Asian Journal of Control, Vol. 7, No. 2, pp. 91-98, June 2005

MODELING AND CONTROL OF THE ACTIVE SUSPENSION SYSTEM USING PROPORTIONAL INTEGRAL SLIDING MODE APPROACH

Yahaya Md. Sam and Johari Halim Shah Bin Osman

ABSTRACT

The purposes of this paper are to present a new method in modeling an active suspension system for half-car model in state space form and to develop a robust strategy in controlling the active suspension system. Proportional integral sliding mode control strategy is proposed for the system. A simulation study is performed to prove the effectiveness and robustness of the control approach and performance of the controller is compared to the linear quadratic regulator and the existing passive suspension system.

KeyWords: Active suspension systems, sliding mode control, mismatched condition.

I. INTRODUCTION

An ideal suspension should isolate the car body from road disturbances and inertial disturbances associated with cornering and braking or acceleration [1]. Furthermore, the suspension must be able to minimize the vertical force transmitted to the passengers for passengers comfort. These objectives can be achieved by minimizing the vertical car body acceleration. An excessive wheel travel will result in non-optimum attitude of tyre relative to the road that will cause poor handling and adhesion. Furthermore, to maintain good handling characteristic, the optimum tyre-to-road contact must be maintained on four wheels.

An early design for automobile suspension systems focused on unconstrained optimizations for passive suspension system which indicate the desirability of low suspension stiffness, reduced unsprung mass, and an optimum damping ratio for the best controllability [2]. Thus the passive suspension system, which approach optimal characteristics, had offered an attractive choice for a vehicle suspension system and had been widely used for passengers. However, the suspension spring and damper do not provide energy to the suspension system and control only the motion of the car body and wheel by limiting the suspension velocity according to the rate determined by the designer. To overcome the above problem, active suspension systems have been proposed by various researchers [3-5]. Active suspension systems dynamically respond to changes in the road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel. Typically, active suspension systems include sensors to measure suspension variables such as body velocity, suspension displacement, wheel velocity and wheel or body acceleration. An active suspension is one in which the passive components are augmented by actuators that supply additional forces. These additional forces are determined by a feedback control law using data from sensors attached to the vehicle. Various control strategies such as optimal state-feedback [3,5], backsteeping method [4], fuzzy control [6] and sliding mode control [7] have been proposed in the past years to control the active suspension system.

In this paper, we will consider a new modeling technique to represent a half-car motion equation in the state space form. Usually, only one highest degree of differential variable exists in the motion equation. In the previous modeling, it is difficult to represent the motion equation into the state space form if more than one highest degree of differential variable occurred in the motion equation. This new technique is capable to overcome such situation. Then, we discuss a control scheme that will improve further the ride comfort and road handling of the active suspension system. The proposed control scheme differs from the previous sliding mode techniques in the sense that the sliding surface is based on the proportional-integral sliding mode control (PISMC) strategy. The additional integral in the proposed sliding surface provides one more degree of

Manuscript received April 1, 2003; revised July 29, 2003; accepted May 24, 2004.

The authors are with the Faculty of Electrical Engineering, University Technology of Malaysia, 81310 UTM Skudai, Johor, Malaysia (e-mail: yahaya@fke.utm.my).

freedom and also reduce the steady state error. In the conventional sliding mode, the sliding mode gain is determined solely by the desired closed loop poles. Therefore the sliding surface is fully dependent on the sliding mode gain. In the PISMC, the sliding surface gain is determined by the desired closed loop gain and the design parameter that can be adjusted to fulfill the sliding surface requirement. To demonstrate the effectiveness and robustness of the proposed control scheme, computer simulation was performed and presented in this paper.

II. DYNAMIC MODEL OF THE ACTIVE SUSPENSION SYSTEM

Consider the model of a passenger's car subject to irregular excitation from a road surface as shown in Fig. 1. This model has been used by [6] in the design of active suspension system. The motion equations of the half-car model is represented in a vector matrix form as follows:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{S}\dot{\mathbf{X}} + \mathbf{T}\mathbf{X} = \mathbf{D}\mathbf{f} + \mathbf{E}\mathbf{w} \tag{1}$$

where the state, active control and excitation vectors are, respectively, given by

$$\mathbf{X} = (x_{bf}\ x_{wf}\ x_{br}\ x_{wr})^T, \quad \mathbf{f} = (f_f\ f_r)^T,$$
$$\mathbf{w} = (\dot{w}_f\ w_f\ \dot{w}_r\ w_r)^T$$

and the matrices **M**, **S**, **T**, **D**, and **E**, respectively, are given as follows:

$$\mathbf{M} = \begin{bmatrix} L_r m_b / L & 0 & L_f m_b / L & 0 \\ I_b / L & 0 & -I_b / L & 0 \\ 0 & m_{wf} & 0 & 0 \\ 0 & 0 & 0 & m_{wr} \end{bmatrix},$$

$$\mathbf{S} = \begin{bmatrix} c_{bf} & -c_{bf} & c_{br} & -c_{br} \\ L_f c_{bf} & -L_f c_{bf} & -L_r c_{br} & L_r c_{br} \\ -c_{bf} & c_{bf} & 0 & 0 \\ 0 & 0 & -c_{br} & c_{br} \end{bmatrix},$$

$$\mathbf{T} = \begin{bmatrix} k_{bf} & -k_{bf} & k_{br} & -k_{br} \\ L_f k_{bf} & -L_f k_{bf} & -L_r k_{br} & L_r k_{br} \\ -k_{bf} & k_{bf} + k_{wf} & 0 & 0 \\ 0 & 0 & -k_{br} & k_{br} + k_{wr} \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} 1 & 1 \\ L_f & -L_r \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \text{ and } \mathbf{E} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k_{wf} & 0 & 0 & 0 \\ 0 & 0 & k_{wr} & 0 \end{bmatrix}.$$

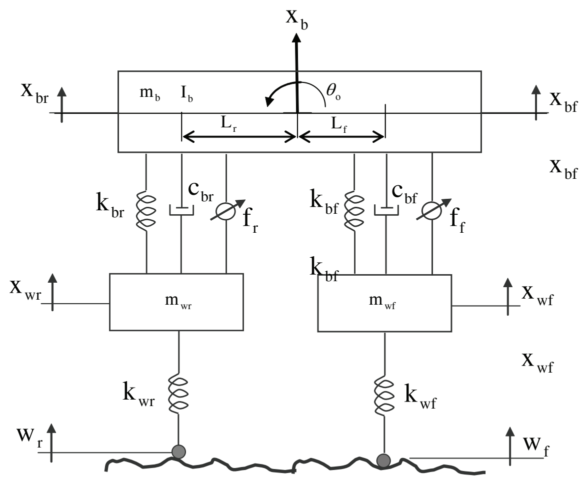

Fig. 1. A half car active suspension model.

I_b is the mass moment of inertia for the vehicle body, m_b is the mass for the vehicle body, m_{wf} and m_{wr} are the mass of the front and rear wheels, respectively, x_c is the vertical displacement of the vehicle body at the center of gravity, x_{bf} and x_{br} are the vertical displacements of the vehicle body at the front and rear suspension locations, respectively, x_{wf} and x_{wr} are the vertical displacements of the vehicle body at the front and rear wheels, respectively, θ_0 is the rotary angle of the vehicle body at the center of gravity, f_f and f_r are the active controls at the front and rear suspensions, respectively, w_f and w_r are the irregular excitations from the road surface, L_f and L_r are the distances of the front and rear suspension locations, respectively, with reference to the center of gravity of the vehicle body, and $L_f + L_r = L$.

Define the N state variables for the system as $x_i = \mathbf{X}$ and $x_{i+\frac{N}{2}} = \dot{\mathbf{X}}$, where $i = 1, 2, 3, \cdots, N/2$, the half-car model in Eq. (1) can be rewritten in state space form as follows:

$$\dot{\mathbf{x}}_i(t) = \mathbf{A}\mathbf{x}_i(t) + \mathbf{B}u(t) + \mathbf{B}_p z(t) \tag{2}$$

where $\mathbf{A} = \begin{bmatrix} 0_{\frac{N}{2}\times\frac{N}{2}} & I_{\frac{N}{2}\times\frac{N}{2}} \\ \mathbf{A}_{p1} & \mathbf{A}_{p2} \end{bmatrix}$, $\mathbf{A}_{p1} = -\mathbf{M}^{-1}\mathbf{T}$, $\mathbf{A}_{p2} = -\mathbf{M}^{-1}\mathbf{S}$, $\mathbf{B} = \begin{bmatrix} 0_{\frac{N}{2}} \\ \mathbf{M}^{-1}\mathbf{D} \end{bmatrix}$, $\mathbf{B}_p = \begin{bmatrix} 0_{\frac{N}{2}} \\ \mathbf{M}^{-1}\mathbf{E} \end{bmatrix}$, $u(t)$ and $z(t)$ are the control input and the disturbance input, respectively.

Appendix 1 details the state equation of the half-car model in the state space form with the state variables defined as $x_1 = x_{bf}$, $x_2 = x_{wf}$, $x_3 = x_{br}$, $x_4 = x_{wf}$, $x_5 = \dot{x}_{bf}$, $x_6 = \dot{x}_{wf}$, $x_7 = \dot{x}_{br}$, and $x_8 = \dot{x}_{wr}$. The state equation shows that the disturbance input is not in phase with the actuator input, *i.e.*, $rank[B] \neq rank[B, B_{p2}]$, therefore the system does not satisfying the matching condition.

Let start the analysis with rewriting Eq. (2) into the following form,

$$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{A}\mathbf{x}(\mathbf{t}) + \mathbf{B}u(t) + f(t) \tag{3}$$

where $\mathbf{x}(\mathbf{t}) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control input, and the continuous function $f(t)$ represents the uncertainties with the mismatched condition *i.e.*, $rank[\mathbf{B}, f(t)] \neq rank[\mathbf{B}]$. The following assumptions are taken as standard:

Assumption i: There exists a known positive constant β such that $\| f(t)\| \leq \beta$, where $\|\bullet\|$ denotes the standard Euclidean norm.

Assumption ii: The pair $(\mathbf{A}, \mathbf{B})$ is controllable and the input matrix $\mathbf{B}$ has full rank.

III. SWITCHING SURFACE AND CONTROLLER DESIGN

In this study, we utilized the PI sliding surface define as follows:

$$\sigma(t) = \mathbf{C}\mathbf{x}(\mathbf{t}) - \int_0^t (\mathbf{CA} + \mathbf{CBK})\, x(\tau)\, d\tau \tag{4}$$

where $\mathbf{C} \in \Re^{m\times n}$ and $\mathbf{K} \in \Re^{m\times n}$ are constant matrices. The matrix $\mathbf{K}$ satisfies $\lambda(\mathbf{A} + \mathbf{BK}) < 0$ and $\mathbf{C}$ is chosen so that $\mathbf{CB}$ is nonsingular. It is well known that if the system is able to enter the sliding mode, hence $\sigma(t) = 0$. Therefore the equivalent control, $u_{eq}(t)$ can thus be obtained by letting $\dot{\sigma}(t) = 0$ [9] *i.e.*,

$$\dot{\sigma}(t) = \mathbf{C}\dot{\mathbf{x}}(\mathbf{t}) - \{\mathbf{CA} + \mathbf{CBK}\}\, \mathbf{x}(\mathbf{t}) = 0 \tag{5}$$

If the matrix $\mathbf{C}$ is chosen such that $\mathbf{CB}$ is nonsingular, this yields

$$u_{eq} = \mathbf{K}\mathbf{x}(\mathbf{t}) - (\mathbf{CB})^{-1}\mathbf{C}f(t) \tag{6}$$

Substituting Eq. (6) into Eq. (3) gives the equivalent dynamic equation of the system in sliding mode as:

$$\dot{\mathbf{x}}(\mathbf{t}) = (\mathbf{A} + \mathbf{BK})\, \mathbf{x}(\mathbf{t}) + \{\mathbf{I_n} - \mathbf{B}(\mathbf{CB})^{-1}\mathbf{C}\} f(t) \tag{7}$$

Theorem 1. If $\|\tilde{\mathbf{F}}(\mathbf{t})\| = \|\mathbf{I_n} - \mathbf{B}(\mathbf{CB})^{-1}\mathbf{C}\|\beta \leq \beta_1$, the uncertain system in Eq. (7) is boundedly stable on the sliding surface $\sigma(t) = 0$.

Proof. For simplicity, we let

$$\tilde{\mathbf{A}} = (\mathbf{A} + \mathbf{BK}) \tag{8}$$

$$\tilde{\mathbf{F}}(\mathbf{t}) = \{\mathbf{I_n} - \mathbf{B}(\mathbf{CB})^{-1}\mathbf{C}\} f(t) \tag{9}$$

and rewrite (7) as

$$\dot{\mathbf{x}}(\mathbf{t}) = \tilde{\mathbf{A}}\mathbf{x}(\mathbf{t}) + \tilde{\mathbf{F}}(\mathbf{t}) \tag{10}$$

Let the Lyapunov function candidate for the system is chosen as

$$\mathbf{V}(\mathbf{t}) = \mathbf{x}^{\mathbf{T}}(\mathbf{t})\, \mathbf{P}\mathbf{x}(\mathbf{t}) \tag{11}$$

Taking the derivative of $\mathbf{V}(\mathbf{t})$ and substituting Eq. (7), gives

$$\begin{aligned}\dot{\mathbf{V}}(\mathbf{t}) &= \mathbf{x}^{\mathbf{T}}(\mathbf{t})\,[\tilde{\mathbf{A}}^{\mathbf{T}}\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}}]\mathbf{x}(\mathbf{t}) + \tilde{\mathbf{F}}^{\mathbf{T}}(\mathbf{t})\,\mathbf{P}\mathbf{x}(\mathbf{t}) + \mathbf{x}^{\mathbf{T}}(\mathbf{t})\,\mathbf{P}\tilde{\mathbf{F}}(\mathbf{t}) \\ &= -\mathbf{x}^{\mathbf{T}}(\mathbf{t})\,\mathbf{Q}\mathbf{x}(\mathbf{t}) + \tilde{\mathbf{F}}^{\mathbf{T}}(\mathbf{t})\,\mathbf{P}\mathbf{x}(\mathbf{t}) + \mathbf{x}^{\mathbf{T}}(\mathbf{t})\,\mathbf{P}\tilde{\mathbf{F}}^{\mathbf{T}}(\mathbf{t})\end{aligned} \tag{12}$$

where $\boldsymbol{P}$ is the solution of $\tilde{\mathbf{A}}^{\mathbf{T}}\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}} = -\mathbf{Q}$ for a given positive definite symmetric matrix $\mathbf{Q}$. It can be shown that Eq. (12) can be reduced to:

$$\dot{\mathbf{V}}(\mathbf{t}) \leq -\lambda_{\min}(\mathbf{Q})\,\|\mathbf{x}(\mathbf{t})\|^2 + 2\beta_1 \|\mathbf{P}\|\,\|\mathbf{x}(\mathbf{t})\| \tag{13}$$

Since $\lambda_{\min}(\mathbf{Q}) > 0$, consequently $\dot{\mathbf{V}}(\mathbf{t}) < 0$ for all t and $\mathbf{x} \in B^c(\eta)$, where $B^c(\eta)$ is the complement of the closed ball $B(\eta)$, centered at $x = 0$ with radius $\eta = \dfrac{2\beta_1 \|\mathbf{P}\|}{\lambda_{\min}(\mathbf{Q})}$. Hence, the system is boundedly stable. ■

Remark. For systems with uncertainties satisfying the matching condition, *i.e.*, $rank[\mathbf{B}, f(t)] = rank[\mathbf{B}]$, Eq. (7) can be reduced to $\dot{\mathbf{x}}(\mathbf{t}) = (\mathbf{A} + \mathbf{BK})\mathbf{x}(\mathbf{t})$ [10]. Thus asymptotic stability of the system during sliding mode is always assured.

Now the design of the control scheme that drives the state trajectories of the system in Eq. (3) onto the sliding surface $\sigma(t) = 0$ and the system remains in it thereafter is presented. For the uncertain system in Eq. (3) satisfying assumptions (i) and (ii), the following control law is proposed:

$$u(t) = -(\mathbf{CB})^{-1}[\mathbf{CA}\mathbf{x}(\mathbf{t}) + \phi\sigma(t)] - k(\mathbf{CB})^{-1}\frac{\sigma(t)}{|\sigma(t)| + \delta} \tag{14}$$

where $\phi \in \Re^{m\times m}$ is a positive symmetric design matrix, k and δ are positive constants.

Theorem 2. The hitting condition of the sliding surface (4) is satisfied if

$$\|\mathbf{A} + \mathbf{BK}\|\,\|\mathbf{x}(\mathbf{t})\| \geq \|f(t)\| \tag{15}$$

Proof. In the hitting phase $\sigma^T(t)\sigma(t) > 0$. Using the Lyapunov function candidate $\mathbf{V}(\mathbf{t}) = \frac{1}{2}\sigma^T(t)\sigma(t)$, it can be shown that:

$$\dot{\mathbf{V}}(\mathbf{t}) = \sigma^T(t)\,\dot{\sigma}(t)$$

$$= \sigma^T(t)\left[-(\mathbf{CA}+\mathbf{CBK})\,\mathbf{x}(\mathbf{t}) - \phi\sigma(t) - \frac{k\sigma(t)}{|\sigma(t)|+\delta} + \mathbf{C}f(t)\right]$$

$$\leq -\left[\left\{\|\phi\| + \left\|\frac{k}{|\sigma(t)|+\delta}\right\|\right\}\|\sigma(t)\|^2\right.$$

$$\left. + \{\|\mathbf{C}\|\|\mathbf{A}+\mathbf{BK}\|\|\mathbf{x}(\mathbf{t})\| - \|\mathbf{C}\|\|f(t)\|\}\|\sigma(t)\|\right] \quad (16)$$

It follows that $\dot{\mathbf{V}}(\mathbf{t}) < 0$ if condition (15) is satisfied. Thus, the hitting condition is satisfied. ■

IV. SIMULATION AND DISCUSSION

The mathematical model of the system as defined in Eq. (3) and the proposed proportional integral sliding mode controller (PISMC) in Eq. (14) were simulated on computer. For comparison purposes, the performance of the PISMC is compared to the linear quadratic regulator (LQR) control approach. We assume a quadratic performance index is in the form of:

$$J = \frac{1}{2}\int_0^\infty \left(\mathbf{x}^\mathrm{T}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^\mathrm{T}(t)\mathbf{R}\mathbf{u}(t)\right) dt \quad (17)$$

where $\mathbf{Q}$ is a symmetric positive semi-definite matrix and $\mathbf{R}$ is a symmetric positive definite matrix. Then the optimal linear feedback control law is obtained as

$$u = -\mathbf{K}\mathbf{x}\,(\mathrm{t}) \quad (18)$$

where $\mathbf{K}$ is the designed matrix gain.

The numerical values for the model parameters are taken from [6], and are as follows:

$m_b = 430$kg, $I_b = 600$kgm^2, $m_{wf} = 30$kg, $m_{wr} = 25$kg,

$k_{bf} = 10000$kN/m,

$k_{br} = 6666.67$kN/m, $k_{wf} = k_{wr} = 152$kN/m,

$c_{bf} = 500$N/(m/s), $c_{br} = 400$N/(m/s),

$L_f = 0.871$m, $L_r = 1.469$m, $v = 20$m/s.

In the study, the following typical road disturbance is used: where a denotes the bump amplitude (see Fig. 2). This type

$$w(t) = \begin{cases} a(1-\cos(8\pi t))/2 & \text{if } 0.50 \leq t \leq 0.75 \text{ and } 3.00 \leq t \leq 3.25 \\ 0 & \text{otherwise} \end{cases}$$

of road disturbance has been used by [4,11] in their studies. Furthermore, the maximum travel distance of the suspension travel used is ±8cm as suggested by [4].

In the design of the LQR controller, the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ are selected as follows: $\mathbf{Q} = diag[q_1, q_2,$

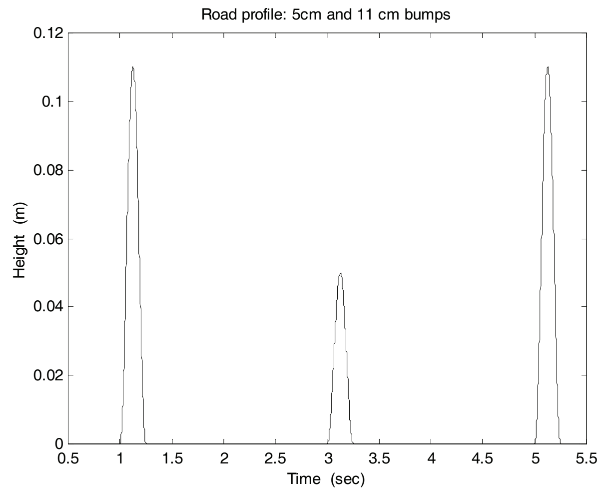

Fig. 2. Typical road disturbance.

$q_3, q_4, q_5, q_6, q_7, q_8]$ where $q_1 = q_2 = q_3 = q_4 = q_5 = q_6 = q_7 = q_8 = 100$ and $\mathbf{R} = diag(r_1, r_2)$ where $r_1 = r_2 = 1 \times 10^{-2}$. Thus, the designed gains for the LQR controller for the front and rear suspensions can be calculated as $k_{1f} = 49.8$, $k_{1r} = 30.8$, $k_{2f} = -210.4$, $k_{2r} = 1.4$, $k_{3f} = -20.7$, $k_{3r} = 74.6$, $k_{4f} = -4.2$, $k_{4r} = -367.1$, $k_{5f} = 2654.7$, $k_{5r} = -80$, $k_{6f} = -229.3$, $k_{6r} = 6.1$, $k_{7f} = -69.1$, $k_{7r} = 6.1$, $k_{8f} = 3.2$ and $k_{8r} = -305.1$. The values of the matrix K used for the PISMC is similar to the values calculated for the LQR controller, *i.e.*,

$$\mathbf{K} = \begin{bmatrix} -49.8 & 210.4 & 20.7 & 4.2 & -2654.7 & 229.3 & 69.1 & -3.2 \\ -30.8 & -1.4 & -74.6 & 367.1 & 80 & -6.1 & -6.1 & 305.1 \end{bmatrix}$$

and thus $\lambda(\mathbf{A}+\mathbf{BK}) =$

$\{-28.37, -35.64, -13.27, -8.9, -0.1 \pm j0.87, -0.05 \pm 0.69\}$.

In this simulation the following values are selected for the PISMC: $\mathbf{C} = \begin{bmatrix} 10 & 2 & 1 & 2 & 1 & 1 & 1 & 5 \\ 1 & 2 & 20 & 2 & 0.1 & 5 & 0.4 & 0.1 \end{bmatrix}$,

$\phi = diag\,[1000, 1000]$, $k_1 = k_2 = 100$ and $\delta_1 = \delta_2 = 1$.

Figures 3(a) and 3(b) show that the state trajectories slide onto the sliding manifold and remains in it thereafter. Therefore the hitting condition as in Theorem 2 is satisfied. The controller input signals that drives the state trajectories onto the sliding manifold are shown in Figs. 4(b) and 4(c). The results imply that the proposed control strategy is robust to the mismatch problem inherent in the system.

In order to fulfill the objective of designing an active suspension system, *i.e.*, to increase the ride comfort and road handling, there are two parameters to be observed in the simulations. The two parameters are the car body acceleration and the wheel deflection. Figures 5(a) and 5(b) show the travel distance for the front and rear suspensions for an active suspension system using both controllers and a passive suspension system for comparison purposes. The results show that the suspension travel within the travel

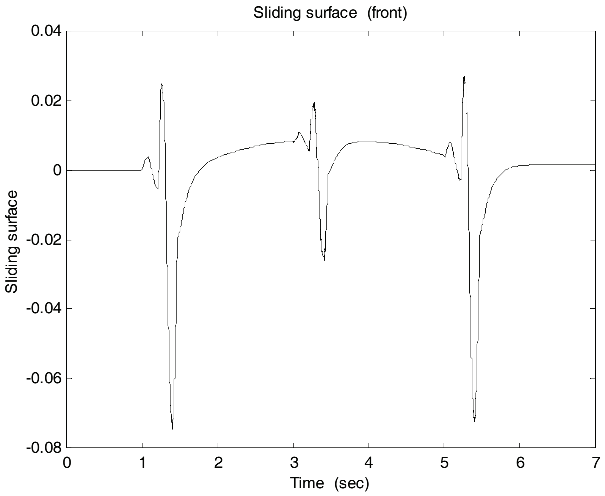

Fig. 3(a). Sliding surface for front PISMC.

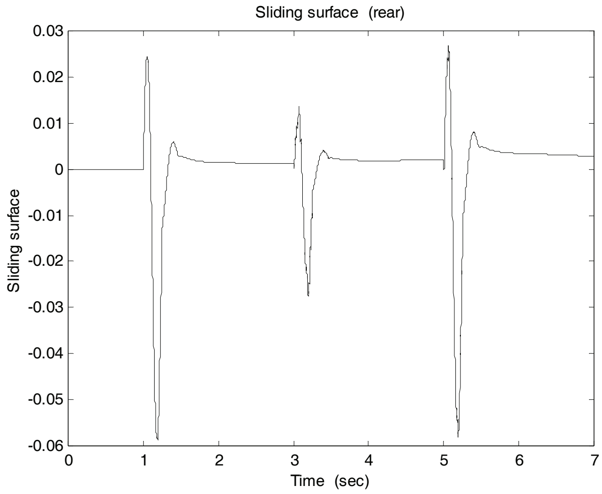

Fig. 3(b). Sliding surface for rear PISMC.

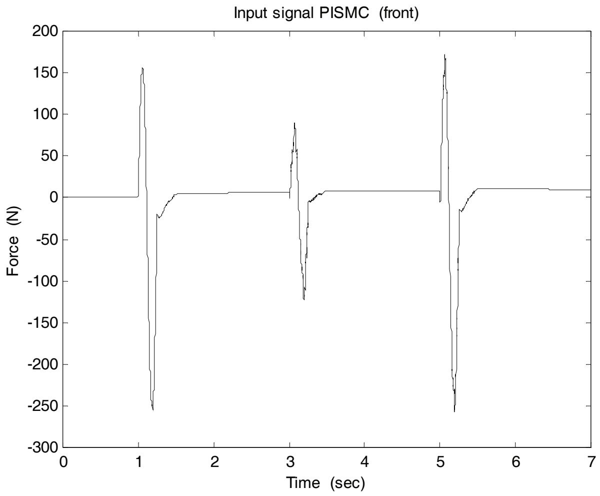

Fig. 4(a). Input signal for front PISMC.

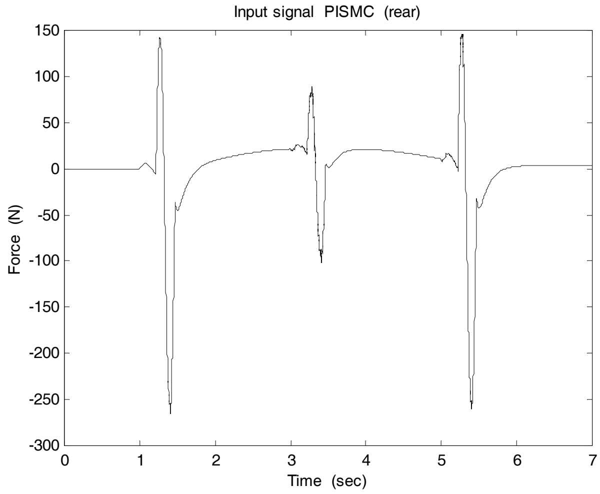

Fig. 4(b). Input signal for rear PISMC.

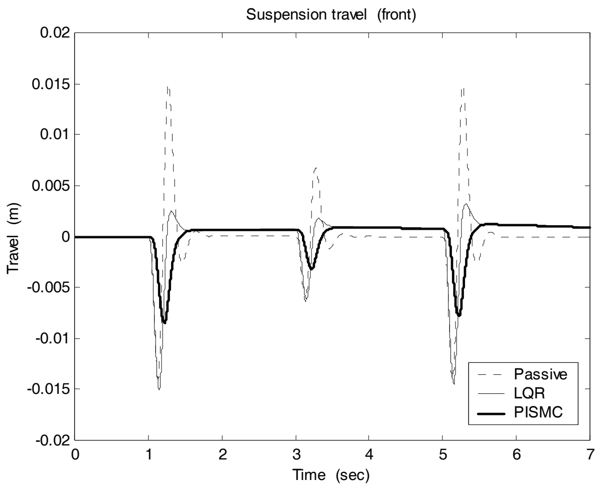

Fig. 5(a). Travel distance for front suspension.

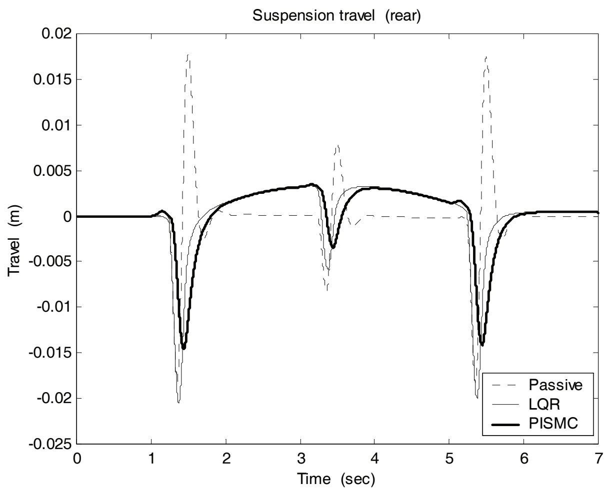

Fig. 5(b). Travel distance for rear suspension.

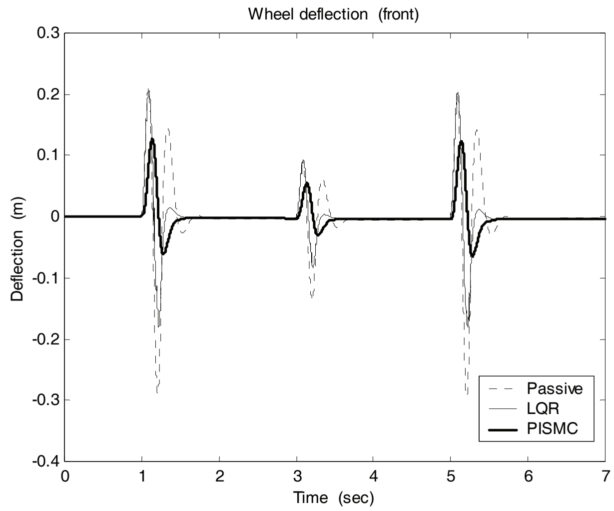

Fig. 6(a). Deflection of front wheel.

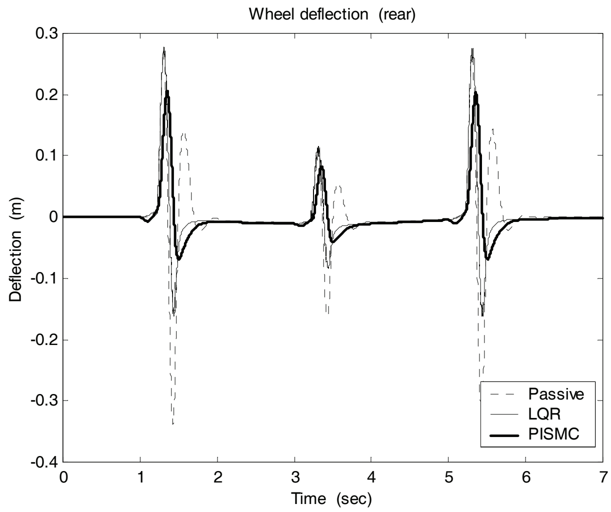

Fig. 6(a). Deflection of rear wheel.

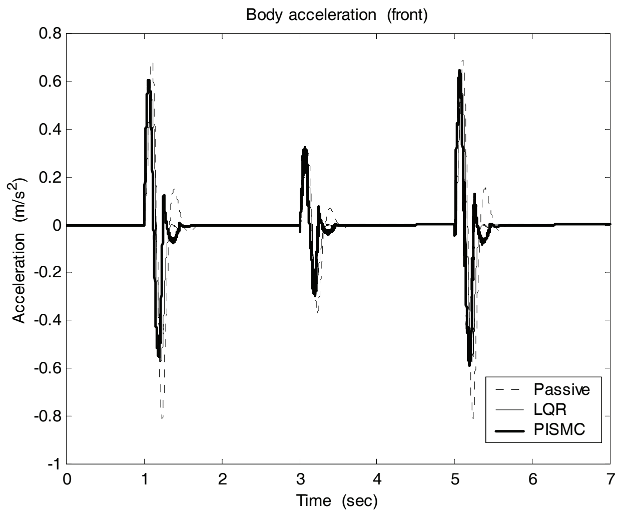

Fig. 7(a). Acceleration of front car body.

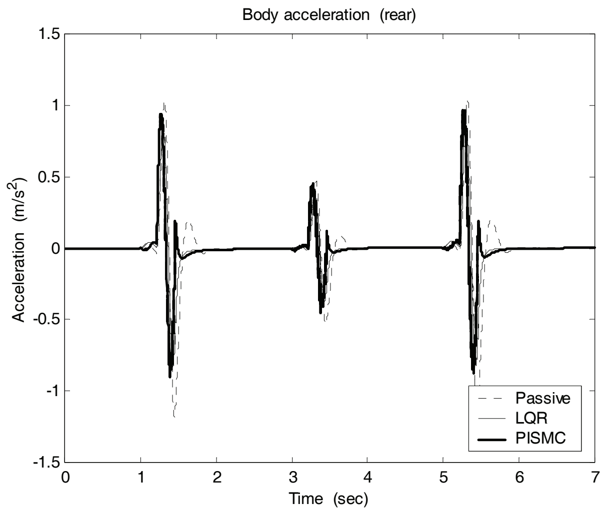

Fig. 7(b). Acceleration for rear car body.

limit, *i.e.*, ±8cm, and the results also show that the active suspension utilizing the PISMC technique perform better as compared to the others. Figures 6(a) and 6(b) illustrate clearly how the PISMC can effectively increase the tyre to road contact in comparisons to the LQR method and the passive system. Increase the 'on' road contact will directly improve the vehicle handling which can avoid the car from skidding. Moreover the body acceleration for the half-car model using the proposed PISMC technique as in Figs. 7(a) and 7(b) is slightly reduced, this guarantees better ride comfort. Therefore it is concluded that the active suspension system with the PISMC strategy will improve the ride comfort while retaining the road handling characteristics, as compared to the LQR method and the passive suspension system.

V. CONCLUSION

The paper presents a new method to transform the motion equation into the state space form. Furthermore, the paper also presents a robust strategy in designing a controller for an active suspension system which is based on the variable structure control theory. The proposed controller is capable of satisfying all the pre-assigned design requirements within the actuators limitation. A detailed study of the proportional integral sliding mode control algorithm is presented. The performance characteristics and robustness of the active suspension system under the proposed method are evaluated and compared with an LQR control strategy and a passive suspension system. The result shows that the use of the proposed proportional integral sliding mode control technique proved to be effective in controlling the vehicle and is more robust as compared to the linear quadratic regulator method and the passive suspension system.

REFERENCES

1. Appleyard, M. and P.E. Wellstead, "Active Suspension: Some Background," *Proc. Contr. Theory App.*, Vol. 142, pp. 123-128 (1995).
2. Thompson, A.G., "Design of Active Suspension," *Proc. Inst. Mech. Engrs*, Vol. 185, pp. 553-563 (1971).
3. Alleyne, A. and J.K. Hedrick, "Nonlinear Adaptive Control of Active Suspensions," *IEEE Trans. Contr. Syst. Technol.*, Vol. 3, pp. 94-101 (1997).
4. Lin, J.S. and I. Kanellakopoulos, "Nonlinear Design of Active Suspension," *IEEE Contr. Syst. Mag.*, Vol. 17, pp. 45-59 (1997).
5. Esmailzadeh, E. and H.D. Taghirad, "Active Vehicle Suspensions with Optimal State-Feedback Control," *J. Mech. Sci.*, pp. 1-18 (1996).
6. Yoshimura, T., K. Nakaminami, M. Kurimoto, and J. Hino, "Active Suspension of Passengers Cars using Linear and Fuzzy-Logic Controls," *Contr. Eng. Prac.*, Vol. 7, pp. 41-47 (1991).
7. Yoshimura, T., A. Kume, M. Kurimoto, and J. Hino, "Construction of an Active Suspension System of a Quarter Car Model using the Concept of Sliding Mode Control," *J. Sound Vib.*, Vol. 239, pp. 187-199 (2001).
8. Osman, J.H.S., "Decentralized and Hierarchical Control of Robot Manipulators," Ph.D. Dissertation, City University, London, U.K. (1991).
9. Itkis, U., *Control System of Variable Structure*, Wiley, New York, U.S.A. (1976).
10. Edwards, C. and S.K. Spurgeon, *Sliding Mode Control: Theory and Applications*, Taylor and Francis, London U.K. (1998).
11. D'Amato, F.J., and D.E. Viasallo, "Fuzzy Control for Active Suspensions," *Mechatronics,* Vol. 10, pp. 897-920 (2000).

Yahaya Md. Sam received the B.E. degree in Electrical Engineering from University Technology of Malaysia in 1986, M.Sc. degree in Control Systems Engineering from Sheffield University, United Kingdom, in 1988, and the Ph.D. degree in Control Engineering from University Technology of Malaysia in 2004.

He is currently an Associate Professor with the Faculty of Electrical Engineering, University Technology of Malaysia. His research interests include an optimal control, robust control and sliding mode control and application of these ideas to the automotive systems.

Johari Halim Shah Bin Osman obtained his Bachelor of Science degree in Physics and Master of Science Degree in Electrical Engineering from Southern Illinois University, Carbondale, Illinois, USA, in 1983 and 1985, respectively. He received his Ph.D. degree in Control Engineering in 1991 from City University, London, UK.

Currently, he is attached to the Fakulti Kejuruteraan Elektrik, Universiti Teknologi Malaysia, as an academic staff since 1986, where he teaches Control Engineering Theory, Digital Control Theory, and Robotics.

His research interests include Robotics, Robust Control of Uncertain Systems, Adaptive Control, Variable Structure Control Theory, and Control of Large Scale Systems.

APPENDIX A

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{O}_0 & \mathbf{I} \\ \mathbf{A}_{p1} & \mathbf{A}_{p2} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{L}{m_b(L_r + L_f)} + \frac{L_f^2 L}{I_b(L_r + L_f)} & \frac{L}{m_b(L_f + L_r)} - \frac{L_f L L_r}{I_b(L_f + L_r)} \\ \frac{-1}{m_{wf}} & 0 \\ \frac{L}{m_b(L_f + L_r)} - \frac{L_f L L_r}{I_b(L_f + L_r)} & \frac{L}{m_b(L_f + L_r)} + \frac{L_r^2 L}{I_b(L_f + L_r)} \\ 0 & \frac{-1}{m_{wr}} \end{bmatrix} \begin{bmatrix} f_f \\ f_r \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{k_{wf}}{m_{wf}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{k_{wr}}{m_{wr}} & 0 \end{bmatrix} \begin{bmatrix} \dot{w}_f \\ w_f \\ \dot{w}_r \\ w_r \end{bmatrix}$$

where $\mathbf{O}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

$$\mathbf{A}_{p1} = \begin{bmatrix} \frac{-Lk_{bf}}{m_b(L_f + L_r)} - \frac{L_f^2 Lk_{bf}}{I_b(L_f + L_r)} & \frac{Lk_{bf}}{m_b(L_f + L_r)} + \frac{L_f^2 Lk_{br}}{I_b(L_f + L_r)} & \frac{-Lk_{br}}{m_b(L_f + L_r)} + \frac{L_f L L_r k_{br}}{I_b(L_f + L_r)} & \frac{Lk_{br}}{m_b(L_f + L_r)} - \frac{L_f L L_r k_{br}}{I_b(L_f + L_r)} \\ \frac{k_{bf}}{m_{wf}} & \frac{-(k_{bf} + k_{wf})}{m_{wf}} & 0 & 0 \\ \frac{-Lk_{bf}}{m_b(L_f + L_r)} + \frac{L_f L_r Lk_{bf}}{I_b(L_f + L_r)} & \frac{Lk_{bf}}{m_b(L_f + L_r)} - \frac{L_f L_r Lk_{bf}}{I_b(L_f + L_r)} & \frac{-Lk_{br}}{m_b(L_f + L_r)} - \frac{L_r^2 Lk_{br}}{I_b(L_f + L_r)} & \frac{Lk_{br}}{m_b(L_f + L_r)} + \frac{L_r^2 Lk_{br}}{I_b(L_f + L_r)} \\ 0 & 0 & \frac{k_{br}}{m_{wr}} & \frac{-(k_{br} + k_{wr})}{m_{wr}} \end{bmatrix}$$

and

$$\mathbf{A}_{p2} = \begin{bmatrix} \frac{-Lc_{bf}}{m_b(L_f + L_r)} - \frac{L_f^2 Lc_{bf}}{I_b(L_f + L_r)} & \frac{Lc_{bf}}{m_b(L_f + L_r)} + \frac{L_f^2 Lc_{br}}{I_b(L_f + L_r)} & \frac{-Lc_{br}}{m_b(L_f + L_r)} + \frac{L_f L L_r c_{br}}{I_b(L_f + L_r)} & \frac{Lc_{br}}{m_b(L_f + L_r)} - \frac{L_f L L_r c_{br}}{I_b(L_f + L_r)} \\ \frac{c_{bf}}{m_{wf}} & \frac{-c_{bf}}{m_{wf}} & 0 & 0 \\ \frac{-Lc_{bf}}{m_b(L_f + L_r)} + \frac{L_f L_r Lc_{bf}}{I_b(L_f + L_r)} & \frac{Lc_{bf}}{m_b(L_f + L_r)} - \frac{L_f L_r Lc_{bf}}{I_b(L_f + L_r)} & \frac{-Lc_{br}}{m_b(L_f + L_r)} - \frac{L_r^2 Lc_{br}}{I_b(L_f + L_r)} & \frac{Lc_{br}}{m_b(L_f + L_r)} + \frac{L_r^2 Lc_{br}}{I_b(L_f + L_r)} \\ 0 & 0 & \frac{c_{br}}{m_{wr}} & \frac{-c_{br}}{m_{wr}} \end{bmatrix}$$