Matrix : A matrix is a rectangular array of numbers or functions which enclose in bracket. For example,

$$\begin{bmatrix} 0.3 & 1 & -5 \\ 0 & -0.2 & 16 \end{bmatrix}, \begin{bmatrix} e^{-x} & 2x^2 \\ e^{6x} & 4x \end{bmatrix}, [a_1 \; a_2 \; a_3], \begin{bmatrix} 4 \\ \frac{1}{2} \end{bmatrix}$$

The numbers (or functions) are called entries (elements) of the matrix.
The first matrix has two rows, which are the horizontal lines of entries. It has three columns, which are the vertical lines of entries.
The second matrix has 2 rows and 2 columns, so it is called Square Matrix. (same no. of rows and columns)

Matrices having just a single row or column are called vectors. So, third & fourth matrices (above) are called row vector and column vectors respectively.

General Notation of a matrix : a general $m \times n$ matrix A has the form

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \text{ or } [a_{ij}]_{m\times n}$$

Each entry has two subscripts. The first is the row number and second is the column number. So, a_{21} is the entry in Row 2 and column 1.

If $m=n$, call A an $n\times n$ square matrix. Then its diagonal containing the entries $a_{11}, a_{22}, a_{33}, \ldots, a_{nn}$ is called the main diagonal of A.

A diagonal matrix all of whose diagonal entries are the same is called a scalar matrix.
If the scalar (entry) on the diagonal is 1, the scalar matrix is called an identity matrix.

$$\begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Two matrices are equal if they have the same size and if their corresponding entries are equal. Thus, if $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{r\times s}$, then $A = B$ if and only if $m = r$ and $n = s$ and $a_{ij} = b_{ij}$ $\forall$ i and j.

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are $m\times n$ matrices, their Sum $A+B$ is the $m\times n$ matrix obtained by adding the corresponding entries.

$$A + B = [a_{ij} + b_{ij}]$$

Scalar Multiplication : If A is an $m\times n$ matrix and c is a scalar, then the scaler multiple cA is the $m\times n$ matrix obtained by multiplying each entry of A by c.

$$cA = c[a_{ij}] = [ca_{ij}]$$

The matrix $(-1)A$ is written as $-A$ and called the negative of A. If A and B are the same size, then $A - B = A + (-B)$.

A matrix all of whose entries are zero is called a zero matrix and denoted by O ($O_{m\times n}$).

Matrix Multiplication : If A is an $m\times n$ matrix and B is an $n\times r$ matrix, then the product $C = AB$ is an $m\times r$ matrix. The c_{ij} entry of the product is computed as

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj} = \sum_{k=1}^{n} a_{ik}b_{kj}$$

[i.e. the dot product of ith row and jth column give c_{ij} entry of matrix C]

$$A_{m\times n} \; B_{n\times r} = C_{m\times r}$$

same

size of AB

Transpose of a matrix: The transpose of an $m \times n$ matrix A is the $n \times m$ matrix A^T obtained by interchanging the rows and columns of A.
So, transpose of a column matrix (vector) becomes a row matrix (vector) and vice-versa.

$$(A^T)_{ij} = A_{ji} \quad \forall i \& j$$

A square matrix A is symmetric if $A^T = A$ i.e., if A is equal to its own transpose. e.g. $[A_{ij} = A_{ji}]$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} ; \; A^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \rightarrow A \text{ is symmetric}$$

$$B = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} ; \; B^T = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \; B \neq B^T \Rightarrow B \text{ is not symmetric}$$

A symmetric matrix has the property that it is its own mirror image across its main diagonal.

A square matrix A is skew-symmetric if $A^T = -A$ i.e, if A^T is equal to minus of the matrix A. $[a_{ij} = -a_{ij} \Rightarrow a_{ii} = 0]$.

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ is skew-symmetric}$$

Triangular Matrices: Upper triangular matrices that can have nonzero entries only on and above the main diagonal, whereas any entry below the diagonal must be zero.
Similarly, lower triangular matrices can have nonzero entries only on and below the main diagonal.
Any entry on the main diagonal of a triangular matrix may be zero or not.

$$\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} \quad \begin{bmatrix} 3 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix}$$

Upper triangular — Lower triangular

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Diagonal matrix — upper — lower

Orthogonal matrix : A matrix (square) A is called orthogonal if

$AA^T = A^T A = I$.

Also, A square matrix with the property $A^{-1} = A^T$ is said to be an Orthogonal matrix.

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Determinant : The determinant of a matrix (square) is a number that can be computed from the elements of a square matrix through a particular process as

$$D = \sum_{k=1}^{n} (-1)^{j+k} a_{jk} M_{jk} \quad (j = 1, 2, \ldots, \text{ or } n)$$

$$D = \sum_{j=1}^{n} (-1)^{j+k} a_{jk} M_{jk} \quad (k = 1, 2, \ldots, \text{ or } n)$$

where M_{jk} is minor of a_{jk} in A.