Problem 1: The Suspicious Card Game

Assigned: 12 October
Due: 28 October
Maximum Mark: 10
Maximum Submission Length: 4 pages

You're playing a simple game of chance against a stranger. In each round of the game, each player draws two cards from what appears to be a standard playing-card deck (which normally contains 52 cards, four each from the set [A,2,3,4,5,6,7,8,9,10,J,Q,K]). The player with the highest card in their hand wins, and the losing player must pay them £10. Each player draws new cards each round, returning cards from the previous round to the deck. Aces are low.

After the first five rounds you obtain the following results:

Round 1:   You = [10][2]   Opponent = [K][5]   (You LOSE)
Round 2:   You = [9][6]   Opponent = [5][K]   (You LOSE)
Round 3:   You = [2][K]   Opponent = [K][9]   (You DRAW)
Round 4:   You = [Q][1]   Opponent = [K][7]   (You LOSE)
Round 5:   You = [7][2]   Opponent = [10][10]   (You LOSE)

Without a win to your name, you begin to suspect that your opponent may be cheating.

1. Develop and describe a mathematical model for the game that enables you to address your suspicions using statistical inference, and use it to argue for or against your suspicion. Specifically, do the following:

(a) Define a *test statistic*: a value that can be calculated for any potential outcome of the game that encapsulates how unusual (or not) the result is. Determine the value of the test statistic for the result that actually occurred, using the data at hand.

(b) Define a *null hypothesis*: a scenario, expressed mathematically, that describes a specific uninteresting configuration of the system. Also define an *alternative hypothesis* that describes the scenario or scenarios that would be considered of interest.

(c) Calculate the probability that a result (test statistic) "as or more extreme" than the actual result would result from the scenario under the null hypothesis. You may use an analytic method, or a simulation.

(d) Based on the previous calculation, draw a conclusion about the null hypothesis and/or alternative hypothesis.

2. In practice, did the procedure you carried out follow all rules of statistical hypothesis-testing, such that it constitutes a formal and unbiased means of evaluating whether your opponent might be cheating? Describe any important deviations from the assumptions of hypothesis-testing, if present. (Note: this is a question about the procedure you followed, not about the quantity or quality of data.)

3. What steps could you take next to make your conclusion more secure?