Basics of MATRICES

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The word matrix was introduced first by Cayley in 1860. It means rectangular arrangement of any numbers arranged in m rows and n column.

$$A = [a_{ij}]_{m \times n}$$

entries or elements.

* **Application** – In solution of system of linear equ. Probability, Mathematical economics, Electrical network, Quantum mechanics, Transportation problems, matrix are very amenable for computers.

* **Sum (Difference)** → $C = A \pm B$, where $[c_{ij}] = [a_{ij}] \pm [b_{ij}]$ where A and B are of same order

Product of matrices → $[A]_{m\times n}[B]_{n\times r} = [C]_{m\times r}$.

* **Equality** – Two matrices A and B are equal if they are of same order. $a_{ij} = b_{ij}$.

* **Transpose of matrix** $A_{m\times n}$ is denoted by $A^T_{n\times m}$ obtained by interchanging rows and column. $(AB)^T = B^T A^T$

* **Square Matrix** – If $m = n$ (No. of rows = No. of column) then it is said to be square matrix. The elements a_{ii} are known as diagonal elements.

* **Null or Zero matrix** is a matrix with all elements zero.

* Trace $\sum_{i=1}^{n} a_{ii}$ = Sum of diagonal elements.
* Singular Matrix ; if $|A| = 0$
* Nonsingular Matrix ; if $|A| \neq 0$
* Upper triangular Matrix, if $a_{ij} = 0 \quad i > j$
* Lower triangular Matrix, if $a_{ij} = 0 \quad i < j$
* Diagonal Matrix ; $a_{ij} = 0$ where $i \neq j$
* Scalar Matrix ; A diagonal matrix with $a_{ii} = k$ for every i and k is constant.
* Identity Matrix ; with all diagonal elements unity.

For Square Matrix

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* Row Matrix – Row matrix is a ... one row.

* Column matrix – column matrix ... having one column

* Power of matrix; A^n is a matrix ... multiplying A by

INVERSE OF MATRIX –

Inverse of n-square matrix A is de... is defined such that $AA^{-1} = A^{-1}A = ...$

* Inverse of A exists only if $|A| \neq 0$
* Inverse of matrix is unique.
* Inverse of a product is the prod... in reverse order.

  $(AB)^{-1} = B^{-1}A^{-1}$

* For Diagonal matrix D with d_{ii} as ... D^{-1} is a diagonal matrix with recip... as diagonal elements.
* Transposition and Inverse are Comm...
* $(A^{-1})^{-1} = A$

ADJOINT OF MATRIX –

Adjoint of a matrix is denoted by adj A is ... transpose of n square matrix $[A_{ij}]$ where ... elements A_{ij} are the cofactors of a_{ij} of A

$$\text{adj } A = \begin{bmatrix} A_{11} & \cdots & A_{n1} \\ A_{12} & \cdots & A_{n2} \\ \vdots & & \vdots \\ A_{1n} & \cdots & A_{nn} \end{bmatrix}$$

$\text{adj } AB = (\text{adj } A)(\text{adj } B)$

$A^{-1} = \dfrac{\text{adj } A}{|A|}$

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potent matrix – $A^2 = A$

Ex. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

olutory matrix – $A^2 = I$

Ex. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

hogonal matrix – $A'A = AA' = I$

itary matrix – A square matrix is said to be unitary matrix if $AA^* = A^*A = I$

Show that every square matrix can be uniquely expressed as sum of a symmetric and skew symm. matrices.

Let $A = [a_{ij}]$

$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$

Let $P = \frac{1}{2}(A+A')$, $Q = \frac{1}{2}(A-A')$

$A = P + Q$

$P' = \frac{1}{2}(A+A)' = \frac{(A'+A)}{2} = P$

$Q' = [\frac{1}{2}(A-A')]' = \frac{1}{2}(A'-A) = -\frac{1}{2}(A-A')$
$= -Q$

$\Rightarrow$ $\boxed{A = \text{Sym. matrix} + \text{skew sym. matrix}}$