P & Q = ? P = [illegible] Q = [illegible]

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Solution of System of Linear Equations.

A system of non-homogeneous linear Eq^n
$AX = B$

↓ find $R(A)$ and $R(C)$ Augment matrix

↓ if $R(A) = R(C)$

System is consistent

↓ if $R(A) = R(C) = n$

Unique soln.

↓ $R(A) = R(C) < n$ (No. of unknowns)

Infinite no. of soln.

↓ if $R(A) \neq R(C)$

No solution, System is inconsistent

Discuss the consistency of the following system of Eq^ns

$$2a + 3b + 4c = 11$$
$$a + 5b + 7c = 15$$
$$3a + 11b + 13c = 25$$

If found consistent, then solve it.

Ans $a = 2, b = -3, c = 4$.

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A system of homogenous linear equations $AX = 0$

↓

Always has a solution

↓ Find $R(A)$

$R(A) = n$ (no. of unknowns)

↓

Unique or trival soln ($x=0, y=0, z=0$)

Q. Find the value of λ such that the equations $2a+3b-2c=0$, $3a-b+3c=0$ and $7a+\lambda b-c=0$ has nontrival solution. Find the solution.

Ans

$x =$

Soln. $C = [A:B] = \begin{bmatrix} 2 & 3 & -2 & ; & 0 \\ 3 & -1 & 3 & ; & 0 \\ 7 & \lambda & -1 & ; & 0 \end{bmatrix}$

$(2R_3 - 7R_1)$ $(2R_2 - 3R_1)$

$\sim \begin{bmatrix} 2 & 3 & -2 & ; & 0 \\ 0 & -11 & 12 & ; & 0 \\ 0 & (2\lambda - 21) & 12 & ; & 0 \end{bmatrix}$

$(R_3 \leftrightarrow R_2)$

$\sim \begin{bmatrix} 2 & 3 & -2 & ; & 0 \\ 0 & -11 & 12 & ; & 0 \\ 0 & (2\lambda - 10) & 0 & ; & 0 \end{bmatrix}$

For nontrival soln. $2\lambda - 10 = 0$

$\Rightarrow \boxed{\lambda = 5}$

Q. Determine the values of k for which the following system of equations has non trivial soln. and find them.

$(k-1)x + (4k-2)y + (k+3)z = 0$,
$(k-1)x + (3k+1)y + 2kz = 0$
$2x + (3k+1)y + 3(k-1)z = 0$

Soln

$$A = \begin{bmatrix} (k-1) & (4k-2) & (k+3) \\ (k-1) & (3k+1) & 2k \\ 2 & (3k+1) & 3(k-1) \end{bmatrix}$$

$\{(k-1)R_3 - 2R_1\}$ $(R_2 -$

$$\sim \begin{bmatrix} (k-1) & (4k-2) & \\ 0 & (-k+3) & \\ 0 & (k-3)(3k-1) & \end{bmatrix}$$

$S1$ $\{R_3 -$

$$\begin{bmatrix} (k-1) & (4k-2) & \\ 0 & (3-k) & \\ 0 & 0 & \end{bmatrix}$$

for Non trivial soln. $6k(k-$

$\Rightarrow k = 3$, $\Rightarrow k = 0$ or

$(3k-3)(k-1) - 2k - 6$
$3k^2 - 6k + 3 - 2k - 6$
$3k^2 - 8k - 3$
$3k^2 - 9k + k - 3$
$3k(k-3) + 1(k-3)$
$(k-3)(3k+1)$

$(3k+1)(k-3) - (k-3)(1-3k)$
$(k-3)\{(3k+1) + (3k-1)\}$
$6k(k-3)$

$(3k+1)(k-1) - 8k + 4$
$3k^2 + 2k - 1 - 8k + 4$
$3k^2 - 10k + 3$
$3k^2 - 9k - k + 3$
$3k(k-3) - 1(k-3)$
$(k-3)(3k-1)$