Question Bank-Unit 1
(Propositional Logic)

S. No.	Questions	CO	Bloom's Taxonomy Level	Difficulty Level	Competitive Exam Question Y/N	Area	Topic	Unit	Marks
1	Which of these sentences are **propositions**? What are the **truth values** of those that are propositions? **a)** 2 + 3 = 5 (**True implies proposition**). **b)** Answer this question. **(not declarative implies not a proposition)**	1	K1	L	N			1	2
2	Check whether these statements are wff (well formed formula) or not:(a) $(p \vee q) \wedge \sim r$ (b) $p \vee q \wedge r$ Ans. well formed formula Example: $pq\wedge$, $p\text{-}\wedge q, -\wedge pq$ not well formed	1	K2	L	N			1	2
3	Check whether these statements are wff or not: (a) $(p\wedge(q\wedge r))\rightarrow s$ (b) $((p\vee q)\wedge r\rightarrow q)$ Ans. well formed formula	1	K2	L	N			1	2
4	Determine whether these **biconditionals** are true or false. a) 2 + 2 = 4 if and only if 1 + 1 = 2. b) 1 + 1 = 2 if and only if 2 + 3 = 4. **Sol. a. T and T= T b. T and F= F**	1	K2	L	N			1	2
5	Determine whether these **biconditionals** are true or false. a) 1 + 1 = 3 if and only if monkeys can fly. b) 0 >1 if and only if 2 >1. (16/14/KR) **Sol. a. F and F=T b. F and T=F**	1	K2	L	N			1	2
6	Determine whether each of these **conditional statements** is true or false. a) If 1 + 1 = 2, then 2 + 2 = 5. b) If 1 + 1 = 3, then 2 + 2 = 4.	1	K2	L	N			1	2

	(17/14/KR) **Sol. a. T and F= F** **b. F and T=T**								
7	Determine whether each of these **conditional statements** is true or false. a) If 1 + 1 = 3, then 2 + 2 = 5. b) If monkeys can fly, then 1 + 1 = 3. (17/14/KR) **Sol. a. F and F=T** **b. F and F=T**	1	K2	L	N			1	2
8	State the **converse**, **contrapositive**, and **inverse** of the conditional statements. If it snows today, then I will ski tomorrow. (27/15/KR) **Sol.** If it snows today, then I will ski tomorrow. Converse: If I will ski tomorrow, then it snows today Contra-positive: If I will not ski tomorrow, then It does not snow today Inverse: If it does not snow today, then I will not ski tomorrow	1	K2	L	N			1	2
9	State the **converse**, **contrapositive**, and **inverse** of each of these conditional statements. I come to class whenever (if) there is going to be a quiz. (27/15/KR) **Sol.** I come to class whenever (if) there is going to be a quiz. **OR we can re-write above statement as** **If there is going to be a quiz, then I come to class.** **same as above**	1	K2	M	N			1	6
10.	State the **converse**, **contrapositive**, and **inverse** of the conditional statements. A positive integer is a prime only if it has no divisors other than 1 and itself. (27/15/KR) **Sol. OR we can re-write above statement as** **If a positive integer is a prime, then it has no divisors other than 1 and itself.**	1	K2	M	N			1	6
11	Let *P(x)* denote the statement "$x \le 4$." What are these **truth values**? **a)** *P*(4) **b)** *P*(6) (1/53/KR)	1	K2	L	N			1	2

	Sol. P(x): $x \le 4$ (given) Then, P(4): $4 \le 4$-----(True) P(6): $6 \le 4$-----(False)								
12	Let *P(x)* be the statement “$x = x^2$.” If the domain consists of **the integers**, what are these truth values? **a)** *P(0)* **b)** ∀*x P(x)* (11/53/KR) **Sol**. $P(x): x = x^2$. Then, $P(0)$: $0 = 0^2$ True b. Since P(x) is only true for 0 and 1 this implies ∀*xP(x) is false* ∀-for each or for all values	1	K2	L	N			1	2
13	Let *P(x)* be the statement “$x = x^2$.” If the domain consists of the integers, what are these truth values? **a)** *P(1)* **b)** ∃*x P(x)* (11/53/KR) ∃-there exists **Sol**. $P(x): x = x^2$. Then, P(1) is True $1 = 1^2$ b. ∃-existantential quantifier ∃*xP(x) is true since it is true for 0 and 1.*	1	K2	L	N			1	2
14	Determine the truth value of each of these statements if the domain consists of all integers. **a)** $\forall n(n + 1 > n)$ **b)** $\exists n\ (2n = 3n)$ (13/53/KR) ∀-universal quantifier	1	K2	L	N			1	2
15	Determine the truth value of each of these statements if the domain consists of all integers. **a)** $\exists\ n\ (n = -n)$ **b)** $\forall n\ (3n \le 4n)$ (13/53/KR)	1	K2	L	N			1	2

16	Define Skolemization. **(Not in syllabus)**	1	K1	L	N			1	2
17	Define Skolem constant. **(Not in syllabus)**	1	K1	L	N			1	2
18	Define skolem function. **(Not in syllabus)**	1	K1	L	N			1	2
19	Let p, q, r denote the statements "It is raining", "It is cold", and "It is pleasant", respectively. Then the statement "It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold" is represented by (a) $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$ (b) $(\neg p \wedge r) \wedge ((p \wedge q) \rightarrow \neg r)$ (c) $(\neg p \wedge r) \vee ((p \wedge q) \rightarrow \neg r)$ (d) $(\neg p \wedge r) \vee (r \rightarrow (p \wedge q))$ (GATE 2017, 1 mark) **Sol.** p: It is raining, q: It is cold, r: It is pleasant. Then the statement "It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold" Logic: $(-p \wedge r) \wedge (-r \rightarrow (p \wedge q))$ (option (a))	1	K3	H	Y			1	6
20	Which one of the following is not equivalent to $p \leftrightarrow q$? (a) $(\neg p \vee q) \wedge (p \vee \neg q)$ (b) $(\neg p \vee q) \wedge (q \rightarrow p)$ (c) $(\neg p \wedge q) \vee (p \wedge \neg q)$ (d) $(\neg p \wedge \neg q) \vee (p \wedge q)$ (GATE 2015, 1 mark) Sol. Exercise Ans. c	1	K2	M	Y			1	2
21	Let p, q, and r be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction. Then the expression $(r \rightarrow p) \rightarrow q$ is (a) a tautology (b) a contradiction (c) always TRUE when p is FALSE (d) always TRUE when q is TRUE (GATE 2017, 2 mark) **Sol. In conditional proposition f**alse implies 1st is T and 2nd is F. This implies ($p \rightarrow q$ **(1st is not T and 2nd is not F))** always True and r always false. r is false this implies that **r→ p** is always true.	1	K2	M	Y			1	2

	(r→p) always ture and if **q** is true=true (r→p) always ture and if **q** is False=false Ans. d								
22.	The statement (¬p) → (¬q) is logically equivalent to : I. p → q II. q → p III. (¬q)V p IV. (¬p)V q (a) I only (b) I and IV only (c) II only (d) II and III only (GATE 2017, 1 mark) Sol. **Will continue in next class i.e., 17th July 2020**	1			Y			1	2
23.	Consider the following two statements: S1: If a candidate is known to be corrupt, then he will not be elected. S2: If a candidate is kind, he will be elected. Which one of the following statements follows from S1 and S2 as per sound inference rules of logic? (a) If a person is known to be corrupt, he is kind. (b) If a person is not known to be corrupt, he is not kind. (c) If a person is kind, he is not known to be corrupt. (d) If a person is not kind, he is not known to be corrupt. (GATE2015, 1)	1	K3	H	Y			1	6
24	Let p, q, r, s represent the following propositions: $p: x \in \{8,9,10,11,12\}$, q: x is a composite number, r: x is a perfect square, s: x is a prime number The integer $x \geq 2$ which satisfies ¬((p→q) ∧ (¬r V¬s)) is...................... (GATE2016, 1)	1	K2	H	Y			1	2

	Sol. $p : x \in \{8,9,10,11,12\}$ q: x is a composite number={8,9,10,12} r: x is a perfect square={9} -r={8,10,11,12} s: x is a prime number={11} -s={8,9,10,12} (¬r ∨¬s)=union={8,9,10,11,12} p→q =={8,9,10,12} (p→q) ∧ (¬r ∨¬s) =intersection or common member={8,9,10,12} -((p→q) ∧ (¬r ∨¬s))=11 Ans								
25	Which one of the following well formed formula in predicate calculus is NOT valid? (a) $(\forall x\, p(x) \rightarrow \forall x\, q(x)) \rightarrow (\exists x \neg p(x) \vee \forall x\, q(x))$ (b) $(\exists x\, p(x) \vee \exists x\, q(x)) \rightarrow \exists x\, (p(x) \vee q(x))$ (c) $\exists x\, (p(x) \wedge q(x)) \rightarrow (\exists x\, p(x) \wedge \exists x\, q(x))$ (d) $\forall x(p(x) \vee q(x)) \rightarrow (\forall x p(x) \vee \forall x\, q(x))$ (GATE 2016, 2 mark)	1	K2	M	Y			1	2
26	What is the logical translation of the following statements? " None of my friends are perfect" (a) $\exists x(F(x) \wedge \neg P(x))$ (b) $\exists x(\neg F(x) \wedge P(x))$ (c) $\exists x(\neg F(x) \wedge \neg P(x))$ (d) $\neg \exists x(F(x) \wedge P(x))$ (GATE 2013, 2 mark)	1	K2	M	Y			1	2
27	Consider the statement:" Not all that glitters is gold" Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formula represents the above statement? (a) $\forall x$:*glitters(x)* → ¬gold(x) (b) $\forall x$: gold (x) → *glitters(x)*	1	K2	M	Y			1	2

	(c) $\exists x$: gold(x) $\wedge\neg$*glitters(x)* (d) $\exists x$: *glitters* (x) $\wedge\neg$gold *(x)* (GATE 2014, 1 mark)								
28	Construct a **truth table** for each of these compound propositions. a) $(p \to q) \leftrightarrow (\neg q \to \neg p)$ b) $(p \to q) \to (q \to p)$ (31/15/KR)	1	K1	L	N			1	6
29	Construct a **truth table** for each of these compound propositions. a) $(p \vee q) \to (p \oplus q)$ b) $(p \oplus q) \to (p \oplus \neg q)$ (33/15/KR)	1	K2	L	N			1	6
30	Use **De Morgan's laws** to find the negation of each of the following statements. **a)** Jasbir is rich and happy Sol. Let p: Jasbir is rich q: he is happy Then, p $\wedge$ q (Given). Therefore, $-(p \wedge q) = -p \vee -q$ Ans: Neither Jasbir is rich nor happy. **b)** Rajan will bicycle or run tomorrow. (7/34/KR)	1	K2	L	N			1	6
31	Use **De Morgan's laws** to find the negation of each of the following statements. **a)** Shakila walks or takes the bus to class. **b)** Ibrahim is smart and hard working. (7/34/KR)	1	K2	L	N			1	6
24	Show that each of these conditional statements is a **tautology** by using truth tables. (And then without using the truth tables.) **a)** $(p \wedge q) \to p$ **b)** $p \to (p \vee q)$ **c)** $\neg p \to (p \to q)$ (9, 11/35/KR)	1	K2	M				1	6

	Sol. a. *(p ∧ q)* → *p* . *T and F=F not permissible here for true value for Tautology.* *Therefore, if p-true then (p ∧ q) → p –is also true (Tautology).* *If p-false then, (p ∧ q)-is also false implies (p ∧ q) (F)→ p (F) –is true (Tautology).*								
25	Show that each of these conditional statements is a **tautology** by using truth tables. (And then without using the truth tables.) **(a)** $(p \wedge q) \rightarrow (p \rightarrow q)$**(b)**$\neg(p \rightarrow q) \rightarrow p$ **(c)** $\neg(p \rightarrow q)\rightarrow\neg q$ (9, 11/35/KR)	1	K2	M	N			1	6
26	Show that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a **tautology**. (29/35/KR)	1	K2	M	N			1	6
27	Show that two compound propositions are **logically equivalent**: a) $\neg(p \leftrightarrow q)$ and $p \leftrightarrow\neg q$ b). $\neg p \leftrightarrow q$ and $p \leftrightarrow\neg q$ c) $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$ (17,19,21,23/35/KR)	1	K2	L	N			1	2
28	Show that two compound propositions are logically equivalent: $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ (17,19,21,23/35/KR)	1	K2	M	N			1	6
29	Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent. (31/35/KR)	1	K2	M	N			1	6
30.	Show that two compound propositions are **logically implied**:	1	K2	M	N			1	6
31	Show that two compound propositions are **logically implied**:	1	K2	M	N				
32	Determine whether each of these compound propositions is satisfiable. **a)** $(p \vee\neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee\neg q)$- *R*	1	K2	M	N			1	6

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<tr><td>p</td><td>q</td><td>-p</td><td>-q</td><td>$p \lor \neg q$</td><td>$\neg p \lor q$</td><td>$\neg p \lor \neg q$</td><td>R</td></tr>
<tr><td>T</td><td>T</td><td>F</td><td>F</td><td>T</td><td>T</td><td>F</td><td>F</td></tr>
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<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>Satisfiable</td></tr>
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b) $(p \to q) \land (p \to \neg q) \land (\neg p \to q) \land (\neg p \to \neg q)$
c) $(p \leftrightarrow q) \land (\neg p \leftrightarrow q)$
(61,62/36/KR)
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Find the dual of each of these compound propositions.
a) $p \land \neg q \land \neg r$ b) $(p \land q \land r) \lor s$ c) $(p \lor \mathbf{F}) \land (q \lor \mathbf{T})$
(35/35/KR)
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<td>1</td><td>K3</td><td>M</td><td>N</td><td></td><td></td><td>1</td><td>6</td>
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Find the Disjunctive Normal Form (DNF) of $(p \lor q) \to \neg r$.
Disjunctive Normal Form (DNF): The formulation of conjuctive propositions into disjunction propositions.
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<tr><td>p</td><td>q</td><td>$p \lor q$</td><td>-r</td><td>$(p \lor q) \to \neg r$</td></tr>
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<td>1</td><td>K2</td><td>H</td><td>N</td><td></td><td></td><td>1</td><td>6</td>
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DNF: $(p \wedge q \wedge -r) \vee (p \wedge -q \wedge -r) \vee (-p \wedge q \wedge -r) \vee (-p \wedge -q \wedge r) \vee (-p \wedge -q \wedge -r)$</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>35</td><td>Find the Disjunctive Normal Form (DNF) of $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$.</td><td>1</td><td>K3</td><td>H</td><td>N</td><td></td><td></td><td>1</td><td>6</td></tr>
<tr><td>36</td><td>Find the Disjunctive Normal Form (DNF) of $p \leftrightarrow (\sim p \vee \sim q)$. $=R$.
<table><tr><td>p</td><td>q</td><td>-p</td><td>-q</td><td>~p ∨ ~q</td><td>R</td></tr><tr><td>T</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td></tr><tr><td>T</td><td>F</td><td>F</td><td>T</td><td>T</td><td>T</td></tr><tr><td>F</td><td>T</td><td>T</td><td>F</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>T</td><td>T</td><td>T</td><td>F</td></tr></table>
Dnf: $T \vee (p \wedge -q) \vee T$</td><td>1</td><td>K3</td><td>H</td><td>N</td><td></td><td></td><td>1</td><td>6</td></tr>
<tr><td>37</td><td>Put the following into Conjunctive Normal form $\neg(p \rightarrow q) \vee (r \rightarrow p)$.(42/35/KR)

CNF= dual of DNF (replace symbol ∨ by ∧ and vice versa)</td><td>1</td><td>K3</td><td>H</td><td>N</td><td></td><td></td><td>1</td><td>6</td></tr>
<tr><td>38</td><td>Put the following into Conjunctive Normal form$(p \wedge q) \vee (\sim p \wedge q \wedge r)$.(Ex 4.26/4.25/SS)</td><td>1</td><td>K3</td><td>H</td><td>N</td><td></td><td></td><td>1</td><td>6</td></tr>
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39	Put the following into **Conjunctive Normal form** $(p \vee \sim q) \rightarrow q$ *(Ex 4.27/4.25/SS)*	1	K3	H	N			1	6
40	Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. **a)** No one is perfect. **b)** Not everyone is perfect. **c)** All your friends are perfect. **d)** At least one of your friends is perfect. **e)** Everyone is your friend and is perfect. **f)**Not everybody is your friend or someone is not perfect. (25/54/KR)	1	K2	H	N			1	6
41	Find the argument form for the following argument and determine whether it is **valid**. Can we conclude that the conclusion is true if the premises are true? If Socrates is human, then Socrates is mortal. Socrates is human. ∴ Socrates is mortal. (1/78/KR)	1	K2	H	N			1	6
42	Check the validity of the following arguments: Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset." Sol. p: It is sunny this afternoon, q: it is colder than yesterday, r: We will go swimming, s: we take a canoe trip, t: we will be home by sunset Premises (or assumptions): $-p \cap q$, $r \rightarrow p$, $-r \rightarrow s$, $s \rightarrow t$ Conclusion: t (to show or prove) Proof: $-p \cap q$ premises -p Simplification $r \rightarrow p$ or $-p \rightarrow -r$ premises -r by modus ponens $-r \rightarrow s$ premise s by modus ponens $s \rightarrow t$ premise t result. Therefore conclusion is valid by rules of inferences.	1	K2	H	N			1	9

	Thus, the conclusion is valid								
43	Consider the argument: “If you invest in the stock market, then you will get rich” “if you get rich, then you will be happy” therefore” if you invest in the stock market, then you will be happy” check whether the given argument is valid. Sol. premises : $p \to q,\ \ q \to r,$ Conclusion: $p\backslash to\ r$ Proof: $p \to q,\ \ q \to r$ premises $p \to \ r$ Result by Hypothetical syllogism Conclusion is valid	1	K2	H	N			1	6
44	Use **rules of inference** to show that the hypotheses “Randy works hard(p),” “If Randy works hard, then he is a dull boy(q),” and “If Randy is a dull boy, then he will not get the job” imply the conclusion “Randy will not get the job.” (5/78/KR) Sol.Premises: $p,\ p \to q,\ q \to -r$ Conclusion: $-r$ Proof: p premises $p \to q \quad premises$ q by modus ponens $q \to -r \quad premise$ -r result by modus ponens	1	K2	M	N			1	6

	Thus, the conclusion is valid								
45	For each of these arguments, explain which **rules of inference** are used for each step:“Danish, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job.” Sol. C- represents class x- represents “is a student in this class” W- write programs in JAVA J- get a high-paying job Then, the premises are: C(Danish), W(Danish), $\forall x\,(W(x) \rightarrow J(x))$ Conclusion: someone in this class can get a high-paying job. Proof: C(Danish), W(Danish), $\forall x\,(W(x) \rightarrow J(x))$ premises Use 3rd one : $W(Danish) \rightarrow J(Danish)$ Universal instatiation Combine 2nd with above one: J(Danish) By modus ponens C(Danish) $\cap J(Danish)$ Conjunction $\exists x($ C(x) $\cap J(x)$) Result Conclusion is valid.	1	K2	M	N			1	6
46	Determine whether each of these **arguments is valid**. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs? a) If n is a real number such that $n>1$, then $n^2>1$. Suppose that $n^2>1$. Then, $n>1$. Sol. Let assume p: n>1, q: $n^2 > 1$.	1	K2	M	N			1	6

Premises are: $\{p \to q \; or \; -q \to -p\}$ and q

Conclusion: p

Conclusion is not valid or Fallacy(or failure) of rule of inferences.

Example: if n=-1.5, n^2=2.25>1. But n<1. Not valid.

b) If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \leq 9$. Then, $n \leq 3$.

Sol. Assume p: n>3, q: $n^2 > 9$.

Premises are: $\{p \to q \; or -q \to -p \; contrapositive\ since\ conditional\ and$

$contapositive\ are\ equaivalent\} \; and$ -q

Conclusion: -p (by modus tollens)

Conclusion is valid.

c) If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then, $n^2 \leq 4$.

Sol. Assume: p: $n > 2$, q: $n^2 > 4$

Premises (hypothesis or assumptions) are: $\{p \to q \; or \; -q \to -p\} \; and \; -p$

Conclusion : -q

	Conclusion is not valid or Fallacy (or failure) of hypothesis or rule of inferences.								
47	Check the following systems for consistency: a) $p \to q, p \to r, q \to \sim r, p$ (b) $p \to (q \to r), q \to (r \to s), p \wedge q \wedge \sim s$	1	K2	M	N			1	6
48	Test the validity of the following argument: All integers are irrational numbers. Some integers are powers of 2. Therefore, some irrational number is a power of 2.	1	K3	H	N			1	9
49	Test the validity of the following argument: If philosopher are not money-minded and some money-minded persons are not clever, then there are some persons who are neither philosopher nor clever.	1	K3	H	N			1	9
50	Test the validity of the following argument: It is not the case that if the price of petrol goes up, then the demand for two wheelers goes down. It is not true that either an alternative source of energy will be invented or the income of Indian Railways will not increase..Therefore the demand for two wheelers will not go down and the income of Indian Railways will increase.	1	K3	H	N			1	9
51	Prove that if n is an integer and 3n+2 is odd, then n is odd. (Ex3/76/KR) Proof: Given n is an integer and 3n+2 is odd. Aim: we have to prove that n is also odd. NOTE: 1. A number p is an even number if p=2m, p is multiple of 2, m is a number. 2. A number p is an odd number if p=2m+1, m is a number.	1	K2	M	N			1	6

	Since 3n+2 is an odd number implies there is a number m such that **3n+2=2m+1 By definition of an odd number.** **Substrate "1" from both the sides, we get** **This implies 3n+1=2m** **again subtracting 1 from both the sides, we get** **This implies** $n=\frac{2m-1}{3}$ $\boldsymbol{which\ is\ not\ an\ integer\ in\ general.}$ **There is no possible direct way to prove our conclusion. So direct proof is not valid.** **Now we prove our result by a contradiction:** **Assume n is even.** **Then, there is a number r such that n=2r by def of even number** **Substitute value of n, we get** **Then, 3n+2= 3(2r)+2= 2(3r+1)= multiple of 2 implies 3n+2 is even.** **A contradiction that 3n+2 is odd(given).** **So our assumption is wrong i.e., n is even is wrong.** **Thus, n is odd. Proved.**								
52	Give a direct proof of the theorem "If n is an odd integer, then n^2 is odd." (Ex1/75/KR) Proof: Given n is odd integer this implies there is a number s such that n=2s+1 by def of odd number. Squaring on both the sides $\boldsymbol{n^2 = (2s+1)^2 = 4s^2 + 4s + 1 = 2(2s^2 + 2s) + 1 = 2p + 1,}$ $\boldsymbol{where\ p = 2s^2 + 2s}$ $\boldsymbol{since\ n^2 = 2p + 1\ \ this\ implies\ n^2\ is\ odd.}$	1	K2	M	N			1	6

53	Prove that if n is an integer and n^2 is odd, then n is odd. (Ex8/78/KR) Proof: Given n is an integer and n^2 is odd. Aim: n is odd Prove this by contradiction Assume n is even. This implies n=2m, where m is number This implies $n^2 = 2(\ 2m^2) = multiple\ of\ 2$ Implies n^2 is even. A contradiction for n^2 is odd (Given). Our assumption that n is even is wrong. Thus, n is odd. Proved	1	K2	M	N			1	6
54	Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. (Ex10/78/KR) Proof: Let $\sqrt{2}$ is not irrational i.e., $\sqrt{2}$ is rational number $\sqrt{2} = \frac{p}{q},\ q \neq 0$ where p and q are relatively prime integers i.e., p and q have not any common factor except 1. {by def. of rational number} Squaring both the sides	1	K2	M	N			1	6

	$2=\frac{p^2}{q^2}$ implies $p^2=2q^2$ implies p^2 is even implies p is even. This implies p=2m ....(1) This implies $q^2=2m^2$ after substitution p=2m implies q^2 is even implies q is even q=2n. .... (2) From (1) and (2) p and q have 2 as a common factor. A contradiction for assumption that p and q are relatively prime is wrong. $\sqrt{2}$ is a irrational number.								
55	Give a proof by contradiction of the theorem “If 3n+2 is odd, then n is odd”. (Ex11/79/KR)	1	K2	M	N			1	6
56	Skolemize: ”Every philosopher writes at least one book.” **Not in syllabus**	1	K3	H	N			1	6
57	Skolemize” $(\forall x)(\exists y)(P(x,y))$ ” **Not in syllabus**	1	K3	H	N			1	2
58	Skolemize” $(\forall x)(\exists y)(\forall z)(P(x)\wedge Q(y,z))$ ” **Not in syllabus**	1	K3	H	N			1	2
59	Skolemize” $(\forall w)(\forall x)(\exists y)(\forall z)(P(x)\wedge Q(w,y,z))$ ” **Not in syllabus**	1	K3	H	N			1	6
60	Translate these statements into English, where *C(x)* is “*x* is a comedian” and *F(x)* is “*x* is funny” and the domain consists of all people. **a)** ∀*x*(*C*(*x*) → *F*(*x*)) **b)** ∀*x*(*C*(*x*) ∧*F*(*x*)) **c)** ∃*x*(*C*(*x*) → *F*(*x*)) **d)** ∃*x*(*C*(*x*) ∧*F*(*x*)) (7/53/KR)	1	K2	H	N			1	2

Q. If 2+2=4 True or 6 False is a composite number. True/false Ans. True