Problem 10: X-ray Flares

Assigned: 23 November
Due: 17 December
Maximum Mark: 10 Points
Maximum Submission: 4 pages

You are analysing an object that normally does not produce X-rays, but every once in a while it flares up, emits a huge number (S) of X-rays randomly in all directions, and then fades away. Each flare is thought to be basically identical. We have placed a detector near the source, which is expected to intercept one percent of all the X-ray photons emitted during the flare *on average*. However, the actual number of X-ray photons received during each flare may differ due to counting statistics.

1. After a long wait, the source finally flares and we count a total of n = 6 X-ray photons in our detector. Based on this measurement, and knowing that the detector intercepts 1% of photons on average, estimate the value of S and provide a (non-approximate) 95% frequentist confidence interval.

2. Write down the equation for the likelihood function L(S; n) for this scenario (for arbitrary values of S and n). Then, make a (1D) plot of the likelihood L as a function of S given our data (n=6).

3. What is the maximum-likelihood value of S, if n=6? (You can calculate numerically based on the plotted curve in #2, or prove an exact result analytically if you prefer.)

4. What is an approximate 95% credible interval on S, if n=6? (Assume flat priors. You can estimate the limits of the interval numerically.)