OABC.

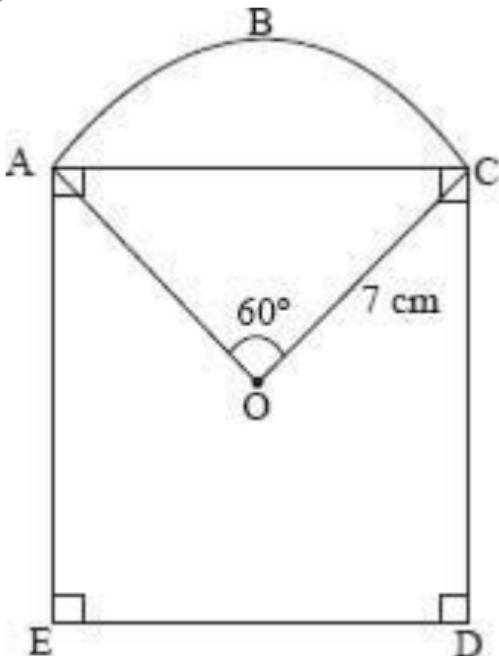
- (c) the area of the segment ABCD.
5. The figure below shows an arc ACE of a circle of radius 6 cm. If $BC = CD = 4$ cm, calculate the area of the shaded region.



6. In the figure below, ABC is an arc of a circle with centre O and radius 7 cm. The arc subtends an angle of

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60° at the centre and $AE = DC = AC = ED = 7 \text{ cm}$. Calculate the area of the figure ABCDE.



12.3: Area of the Common Region between Two Intersecting Circles

Example 5

Figure 12.5 shows two circles of radii 8 cm and 6 cm with centres O_1 and O_2 respectively. The circles intersect at points A and B. The lines O_1O_2 and AB are perpendicular to each other. If the common chord AB is 9 cm, calculate the area of the shaded region.

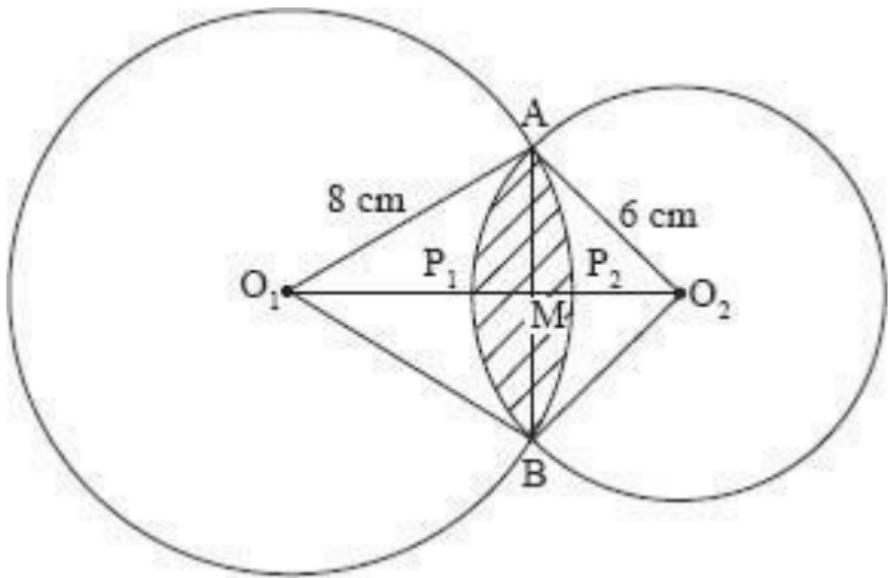


Fig. 12.5

Solution

From ΔAO_1M ;

$$\begin{aligned}O_1M &= \sqrt{8^2 - 4.5^2} \\&= \sqrt{43.75} \\&= 6.14 \text{ cm}\end{aligned}$$

From ΔAO_2M

$$\begin{aligned}O_2M &= \sqrt{6^2 - 4.5^2} \\&= \sqrt{15.75} \\&= 3.969 \text{ cm}\end{aligned}$$

The area of the shaded region is the sum of the areas of segments AP_1B and AP_2B .

Area of segment AP_1B = area of sector C_2AP_1B – area of ΔO_2AB .

Using trigonometry, $\sin \angle AO_2M = \frac{AM}{AO_2}$
 $= \frac{4.5}{6}$
 $= 0.75$

$$\angle AO_2M = 48.59^\circ$$

$$\angle AO_2B = 2\angle AO_2M$$

$$= 2 \times 48.59$$

$$= 97.18^\circ$$

Area of segment AP_1B =

$$\begin{aligned}&= \frac{97.18}{360} \times 3.142 \times 6^2 - \frac{1}{2} \times 9 \times 3.969 \\&= 30.53 - 17.86 \\&= 12.67 \text{ cm}^2\end{aligned}$$

Area of segment AP_2B = area of sector O_1
 AP_2B – area of ΔO_1AB

Using trigonometry, $\sin \angle AO_1M =$

$$= \frac{AM}{AO_2}$$

$$= \frac{4.5}{8}$$

$$= 0.5625$$

$$\angle AO_1M = 34.23^\circ$$

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$$\angle AO_1B = 2 \angle AO_1M$$

$$= 2 \times 34.23^\circ$$

$$= 68.46^\circ$$

Area of segment AP_2B

$$= \frac{68.46}{360} \times 3.142 \times 8^2 - \frac{1}{2} \times 9 \times 6.614$$

$$= 38.24 - 29.76$$

$$= 8.48 \text{ cm}^2$$

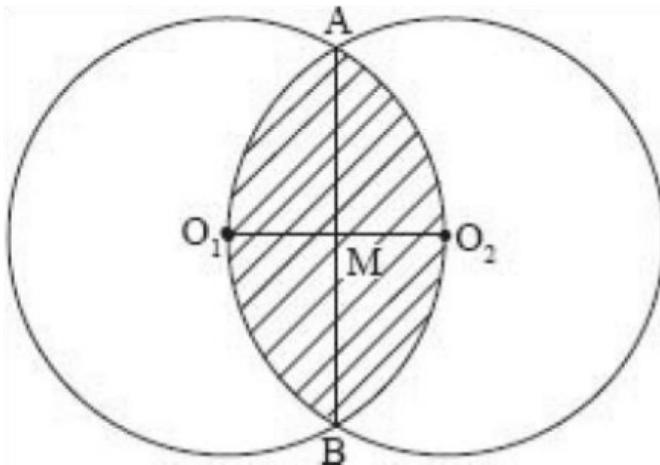
Therefore, area of the shaded region is given by; area of segment AP_1B + area of segment AP_2B = 12.67 + 8.48

$$21.15 \text{ cm}^2$$

Exercise 12.3

(Take $\pi = 3.142$)

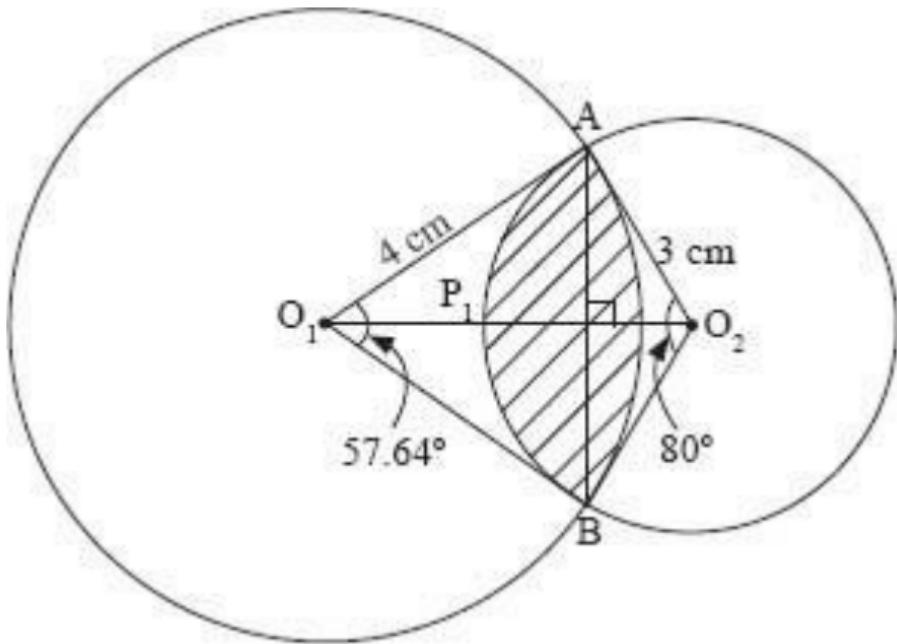
1. The figure below shows two intersecting circles of radius 9 cm each with centre O_1 and O_2 .



Find:

- (a) the length of the common chord AB.
- (b) the area common to the two circles.

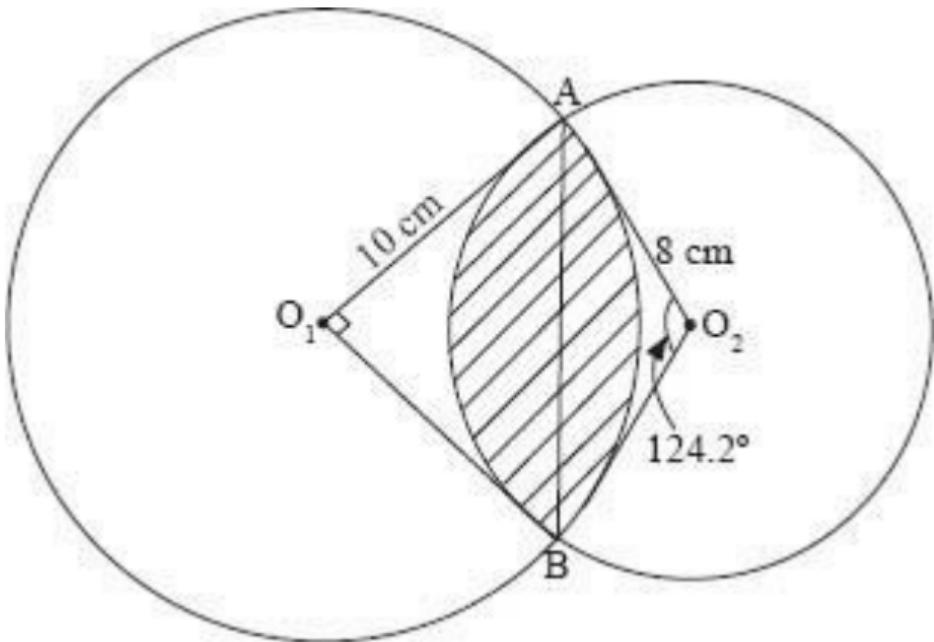
2. The figure below shows two intersecting circles with centres O_1 and O_2 having radii 4 cm and 3 cm respectively. $\angle AO_1B = 57.64^\circ$ and $\angle AO_2B = 80^\circ$. O_1O_2 is a perpendicular bisector of AB.



Calculate:

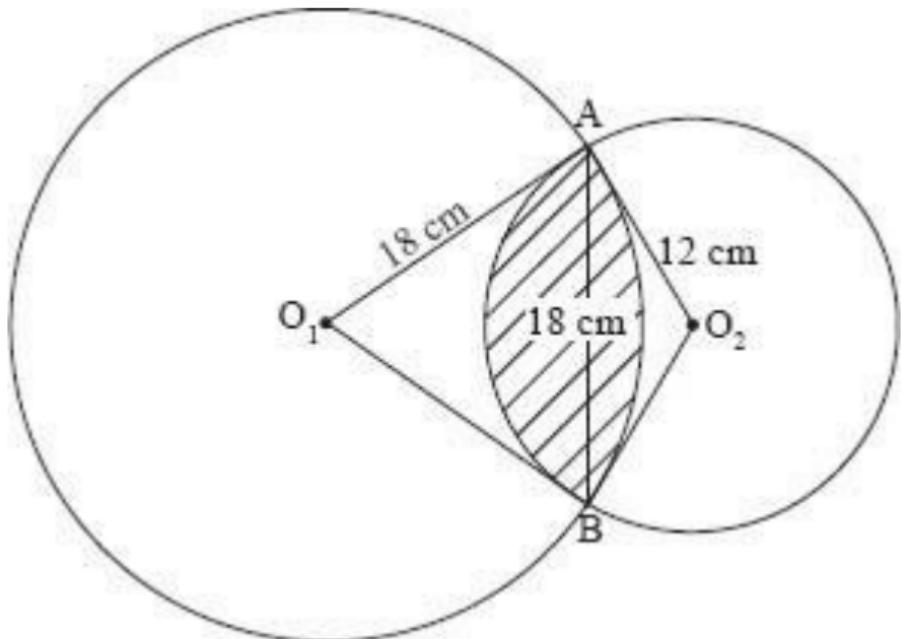
- (a) the length of AB .
- (b) the area of the shaded region.

3. Find the area of the shaded region in the figure below, given that the two circles with centres O_1 and O_2 have radii 10 cm and 8 cm respectively.
 $\angle AO_1B = 90^\circ$ and $\angle AC_2B = 124.2^\circ$.



4. Find the common area between the two intersecting circles in the figure below. The circle centres O_1 and O_2 have radii 18 cm and 12 cm respectively and the chord AB is 18

cm long.



Chapter 13

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SURFACE AREA OF SOLIDS

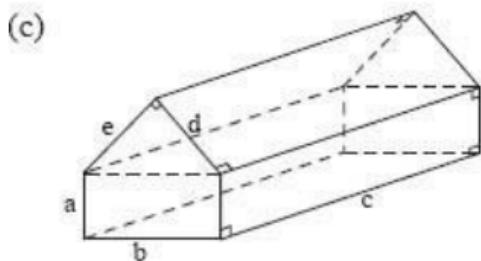
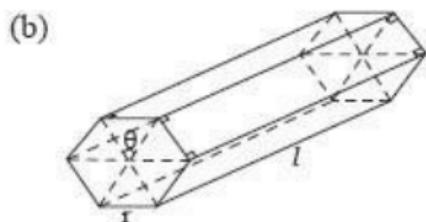
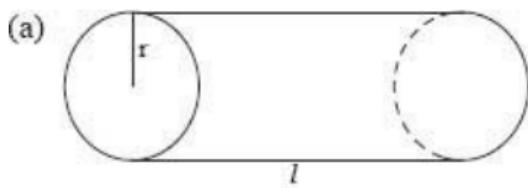
13.1: Introduction

A solid is an object with definite shape and size. This topic deals with the surface areas of prisms, pyramids, frustums and spheres.

The surface area of a solid is the sum of the areas of all its faces.

13:2: Surface Area of a Prism

A prism is a solid with uniform crosssection. Figure 13.1 shows prisms with different cross-sections.



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Fig. 13.1

Example 1

Find the surface area of each of the prisms in figure 13.1 if:

- (a) $r = 2.8 \text{ cm}$, $l = 13 \text{ cm}$.
- (b) $r = 6 \text{ cm}$, $\theta = 60^\circ$, $l = 15 \text{ cm}$.
- (c) $a = 4 \text{ cm}$, $b = 5 \text{ cm}$, $c = 8 \text{ cm}$, $d = 4 \text{ cm}$

Solution

- (a) Surface area = area of two circular ends + area of the curved surface

$$\begin{aligned}&= 2\pi r^2 + 2\pi rl \\&= \left(2 \times \frac{22}{7} \times 2.8 \times 2.8\right) + \left(2 \times \frac{22}{7} \times 2.8 \times 13\right) \\&= 49.28 \text{ cm}^2 + 228.8 \text{ cm}^2 \\&= 278.08 \text{ cm}^2\end{aligned}$$

(b) Surface area = area of the hexagonal ends + area of the six rectangular faces

$$\begin{aligned}
 &= [6\left(\frac{1}{2}ab\sin\theta\right)] \times 2 + 6(l \times w) \\
 &= 2[6\left(\frac{1}{2} \times 6 \times 6 \times \sin 60^\circ\right)] + 6(6 \times 15) \\
 &= 2[6\left(\frac{1}{2} \times 36 \times 0.8660\right)] + 6 \times 90 \\
 &= (187.06 + 540) \text{ cm}^2 \\
 &= 727.06 \text{ cm}^2
 \end{aligned}$$

(c) Surface area = area of the ends + area of the five rectangular faces

$$\begin{aligned}
 &= 2[(a \times b) + (\frac{1}{2} \times e \times d)] + 2(c \times a) + (c \times b) + (c \times d) + (c \times e) \\
 &= 2[(4 \times 5) + (\frac{1}{2} \times 3 \times 4)] + 2(8 \times 4) + (8 \times 5) + (8 \times 4) + (8 \times 3) \\
 &= 2(20 + 6) + 64 + 40 + 32 + 24 \\
 &= 52 + 64 + 40 + 32 + 24 \\
 &= 212 \text{ cm}^2
 \end{aligned}$$

13.3: Surface Area of a Pyramid

A pyramid is a solid with a polygonal base and slanting sides that meet at a common apex. The surface area of a pyramid is the sum of the area of the slanting faces and the area of the base.

Figure 13.2 shows a right pyramid VABCD with a square base of side x cm, a slanting height t cm and a perpendicular height h cm. The

perpendicular height is usually referred to as the 'height'.

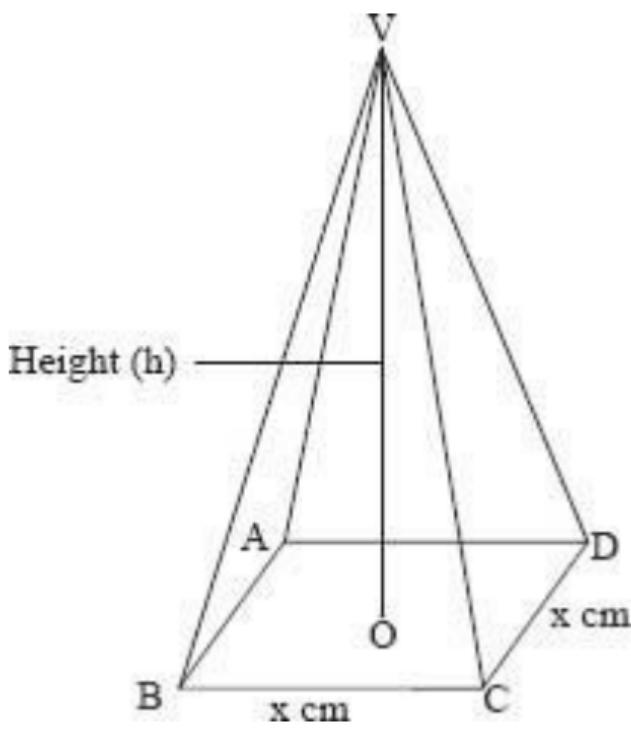


Fig. 13.2

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Example 2

Figure 13.3 is a right pyramid with a square base of 4 cm and a slanting edge of 8 cm. Find the surface area of the pyramid.

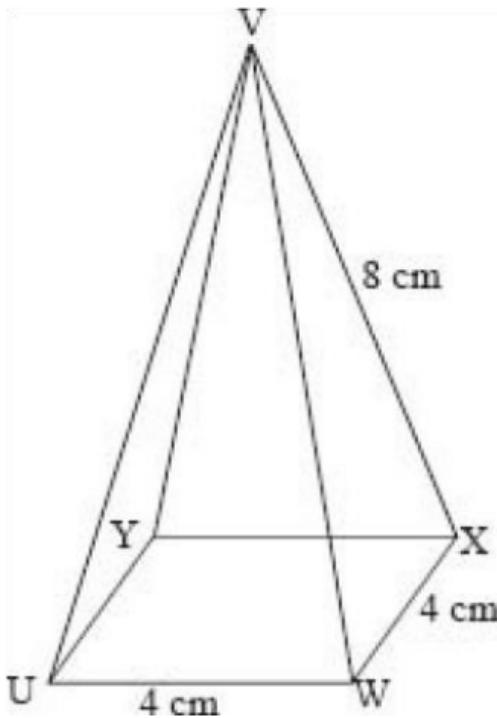


Fig. 13.3

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Solution

Surface area = base area + area of the four triangular faces.

$$\begin{aligned} &= (l \times w) + 4\left(\frac{1}{2}bh\right) \\ &= (4 \times 4) + 4 \times \frac{1}{2} \times 4 \times \sqrt{(8^2 - 2^2)} \\ &= 16 + 2 \times 4 \times \sqrt{60} \\ &= 16 + 2 \times 4 \times 7.746 \\ &= (16 + 61.97) \text{ cm}^2 \\ &= 77.97 \text{ cm}^2 \end{aligned}$$

Example 3

Find the surface area of a right pyramid with a rectangular base of 6 cm by 4 cm and a height of 9 cm.

Solution

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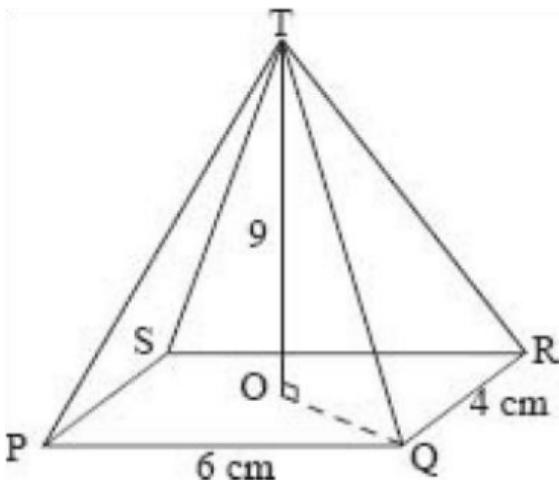


Fig. 13.4

$$SQ = \sqrt{(6^2 + 4^2)}$$

$$= \sqrt{52}$$

$$= 7.211$$

$$OQ = \frac{1}{2}$$

$$= \frac{1}{2} \times 7.211$$

Slanting edge = 3.606

$$TQ^2 = TO^2 + OQ^2$$

$$= 9^2 + 3.606^2 \quad TQ = 81.13.00$$

$$= 9.695 \text{ cm.}$$

Height of the triangular faces with a base of 4 cm; $h = \sqrt{(9.695^2 - 2^2)}$

$$= \sqrt{(94 - 4)}$$

$$= \sqrt{90}$$

$$= 9.487 \text{ cm}$$

Height of triangular faces with a base of

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$$6 \text{ cm}; h =$$

$$\sqrt{(9.695^2 - 3^2)}$$

$$= \sqrt{(94 - 9)}$$

$$= \sqrt{85}$$

$$= 9.220 \text{ cm}$$

Surface area = area of base + area of the slanting faces

$$= (6 \times 4) + 2\left(\frac{1}{2} \times 4 \times 9.487\right) + 2\left(\frac{1}{2} \times 6 \times 9.220\right)$$

$$= (24 + 37.95 + 53.32) \text{ cm}^2$$

$$= 117.3 \text{ cm}^2$$

13.4: Surface Area of a Cone

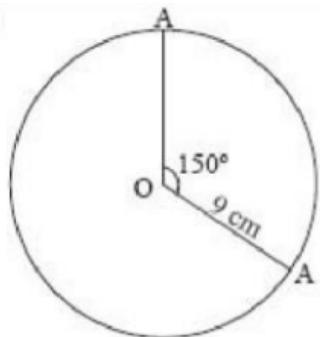
A cone is a pyramid with a circular base.

Project

- (i) Draw a circle of radius 9 cm and cut it out.
- (ii) Cut out a sector whose arc subtends an angle of 150° at the centre, see figure 13.5 (a).
- (iii) Find the area of the sector.

(iv) Fold the sector to form the curved surface of an open cone, see figure 13.5(b).

(a)



(b)

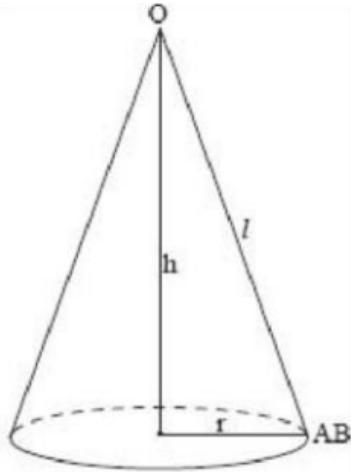


Fig. 13.5

(v) Deduce the slant height l of the cone.

- (vi) Deduce the radius (r) of the circular base of the cone.
- (vii) Find the area of the curved surface of the cone.
- (viii) Compare the area of the curved surface of the cone with the product $\pi r l$.

Note that:

- (i) The area of the curved surface of the cone is equal to the product of $\pi r l$.
- (ii) The surface area of the closed cone is $\pi r^2 + \pi r l$, where πr^2 is area of the base and $\pi r l$ the area of the curved surface.

Example 4

Calculate the surface area of a solid cone of base radius 7 cm and height 13 cm.

Solution

$$\text{Surface area} = \pi r^2 + \pi r l$$

$$\begin{aligned}\text{But } l &= \sqrt{(13^2 + 7^2)} \\ &= \sqrt{218} \\ &= 14.76 \text{ cm}\end{aligned}$$

Therefore, surface area =

$$\begin{aligned}&= \left(\frac{22}{7} \times 7 \times 7\right) + \left(\frac{22}{7} \times 7 \times 14.76\right) \\ &= 154 + 324.72 \\ &= 478.7 \text{ cm}^2\end{aligned}$$

13.5: Surface Area of a Frustum

If a cone or a pyramid is cut through on a plane parallel to the base, then the top

part forms a smaller cone or pyramid and the bottom part forms a frustum (plural frusta).

Example 5

Find the surface area of the fabric required to make a lampshade in the shape of a frustum whose top and bottom diameters are 20 cm and 30 cm respectively and height 12 cm.

Solution

The frustum is made from an original cone of height H as shown in fig. 13.6.

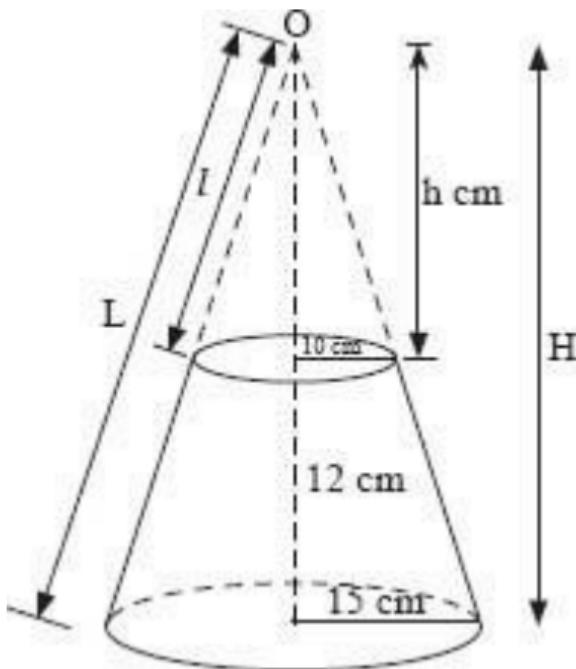


Fig. 13.6

From knowledge of similar triangles;

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$$\frac{h}{10} = \frac{h+12}{15} \quad \frac{h}{r} = \frac{H}{R}$$

$$15h = 10h + 120$$

$$15h - 10h = 120$$

$$5h = 120 \quad h = 24$$

Surface area of = area of curved – area
of the the frustum surface of curved
surface

bigger cone of smaller cone

Surface area = $\pi RL - \pi rl$

$$\begin{aligned}&= \frac{22}{7} \times 15 \times \sqrt{(36^2 + 15^2)} - \frac{22}{7} \times 10 \times \sqrt{(24^2 + 10^2)} \\&= \frac{22}{7} \times 15 \times 39 - \frac{22}{7} \times 10 \times 26 \\&= 1838.57 \text{ cm}^2 - 817.14 \text{ cm}^2 \\&= 1021 \text{ cm}^2 \text{ (4 s.f.)}\end{aligned}$$

13.6: Surface Area of a Sphere

A sphere is a solid that is entirely round with every point on the surface at an equal distance from the centre.

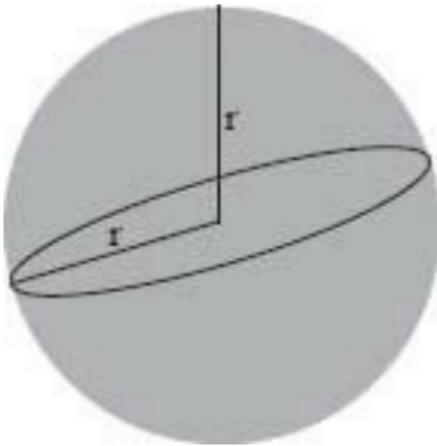


Fig. 13.7

Figure 13.7 is a sphere of radius r cm. The surface area of a sphere of radius r cm is four times the area of a circle of the same radius as the sphere, i.e., surface area of a sphere is $4\pi r^2$.

Example 6

Find the surface area of a sphere whose diameter is 21 cm.

Solution

$$\text{Surface area} = 4\pi r^2$$

$$\begin{aligned}&= 4 \times \frac{1}{2} \times 10.5 \times 10.5 \\&= 1386 \text{ cm}^2\end{aligned}$$

13.7: Surface Area of a Hemisphere

A hemisphere is a half of a sphere. The surface area of a hemisphere is the sum

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of the curved area plus the area of the circular top of the hemisphere.

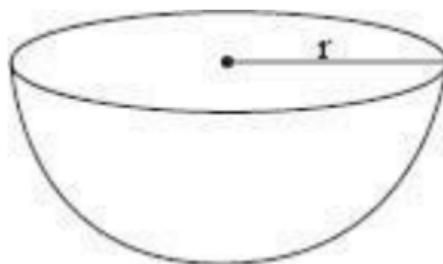


Fig. 13.8

$$\text{Surface Area} = \pi r^2 + \frac{4\pi r^2}{2}$$

+

$$= \pi r^2 + 2\pi r^2$$

$$= 3\pi r^2$$

Example 7

Find the surface area of a hemisphere whose radius is 15 cm².

Solution

$$\text{Surface area} = 3\pi r^2$$

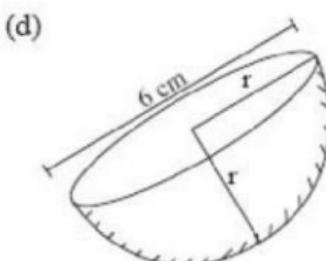
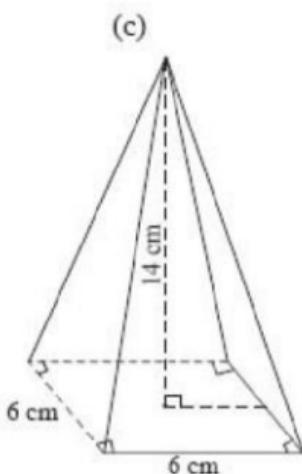
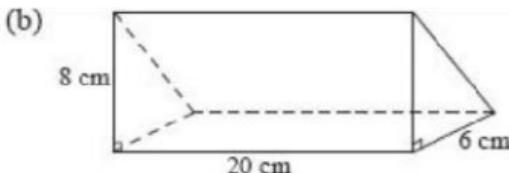
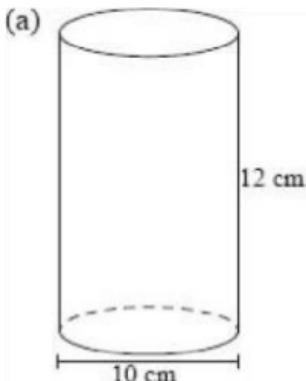
$$= 3 \times \pi \times 15^2$$

$$= 2120.6 \text{ cm}^2$$

Exercise 13.1

1. Find the surface area of each of the following solids:

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2. Find the surface area of a hollow cylindrical iron pipe 2.1 m long and external diameter 15 cm, given that

the iron is 1 cm thick. (Give your answer in m^2)

3. A metal storage water tank is in the form of a cylinder with one hemispherical end of radius 2 m. If the other end is flat and the total height of the tank is 8 m, calculate the outer surface area of the tank.
4. A right cone of base diameter 8 cm and height 13 cm is slit and opened out into a sector. Find the angle

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formed by the two radii of the sector.

5. A right pyramid 6 cm high stands on a rectangular base 6 cm by 4 cm. Calculate the surface area of the pyramid.
6. A spherical ball is 15 cm in diameter. Find its surface area.
7. A right pyramid on a square base 8 metres is 15 metres high.
 - (a) Find its surface area.

- (b) If the pyramid is cut at a height of 6 metres from the bottom, find the surface area of the frustum formed.
8. A solid frustum of a right pyramid with a square bottom and a square top of side 2 m and 1 m respectively. If the height of the frustum is 1.5 metres, find its surface area.

9. A water container is a frustum of a cone whose upper and lower radii are 3.5 cm and 26 cm respectively. If the height of the container is 60 cm, calculate the outer surface area of the material used to make the container.
10. A scientist using a telescope observed a spherical object moving in space. She established that the object has a radius of 17.5 m. Find the surface area of the object.

Chapter 14

VOLUME OF SOLIDS

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Volume is the amount of space occupied by matter. The volume of an object is expressed in cubic units.

14.1: Volume of a Prism

A prism is a solid with a uniform crosssection. The volume V of a prism with cross section area A and length l is given by $V = Al$.

Example 1

Find the volume of a triangular-based prism of sides 10 cm, 7 cm and 13 cm and length 25 cm.

Solution

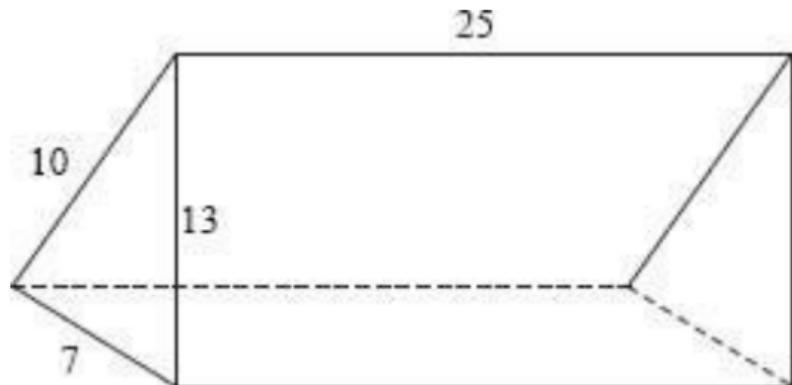


Fig. 14.1

Area of the cross-section, $A = \sqrt{s(s-a)(s-b)(s-c)}$

But $s = \frac{1}{2}(10 + 7 + 13)$

$$= 15$$

Therefore, $A = \sqrt{15(15-13)(15-10)(15-7)}$

$$= \sqrt{15(2)(5)(8)}$$
$$= \sqrt{1200}$$
$$= 34.64$$

Volume of the prism = $A \times l$

$$= 34.64 \times 25$$
$$= 866 \text{ cm}^2$$

Example 2

Find the volume of a regular hexagonal nut of side 5 cm and height 3 cm.

Solution

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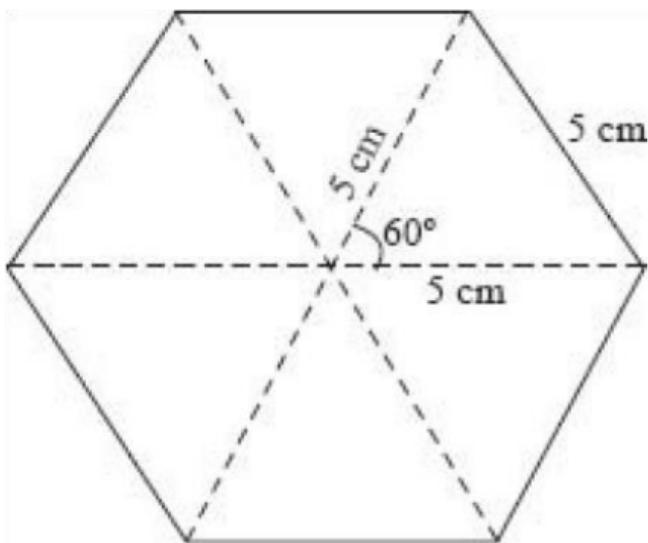


Fig. 14.2

Area of cross section

$$= 6 \times \left(\frac{1}{2} \times 5 \times 5 \sin 60^\circ \right)$$

$$= 6 \times \frac{1}{2} \times 25 \times \sin 60^\circ$$

$$= 64.95$$

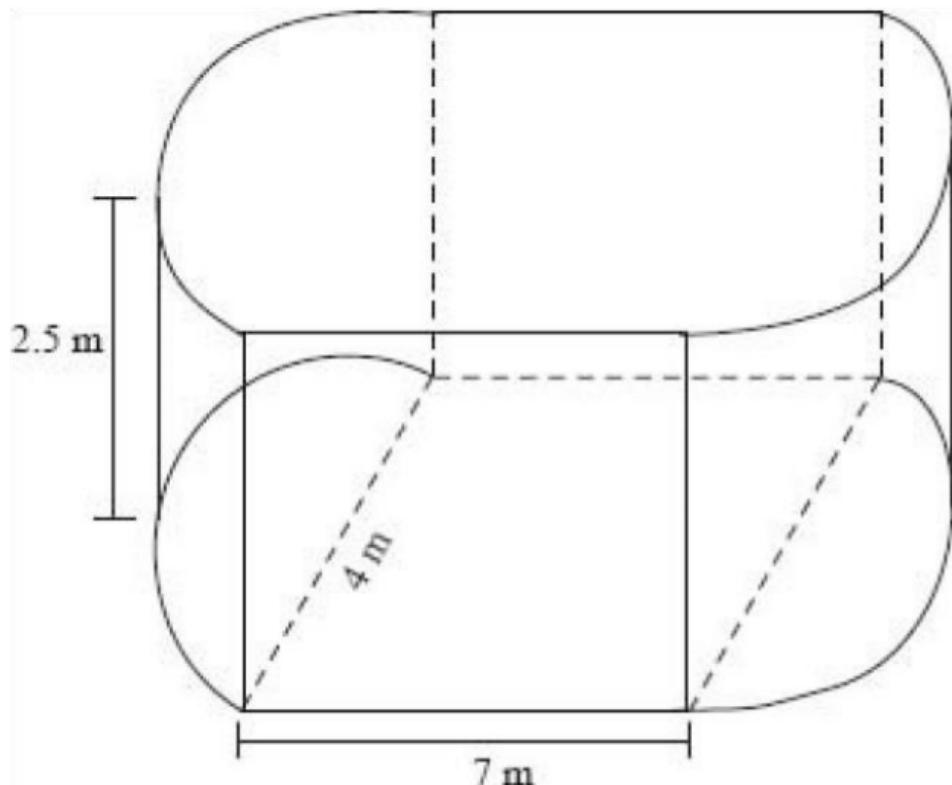
Volume of the nut = 64.95×3

$$= 194.9 \text{ cm}^3$$

Exercise 14.1

1. A path is 300 m long and 1.8 m wide. Find the volume of gravel needed to cover it to a depth of 5 cm.
2. Calculate the volume of a metal tube whose internal diameter is 70 mm and thickness 7 mm, if it is 15 cm long.

3. A water tank of height 2.5 m has a uniform cross-section in the shape of a rectangle with semi-circular ends as shown in the figure below. The inside length of the tank is 7 m and its width 4 m. Calculate the:
- (a) area of the base.
 - (b) volume of the tank.
 - (c) capacity of the tank in litres.

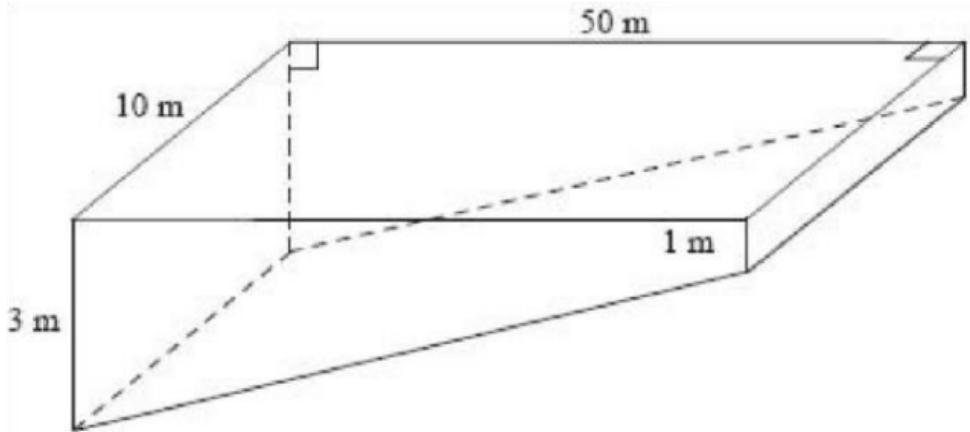


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4. A steel bar has the shape of a regular hexagon of side 2 cm, and it is 20 m long. Calculate:
- (a) the cross-section area of the bar.
 - (b) the volume of the bar.
 - (c) the mass in kilograms of the bar if 1 m^3 of steel weighs 7 800 kg.
5. A concrete tower on which a water tank is to be placed has a uniform octagonal cross-section of side 3 m. If its height is 24 m, find the volume of concrete used to make it.

6. A metal rod of 20 m length has an isosceles triangular base, where the equal side are 12 cm each. If the included angle of the base is 40° , calculate:
- (a) the area of the cross-section.
 - (b) the volume of the metal rod.
 - (c) the mass of the metal rod, if it is made of copper, whose density is $9\ 000\ \text{kg/m}^3$.

7. The cross-section of a container is in the shape of a trapezium. The two parallel sides of the container are 3.6 m and 2.1 m long, while the perpendicular height between them is 2.4 m. If its length is 6 m, find the volume of the container.
8. The figure below represents a swimming pool. Calculate:
- (a) the volume of the pool in cubic metres.
 - (b) the capacity of the pool in litres.



14.2: Volume of a Pyramid

Project

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Figure 14.3 shows the net of a square based pyramid. (All measurements are in centimetres.)

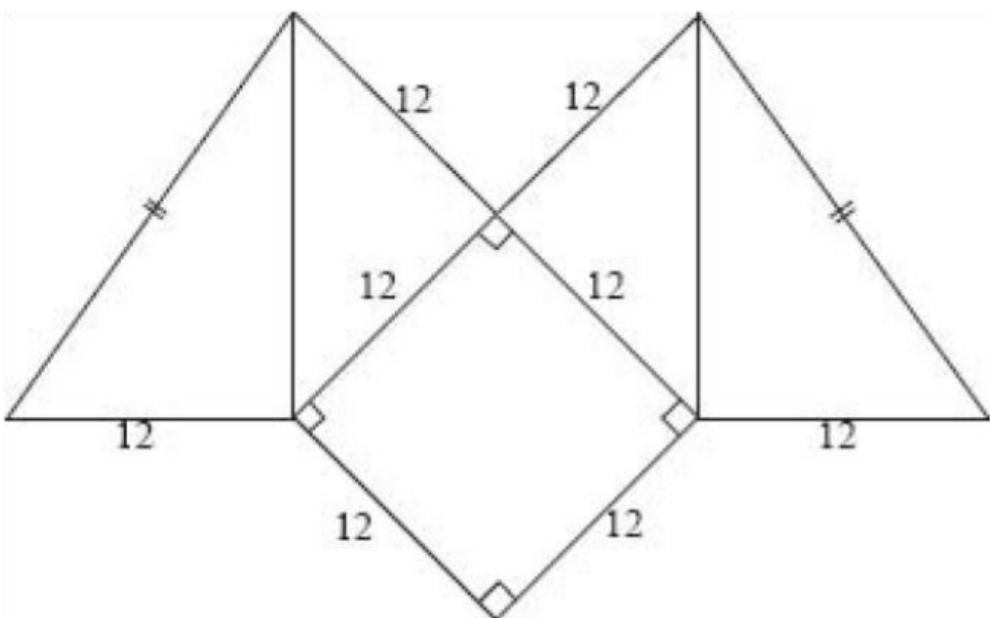


Fig. 14.3

Construct three such nets and fold them to form three pyramids. Assemble them to make a cube.

- (i) What is the volume of the cube you have made from the pyramids?
- (ii) What is the volume of one pyramid?

You should notice that the:

- (i) volume of the cube is $(12 \times 12 \times 12) \text{ cm}^3 = 1728 \text{ cm}^3$.

(ii) volume of the each pyramid $\frac{1}{3}$ is of the volume of the cube. That is;

$$\begin{aligned}\text{Volume of pyramid} &= \frac{1}{3} (12 \times 12 \times 12) \text{ cm}^3 \\ &= 576 \text{ cm}^3\end{aligned}$$

Therefore, the volume of each pyramid is given by;

$$\frac{1}{3}(\text{base area of pyramid} \times \text{height})$$

In general, the volume (V) of any pyramid is given by;

$$\begin{aligned}V &= \frac{1}{3}(\text{base area}) \times \text{height} \\ &= \frac{1}{3}Ah.\end{aligned}$$

where A is the area of the base and h the perpendicular height of the pyramid

Example 3

Find the volume of the pyramid in figure 14.4:

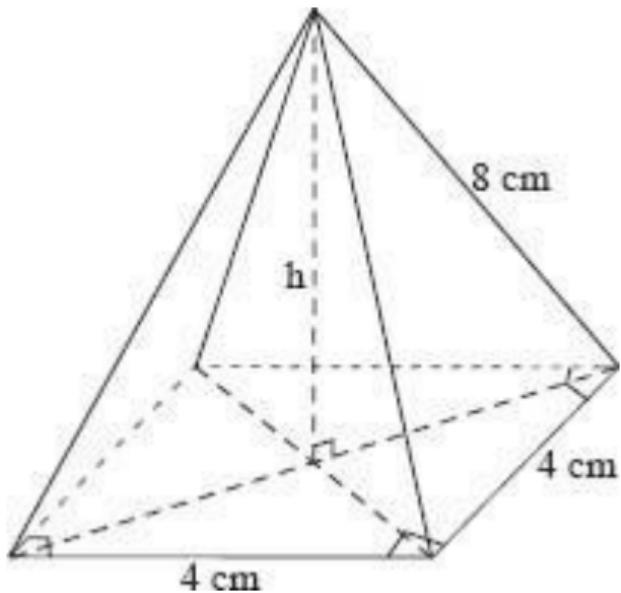


Fig. 14.4

Solution

$$\text{Area of base} = 4 \times 4 = 16 \text{ cm}^2$$

To find the height of the pyramid,
proceed as follows:

Half the length of the diagonal

$$= \frac{1}{2} \times \sqrt{4^2 + 4^2}$$

$$= \frac{1}{2} \sqrt{32}$$

$$= \frac{1}{2} \times 5.657$$

$$= 2.829 \text{ cm}$$

The height (h) of the pyramid

$$= \sqrt{(8^2 - 2.829^2)}$$

$$= \sqrt{(64 - 8.003)}$$

$$= 7.483 \text{ cm}$$

Therefore, the volume of the pyramid

$$= \frac{1}{3} \times 16 \times 7.483$$

$$= \frac{1}{3} \times 119.7$$

$$= 39.91 \text{ cm}^3$$

14.3: Volume of a Cone

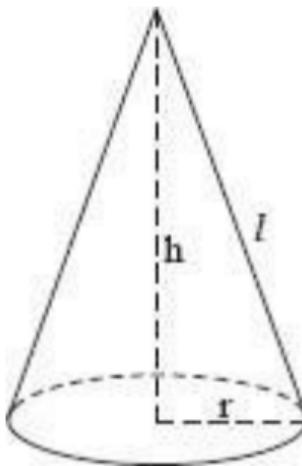
Since a cone is a right pyramid with a circular base, its volume V is given by;

$$\begin{aligned}V &= \frac{1}{3} (\text{base area}) \times \text{height} \\&= \frac{1}{3} \pi r^2 h\end{aligned}$$

Example 4

Calculate the volume of a cone whose height is 12 cm and length of the slant height 13 cm. (Take $\pi = \frac{22}{7}$)

Solution



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Fig. 14.5

$$\begin{aligned}\text{Volume} &= \frac{1}{3} (\text{base area}) \times \text{height} \\ &= \frac{1}{3} \pi r^2 h\end{aligned}$$

$$\begin{aligned}\text{But, base radius } r &= \sqrt{13^2 - 12^2} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{Therefore, volume} &= \frac{1}{3} \times \frac{22}{7} \times 25 \times 12 \text{ cm} \\ &= 314.3 \text{ cm}^3\end{aligned}$$

Example 5

Figure 14.6 represents a frustum of base radius 2 cm and height 3.6 cm. If the height of the cone from which it was cut was 6 cm, calculate:

- (a) the radius of the top surface.
(b) the volume of the frustum.

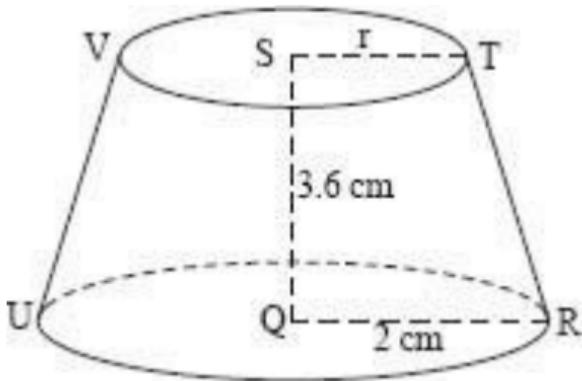


Fig. 14.6

Solution

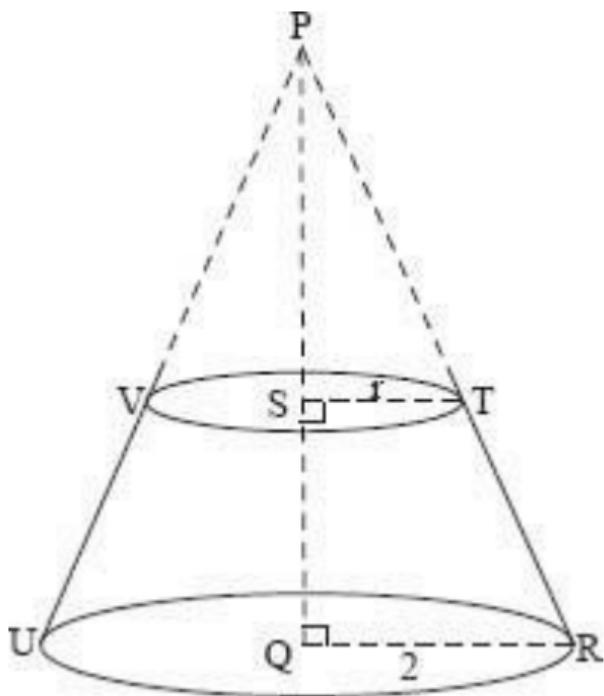


Fig. 14.7

(a) Triangles PST and PQR are similar.

Therefore, $\frac{PQ}{PS} = \frac{QR}{ST} = \frac{PR}{PT}$

$$S \quad \frac{6}{2.4} = \frac{2}{ST} \quad O,$$

$$ST = 0.8 \text{ cm}$$

Therefore, the radius of the top surface is 0.8 cm.

- (b) Volume of the frustum = volume of large cone – volume of smaller cone.

$$\begin{aligned} &= \frac{1}{3} \times 3.142 \times 4 \times 6 - \frac{1}{3} \times 3.142 \times (0.8)^2 \times 2.4 \\ &= 25.14 - 1.61 \\ &= 23.53 \text{ cm}^3 \end{aligned}$$

Alternatively

Using volume scale factor;

a

$$\text{Line r scale factor (LSF)} \quad \frac{2}{0.8}$$

=

$$= \left(\frac{5}{2}\right)^3$$

2.5

$$= \frac{125}{8}$$

Volume scale factor
(VSF)

$$= 25.14 \times \frac{8}{125}$$

$$= 1.61 \text{ cm}^3$$

Volume of the large cone = 25.14 cm^3

Volume of the small cone

Volume of the frustum = volume of large cone – volume of small cone

$$= 25.140 - 1.61 =$$

23.53 cm³

Exercise 14.2

1. Find the volume of each of the following pyramids:
 - (a) Triangular base, sides 5 cm, 12 cm and 13 cm, height 24 cm.
 - (b) Square base, side 6 cm, height 10 cm.

c i

- (c) Rectangular base, 8 cm by 10 cm, height 12 cm.

2. Find the volume of each of the following right pyramids:

- (a) Rectangular base, 18 cm by 24 cm, slant edge 39 cm.

- (b) Square base, side 8 cm edge 8 cm.

- (c) Triangular base, sides 10 cm, 12 cm and 4 cm, height 24 cm.

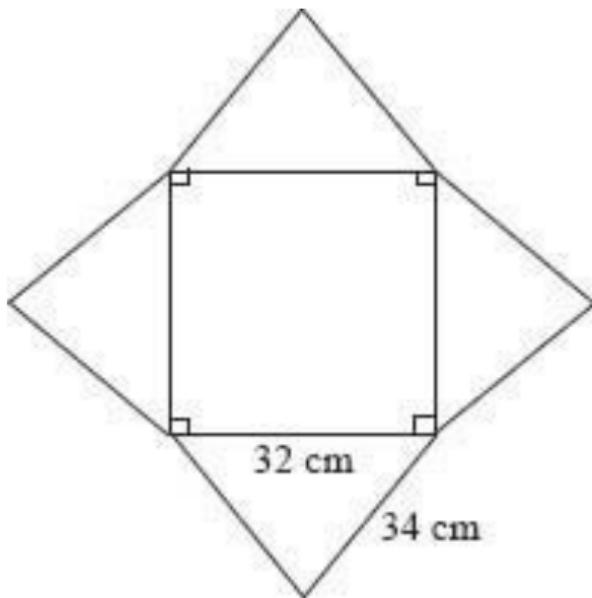
3. (a) The volume of a pyramid is 120 m^3 . Calculate its height and the surface area if the base is a square of side 6 cm.
- (b) Find the height of a pyramid with base area 24 cm^2 and volume 96 cm^3 .
4. Calculate the height and the length of the slant edge of a right pyramid whose base is a square of side 5 cm and volume 125 cm^3 .

e i

5. The volume of a right pyramid of height 16 cm on a square base is 192 cm^3 . Calculate the length of the side of the base.
6. VABCD is a right pyramid with a square base ABCD. Calculate its volume if the height is 12 cm and the slant edge is 16 cm.
7. The figure below is a net of a pyramid with a square base of side 32 cm. The other four faces are

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isosceles triangles whose equal sides are 34 cm each. Calculate the volume of the pyramid.



g i

8. Calculate the volume of a pyramid of height 14 cm with a square base of side 24 cm.
9. A cone has a base radius of 9.44 cm and a volume of 113.6 cm^3 . Calculate the height of the cone.
10. Find the volume of each of the following cones:
 - (a) Base area 18 cm^2 , height 6 cm.
 - (b) Slant height 15 cm. height 9 cm.

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- (c) Base perimeter 16 cm, height 10 cm.
- (d) Slant height 17 cm, height 7 cm.
- (e) Base radius 9 cm, height 24 cm.

11. Calculate the total surface area and volume of a frustum of a pyramid whose ends are squares of 10 cm and 12 cm and the height between the two ends is 4 cm.
12. A toy consists of a conical top and a cylindrical base. The diameter of the base is 5 cm and the height of

i i

the cylindrical part is 4 cm. If the total height of the toy is 9 cm, calculate:

- (a) its total surface area.
- (b) its total volume.

13. A frustum is cut from a cone whose radius is r cm and height h cm. If the height of the frustum is t centimetres, express the volume of

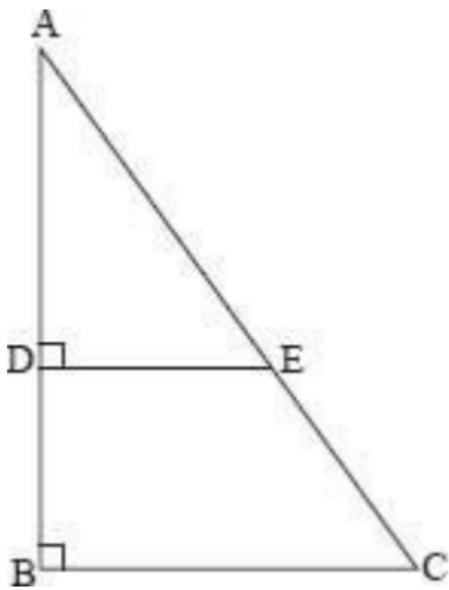
the frustum as a fraction of the volume of the cone.

14. The figure below shows a triangle in which $AB = 18 \text{ cm}$, $BC = 6 \text{ cm}$, BD

i

= 7 cm and DE is parallel to BC. If the triangle is rotated about AB, calculate:

- (a) the surface area of the cone formed.
- (b) the volume of the cone.
- (c) the volume of the frustum formed by the portion BCED.



14.4: Volume of a Sphere

e

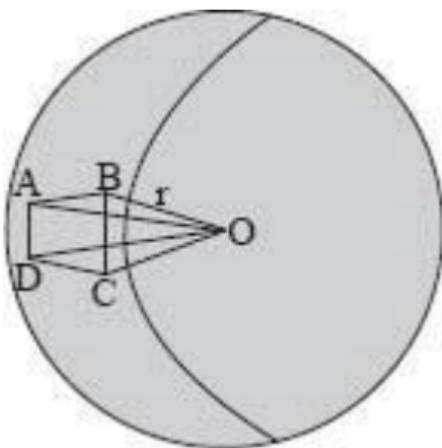


Fig. 14.8

Figure 14.8 shows a sphere with radius r cm and a unit square ABCD. Let the volume of pyramid OABCD be V cm³.

$$\begin{aligned}\text{Therefore, } V &= \frac{1}{3}(\text{base area} \times \text{height}) \\ &= \frac{1}{3} \times l \times r \\ &= \frac{r}{3} \text{ cm}^3\end{aligned}$$

Surface area of the sphere = $4 \pi r \text{ cm}^2$

Therefor , volume of the sphere =
volume of each \times surface area of
pyramid the sphere

$$\begin{aligned}&= \frac{r}{3} \times 4\pi r^2 \\ &= \frac{4\pi r^3}{3} \text{ cm}^3\end{aligned}$$

Exercise 14.3

e

1. Calculate the volume of a sphere whose radius is:
 - (a) 4.8 cm
 - (b) 21 cm.
2. A solid metal cylinder is 60 cm long and 18 cm in diameter. Determine the number of solid spheres of diameter 8 cm that can be moulded from the metal of the cylinder.

3. The internal and external radii of a spherical shell are 8 cm and 9 cm respectively. Calculate the volume of the material of the shell to the nearest cm^3 .
4. A cylindrical tank of radius 0.6 m contains water to a depth of 1 m. A solid metal sphere of radius 0.5 m is placed in it. Calculate the rise of the water level in centimetres.
5. The volume of water in a measuring cylinder is 25.2 cm^3 . After a solid metal sphere is immersed in it, the

e

measuring cylinder reads 29.4 cm^3 .
Calculate the radius of the sphere.

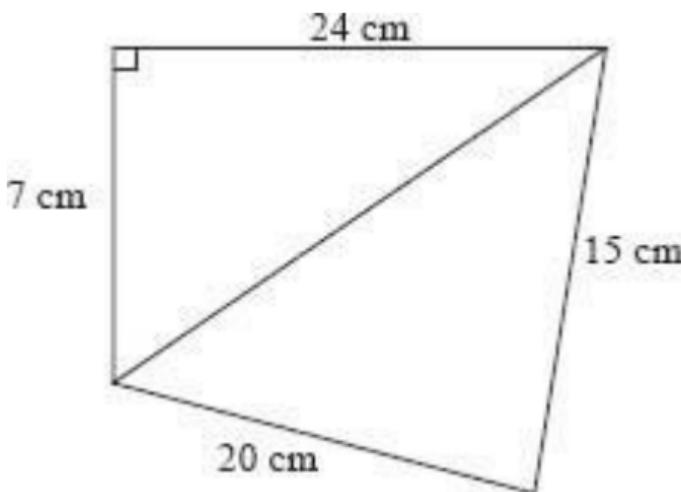
6. The mass of 1 cm^3 of lead is 11.4 g .
Calculate the mass of a lead sphere
whos radius is 15 cm .
7. If 1 cm^3 of copper metal has a mass
of 8.88 g , calculate the mass of a
quarter sphere of copper whose
radius is 24 cm .

8. A spherical container which is 30 cm in diameter is $\frac{4}{3}$ full of water. The water is emptied into a cylindrical container of diameter 12 cm. Find the depth of the water in the cylindrical container.
9. A solid copper sphere of 36 cm diameter is to be moulded from a copper wire 0.8 mm in diameter. Find the length of copper wire required to make the sphere?

Mixed Exercise 2

e

- Find the area of the quadrilateral below.



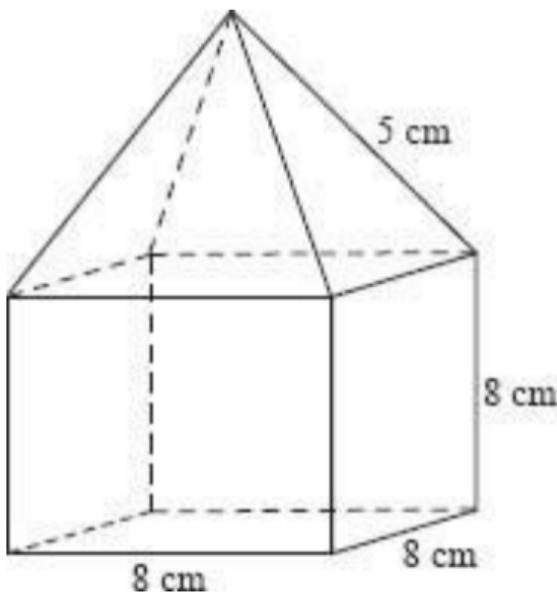
- The length of an arc of a circle is $\frac{1}{10}$ of its circumference. If the area of

the circle is 13.86 cm^2 , find:

- (a) the angle subtended by the arc at the centre of the circle.
- (b) the area of the sector enclosed by this arc.

3. A model of a tent consists of a cube and a pyramid on a square base as shown below:

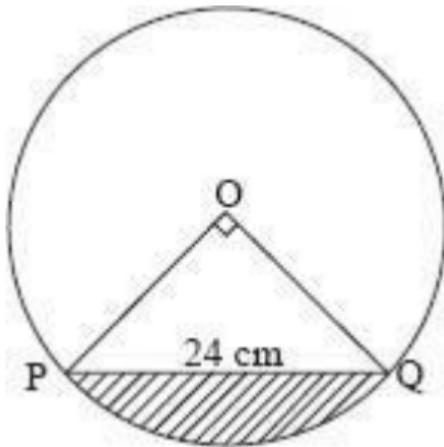
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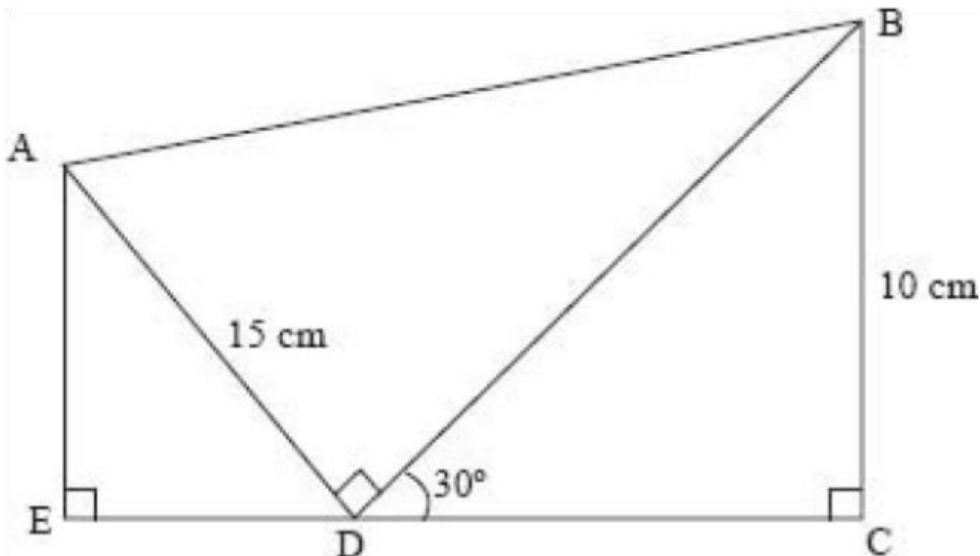
Calculate the total surface area of the model (floor inclusive)

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4. Solve the equation $am + bh = r$, given that $\mathbf{m} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$, $\mathbf{n} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}$, and a and b are scalars.
5. Find the volume of a prism 15 cm long and whose cross-section is a regular heptagon of side 10 cm.
6. In the figure below, O is the centre of the circle, $\angle POQ = 90^\circ$ and $PQ = 24 \text{ cm}$: Find:
- (a) the area of the sector POQ.
 - (b) the area of the shaded region.

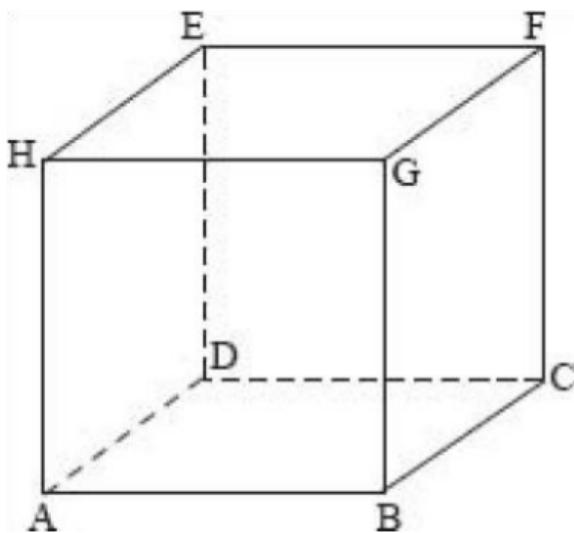


7. Find the area of the figure below if $AD = 15 \text{ cm}$, $BC = 10 \text{ cm}$, $\angle AED = \angle ADB = \angle BCD = 90^\circ$ and $\angle BDC = 30^\circ$:



8. John walks 4.8 km due north from point A to B and then covers a further 6 km due east to C. What is the shortest distance from A to C?

9. The figure below is a cube of side 6 cm:



Calculate the length:

- (a) AC

(b) AG

(c) AF

10. The area of a rhombus is 442 cm^2 .
If one of its diagonal is 34 cm long,
find:

(a) the length of the other
diagonal?

(b) the length of the side of the
rhombus?

11. The base of a solid right pyramid is
a regular hexagon of side 8 cm. If

the slanting edges are 10 cm, find the:

- (a) surface area of the pyramid.
- (b) volume of the pyramid.

12. The cross-section of a prism is a triangle with sides 3 cm, 5 cm and 7 cm. If the prism is 12 cm long, find its volume.

13. Given that $\cos x = \sin (3x + 10)$, find:

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(a) x

(b) $\tan x$

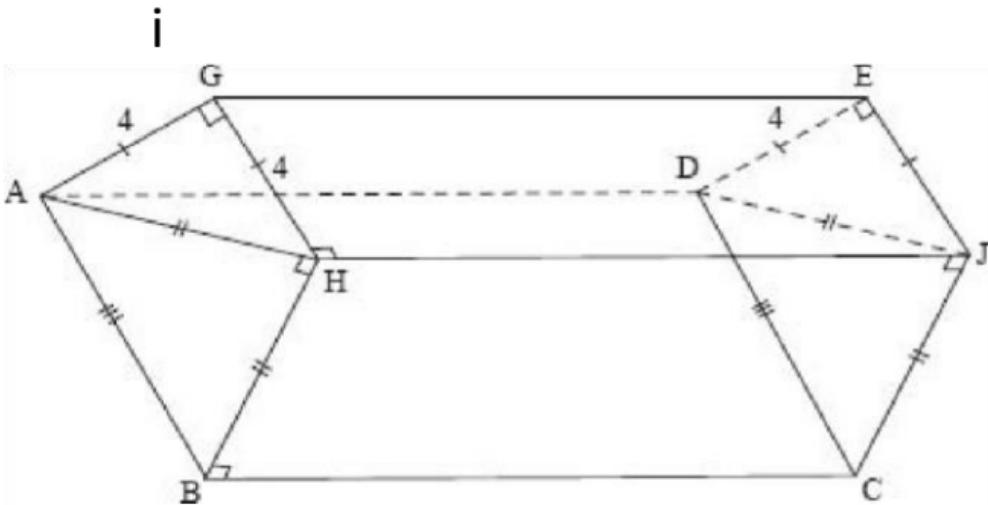
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14. A television aerial is held upright by a wire 16 m long which is fixed to the top of the aerial and to the ground 3 metres from the foot of the aerial. Find the angle of inclination of the wire to the ground.
15. A frustum of height 6 cm is cut from a pyramid whose base is a square of side 20 cm and height 14 cm. Find the volume of the frustum.

16. The radius and height of a cylinder are 3.5 cm and 6.5 cm respectively. Calculate the surface area, the height and the radius of a similar cylinder whose volume is 2 002 cm^3 . (take π

$$= \frac{22}{7}$$

17. In the figure below, $AG = GH = EJ = ED = 4 \text{ cm}$. $AH = BH = CJ = DJ$ and $AB = CD$.

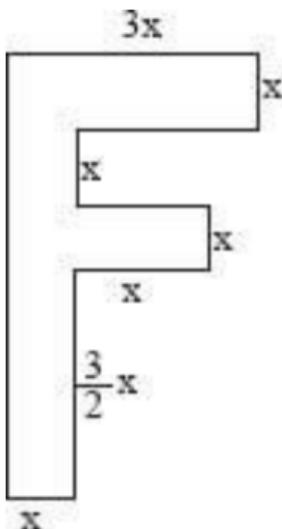


Calculate:

- (a) CJ and DC.
- (b) the ratio of the area of AGH to ABH.

- (c) the ratio of the volume of AHJDEG to ABCDJH.
- (d) the volume and the surface area of the whole block if $GE = BC = 2DJ$.
18. The area of the figure below is A cm^2 and its perimeter is P cm. If $A = P + 8$, find x .

i



19. Find the surface area of a cone whose height is 5.2 cm and volume 49 cm^3 .

20. A boy's shadow is 2.2 m long when
the angle of elevation from the tip
of

i

the shadow to h s head is 30° .
Calculate the height of the boy.

21. Calculate the volume of a cone whose base radius is 5 cm and curved surface area is 109.9 cm^2 .
22. Find the length of a side of a regular pentagon whose vertices lie on the circumference of a circle of radius 5 cm.
23. A man walks 1 km on a bearing of 140° . How far is he;
- (a) east of the starting point?
 - (b) south of the starting point?

24. A triangle ABC is such that $\angle BCA = 90^\circ$, $BC = 48 \text{ cm}$ and $\angle ABC = 60^\circ$. Determine lengths AB and AC.
25. An observer standing 20 m away from a tall building finds that the angle of elevation of the top of the building is 68° and the angle of depression of its foot is 50° . Calculate the height of the building.
26. Calculate the volume of metal required to make a hemispherical bowl with internal and external radii 8.4 cm and 9.1 cm respectively.

i

27. Onyango observed a hawk exactly overhead at the same time as Kamau observed it at an angle of elevation of 50° . If Onyango and Kamau were 25 m apart on level ground, how high was the hawk above the ground, given that Kamau's eye level was 172 cm above the ground?
28. A straight road rises steadily at an angle of 4° to the horizontal. If a car travels 2 km up the road,

i

calculate its vertical and horizontal displacement.

29. A pyramid PQRST has a rectangular base QRST. If the base of the pyramid measures 8 cm by 4 cm and perpendicular height is equal to the diagonal of the base, find its volume and surface area.
30. A ladder 8 m long is leaning against a vertical wall. The ladder makes an angle of 70° with the level ground. Calculate the distance of the top of the ladder from the ground.

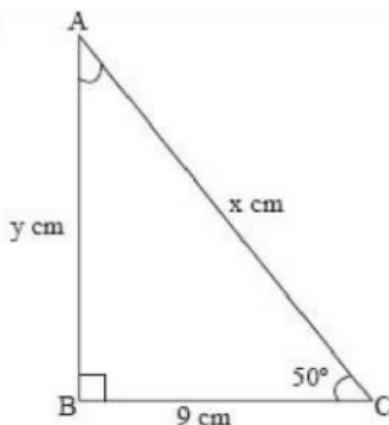
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31. Find the value of the unknown

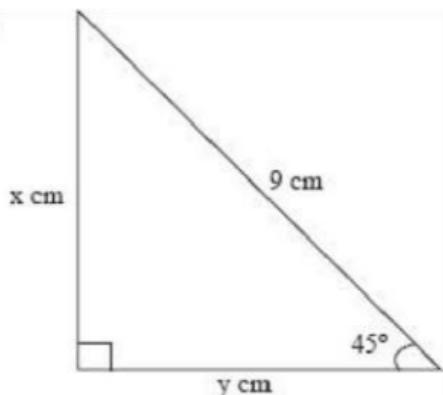
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lengths in each of the following diagrams:

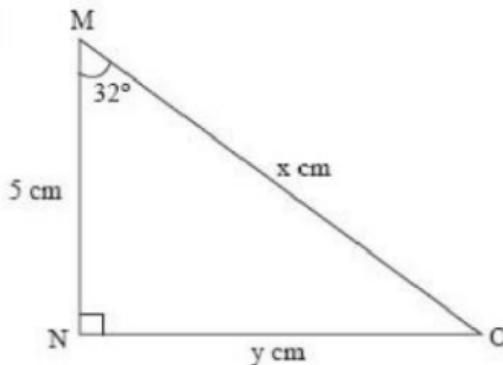
(a)



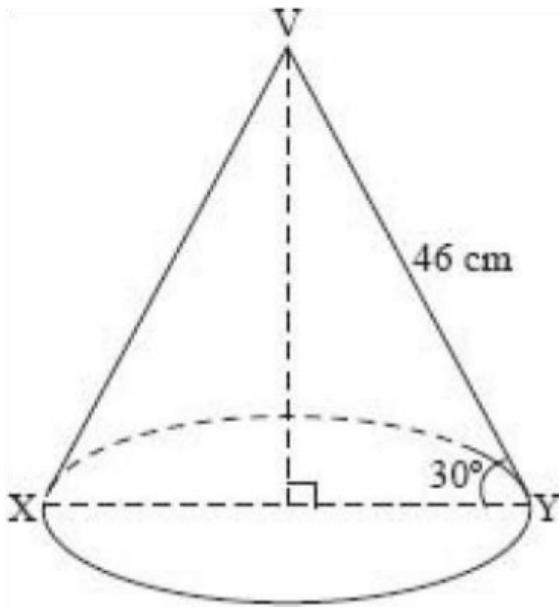
(b)



(c)



32. The figure below shows a cone in which angle $VYX = 30^\circ$ and $VY = 46$ cm. Calculate the height and diameter of the cone.

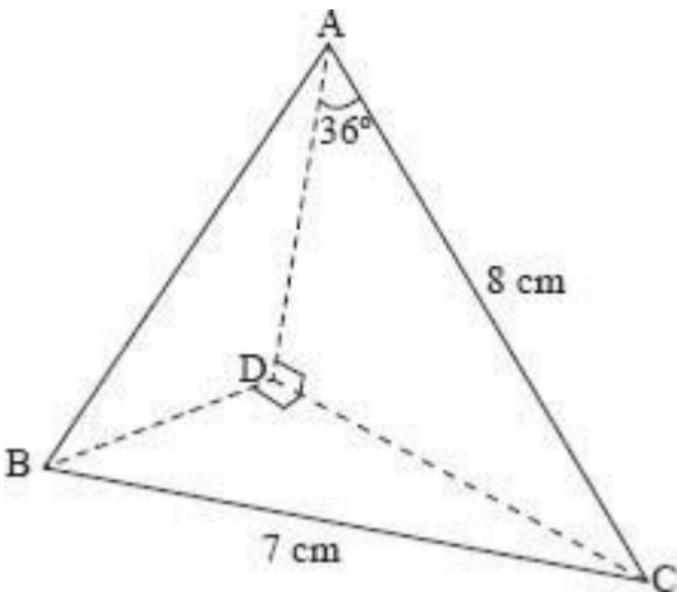


33. AB is a chord of a circle with centre O and radius 12.3 cm. If $\angle OAB = 70^\circ$, calculate the length AB and the distance of O and AB.

34. Find the radius of a cone whose height is 8 cm and volume 115 cm^3 .
35. The two parallel sides of a trapezium measure 12 cm and 9 cm. A third side measures 10.4 cm and makes an angle of 50° with the longest side. Calculate the area of the trapezium.
36. A bucket is 44 cm in diameter at the top and 24 cm in diameter at the

bottom. Find its capacity in litres if it is 36 cm deep.

37. The figure below shows a tetrahedron ABCD in which $AC = 8$ cm and $BC = 7$ cm. AD and BD are perpendicular to CD and $\angle CAD = 36^\circ$.



(a) Calculate:

(i) AD

(ii) CD

(iii) BD

(iv) $\angle BCD$

(b) Find AB, given that $\angle ADB = 120^\circ$ and $\angle DAB = 20^\circ$

38. Find the radius of a hemispherical bowl which can hold 0.5 litres of liquid.

39. The dimensions, in centimetres, of a rectangle are $(2n + 3)$ by $(n + 1)$

and its area is 817 cm^2 . Determine the length of the diagonal.

40. A prism has a regular heptagonal cross-section of radius 91 cm. If prism has a length of 120 cm, find its surface area.

Chapter 15

QUADRATIC EXPRESSIONS AND EQUATIONS

15.1: Expansion

Expansion is the removal of brackets, for example, $a(x + y) = ax + ay$ The same idea can be used to remove brackets in expressions such as

$$(x + 5)(3x + 2)$$

$$\begin{aligned}(x + 5)(3x + 2) &= x(3x + 2) + 5(3x + 2) \\&= 3x^2 + 2x + 15x + 10 \\&= 3x^2 + 17x + 10\end{aligned}$$

Thus, $3x^2 + 17x + 10$ is the expanded form of the expression $(x + 5)(3x + 2)$.

Example 1

Expand $(m + 2n)(m - n)$.

Solution

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$$\begin{aligned}
 (m + 2n)(m - n) &= m(m - n) + 2n(m - n) \\
 &= m^2 - mn + 2mn - 2n^2 \\
 &= m^2 + mn - 2n^2
 \end{aligned}$$

Example 2

Expand $(\frac{1}{4} - \frac{1}{x})^2$

Solution

$$\begin{aligned}
 \left(\frac{1}{4} - \frac{1}{x}\right)^2 &= \left(\frac{1}{4} - \frac{1}{x}\right)\left(\frac{1}{4} - \frac{1}{x}\right) \\
 &= \frac{1}{4}\left(\frac{1}{4} - \frac{1}{x}\right) - \frac{1}{x}\left(\frac{1}{4} - \frac{1}{x}\right) \\
 &= \frac{1}{16} - \frac{1}{4x} - \frac{1}{4x} + \frac{1}{x^2} \\
 &= \frac{1}{16} - \frac{1}{2x} + \frac{1}{x^2}
 \end{aligned}$$

Exercise 15.1

Expand and simplify each of the following expressions:

1. (a) $(6x + 2)(4x + 3)$

(b) $(3x - 2)(x + 5)$

(c) $(x - 1)(x + 2)$

(d) $(2x - 3)(x + 2)$

(e) $(6x - 2)(2x + 8)$

(f) $(2 - x)(x + 4)$

2. (a) $(x + 7)(x + 8)$

(b) $(3x - 2)(2x + 3)$

(c) $(5x + 3)(3x + 5)$

(d) $(7x - 4)(4x - 7)$

(e) $(m - n)(n - m)$

(f) $(2x - 3)(x + 3)$

3. (a) $(ax + b)(2ax - 3b)$

(b) $(4 - 2x)\left(\frac{1}{2} - x\right)$

(c) $(6x - 5)(5x + 6)$

(d) $(y - 2)(y + 2)$

(e) $(x + 2a)(x + 2a)$

$$(f) \quad (3x - 4)(5x - 6)$$

4. (a) $(x + 2a)(x - 2a)$

(b) $(x - 7)(x + 7)$

(c) $(x + 2)^2$

(d) $(4x + 2)^2$

5. (a) $\left(\frac{1}{2} + x\right)^2$

(b) $(ax + d)^2$

$$(c) \left(\frac{1}{4} + \frac{1}{x}\right)^2$$

$$(d) (x - 3)^2$$

$$\left(4x - \frac{3}{4}\right)^2 \quad (e)$$

$$\left(1 - \frac{1}{x}\right)^2 \quad (f)$$

15.2: The Quadratic Identities

From the previous exercise, note that:

$$(x + 2)^2 = x^2 = 4x + 4$$

$$(x - 3)^2 = x^2 - 6x + 9, \text{ and}$$

$$(x + 2a)(x - 2a) = x^2 - 4a^2 \text{ In general:}$$

$$(i) \quad (a + b)^2 = a^2 + 2ab + b^2$$

$$(ii) \quad (a - b)^2 = a^2 - 2ab + b^2$$

$$(iii) \quad (a + b)(a - b) = a^2 - b^2 \text{ (difference of two squares)}$$

The three identities are useful in expansion and factorisation of many quadratic expressions.

Exercise 15.2

Use the quadratic identities to write down the expansions of each of the following expressions:

1 (a) $(x + 5)^2$

(b) $(x - 5)^2$

(c) $(4x + 3)^2$

(d) $(2x + 3)^2$

2 (a) $(3 - x)^2$

$$(b) \left(\frac{1}{2} + x\right)^2$$

$$(c) \left(\frac{1}{x} - 1\right)^2$$

$$(d) \left(\frac{1}{4} + \frac{3}{4}b\right)^2$$

3 (a) $\left(\frac{1}{2}a - \frac{1}{3}b\right)^2$

$$(b) \left(1 - \frac{1}{a}\right)\left(1 + \frac{1}{a}\right)$$

$$(C) \left(\frac{1}{x} + \frac{1}{y}\right)\left(\frac{1}{x} - \frac{1}{y}\right)$$

$$(d) (-2x - 3y)(-2x + 3y)$$

4. (a) $\left(\frac{1}{3y} + \frac{1}{4x}\right)^2$
(c) $\left(\frac{1}{5}a + \frac{4}{5}b\right)^2$

(b) $\left(-\frac{1}{3}x - 1\right)^2$
(d) $\left(\frac{3}{7}x + \frac{1}{2}y\right)^2$

15.3: Factorisation

When the Coefficient of x^2 is One

The expression $ax^2 + bx + c$, where a , b and c are constants and $a \neq 0$, is known as a quadratic expression. In this expression, a is called the coefficient of x^2 , b the coefficient of x and c the constant term. Consider the following expressions:

(i) $(x + 2)(x + 5) = x^2 + 2x + 5x + 2 \times 5$

$$= x^2 + 7x + 10$$

$$\begin{aligned}\text{(ii)} \quad (x+4)(x+3) &= x^2 + 3x + 4x + 4 \times 3 \\ &= x^2 + 7x + 12\end{aligned}$$

$$\begin{aligned}\text{(iii)} \quad (x+5)(x-2) &= x^2 + 5x - 2x + 5 \times (-2) \\ &= x^2 + 3x - 10\end{aligned}$$

$$\begin{aligned}\text{(iv)} \quad (x-5)(x-2) &= x^2 - 5x - 2x + (-5) \times (-2) \\ &= x^2 - 7x + 10\end{aligned}$$

From the examples, note that:

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(i) $x^2 + 7x + 10$ can be factorised into
 $(x + 2)(x + 5)$

(ii) $x^2 + 7x + 12$ can be factorised into
 $(x + 4)(x + 3)$

In each case:

- (i) the sum of the constant terms in the factors is equal to the coefficient of x in the expression.
- (ii) the product of the constant terms in the factors is equal to the constant term of the expression.

Example 3

Factorise the expressions:

(a) $x^2 + 6x + 5$

(b) $x^2 + x - 12$

Solution

In both cases, the factors will be of the form $(x + a)(x + b)$.

(a) $x^2 + 6x + 5$

Look for two numbers a and b such
that;

$$\begin{aligned}a + b &= 6 \text{ (coefficient of } x) \\a \times b &= 5 \text{ (constant term in the expression)}\end{aligned}$$

In this case, the numbers are 1 and

$$5x^2 + 6x + 5 = x^2 + x + 5x + 5$$

$$= x(x + 1) + 5(x + 1)$$

$$= (x + 1)(x + 5)$$

(b) $x^2 + x - 12$

Look for two numbers a and b such that $a + b = 1$ and $a \times b = -12$.
The numbers are 4 and -3.

$$\begin{aligned} \text{Then, } x^2 + x - 12 &= x^2 - 3x + 4x - 12 \\ &= x(x - 3) + 4(x - 3) \\ &= (x - 3)(x + 4) \end{aligned}$$

Find the numbers whose sum is the first number of the pair and whose product is second in each of the following:

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- (i) (5, 6)
- (ii) (13, 42)
- (iii) (1, -20)
- (iv) (5, -6)
- (v) (-5, 6)
- (vi) (-7, 6)
- (vii) (3, -40)
- (viii) (8, -20)
- (ix) (1, -12)
- (x) (-12, 35)

Exercise 15.3

Factorise each of the following expressions:

1. (a) $x^2 + 6x + 8$

(b) $x^2 - 5x + 6$

(c) $x^2 + 4x - 21$

(d) $x^2 + x - 2$

(e) $x^2 + 2x - 35$

$$(f) \quad x^2 - 10x + 24$$

2. (a) $x^2 - 4x - 32$

(b) $x^2 + 3x - 54$

(c) $x^2 + 2x + 1$

(d) $x^2 + 4x + 4$

(e) $x^2 - 2x + 1$

(f) $x^2 - 14x + 49$

3. (a) $x^2 - 16x + 64$

(b) $x^2 - 4$

(c) $1 - x^2$

(d) $(x^2 - 16)$

(e) $x^2 - 5x + 6$

(f) $x^2 - 5x - 6$

4. (a) $x^2 + 2x + ax + 2a$

(b) $t^2 + 8t + 12$ when the

Coefficient of x^2 is not One

In this section, the general quadratic expression of the form $ax^2 + bx + c$, where a is not equal to 1 is discussed. Consider the following expansions:

$$(i) \quad (4x + 3)(2x + 1) = 8x^2 + 4x + 6x + 3 \\ = 8x^2 + 10x + 3$$

$$(ii) \quad (3x + 2)(x - 3) = 3x^2 - 9x + 2x - 6 \\ = 3x^2 - 7x - 6$$

$$(iii) \quad (3x + 2)(2x - 5) = 6x^2 - 15x + 4x - 10 \\ = 6x^2 - 11x - 10$$

$$\begin{aligned}(iv) \quad (3x - 2)(2x - 3) &= 6x^2 - 9x - 4x + (-2x \\ &\quad - 3) \\ &= 6x^2 - 13x + 6\end{aligned}$$

In the examples above, factors are multiplied to get the final expressions on the right hand side. Given the expressions on the right hand side, how can the factors be obtained?

Consider the expression $8x^2 + 10x + 3$ [(i) above]. It can be factorised as follows:

- Look for two numbers such that:
 - their product is $8 \times 3 = 24$.

Where 8 is the coefficient of x^2 and 3 is the constant term in the given expression.

- Their sum is 10, where 10 is the coefficient of x.

The numbers are 4 and 6.

- Rewrite the term $10x$ as $4x + 6x$.

Thus, $8x^2 + 10x + 3 = 8x^2 + 4x + 6x$
+ 3

a

- Factorise the expression.

$$\begin{aligned} \text{Thus, } 8x^2 + 4x + 6x + 3 &= 4x(2x + 1) \\ &\quad + 3(2x + 1) \\ &= (4x + 3)(2x + 1) \end{aligned}$$

In the examples (ii), (iii) and (iv), for each give the product and the sum of the two required numbers.

Example 4

Factorise $6x^2 - 13x + 6$

Solution

To factorise the expression $6x^2 - 13x + 6$,
look for two numbers such that:

- (i) their product is $6 \times 6 = 36$.
- (ii) and their sum is -13 .

The numbers are -4 and -9 .

$$\text{Therefore, } 6x^2 - 13x + 6 = 6x^2 - 4x - 9x + 6$$

$$= 2x(3x - 2) -$$

$$3(3x - 2)$$

. (a)

(b)

$$= (2x - 3) (3x - 2)$$

Generally, to factorise the expression $ax^2 + bx + c$, look for two numbers such that their product is ac and their sum is b .

Split the middle term into two terms such that their coefficients are the numbers so determined.

Exercise 15.4

Factorise each of the following expressions:

$$1 \quad 2x^2 + 3x - 2$$

$$3x^2 - 2x - 8$$

(c) $4x^2 + 7x + 3$

(d) $5x^2 - 21x + 4$

2. (a) $14x^2 - 16x + 2$

(b) $3x^2 + 11x + 6$

(c) $8 - 2x - 3x^2$

. (a)

(b)

(d) $16x + 24 + 2x^2$

3. (a) $35x^2 + 43x + 12$

(b) $-21x^2 + 58x - 21$

(c) $150x^2 - 25x - 21$

(d) $17x^2 + 82x - 15$

. (a)

(b)

4 $8x^2 + 6x + 1$

$$x^2 - 4x - 117$$

(c) $9x^2 + 48x + 64$

(d) $75x^2 + 10x - 21$

5. (a) $1 + 7x - 30x^2$

(b) $19x^2 - 22x + 3$

(c) $63x^2 + 130x + 63$

(d) $x^2 - \frac{2}{15}x - \frac{1}{15}$

6. (a) $4x^2 - 12x + 9$

. (a)

(b)

(b) $9x^2 + 6x + 1$

(c) $2x^2 - 32y^2$

(d) $1 - \frac{2}{y} + \frac{1}{y^2}$
7 $t^8 - t^4$

$35x^2 - x - 12$

(c) $4x^2 + 2x + \frac{1}{4}$

(d) $2x^2 + 4x - 6$

8. (a) $21x^2 + 17x - 4$

(b) $25x^2 - \frac{5}{2}x + \frac{1}{16}$

(a)

(b)

(c) $26x - 29x^2 + 3$

(d) $\frac{1}{16}x^2 - \frac{7}{2}x + 49$

9. (a) $24x^2 - 43x - 56$

(b) $33x^2 - 16x - 4$

(c) $\frac{1}{x^2} - \frac{4}{x} + 4$

(d) $w^4 - t^4 + t^2 + w^2$

10. $25x^2 + 10xy + y^2$

$(x + y)^2 + (x + y) + 1$

. (a)

(b)

(c) $4x^2 + 12x + 9$

(d) $6x^2 + 7x + 1$

16.4: Quadratic Equations

Any equation of the form $ax^2 + bx + c = 0$ where a , b and c are constants and $a \neq 0$ is known as a quadratic equation. In this section, solutions of quadratic equations using the factor method is discussed.

Example 5

Solve $x^2 + 3x - 54 = 0$

(a)

(b)

Solution

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Factorising the L.H.S.

$$x^2 + 3x - 54 = 0$$

$$x^2 - 6x + 9x - 54 = 0$$

$$x(x - 6) + 9(x - 6) = 0$$

$$(x + 9)(x - 6) = 0$$

In general, if $a \times b = 0$, then either $a = 0$ or $b = 0$

Therefore, $x + 9 = 0$ or $x - 6 = 0$

Thus, $x = -9$ or $x = 6$

Check the answers by substituting into the given equation.

Example 6

Solve the equation $4x^2 + 7x + 3 = 0$

Solution

Factorising the L.H.S $4x^2 + 7x + 3 = 0$

$$4x^2 + 4x + 3x + 3 = 0$$

$$4x(x + 1) + 3(x + 1) = 0$$

$$(x + 1)(4x + 3) = 0$$

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(c)

(d)

Either $(x + 1) = 0$ or $(4x + 3) = 0$

Therefore, $x = -1$ or $x = -\frac{3}{4}$

Numbers that satisfy an equation (its solutions) are called the roots of the equation. In Example 5, the roots of $x^2 + 3x - 54 = 0$ are -9 and 6 .

Exercise 15.5

Solve each of the following equations:

1. (a) $x^2 + 6x + 8 = 0$

$$(b) 2x^2 + 3x - 2 = 0$$

$$14x^2 - 16x + 2 = 0$$

$$4x^2 + 7x - 5 = x^2 - 9$$

2. (a) $x^2 - 25 = 0$

(b) $3x^2 - 2x = 5$

(c) $3x^3 + x^2 - 4x = 0$

(d) $1 - 6x^2 = x$

(c)

(d)

3. (a) $4 \times 2 - x - \frac{1}{2}$

$x + \frac{20}{x} = 9$ (b)

(c) $16x + 24 + 2x^2 = 0$

$\frac{1}{4} - \frac{1}{9}x^2 = 0$ (d)

4. $\frac{3}{x^2} - \frac{8}{x} = 16$

(a)

(b) $(2x - 1)^2 - 1 = 3$

(c)

(d)

$$x^2 + \frac{5}{6}x = \frac{1}{6}$$

$$\left(1 - \frac{1}{x}\right)^2 - 1 = 35$$

5. (a) $\frac{1}{3}x^2 = 2x - 3$

(b) $x^2 + \frac{1}{25} = \frac{2}{5}$

(c) $-\frac{12}{x} = -x + 4$

(d) $x^2y^2 - x^2 - y^4 + y^2 = 0$

(c)

(d)

6. (a) $7x^2 - 15x + 8 = 0$

(b) $8x^2 + 6x + 1 = 0$

(c) $x^2 - \frac{2}{15}x - \frac{1}{15} = 0$

(d) $\frac{2x^2 - 15}{x} = 7$

7 (a) $\frac{x^2}{2} + 1 = \frac{9x}{4}$

(b) $1 - x = \frac{1}{x} + 3$

$$x^2 - \frac{1}{18} = \frac{x}{6}$$

$$4\left(\frac{1}{y^2}\right) - 4\left(\frac{1}{y}\right) + 1 = 0$$

8 (a) $\frac{6}{x-4} = \frac{x+4}{x}$

(b) $\frac{6x+8}{2} + \frac{1}{x} = 0$

15.5: Formation of Quadratic Equations

From Given Roots

(c)

(d)

Given that the roots of a quadratic equation are $x = 2$ and $x = -3$, the quadratic equation can be obtained as follows:

If $x = 2$, then $x - 2 = 0$

If $x = -3$, then $x + 3 = 0$

Therefore, $(x - 2)(x + 3) = 0$

Expand the L.H.S, i.e., $x(x + 3) - 2(x + 3) = 0$
 $x^2 + 3x - 2x - 6 = 0$
 $x^2 + x - 6 = 0$ the required equation.

Exercise 15.6

For each of the following pairs of roots, obtain a corresponding equation in the form $ax^2 + bx + c = 0$, where a, b and c are constants.

1. (a) $(-2, 2)$

$$\left(\frac{1}{2}, \frac{1}{3}\right) \text{ (b)}$$

$$\left(\frac{1}{4}, -3\right) \quad 2. \text{ (a)}$$

(b) $(-3, -5)$

3. (a) $(0.5, 0.75)$

(b) $(0, -3)$

4. (a) $(-3, -3)$

$\left(-2\frac{1}{2}, 3\frac{1}{2}\right)$ (b) ()

5. (a) (p, q)

$\left(\frac{1}{p}, \frac{1}{q}\right)$

(b)

From Given Situations

Example 7

A rectangular room is 4 metres longer than it is wide. If its area is 12 m^2 , find its dimensions.

Solution

Let the width of the room be x metres.
Its length will be $x + 4$ metres.

The area of the room is $x(x + 4) \text{ m}^2$

$$\text{Therefore, } x(x + 4) = 12 \\ x^2 + 4x = 12$$

$$\text{Therefore, } x^2 + 4x - 12 = 0$$

$$(x + 6)(x - 2) = 0$$

Either $x + 6 = 0$ or $x - 2 = 0$

Therefore, $x = -6$ or $x = 2$

One of the answers, $x = -6$ is ignored because the width cannot be negative.

The width of the room is, therefore, 2 m and the length of the room is; $x + 4 = 2 + 4$

$$= 6 \text{ m}$$

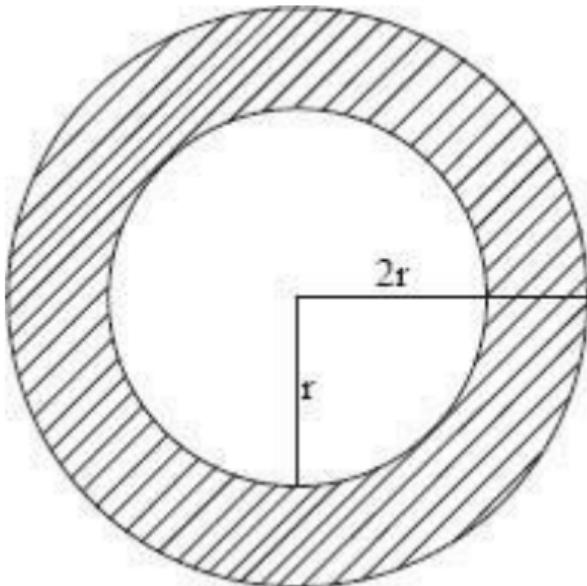
Exercise 15.7

1. The square of a number is 4 more than three times the number. Find the number.
2. Given that the lengths of the three sides of right-angled triangle are x , $x + 1$ and $x + 2$ units, find the value of x .
3. A triangle ABC has a base of $(x + 3)$ cm and a height of x cm. If its area is

5 cm^2 , calculate the length of its base.

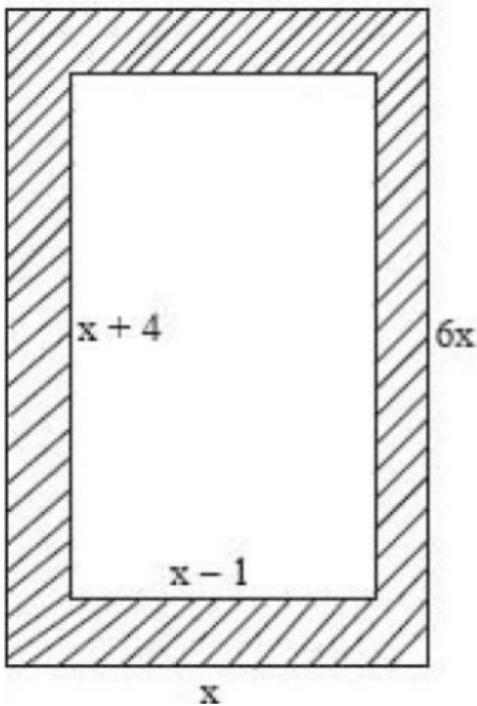
4. After reducing by an equal amount the length and width of a rectangle which were originally 8 cm and 5 cm respectively, the area of the new rectangle is 18 cm^2 . Find the dimensions of the new rectangle.
5. A number of people bought a 300 hectare farm which they shared equally. If the number of hectares per person was 5 less than their number, find the number of people.

6. The figure below shows two concentric circles. If the ratio of their radii is 1:2 and the area of the shaded region is 75 cm^2 , calculate the area of the larger circle.



7. A picture measuring 5 cm by 8 cm is mounted on a frame so as to leave a uniform margin of width x cm all round. If the area of the margin is 30 cm^2 , find x .

8. The figure below shows a rectangular lawn of dimensions $6x$ metres by x metres. In the centre is a rectangular flower bed of length $(x + 4)$ m and width $(x - 1)$ m. If the area of the shaded region is 40 m^2 , calculate the area of the flower bed.



9. There are 240 exercise books to be given to a class. Thirty-four students in the class did not receive the books for various reasons. If the

books were then shared equally among the remaining members of the class, the number of books given to each student is equal to the number of students in the class. Find the number of students in the class.

10. Members of a group decided to raise K£100 towards a charity by contributing equal amounts. Five of them were unable to contribute.

The rest had, therefore, to pay K£1 more each to realise the same amount. How many members were in the group originally?

11. ABC is an isosceles triangle in which AB = AC. The size of angles ABC and ACB are $(3x^2 - 2x + 4)^\circ$ and $(9x - 6)^\circ$ respectively. Calculate the two possible sets of values of the three angles of triangle ABC.

Chapter 16

LINEAR INEQUALITIES

16.1: Inequality Symbols

Recall the following symbols:

< means greater than, e.g., $5 > 2$.

> means less than, e.g., $4 < 7$.

\geq means greater than or equal to.

\leq means less than or equal to.

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Statements connected by these symbols are called inequalities.

Simple Statements

Whereas $x = 2$ represents a specific number, $x > 2$ does not. It represents all numbers to the right of 2, as illustrated on the number line below:

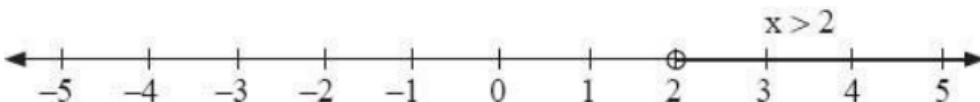


Fig. 16.1

In figure 16.1, the unshaded circle around the number 2 means that 2 is

not included in the list of numbers to the right of it.

The expression $x \geq 2$ on the other hand means that 2 is included in the list of numbers to the right of it. This information can be illustrated on the number line as in figure 16.2:

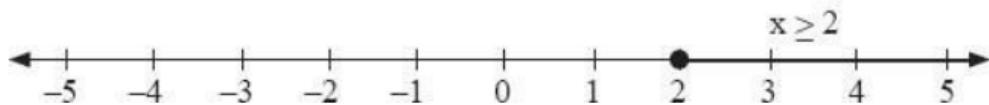


Fig. 16.2

Note that the circle around 2 is shaded to show that 2 is also included.

Statements such as $x > n$, $x \geq n$, $x < n$ or $x \leq n$ where n is real number are called simple statements.

Exercise 16.1

Illustrate each of the following inequalities on a number line:

1. (a) $x < 7$
(b) $x > -3$
(c) $x \leq 0$

(d) $x \leq -5$

2. (a) $x < -10$

(b) $x < -4$

(c) $x \geq -6$

(d) $x < 2.5$

3. (a) $x \leq -\frac{1}{2}$

(d) $x \leq -2.3$

Compound Statements

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Sometimes it is necessary to represent on a number line all values of x which satisfy two conditions simultaneously, e.g., $x > 2$ and $x \leq 8$.

The two statements can be combined into one statement as $2 < x \leq 8$. The combined statement is called the compound statement. Combined into a one compound statement, the two become; $2 < x \leq 8$.

This is illustrated in figure 16.3. The compound statement $2 < x \leq 8$

represents an interval with 2 and 8 as its endpoints, but 2 is not included.

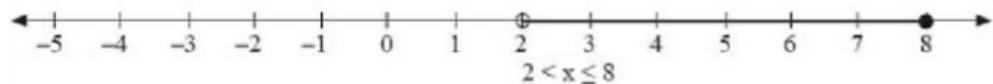


Fig. 16.3

Example 1

Write each of the following pairs of simple statements into compound statements and illustrate them on a number line: (a) $x > -4$, $x \leq 2$

(b) $x < 0.5, x > 0$

Solution

(a) $-4 < x \leq 2$



Fig. 16.4

(b) $0 < x < 0.5$



Fig. 16.5

Exercise 16.2

1. Write each of the following pairs of simple statements as a compound statement:
 - (a) $x > 2, x < 5$
 - (b) $x \geq 8 \text{ and } x < 6$
 - (c) $x \geq 8 \text{ or } x < 7$
 - (d) $x > -4, x \leq 0$
 - (e) $x \geq -3, x \leq -1$

2. Write each of the following pairs of simple statements into a compound statement and illustrate the answer on a number line:
- (a) $x > 2, x < 6$
 - (b) $x > -4, x \leq 6$
 - (c) $x \leq 0.5, x \geq 0.1$
 - (d) $x \leq 4, x \geq 6$
 - (e) $x > 2, x \leq -3$
3. Illustrate the following compound statements on a number line:
- (a) $-1 \leq x < 4$

(b) $-2 < x < 0$

(c) $3 \leq x < 7$

(d) $-5 < x < 5$

(e) $-5 \leq x \leq 3$

(f) $-2 < x < 2$

(g) $-4 < x \leq -1$

(h) $-9 \leq x \leq 15$

(i) $0 \leq x \leq 10$

$$(j) \quad 4 > x \leq 1$$

$$\frac{1}{5} < x < \frac{1}{3} \quad (k)$$

$$2\frac{1}{2} \leq x < 3\frac{1}{5} \quad (l)$$

$$(m) \quad -0.75 < x \leq 0.75$$

$$(n) \quad -15 \leq x \leq -3$$

$$(p) \quad -\frac{1}{2} \leq x \leq 5\frac{1}{2}$$

16.2: Solution of Simple Inequalities

Example 2

Solve the inequality $x - 1 > 2$.

Solution

Add 1 to both sides.

$$x - 1 + 1 > 2 + 1$$

Therefore, $x > 3$

If any number greater than 3 is substituted in the inequality, the inequality remains true.

Example 3

Solve the inequality $x + 3 < 8$.

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Solution

Subtract 3 from both sides

$$x + 3 - 3 < 8 - 3 \\ x < 5$$

Example 4

Solve the inequality $2x + 3 \leq 5$

Solution

Subtract 3 from both sides

$$2x + 3 - 3 \leq 5 - 3$$

$$2x \leq 2$$

Divide both sides by 2

$$\frac{2x}{2} \leq \frac{2}{2}$$

Therefore, $x \leq 1$

Check the answer by substituting back in the inequality any number less than or equal to 1.

Example 5

Solve the inequality $\frac{1}{3}x - 2 \geq 4$

Solution

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Add 2 to both sides;

$$\begin{aligned}\frac{1}{3}x - 2 + 2 &\geq 4 + 2 \\ \frac{1}{3}x &\geq 6\end{aligned}$$

Multiply both sides by 3;

$$\begin{aligned}\frac{x}{3} \times 3 &\geq 6 \times 3 \\ x &\geq 18\end{aligned}$$

Multiplication and Division of an Inequality by a Negative Number.

Consider the simple statement $5 > 2$.

Multiply both sides by -3 ;

L.H.S. = -15 , R.H.S. = -6 . But -6 is greater than -15 .

The correct statement after multiplication is $-15 < -6$.

The inequality symbol has been reversed.

Also consider dividing both sides of the inequality by -2 .

$$\begin{aligned}\text{L.H.S} &= \frac{5}{2} \text{ and R.H.S} = \frac{2}{-2} \\ &= -2.5 & &= -1\end{aligned}$$

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But -1 is greater than -2.5

The correct statement after division is –
 $2.5 < -1$

Generally:

- (i) Multiplying or dividing both sides of an inequality by a positive number leaves the inequality sign unchanged.
- (ii) Multiplying or diving both sides of an inequality by a negative number reverses the inequality sign.

Example 6

Solve the inequality $1 - 3x < 4$

Solution

Subtract 1 from both sides;

$$1 - 3x - 1 < 4 - 1$$

$$-3x < 3$$

Divide both sides by -3 ;

$$\frac{-3x}{-3} > \frac{3}{-3}$$

(Note that the sign is reversed)

Therefore, $x > -1$

It is advisable to check your answer by substituting back in the original inequality.

Exercise 16.3

Solve each of the following inequalities and represent your solution on a number line.

1. (a) $2x + 4 > 10$

(b) $3x - 5 < 2$

$$2. \text{ (a)} \quad 5x + 3 > 4$$

$$\text{(b)} \quad 3x - 4 \leq -13$$

$$3. \text{ (a)} \quad 3x - 7 \geq 5$$

$$\text{(b)} \quad 1 - 4x \geq 9$$

$$4. \text{ (a)} \quad 6 - \frac{1}{2}x > 12$$

$$\text{(b)} \quad 3 - 2x < 17$$

$$5. \text{ (a)} \quad \frac{1}{3} - 2x \leq -8\frac{1}{3}$$

$$\text{(b)} \quad 3(1 - x) + 4(x + 3) \geq 30$$

$$6. \text{ (a)} \quad 2x + 3 < -1$$

$$\text{(b)} \quad -3x - 4 \geq 2$$

$$-4 - \frac{2}{3}x \leq 0$$

$$\frac{x-7}{-49} \leq \frac{-1}{7}$$

$$7. \text{ (a)}$$

$$\text{(b)}$$

16.3: Simultaneous Inequalities

Example 7

Solve the following pair of simultaneous inequalities: $3x - 1 > -4$

$$2x + 1 \leq 7$$

Solution

Solve the first inequality;

$$3x - 1 > -4$$

$$3x > -3 \quad x > -$$

$$1$$

Solve the second inequality;

$$2x + 1 < 7$$

$$2x \leq 6$$

$$x \leq 3$$

The solution which satisfies both inequalities is $-1 < x \leq 3$. This is shown on the number line in figure 16.6.



Fig. 16.6

Simultaneous inequalities can be written as a compound statement. For example, consider the statement $x - 4 <$

$4x < 4$; the statement can be broken into two as:

(i) $x - 4 < 4x$

(ii) $4x < 4$

Solve the first inequality

$$x < 4x + 4$$

$$-3x < 4$$

$$x > -\frac{4}{3}$$

$$4x < 4$$

$$x < \frac{4}{4}$$

S_{x < 1} solve the second inequality;

The solution which satisfies both inequalities is,

$$-\frac{4}{3} < x < 1$$

Exercise 16.4

Solve each of the following simultaneous inequalities and illustrate your answers on a number line:

1. (a) $x + 3 > 5$

$$x - 4 < 4$$

(b) $x + 10 > 6$

$$x - 2 < 3$$

2. (a) $-\frac{1}{2}x - 2 \leq 1$

$$-3x - 9 > -6$$

(b) $-5x + 7 < 12$

$$\frac{1}{3}x + 2 \leq 5$$

3. (a) $-7x - 1 < 6$

$$-\frac{x}{3} + 1 < \frac{4}{3}$$

(b) $3x - \frac{1}{2} > 4$

$$x - \frac{1}{5} < \frac{2}{5}x + 1$$

4. (a) $\frac{x}{5} + \frac{1}{3} < 1$

$$x - \frac{4}{5} > \frac{1}{8}x$$

$$(b) 5 \leq 3x + 2$$

$$3x - 14 < -2$$

$$\frac{x}{3} + \frac{2}{3} < 2 \quad 5. \quad (a)$$

$$-\frac{1}{2}x + 1 < 2$$

$$\frac{x-3}{-3} > 1 \quad (b)$$

$$3x + 1 < -17$$

$$\frac{x+2}{2} < 5 \quad 6. \quad (a)$$

$$\frac{-x+6}{3} < 4$$

$$(b) 9 - 2x \leq 3$$

$$1 < 16 - x$$

$$7. (a) \frac{1}{2} - \frac{1}{4}x \leq x \leq 2$$

$$(b) 12 - x \geq 5 \leq 2x - 2$$

$$8. (a) -4x < 6 \leq 8x$$

$$(b) 3x - 2 > -4 < -1 - 2x$$

$$9. (a) 6x - 13 \leq 17 < 8x - 7$$

$$(b) 2x + 3 > 5x - 3 > -8$$

16.4: Graphical Representation of Inequalities

Figure 16.7 shows the graph of $x = 2$:

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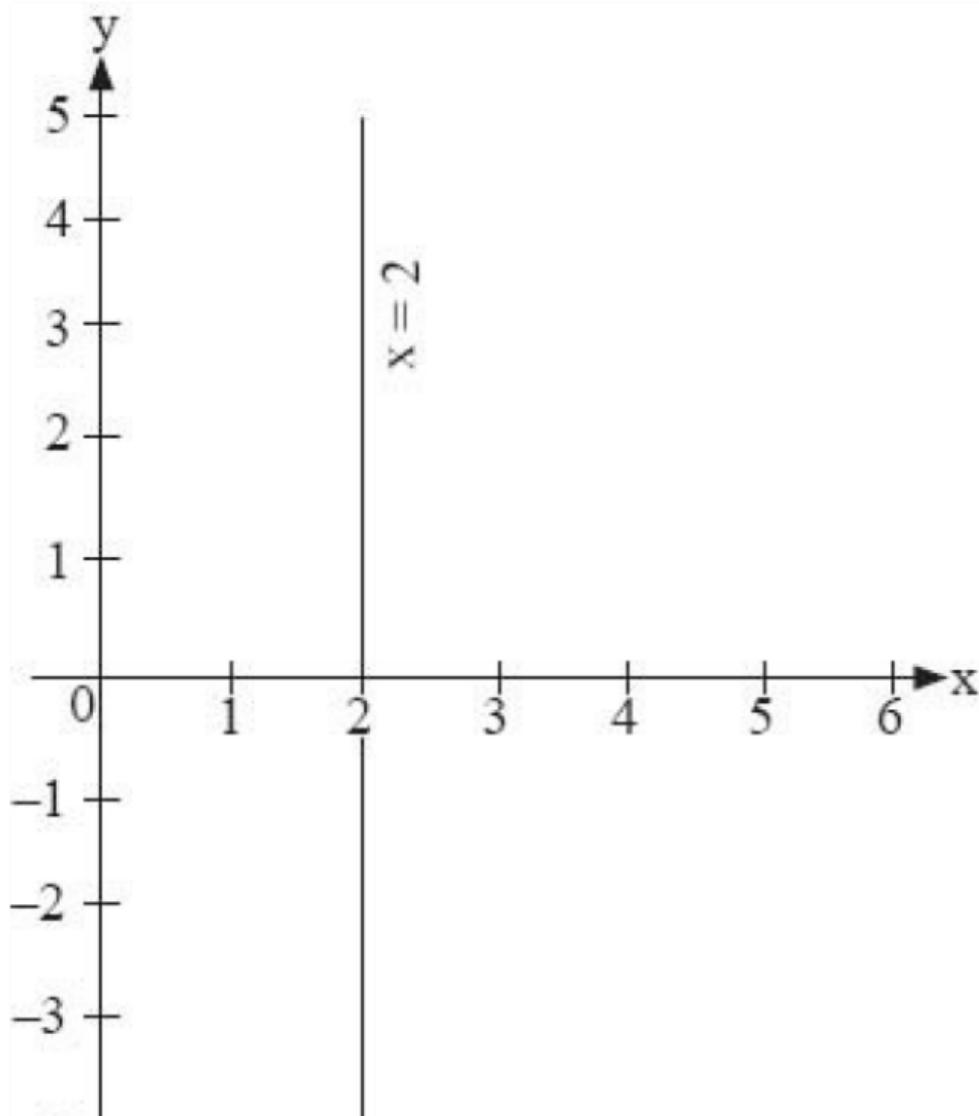


Fig. 16.7

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Give the co-ordinates of any four points:

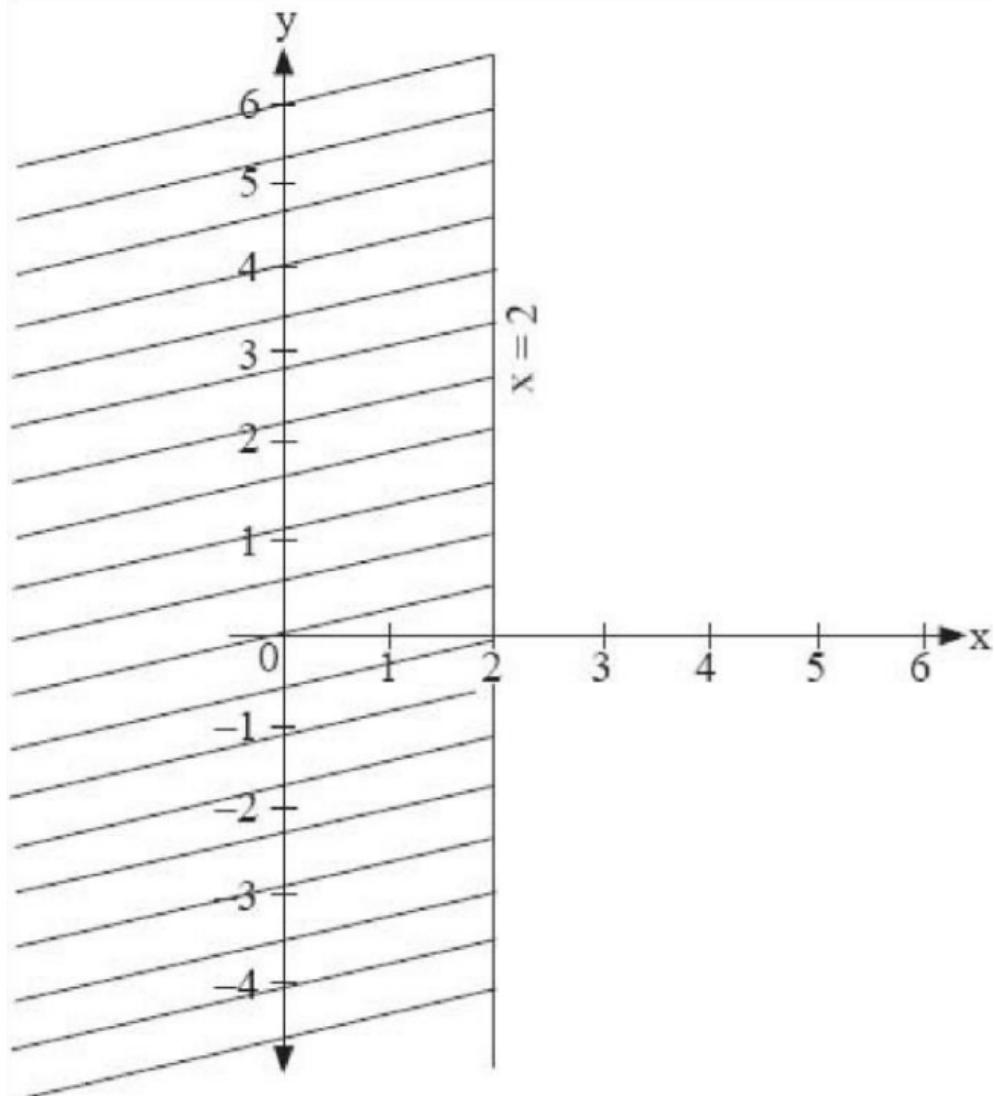
- (i) On the line $x = 1$.
- (ii) to the right of the line $x = 2$.
- (iii) to the left of the line $x = 2$.

Note that all points lying in the region to the right of $x = 2$ satisfy the inequality $x > 2$.

Write down an inequality for those points lying in the region to the left of $x = 2$.

t

Points which either lie on the line $x = 2$ or to the right of it are represented by the inequality $x > 2$. Figure 16.8 shows the unshaded region satisfied by the inequality $x > 2$.



t

Fig. 16.8 Note:

- (i) Always shade the region which does not satisfy the inequality (unwanted region).
- (ii) The line $x = 2$ has been drawn continuously (unbroken) because it forms part of the region (all points on the line satisfy the inequality $x > 2$).

The region $x < 2$ is shown in figure 16.9.

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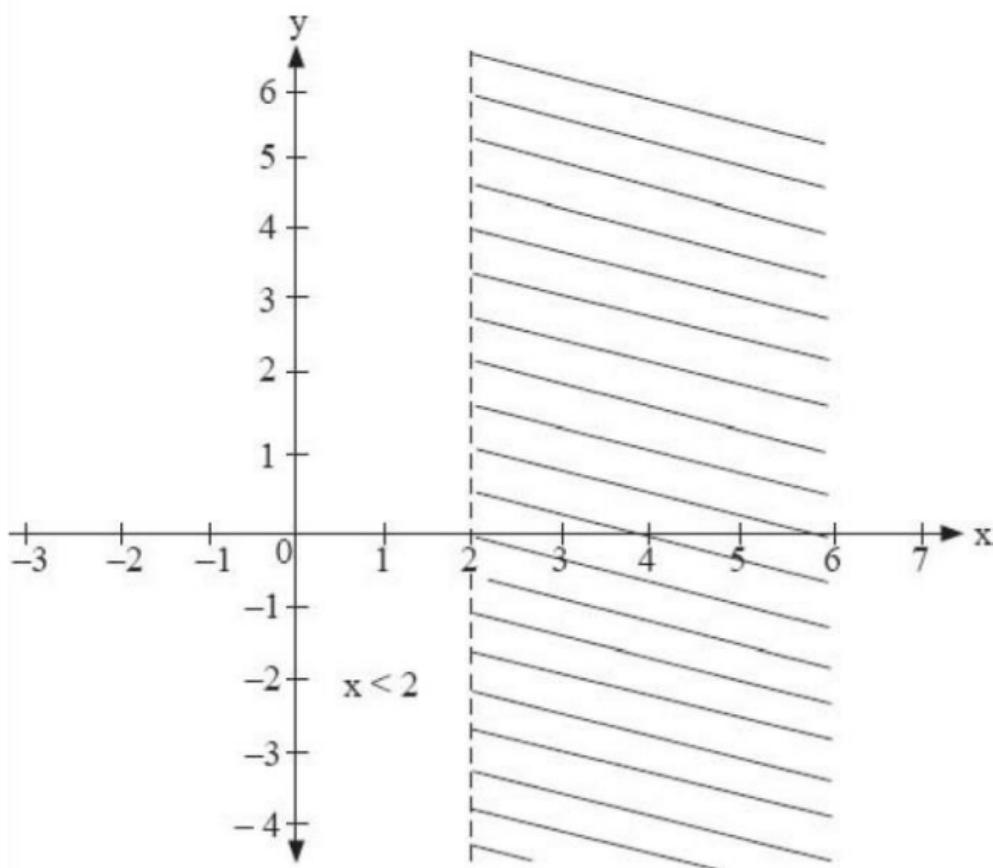


Fig. 16.9 Note:

t

In this case, the line $x = 2$ is broken. This is because points on the line do not form part of the region (they do not satisfy the inequality $x < 2$).

Exercise 16.5

Show the regions that satisfy each of the following inequalities.

1. (a) $x \leq 4$
(b) $x > -2$

$$2. \quad (a) x < -1$$

$$(b) y \leq 3$$

$$3. \quad (a) y \geq -4$$

$$(b) y \leq 0$$

$$4. \quad (a) y + 2 < -5$$

$$(b) x + 2 \geq -1$$

$$5. \quad (a) 3 - x > 7$$

$$(b) \frac{x+1}{3} < 6$$

$$\frac{y-1}{4} \geq 5 \quad 6. \quad (a)$$

$$\frac{y+3}{5} \leq 2 \quad (b)$$

t

7. (a) $-2 \leq x < 4$

(b) $-4 < y \leq 3$

8. (a) $-2 \leq \frac{1}{2}x < 7$

(b) $4 < y < 6$

9. (a) $-1 \leq 3x - 1 < 5$

$$(b) \frac{x-3}{4} > \frac{x+5}{2}$$

$$10. (a) \frac{1}{3} - \frac{1}{5}x < \frac{1}{2}x + 1$$

$$(b) \frac{2}{3}x - 7 + \frac{1}{5}x \geq -1$$

$$11. (a) \frac{1}{2}x + \frac{1}{2} > \frac{1}{4}$$

$$(b) x^2 - 4x \geq x(x - 1) - 18$$

$$12. \text{ (a)} \quad \frac{1}{7}x - \frac{1}{3} < \frac{1}{5} + \frac{x}{3}$$

$$\text{(b)} \quad x - 2 > 4 + 3x$$

16.5: Linear Inequalities in Two Unknowns

In the previous section, regions representing inequalities of the form $x \geq d$ or $x \leq e$, where d and e are constants were discussed. This section deals with how to represent inequalities of the form $ax + by \geq c$ graphically (where a , b and c are constants).

For example, the region represented by $x + y \geq 8$ is shown in figure 16.10. The boundary is the line whose equation is $x + y = 8$.

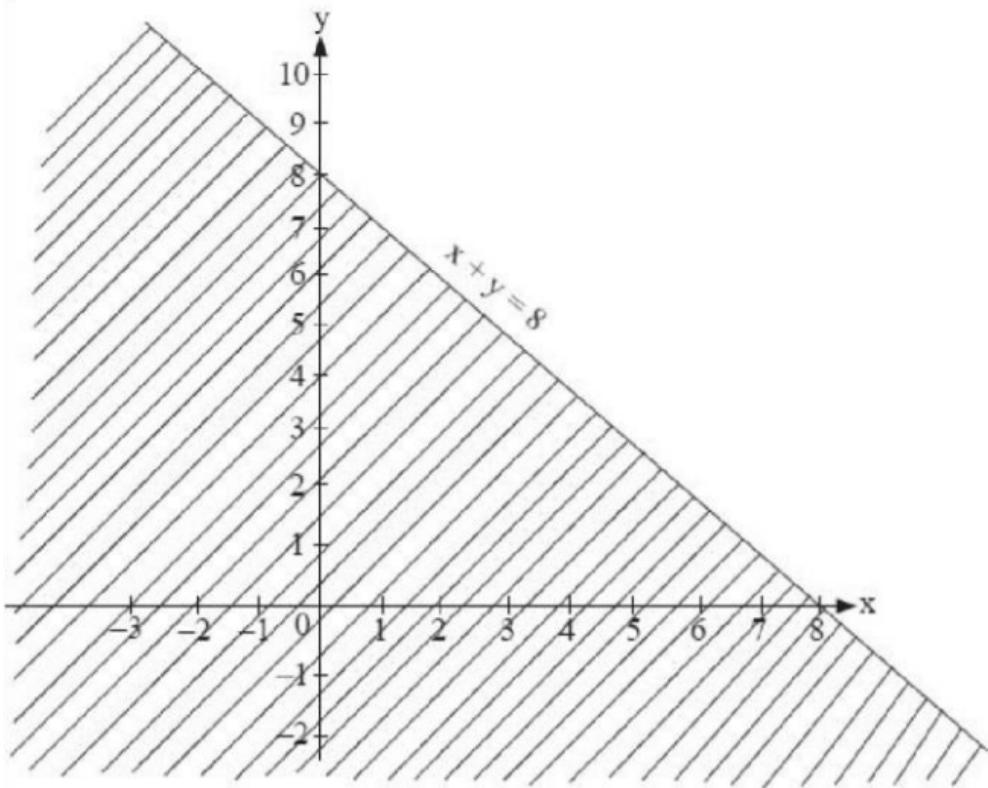


Fig. 16.10

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If a point above the line, e.g., $(10, 10)$ is picked, then $x + y = 10 + 10 = 20$. Since $20 > 8$, the point $(10, 10)$ lies in the desired region. On the other hand, if a point below the line, $(2, 3)$ is picked, e.g., $x + y = 2 + 3 = 5$. Then $5 < 8$, this point does not satisfy the inequality. The region below the line, therefore, does not satisfy the inequality and that is why it is shaded. Show on the graph the region $x + y < 8$.

Given a region satisfied by an inequality, the inequality can be found. For

example, consider the region in fig 16.11.

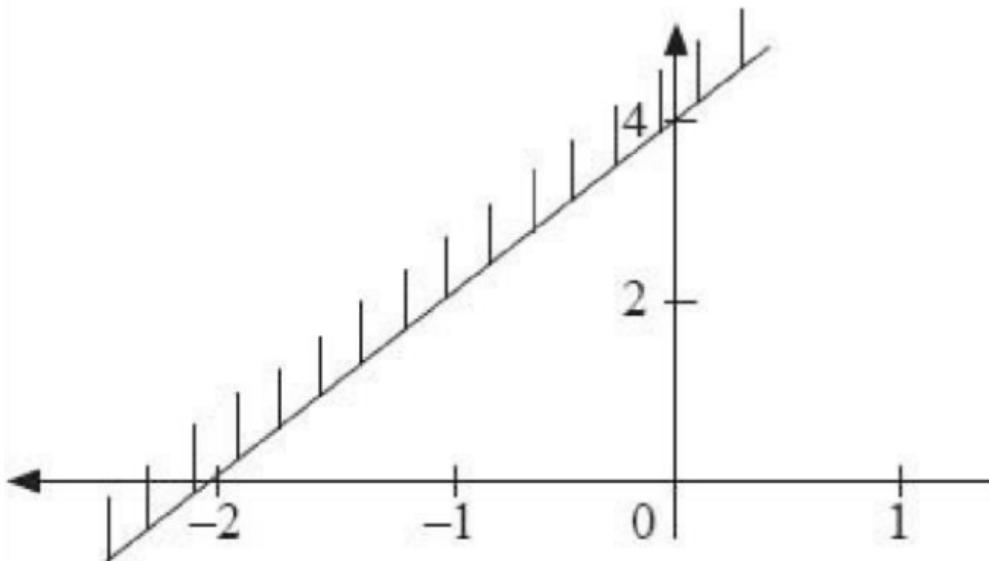


Fig. 16.11

The equation of the line is $y = 2x + 4$.

Consider a point in the desired region such as the origin $(0, 0)$. The x-coordinate and the y-coordinate are both zero. Substituting these values in the equation $y = 2x + 4$, gives zero on the left hand side and four on the right hand side. Since $0 < 4$ and the line $y = 2x + 4$ is continuous, the required inequality is $y < 2x + 4$.

Exercise 16.6

Graph each of the following inequalities:

1. (a) $2x + y > 3$

$$(b) x - y < 4$$

()

2. a $3x + 2y > 12$

(b) $3y + x \leq -5$

3. (a) $y + 4x < 3$

(b) $y - \frac{1}{2}x \geq 1$

4. (a) $2x > y + 4$

(b) $\frac{1}{5}x + \frac{1}{3}y \leq \frac{1}{4}$

5. (a) $2y - 3x - 5 \leq 0$

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(b)

$$(b) \frac{1}{6}x + \frac{1}{3}y \leq -1$$

6. (a) $2y - 5x \geq 7$

$$(b) \frac{1}{2}x - y \geq 1$$

7. (a) $3\left(\frac{1}{2}x - \frac{1}{3}y\right) \leq -1$

$$(b) 3y - 8 \geq -6x$$

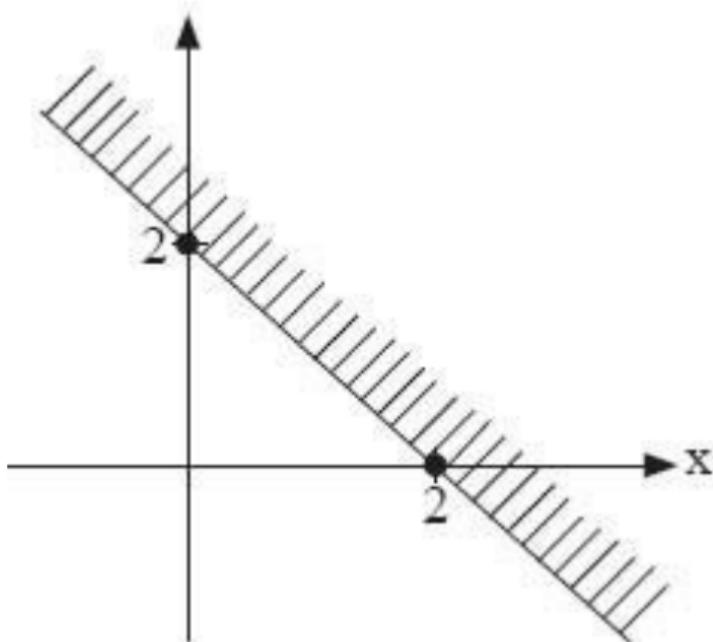
8. (a) $1 < x + y < 8$ $2y \geq 0.75 - x$

9. $0.5x + 0.3y > -1$

10. Determine the inequalities which satisfy the following unshaded regions:

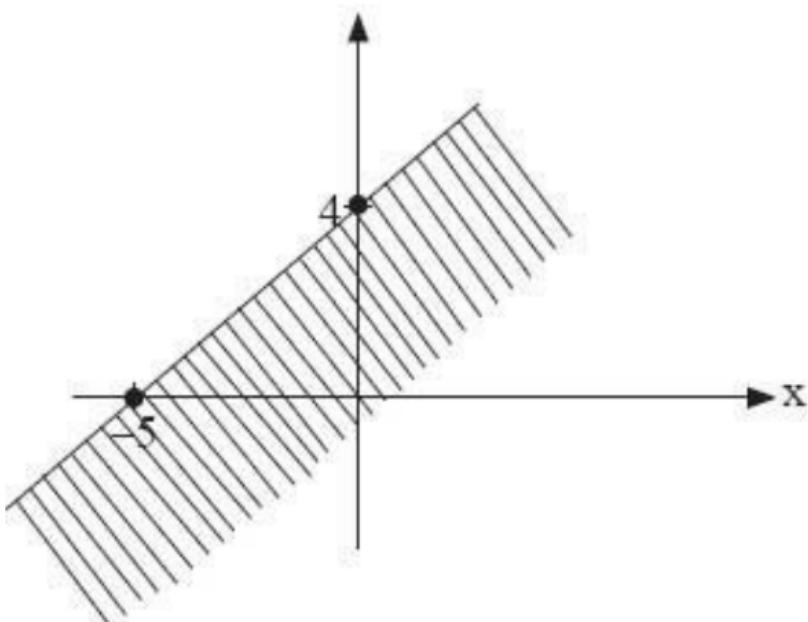
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(a)

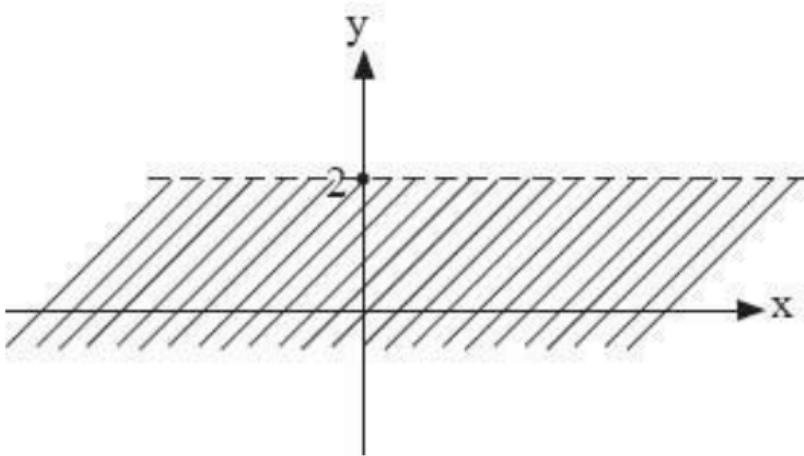


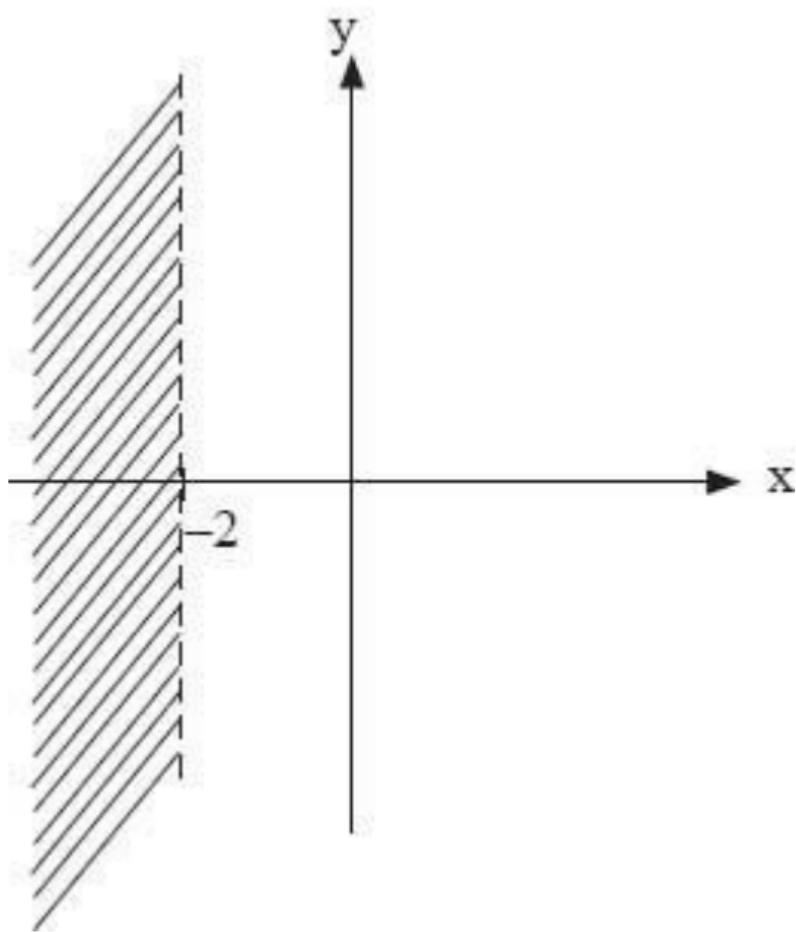
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(b)

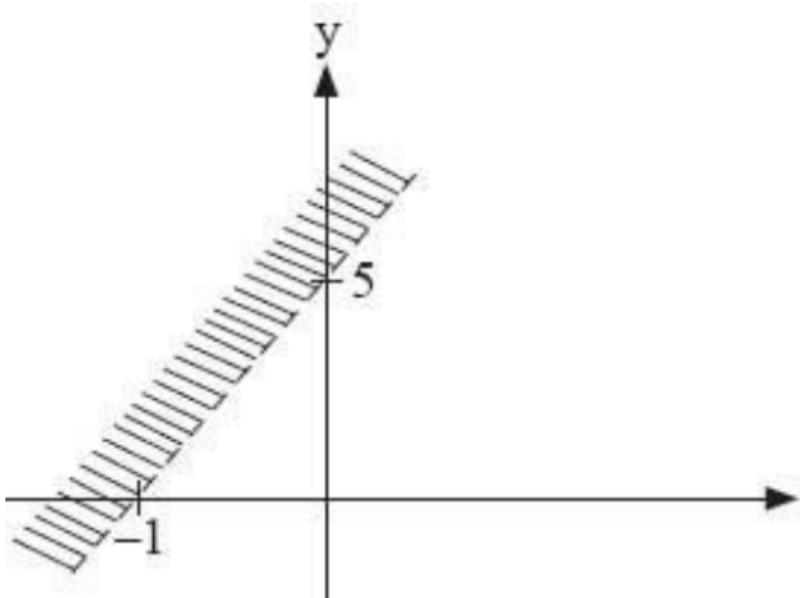


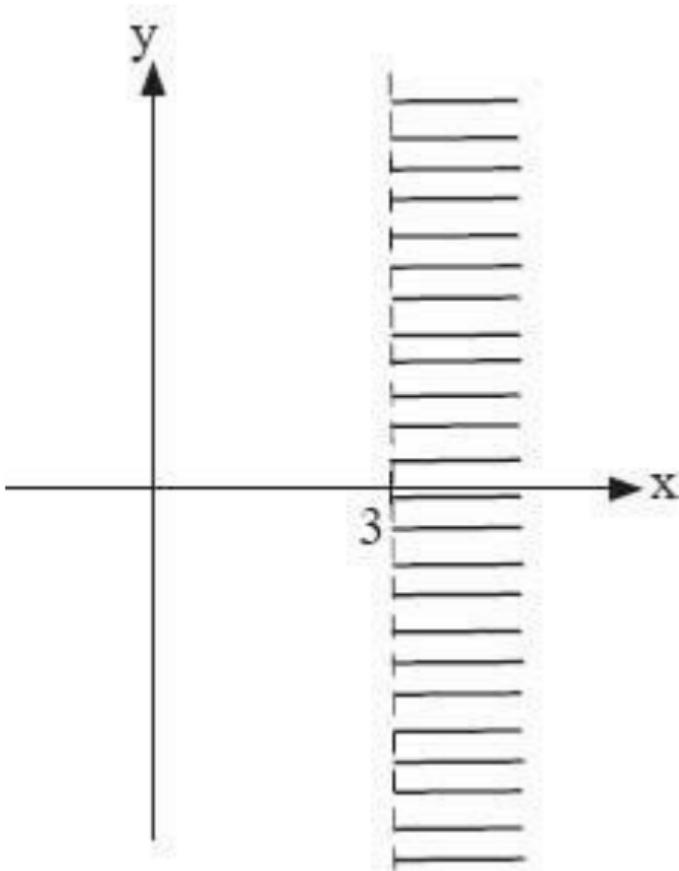
(c)





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(h)

16.6: Intersecting Regions

So far, only regions which satisfy one inequality have been discussed. Situations arising in regions which satisfy more than one inequality simultaneously need to be identified.

Consider the following examples.

Example 8

Draw a region which satisfies both the inequalities $x + y \leq 8$ and $x > 2$.

Solution

The solution is shown in figure 16.12.
The required region is unshaded.

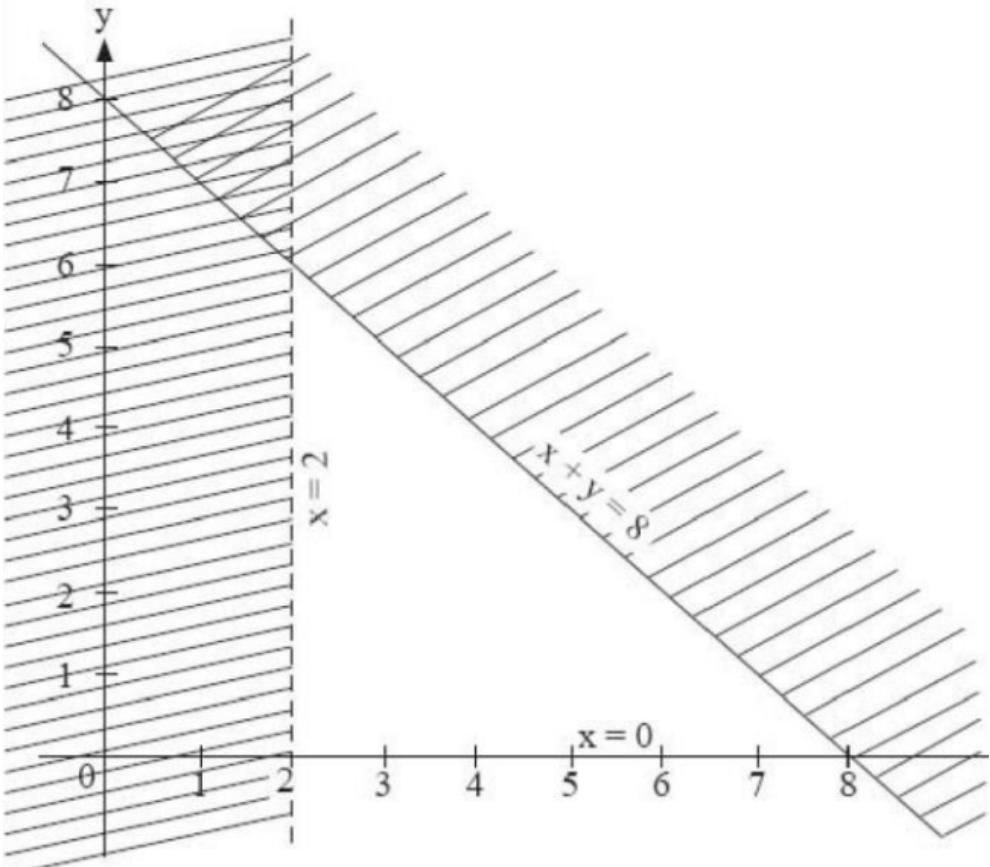


Fig. 16.12

Example 9

Show the region represented by $2x + y > 3$, $x - y \geq 4$ and $y \leq 3$.

Solution

The solution is shown in figure 16.13.

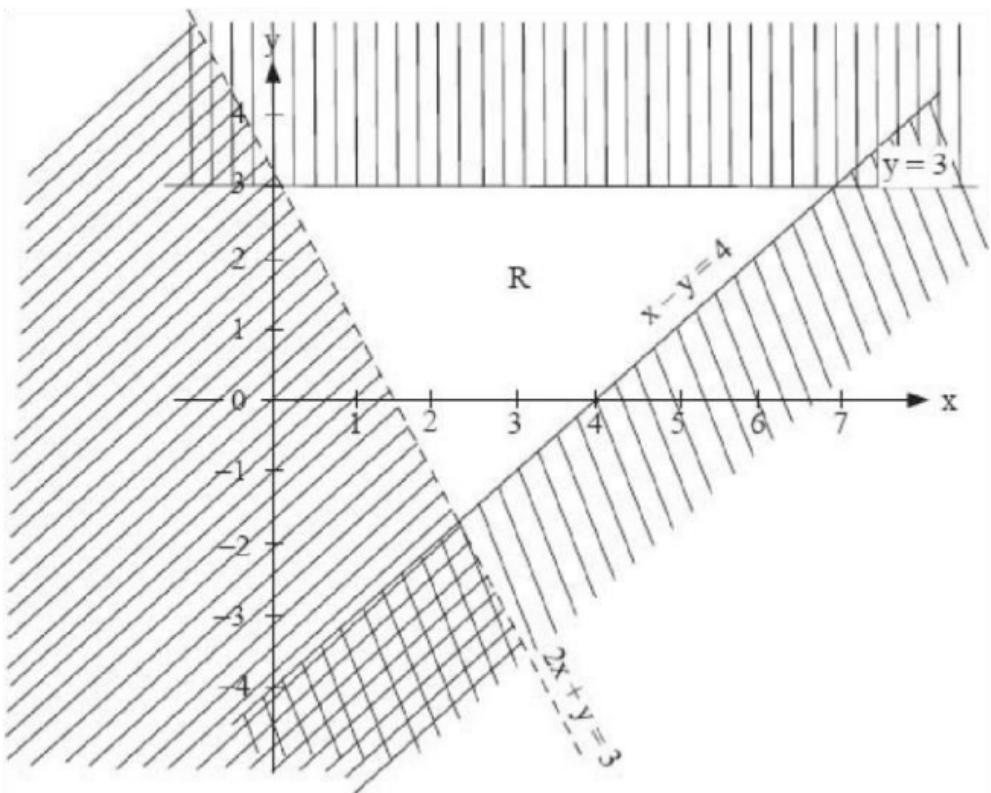


Fig. 16.13

Note that the required region (R) is unshaded.

Exercise 16.7

In each of the questions 1 – 5, draw the regions which satisfy all the inequalities.

1. (a) $x + y > 0$ $x < 2$ $y > 0$

(b) $2x + y \geq 6$

$x < 3$ $y <$

6

2. (a) $4x - 3y \leq 12$

$$x > 0$$

$$y > 0$$

(b) $4x - 3y <$

$$12 \quad y \geq 0$$

$$y \leq 6$$

3. (a) $y + 2x \leq 5 \quad y < 4x + 12$

(b) $y - x < 0 \quad x$

$$\leq 5 \quad y \geq 0$$

4. (a) $2x + y > 4$

$$6x + 2y \leq 12$$

$$(b) x + y \leq 6$$

$$x > 4$$

$$y > 0$$

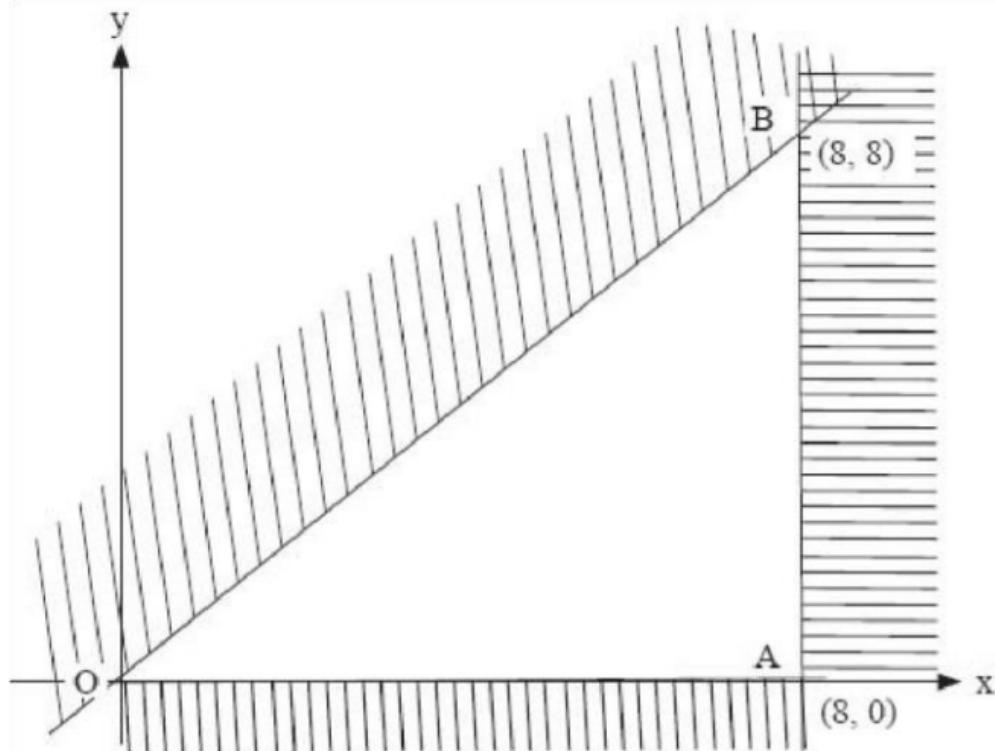
$$5. \quad (a) 5x + 3y \geq 15 \quad 6y + 5x < 30 \quad y \geq 0$$

$$(b) 0 \leq y < 3$$

$$0 \leq x < 4$$

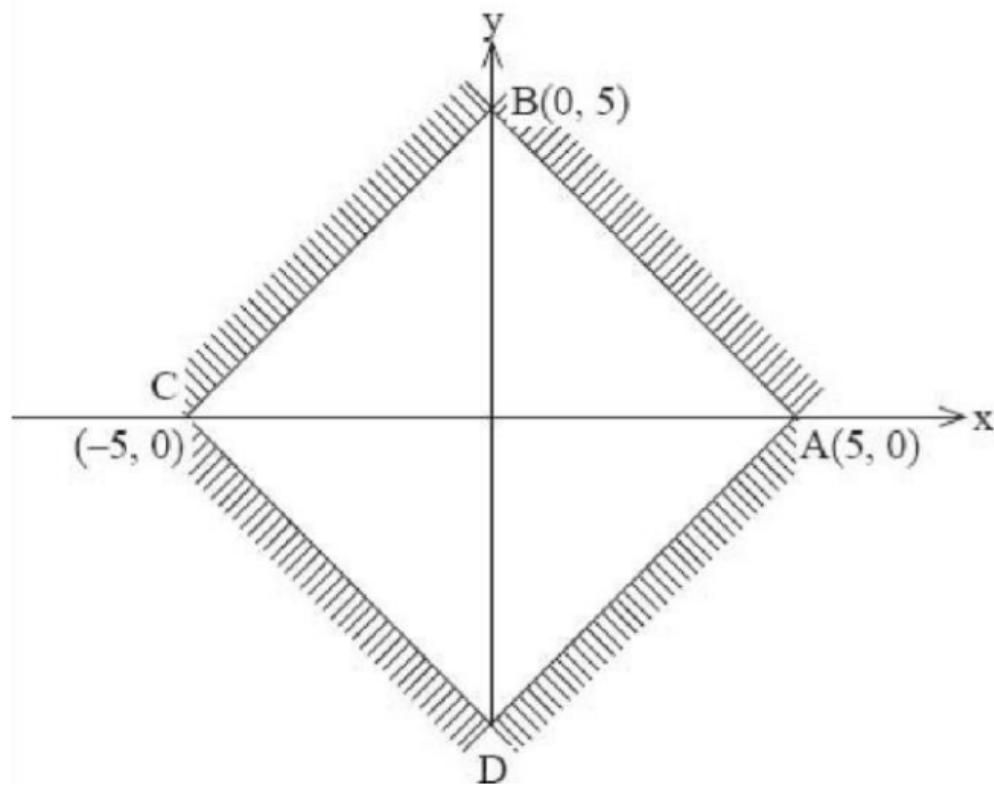
6. Find the area of the required regions in questions 5 (a) and (b).

7. Write down the co-ordinates of any four points with integral values which lie in the region obtained in question 4 (a).
8. The vertices of the unshaded triangular region in the figure below are $O(0,0)$, A $(8, 0)$ and B $(8, 8)$. Write down the inequalities which are satisfied by the region.

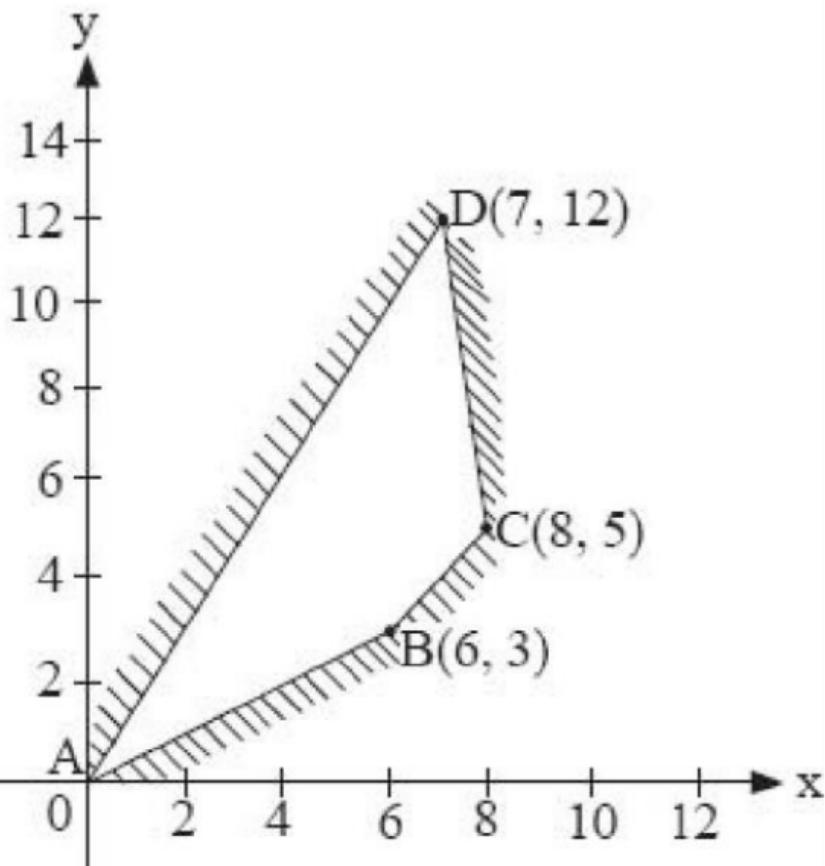


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9. The figure below shows a square ABCD with vertices A (5, 0), B(0,5) C(-5, 0) and D.



- (a) Determine the co-ordinates of point D.
- (b) Write down the equations of lines AB, CB, CD and AD.
- (c) Write down the inequalities which determine the square.
10. The figure below shows the vertices of a region bounded by lines AB, BC, CD and DA.



Write down the inequalities that determine the region ABCD.

11. Given the inequalities $y - 2x \leq 4$ and $x + y \leq 1$, find two other inequalities which, together with these two, will complete a parallelogram with $(-2, 0)$ and $(1, 0)$ as two of its vertices.

Chapter 17

LINEAR MOTION

17.1: Definitions

The distance between two points is the length of the path joining them. The SI unit of distance is the metre.

Displacement is distance in a specified direction.

Speed is the rate of change of distance with time. The SI unit of speed is m/s, though speed is commonly given in km/h. Velocity is speed in a specified direction or the rate of change of displacement with time.

Acceleration on the other hand is the rate of change of velocity with time. The SI unit of acceleration is m/s².

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17.2: Speed

Over a time interval, the speed normally varies and what is often used is the average speed. Average speed = $\frac{\text{distance covered}}{\text{time taken}}$

If a motorist covers a distance of 180 km in 3 hours, his average speed is $\frac{180}{3}$ km/h = 60 km/h.

Example 1

A man walks for 40 minutes at 6 km/h. He then travels for two hours in a minibus at 80 km/h. Finally, he travels

by bus for one hour at 60 km/h. Find his speed for the whole journey.

Solution

Average speed $\frac{\text{distance covered}}{\text{time taken}}$

$$\begin{aligned}\text{Total distance} \\ &= \left(\frac{40}{60} \times 6 \text{ km}\right) + (2 \times 80) \text{ km} + (1 \times 60) \text{ km} \\ &= 224 \text{ km}\end{aligned}$$

$$\begin{aligned}\text{Total time taken} &= \frac{4}{6} + 2 + 1 \\ &= 3\frac{2}{3} \text{ hrs}\end{aligned}$$

Average speed =

$$\frac{224}{3\frac{2}{3}} = \frac{224 \times 3}{11} = 61.09 \text{ km/h}$$

Exercise 17.1

1. Juma cycled for $2\frac{1}{2}$ hours to Mula Trading Centre, 30 km away from his home. He then took his business van and drove at an average speed of 80 km/h to a town 180 km away and back to Mula. He finally cycled home at an average speed of 15 km/h. Find the

average speed for the whole journey.

2. A motorist travelled for 2 hours at a speed of 80 km/h before his vehicle broke down. It took him half an hour to repair the vehicle. He then continued with his journey for $1\frac{1}{2}$ hours at a speed of 60 km/h. Determine his average speed.

3. John travelled from town A to town B, a distance of 30 km. He cycled for $1\frac{1}{2}$ hours at a speed of 16 km/h and then walked the remaining distance at a speed of 3 km/h. Find his average speed for the journey.
4. Anne takes two hours to walk from home to her place of work, a distance of 8 km. On a certain day, after walking for 30 minutes, she stopped for ten minutes to talk to a friend. Determine the average

speed she should walk to reach her work on time.

5. A motorist drove for 1 hour at 100 km/h. She then travelled for $1\frac{1}{2}$ hours at a different speed. If the average speed for the whole journey was 88 km/h, find the average speed for the latter part of the journey.

6. A commuter train moves from station A to station D via stations B and C in that order. The distance from A to C via B is 70 km and that from B to D via C is 88 km. Between the stations A and B, the train travels at an average speed of 48 km/h and takes 15 minutes. Between the stations C and D, the average speed of the train is 45 km/h. Find:
- (a) the distance from B to C.
 - (b) the time taken between C and D.

(c) If the train halts at B for 3 minutes and at C for 4 minutes and its average speed for the whole journey is 50 km/h, find its average speed between B and C.

17.3: Velocity and Acceleration

Consider two cars moving in opposite directions, each at a speed of 65 km/h. Their velocities are different because they are travelling in different

directions. For motion under constant acceleration;

Average velocity =

$$\frac{\text{initial velocity} + \text{final velocity}}{2}$$

Consider a car moving in a given direction under constant acceleration. If its velocity at a certain time is 75 km/h and 10 seconds later it is 90 km/h, then;

$$\begin{aligned}\text{Acceleration} &= \frac{\text{change in velocity}}{\text{time taken}} \\&= \frac{(90 - 75) \text{ km/h}}{10 \text{ s}} \\&= \frac{(90 - 75) \times 1000}{10 \times 60 \times 60} \text{ m/s}^2 \\&= \frac{5}{12} \text{ m/s}^2\end{aligned}$$

Now, consider a car moving with a velocity of 50 km/h then the brakes are applied so that it stops after 20 seconds. In this case, the final velocity is 0 km/h and initial velocity = 50 km/h.

Therefore, acceleration =

$$= \frac{(0 - 50) \times 1\,000}{20 \times 60 \times 60} \text{ m/s}^2$$
$$= \frac{-25}{36} \text{ m/s}^2$$

The acceleration is negative.

Negative acceleration is usually referred to as deceleration or retardation. Here, the retardation is $\frac{25}{36}$ m/s².

Exercise 17.2

1. The initial velocity of a car is 10 m/s. The velocity of the car after 4

seconds is 30 m/s. Find its acceleration.

2. A bus accelerates from a velocity of 12 m/s to a velocity of 25 m/s. Find the average velocity during this interval.
3. A car moves with a constant acceleration of 8 m/s^2 for 5 seconds. If the final velocity is 40 m/s, find the initial velocity.

4. A train driver moving at 40 km/h applies the brakes so that there is a constant retardation of 0.5 m/s^2 . Find the time taken before the train stops.
5. The velocity of a particle increases uniformly from 27 m/s to 57 m/s in one minute. Find the acceleration.
6. The driver of a minibus which is moving at 80 km/h notices a road block a short distance ahead. He applies the brakes immediately so that the minibus moves with

constant deceleration for 5 seconds before coming to rest at the road block. Find:

- (a) the average velocity of the minibus during the 5 seconds.
- (b) the deceleration.

17.4: Distance-Time Graph

When distance is plotted against time, a distance-time graph is obtained.

Example 2

Table 17.1 shows the distance covered by a motorist from Limuru to Kisumu:

Table 17.1

Time	9.00 am	10.00 am	11.00 am	11.30 am	12 no
------	------------	-------------	-------------	-------------	----------

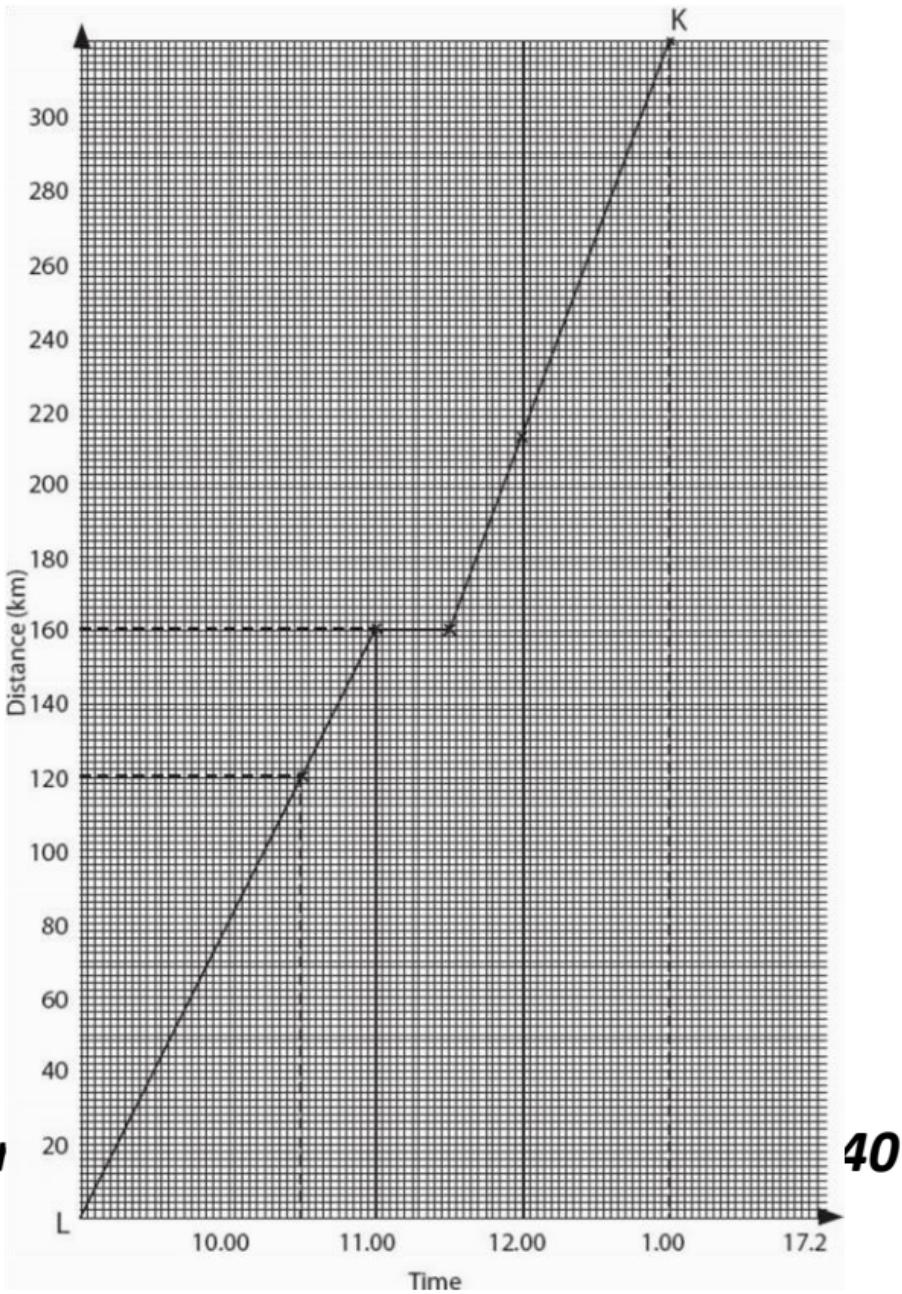
Distance (km)	0	80	160	160	2
------------------	---	----	-----	-----	---

- (a) Draw the distance-time graph
- (b) Use the graph to answer the following questions:
 - (i) How far was the motorist from Limuru at 10.30 a.m?
 - (ii) What was the average speed during the first part of the journey?
- (c) What was the average speed for the whole journey?

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Solution

(a)



Coi

40

Fig. 17.1

(b) (i) 120 km

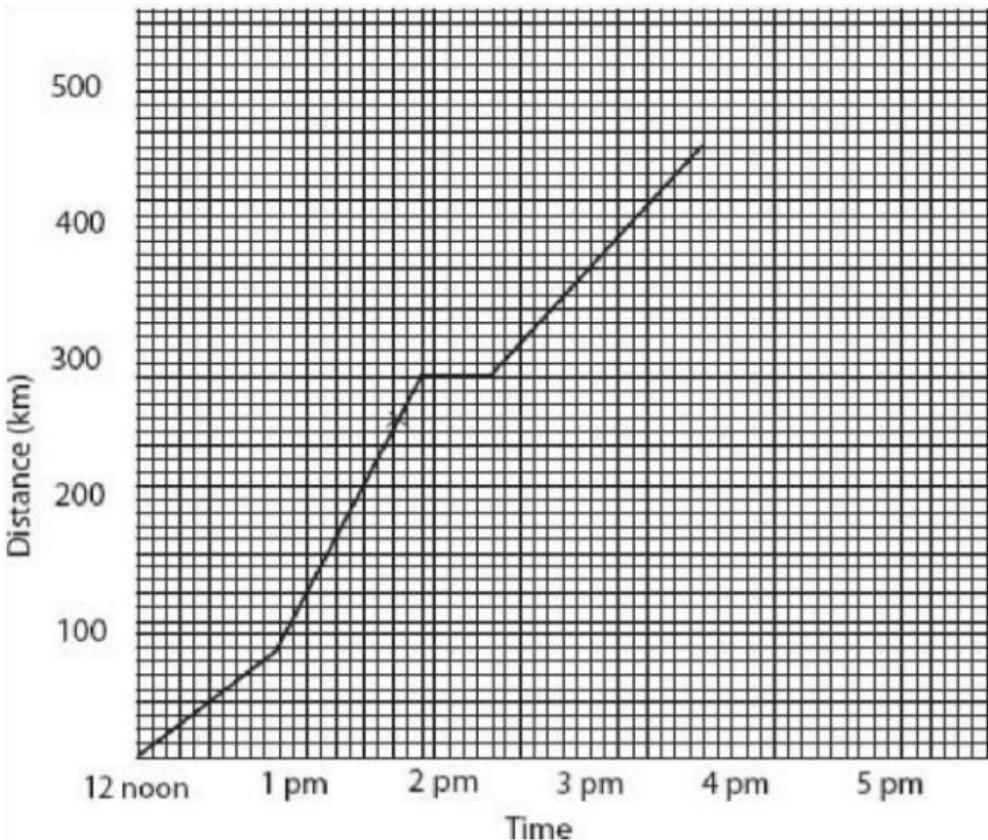
$$\text{(ii)} \frac{160 \text{ km}}{2 \text{ h}} = 80 \text{ km/h}$$

(c) Average speed =

$$\frac{\text{total distance}}{\text{total time}} = \frac{310 \text{ km}}{4 \text{ h}} = 77.5 \text{ km/h}$$

Exercise 17.3

1. The figure below shows a travel graph for a motorist who starts his journey at 12 noon and drives to a town 450 kilometres away.



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Use the figure to answer the questions below:

- (a) Find the average speed of the motorist:
- (i) between 12 noon and 1 pm.
 - (ii) between 1 p.m and 2 pm.
 - (iii) between 1 p.m and 3 pm.
 - (iv) for the whole journey.
 - (v) for the first 200 kilometres.
 - (vi) for the last 200 kilometres

- (b) Find the distance covered between:
- (i) 12 noon and 2 pm.
 - (ii) 1.30 pm and 3.30 pm.
- (c) Between what times was the average speed greatest?
2. A man leaves home at 9.00 am and walks to a bus stop 6 km away at an average speed of 4 km/h. He then

waits at the bus stop for 25 minutes before boarding a bus to a town 105 km. Draw a distance time-graph for the journey and use it to answer the following questions:

- (a) State the time the man was 100 km from home.
 - (b) How far away from home was he at 10.15 am?
3. A motorist left Eldoret at 11.00 am. for Nairobi, a distance of 320 km. She took $1\frac{1}{2}$ hours to reach Nakuru

half-way between Eldoret and Nairobi where it took her 20 minutes to fuel the car. She then set off for Nairobi, but stopped after 25 minutes at Keekopey, 30 km from Nakuru for 30 minutes. She then proceeded with the journey to Nairobi and arrived at 5.00 pm.

- (a) Draw the distance-time graph to show the journey.
- (b) Use the graph to find:

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- (i) the average speed for the whole journey.
- (ii) the average speed of the last part of the journey.

17.5: Velocity-Time Graph

When velocity is plotted against time, a velocity time graph is obtained.

Consider a particle which starts from rest and moves in a straight line. For the first four seconds, the acceleration is 10 m/s^2 . The velocity then remains constant for the next two seconds before the

particle undergoes a deceleration of 8 m/s^2 and eventually comes to rest. This information is represented in table 17.2.

Table 17.2

Time in seconds	0	1	2	3	4	5	6	
Velocity in m/s	0	10	20	30	40	40	40	

The velocity-time graph is given in figure 17.2

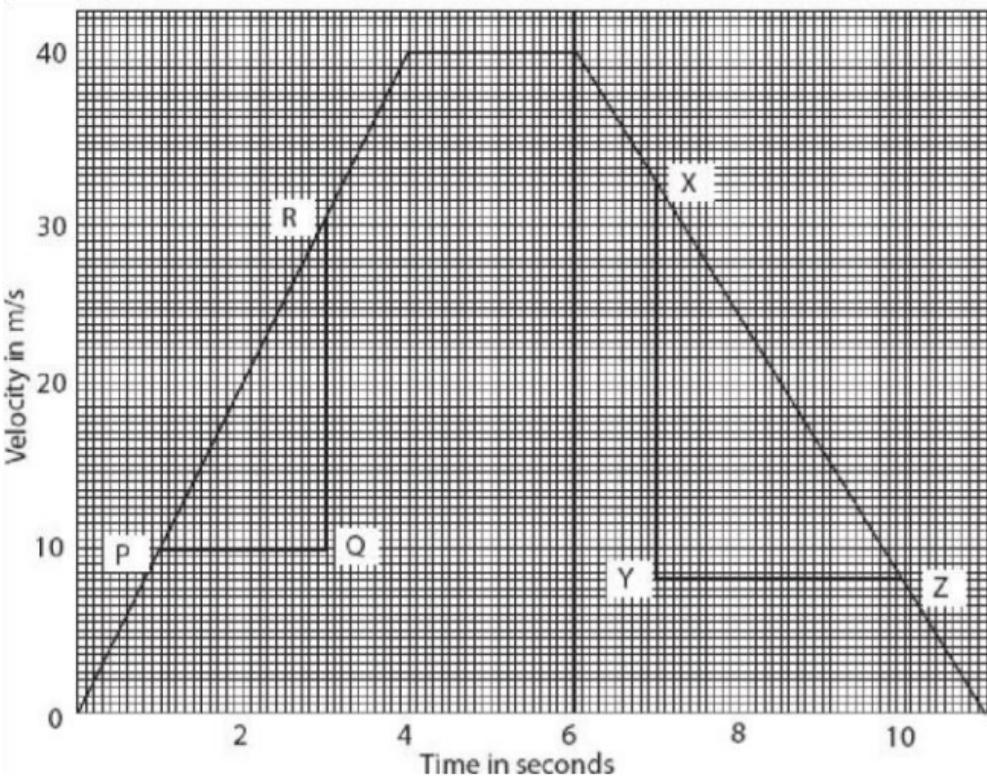


Fig 17.2

Note that in the figure, the ratio;

$$\frac{\text{change in velocity}}{\text{corresponding change in time}} = \text{acceleration}$$

For example, the acceleration in the first 4 seconds is given by;

$$\begin{aligned}\frac{QR}{PQ} &= \frac{(30 - 10) \text{ m/s}}{(3 - 1) \text{ s}} \\ &= 10 \text{ m/s}^2\end{aligned}$$

Similarly; in the last five seconds, the retardation is given by;

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$$\frac{XY}{YZ} = \frac{(8 - 32) \text{ m/s}}{(10 - 7) \text{ s}}$$
$$= -8 \text{ m/s}$$

In general, the slope of the graph in a velocity-time graph is the acceleration.

To see how distance covered can be obtained from a velocity-time graph, consider a car moving for two hours at a constant velocity of 80 km/h. The velocity-time graph is shown in figure 17.3

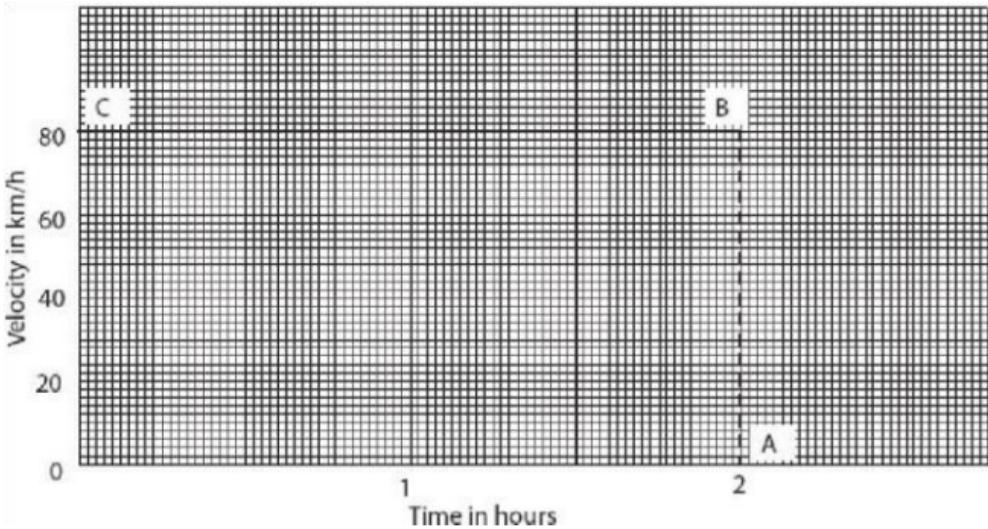


Fig. 17.3

Distance covered in 2 hours is given by;

$$80\text{km/h} \times 2\text{h} = 160 \text{ km}$$

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It should be noted that the distance is equal to the area of the rectangle OABC.

In general, the area under a velocity-time graph represents the total distance covered. Thus, in figure 17.4, the shaded area represents the total distance covered between the times t_1 and t_2 .

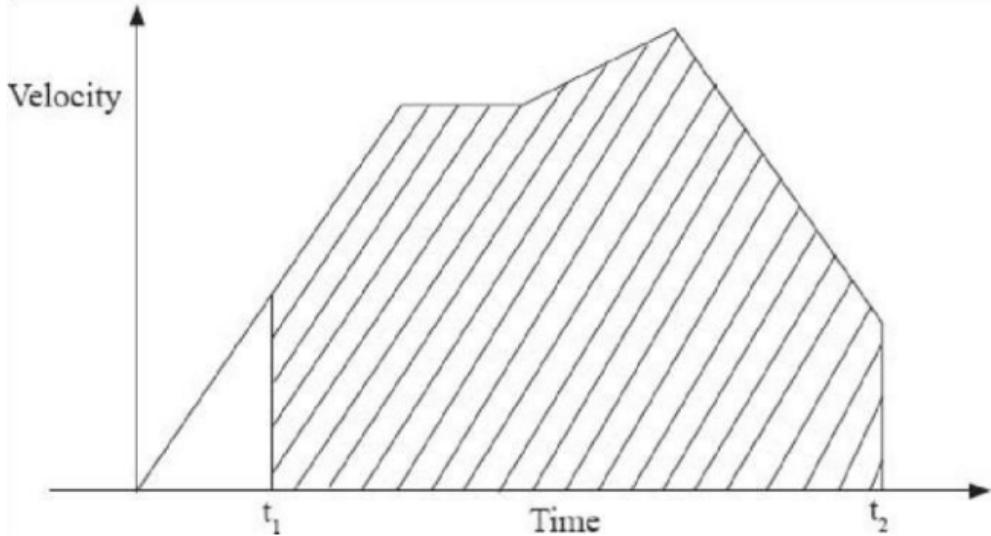


Fig. 17.4

Table 17.3 gives velocity of a train moving with a constant acceleration of 5 m/s^2 .

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Table 17.3

Time in seconds	0	1	2	3	4	5
Velocity in m/s	0	5	10	15	20	25

Figure 17.5 is he velocity-time graph.

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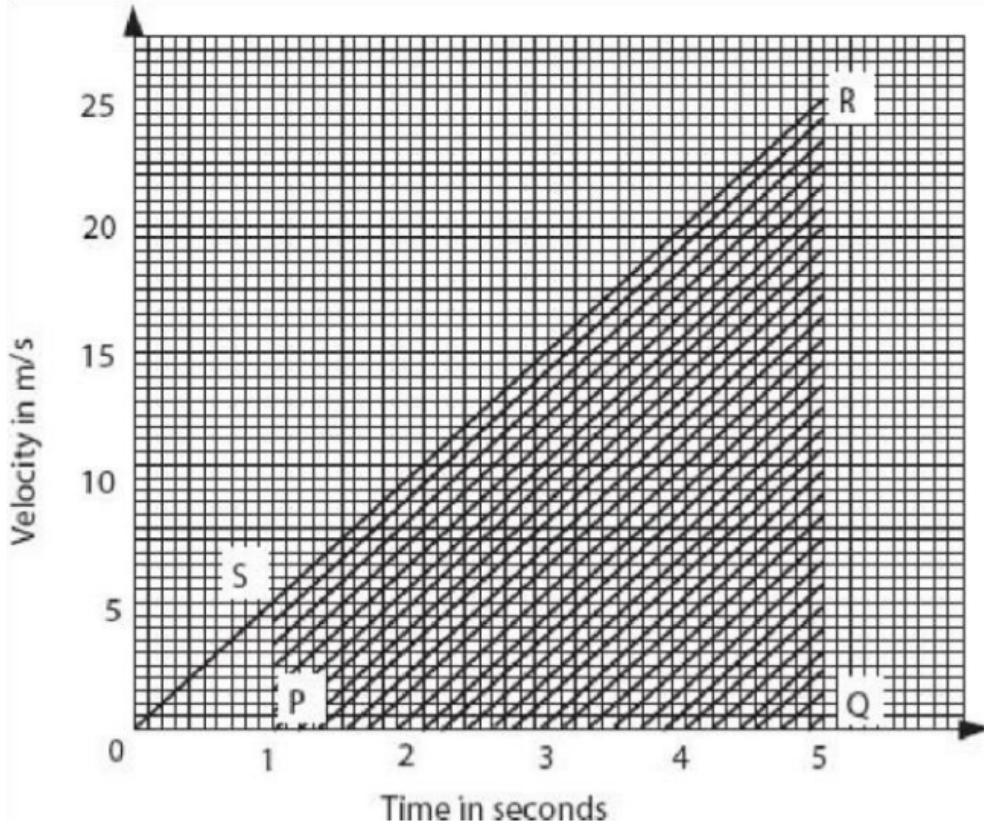


Fig 17.5

The shaded area, which is a trapezium, gives the distance covered between the

first and the fifth seconds (i.e. in 4 seconds).

Distance = Area of a trapezium

$$= \frac{1}{2} \times 4 (5 + 25)$$

$$= 2 \times 30$$

$$= 60 \text{ km}$$

This is equal to the distance covered within 4 seconds at an average velocity of; $\frac{5 \text{ m/s} + 25 \text{ m/s}}{2} = 15 \text{ m/s}$

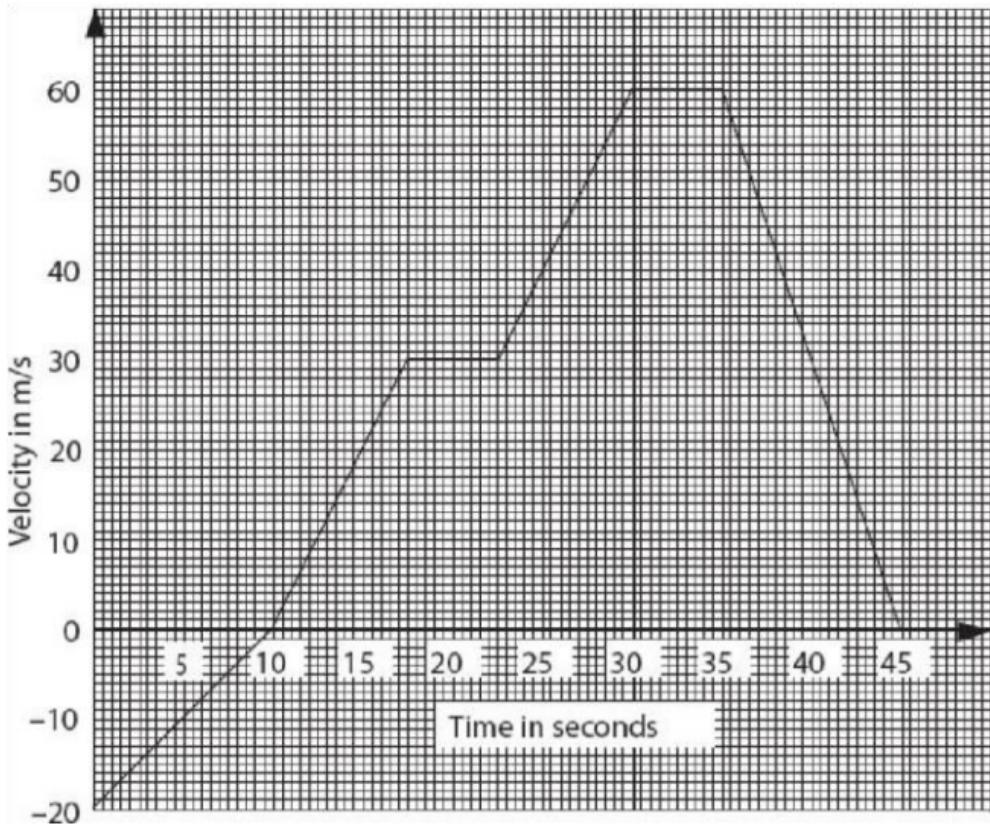
t

Exercise 17.4

1. A car is travelling at 40 m/s. Its brakes are applied and it then decelerates at 8 m/s^2 . Use a velocitytime graph to find the distance it travels before stopping.
2. A particle is projected vertically upwards with a velocity of 30 m/s. If the retardation to motion is 10 m/s^2 , use a graphical method to find the maximum height reached by the particle.

3. The figure below shows a velocitytime graph of a particle moving along a West-East straight line, with velocity considered positive eastwards.

t



Use the graph to answer the following questions:

- (a) Find the velocity of the particle:

- (i) after 5 seconds.
 - (ii) after 15 seconds.
 - (iii) after 20 seconds.
 - (iv) after 40 seconds.
- (b) Find the acceleration of the particle:
- (i) during the first 5 seconds.
 - (ii) during the 15th second of the motion.

t

(iii) between the 18th and the 20th second.

(iv) between the 37th and the 41st second.

(c) Use area under the graph to find the distance covered by the particle:

(i) between the 10th and the 15th second.

(ii) between the 25th and the 35th second.

(iii) between the 20th and the 25th second.

(d) Find the average velocity:

(i) during the period from the 10th to 15th second.

(ii) during the period from 35th to the 40th second.

(iii) during the period from 30th to the 35th second.

t

4. A particle moving with a velocity of 25 m/s along a North-South straight line passes through a point P at 11.00 am. If the acceleration is 5 m/s²

southwards:

- (a) Find the velocity of the particle
(give direction):
- (i) $4\frac{1}{2}$ seconds after 11.00 am.
 - (ii) 6 seconds after 11 am.
 - (iii) 2 seconds before 11 am.
- (b) Find the average velocity of the particle:

- (i) during the first 10 seconds after 11 am.
 - (ii) during the first 12 seconds after 11 am.
- (c) Use the graphical method to find the distance of the particle from P at 20 seconds past 11.00 am.

17.6: Relative Speed

Consider two bodies moving in the same direction at different speeds. Their relative speed is the difference between the individual speeds.

On the other hand, the relative speed when two bodies are approaching each other is the sum of their speeds.

Example 3

A van left Nairobi for Kakamega at an average speed of 80 km/h. After half an hour, a car left Nairobi for Kakamega at a speed of 100 km/h.

- (a) Find the relative speed of the two vehicles.

(b) Find how far from Nairobi the car overtook the van.

Solution

(a) Relative speed = difference between the speeds

$$= 100 - 80$$

$$= 20 \text{ km/h}$$

(b) Distance covered by van in 30

minutes;

$$\begin{aligned}\text{Distance} &= \frac{30}{60} \times 80 \\ &= 40 \text{ km}\end{aligned}$$

$$= \frac{40}{20}$$

$$T = 2 \text{ hours}$$

ime taken for car
to overtake matatu

$$\text{Distance from Nairobi} = 2 \times 100$$

$$= 200 \text{ km}$$

Example

4

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A truck left Nyeri at 7.00 am. for Nairobi at an average speed of 60 km/h. At 8.00 am., a bus left Nairobi for Nyeri at a speed of 120 km/h. Find how far from Nyeri did the vehicles meet if Nyeri is 160 km from Nairobi.

Solution

$$\begin{aligned}\text{Distance covered by lorry in 1 hour} &= 1 \times \\ 60 &= 60 \\ \text{km} &\end{aligned}$$

Distance between the vehicles at 8.00 am. = $(160 - 60)$ km

=

100 km

Relative speed = $60 \text{ km/h} + 120 \text{ km/h}$
= 180 km/h

Time taken for the vehicles to meet

$$\begin{aligned}&= \frac{100}{180} \\&= \frac{5}{9} \text{ hours}\end{aligned}$$

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$$\begin{aligned}\text{Distance from Nyeri} &= 60 + \frac{5}{9} \times 60 \\ &= 60 + 33.3 \\ &= 93.3 \text{ km}\end{aligned}$$

Exercise 17.5

1. A heavy commercial truck left Murang'a at 8.00 am. for Nairobi, a distance of 60 km, at a speed of 40 km/h. If 30 minutes later, a matatu left Murang'a for Nairobi at a speed of 80 km/h. Find the time the matatu overtook the truck.
2. How long will it take a car 3 metres long moving at 100 km/h to overtake

a truck 10 m long moving at 50 km/h
if the car is 10 m behind the truck?

3. A motorist left Embu for Nairobi, a distance of 240 km, at 8.00 am. and travelled at an average speed of 90 km/h. Another motorist left Nairobi for Embu at 8.30 am. and travelled at 100 km/h. Find:
- (a) the time they met.
 - (b) how far from Nairobi they met

4. A cyclist travelling at 15 km/h left Kitale at 7.00 a.m. for Kakamega, a distance of 70 km. Another cyclist left Kakamega for Kitale at 8.00 am. If they met after $2\frac{1}{2}$ hours, find the

speed of the second cyclis .

5. A matatu left town A at 7 am. and travelled towards a town B at an average speed of 60 km/h. A second matatu left town B at 8 am. and travelled towards town A at 60 km/h. If the distance between the two towns is 400 km, find:
- (a) the time at which the two matatus met.

t

- (b) the distance of the meeting point from town A.

Chapter 18

STATISTICS

18.1: Introduction

Statistics is a branch of mathematics that deals with collection, organisation,

representation and interpretation of data. Data is basic information, usually in number form. Statistics is used in management where data is involved.

Consider the marks obtained by students in a test, as recorded below

70, 60, 55, 70, 41, 30, 60, 70, 55, 70, 30,
41, 70, 55, 41, 30, 55, 41.

This data can be arranged in rank order, i.e.,

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70, 70, 70, 70, 70, 60, 60, 55, 55, 55, 55,
41, 41, 41, 41, 30, 30, 30.

It is now easy to state how many students attained a certain mark at a glance.

Data can be obtained for:

- (i) masses, in kilogrammes, of students in a school.
- (ii) the age, in years, of the students in a class.

- (iii) the height, to the nearest centimetre, of the students in a class.
- (iv) the marks obtained in a recent test.
- (v) the number of children in a number of families.
- (vi) the favourite subjects of students in a class.
- (vii) the favourite games of students in a class.

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- (viii) the shoe sizes worn by the students in a class.
- (ix) the time taken in a 100 metres race by students in a class.
- (x) the daily temperature recorded at a school for one month.

18.2: Frequency Distribution Table

The following data represents the shoe sizes worn by 20 Form Two students:

7, 9, 6, 10, 8, 8, 9, 11, 8, 7, 9, 6, 8, 10, 9, 8, 7, 7, 8, 9.

The data can be represented in a frequency distribution table as shown in table 18.1. Table 18.1

Shoe size	Tally	No. of students (frequency)
6	//	2
7	///	4
8	/// /	6

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9	///	5
10	//	2
11	/	1
20		

Tally

Each stroke is a tally and represents a quantity. For example, four students wear shoe size 7. This is represented by 4 tallies.

Frequency

This is the number of times an item or value occurs. For example, in the table 8 appears 6 times. Therefore, the frequency of 8 is 6. Use the same table to identify the frequency of shoe size 6,

9, 10, 11.

Example 1

The data below shows marks obtained by 20 students for an essay:

9, 5, 5, 4, 5, 3, 5, 11, 6, 3, 6, 8, 9, 6, 13, 8, 8, 13, 5, 10.

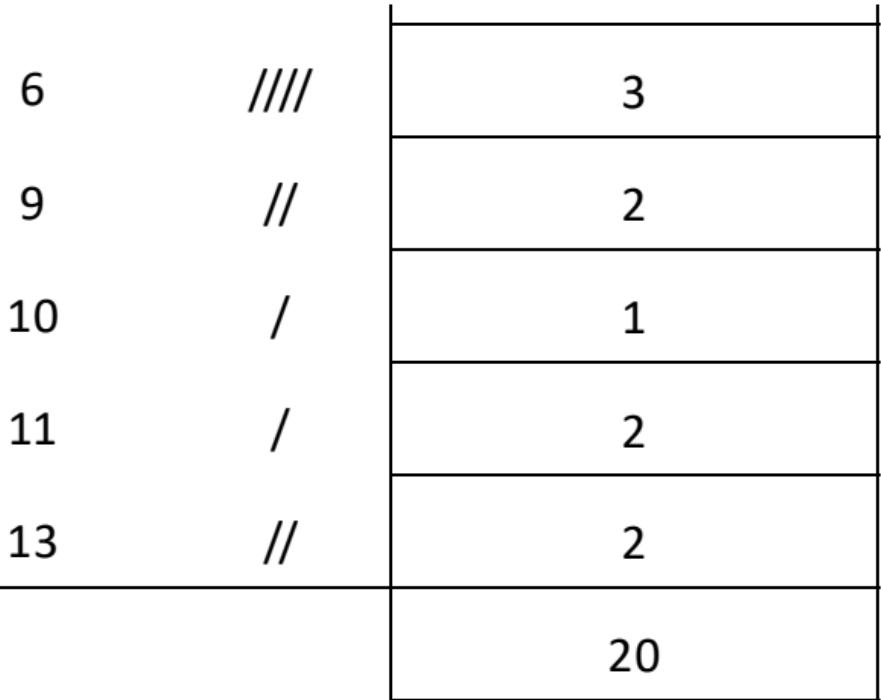
Prepare a frequency distribution table:

Solution

Table 18.2

Marks	Tally	Frequency (f)
3	//	2
4	/	1
5	///	5

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18.3: Mean, Mode and Median

Mean, mode and median are measures of central tendency. They are values around which data tends to cluster.

Mean

This is usually referred to as the arithmetic mean, and an average value for the data.

Example 2

Find the mean of the following marks:

9, 5, 5, 4, 5, 3, 5, 11, 6, 3, 6, 8, 9, 6, 13, 8, 8, 13, 5, 10

Solution

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The mean mark = $\frac{\text{total marks scored}}{\text{total number of students}}$

$$\begin{aligned}\text{Sum} &= 9 + 5 + 5 + 4 + 5 + 3 + 5 + 11 + 6 \\&+ 3 + 6 + 8 + 9 + 6 + 13 + 8 + 8 + 13 + 5 + \\&10\end{aligned}$$

$$\text{Total marks} = 142$$

$$\begin{aligned}\text{Therefore the mean mark} &= \frac{142}{20} \\&= 7.1\end{aligned}$$

Frequency distribution table can be used to find the mean as follows:

Table 18.3

<i>Marks (x)</i>	<i>Frequency (f)</i>	<i>fx</i>
3	2	6
4	1	4
5	5	25
6	3	18
8	3	24
9	2	18
10	1	10
13	2	26

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$$\sum f = 20$$

$$\sum fx = 142$$

The symbol Σ is Greek (sigma). It means 'sum of'.

Sum of the frequencies 'f' is written as Σf . Sum of products of x and f, is written as Σfx . The mean is denoted as \bar{x} .

Thus, $\bar{x} = \frac{\sum fx}{\sum f}$

$$\begin{aligned}\bar{x} &= \frac{\sum fx}{\sum f} = \frac{142}{20} \\ &= 7.1\end{aligned}$$

U **Mode** sing table 18.3, the mean mark, \bar{x} is;

This is the most frequently occurring item or value in a data. For example, in table 18.3, the mark 5 is the most

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frequent. It is called the mode. Its corresponding frequency is called the modal frequency. A data set can have more than one mode.

Example 3

The following are the times in seconds taken by athletes in a 100 m race:

10.0, 9.9, 9.9, 11.0, 10.0, 10.1, 12.0,
11.0, 10.3, 11.3, 11.0.

State the mode.

Solution

The most frequent time is 11.0 seconds.
The mode is 11.0 seconds

Median

When data is arranged in ascending or descending order, the middle item is the median for example:

The following were the marks obtained by 11 students in a mathematics test:

40, 35, 37, 50, 54, 39, 60, 57, 56, 51, 65.

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To find the median, the marks are arranged in either ascending or descending order, that is; 35, 37, 39, 40, 50, 51, 54, 56, 57, 60, 65, or 65, 60, 57, 56, 54, 51, 50, 40, 39, 37, 35.

The one in the middle is the median.

Thus, 51 is the median of this data set.

If there is an even number of items, the average of the two in the middle is taken as the median. Consider the following ages in years of 6 From Two students:

15, 14, 16, 14, 17 and 18.

To find the median, arrange the ages in either ascending or descending order, i.e., 14, 14, 15, 16, 17, 18.

Then, the median age is

$$\frac{15 + 16}{2} = \frac{31}{2} \text{ years}$$
$$= 15\frac{1}{2} \text{ years}$$

If there are N items and N is odd, the item occupying $\left(\frac{N+1}{2}\right)$ position is the median.

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If N is even, the average of the items in position $\frac{N}{2}$ and $\frac{N}{2} + 1$ is the median.

The measures of central tendency are useful in our everyday life. Below are some examples of situations in which these measures are applied:

- (i) In our industries, e.g. shoe production firms, the modal size in use is manufactured in larger quantities.
- (ii) In the tourist industry, there is extensive preparation in the month

when the highest number of tourists visit the country.

- (iii) For a shopkeeper, favourite items are stocked in quantity.
- (iv) For a hotelier, customers favourite dish will be important in preparing a menu.
- (v) In a country where the mean age of

th population is below 20 years, the government may find it necessary to plan for more schools, colleges and jobs.

- (vi) The mean monthly rainfall of a particular region may determine the type of crops that can be grown.

Exercise 18.1

1. The data below represents marks scored by 10 students in a test. Find the mean, the mode and the median of the marks.

e

7, 8, 9, 14, 13, 5, 10, 9, 11, 6

2. Forty eight pupils gave the number of children in their homes as follows:

3 4 1 2 3 2 2 1 3 5 6 8 6 1 4 2

3 1 5 7 8 9 1 2 1 2 5 6 5 5 7 8

2 3 1 3 2 1 1 1 2 4 2 3 3 4 2 1 3

- (a) Make a frequency distribution table for the data.

- (b) Determine the mode.
- (c) Calculate the mean number of children.
3. Twenty pupils in a class measured their masses and recorded them to the nearest kilogramme, as below:
- 40 48 56 52 49 57 56 52 53 48
38 39 43 47 41 60 63 59 45 51
- Make a frequency distribution table and find the mean and the mode.
4. The classes in a primary school had the following number of pupils:

e

42 45 48 40 46 42 44 48 39 40 42

41 47 46 45 49 45 42 40 38 39

46 47 42 40 41 43 44 45 46 48

- (a) Make a frequency distribution table.
- (b) Determine the mode.
- (c) Calculate the mean.

5. In an experiment to measure the length, in centimetres, of an

exercise book, 16 students recorded the following:

12.8 12.5 13.4 13.6 13.5 14.5

14.5 13.9 13.7 14.4 14.4 14.3

Construct a frequency table and use it to determine the mean and the mode.

6. A sample of 40 nuts were picked at random from a factory and their diameters measured to the nearest 0.1 mm as below:

20.4 20.5 10.1 10.2 20.3 20.0

e

20.6	20.5	20.4	20.3	20.1	20.9
20.3	20.4	20.1	20.1	20.8	20.7
20.0	20.1	20.5	20.6	20.7	20.1
20.3	20.4	19..9	20.0	21.0	20.9

- (a) Prepare a frequency distribution table for the information.
- (b) Find the mode, mean and median of the data.

7. In an athletics competition held in

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K richo District, 12 athletes ran the 400 m race and their times in seconds were recorded as below:

58.2 54.2 49.8 52.3 55.6 50.8
53.8 51.9 50.7 52.3 57.2 52.3

Find the mean, mode and median.

8. The height in metres of 50 people were recorded as follows:

1.21 1.25 1.36 1.42 1.53 1.67
1.34 1.42 1.52 1.44 1.53 1.42
1.30 1.24 1.46 1.38 1.39 1.42

e

1.33	1.44	1.28	1.35	1.40	1.70
1.40	1.36	1.38	1.39	1.50	1.61

Make a frequency table and find the mean, the mode and the median.

In numbers 9 to 11, find the mean, mode and median

9. The following frequency distribution table represents masses in kilogrammes of children in a class:

Mass of children (kg)	35.5	36.8	38.2	40.0	40.2	
No. Of children	4	3	2	2	2	

4

10. The volume of milk (to the nearest 0.01) in 24 tetrapaks was measured and recorded as below:

0.50 0.50 0.49 0.48 0.54 0.50

e

0.49 0.51 0.53 0.52 0.52 0.50

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0.48 0.48 0.50 0.50 0.50 0.49

11. The table below shows the duration, to the nearest minute, of telephone calls through an exchange.

Duration in min.	0	1	2	3	4	5	6	7
Frequency	2	2	11	20	14	9	9	7

12. In an examination, the mean mark for Form 2B, in which there were 34 students, was 51. Form 2A, in which there were 36 students, had a mean

of 42 marks. Calculate (to 1 dec. place) the mean mark for the two classes combined.

13. The mean of the numbers 3, 4, a, 5, 7, a, 5, 8, 5 and 9 is equal to the mode. Find the value of a and hence the median of the data.
14. The mean age in Form II class of 36 boys was 16 years 8 months. If an 18-year old boy later left the class,

what was the new mean age for the class?

15. Forty five pupils in a mixed class sat for a mathematics test. The mean mark for the 25 girls in this class was 58.5 while the mean mark for all the pupils was 60.2. Calculate the mean mark for the boys.
16. The mean height of five girls was found to be 148 cm. When a sixth girl joined the group, the mean height was reduced by 3 cm. Find the height of the sixth girl.

18.4: Grouped Data

When a large number of items is being considered, it is necessary to group the data. Consider the masses in kilogrammes of 50 women which were recorded as follows:

63.7 50.0 58.0 73.4 76.3 42.3 58.0 45.0 58.0
59.5 45.2 79.6 69.1 42.8 49.8 84.2 52.1 63.5
58.0 45.5 58.7 59.5 40.6 61.4 72.9 49.2 74.2
60.2 43.9 73.4 53.3 67.3

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62.3 46.8 66.2 52.1 59.7 60.5 60.3 69.6

The masses vary from 40.6 kg to 84.2. The difference between the smallest and the biggest values in a set of data is called the range. In this case, the range is $84.2 - 40.6 = 43.6$.

The data can be grouped into a convenient number of groups called classes. Usually, the convenient number of classes varies from 6 to 12. In this example, a class size of 4 will give; $\frac{43.6}{4} =$
11 classes.

A class size of 5 will give = 9 $\frac{43.6}{5}$
classes.

A class size of 6 will give = 8 $\frac{43.6}{6}$
classes.

A class size of 5 is used to make a grouped frequency table as in table 18.4.

Table 18.4

6

6

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Class	40–44	45–49	50–54	55–59	60–64	
Frequency	5	5	7	6	13	

Note:

A 40–44 class includes all values equal to or greater than 39.5 but less than 44.5.

40 and 44 are called class limits while 39.5 and 44.5 are called class boundaries. The class interval (width) is obtained by getting the difference between the class boundaries. In this case, $44.5 - 39.5 = 5$ which is the class size or class interval.

The class with the highest frequency is called the modal class. In this case it is 60–64 class, with a frequency of 13.

For grouped data with N items, the median position is taken as $\frac{N}{2}$ regardless of whether N is even or odd.

There are 50 women in the above grouped data. The position of the median is $\frac{50}{2} = 25$, therefore, the mass

of the woman in the 25th position is the median.

The 25th woman falls in the class 60–64 in which there are 13 women. Up to the class 55–59, there are 23 women, two additional women from the class of 60–64 are required to give 25 (the median position).

$$\begin{aligned}\text{Thus, the median mass, } &= 59.5 + \frac{2}{13} \times 5 \\ &= 60.269 \text{ kg}\end{aligned}$$

To calculate the mean of a grouped data, the midpoint of each class is taken

to represent each item. For example, the midpoint of 40–44 is;

$$\left(\frac{40 + 44}{2} \right) = 42.0$$

The calculation is as shown in the table below.

Table 18.5

Class	Midpoint (x)	Frequency (f)	xf

44– 44	42.0	5	210
45– 49	47.0	5	235
50– 54	52.0	7	364
55– 59	54.0	6	324
60– 64	62.0	13	806

65– 69	67.0	3	201
70– 74	72.0	5	360
75– 79	77.0	3	231
80–			

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84	82.0	3	246
		$\sum f = 50$	$\sum fx = 2977$

$$\begin{aligned}\text{Mean } \bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{2977}{50} \\ &= 59.54\end{aligned}$$

Therefore, the mean mass is 59.54 kg

Grouping the data makes it less accurate. Therefore, the mean and the median obtained from grouped data are estimates.

Exercise 18.2

1. The examination marks in a mathematics test for 72 students were as follows:

36	62	15	28	60	30	25	35	75
14	16	33	58	72	80	92	44	57
55	56	70	34	40	18	15	28	32
60	57	68	83	30	32	40	38	45
48	52	58	62	65	75	84	63	58

47	55	60	38	25	18	29	15	32
35	38	45	43	46	53	58	64	68
54	59	60	61	64	65	70	78	90

Using class interval of 10 and starting from 10–19, make a frequent distribution table. Use the table to:

(a) Find the modal class.

(b) Estimate:

(i) the mean.

(ii) the median.

2. The heights, to the nearest centimetre, of 50 students in a particular school were as follows:

165	170	182	169	165	180	182
180	184	174	186	175	175	186
174	183	172	167	168	158	188
135	148	159	148	182	163	140
156	158	155	147	143	142	138

160 156 140 148 158 165 180
184 168

- (a) Using class interval of 10 and 135 – 144 as the first class, make a frequency distribution table.
- (b) From the table:
- (i) state the modal class.
 - (ii) estimate the median and the mean.
3. The population in thousands of 42 districts were recorded as follows:
- 155 182 143 88 89 162 190

95	172	132	104	125	116	142
120	116	136	156	159	170	182
128	160	158	172	178	90	86
98	88	95	118	132	140	153
112	170	163	171	185	189	180

Make a grouped frequency distribution table with the first class as 85–94 and from it:

- (a) find the modal class.

- (b) estimate the mean and the median.
4. In a Kiswahili test, 40 students scored the following marks:

43	39	59	56	58	63	71	40
72	66	47	38	51	50	61	64
32	78	29	32	45	80	70	57
52	46	45	39	58	72	41	55
56	53	66	63	61	46	82	64

Group the above data using class intervals of size 5 and 25–29 as the

first class. Hence, estimate the mean and the median. State also the modal class.

In questions 5 to 8, estimate the mean and the median, and state the modal class.

5. Masses of fish in kilogrammes caught by fishermen in one day.

Table 18.6

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Mass in kg	0– 4	5– 9	10– 14	15– 19	20– 24	25– 29	3 3
No. of fish	2	6	20	12	10	5	

0

4

6

6. The number of words which could be read per minute by 80 candidates in an examination.

Table 18.7

Words read	0– 9	10– 19	20– 29	30– 39	40– 49	50– 59
Frequency	20	5	5	6	10	17

7. The diameters, to the nearest millimetre, of 78 bolts.

Table 18.8

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Diameter (mm)	25– 26	27– 28	29– 30	31– 32	33– 34	3 3
No. of bolts	2	4	7	18	20	1

5

8. The amount in shillings, of pocket money given to students of a particular school.

Table 18.9

Pocket money in Ksh	100– 190	200– 290	300– 390	400– 490	500 59 0
No. Of students	5	14	40	38	24

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9. The information below represents scores of 40 candidates in an aptitude test:

11.3 23.6 41.2 25.6 50.0 51.2 55.3 47.2 21.2
21.6 11.5 53.1 35.6 39.2 39.9 26.1 22.4 23.5
42.1 57.2 11.5 31.3 11.7 27.0 11.3 11.8 54.2
33.4 37.2 39.6 56.7 47.2

By using groups 10–14, 15–19, 20–
24,:

(a) determine the modal class.

(b) estimate the mean and the median.

10. The data below represents times in seconds recorded in the heats of a 100 m race during an athletics meeting:

14.7 13.8 14.6 15.2 15.0 14.5 15.0 14.9 11.8
14.0 12.7 13.2 15.0 15.0 15.2 15.7 14.5 15.5
11.9 12.5 15.1 15.2 15.4 12.1
14.8 14.9 15.6 15.0 13.2 13.4 14.5 15.0

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12.0 14.7 12.4 13.4 13.6 12.8 13.3 11.8

By using groups of 11.5–11.9, 12.0–12.4 ..., make a frequency distribution table for the above data and hence:

(a) determine the modal class.

(b) estimate the mean and median.

18.5: Representation of Data

The main purpose of representation of statistical data is to make collected data more easily understood. Methods commonly used in representation are:

- (i) Bar graphs.
- (ii) Pictograms.
- (iii) Pie charts.
- (iv) Line graphs.
- (v) Histograms.
- (vi) Frequency polygons.

Bar Graphs

A bar graph consists of a number of spaced rectangles which generally have

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their major axes vertical. Bars are of uniform width. The axes must always be labelled and scales indicated. For example, Kamau's performance in five subjects was as follows:

Maths 80

English 75

Kiswahili 60

Biology 70

Geography 75

This information can be represented on a bar graph as shown below:

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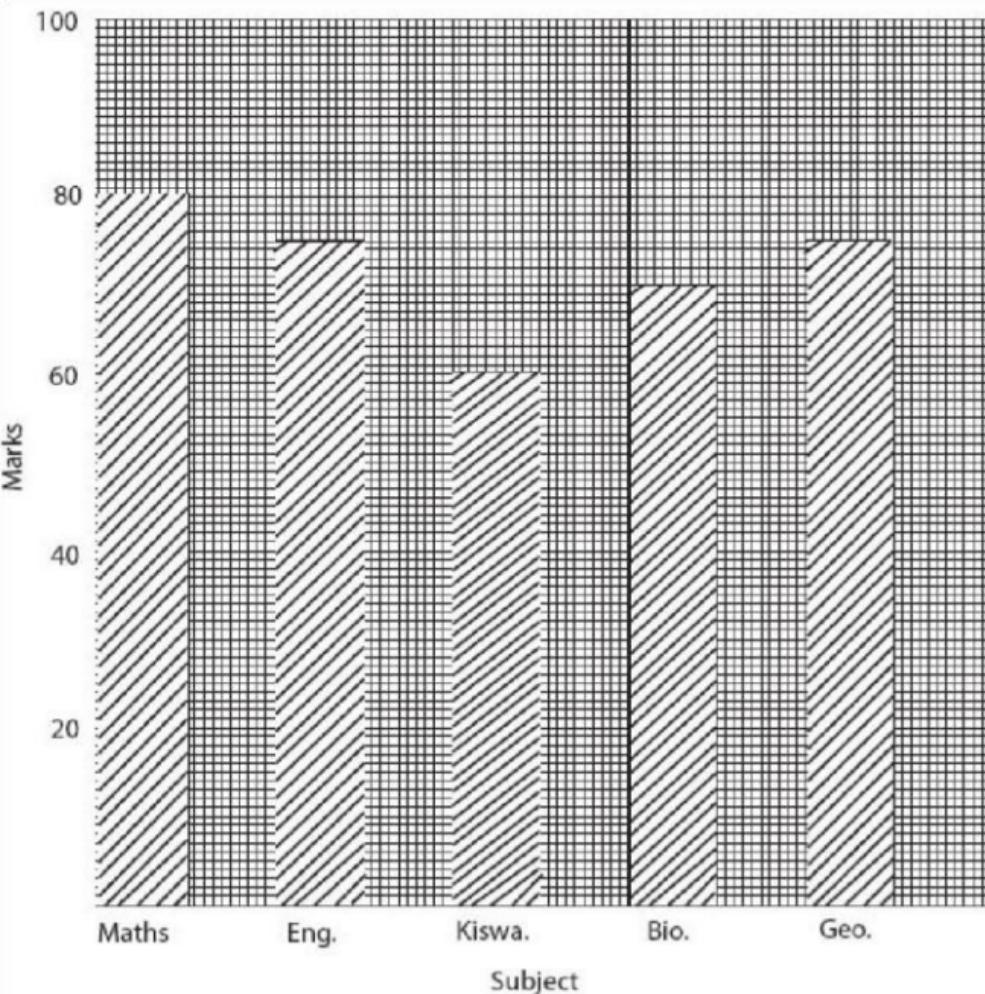


Fig. 18.1

Pictograms

In a pictogram, data is represented using pictures. Each picture represents a given quantity. Consider the following table which shows the different makes of cars imported over a certain period:

Toyota 2 000

Nissan 3 500

Peugeot 1 500

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This information can be represented by a pictogram using the following key:

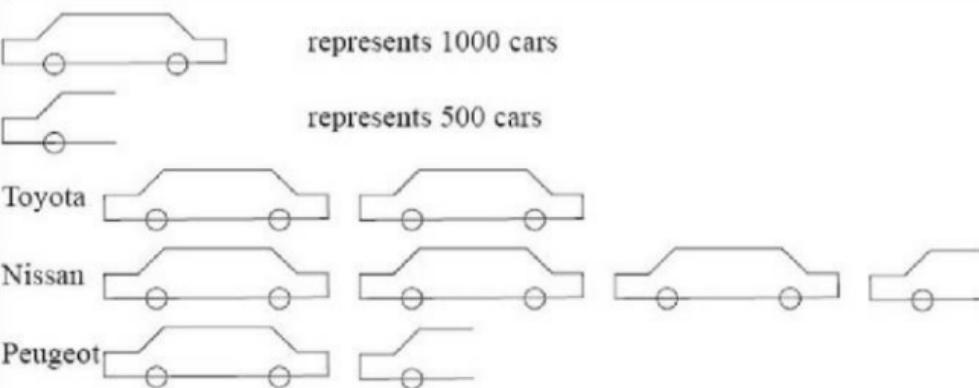


Fig. 18.2
Pie Chart

A pie chart is a circle divided into various sectors. Each sector represents a certain quantity of the item being considered.

The size of the sector is proportional to the quantity it represents.

Consider the total population of animals in a farm given as 1 800. Out of these 1 200 are chicken, 200 cows, 300 goats and 100 ducks. This information can be represented in a pie chart as follows:

Chicken

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Angle = $\frac{\text{Number of chickens}}{\text{total population}} \times 360$

$$= \frac{1\,200}{1\,800} \times 360$$
$$= 240^\circ$$

Cows

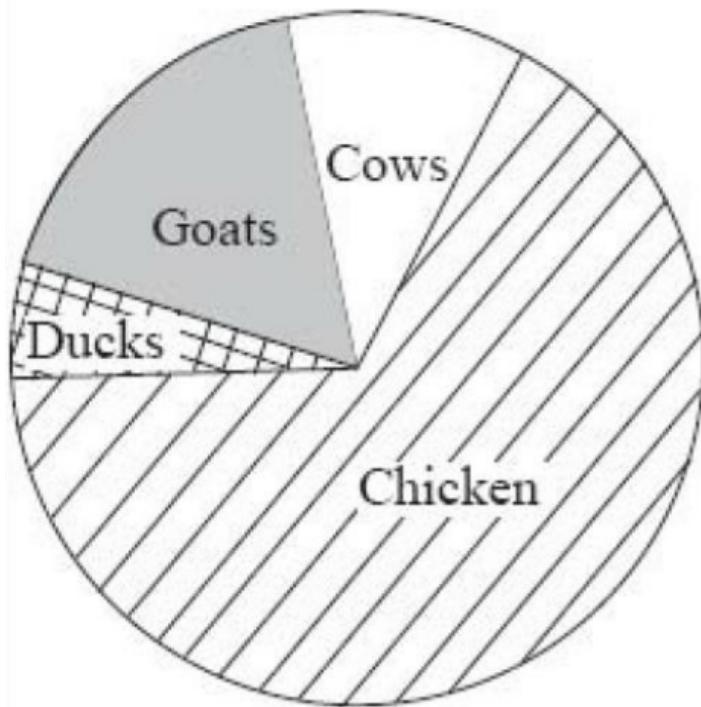
$$\begin{aligned}\text{Angle} &= \frac{200}{1\,800} \times 360 \\ &= 40^\circ\end{aligned}$$

Goats

$$\begin{aligned}\text{Angle} &= \frac{300}{1\,800} \times 360 \\ &= 60^\circ\end{aligned}$$

Ducks

$$\text{Angle} = \frac{100}{1800} \times 360 \\ = 20^\circ$$



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Fig. 18.3

Line Graphs

In line graphs, data is represented using lines. Examples of line graphs are a graph showing temperature of a patient at various times of the day or a travel graph.

Consider the following table which shows the temperature of a patient at different times of the day.

Table 18.10

Time	8 am	9 am	10 am	11 am	12 noon	p
Temp. (°C)	35.6	36.4	37.0	37.2	36.8	3

1

m

5

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The data can be represented with a line graph as shown below:

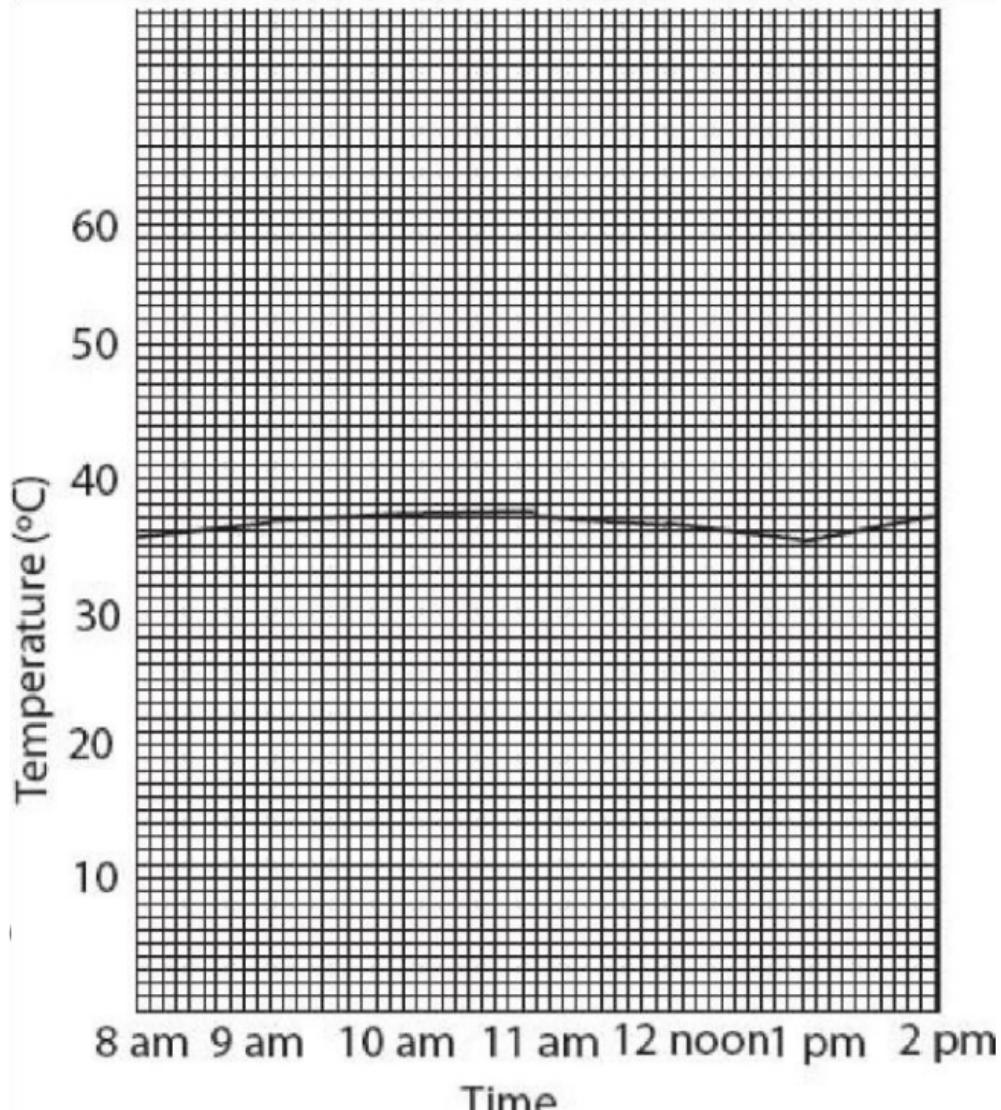


Fig. 18.4

Histograms

t

In his, the frequency in each class is represented by a rectangular bar whose area is proportional to the frequency. When the bars are of same width, the height of the rectangle is proportional to the frequency. Note that the bars are joined together.

For example, the following data represents masses to the nearest kilogramme of fish caught by a fisherman in a day:

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Table 18.11

fish							
------	--	--	--	--	--	--	--

The above data can be represented in a histogram as shown in figure 18.5.

Mass (kg)	5– 9	10– 14	15– 19	20– 24	25– 29	30– 34	3 3
No. of	6	20	12	10	5	6	

t

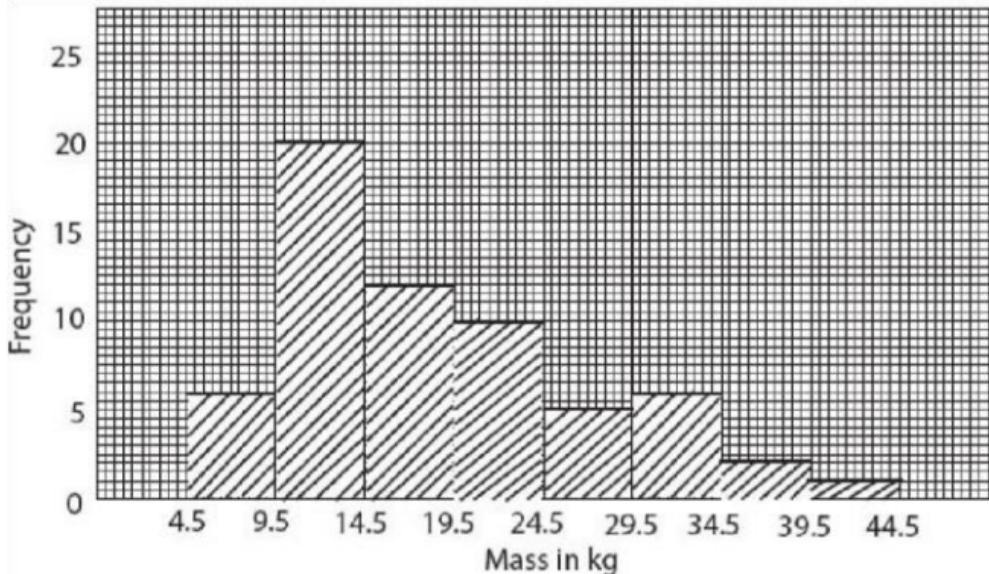


Fig. 18.5

Note:

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The class boundaries mark the boundaries of the rectangular bars in the histogram.

Histograms can also be drawn when the class intervals are not the same. For example, the following table shows the marks obtained in a maths test:

Table 18.12

Marks	10– 14	15– 24	25– 29	30– 44
-------	-----------	-----------	-----------	-----------

t

No. of students	5	16	4	15
-----------------	---	----	---	----

The above information can be represented in a histogram as shown below;

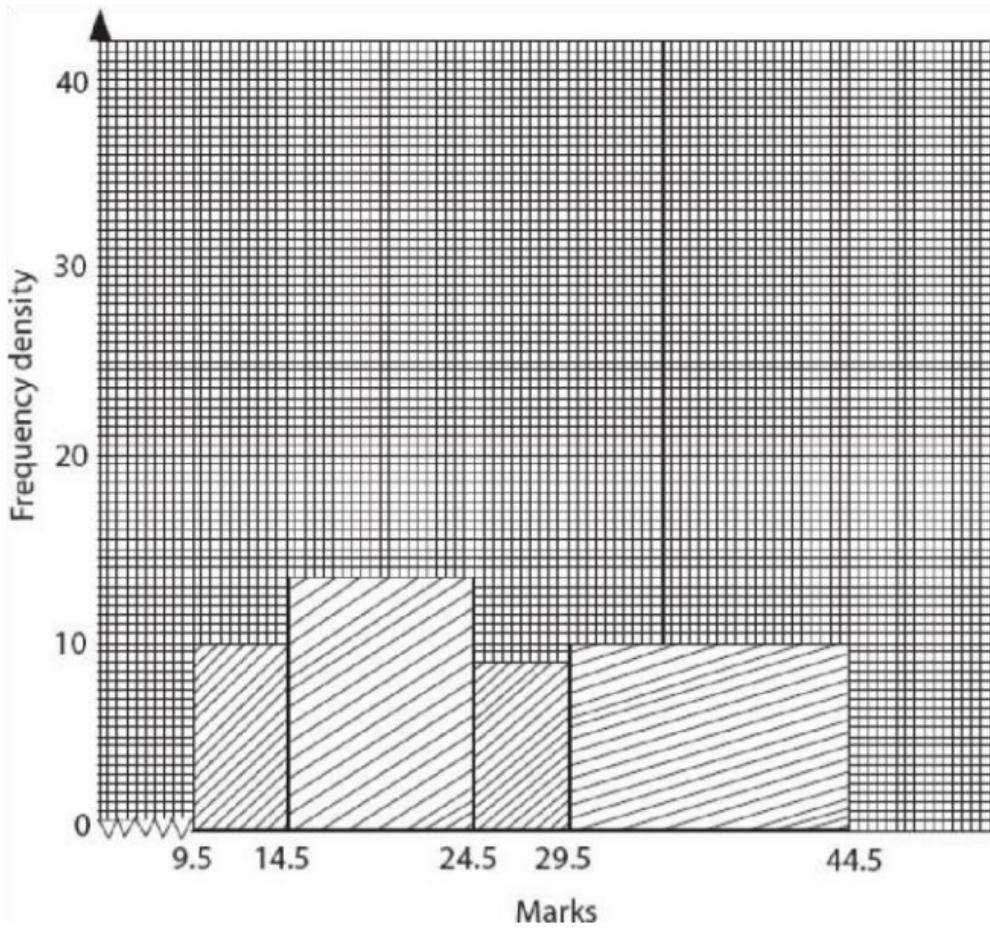


Fig. 18.6

t

The height of each bar in this case is the

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

ot

N e:

Histograms can also be drawn with heights other than the frequency density as long as the area of each bar is proportional to the frequency of that class.

Frequency Polygon

When the midpoints of the tops of the bars in a histogram are joined with straight lines, the resulting graph is called a frequency polygon. However, it might not be necessary to draw bars to get a frequency polygon. A frequency polygon can be obtained by plotting the

o

frequency against the midpoints of the classes.

Figure 18.7 represents a frequency polygon for the data in table 18.11.

O

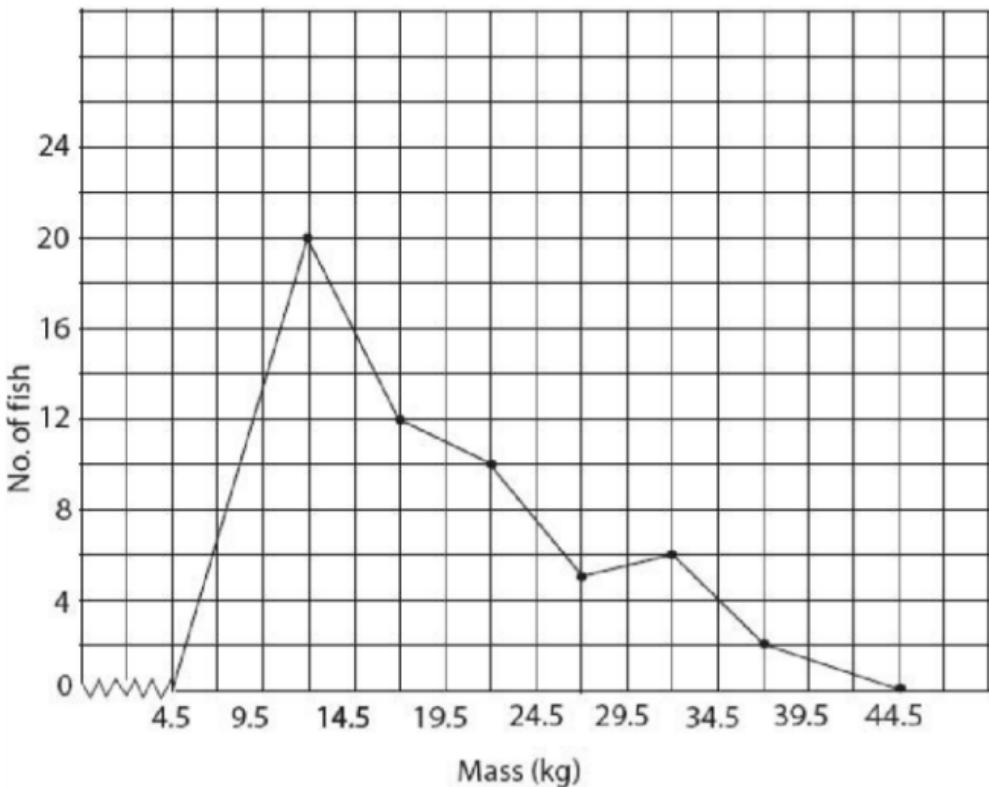


Fig. 18.7

Note:

o

The frequency polygon is joined to the xaxis.

More than one frequency polygon can be drawn on the same axes for the purpose of comparison. For example, in an examination, the marks in Mathematics and Physics were grouped as given below. Draw frequency polygons to represent the data as in figure 18.8.

Table 18.13

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O

	Marks	10– 19	20– 29	30– 39
Frequency	Mathematics	2	6	7
	Physics	3	9	10

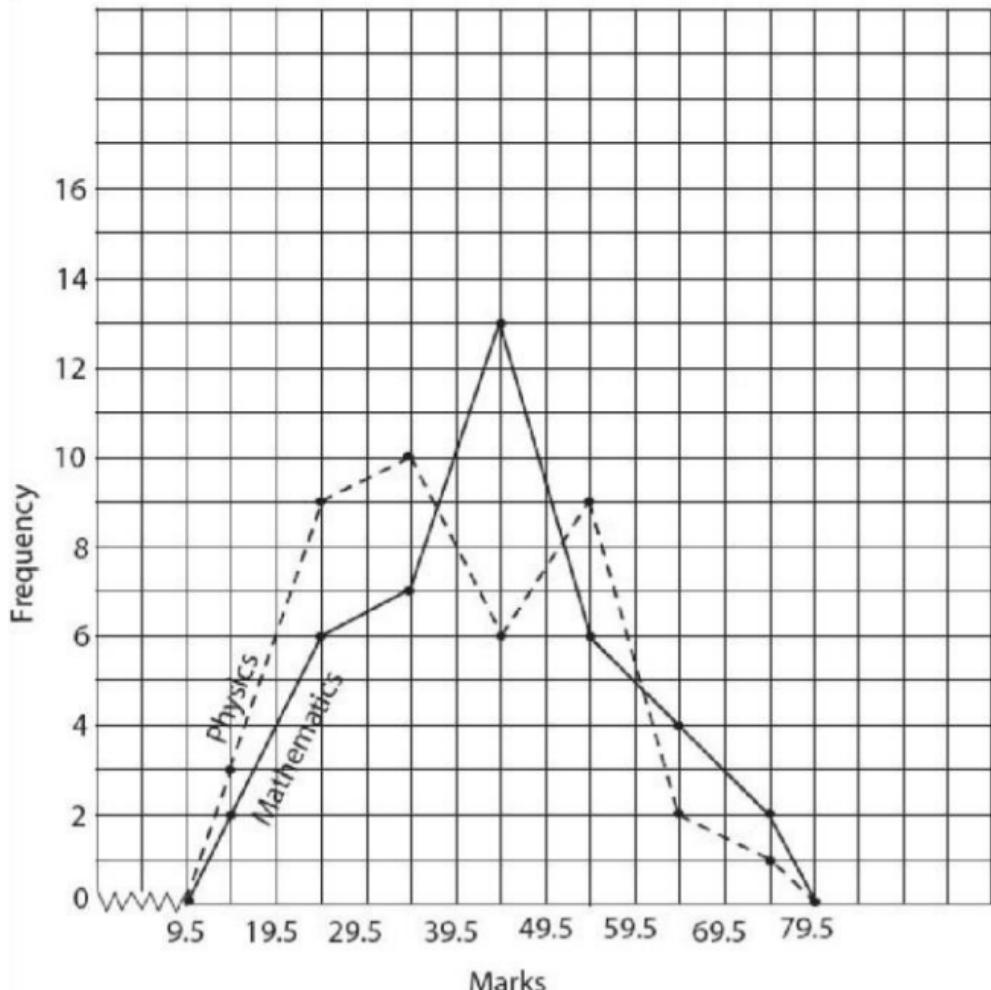


Fig. 18.8

From the above frequency polygon, notice that:

- (i) there were more students in Physics than in Mathematics with less than 39.5 marks.
- (ii) there were more students in Mathematics than in Physics with more than 39.5 marks. This implies that more students had relatively higher marks in Mathematics than in Physics.

From the two observations, we can conclude that the performance in

Mathematics was better than that in Physics.

Exercise 18.3

1. A 24-hectare farm is divided as follows:

Coffee	4 ha
Grass	3 ha
Maize	7 ha
Bananas	5 ha

Homestead 0.5 ha

Vegetables and paths 4.5 ha

Show the above information in a pie chart.

2. A test given to 40 students produced the following results:
 - 5 students got grade A.
 - 12 students got grade B.
 - 15 students got grade C.
 - 5 students grade D.
 - 3 students got grade E.

- (a) Show the above information on a pie chart.
- (b) Find the percentage which represents:
- (i) those with grade A.
 - (ii) those with grade C and D.

3. The following table shows the number of trees planted in a certain farm:

Year	1998	1999	2000	2001	2002
No. of trees	7 400	11 200	10 700	5 600	9 800

Represent the information on a bar graph.

4. The data below shows the population of Matunya Secondary School.

Year	1996	1997	1998	1999	200
No. of students	200	300	275	400	35 0

Draw a pictogram to represent this information.

5. The table below shows the importation of vehicles (in thousands) for the year 1994 to 2002.

Year	1994	1995	1996	1997	1998
No of vehicles (in thousands)	15	24	29	42	5

Draw a line graph to represent the information.

6. The heights, to the nearest centimetre, of 20 boys and 20 girls were measured and recorded as follows:

Boys:	119	122	123	128
	129	130	131	132
	132	135	136	137
	138	140	142	145
	150	152	154	160
Girls:	116	120	121	125

Compiled by Lec Rakdfha :0770112040

126	127	129	130
130	132	135	135
136	138	141	144
148	150	152	159

- (a) Construct a histogram for each of the above data, using 115 – 119 as the first class for both boys and girls.
- (b) Construct frequency polygons of the data on the same axes.
7. The table below shows the number of letters collected from the post

office by a school driver during a school year:

Letters per day	6–10	11–15	16–20	21–25	26–30	31–35
Frequency	5	19	21	23	25	2

5

7

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Draw a frequency polygon to represent the data.

8. The number of pupils present in a class for 20 school days were:

43 38 39 37 33

31 28 35 27 32

29 30 34 44 24

32 34 41 36 38

- (a) Make a grouped frequency table with 24–26 as the first class.

- (b) Draw a bar-graph to represent the information.
- (c) Draw a histogram to represent the information.
9. Pupils in a class on a certain day school were asked how they travelled to school. They gave the following information:

Mean of travel	Walk	Cycle	Matatu	Bus
----------------	------	-------	--------	-----

No. of pupils	17	5	18	2
---------------	----	---	----	---

Draw a pie-chart to represent this data.

10. In a day school, the distances in kilometres travelled to school by some pupils were recorded as below:

Distance in km	0– 2	3– 5	6– 8	9– 11	12– 14	15– 17	1
No. of pupils	20	6	9	8	7	4	

Draw a histogram to represent this data.

11. The data below shows the total number of KCSE subjects passed in two schools:

No. of subjects		0	1	2	3	4
No. of candidates	Schoool A	1	1	1	10	12
	School B	3	2	3	5	6

1

- (a) On the same axes, draw frequency polygons for the two schools.
- (b) If anyone with 6 or more passes qualified for higher education,

how many candidates qualified in each school?

- (c) If anyone with 4 or more passes could be admitted to a training college, how many qualified for a place in a training college from each school?

Chapter 19

ANGLE

PROPERTIES OF A

CIRCLE

19.1: Arc, Chord and Segment of a

Circle

Arc

Figure 19.1 shows circle KLMN with centre O. Any part of the circumference of a circle is called an arc. Arc LMN is less than half the circumference of the circle and is therefore the minor arc. The remaining part, LKN, which is greater than half the circumference of the circle, is the major arc.

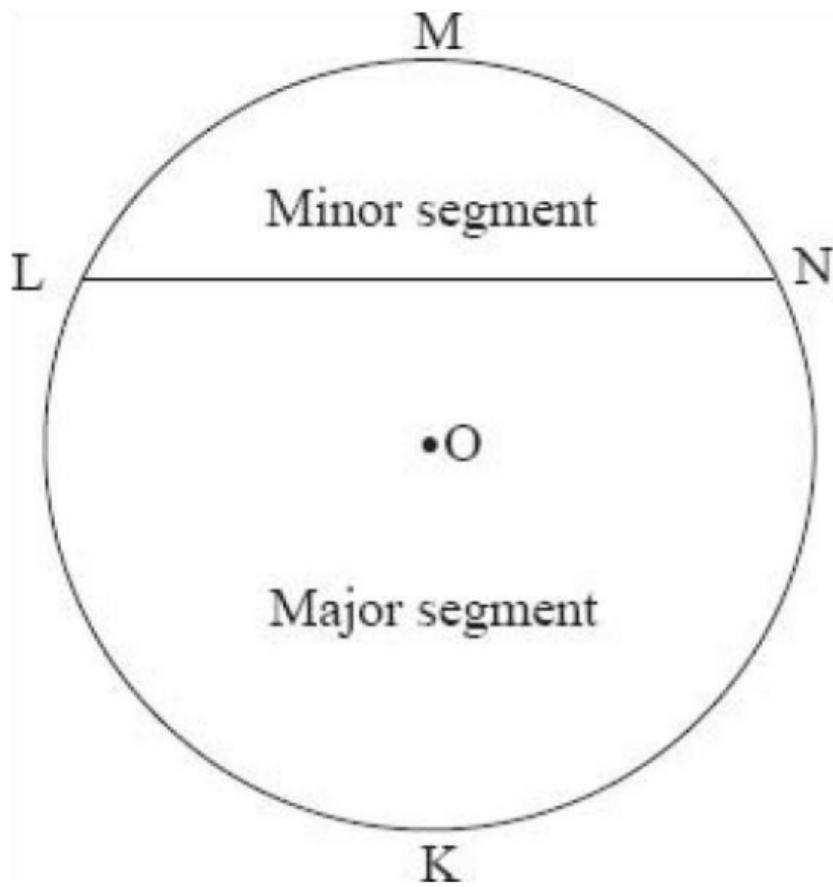


Fig. 19.1

Chord

A line joining any two points on the circumference is called a chord. For example, LN in figure 19.1 is a chord. The diameter of a circle is its longest chord.

Segment

Any chord divides a circle into two regions called segments. The smaller one is called the minor segment and the

larger one is the major segment, see figure 19.1.

19.2: Angle at the Centre and Angle at the Circumference

A chord or an arc can subtend an angle at any given point. For example, in figure 19.2, arc NML subtends angle NCL at the circumference and NOL at the centre of the circle.

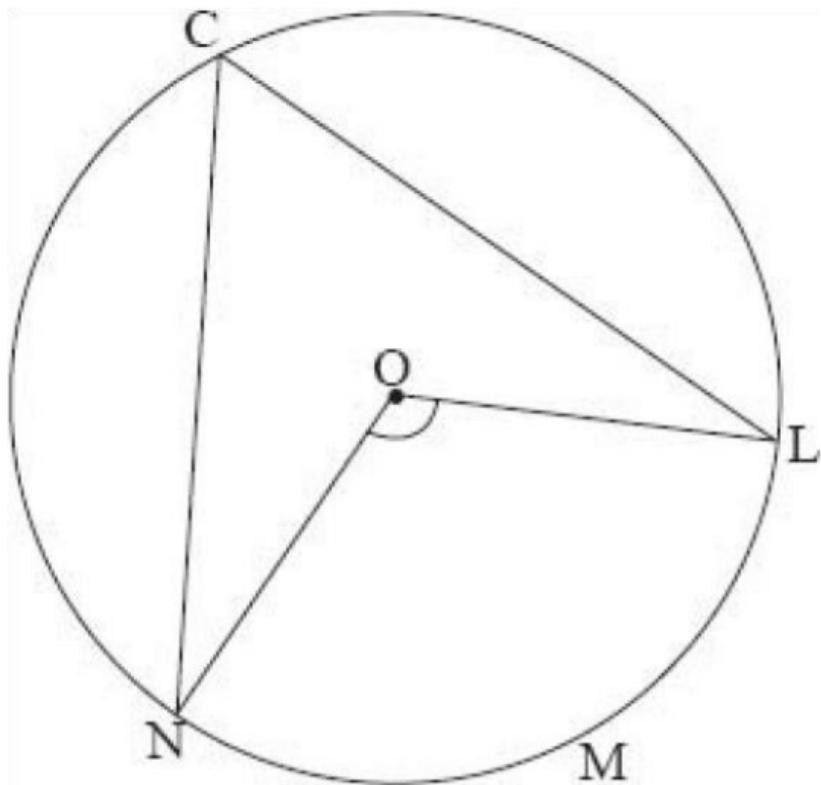


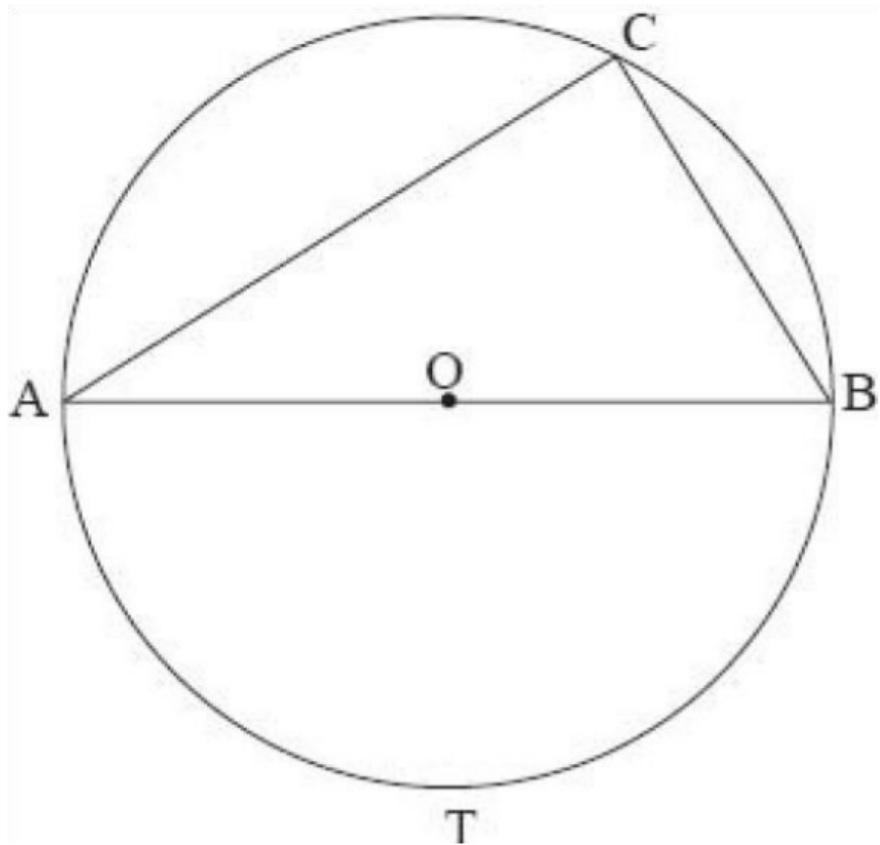
Fig. 19.2

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Draw a circle of any reasonable radius with an arc subtending an angle at the centre and several angles at the circumference. Compare the sizes of angles at the circumference and at the centre.

Note that the angle which an arc subtends at the centre is twice that which it subtends at any point on the circumference of the circle.

In figure 19.3, AB is the diameter of the circle with centre O.



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Fig. 19.3

The arc ATB subtends an angle of 180° at the centre of the circle. The same arc subtends $\angle ACB$ at the circumference.

But, the angle subtended at the centre, is twice the angle subtended at the circumference.

Therefore,

$$\angle AOB = 2 \times \angle ACB$$

$$180 = 2 \angle ACB$$

$$\angle ACB = \frac{180^\circ}{2} = 90^\circ$$

Hence, the diameter of a circle subtends a right angle at any point on the circumference of the circle. Conversely, if a chord subtends a right angle at any point on the circumference of the circle, then the chord must be a diameter of the circle.

Figure 19.4 is a circle centre O. L, M, N, P, Q, R_1 and S are points on the circumference.

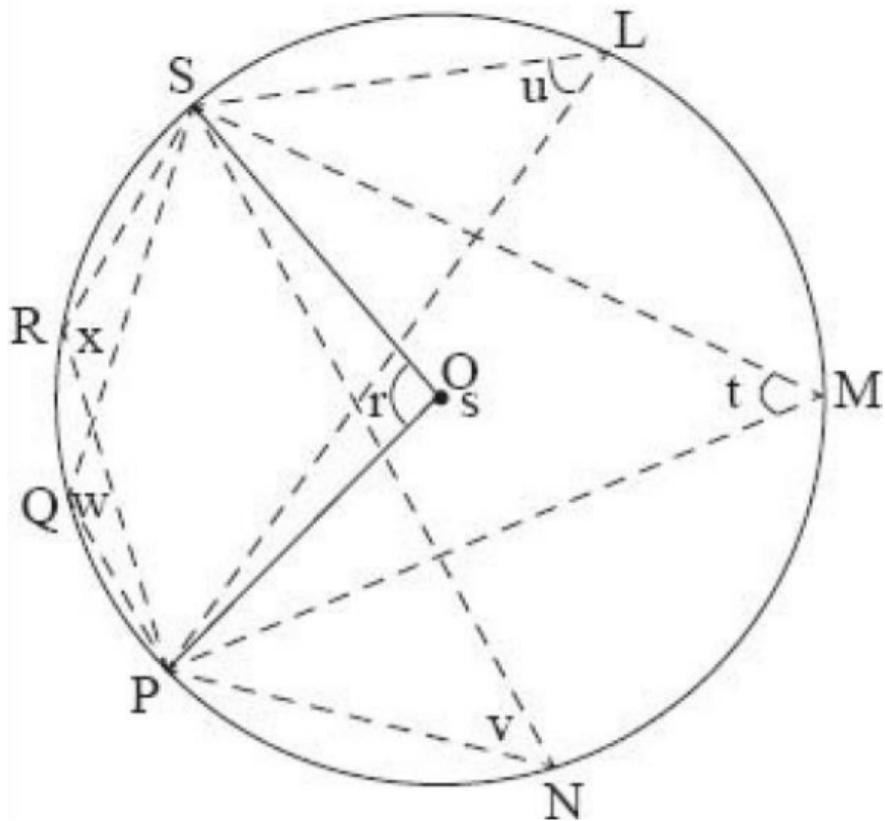


Fig. 19.4

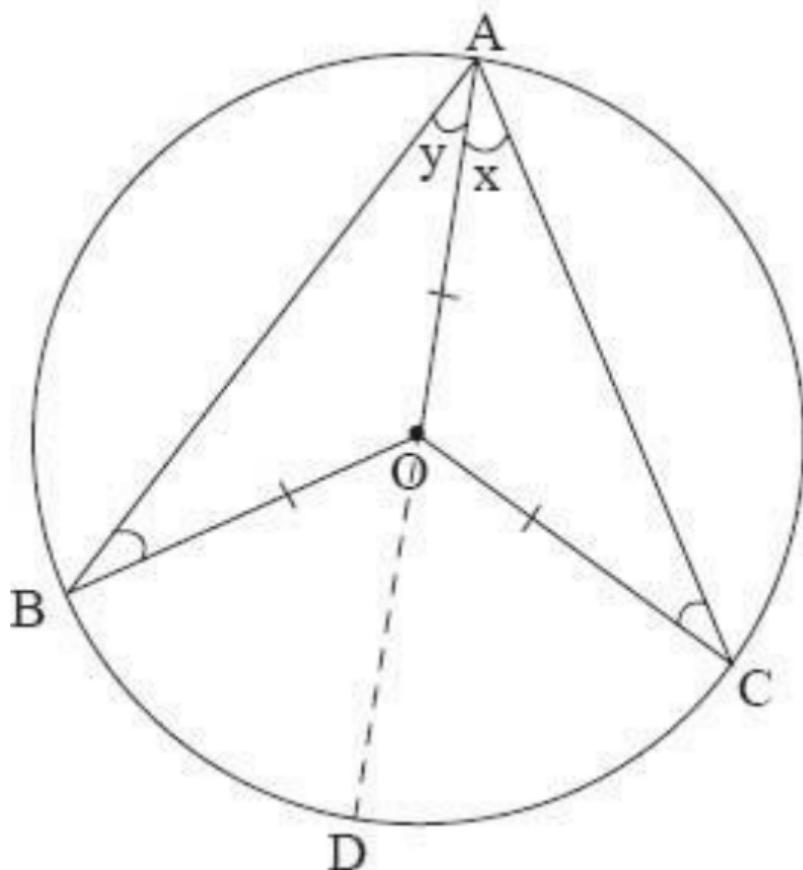
- (i) Name the arcs that subtend the angles marked t, u, v, w and x on the circumference.
- (ii) Name the arcs that subtend the angles marked r and s at the centre O.
- (iii) If $r = 120^\circ$, find u, t, v, w and x.

Example 1

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In the figure 19.5, A, B, D and C are points on the circumference of a circle centre O.

Angles $\angle BAO = y^\circ$ and $\angle CAO = x^\circ$. Use x and y to show that $\angle BOC = 2 \angle BAC$



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Fig. 19.5

Solution

In the figure, $\angle ABO = \angle BAO = y$ (base angles of an isosceles $\triangle ABO$).

$\angle ACO = \angle CAO = x$ (base angles of isosceles $\triangle ACO$)

$\angle BOD = \angle ABO + \angle BAO = 2y$ (exterior angle of a triangle)

$\angle COD = \angle ACO + \angle CAO = 2x$ (exterior angle of a triangle) $\angle BOC =$

$$2y + 2x = 2(x + y)$$

But $BAC = x + y$

Therefore, $BOC = 2 BAC$

Example 2

Find the values of x , y and z in figure 19.6
if O is the centre of the circle and

$$\angle ABC = 30^\circ$$

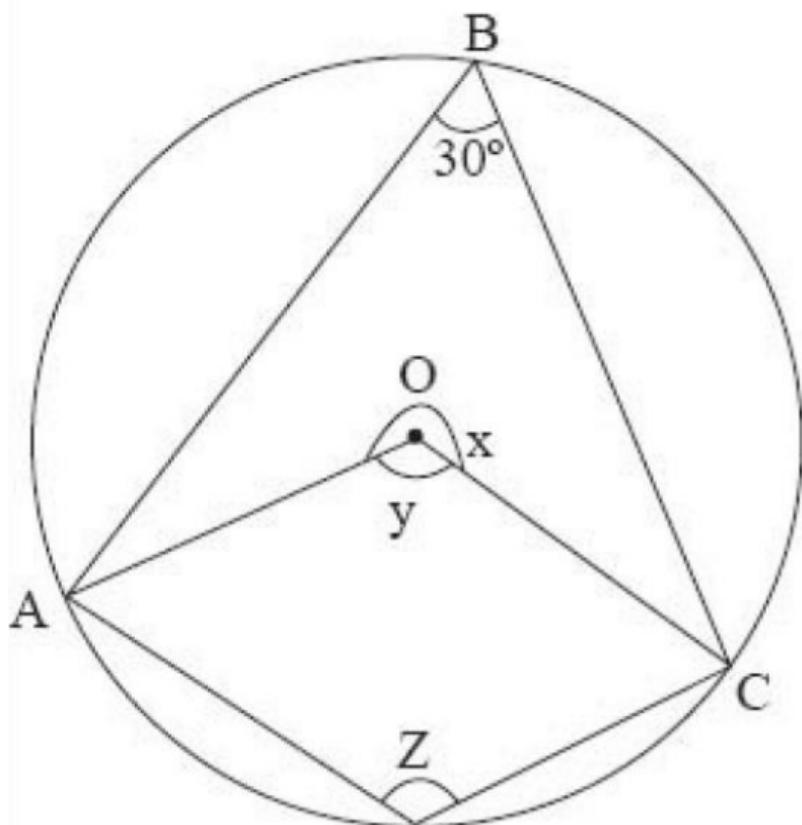


Fig. 19.6

Solution y

$$= 30 \times 2$$

$= 60^\circ$ (angle at the centre is twice the angle on the circumference)

$$x = 360^\circ - 60^\circ$$

$$= 300^\circ \quad z$$

$$= \frac{1}{2} x$$

$$= \frac{1}{2} (300)^\circ$$

$$= 150^\circ$$

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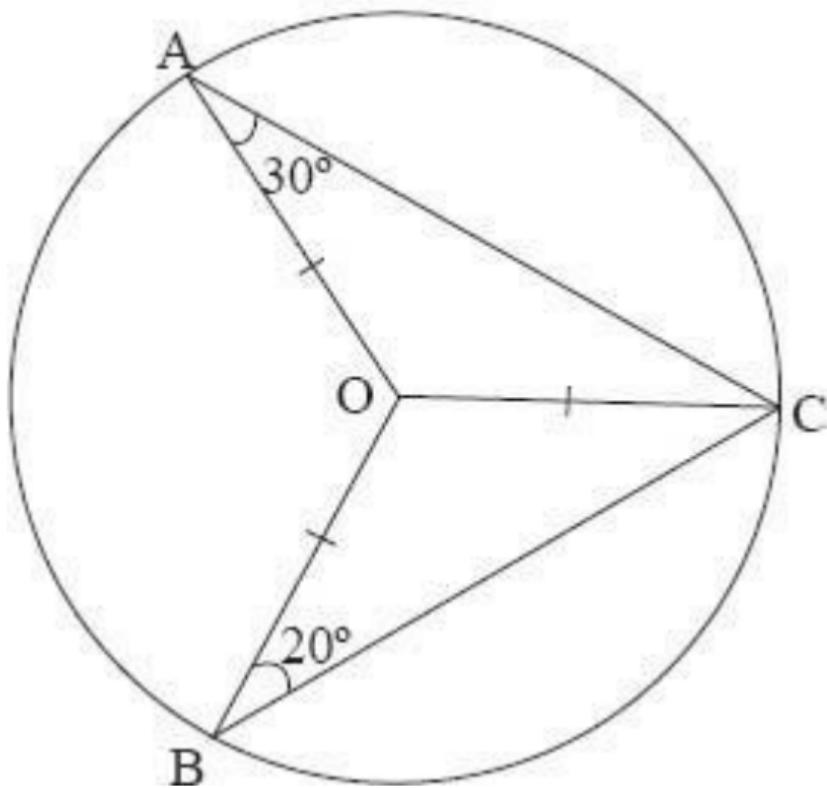
Exercise 19.1

1. Copy and complete the table below.
All angles are subtended by the same arc.

Angle at the centre	80°	64°		122°	
Angle on the circumference		32°	42.5°		2

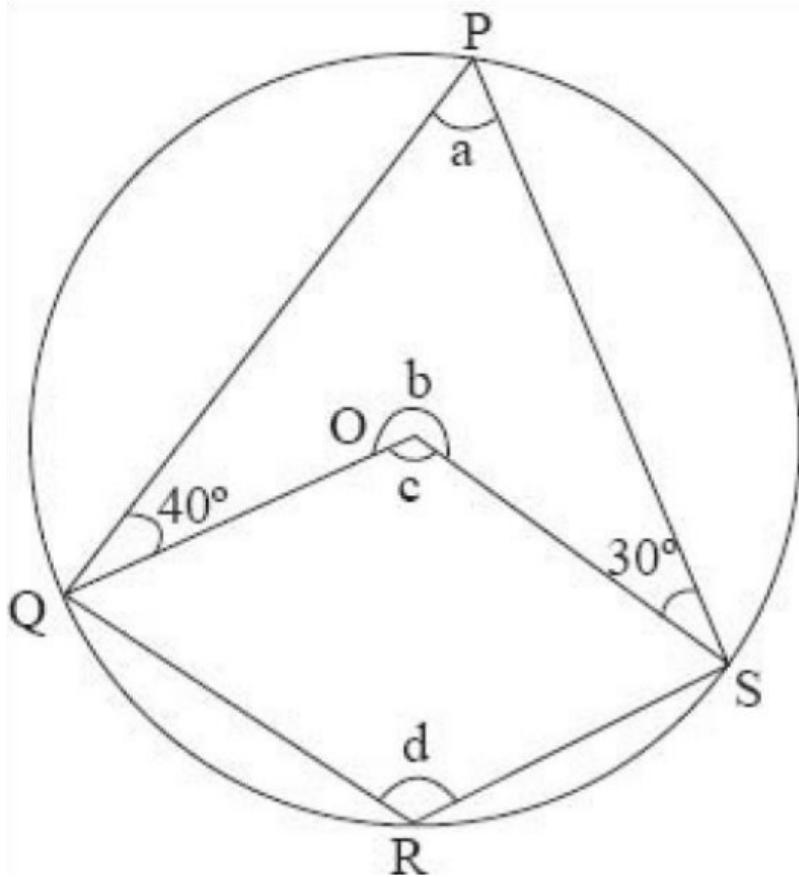
2

2. In the figure below, O is the centre of the circle. Find obtuse AOB.



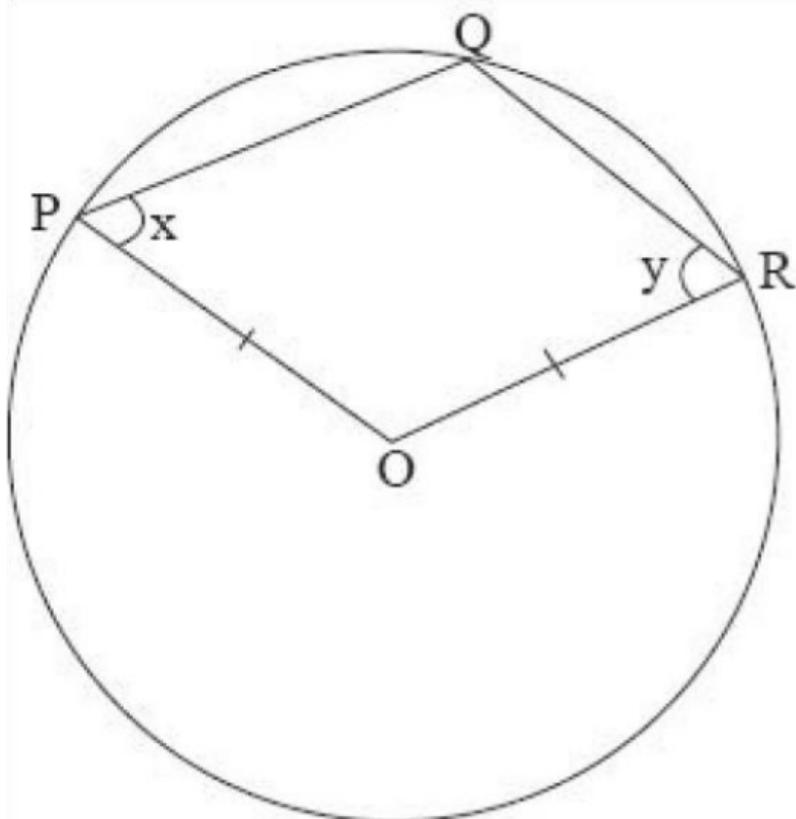
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3. The figure below shows a circle with the centre at O:



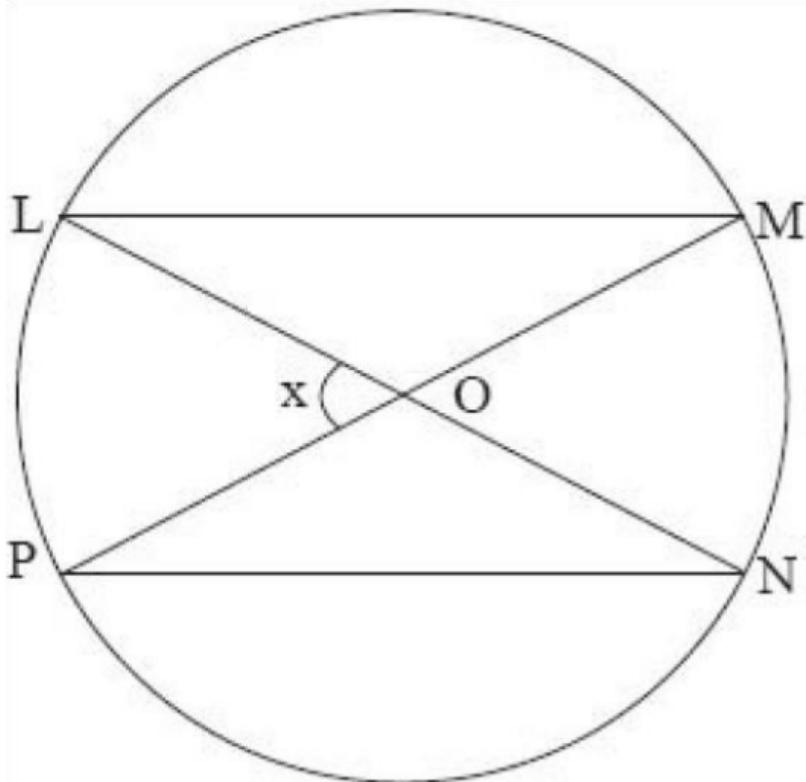
Find the values of a, b, c and d if
 $PQO = 40^\circ$ and $PSO = 30^\circ$

4. In the figure below, O is the centre of the circle. Show that the reflex angle $POR = 2 PQR$.



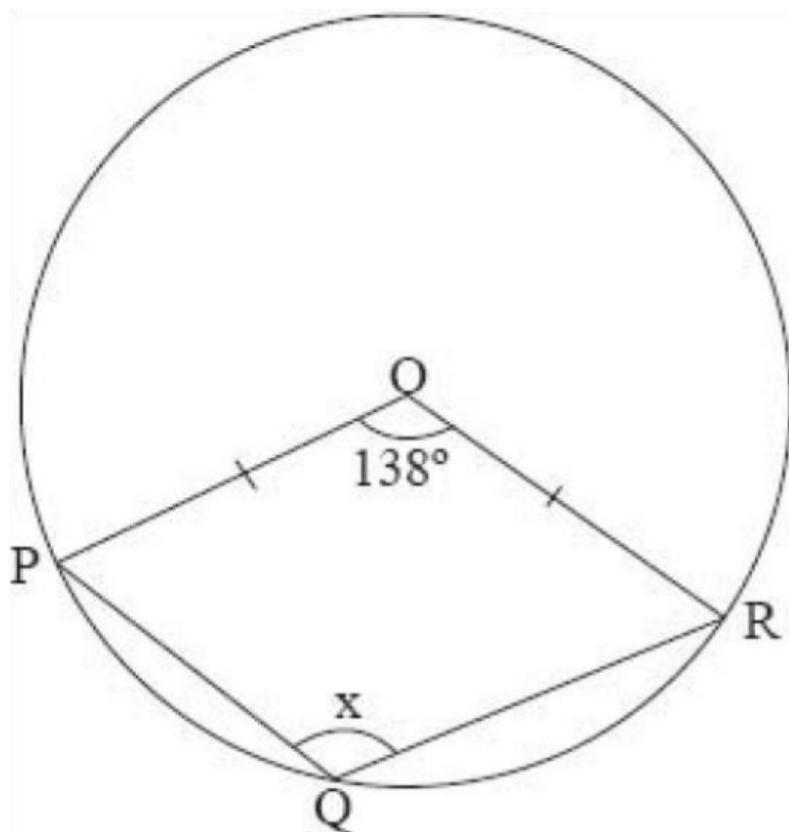
5. In the figure below, O is the centre of the circle. LN and PM are diameters.

Show that LM is parallel to PN:

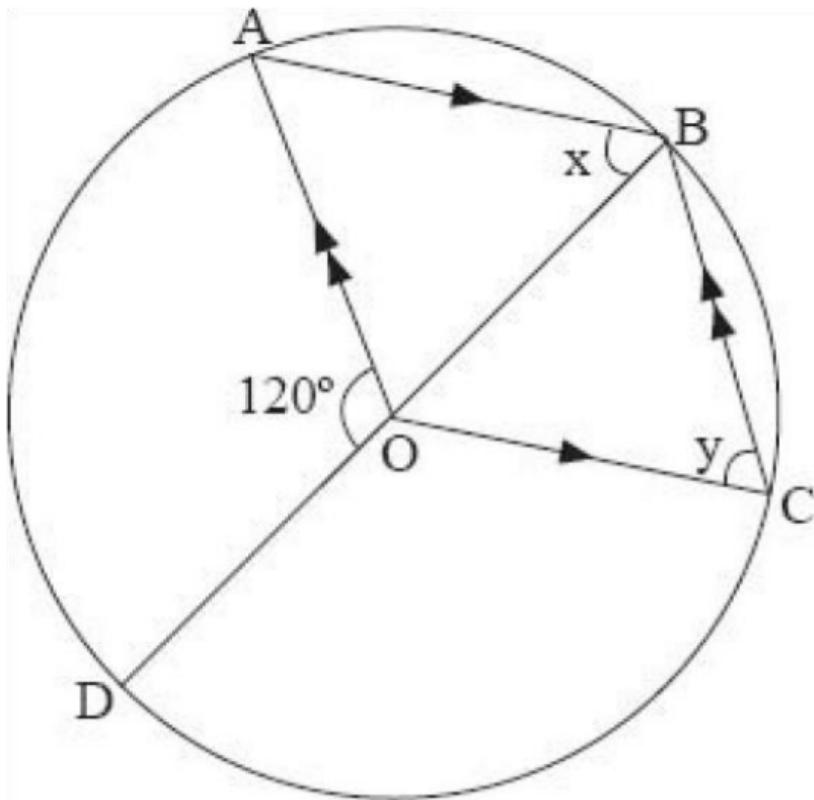


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6. In the figure below, O is the centre of the circle. Calculate x .



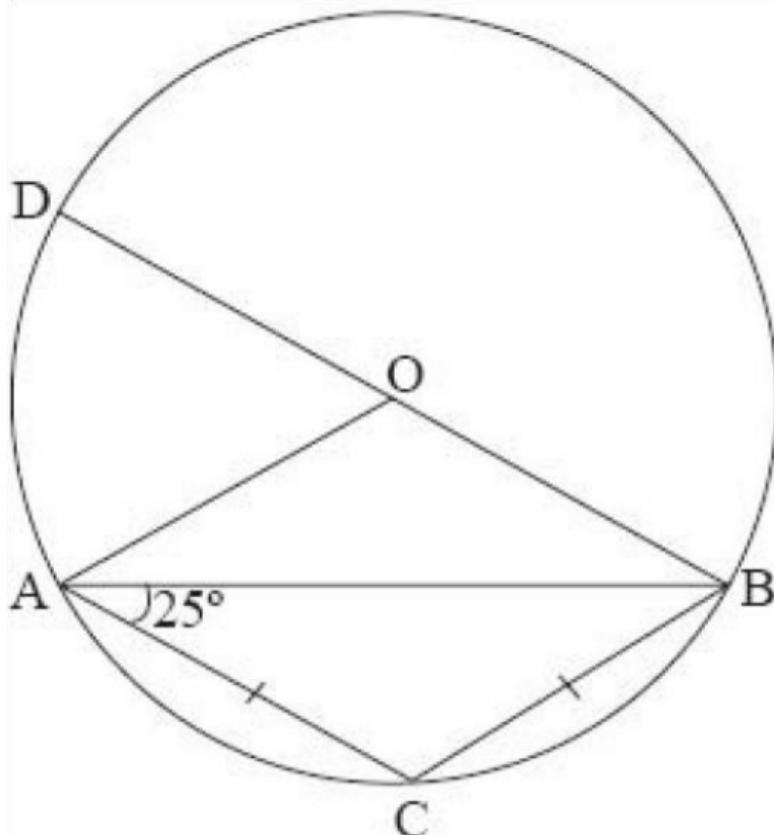
7. In the figure below, O is the centre of the circle and DOB is the diameter. Find x and y if ABCO is a rhombus and $AOD = 120^\circ$.



8. In the figure below, O is the centre of the circle. BOD is the diameter, $AC = BC$ and $BAC = 25^\circ$. Find the size of:

(a) AOD.

(b) BAO.

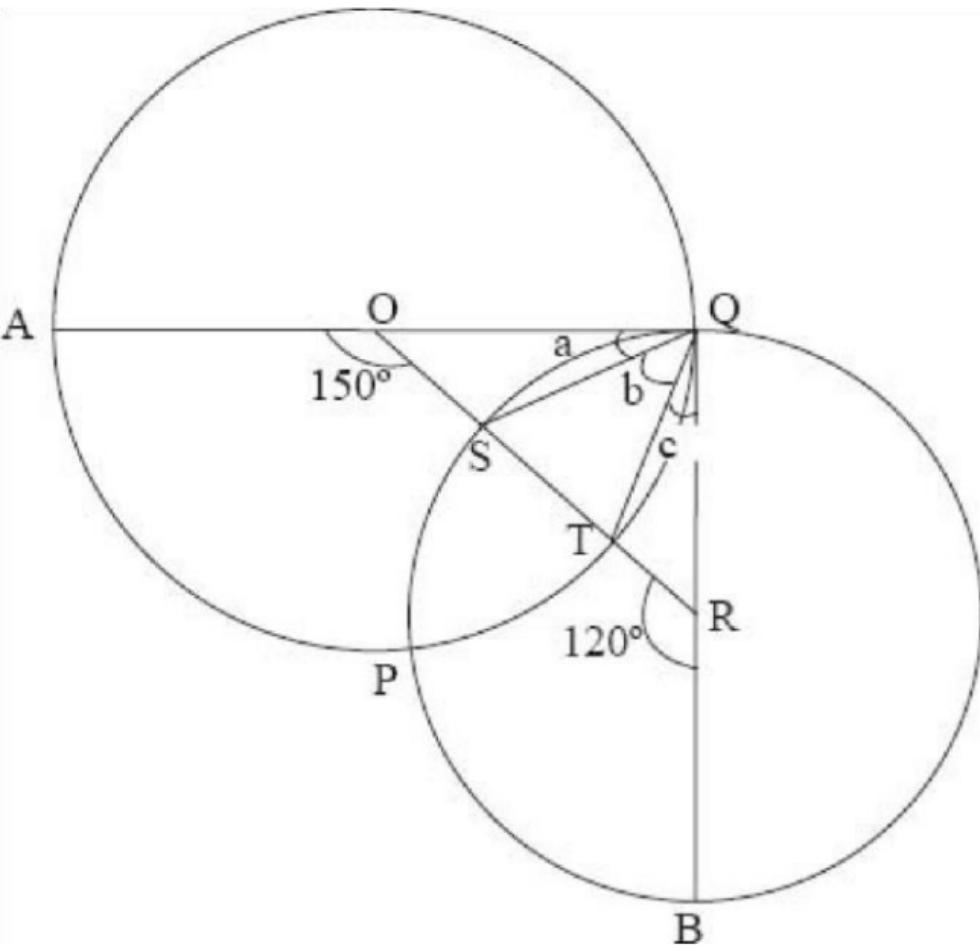


9. AB and CD are intersecting chords of a circle with centre O. If AB is the diameter of the circle, show that:

(a) $AOC = 2 ABC$.

(b) $ADB = 90^\circ$

10. Two circles with centres O and R intersect at P and Q as shown in the figure below. OR cuts the circles at S and T respectively. If $AOR = 150^\circ$ and $BRO = 120^\circ$, find a, b and c.

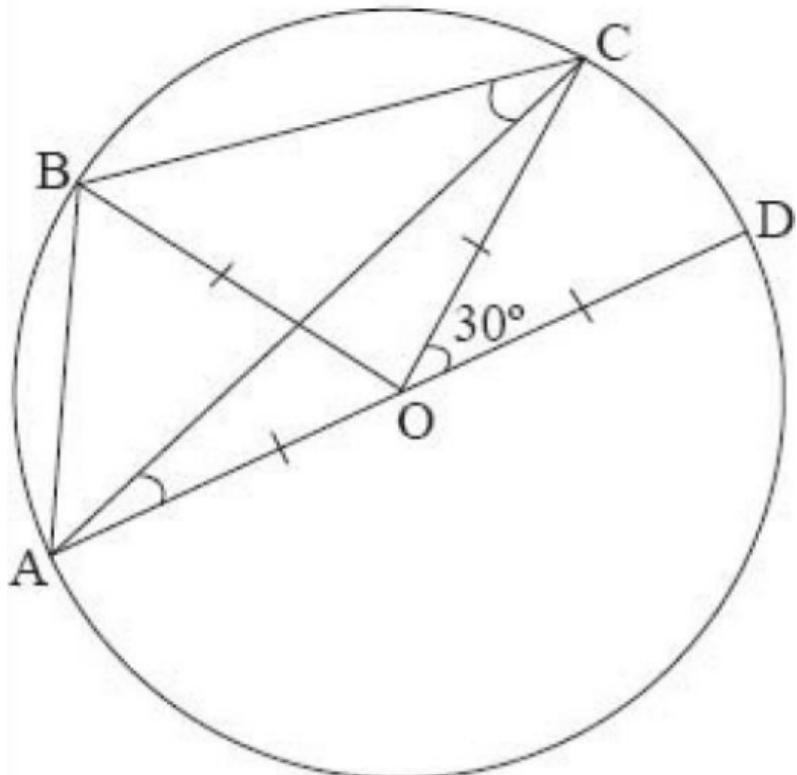


11. In a circle KLMN with centre O, the diagonals of quadrilateral KLMN intersect at R. $\angle LKM = 30^\circ$, $\angle LNK = 64^\circ$ and $\angle KRN = 82^\circ$. Find:
- (a) $\angle KNM$.
 - (b) $\angle LMN$.
 - (c) $\angle KLM$.
12. The figure below shows a circle ABCD, centre O. BO is perpendicular

to the diameter AOD and COD = 30° .
Calculate.

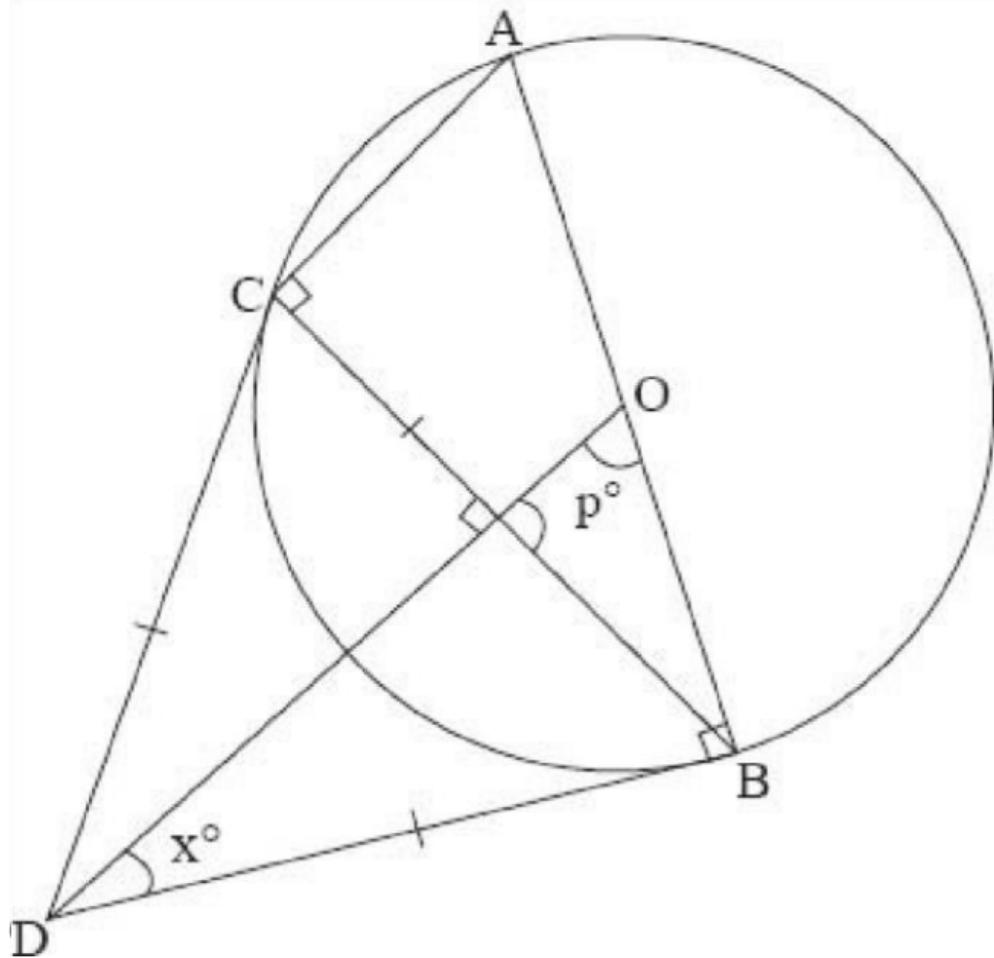
(a) BCA.

(b) CAD.



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13. ACB is a circle, centre O, with diameter AB perpendicular to BD as shown in the figure below. If BC = BD = CD and DO is perpendicular to BC, find p and x.



19.3: Angles in the Same Segment

In figure 19.7, chord PQ divides the circle into two segments.

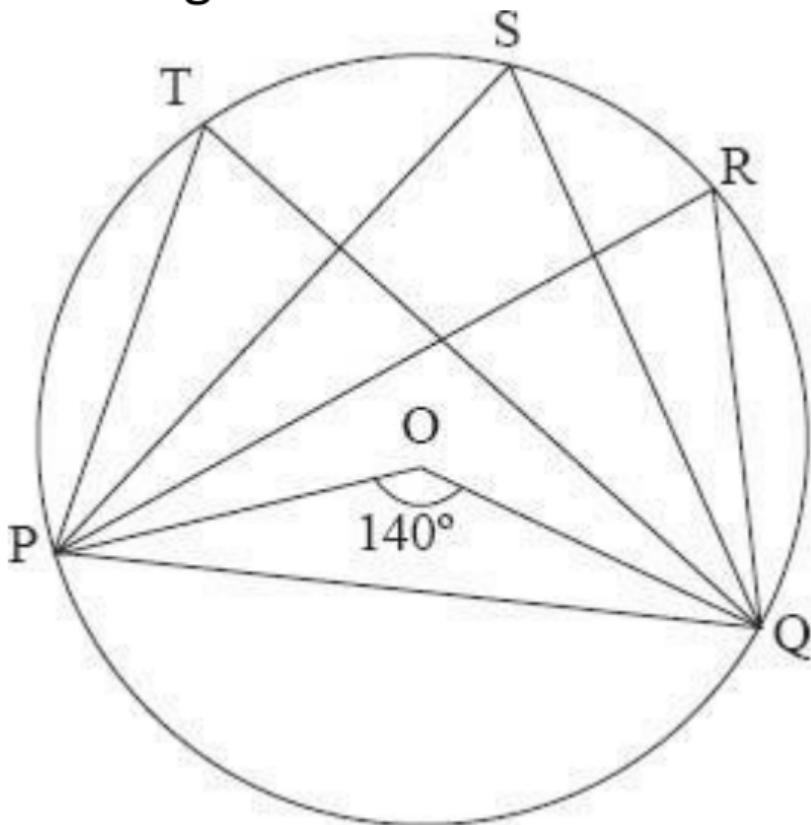


Fig. 19.7

The arc PQ subtends $\angle POQ$ at the centre and angles $\angle PTQ$, $\angle PSQ$ and $\angle PRQ$ on the circumference in the major segment.

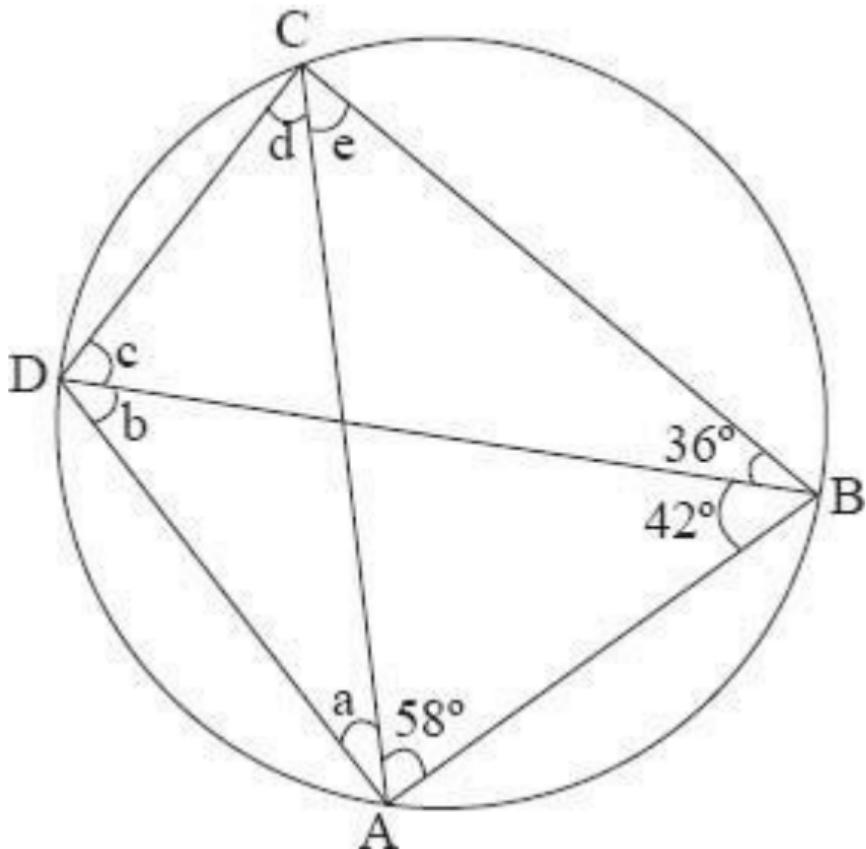
If $\angle POQ$ is 140° , then $\angle PSQ = \angle PRQ = \angle PTQ = 70^\circ$ (angle subtended at the centre is twice angle subtended at the circumference). In general, angles subtended on the circumference by the same arc in the same segment are equal.

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Equal arcs also subtend equal angles at the circumference.

Example 4

Calculate the value of a, b, c, d and e in figure 19.8.



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Fig. 19.8

Solution

a = 36° , since DAC and DBC are subtended by the same arc DC.

c = 58° , since CAB and CDB are subtended by the same arc BC.

d = 42° , since DBA and DCA are subtended by the same arc DA.

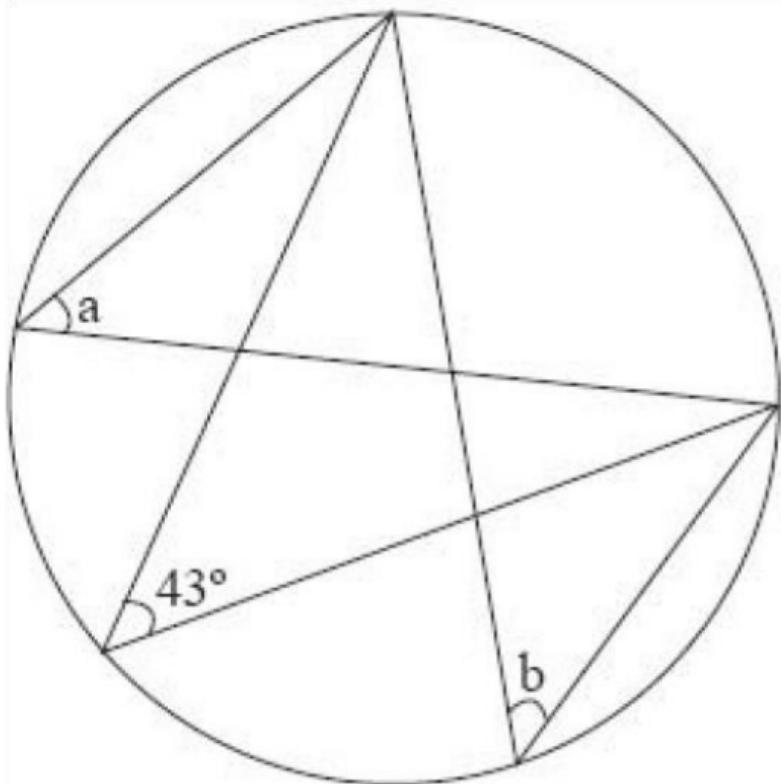
e + $58^\circ + 42^\circ + 36^\circ = 180^\circ$ (angle sum of $\triangle ABC$)
 $e + 136^\circ = 180^\circ$ $e = 44^\circ$

But $e = b$, since $\angle ADB$ and $\angle ACB$ are subtended by the same arc AB or are in the same segment that is major segment $ADCB$.

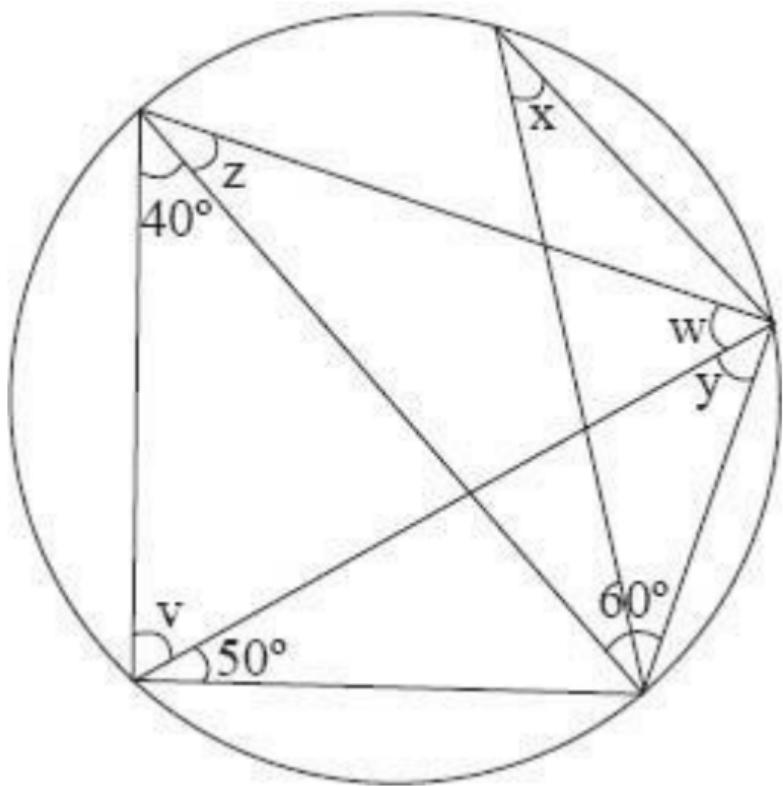
Therefore, $b = 44^\circ$.

Exercise 19.2

1. Find the values of a and b in the figure below.

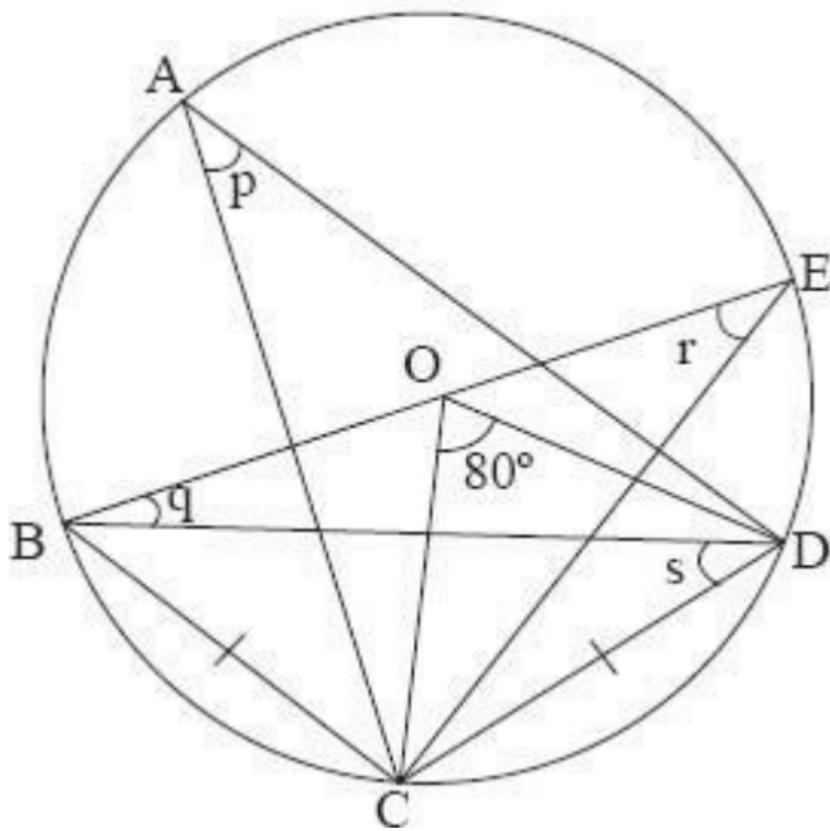


2. In the figure below, calculate the angles marked v, w, x, y and z:



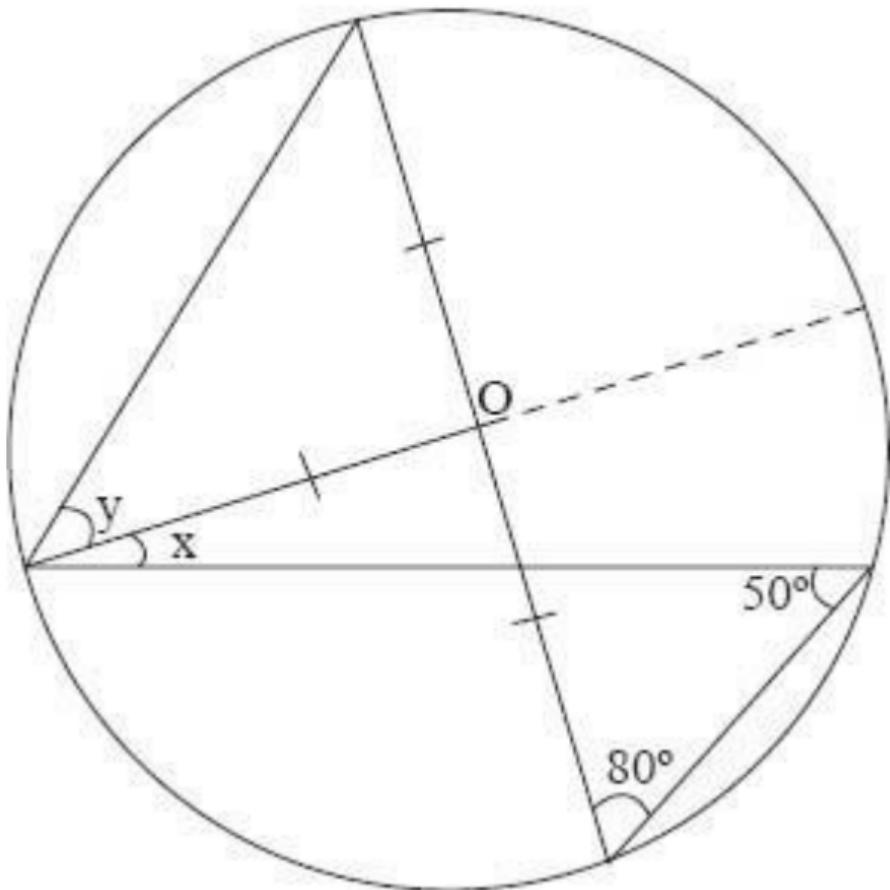
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3. In the figure below, O is the centre of circle ABCDE and BOE is the diameter: $COD = 80^\circ$ and $CB = CD$.



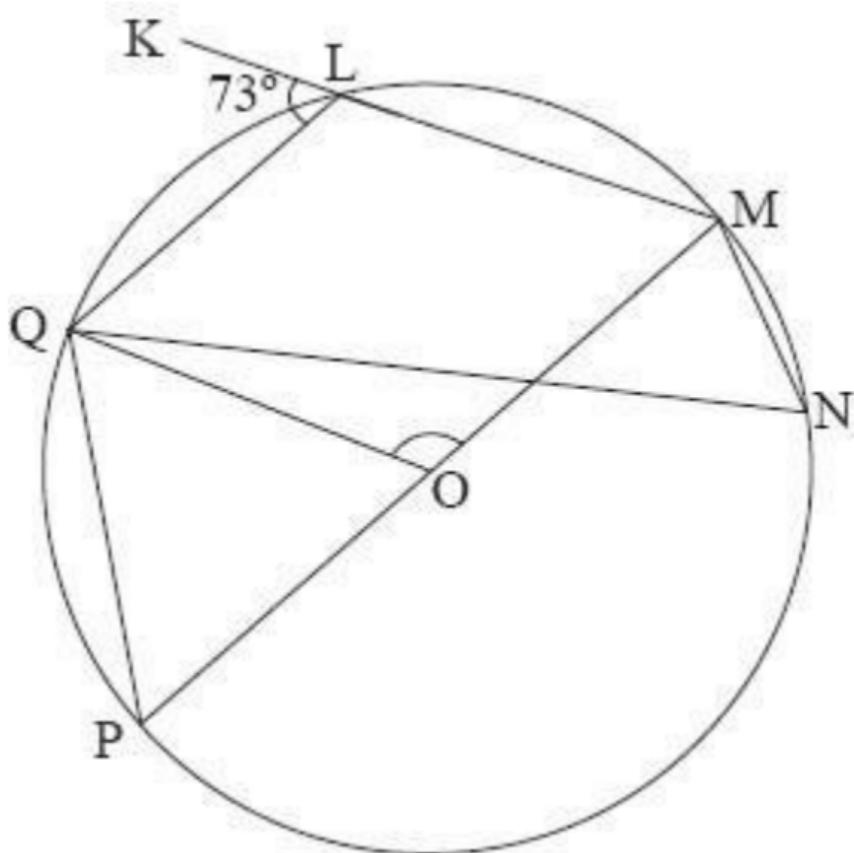
Calculate the angles marked p, q, r and s.

4. In the figure below, O is the centre of the circle. Find the values of x and y.



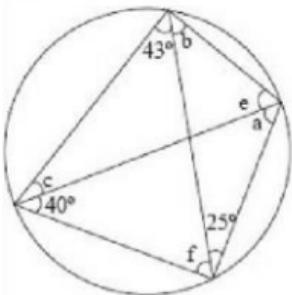
5. In the figure below, O is the centre of the circle and POM is the diameter.

If $QLK = 73^\circ$, calculate QNM:

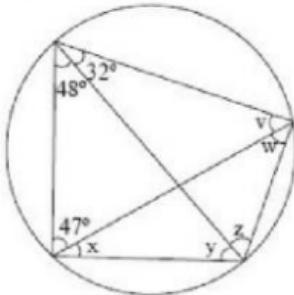


6. Find the angles marked by small letters.

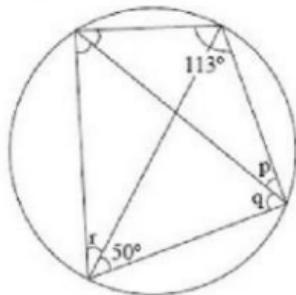
(a)



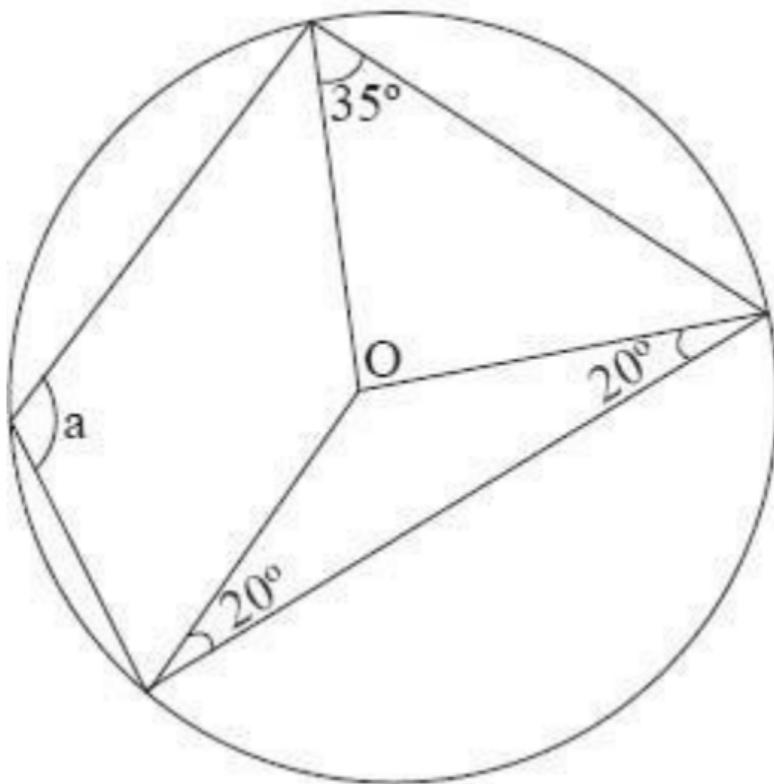
(b)



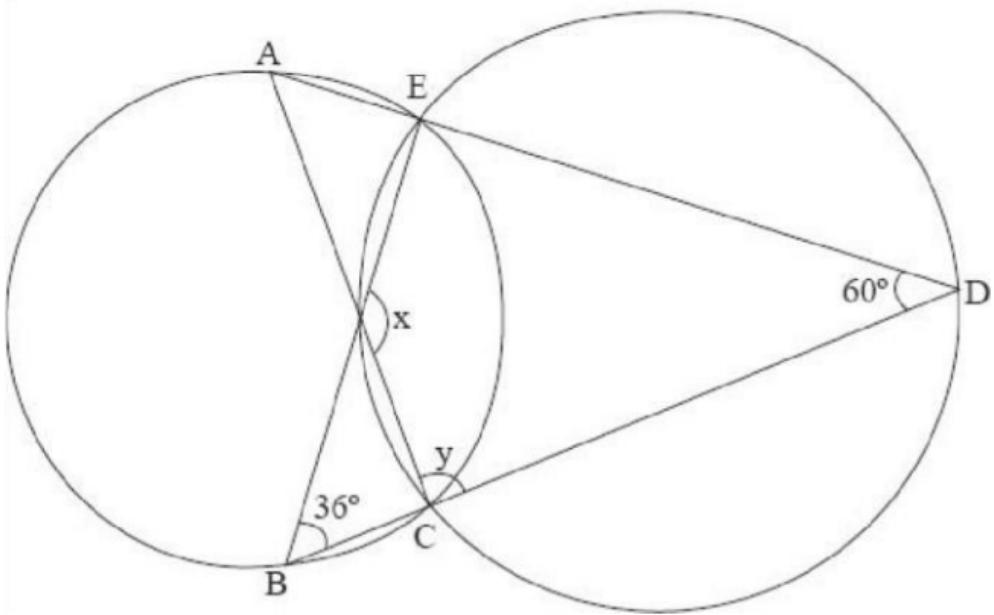
(c)



7. In the figure below, O is the centre of the circle. Calculate the value of angle a .



8. Find the value of x and y .



9. Two equal chords WX and YZ are produced to meet at T . If $\angle WYX = 73^\circ$

and $YZW = 37^\circ$, show that $TZX = XWY = 70^\circ$.

19.4: Cyclic Quadrilaterals

Figure 19.9 shows a quadrilateral ABCD with all its vertices lying on the circumference. Such a quadrilateral is said to be cyclic and the four vertices A, B, C and D are called concyclic points.

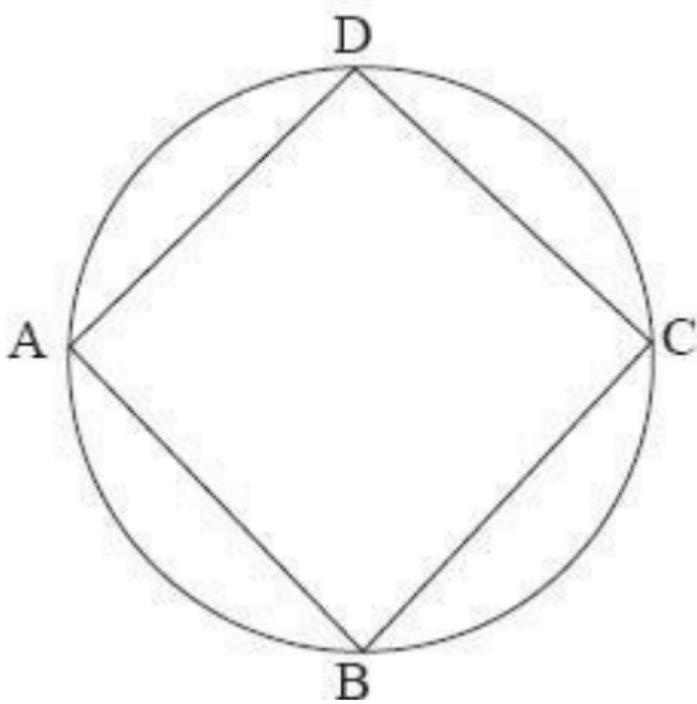


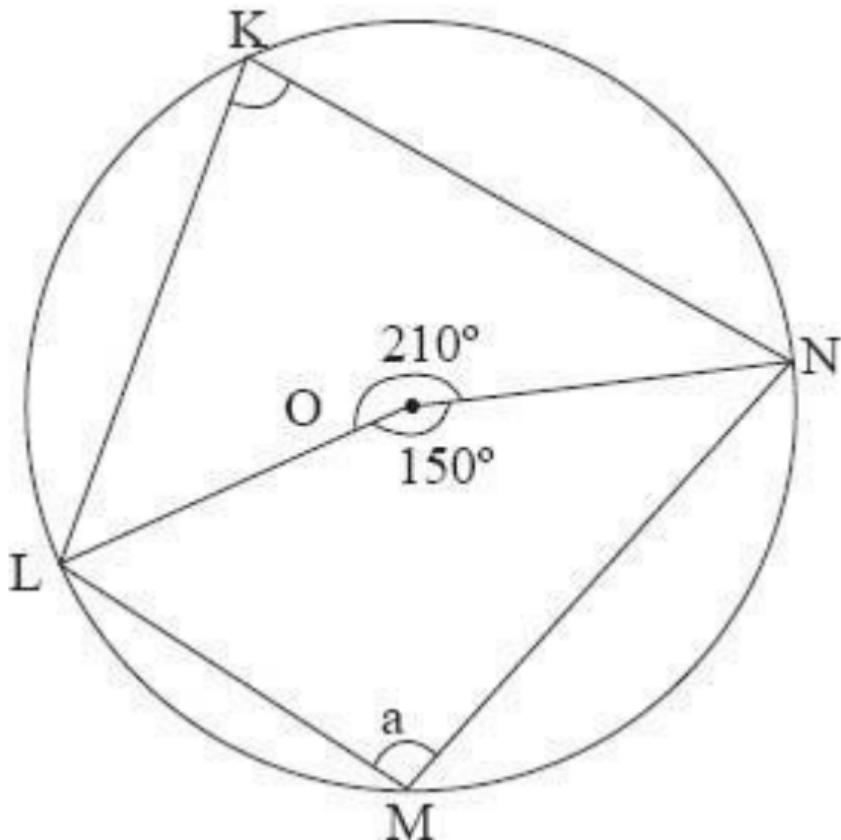
Fig. 19.9

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Angle properties of a cyclic quadrilateral

(a) Supplementary Angles

In figure 19.10, KLMN is a cyclic quadrilateral and O is the centre of the circle.



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Fig 19.10

The obtuse angle LON = 150° .

NKL = 75° (angle at the centre is twice the angle at the circumference). The reflex angle LON = 210° (angles at a point add up to 360°).

Find the size of the angle marked a° .
Determine the sum of LKN and LMN.

Note that LKN and LMN add up to 180° .
Hence angles LKN and LMN are supplementary.

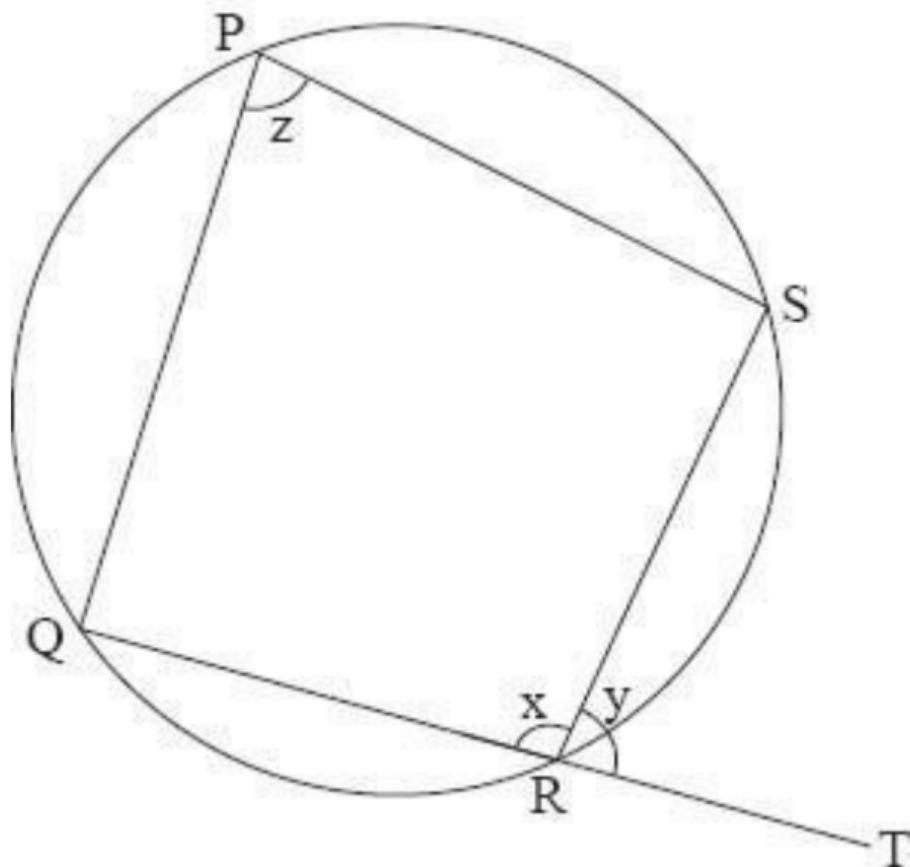
What can you deduce about the sum of angles KLM and MNK?

The above deductions are true for any cyclic quadrilateral. Hence, the opposite angles of a cyclic quadrilateral are supplementary. Conversely, if the opposite angles of a quadrilateral inscribed in a circle add up to 180° , then the quadrilateral is cyclic.

(b) Exterior Angles

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Figure 19.11 shows a cyclic quadrilateral PQRS with QR produced to T.



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Fig. 19.11

Let $\angle QRS = x$, $\angle SRT = y$ and $\angle QPS = z$. $x + y = 180^\circ$ (angles on a straight line) $x + z = (180^\circ)$ (opposite angles of a cyclic quadrilateral)

$$y = 180^\circ - x \quad z =$$

$$180^\circ - x$$

Therefore, $y = z$

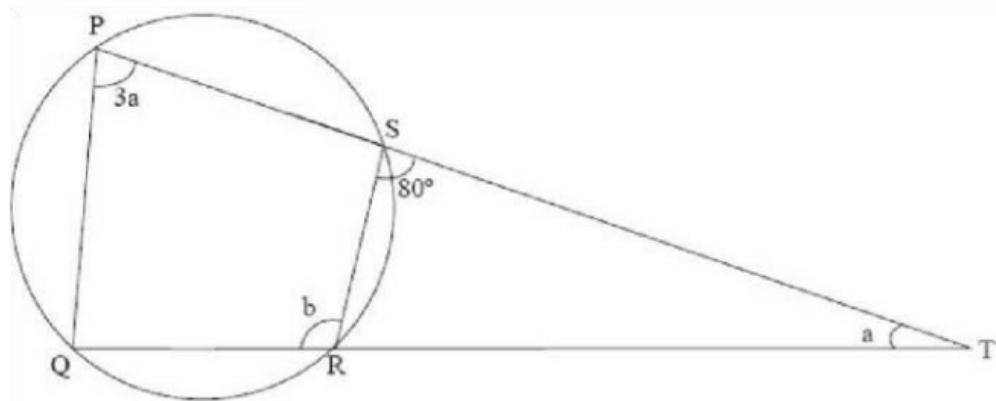
That is, $\angle QPS = \angle SRT$

This shows that if a side of a cyclic quadrilateral is produced, the exterior

angle formed is equal to the interior opposite angle.

Example 4

Calculate the value of a and b in figure 19.12.



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Fig. 19.12

Solution

$QPS = SRT = 3a$ (exterior angle of a cyclic quad.)

Therefore, $3a + a + 80^\circ = 180^\circ$ (angle sum of ΔPQT)
 $4a = 100^\circ$ $a = 25^\circ$

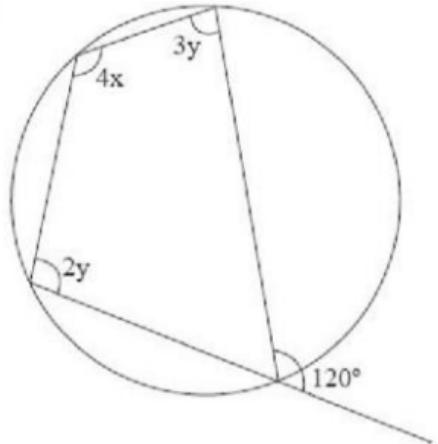
$3a + b = 180^\circ$ (opposite angles of a cyclic quad.)

Therefore, $75^\circ + b = 180^\circ$
 $b = 105^\circ$

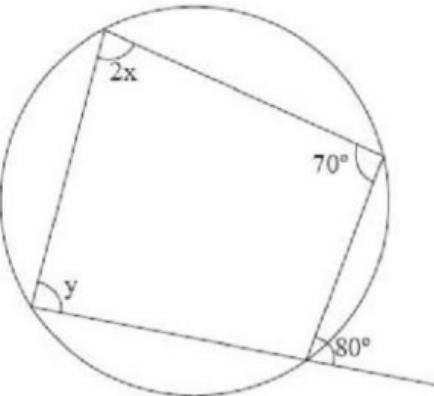
Exercise 19.3

1. Find the values of x and y .

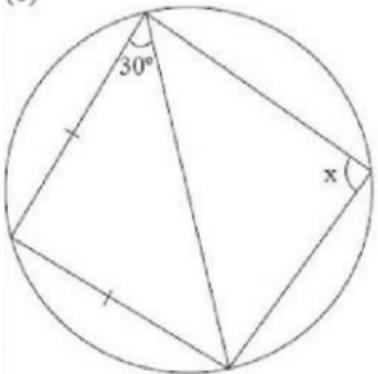
(a)



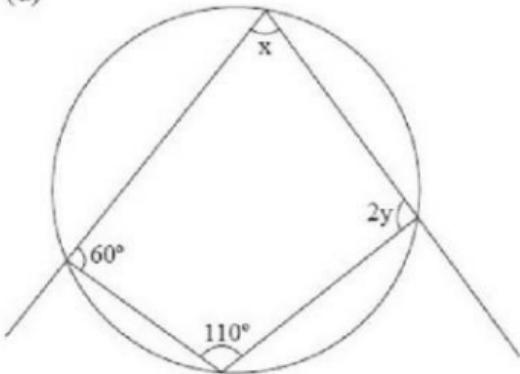
(b)



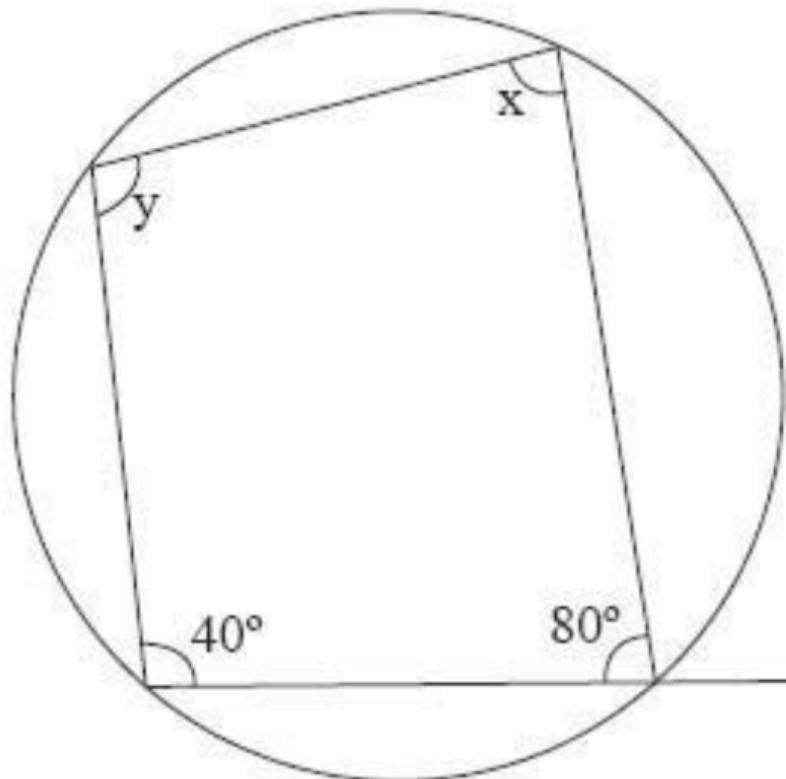
(c)



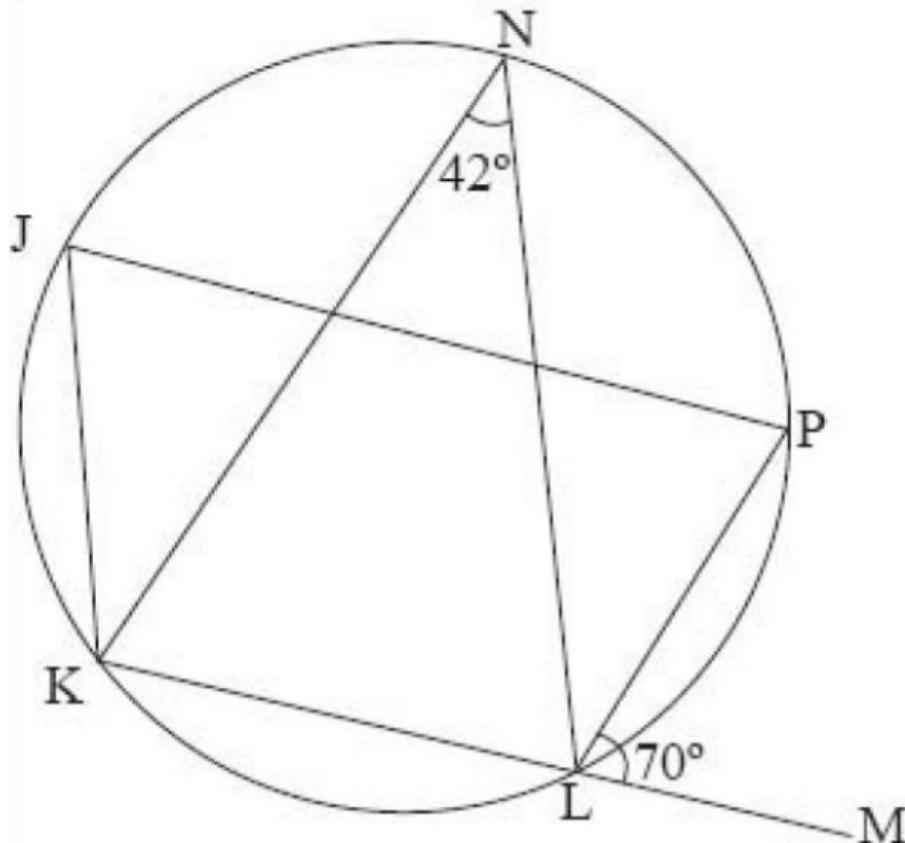
(d)



(e)

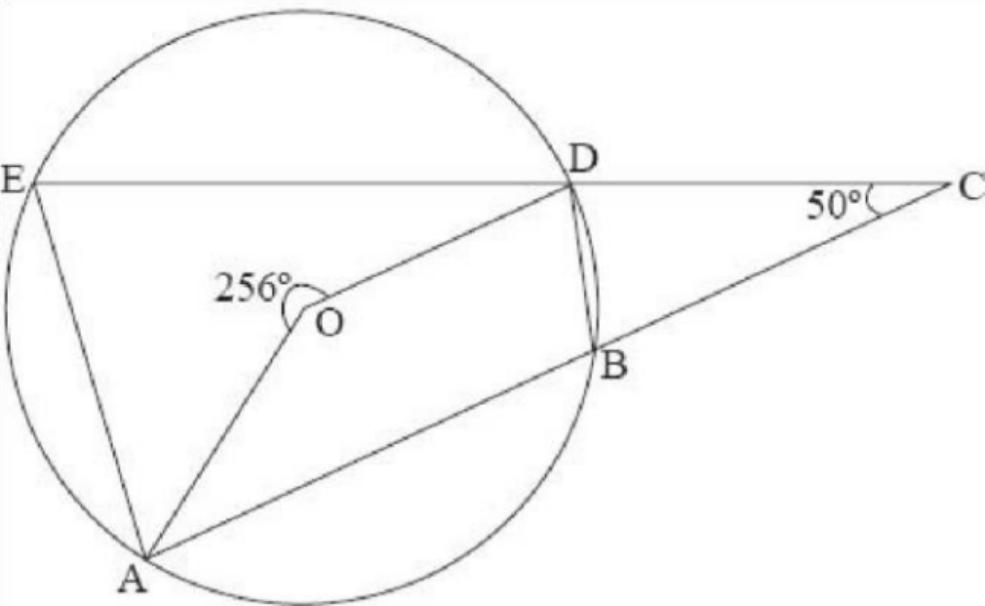


2. In the figure below, calculate LNP if $PLM = 70^\circ$ and $KNL = 42^\circ$

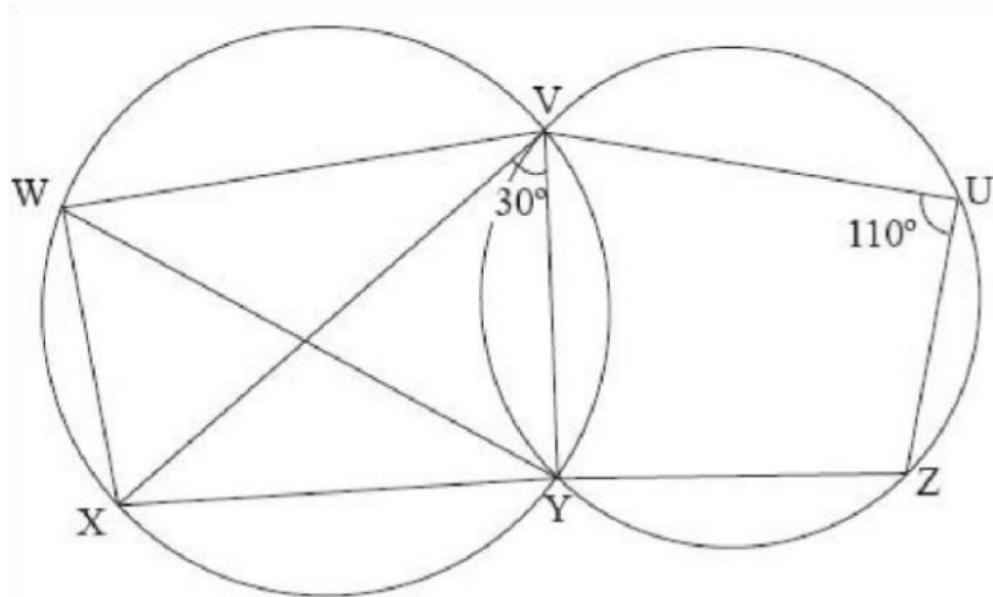


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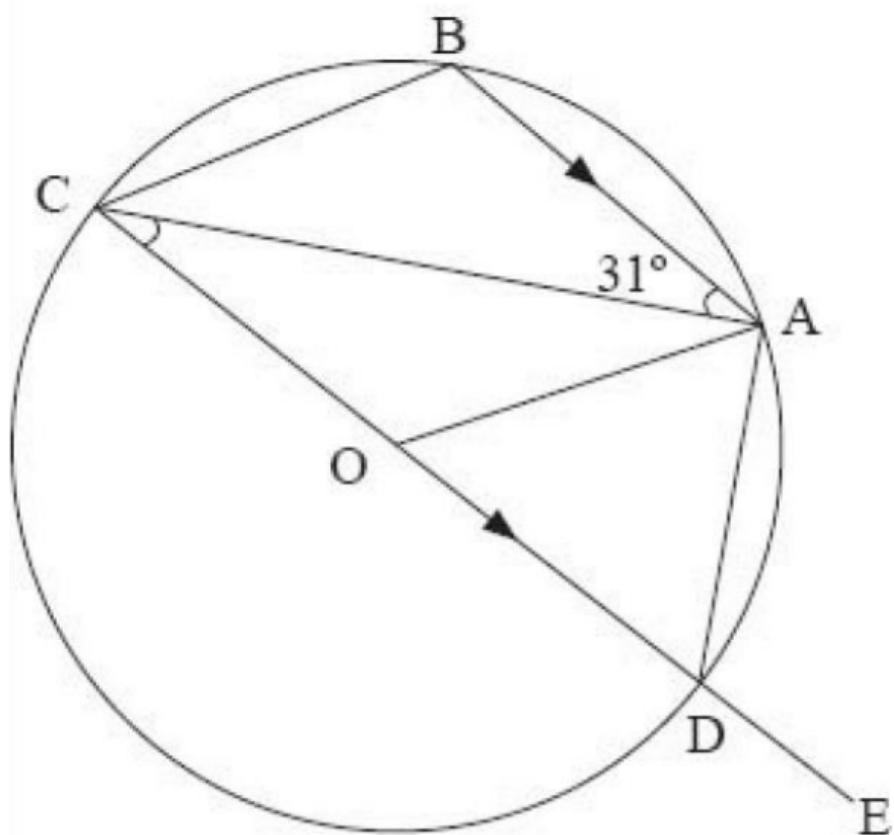
3. In the figure below, O is the centre of the circle. The reflex angle $AOD = 256^\circ$ and $ACE = 50^\circ$. Calculate:
- (a) BDC .
 - (b) BAE .



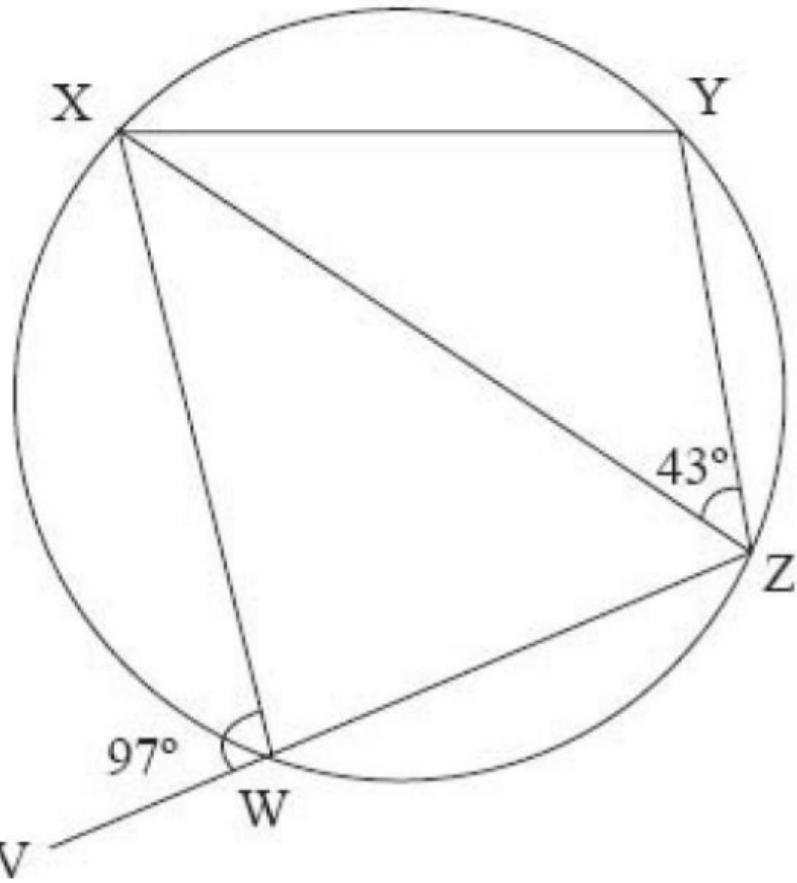
4. In the figure below, $VUZ = 110^\circ$ and $XVY = 30^\circ$. Find the value of VWY .



5. PQRS is a cyclic quadrilateral. $\angle QPR = 32^\circ$, $PS = SR$, $PQ = PR$ and T is a point on the circumference between R and S. Calculate RTS.
6. In the figure below, O is the centre of the circle and BA is parallel to CD. If $\angle BAC = 31^\circ$, calculate $\angle BCO$ and $\angle ADE$.



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7. In the figure below, $WXYZ$ is a cyclic quadrilateral and VWZ a straight line. $\angle VWX = 97^\circ$ and $\angle XZY = 43^\circ$. Calculate the size of angle ZXY .

Chapter 20

VECTORS

20.1: Introduction

Figure 5.1 shows two towns, A and B.

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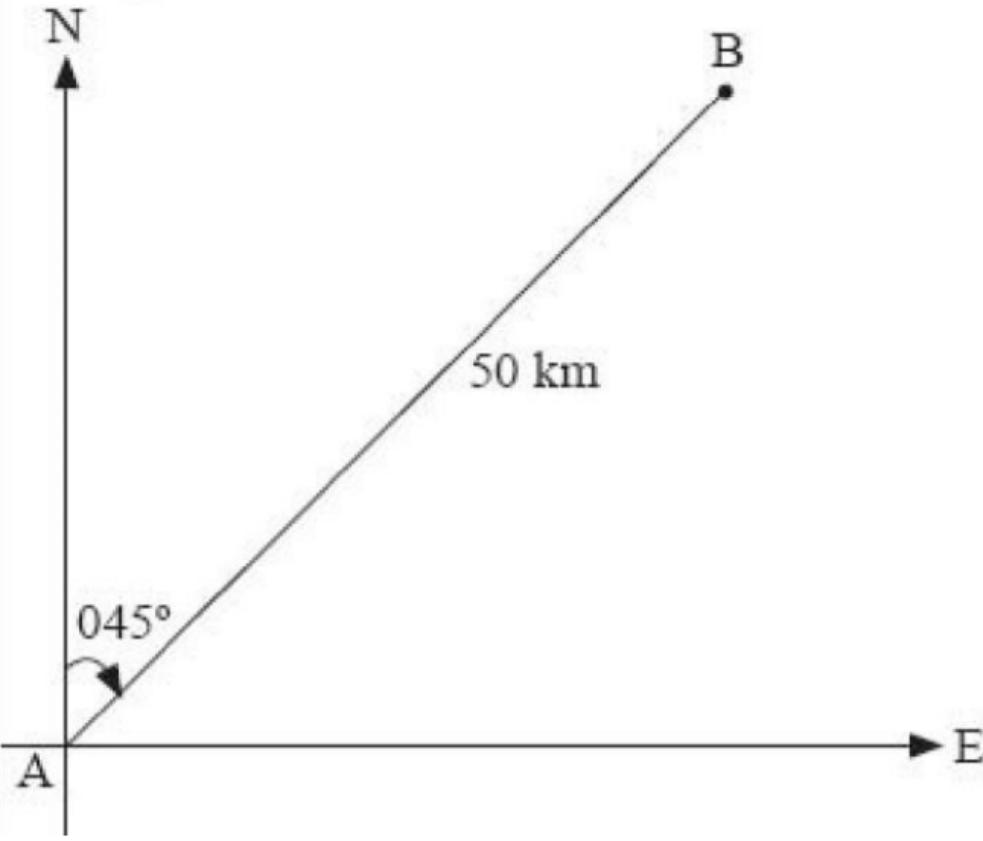


Fig. 20.1

Which of the following statements best describes the position of B relative to A?

- (i) B is 50 kilometres from A.
- (ii) B is north-east of A.
- (iii) B is 50 kilometres north-east of A.

Clearly, the position of B relative to A is best described by (iii), which states not only the distance, but also the direction. A quantity which has both

magnitude and direction is a vector. In the example used, both distance (magnitude) and direction specify the position of B relative to A. Some examples of vector quantities are velocity, acceleration and force.

A quantity which has magnitude only is called scalar. Some examples of scalar quantities are mass, distance, temperature and time.

20.3: Representation of Vectors

A vector can be represented by a directed line, as shown below:



Fig. 20.2

The direction of the vector is shown by the arrow. Its magnitude is represented by the length of AB. Vector AB can be written in short as \underline{AB} or \overrightarrow{AB} or AB . Its magnitude is denoted by $|AB|$. A is called the initial point and B the terminal point. Sometimes a vector can be

denoted by a single small letter a or \underline{a} as shown in figure 20.3:

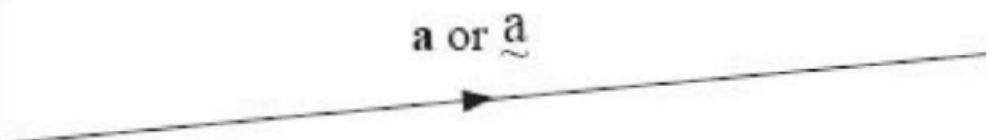


Fig. 20.3

In print, we use bold face a , but in writing, we use the notation \underline{a} to represent the 'vector a '.

20.3: Equivalent Vectors

Two or more vectors are said to be equivalent if they have:

- (i) equal magnitudes.

(ii) the same direction.

(a)

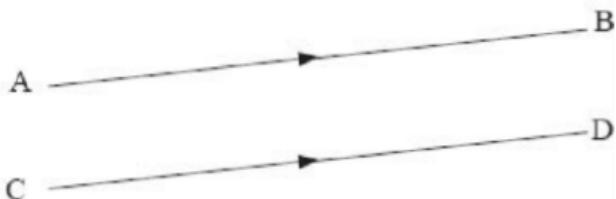


Fig. 20.4 (a)

(b)

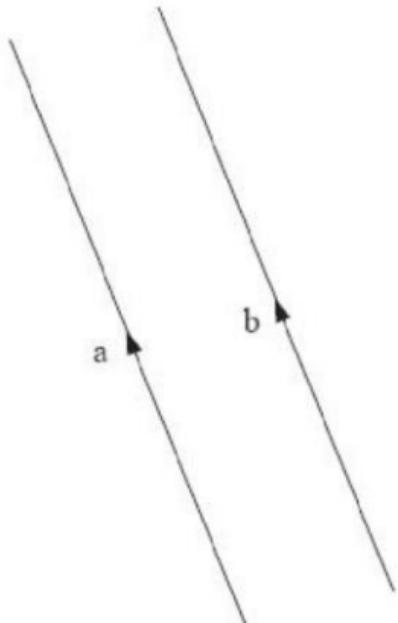


Fig. 20.4 (b)

In figure 20.4 (a), $AB = CD$ because:

- (i) $|AB| = |CD|$.
- (ii) they have the same direction.

Similarly, in figure 20.4 (b), $a = b$ because:

- (i) $|a| = |b|$.
- (ii) they have the same direction.

Figure 20.5 is a cuboid. $EF = HG$. Name the vectors equivalent to CB , CD and CH .

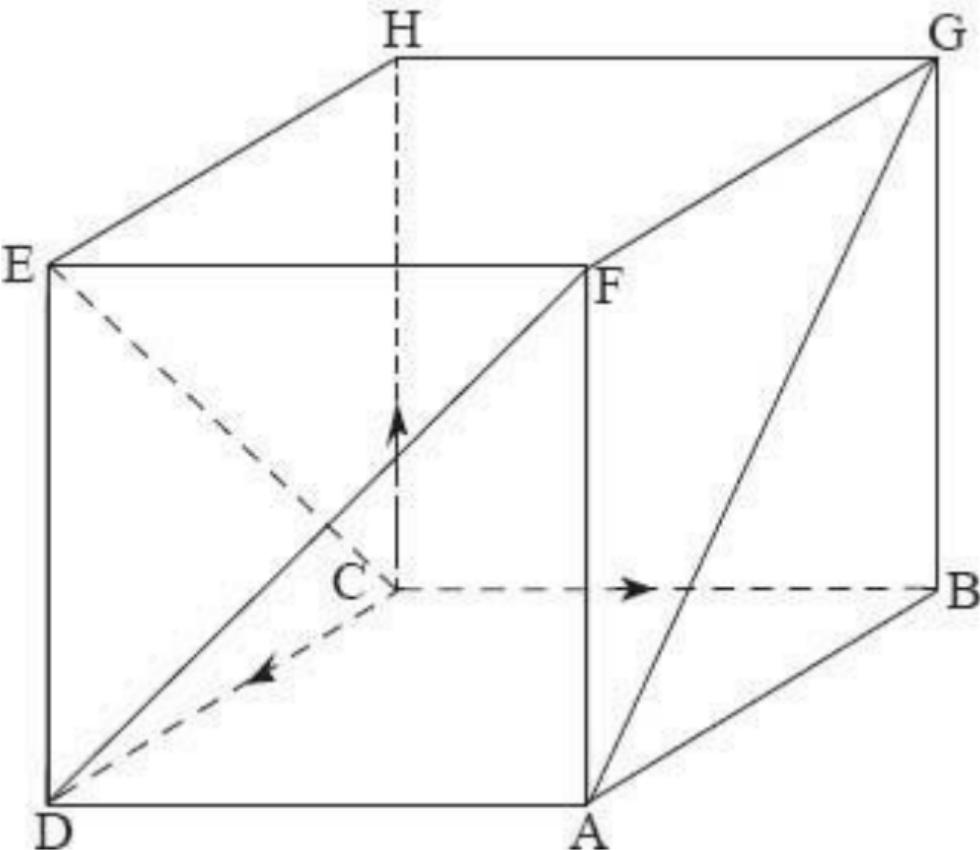


Fig. 20.5

20.4: Addition of Vectors

Movement on a straight line from point A to B can be represented using a vector. This movement can also be referred to as displacement.

Consider the displacement from A to B followed by that from B to C, see figure 20.6.

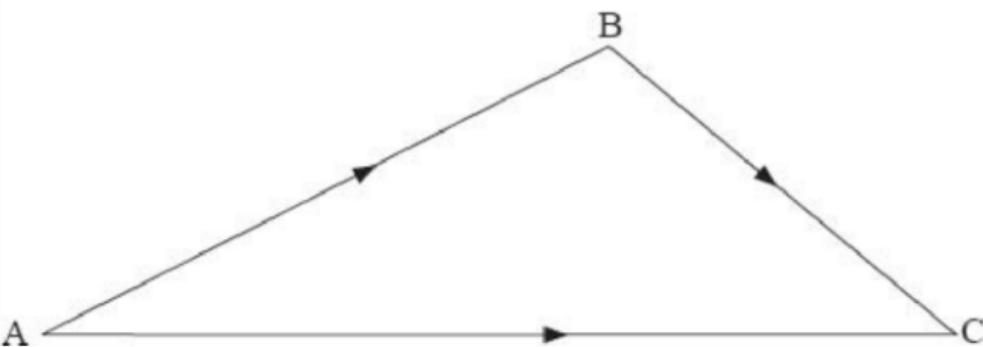


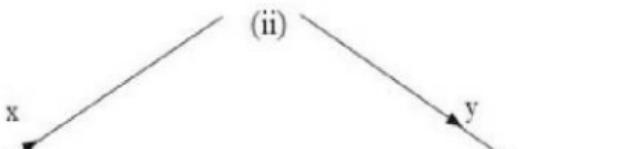
Fig. 20.6

The resulting displacement is AC. This is usually written as $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$.

In order to add any two vectors, a rearrangement may be necessary. The initial point of either vector is placed in the terminal point of the other, as illustrated in the following figures.

VECTORS

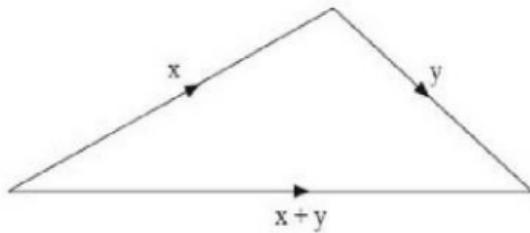
(a) (i)



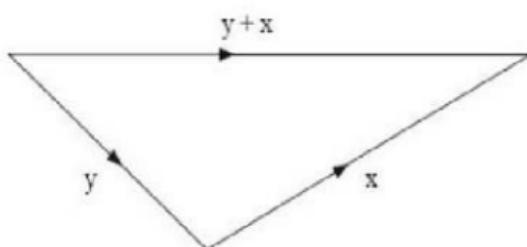
(ii)



(b)



(c)



(d)

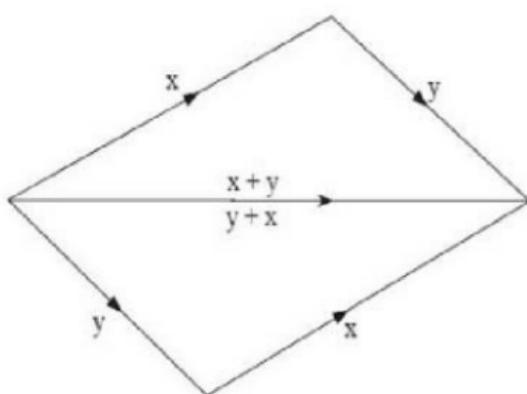


Fig. 20.7

The two ways of adding vectors \underline{x} and \underline{y} in figure 20.7 (a) are shown in figure 20.7 (b) and (c). In figure 20.7 (b), the initial point of \underline{x} is placed on the terminal point of \underline{y} to obtain \underline{z} . In figure 20.7 (c), the initial point of \underline{y} is placed on the terminal point of \underline{x} to obtain \underline{z} .

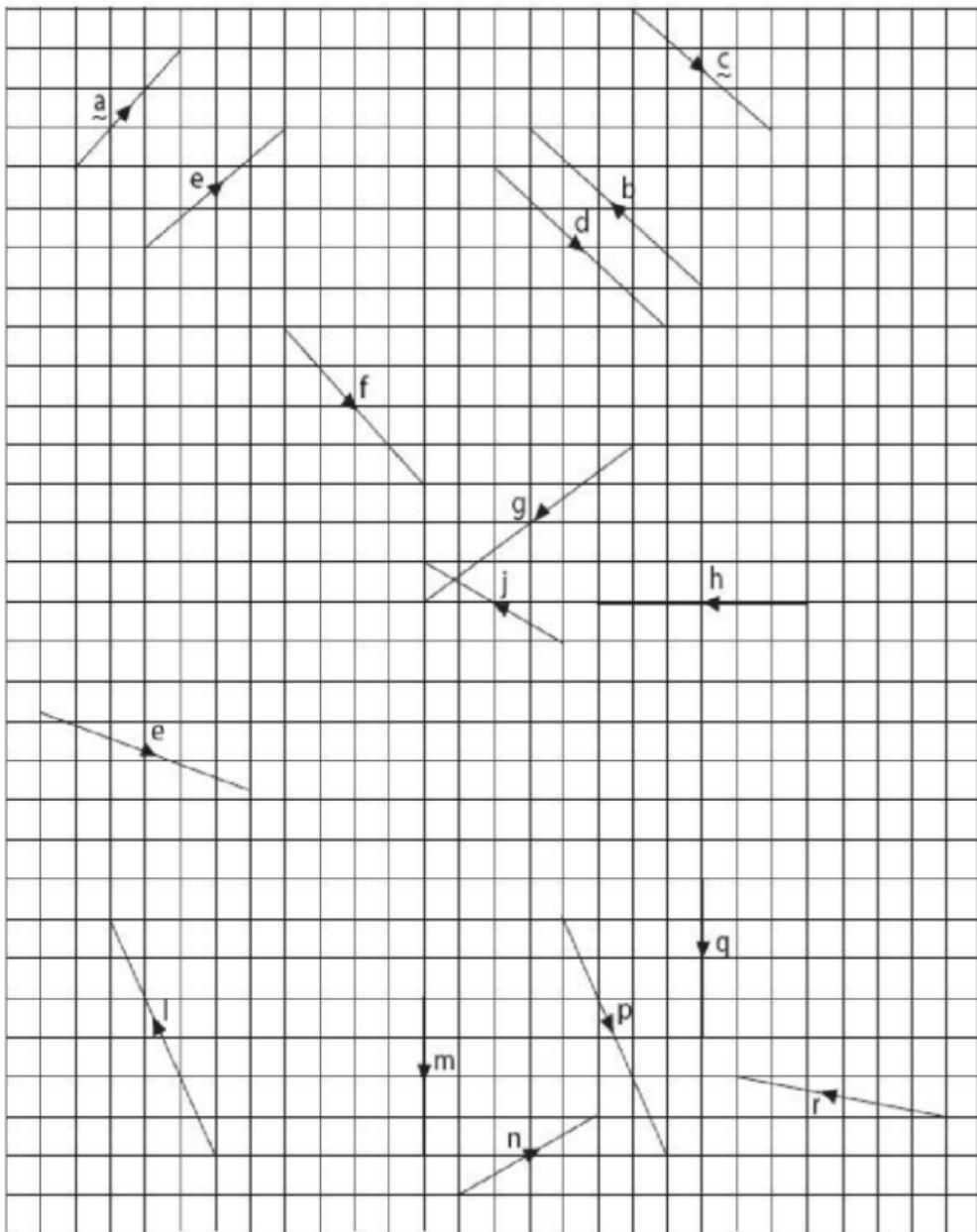
\underline{x} \underline{y} \underline{y} \underline{x}

Note that; $\underline{z} = \underline{y} + \underline{x}$ as illustrated in figure 20.7 (d).

Exercise 20.1

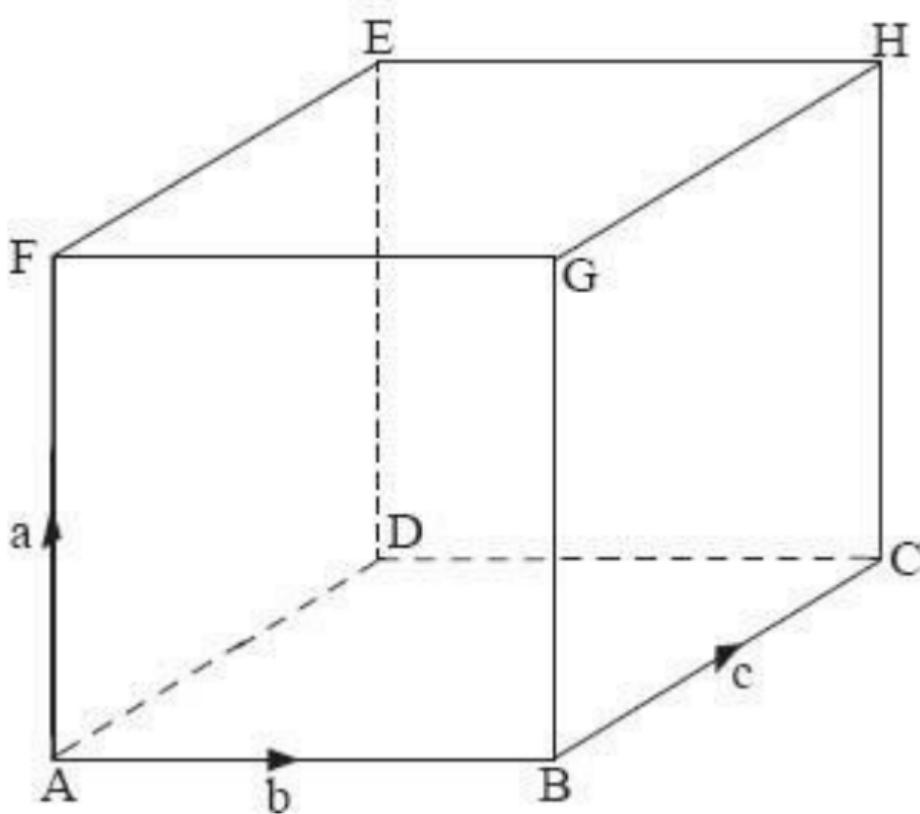
Use the figure below to answer questions 1, 2 and 3.

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1. Name pairs of equal vectors.
2. Name pairs of vectors with equal magnitudes but opposite directions.
3. Use graph paper to illustrate the sums of the following vectors:
 - (a) $a + b$
 - (b) $b + e$
 - (c) $e + h$

- (d) $j + f$
 - (e) $c + i + r$
 - (f) $f + g + m$
4. ABCDEFGH is a cuboid with vectors AF, AB and BC given as a, b and c respectively, as in the figure shown. Express all the diagonal vectors in terms of a, b and c.



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The Zero Vector



Fig. 20.8

Consider a displacement from A to B and back to A. The total displacement is zero. A zero displacement is represented by a vector of zero magnitude, denoted by O. This vector is called a zero or null vector. In this case, $AB + BA = O$.

The vector BA has the same magnitude as the vector AB, but its direction is

opposite to that of AB , i.e., $BA = -AB$.
Generally, if $a + b = 0$, then;

$$\underline{b} = -a, \text{ or, } a = -\underline{b}$$

20.5: Multiplication of a Vector by a Scalar

(a) Positive Scalar

In figure 20.9, $AB = BC = CD = a$.

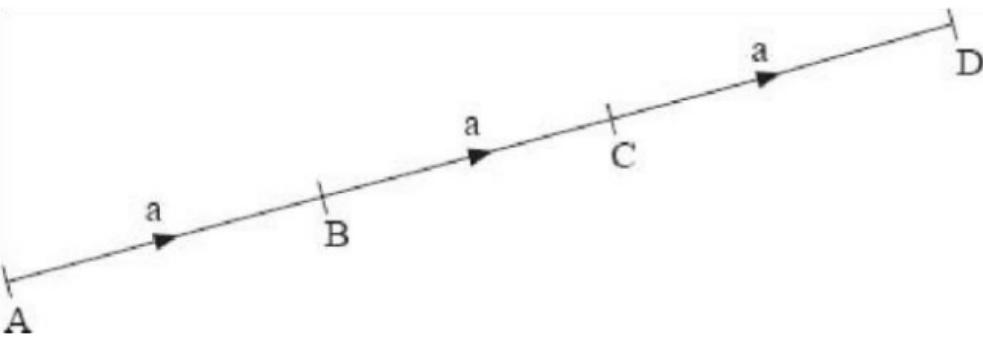


Fig. 20.9

Therefore, $AD = a + a + a = 3a$

This vector has the same direction as a and a magnitude three times that of a . In general, ka , where k is a positive number, is a vector which has the same direction as a and magnitude k times that of a .

(b) Negative Scalar

In figure 20.10, $DA = (-a) + (-a) + (-a)$
 $= -3a$

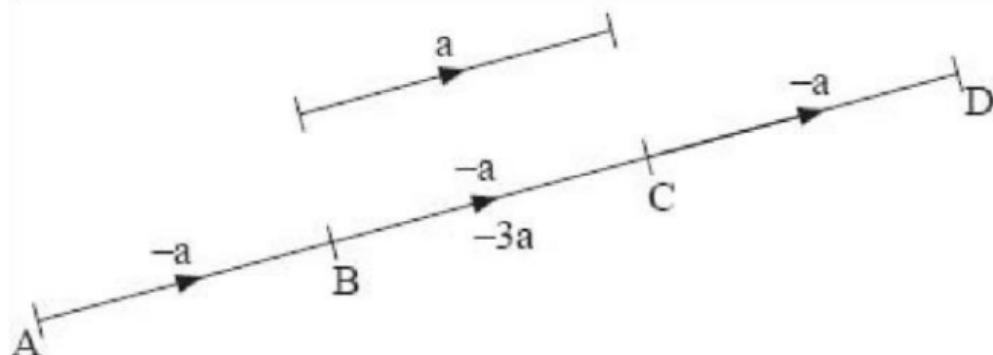


Fig. 20. 10

The vector $-3a$ has a magnitude three times that of a and its direction is opposite that of a .

The vector ka (k negative) has direction opposite that of a and its magnitude is k times that of a . The negative sign does not affect the magnitude. It only affects the direction of the vector.

(c) The Zero Scalar

When a vector a is multiplied by 0, its magnitude is zero times that of a . The result is a zero vector. $a \cdot 0 = 0 \cdot a = 0$

Example 1

Vectors a and b are given below. Use the grid to construct:

- (a) $b - a$.
- (b) $3a - 2b$.

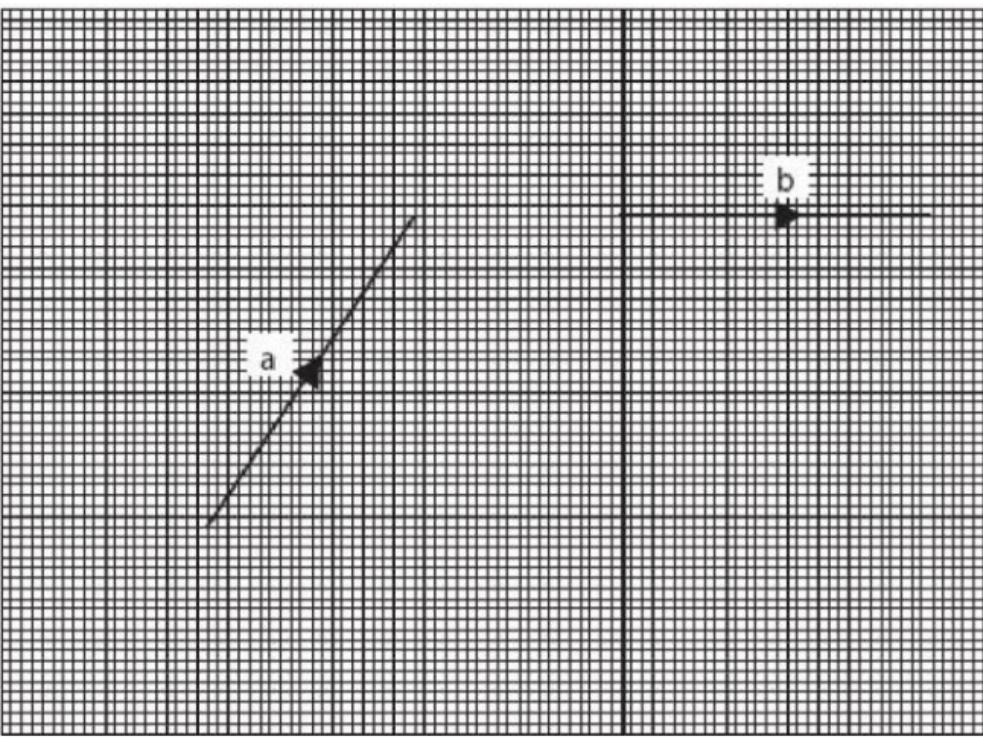


Fig. 20.11

Solution

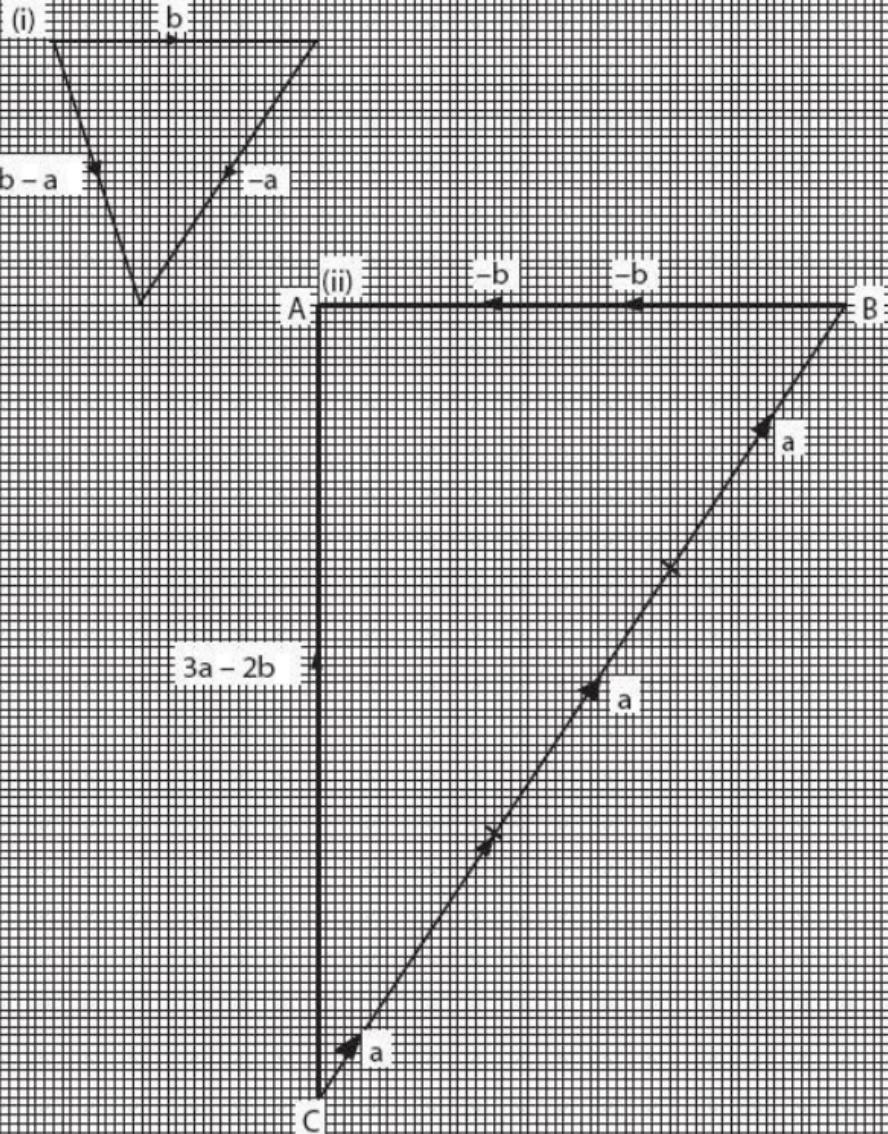
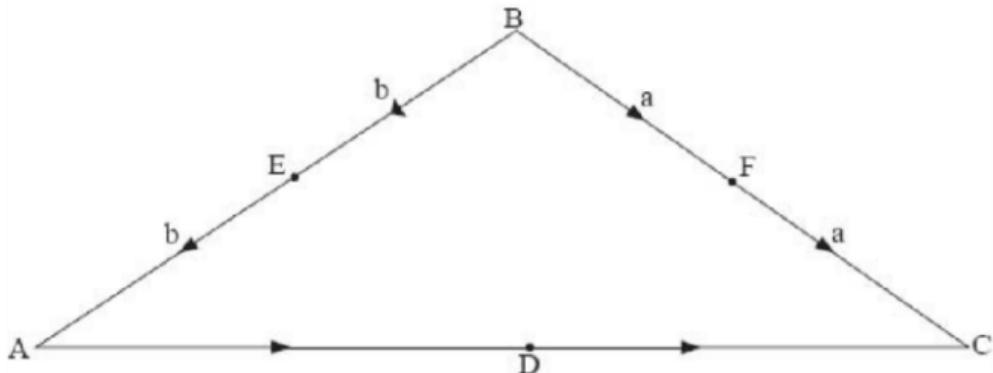


Fig. 20.12

Exercise 20.2

1. The figure below shows ABC a triangle in which the midpoints of AB, BC and AC are E, F and D respectively. Vector $\overrightarrow{AB} = -2\mathbf{b}$ while $\overrightarrow{BC} = 2\mathbf{a}$.

VECTORS



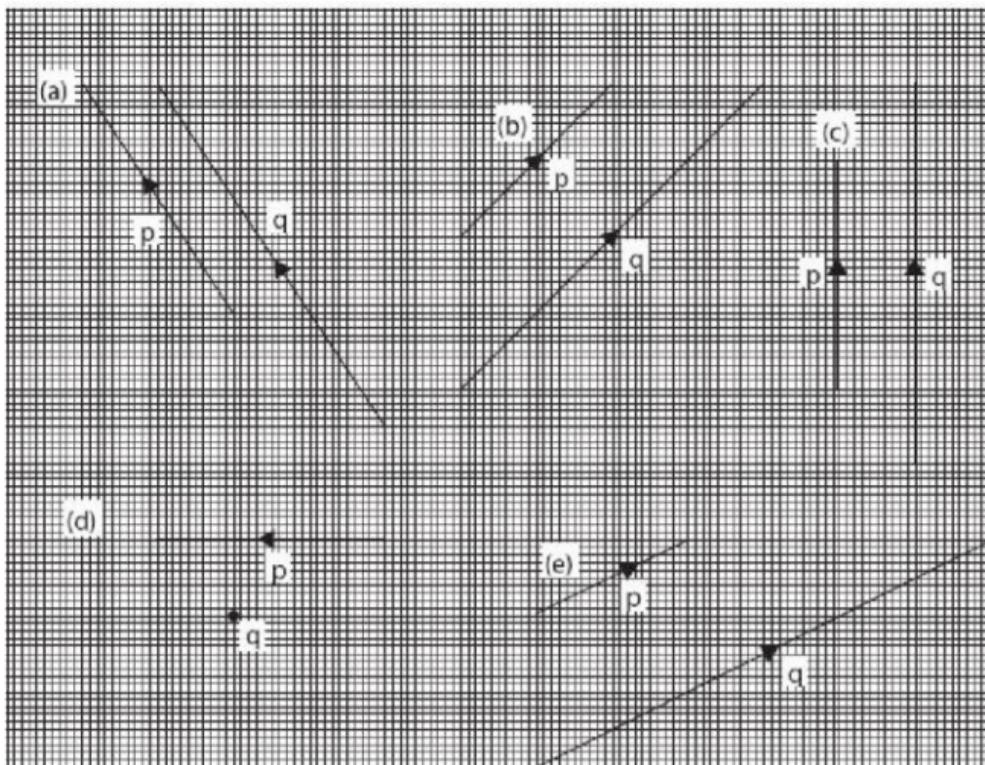
Rewrite each of the following vectors
in terms of a and b :

(a) \overrightarrow{BF}

(b) \overrightarrow{AF}

- (c) AC
 - (d) DC
 - (e) DA
 - (f) BD
2. For each of the pairs of vectors shown in the figure below, write, where possible, the relationship between p and q in the form:
- (i) $q = mp$.
 - (ii) $p = nq$.

Where m and n are scalars.



3. Given that $t = 2a - b$ and $r = b + 3a$, find, in terms of a and b , the following vectors:

(a) $2t$

(b) $\frac{1}{2}r$

(c) $-t$

(d) $-2r$

(e) $2t - 2r$

(f) $-t + \frac{1}{2}r$

(g) $2r - t$

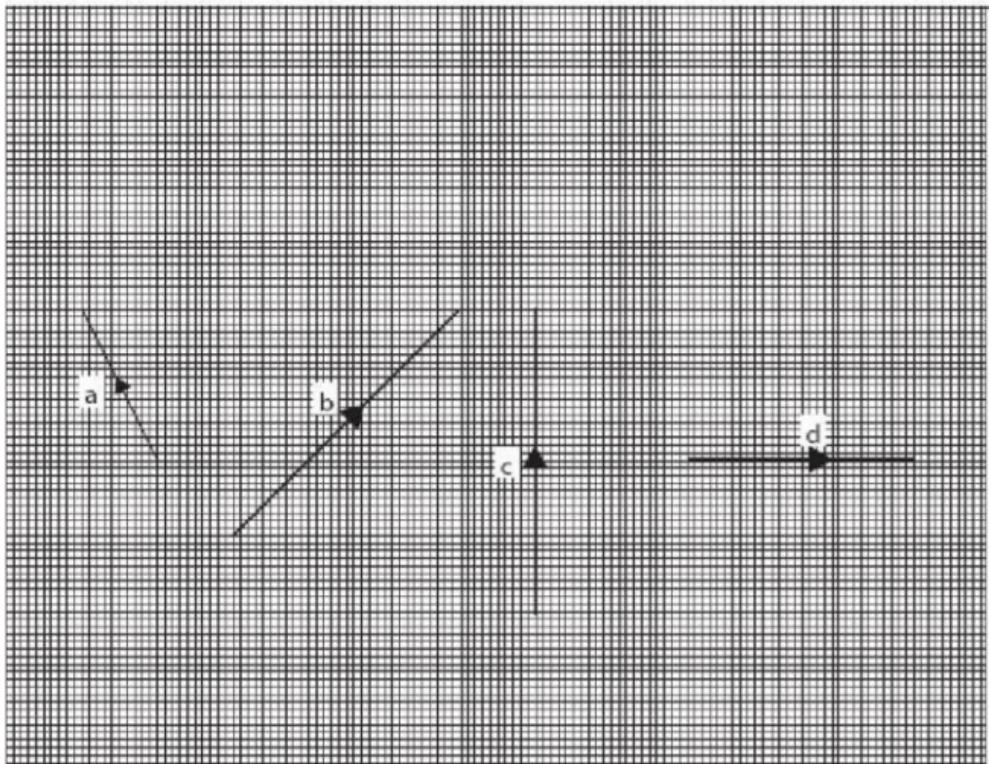
4. Copy vectors a , b , c and d in the figure below on graph paper and use your copy to construct the following vectors:
- (a) $a + b$
 - (b) $a - b$
 - (c) $c + d$
 - (d) $c - d$
 - (e) $b + c$

(f) $b - c$

(g) $a + c$ (h) $a - c$

(i) $a + d$

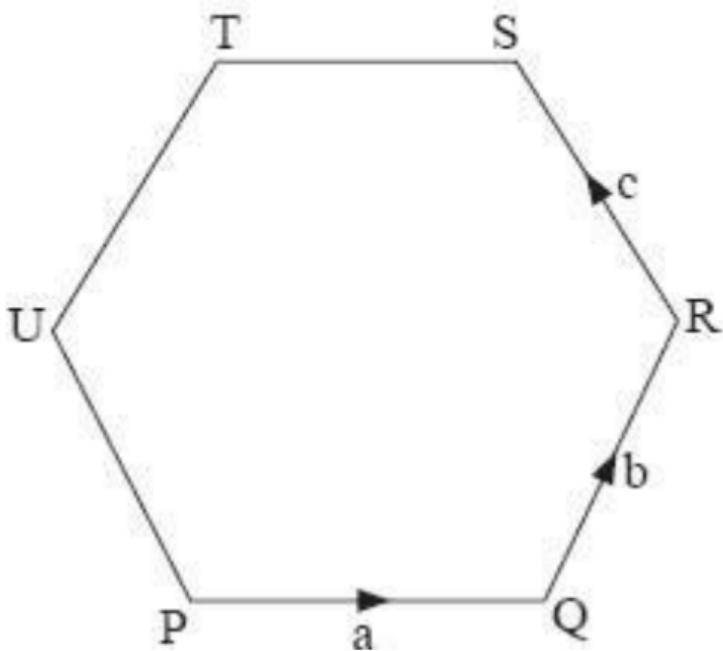
(j) $a - d$



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5. Use vectors a , b , c and d in the figure above to construct:
- (a) $2a - \frac{1}{3}b$
 - (b) $b - 2.5a$
 - (c) $\frac{1}{3}b + c - \frac{1}{4}a$
 - (d) $d + c - 2a$
 - (e) $-2a - b - d$
6. The figure below shows a regular hexagon PQRSTU. $PQ = a$, $QR = b$ and $RS = c$

VECTORS



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Write in terms of a, b and c each of
the following:

(a) QP (b)

PS

(c) T

Q

(d) T

P (e) RT

(f) UQ

(g) US

7. Simplify:

(a) $3(a - 2b) - 4(5a - b)$

(b) $\frac{1}{3}(a - b) + \frac{1}{4}(3b - 4a)$

8. If $\frac{1}{3}\mathbf{a} + \frac{1}{5}\mathbf{b} = 0$, write \mathbf{a} as a multiple of \mathbf{b} .

9. Simplify:

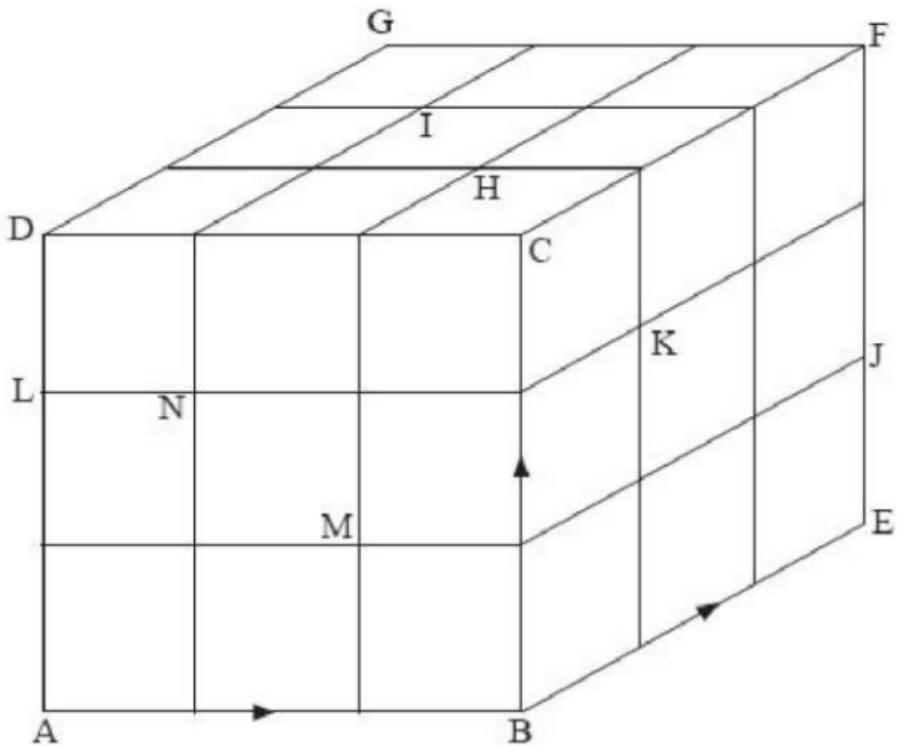
(a) $3\mathbf{a} + 2\mathbf{b} - \mathbf{c} + 4(\frac{1}{2}\mathbf{a} - \frac{3}{2}\mathbf{c}) + \frac{1}{3}(6\mathbf{a} - 9\mathbf{b})$

(b) $2u - 3v + 2(w - u) + 3(u + v)$

$$(c) (p - q) + (r - p) + (q - r)$$

10. The figure below shows a large cube divided into 27 smaller ones.

$$AB = \frac{3}{2}a \text{ and } BE = b$$



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Write each of the following in terms of a, b and c:

(a) AE

(b) BD

(c) BF

(d) AG

(e) AJ

(f) AK

(g) AI

(h) MN

(i) LN

(j) NJ

(k) MI

(l) 2NL + HD

(m) MK + KI + IM

(n) JN – NM

(p) NK + IF

(q) 3MK – 2NH

20.6: Column Vectors

Consider points A(2,1) and B(4, 7) in figure 20.13. The vector AB represents a displacement from A (2, 1) to B (4, 7). This displacement consists of two components AE and EB.

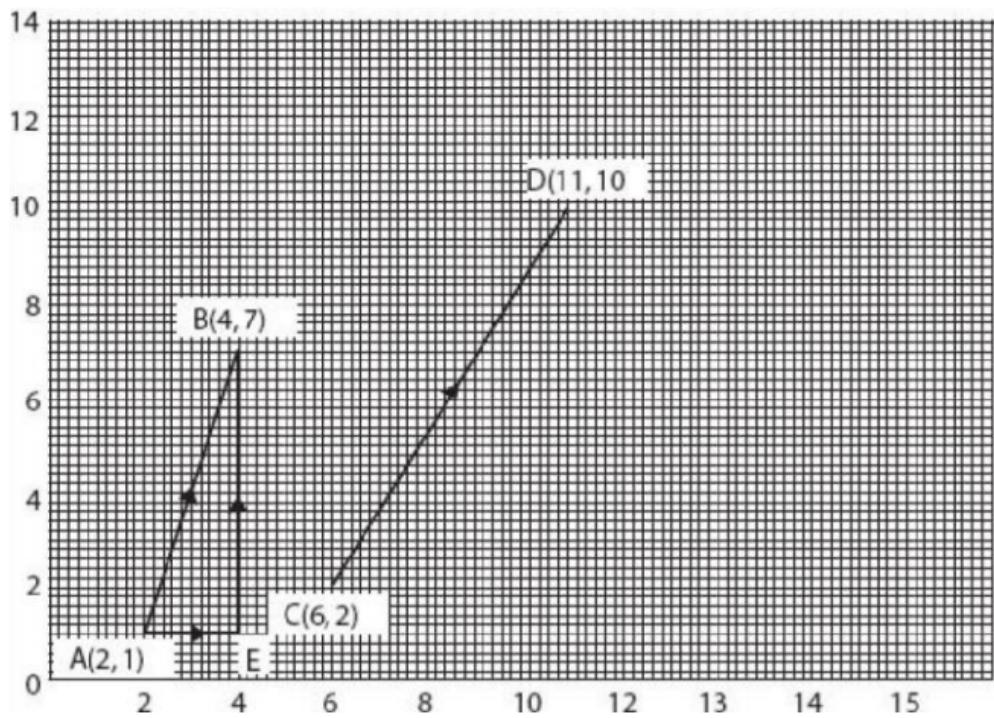


Fig. 20.13

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For vector AB, the components are 2 units in the horizontal direction and 6 units in the vertical direction. Vector AB can be represented in terms of its components as $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$

Vectors of the form where a is the horizontal displacement and b the vertical displacement are known as column vectors. Find the column vector of CD in the diagram.

Note:

- (i) Displacements parallel to the x-axis are positive if they are to the right and negative if they are to the left.
- (ii) Displacements parallel to the y-axis are positive if they are upwards and negative if they are downwards.

Example 2

Find the sum of each of the following column vectors and illustrate their solutions graphically:

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$$\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$
$$\mathbf{c} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \text{ and } \mathbf{d} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

(a)

(b)

Solution

(a) Total displacement in x direction is $2 + 7 = 9$.

Total displacement in y direction is $3 + 1 = 4$.

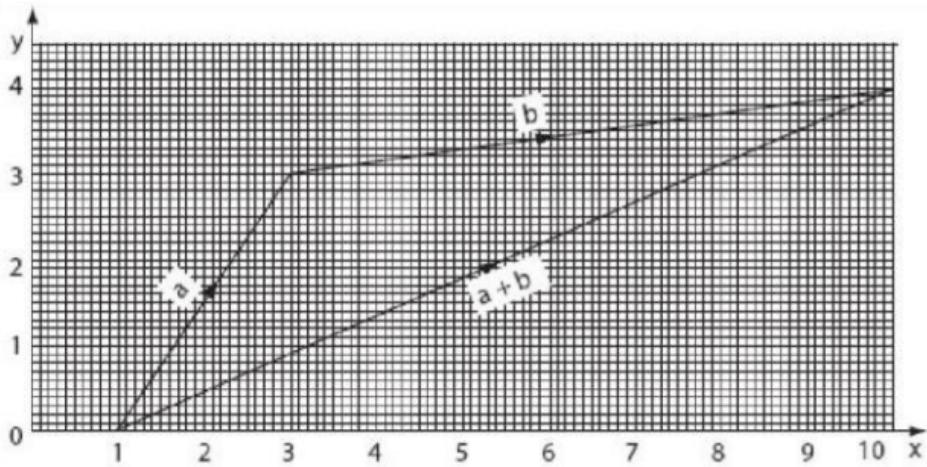


Fig 20.14

$$\begin{aligned}\text{Therefore, } \mathbf{a} + \mathbf{b} &= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 7 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 9 \\ 4 \end{pmatrix}\end{aligned}$$

(b) Total displacement in x direction is $(-4 + 5) = 1$

Total displacement in y direction is
 $(2 + -2) = 0$

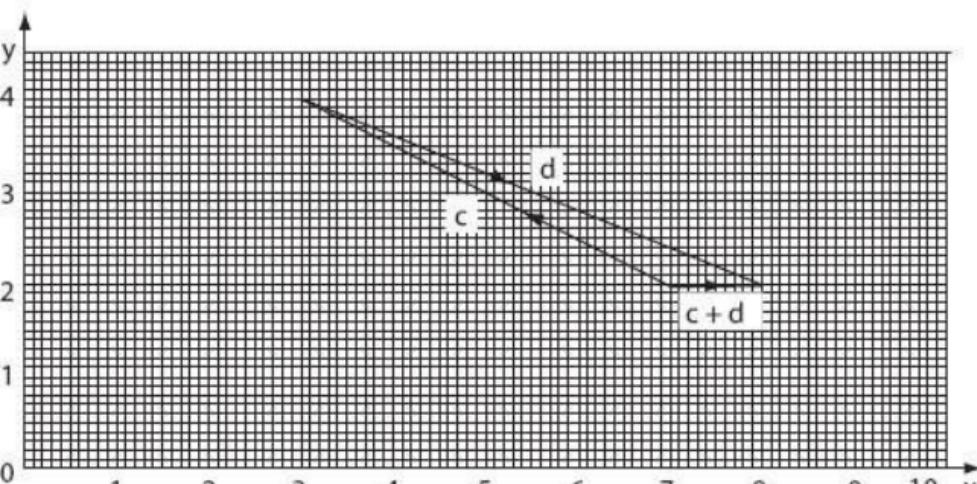


Fig. 20.15

Therefore, $\mathbf{c} + \mathbf{d} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$
 $= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Example 3

If $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, find $3\mathbf{a} + 2\mathbf{b}$.

Solution

$$3\mathbf{a} = 3 \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

$$2\mathbf{b} = 4 \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

$$\text{Therefore, } 3\mathbf{a} + 2\mathbf{b} = \begin{pmatrix} 9 \\ 6 \end{pmatrix} + \begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 11 \\ 14 \end{pmatrix}$$

20.7: The Position Vector

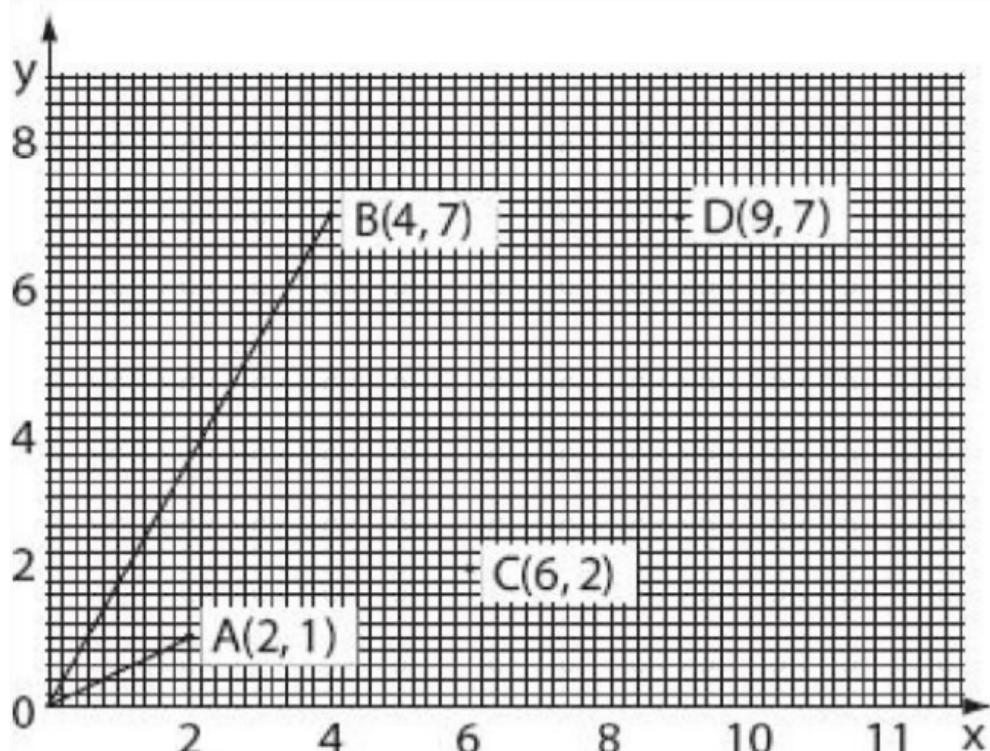


Fig. 20.16

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Each of the points A (2, 1) and B (4, 7) has been located in the plane relative to the origin O. OA is known as the position vector of A relative to the origin.

$$OA = \begin{pmatrix} 2 - 0 \\ 1 - 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 - 0 \\ 7 - 0 \end{pmatrix}$$

$S = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ similarly, the position vector of B is OB

State the position vectors of points C and D in figure 20.16.

For any point A in the plane, its position vector OA is denoted by \underline{a} . In the same way, the position vector OB is denoted by \underline{b} .

VECTORS

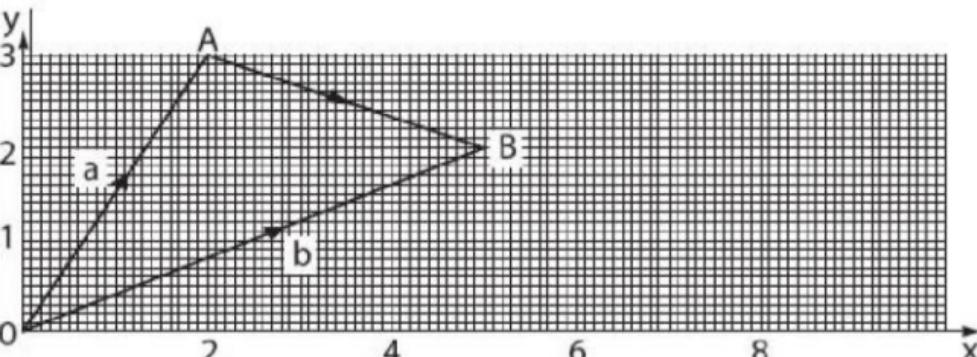


Fig 20.17

In the above figure, $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

$$\begin{aligned}\text{The column vector } \mathbf{AB} &= \mathbf{b} - \mathbf{a} = \begin{pmatrix} 5-2 \\ 2-3 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ -1 \end{pmatrix}\end{aligned}$$

Generally, if \mathbf{a} is the position vector of A

and \mathbf{b} the position vector of B , then the vector $\mathbf{AB} = \mathbf{b} - \mathbf{a}$.

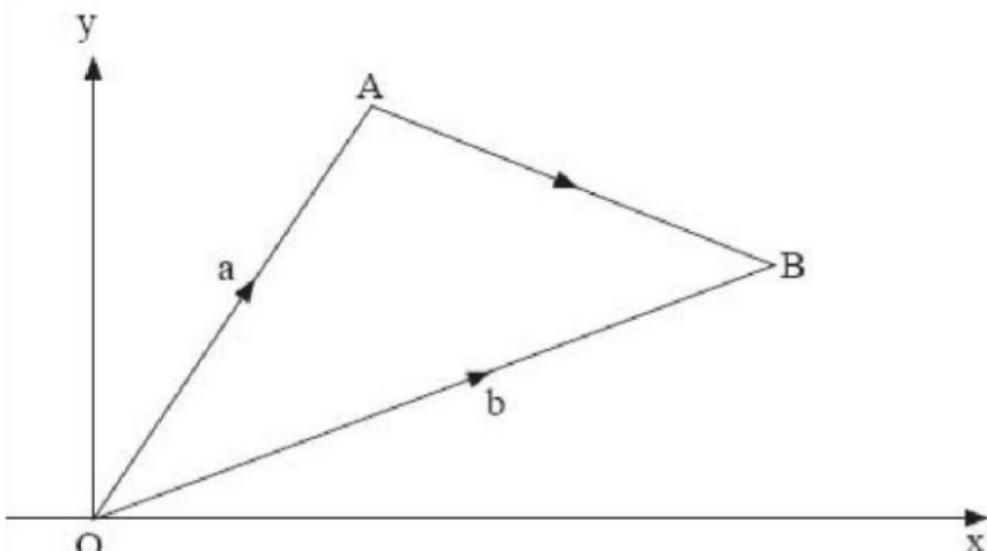


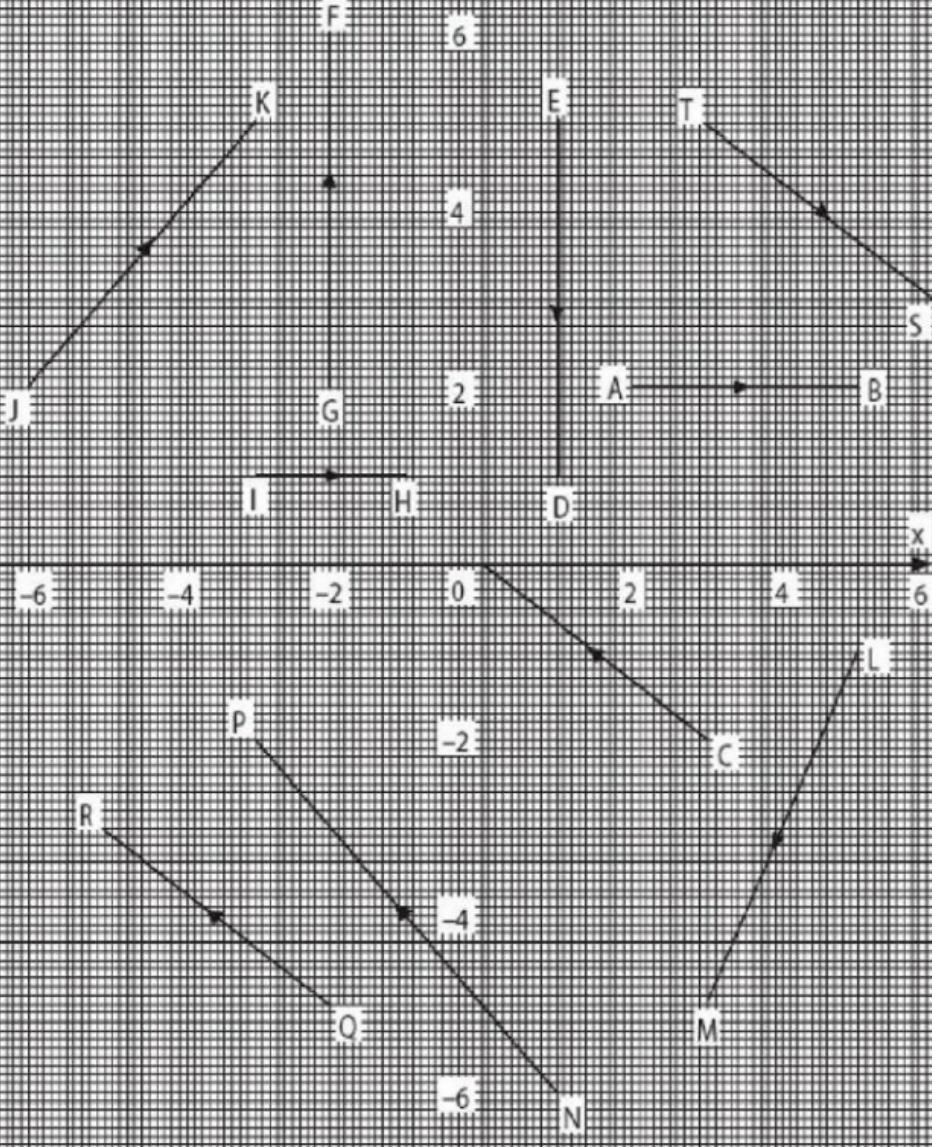
Fig. 20.18

In figure 20.18,

$$\begin{aligned}\underline{\overrightarrow{AB}} &= \underline{\overrightarrow{AO}} + \underline{\overrightarrow{OB}} = -\underline{\overrightarrow{a}} + \underline{\overrightarrow{b}} \\ &= \underline{\overrightarrow{b}} - \underline{\overrightarrow{a}}\end{aligned}$$

Exercise 20.3

1. Write each of the vectors below as a column vector.



2. Draw the following position vectors on squared paper.

$$\mathbf{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\mathbf{f} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\mathbf{g} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$\mathbf{c} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$\mathbf{h} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\mathbf{d} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{e} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

3. Use the figure in question 1 to write down the position vectors of points E, C, Q, L, P and L.

4. If

$\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$, find:

(a) (i) $3\mathbf{a} - 2\mathbf{b}$

$$(ii) 5\mathbf{b} - 3\mathbf{a}$$

$$(iii) \frac{1}{2}\mathbf{c} - \frac{3}{2}\mathbf{d}$$

$$(iv) 3\mathbf{b} + \mathbf{c}$$

$$(v) \frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} + \frac{1}{3}\mathbf{c}$$

$$(vi) 5\mathbf{b} + 7\mathbf{d}$$

$$(vii) 10\mathbf{a} + 15\mathbf{b} + 23\mathbf{d}$$

(b) The scalars r and s such that $r\mathbf{a} + s\mathbf{c} = 9\mathbf{b}$.

5. If $\mathbf{x} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ and $\mathbf{z} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, find $-\mathbf{y}$, given that $\mathbf{x} + \mathbf{y} = \mathbf{z}$.
6. Find the co-ordinates of P if $\mathbf{OP} = \mathbf{OA} + \mathbf{OB} - \mathbf{OC}$ and the coordinates of points A, B and C are $(3, 4)$, $(-3, 4)$ and $(-3, -4)$ respectively.
7. Find scalars m and n such that

$$m \begin{pmatrix} 4 \\ 3 \end{pmatrix} + n \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}.$$

20.8: Magnitude of a Vector

Figure 20.19 shows a right-angled triangle ABC. $AB = \sqrt{3^2 + 4^2}$

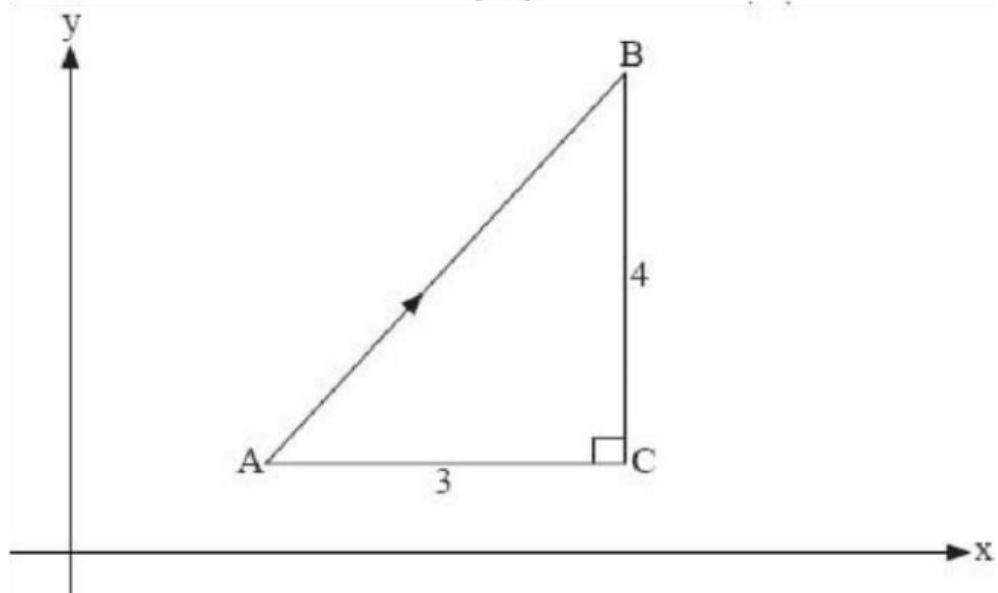


Fig. 20.19

Using Pythagoras' theorem, the length of line AB is:

$$\begin{aligned}\sqrt{3^2 + 4^2} &= \sqrt{25} \\ &= 5\end{aligned}$$

Hence, the magnitude of vector AB is 5. This can also be written as $|AB| = 5$. Generally, if $a = \begin{pmatrix} x \\ y \end{pmatrix}$, then the length or the magnitude of the vector a is given by

$$\sqrt{x^2 + y^2}.$$

Calculate the magnitude of each of the following vectors:

(i)

$$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

(ii)

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

(iii)

$$\begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 8 \\ 15 \end{pmatrix}$$

(iv)

20.9: Midpoints

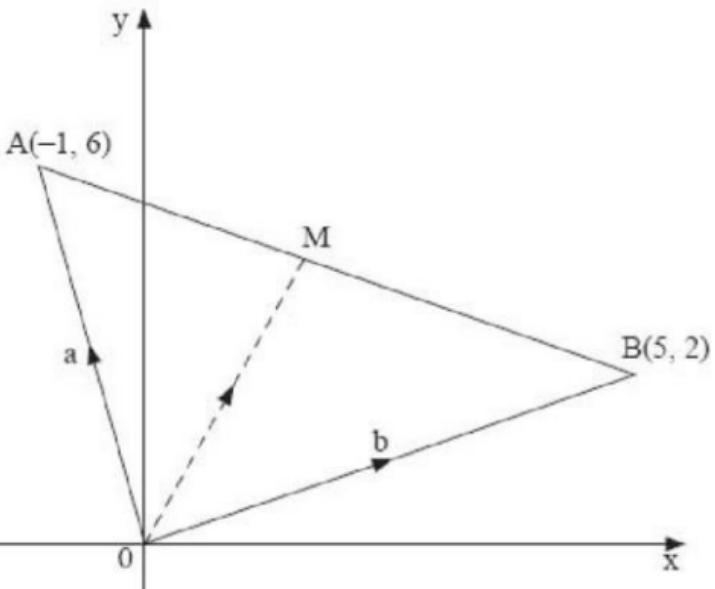


Fig. 20.20

In figure 20.20, M is the midpoint of AB.
We can use the vector method to get
the co-ordinates of M.

$$\begin{aligned}
 OM &= \overline{OA} + \frac{1}{2}\overline{AB} \\
 &= \begin{pmatrix} -1 \\ 6 \end{pmatrix} + \frac{1}{2} \left[\begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \end{pmatrix} \right] \\
 &= \begin{pmatrix} -1 \\ 6 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ -4 \end{pmatrix} \\
 &= \begin{pmatrix} -1 \\ 6 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} \\
 &= \begin{pmatrix} -1+3 \\ 6-2 \end{pmatrix} \\
 &= \begin{pmatrix} 2 \\ 4 \end{pmatrix}
 \end{aligned}$$

Therefore, the co-ordinates of M are (2, 4).

Consider the general case in figure 20.21, where M is the midpoint of AB.

VECTORS

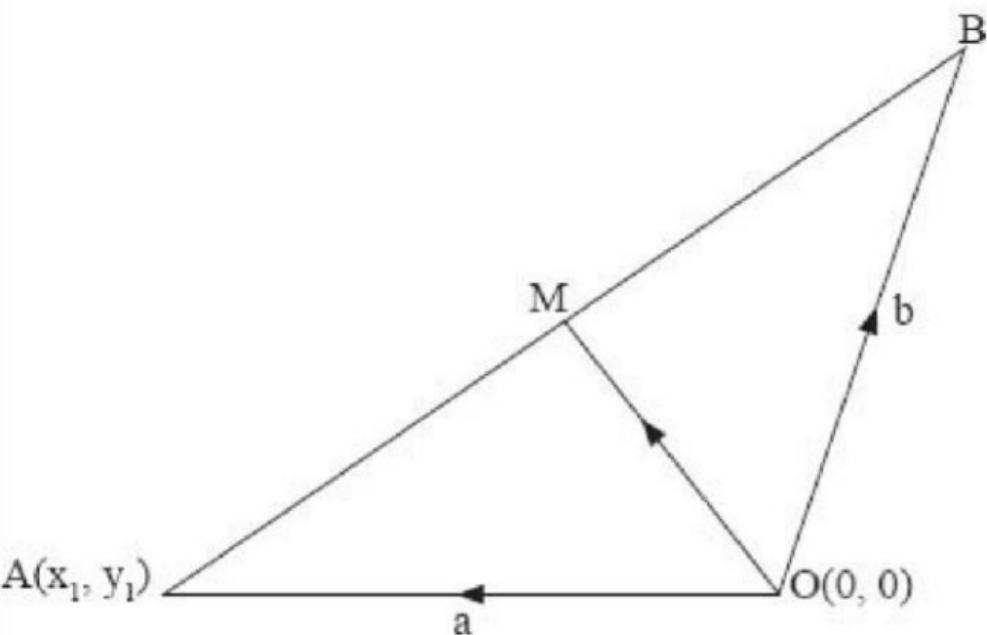


Fig. 20.21

$$\begin{aligned}
 OM &= OA + AM \\
 &= \mathbf{a} + \frac{1}{2}(\mathbf{AB}) \\
 &= \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) \\
 &= \mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} \\
 &= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \\
 &= \frac{1}{2}(\underline{\mathbf{a}} + \underline{\mathbf{b}})
 \end{aligned}$$

But $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

$$\text{Therefore, } \frac{1}{2}(\underline{\mathbf{a}} + \underline{\mathbf{b}}) = \frac{1}{2}\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right)$$

Then, M is the point $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
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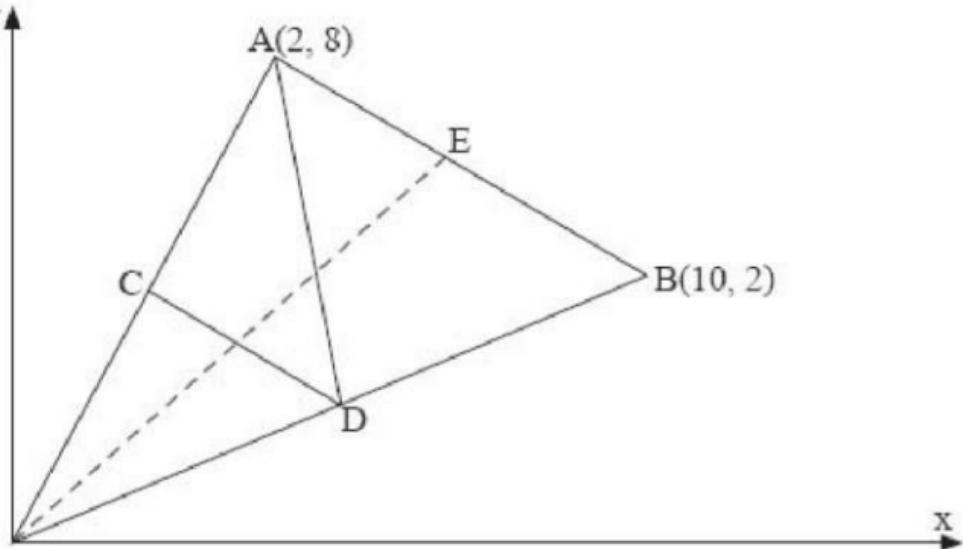
For example, in figure 20.20, since M is

the midpoint, the co-ordinates of M are $\left(\frac{-1+5}{2}, \frac{6+2}{2}\right)$, which is (2, 4).

Exercise 20.4

1. Find the co-ordinates of the midpoint of AB in each of the following cases:
 - (a) A (-4, 3), B(2, 0)
 - (b) A (0, 4), B(0, -2)
 - (c) A(-3, 2), B(4, 2)
 - (d) A(0, 0), B(a, b)

2. In the figure below, OAB is a triangle. A is the point (2, 8) and B the point (10, 2). C, D and E are the midpoints of OA, OB and AB respectively.



- (a) Find the co-ordinates of C and D.
- (b) Find the length of the vectors CD and AB.
3. Triangle OAB is such that $OA = \mathbf{a}$ and $OB = \mathbf{b}$. C lies on OB such that $OC : CB$

= 1:1. D lies on AB such that AD: DB = 1:1 and E lies on OA such that OA:AE = 3:1. Find:

(a) OC

(b) OD

(c) OE (d) CD

(e) DE

20.10: Translation

If a figure is moved in such a way that all its points move the same distance and in the same direction, the figure is said to have undergone a translation. In figure 20.23. $\Delta A'B'C'$ is the image of ΔABC under a translation.

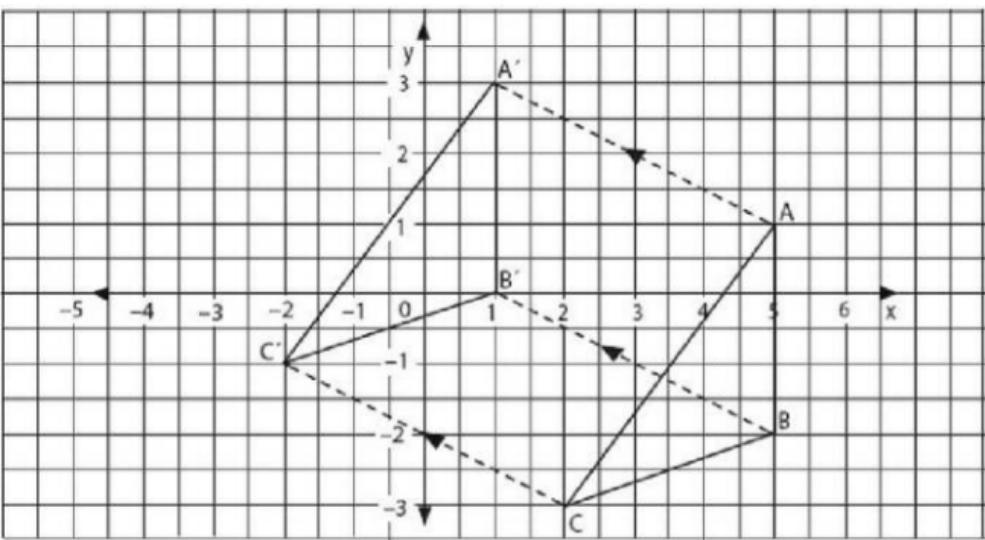


Fig. 20.22

Note:

- (i) All points on ΔABC move equal distances in the same direction. This translation is described by the vector

$$OA' - OA \text{ or } OB' - OB \text{ or } OC' - OC = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

- (ii) Under a translation, a figure and its image are directly congruent. If letter

T is used to denote a translation, the $T(P)$ represents translation T on P .

For example, if T is translation $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and P is the point $(3, 1)$, we can get P' , the image of P under T as follows:

The position vector of P is $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$. The position vector of P' , the image of P , is;

$$OP' = OP + T(P)$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -1 \end{pmatrix}$$

Therefore, the co-ordinates of P', are (8, -1).

Example 4

The points A (- 4, 4), B(-2, 3), C(- 4, 1) and D(-5, 3) are vertices of a quadrilateral.

If the quadrilateral is given the translation T defined by the vector $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$, draw the quadrilateral ABCD and its image under T.

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Solution

$$OA' = \begin{pmatrix} -4 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{ so } A' \text{ is } (1, 1)$$

$$OB' = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \text{ so } B' \text{ is } (3, 0)$$

$$OC' = \begin{pmatrix} -4 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \text{ so } C' \text{ is } \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$OD' = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \text{ so } D' \text{ is } (0, 0)$$

Quadrilaterals ABCD and A-B-C-D- are shown in figure 20.23.

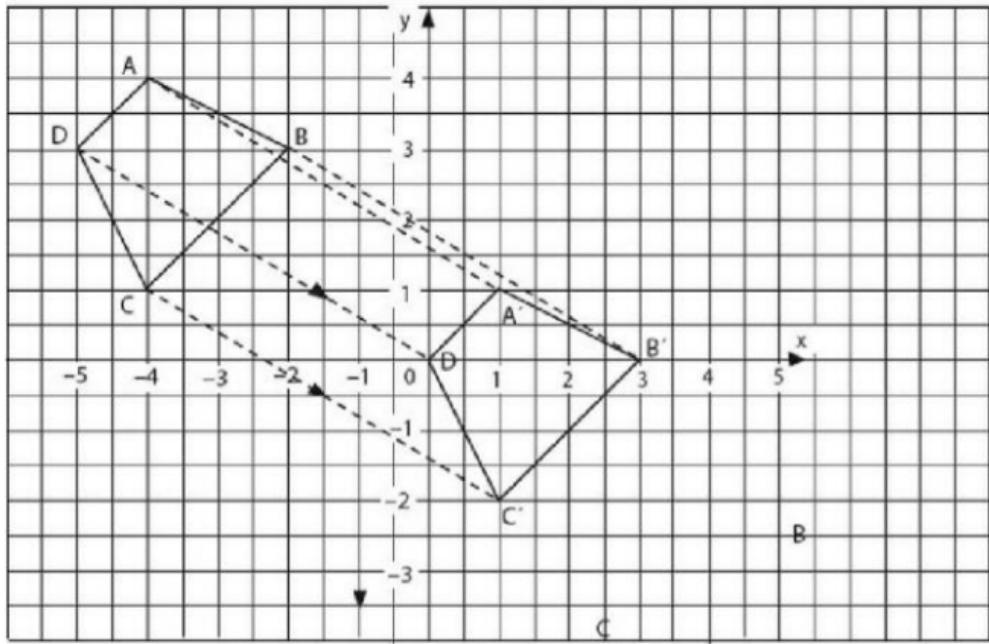


Fig. 20.23

Exercise 20.5

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1. The co-ordinates of A, B, C and D are $(-2, 5)$, $(1, 3)$, $(3, -2)$ and $(2, -4)$ respectively. If A' is $(-5, 6)$ under a translation T, find the co-ordinates of B' , C' and D' under T.
2. Draw ΔPQR with vertices $P(3, 2)$, $Q(5, 0)$ and $R(4, -1)$. On the same axes, plot $P'Q'R'$, the image of ΔPQR under a translation given by $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$.
3. A point $P(-2, 3)$ is given the translation $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$. Find the point P' , the image of P under the

translation. If P' is given a translation $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, find the coordinates of P'' , the image of P' . Find a single translation that maps P onto P'' .

4. The point $A(3, 2)$ maps onto $A'(7, 1)$ under a translation T_1 Find T_1 If A' is mapped onto A'' under translation T_2 given by $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ find the

co-ordinates of A' . Given that $T_3(A) = A''$, find T_3 .

5. For each of the points below, give the image when it is translated by the given vector:

(a) $(2, 3); \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(b) $(0, 0); \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

(c) $(-2, -4); \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(d) $(-4, 3); \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

(e) $(7, 0); \begin{pmatrix} -8 \\ -1 \end{pmatrix}$

(f) $(8, -2); \begin{pmatrix} -10 \\ 4 \end{pmatrix}$

6. Below are images of points which have been translated by the vectors

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indicated. Find the corresponding object points:

(a) $(2, 3); \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(b) $(0, 0); \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

(c) $(9, 17); \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(d) $(5, 6); \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

(e) $(7, -2); \begin{pmatrix} -5 \\ 2 \end{pmatrix}$

(f) $(-2, 4); \begin{pmatrix} 7 \\ 5 \end{pmatrix}$

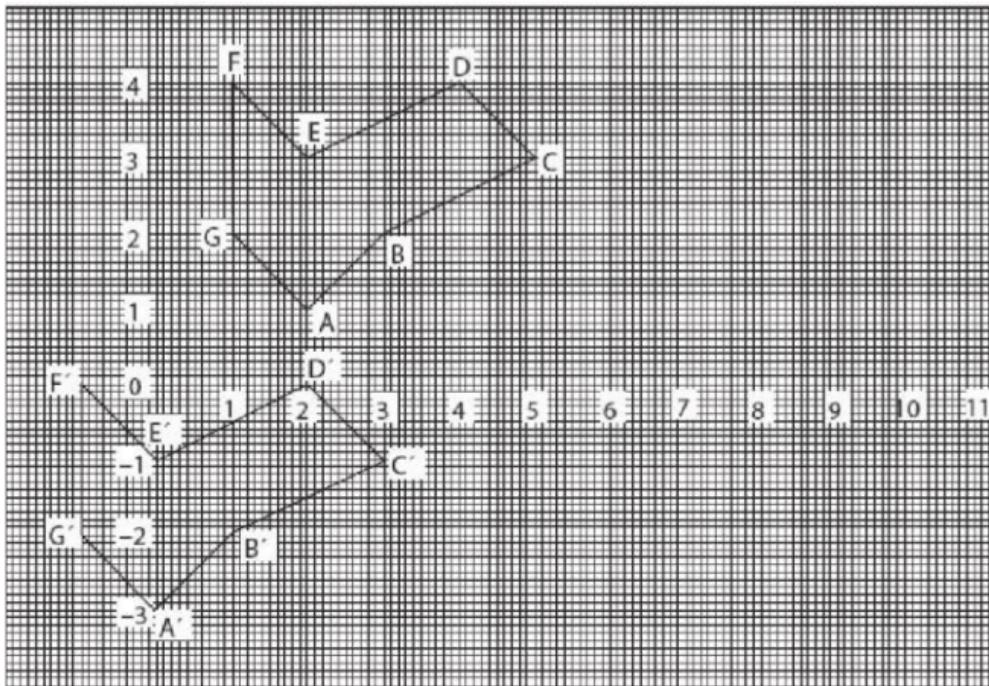
(g) $(3, 3); \begin{pmatrix} -4 \\ -10 \end{pmatrix}$

Compli $(-5, -11); \begin{pmatrix} -12 \\ -0 \end{pmatrix}$ **dfa :0770112040**

(i) $(4, 3); \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

7. The image of point $(6, 4)$ is $(3, 4)$ under a translation. Find the translation vector.

8. In the figure below A' , B' , C' , E' , F' and G are the images of A , B , C , D , E , F , G under a translation. Find the translation vector.



9. (a) Use squared paper to draw the images of PQRS in the figure below

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under the following translation vectors:

(i) $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

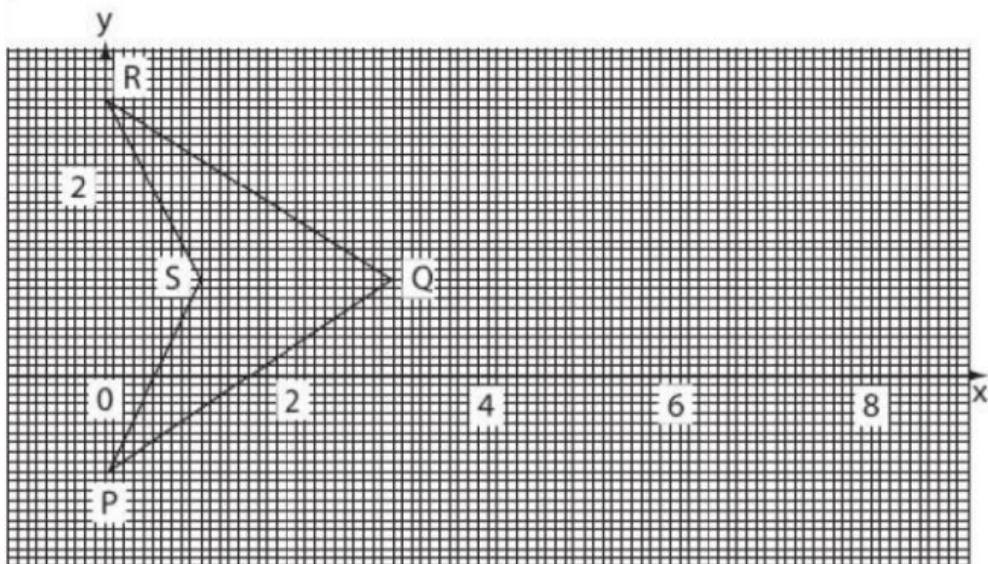
(ii) $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ (iii)

$\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

(iv)

(b) Find the translation vector which would map your image in (a) (i) to the image in (a) (iv) above.



(b) Find the translation vector which would map the image in (a) (iv) to that in (a)(i).

Mixed Exercise 3

1. A particle moves from rest and attains a velocity of 10 m/s after two seconds. It then moves with this velocity for four seconds. It finally decelerates uniformly and comes to rest after another six seconds.
 - (a) Draw a velocity-time graph for the motion of this particle.
 - (b) From your graph find:
 - (i) the acceleration during the first two seconds.

- (ii) the uniform deceleration during the last six seconds.
- (iii) the total distance covered by the particle.
2. The mean of the numbers m , $8m + 1$, 17 and 20 is 14. Calculate:
- the value of m .
 - the mode.
3. Ten casual labourers were hired by a garment factory for one week and

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were paid in shillings, according to the productivity of each, as below:

615, 633, 720, 509, 633, 710, 614,
630, 633, 720

Find the mean, mode and median for the data.

4. In a bakery, the mass of 20 loaves taken at random is 10.03 kg. If the mean mass of the first 13 loaves is 505 g, find the mean mass of the other 7 loaves.

5. The total marks scored in a test by 6 pupils was 420. If the mean mark for the first 5 pupils was 68, find the marks scored by the sixth pupil.
6. At a police check point, the speeds in km/h of the first 50 vehicles were recorded as follows:

Speed	Number of vehicles
10 – 19	3

20 – 29	1
30 – 39	2
40 – 49	5
50 – 59	6
60 – 69	11
70 – 79	9
80 – 89	8
90 – 99	3

100 – 109	2
-----------	---

- (a) Calculate the mean speed.
- (b) Draw a histogram and estimate the median.
- (c) On the same diagram, draw a frequency polygon.

7. The table below shows a record of the number of goals scored in various matches by a football team:

No. of goals	No. of matches
0	8
1	5
2	6
3	5
4	5
5	4
6	3

Calculate the mean number of:

- (a) goals per match.
 - (b) matches per goal.
8. Maina divided his farm among his four sons. The first son got twice as much as the second. The third got 2 hectares more than the first, while the fourth got 2 hectares less than the second. If on a circle graph, the share of the fourth son is represented by a sector of 48° , find the size of the shamba.

9. The heights of 40 pupils in a class were measured to the nearest centimetre and recorded as below:

175 154 157 180 165 150 152 162

173 168 169 181 177 179 175 169

151 153 156 158 163 169 179 180

145 149 150 156 171 175 176 178 169 160 155

174 170 176 182 170

(a) Use class interval of 5 to group the data starting with 145 – 149.

(b) Estimate the mean and the median height of the class.

10. The mean of a set of n numbers is 28. If an extra number, 18, is included in the set, the mean becomes 26. Calculate the value of n .
11. The mean of 100 numbers is 13. Each of the numbers is multiplied by 3 and 5 subtracted from the result, calculate the new mean.

12. The height in centimetres of 25 coffee shrubs in a certain farm was recorded as below:

109 120 114 121 150

149 125 106 148 140

109 108 130 138 142

148 147 107 136 139

132 135 109 116 128

Illustrate this information on a frequency table using a class interval of 5 cm starting with 105 – 109.

- (a) What is the modal class?
(b) Calculate the mean height.

13. Solve the equation: $\frac{1}{5}x^2 + 3x = -10$

14. The co-ordinates of points A, B and C are $(0, -4)$, $(2, -1)$ and $(4, 2)$ respectively.

- (a) Deduce the position vectors of A, B and C.

- (b) Find the lengths of AB and AC.
- (c) Show that the points A, B and C are collinear.
15. The points P (-2, 1), Q(1, 4) and R(3, 1) are vertices of a triangle PQR. If the triangle is given a translation T defined by the vector $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$, draw the triangle PQR and its image P'Q'R'.

16. A pentagon is described by the vectors
 $\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\mathbf{d} = \begin{pmatrix} -3 \\ 11 \end{pmatrix}$
 $\mathbf{e} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ in that order. What is the and relationship between the vectors c and e?

17. Solve the equation $x^2 = 4 (\frac{1}{2}x + 2)$

18. Simplify:

- (a) $\frac{49a^2 - 9b^2}{14a + 7ab + 6b + 3b^2}$
- (b) $\frac{(4x + 2m)^2 (-4m^2 + 16x^2)}{(4m^2 + 16x^2 + 16mx) (-2m + 4x)}$
- (c) $\frac{r^2 - k^2 + r + k}{r - k + 1}$
19. Use vectors to show that the points A (6, 1), B(3, 4) and C(1, 6) lie on a straight line. If O is the origin and E and F are the midpoints of OC and OA respectively, find the coordinates of E and F. Show that AC//EF.
20. The column vectors of two sides of a triangle PQR are given by $\mathbf{PQ} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

and $QR = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$. Write down the column vectors RP and PR. Determine the vectors representing the sides of the image of ΔPQR after a reflection in the:

- (a) x-axis.
- (b) y-axis.
- (c) line $y = 2x$.

21. Find possible values of x if $9^{x^2} =$

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$$27_{(2x+12)}$$

22. The point $P(-2, 5)$ is mapped onto $P'(1, 9)$ by a translation T_1 . If P' is mapped onto P'' by a translation T_2 given by $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$, find the co-ordinates of P'' . Given that $T_3(P) = P''$ find T_3 .
23. If $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 14 \\ 1 \end{pmatrix}$, find:
- $2\mathbf{a} - 3\mathbf{b}$.
 - $\frac{1}{2}\mathbf{a} - \mathbf{b}$.
 - Scalars x and y if $x\mathbf{a} - y\mathbf{b} = \begin{pmatrix} 16 \\ 11 \end{pmatrix}$.

24. Factorise the expressions:

$$(a) (2x + 2y)^2 - (x - y)^2$$

$$(b) x^4 - (2x - 3)^2$$

$$(c) (2x + y)^2 - (x - y)^2$$

$$(d) x^4 - (2x + 3)^2$$

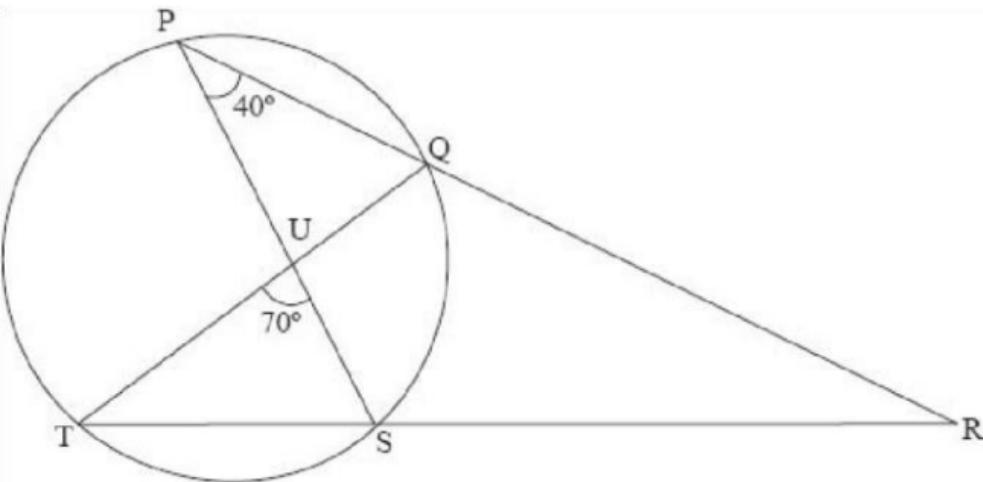
25. Solve the simultaneous equations;

$$2\mathbf{a} + 3\mathbf{b} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

$$-\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

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26. The dimensions of a rectangle in centimetres are $2n - 3$ by $n + 1$ and the area is 817 cm^2 . Determine the dimensions of the rectangle.
27. In the figure below, $\angle SPQ = 40^\circ$ and $\angle TUS = 70^\circ$. Calculate $\angle QRS$.



28. In quadrilateral ABCD, $\angle ADC = 115^\circ$, $\angle ACB = 40^\circ$, $\angle DBC = 35^\circ$ and $\angle BAC = 75^\circ$. Show that A, B, C D are concyclic. Hence, find:

(a) $\angle DAC$.

(b) $\angle DCA$.

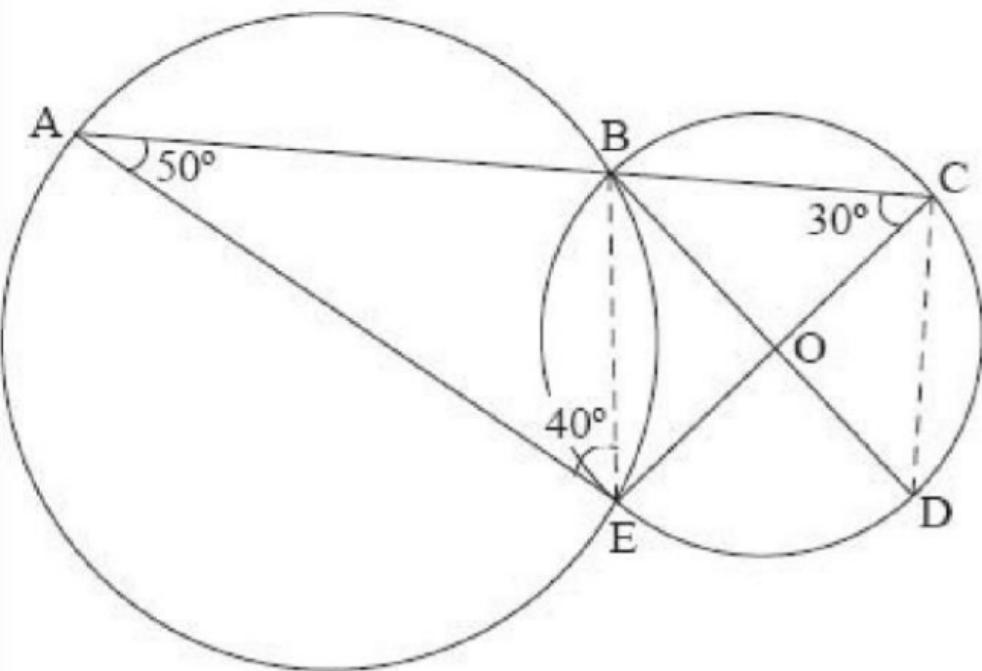
(c) $\angle BDC$.

29. In the figure below, O is the centre of circle BCDE, $\angle BAE = 50^\circ$ and $\angle BCE = 30^\circ$. Find:

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(a) AEB

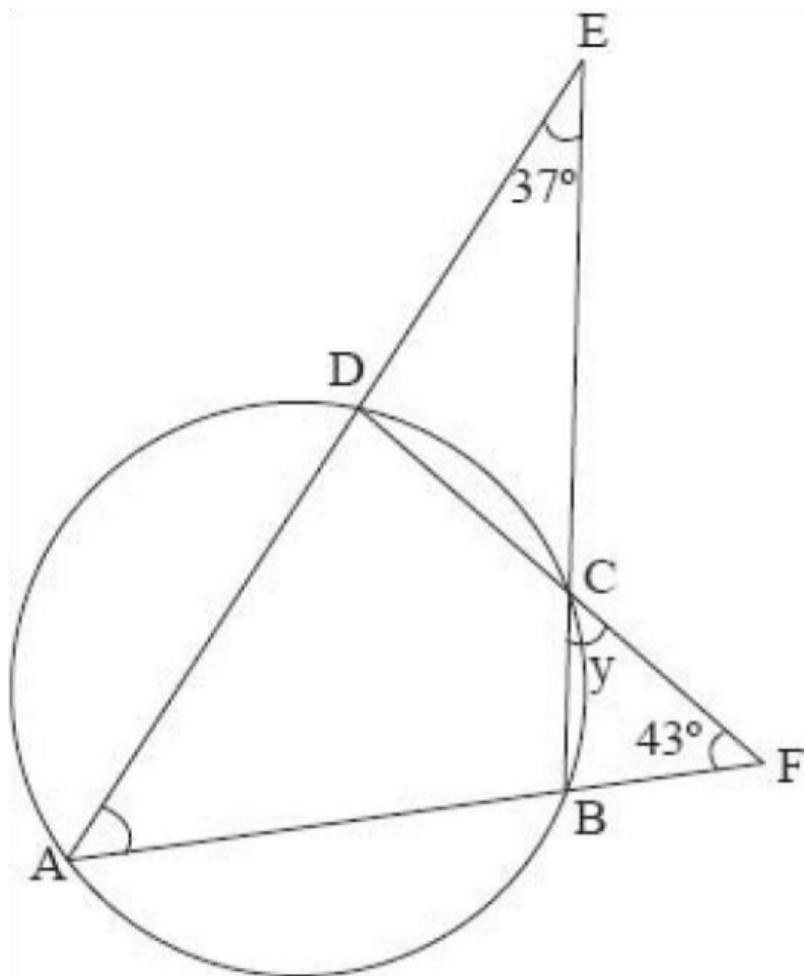
(b) BDC



30. In the figure below, ADE and ABF are straight lines.

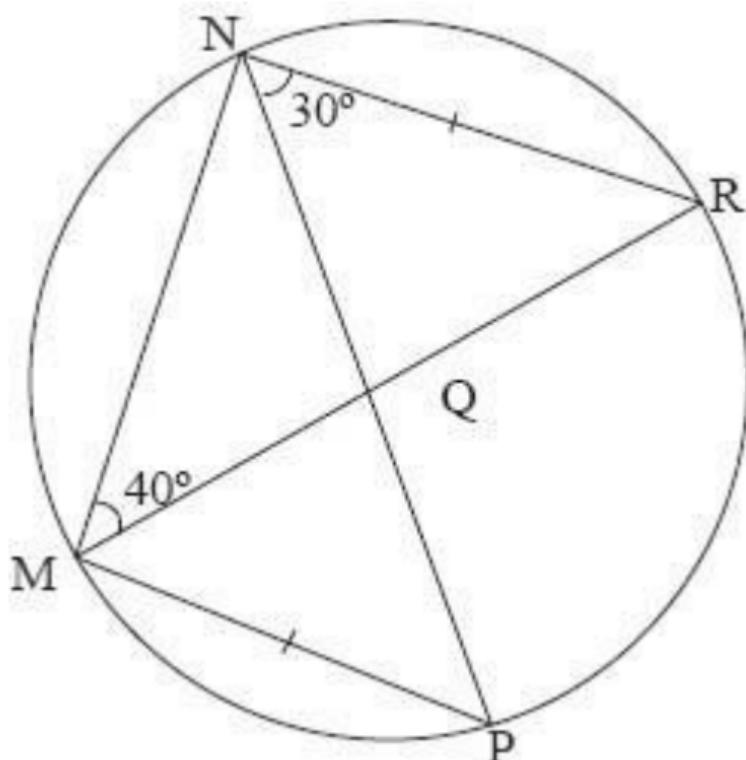
\angle DEC = 37° and BFC = 43°

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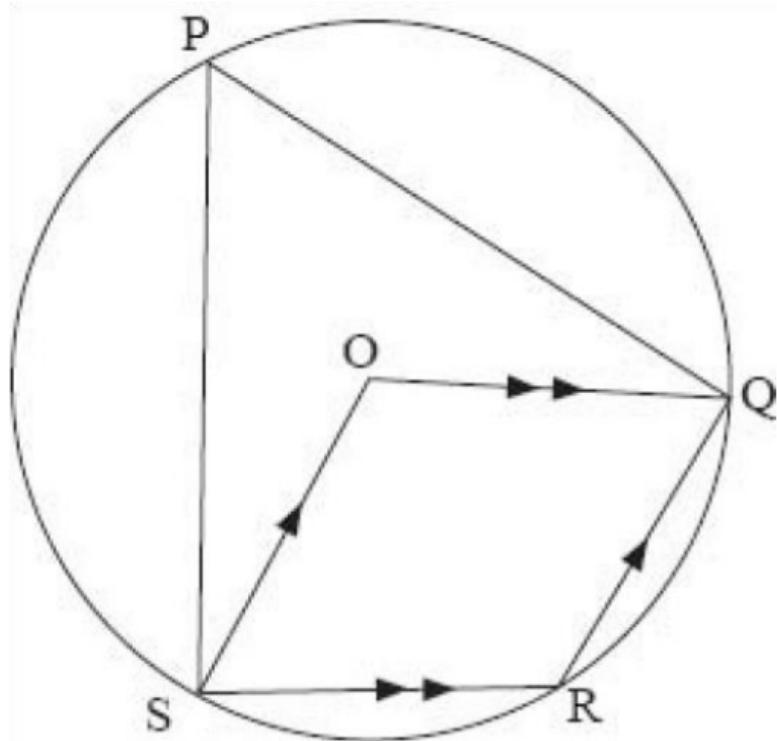


Calculate BCF

31. In the figure below, $QMN = 40^\circ$, $QNK = 30^\circ$, $MP = NR$ and MQR is a straight line passing through the centre of the circle. Find NRM and QNP .

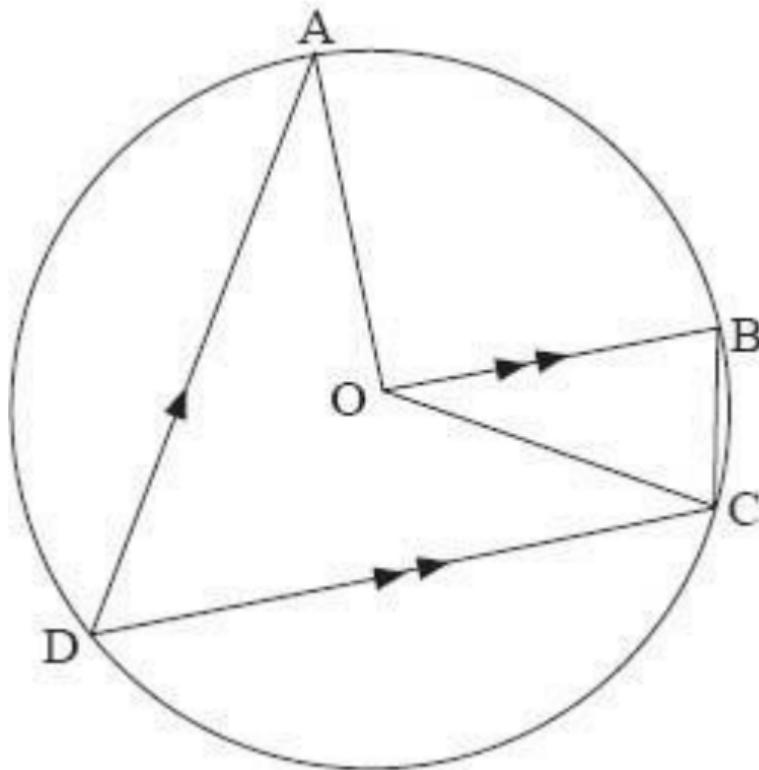


32. The figure below shows a circle PQRS with centre O. If OQ is parallel to SR and SO is parallel to RQ, show that $\angle SPQ = \angle RSO$.

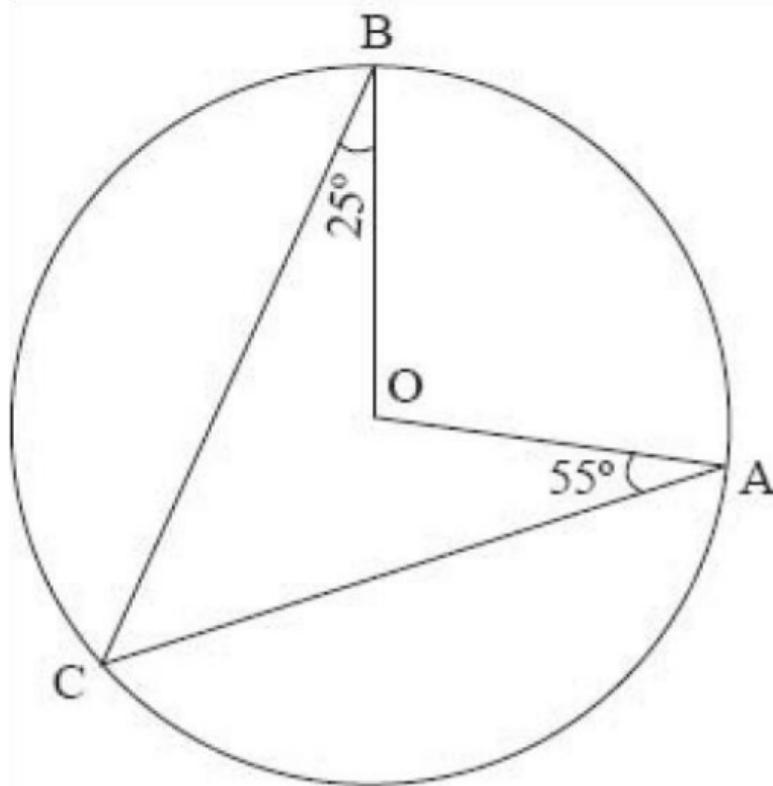


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33. The figure below shows a circle ABCD, centre O. Show that $AOC = 2OBC$.



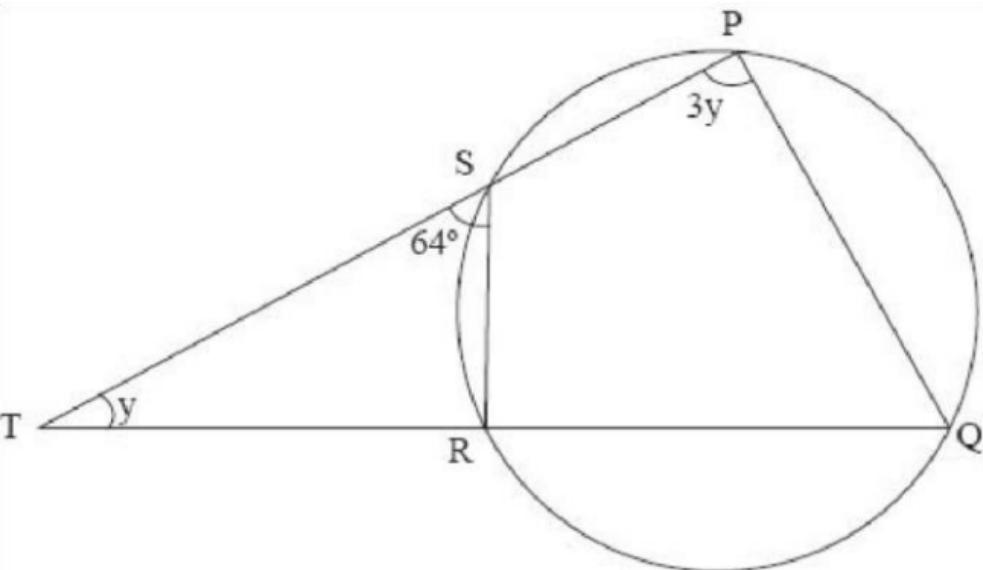
34. In the figure below, O is the centre of circle ABC, $OAC = 55^\circ$ and $OBC = 25^\circ$. Calculate the reflex AOB.



35. The region determined by the inequalities $x + y > 0$, $y \geq x$ and $2y - x \leq 6$ is reflected in the x-axis. Find the

three inequalities that satisfy the image.

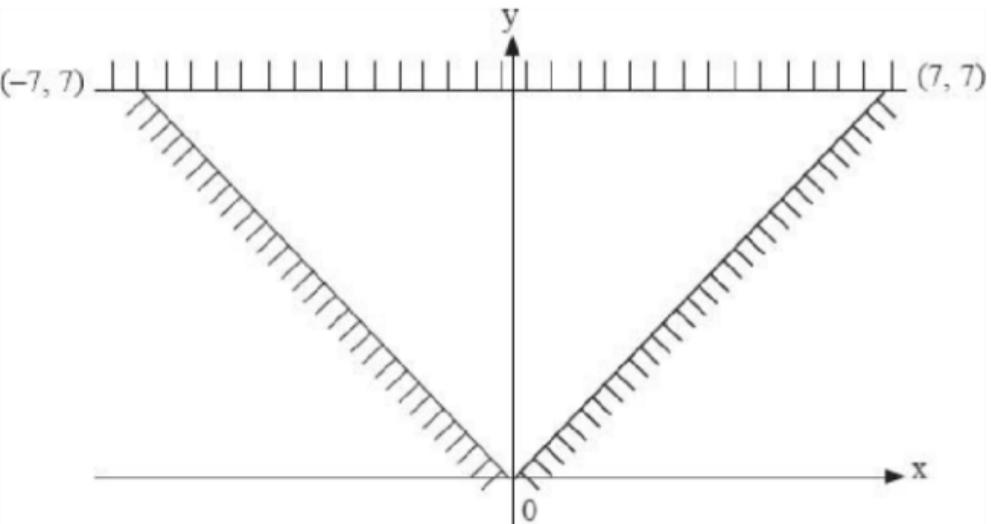
36. In the figure below, calculate the value of y



37. Represent the simultaneous inequalities $\frac{y}{3} + \frac{3}{8} \geq \frac{y}{4}$ and $\frac{y}{2} - 6 < \frac{y}{5}$ on a number line.
38. Represent on a graph the region which satisfies the four inequalities:
 $y \geq -1$, $y \leq x - 1$, $y \geq -x + 9$ and $y +$

$12 \geq 2x$. Determine the area of the region.

39. The unshaded region in the figure below is reflected in the x-axis. Write down the inequalities which satisfy the new region. Find the area of the region.



40. Represent the following inequalities on graph paper by shading out the unwanted region:

$$y \leq 2x + 1,$$

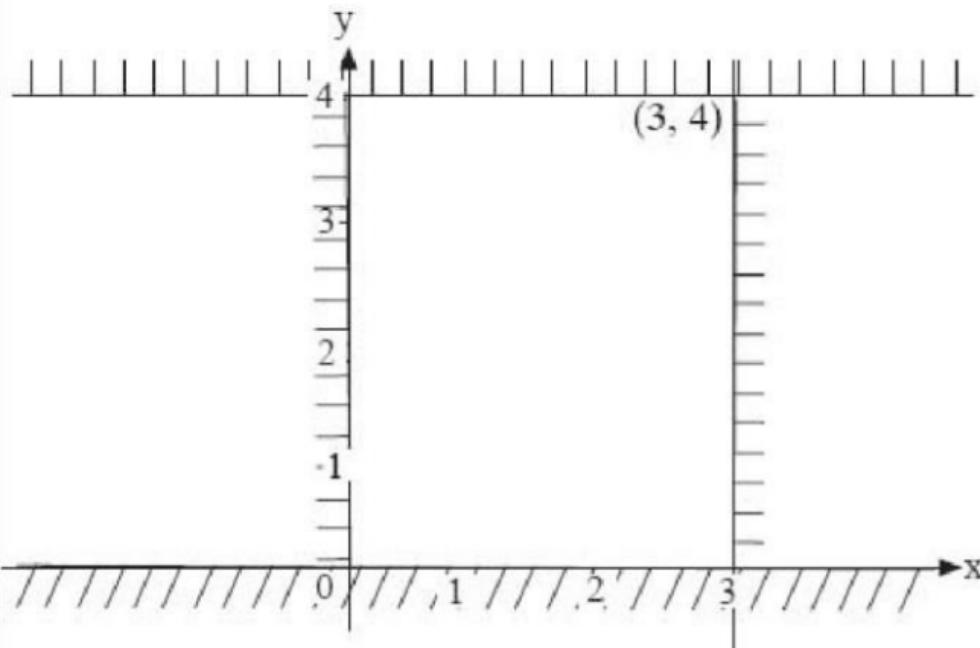
$$4y + x \leq$$

$$22, 2y \leq$$

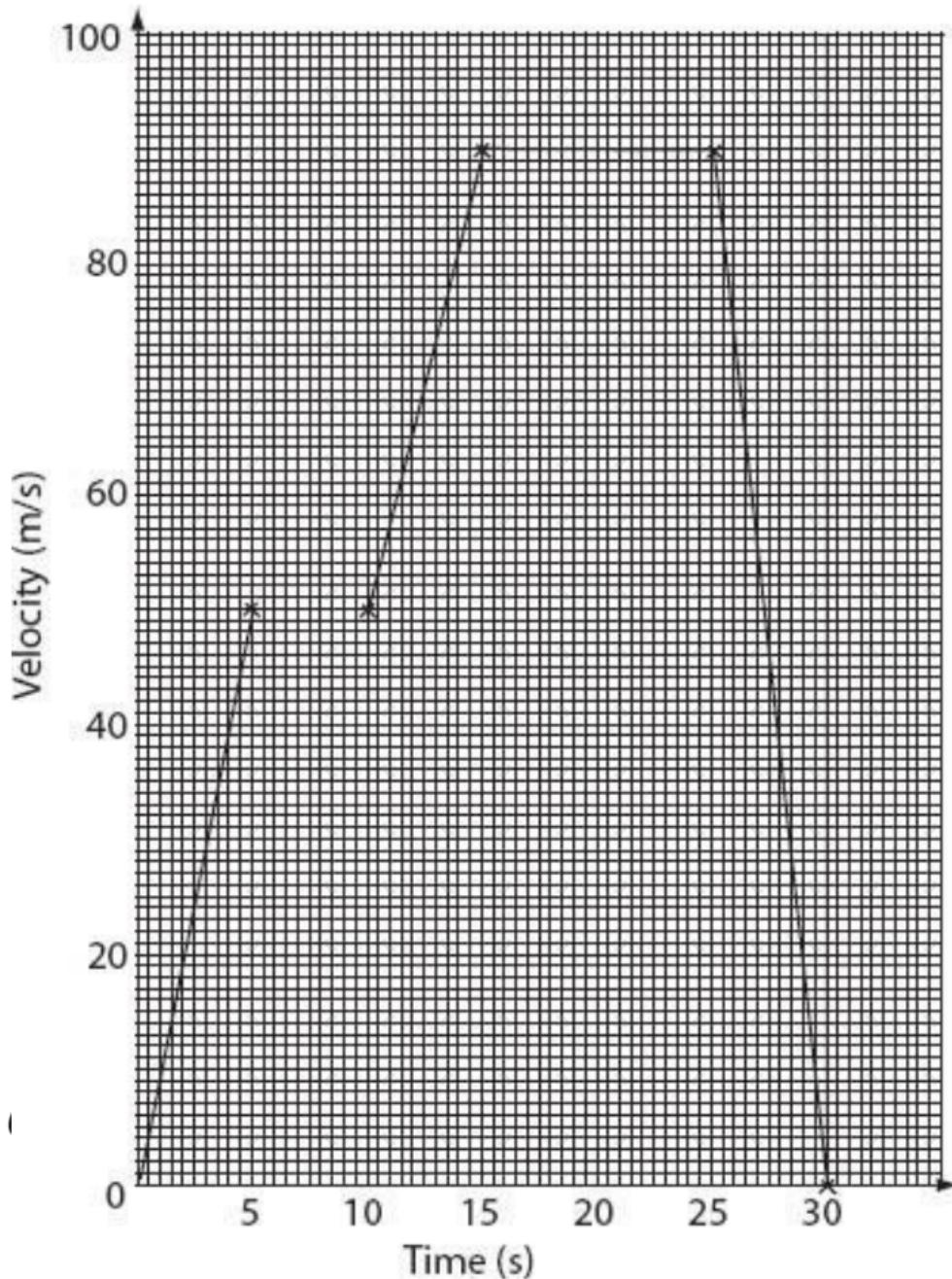
$$x - 5$$

If the straight line through the points $(5, 2)$ and $(6, 4)$ forms the fourth boundary of the enclosed region, write down the fourth inequality satisfied by the region (points on the line are in the region).

41. Write down the inequalities satisfied by the illustrated region in the figure below:



42. The following is a velocity-time graph of a moving object:



Use the graph to calculate the:

- (a) acceleration during the first five seconds.
- (b) distance covered in the first eight seconds.
- (c) total distance travelled.
- (d) acceleration during the 10th and the 15th seconds.

43. Find seven integral co-ordinates that satisfy all the inequalities below:

$$y + 3x \geq 5$$

$$3y + x \leq$$

$$15 - y$$

$$-x \geq 1$$

Find the area and the perimeter of the triangular region bounded by these inequalities

44. Find the area of the trapezoidal region bounded by the inequalities below:

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$$y - 2x \leq 1$$

$$3y - x > -2$$

$$2y + x < 7$$

$$2y + x \geq 2$$

45. A car which accelerates uniformly from rest attains a velocity of 30 m/s after 15 seconds. It travels at this constant velocity for the next 30 seconds before decelerating to stop after another 10 seconds. Use a suitable scale to draw the graph of

velocity against time and use it to determine:

the total distance covered by the car in the first 55 seconds.

- (b) the retardation in the last ten seconds.

Revision Exercises

Revision Exercise 1

1. Evaluate $4\sqrt{\frac{3.45 + 2.62}{786 \times 0.0007}}}$
2. The vertices of a kite are A (0, 0), B(5, 2), C(9, 9) and D (2, 5). Find the equation of the line of symmetry of

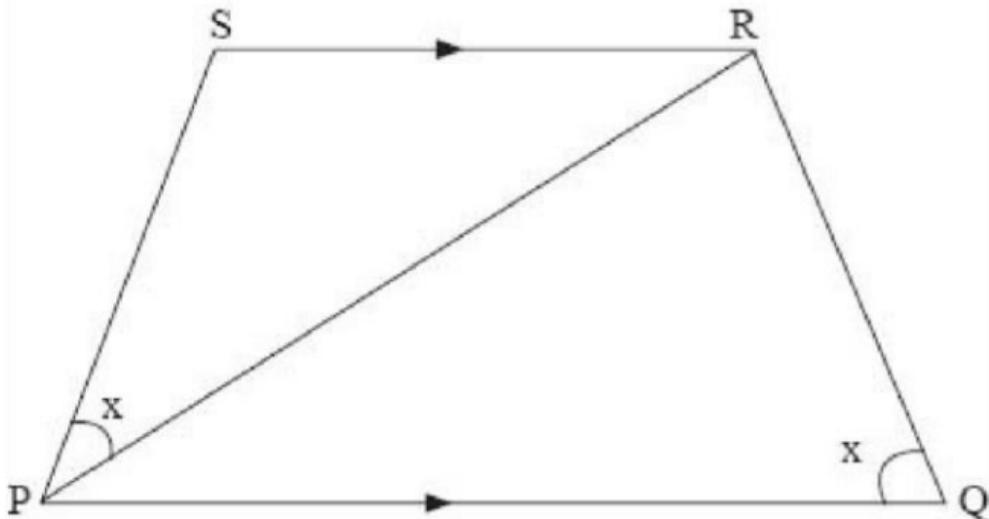
(a)

the kite. If ABCD is reflected in the line $x = 0$, find:

(a) the co-ordinates of the vertices of the image.

(b) the equation of the line of symmetry of the image.

3. In the figure below, PQRS is a trapezium and $\angle SPR = \angle PQR = x$.



- (a) Show that triangles SPR and PRQ are similar.
- (b) If $PQ = 18 \text{ cm}$ and $RS = 8 \text{ cm}$, find PR .
4. If $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, find:

(a)

$$2\mathbf{a} + \frac{1}{2}\mathbf{b}$$

(b) $3\mathbf{a} - 5\mathbf{b}$

(c) $\frac{7}{8}(3\mathbf{a} + \frac{1}{4}\mathbf{b})$

5. A right pyramid has a square base of side 12 cm and slant height of 20 cm. Calculate:

(a) its total surface area.

(b) its volume.

6. Solve the following inequalities and represent the solutions on a number line:

(a) $3x + 7 < -5$

(b) $\frac{t}{3} \leq \frac{5}{6}t - 4$

(c) $5(2r - 1) \leq 7(r + 1)$

(d) $6(x - 4) - 4(x - 1) \geq 52$

(e) $1\frac{3}{4}k - 1 \leq 3k - 2 \leq 6(\frac{2}{3}k + 2)$

7. The vertices of a quadrilateral are A(5, 1) B(4, 4), C(1, 5) and D (2, 2).

(a) Show that:

(a)

- (i) AB is parallel to CD.
- (ii) AD is parallel to CB.
- (iii) AC is perpendicular to DB.

8. The amount of milk produced per day by 20 cows, to the nearest litre, was recorded as below:

No. of cows	2	2	4	3	4	5
-------------	---	---	---	---	---	---

Amount of milk in litres	8	10	7	9	6	5
--------------------------	---	----	---	---	---	---

Find the mean number of litres produced by the cows.

- (b) State the mode of the data.
- (c) Draw a bar graph to represent the data.

9. Simplify:

$$(x + y)^2 + (y + z)^2 + (z + x)^2 - (x + y + z)^2$$

10. Two translations T_1 and T_2 are represented by the vectors $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and

(a) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ respectively. If L is the point (4, 5), find the image of L under:

- (a) T_1
- (b) T_2
- (c) T_1 followed by T_2

Revision Exercise 2

1. Give the co-ordinates of the images of the following points when rotated through -270° about $(4, 5)$:
- (a) $(-4, 0)$
 - (b) $(5, 7)$
 - (c) $(2, -1)$
 - (d) $(-2, -2)$
2. Write down the equations of the following lines, given the gradients and a point on the line:
- (a) $(0, 0), -\frac{1}{2}$

(a)

(b) (3, 0), 0

- (c) $(-1, 0), -1$
- (d) $(3, 2), -2$
3. The sum of two numbers exceeds a third number by four. If the sum of the three numbers is at least 20 and at most 28, find any three integral values satisfying the inequality.
4. Below are records of daily bookings into Karibu Hotel during the month of June 2003:

220 240 190 156 259 260 270
199 189 186 220 270 256 256

()

231 238 240 249 256 248 269

233 245 241 249 257 253 256

Find the mean number of people booked into the hotel each day.

5. Factorise each of the following expressions:

(a) $4x^2 - 6xy + 4xz - 6yz$

(b) $3p^2 - 8p + 4$

(c) $2s^2 + 3st - 2t^2$

(d) $64k^2 - 49n^2$

6. The volume of a right pyramid with a square base is 256 cm^3 . If its height is 16 cm, calculate:
- (a) the base area
 - (b) the side of the square base
7. Draw the following figures and indicate, where possible, their lines of symmetry:
- (a) Regular hexagon
 - (b) Rhombus

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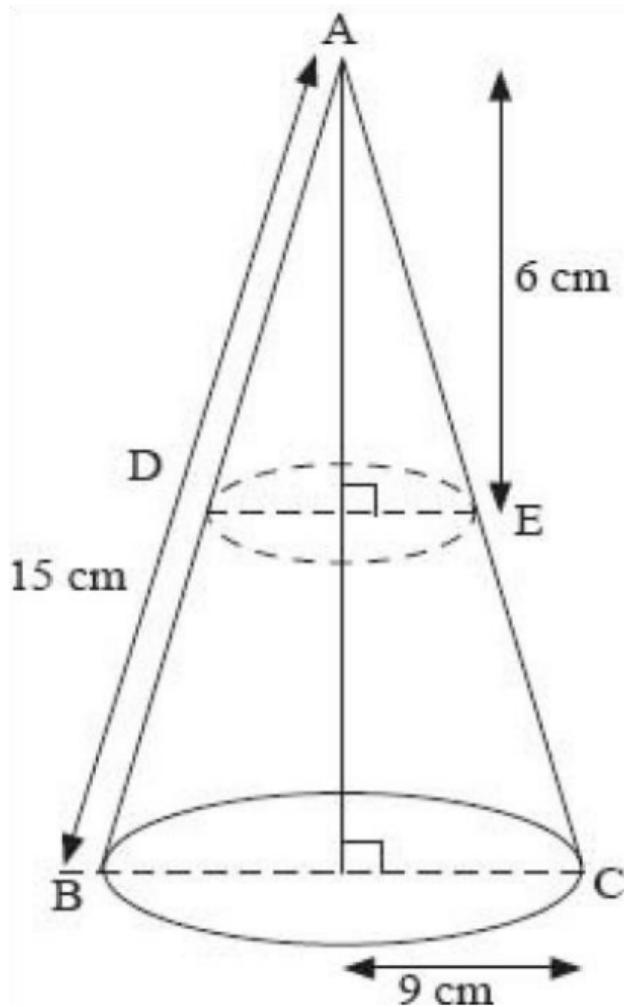
(c) Square

(d) Regular pentagon

(e) Circle

(f) Equilateral triangle

8. The figure below shows a cone with the vertex at A and diameter BC. The cone is cut off along DE: Find the:



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- (a) base radius of the smaller cone.
- (b) volume of the frustum.

9. The total surface area of two

metallic solid spheres is 116 cm^2 and their radii differ by 3 cm. Find the radius of each sphere. If the two spheres are melted and the material moulded into two spheres of equal radii, find the new total surface area.

Revision Exercise 3

1. The data below is a record of the time in seconds taken in a 100-metre race by 30 athletes:

10.5 11.4 11.3 12.6 12.8 10.8

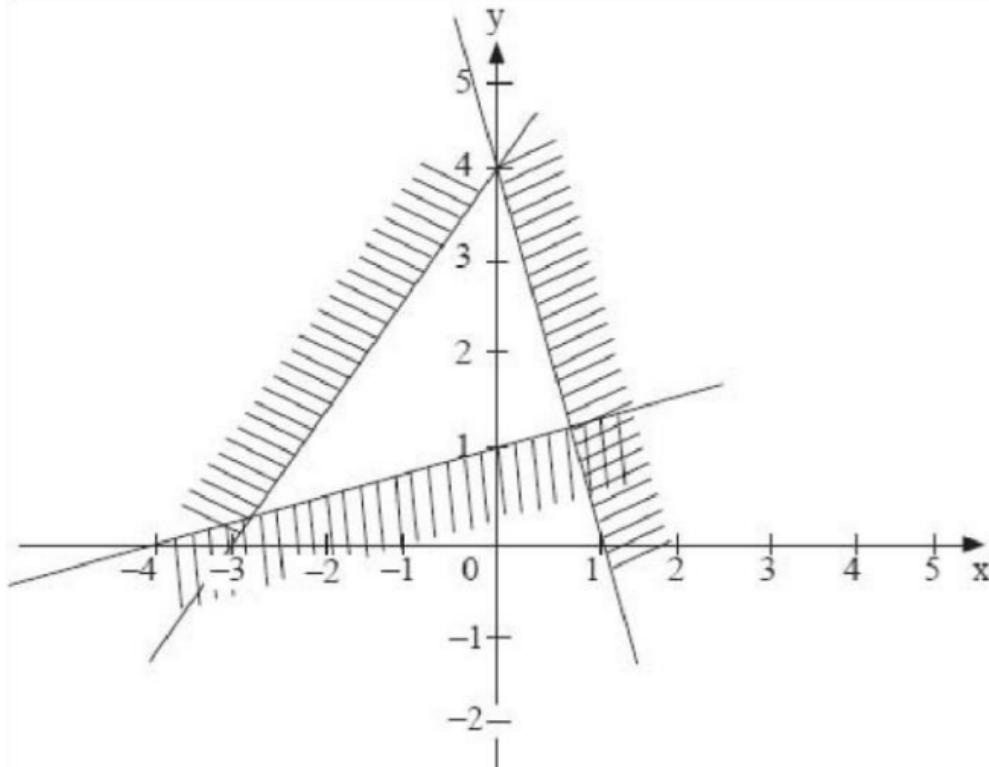
10.4	11.1	12.3	12.5	10.9	10.6
11.6	11.5	11.6	11.6	11.9	11.6
10.7	11.0	11.2	12.2	11.5	11.8
13.2	11.2	11.8	12.7	11.0	11.3

- (a) Group the data using class intervals of 0.4 starting from 10.4 – 10.7.
- (b) Find the mean time taken for the race.

- (c) Estimate the median of the data.
2. A triangle with vertices at $P(2, 3)$, $Q(-3, 3)$ and $R(0, 5)$ is first rotated about $(0, 0)$ through $+90^\circ$, followed by a reflection in the line $y = x$. If it is finally translated through vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$, find the co-ordinates of the vertices of the image.
3. Find the vertices of a triangle defined by the intersection of the lines:

$$y = x, \frac{y-1}{x-5} = -3 \text{ and } 3y - 6 = 2 - x.$$

4. In the figure below, find the inequalities which describe the unshaded region.



5. The LCM of 15, 18 and a third number is 1 260. Find the third number.

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6. A cone has a diameter of 28 cm and a height of 15 cm, calculate:
- (a) its total surface area.
 - (b) its volume.
7. Points A (-6, 10), B(-7, 8), C (-6, 6), D(-4, 6), E(-3, 8) and F(-4, 10) are the vertices of a regular hexagon. The hexagon is reflected in the line $y = x + 4$, followed by a reflection in the line $y = -x$. Find the co-ordinates of the vertices of the final image of the hexagon.

8. A polygon with vertices A(6,2), B(6, 4.5), C(5, 4), D (4, 4.5), E(4, 2) and F(5, 2.5) is enlarged by scale factors (i) 2 (ii) -1 with centre (5, 0). Find the coordinates of its image in each case.
9. Solve the equation:

$$(p + 1)^2 + 3p - 1 = 0$$

Revision Exercise 4

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1. Write down the gradients of the following lines without drawing them:
 - (a) $3x + 2y - 6 = 0$
 - (b) $4x = 6 - 2y$
 - (c) $y + y = 1$
 - (d) $x = -4$
 - (e) $y = 7$
2. If $2^{x+y} = 16$ and $\frac{1}{4}$, find the values of x and y

3. A triangle with the vertices at A (5, -2), B(3, -4) and C (7, -4) is mapped onto a triangle with the vertices at A'(5, -12), B'(7, -10) and C'(3, -10) by an enlargement, Find the linear scale factor and the centre of the enlargement.
4. Points P (2, 3), Q(4, 5) and R (7, 6) are vertices of triangle PQR. Find the coordinates of N, M and D, the

x

midpoints of PQ, QR and PR respectively.

5. Find non-zero scalars k and h such that $ka + hb = c$, given that:

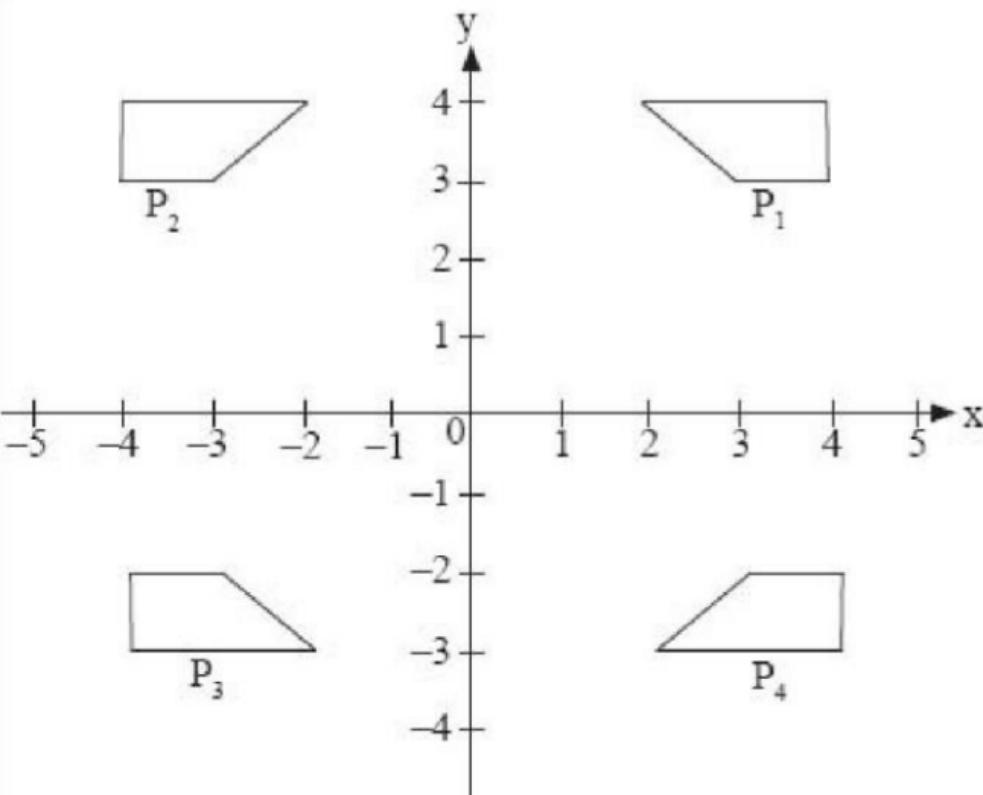
$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \text{ and } \mathbf{c} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}.$$

6. Calculate the surface area and volume of a sphere of radius 14 cm.

e

7. In the figure below, state which of the figures P_1 , P_2 , P_3 and P_4 are:
- (a) directly congruent.
 - (b) indirectly congruent.

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8. Solve the simultaneous inequalities:
 $7x - 8 \leq 342$ and $4 - 1 \geq 135$.

e

9. The masses of 50 babies born in a certain hospital were recorded as below:

Mass kg	2.1– 2.3	2.4– 2.6	2.7– 2.9	3.0– 3.2	3.3– 3.5	3. 3
No. of babies	4	6	11	13	9	6

(a) Find the mean mass of the babies.

(b) Estimate the median mass.

10. AC is the diameter of a circle ABCD. AB = $(x - 2)$ cm, BC = $(x + 5)$ cm and the area of ABC is 30 cm^2 . Find the radius of the circle. If ACD is isosceles, find its area.

Revision Exercise 5

e

1. Illustrate the region defined by the following inequalities on graph paper:
 - (a) $y \leq 2x + 1$, $y \geq 4 - x$ and $x \leq 3$.
 - (b) $-2 < x \leq 5$, $2y < 3x + 6$, $y \leq x - 5$.
2. PQRS is a square. Side QR is extended on either side so that QT = RV. If PSV = SPT, show that TS = PV.
3. The internal and external diameters of a spherical shell are 12 cm and 18

dd
cm respectively. Calculate the volume of the material of the shell.

4. Use vectors to show that a triangle with vertices $P(2, 3)$, $Q(6, 4)$ and $R(10, 5)$ is isosceles.
5. The number of vehicles passing through a check-point were recorded for 20 days as follows:

670	840	560	680	674	589
651	496	645	653	605	673
710	702	317	681	627	731
668	726				

Find the mean number of vehicles which passed through this checkpoint per day.

6. The ratio of the base areas of two cones is 9:16.
 - (a) Find the ratio of their volumes.
 - (b) If the larger cone has a volume of 125 cm^3 , find the volume of the smaller cone.
7. Use gradient to show that a triangle whose vertices are A (1, 3), B(4, -6)

e

and C(4, 3) is right-angled. Find the area of this triangle.

8. Given that $3^{5x-2y} = 243$ and $3^{2x-y} = 3$, calculate the values of x and y.
9. Factorise $9x^2 - 16y^2 + (3x - 4y)^2$.

Revision Exercise 6

1. The following were the marks scored in English and Mathematics tests by 40 pupils:

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Marks	0– 9	10– 19	20– 29	30– 39	40– 49
	9	19	29	39	49

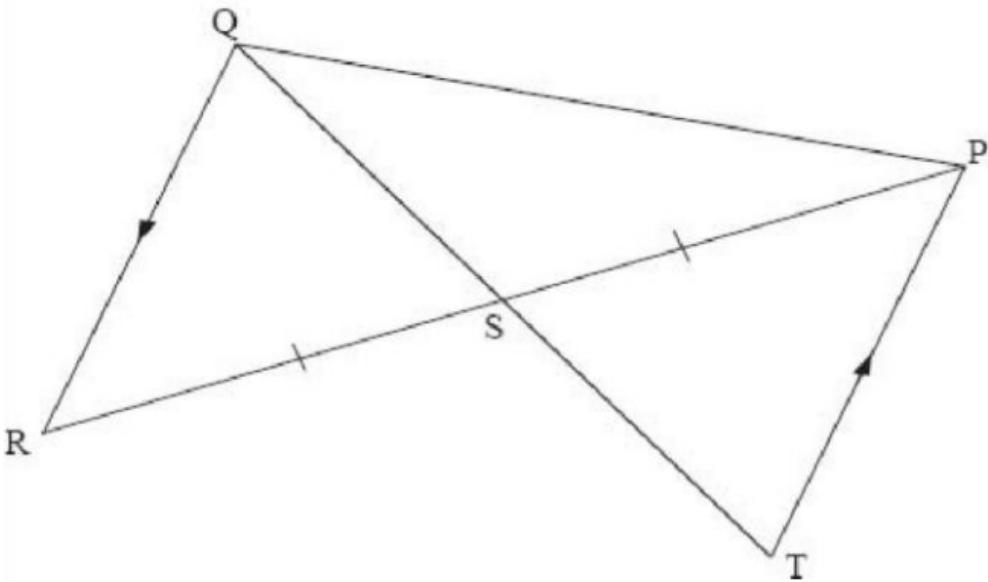
English	2	6	6	4	6
Mathematics	3	3	2	5	7

- (a) Draw two histograms on the same axes, showing the marks scored for the two subjects.
- (b) Draw two frequency polygons on the same axes showing the above data.
- (c) Calculate the mean mark in each case.

e

2. The depths of two similar buckets are 28 cm and 21 cm respectively. If the larger bucket holds 3.1 litres, find the capacity of the smaller one in litres.

3. Find the equation of a line which is parallel and midway between the lines $\frac{y}{x-3} = 1$ and $\frac{y-2}{x-1} = 1$.
4. Find all the points having integral co-ordinates which satisfy the three inequalities: $y \geq 6 - x$, $y \leq x + 4$ and $x < 4$.
5. In the figure below, $RQ \parallel TP$ and $RS = SP$ Show that triangles PST and RSQ are congruent.



6. Find the area of a triangular piece of land whose sides are 7m, 9 m and 14 m.

7. ABCDEFGH is a cube of side 8 cm. If N is the midpoint of AB, find the area of triangle HNG.
8. Three spacecraft in different orbits go around the earth at intervals of 3,6 and 7 hours respectively. An engineer at an observatory on earth first observes the three craft cruising above one another at 6.35 am. At what time will they be next observed in a similar configuration if they all revolve around the earth from east to west?

9. Find a if $a^4 - 15a^2 - 34 = 0$.
10. The vertices of a square are $P(-3, -3)$, $Q(-1, -1)$, $R(-1, -3)$ and $S(-3, -3)$. If the square is first reflected in the line $y = -x$ and followed by a translation of vector $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$, find the co-ordinates of the final image of the square.

Revision Exercise 7

1. Calculate the radius of a sphere

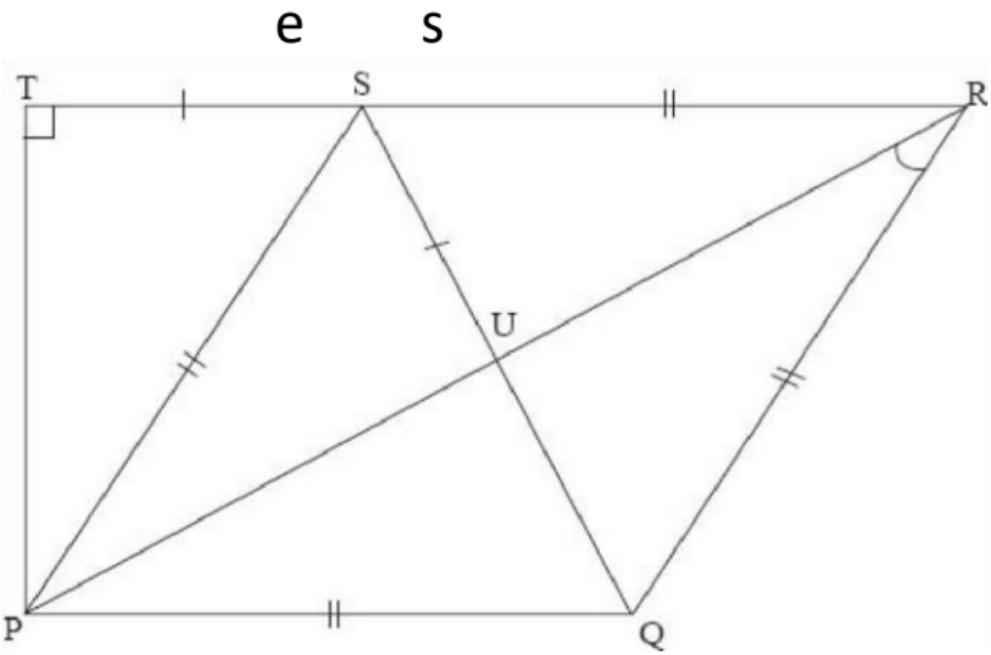
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whos volume i 259 cm^3 . (take $\pi = 3.142$)

2. A translation T maps a point $(4, 1)$ onto the point $(-2, 6)$. R is a halfturn rotation about the point $(-1, 2)$. A triangle with the vertices at $P(2, 2), Q(2, -4)$ and $R(-7, -7)$ is given a translation T followed by the rotation R. Find the coordinates of the first and final images of the triangle.

3. Find the region satisfied by the inequalities; $y \geq 0$, $y + x < 5$ and $2x + y \geq 4$.
- (a) State the points of intersection of the boundary lines.
- (b) Find the area of the region.
4. In the figure below, PQRS is a rhombus. If $TS = SU$, show that $\angle TPS = \angle PRQ$ and that $\triangle SUR$ is congruent to $\triangle QUP$.

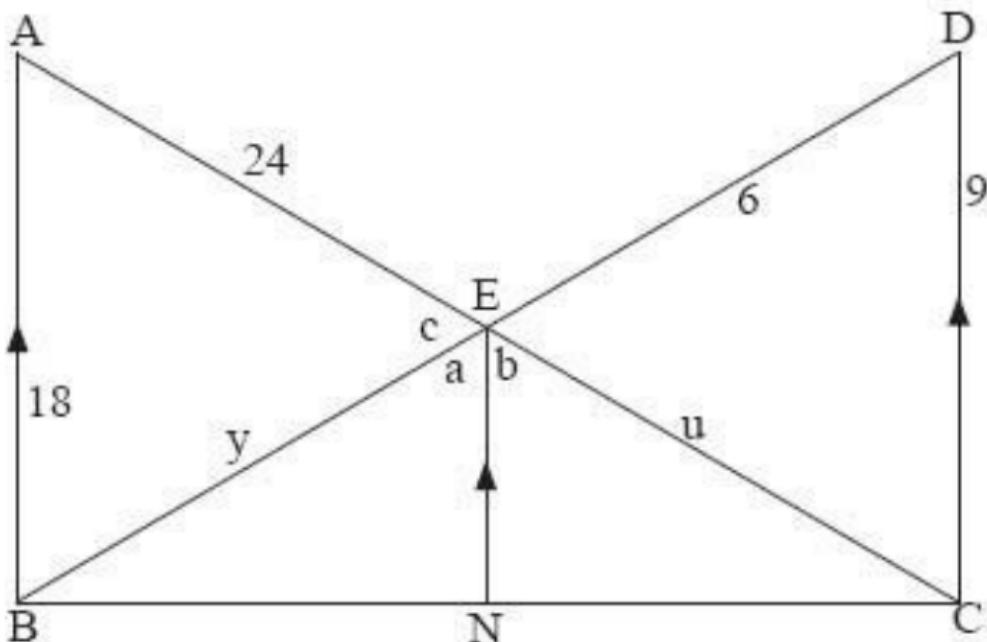


5. Find the area of a parallelogram ABCD in which $AB = 12 \text{ cm}$, $BC = 16 \text{ cm}$ and $\angle ABC = 25^\circ$.
6. The mean mass of x objects is $x \text{ kg}$ and the mean mass of y objects is $y \text{ kg}$. Show

that the mean mass of the remainder is $x + y$.

7. In the figure below, name similar triangles and find the lengths of the unknown sides.

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8. A triangle whose vertices are A (4, 6), B(3, 4) and C (5, 4) is enlarged with scale factor -1 and centre of enlargement (4, 0). The image is then reflected in the line $y = -x$

followed by an enlargement with linear scale factor -1 and centre $(0, 0)$. Find the co-ordinates of the vertices of the final image.

9. A triangle whose vertices are $P(1, 1)$, $Q(2, 1)$ and $R(1.5, 2)$ is first rotated about $(0, 0)$ through 180° followed by an enlargement scale factor 3 with the centre at the origin. Find the co-ordinates of the vertices of the final image.

e s

10. By substituting $x + 2y = p$ in the equation $(x + 2y)^2 + 3(x + 2y) - 4 = 0$, solve the quadratic equation in p .

Hence, find possible values of $x + 2y$. If $x - 2y = 4$, find all the possible values of x and y .

Revision Exercise 8

1. The logarithms of the squares of a and b are 1.204 and 0.954 respectively. Find the logarithm of their product.

2. Find the number of cubes, each of side 19 cm, that can be made from a rectangular sheet of metal measuring 2.28 m by 1.52 m.
3. A trapezium ABCD is such that AD is parallel to BC. Given that $AD = 18$ cm, $BC = 26$ cm and $AB = 9$ cm. Calculate its area.
4. Find the area of the region bounded by the inequalities $y \leq \frac{1}{2}(2x + 4)$, $y <$

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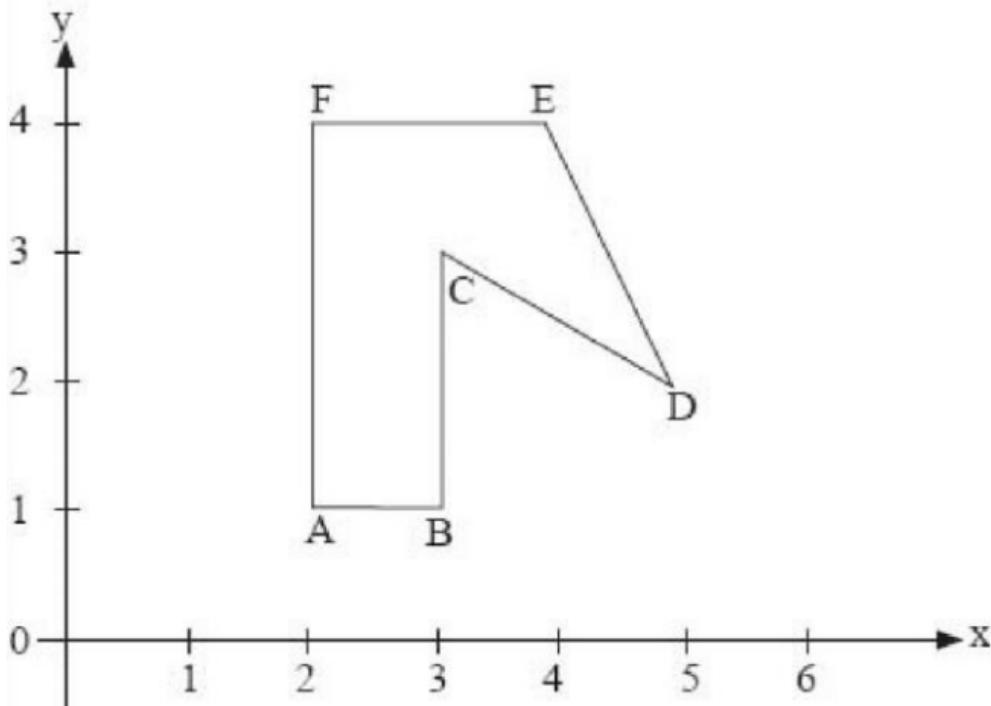
$3(2-x)$ and $y > 0$. If the region is reflected in the line $x = 0$ followed by a reflection in $y = 0$, write down the inequalities satisfying the final image.

5. The length L of a pendulum whose complete swing takes a total time T is given by the formula $L = \frac{T^2 g}{4\pi^2}$. Calculate L when $T = 2.31$, $g = 9.81$ and $\pi = 3.142$.
6. Two spheres have surface areas of 36 cm^2 and 49 cm^2 respectively. If the volume of the smaller sphere is

20.2 cm^3 , calculate the volume of the larger one.

$\frac{1}{3}$

7. Simplify: $\frac{(a - c)^2 - (a + c)^2}{(a^2 + c^2)^2 - (a^2 - c^2)^2}$. Find the value of the expression when $a =$
and $c = \frac{1}{17}$.
8. The figure below shows an irregular hexagon ABCDEF:



T is a translation given by vector

$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, Q is a rotation about (0, 0) through +90° and X is a reflection in the x-axis. Find:

- (a) the gradient of the image of AC under T.
- (b) the equations of CD and ED after T, followed by X, followed finally by Q.

9. A pond holds 27 000 litres of water. Find the number of litres of water a similar pond would hold if its

dimensions were double the first one.

10. (a) A shopkeeper makes a profit of Sh 200 by selling 20 blankets and 50 towels. He makes a loss of Sh 7 if he sells 35 blankets and 28 towels of the same kind. Determine the profit (or loss) on each of the two items sold.

(b) If he sold 12 blankets, find the number of towels he should sell to realise an overall profit of Sh 3012.

This is the second students' title in the KLB Secondary Mathematics series. It has been extensively updated so as to cater adequately for the new Form Two Secondary Mathematics syllabus.

In this edition, Mathematics is presented in a student-friendly manner, with elaborate examples preceding wide-ranging exercises. Mixed exercises are also given to further assist the student in self-assessment on the topics so far covered.

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