Linear Transformation: A Linear Transformation (LT) from a VS V to a VS W is a mapping $T: V \to W$ such that, for all u and v in V and for all scalars c,

1. $T(u+v) = T(u) + T(v)$ 2. $T(cu) = cT(u)$

$T: V \to W$ is a L.T. iff $T(c_1v_1 + c_2v_2 + \dots + c_kv_k) = c_1T(v_1) + c_2T(v_2) + \dots + c_kT(v_k)$ for all $v_1, \dots, v_k$ in V and scalars $c_1, \dots, c_k$.

Ex. Every matrix transformation is a LT. That is, if A is an $m \times n$ matrix, then the transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ defined by $T(x) = Ax$ for x in $\mathbb{R}^n$ is a LT.

Ex. Let D be the differential operator $D: V \to W$ defined by $D(f) = f'$.

Let f and g be diff functions and let c be a scalar.

$D(f+g) = (f+g)' = f' + g' = D(f) + D(g)$

$D(cf) = (cf)' = cf' = cD(f)$.

$\therefore$ D is a LT.

Ex. Let $T: \mathbb{R} \to \mathbb{R}$ defined by $T(x) = 2^x$.

This is not L.T. as $T(x+y) = 2^{x+y} = 2^x \cdot 2^y = T(x) \cdot T(y)$

$T(x) + T(y) = 2^x + 2^y \neq 2^{x+y} = T(x+y)$.

→ Zero Transformation: $T_0: V \to W$ that maps every vector in V to zero vector in W; $T(v) = 0 \quad \forall v \in V$.

→ Identity Transformation: $I: V \to V$ that maps every vector in V to itself; $I(v) = v \quad \forall v \in V$.

Thm: Let $T: V \to W$ be a L.T. Then:

(a) $T(0) = 0$

(b) $T(-v) = -T(v) \quad \forall v \in V$

(c) $T(u-v) = T(u) - T(v) \quad \forall u, v \in V$.