Rank of a matrix: The rank of a matrix is the largest order of a non-zero minor of matrix.

Properties: 1. Rank of A and A^T is same.

2. Rank of null matrix is zero
3. For a rectangular matrix A of order $m \times n$, rank of $A \leq \min(m,n)$ i.e. rank cannot exceed the smaller of m and n.
4. For a n-square matrix, if rank of $A = n$ then $\det(A) \neq 0$ i.e. A is non-singular.
5. For any n-square matrix, if rank of $A < n$ then $\det(A) = 0$ i.e. A is singular.

To find the rank of a matrix, reduce that matrix into Echelon form.

The rank of a matrix will never change if apply elementary row operations on it.

Ex. Find the rank of the matrix $A = \begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -1.5 \end{bmatrix}$

To find rank, apply row operations to convert matrix A into echelon form.

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -1.5 \end{bmatrix} \quad R_2 \to R_2 - 2R_1 \quad \begin{bmatrix} 4 & 2 & 3 \\ 0 & 0 & 0 \\ -2 & -1 & -1.5 \end{bmatrix}$$

$$R_3 \to R_3 + \tfrac{1}{2}R_1 \quad \begin{bmatrix} 4 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ This matrix is in echelon form

Now, the echelon form matrix has one non-zero row.

$\therefore$ The rank of A is 1.

QB21. Find the rank of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Convert into echelon form;

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad R_2 \to R_2 - R_1 \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R_3 \to R_3 - R_1 \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$ This is in echelon form

$\therefore$ The rank of the matrix A is 2.

QB22. Determine the rank of matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$