Q. Find adj and inverse of $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

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Ans. $A^{-1} = \frac{1}{5}\begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix}$.

Elementary matrix – An elementary matrix is that, which is obtained from unit matrix by subjecting it to any of the E.T.

Rank of a Matrix –

A matrix is said to be of rank r when.

(i) It has at least one nonzero minor of order r.

(ii) Every minor of order higher than r vanishes.

[or order of largest non-zero minor]

$\Rightarrow$ Rank of matrix is the largest order of any non-vanishing minor of the matrix.

Elementary Transformation –

To find rank of matrix we use elementary transformation –

① The interchange of any two rows (columns)

② The multiplication of any row (column) by a non-zero number

③ Addition to one row (column) of another row (column) multiplied by any nonzero scalar.

The elementary transformations do not change either the order or the rank of Matrix.

Q. Determine the rank of the following matrix –

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$$

Sol^n. Applying $(R_2 - 2R_1)$ and $(R_3 - 3R_1)$

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & -2 & -2 \\ 0 & -5 & 7 \end{bmatrix} \neq 0$$

Here minor of order 3 is formed. Thus rank of matrix is 3.

Equivalent matrix – Two matrices A and B are said to be equivalent if one can be obtained from other by sequence of E.T.

It have the same order and same rank because elementary transformations do not alter its rank.

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Q. Find rank of $A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 16 & 4 & 12 & 15 \\ 5 & 3 & 3 & 8 \\ 4 & 2 & 6 & -1 \end{bmatrix}$

Sol^n. $C_1 \leftrightarrow C_2$

$$A = \begin{bmatrix} 1 & 6 & 3 & 8 \\ 4 & 16 & 12 & 15 \\ 3 & 5 & 3 & 8 \\ 2 & 4 & 6 & -1 \end{bmatrix}$$

$(R_4 - 2R_1)$ $(R_3 - 3R_1)$ and $(R_2 - 4R_1)$

$$\sim \begin{bmatrix} 1 & 6 & 3 & 8 \\ 0 & -13 & -6 & -16 \\ 0 & -8 & 0 & -17 \\ 0 & -8 & 0 & -17 \end{bmatrix} \begin{matrix} \\ [R_3] \\ [R_2] \\ \\ \end{matrix}$$

$(R_3 \leftrightarrow R_4)$

$$\sim \begin{bmatrix} 1 & 6 & 3 & 8 \\ 0 & -13 & -6 & -16 \\ 0 & -8 & 0 & -17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here is nonsingular minor of order third, thus

$$\begin{vmatrix} 1 & 6 & 3 \\ 0 & -13 & -6 \\ 0 & -8 & 0 \end{vmatrix}$$

rank(A) = 3.

Gauss-Jordan Method of finding the Inverse -

Those elementary row transformations which reduce a given square matrix A to the unit matrix, when applied to unit matrix I give the inverse of A. (Apply row operations only)

Q. Find the inverse of following matrix A -

$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Sol^n. Writing the same Identity matrix side by side, we get -

$$A = \left[\begin{array}{ccc|ccc} 8 & 4 & 3 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \leftrightarrow R_1$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 8 & 4 & 3 & 1 & 0 & 0 \end{array}\right]$$

$(R_3 - 8R_1)$ $(R_2 - 2R_1)$

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & -3 & -1 & 0 & 1 & -2 \\ 0 & -12 & -5 & 1 & 0 & -8 \end{array}\right]$$

$R_3 - 4R_2$

$$\cong \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & -3 & -1 & 0 & 1 & -2 \\ 0 & 0 & -1 & 1 & -4 & 3 \end{array}\right]$$

$(R_1 + R_3)$ & $(R_2 - R_3)$

$$\cong \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & -4 & 4 \\ 0 & -3 & 0 & -1 & 5 & -5 \\ 0 & 0 & -1 & 1 & -4 & 3 \end{array}\right]$$

-12
10
$12-10$

$(3R_1 + 2R_2)$

$$\cong \left[\begin{array}{ccc|ccc} 3 & 0 & 0 & 1 & -2 & 2 \\ 0 & -3 & 0 & -1 & 5 & -5 \\ 0 & 0 & -1 & 1 & -4 & 3 \end{array}\right]$$

$R_1/3$, $R_2/-3$ and $R_3/-1$

$$\cong \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & 2/3 \\ 0 & 1 & 0 & +1/3 & -5/3 & 5/3 \\ 0 & 0 & 1 & -1 & 4 & -3 \end{array}\right]$$

thus inverse of given matrix is –

$$A = \begin{bmatrix} 1/3 & -2/3 & 2/3 \\ 1/3 & -5/3 & 5/3 \\ -1 & 4 & -3 \end{bmatrix}$$

Q. Find Inverse of

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

Correct Ans.

$$\frac{1}{21}\begin{bmatrix} 1 & 10 & -7 \\ 1 & -11 & 14 \\ -3 & 12 & 0 \end{bmatrix}$$

$A = IA$

$[\ \ ] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$A A^{-1} = I$

Reduce the following matrices to column echelon form and find their ranks -

$$A = \begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 4 \\ 4 & -1 & 7 \\ 2 & 1 & 5 \end{bmatrix} \xrightarrow{C_2 - \frac{C_1}{3}} \begin{bmatrix} 3 & 0 & 0 \\ 1 & 5/3 & 5/3 \\ 4 & -7/3 & -7/3 \\ 2 & 1/3 & 1/3 \end{bmatrix}$$

$$\xrightarrow{1.5(C_3 - C_2)} \begin{bmatrix} 3 & 0 & 0 \\ 1 & 5/3 & 0 \\ 4 & -7/3 & 0 \\ 2 & 1/3 & 0 \end{bmatrix}$$

rank = 2

Numericals:

Using elementary row operations, determine ranks of following matrices -

(i) $\begin{bmatrix} 2 & 1 & -2 \\ -1 & -1 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 1 & 3 & -1 \\ 8 & 13 & 14 \end{bmatrix}$.

Ans. (i) 3 (ii) 2.

Using column operations, determine rank -

(i) $\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 3 & 2 \\ -1 & 1 & 3 & -5 \\ 2 & 3 & 4 & 5 \end{bmatrix}$.

Ans. 3

Ans. 2.