The dynamic model and control algorithms for the Active Suspension System

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***Abstract*—The suspension system can be classified as a passive, semi-active, and active suspension, according to its ability to add or extract energy. Road surface roughness is the leading cause of vehicle oscillation and the suspension system is used to dampen these oscillations. The objectives of the active suspension system are to improve the ride quality and handling performance within a given suspension stroke limitation. This paper will analyse the aspects of passive and active suspensions for a quarter-car model and construct an active suspension control for a quarter-car model subject to excitation from a road profile. Algorithms like PID (proportional–integral–derivative) and LQR (linear quadratic regulator) will fit the linear model.**

I. Introduction

VEHICLE suspension main task is to separate passenger and vehicular body interactions from oscillations generated by road abnormalities whilst still maintaining continuous wheel-road contact. Generally, traditional suspension consisting of springs and dampers is referred to as passive suspension, while if the suspension is externally controlled it is known as a semi-active or active suspension. A good suspension system should provide good vibration isolation, i.e. small acceleration of the body mass, and a small suspension travel, which is the maximal allowable relative displacement between the vehicle body and various suspension components.

An active suspension system has the ability to store, dissipate and introduce energy into the system. It may vary its parameters depending on operating conditions. The passive suspension system is an open-loop control system. It is designed to achieve certain conditions only. The characteristic of passive suspension are fixed and cannot be adjusted by any mechanical part. The problem of passive suspension is that it cannot be designed as heavily damped or too hard, otherwise, it will transfer a lot of road input or it could throw around the car due to the unevenness of the road. Additionally, if it is lightly damped or too soft, the suspension will reduce the vehicle's stability in turns, change lane or even swing the car. Therefore, the performance of the passive suspension depends on the road profile.

An active suspension system improves vehicle ride comfort, generating impact force that is transmitted to the sprung and unsprung masses. If the value of this force is small, the system's response will not be good. This means that the car's vibration has yet to be improved. If the impact force is bigger, the ride comfort can be further improved. However, a more significant impact force will cause a change in the dynamic load at the wheel. Once the value of the dynamic force at the wheel is reduced to zero, the wheel may be lifted off the road, and instability will occur. That's why, it is difficult to satisfy both conditions of smoothness and stability when studying control algorithms for active suspension.

The main content of this paper is to implement two different control systems, based on the dynamic equations of the quarter-car model. This paper also compares the active suspension performances, given by different control systems, with the passive suspension when the car hits a speed hump road profile.

II. Suspension performance

The suspension system in a vehicle aims at fulfilling the requirements of both comfort, for passengers, and road handling, for the driver. These two aspects, however, characterise in a general way what the suspension system should provide to the driver and the vehicle. Hence, they must be translated into physical quantities that can be observed, to evaluate the suspension performance. The most important parameters are:

- **Ride comfort**: is the ability of the vehicle suspension system of insulating passengers, and payloads, from vibrations caused by the road profile roughness. It is directly correlated to the acceleration of the sprung mass: $\mathbf{\ddot{Z}_s}$. To provide better comfort, vertical acceleration must be minimised and oscillations must be highly attenuated.
- **Body motion**: which is known as bounce, pitch and roll of the sprung mass created primarily by cornering and braking manoeuvres. It is directly correlated to the motion of the sprung mass: $\mathbf{Z_s}$. To provide better performance the sprung mass should remain in the nominal position in order to avoid oscillations of the vertical force causing oscillations of the longitudinal and lateral forces which means bad performance. This quantity is mainly important in racing cars contrarily in passenger cars.
- **Vehicle handling**: can be interpreted as the capability of the vehicle to correctly respond to the behaviour that the driver is trying to impose, also under critical driving conditions. Thanks to its deflection, the suspension system is capable of guaranteeing continuous wheel-ground contact, required to control the vehicle. Minimising the displacement between sprung and unsprung mass, thus imposing a more rigid behaviour for the suspension system, will prevent tyre-road contact loss and will provide increased road handling, improving safety. This requirement can be related to the physical quantities of suspension travel defined as the difference between the sprung mass motion and the unsprung motion: $\mathbf{Z_s} - \mathbf{Z_{us}}$

III. Physical and mathematical modelling

A quarter-car dynamic model is commonly used in studies of oscillation control for suspension systems. This model,

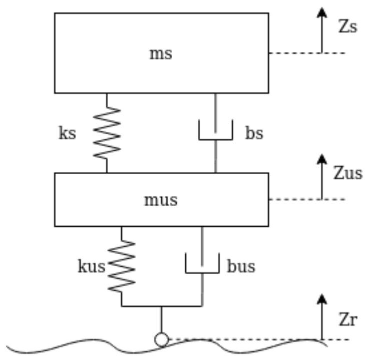

Fig. 1. Passive Suspension Quarter Car Model

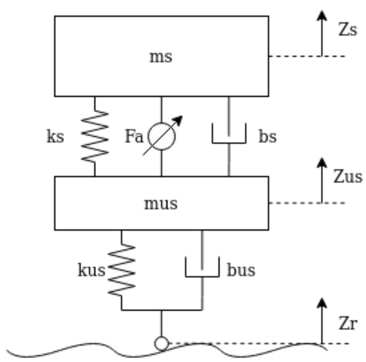

Fig. 2. Active Suspension Quarter car model

Figure 1, includes only two masses: the sprung mass, m_s, and the unsprung mass, m_{us}. The suspension system is modelled as a spring and a shock absorber. The tires are also modelled similarly to the suspension system.

For vehicles with an active suspension system, an actuator is located between the sprung mass and the unsprung mass, Figure 2.

The equations describing the dynamics of a quarter-car model are shown as follows:

$$\begin{aligned} m_s\ddot{Z}_s &= b_s\dot{Z}_{us} - b_s\dot{Z}_s - k_s(Z_s - Z_{us}) + F_a \\ m_{us}\ddot{Z}_s &= -b_s\dot{Z}_{us} - b_{us}\dot{Z}_s + b_s\dot{Z}_s + b_{us}\dot{Z}_r \\ &\quad -k_s(Z_{us} - Z_s) - k_{us}(Z_{us} - Z_r) - F_a \end{aligned} \tag{1}$$

In order to analyse the model, the state-space model describing the active suspension system will be created using the two equations of motion found in (1). The state variable representing the system are:

$x_1 = Z_s - Z_{us}$	suspension travel
$x_2 = \dot{Z}_s$	sprung mass velocity
$x_3 = Z_{us} - Z_r$	wheel's deflection
$x_4 = \dot{Z}_{us}$	wheel's vertical velocity

The state-space model can easily be written in the matrix form shown below:

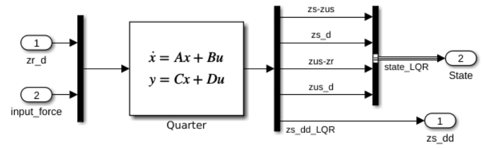

Fig. 3. Active suspension control system

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-k_s}{m_s} & \frac{-b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & \frac{b_s}{m_{us}} & \frac{-k_{us}}{m_{us}} & \frac{-(b_s+b_{us})}{m_{us}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m_s} \\ -1 & 0 \\ \frac{b_{us}}{m_{us}} & -\frac{1}{m_{us}} \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_a \end{bmatrix} \tag{2}$$

To perform the simulations the dynamic system described in (2) has been modelled using *SIMULINK* as shown in Figure 3, where the inputs are the derivative of the road profile and the actuator force, while the outputs are the state space variable and the sprung mass acceleration.

IV. Controller design

A. LQR

The linear time-invariant system (LTI), is described by equation (2). Consider a state variable feedback regulator:

$$u = -Kx \tag{3}$$

where *K* is the state feedback gain matrix. The optimisation procedure consists of determining the control input *u*, which minimises the performance index. The performance index *J* represents the performance characteristic requirements, as well as the controller input limitation. The optimal controller of the given system is defined as controller design, which minimises the following performance index:

$$J = \int_0^\infty (x'Qx + u'Ru)\ dt \tag{4}$$

The matrix $Q \in \mathbb{R}^m$ and $R \in \mathbb{R}^n$, where *m* is the number of states x and *n* is the number of control input u, are weighting matrices, in particular, the matrix Q is used to penalise bad performances, any non-zero state x adds a non-negative amount to J, while the matrix R is used to penalise the control input, any non-zero u adds a non-negative amount to J.

The gain matrix *K* is represented by:

$$K = R^{-1}B'P \tag{5}$$

The matrix *P* must satisfy Riccati's reduced matrix equation:

$$A'P + PA - PBR^{-1}B'P + Q = 0 \tag{6}$$

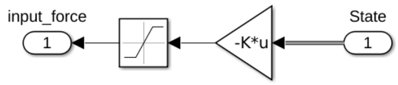

Fig. 4. LQR controller

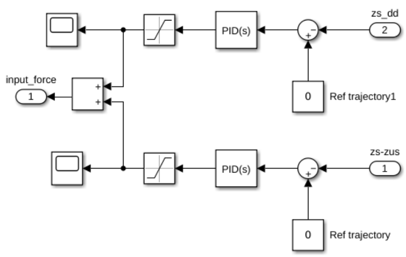

Fig. 5. PID controller

Then the feedback regulator is:

$$u = -(R^{-1}B'P)x \tag{7}$$

The *SIMULINK* model of the LQR controller described in (7) is shown in Figure 4.

B. PID

The PID controller can be written in the following form:

$$u(t) = K_p\ e(t) + K_i \int e(t)\ dt + K_d\ \frac{d}{dt}\ e(t) \tag{8}$$

where: $e(t) = r(t) - y(t)$ is the tracking error.

The desired closed loop dynamics can be obtained by adjusting the three parameters K_p, K_i and K_d, often iteratively with "tuning" and without specific knowledge of a plant model. Stability can often be obtained using only the proportional term. The integral term permits the rejection of a step disturbance. Meanwhile, the derivative term provides damping or shaping of the response.

The *SIMULINK* model of the PID controller described in (8) is shown in Figure 5.

V. Simulation results and discussion

A. Parameters setting

The parameters considered in the simulations are:

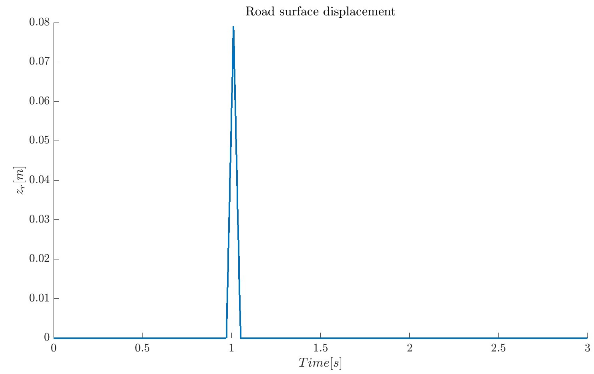

Fig. 6. Speed hump profile

Parameters	Value	Description
m_s	234 [kg]	Total sprung mass
m_{us}	43 [kg]	Total unsprung masses
k_s	26000×4 [N/m]	Parallel of 4 suspension stiffness
b_s	1544×4 [N sec/ m]	Parallel of 4 suspension damping
k_{us}	100000×4 [N/m]	Parallel of 4 tyre stiffness
b_{us}	0×4 [N sec/ m]	Parallel of 4 tyre damping

which represent the parameters of E-AGLE Trento Racing Team's vehicle, *Fenice*.

The response of the system is tested considering a speed hump profile, which corresponds to Figure 9 present in the paper [1]. The speed hump profile has been modelled using the following equation:

$$z_r = \begin{cases} 0.04 \times (1 - cos(7\pi t)) & t_{start} \leq t \leq t_{stop} \\ 0 & otherwise \end{cases}$$

Where the variables t_{start} and t_{stop} are defined depending on the vehicle velocity. In this paper three different simulations have been done considering three different velocities:

- 30 km/h $\rightarrow t_{start} = 1\ [s]\ t_{stop} = 1.09\ [s]$
- 40 km/h $\rightarrow t_{start} = 1\ [s]\ t_{stop} = 1.045\ [s]$
- 72 km/h $\rightarrow t_{start} = 1\ [s]\ t_{stop} = 1.025\ [s]$

The velocities of 30 and 40 km/h simulate a passenger car when hitting a speed hump in the road, while the velocity of 72 km/h simulates a formula-SAE car when hitting a curb in the circuit.

In Figure 6 the speed hump profile considering the vehicle velocity of 72 km/h is represented.

B. Parameters tuning

The control variables used in the two controllers are different, for the PID controller only the suspension travel and the sprung mass acceleration are controlled, while for the LQR controller, all the state space variables are controlled by the feedback regulator.

The tuning of the parameters has been done differently for the two controllers.

For the PID controller, the MATLAB function *pidtune* has been used, which given the transfer function and the type of controller (PID in this case) returns the values of the parameters (K_p, K_i and K_d), then following a "trial and error" procedure a series of simulations have been made in order to set the parameters which maximise the performance parameters. The parameters obtained are described in the following table.

Control variable	K_p	K_i	K_d
$z_s - z_{us}$	3.1953e+05	5.2392e+03	3.8195e+06
$\ddot{z}_s$	160	1.2673e+04	0

Also for the LQR controller, a series of simulations have been made in order to choose the proper weighting matrices Q and R, where Q is a diagonal positive definite and R is a positive constant, starting by setting the elements on the diagonal of the matrix Q and the positive constant R to 1. Using the MATLAB function *lqr* it calculates the gain matrix *K* given the weighting matrices Q, R and the state space matrices A and B. Then as done with the PID controller following a "trial and error" procedure the parameters have been adjusted in order to minimise the quadratic cost function J. The chosen parameters are:

$$Q = \begin{bmatrix} 1 \times 1e10 & 0 & 0 & 0 \\ 0 & 1 \times 1e8 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$R = 1 \tag{10}$$

C. Simulations

The *SIMULINK* scheme used to perform the overall simulation is reported in Appendix B.

To evaluate the performance of the active suspension system, described in Section II, with respect to the passive ones two different performance indices have been defined, one to evaluate the percentage reduction of the oscillations, Equation 11, and one to evaluate the percentage reduction of the overshoot, Equation 12.

$$KPI_{rms} = \left(1 - \frac{|rms(X_{act})|}{|rms(X_{pass})|}\right)\% \tag{11}$$

$$KPI_{max} = \left(1 - \frac{max(|X_{act}|)}{max(|X_{pass}|)}\right)\% \tag{12}$$

Where X_{act} and X_{pass} are the physical quantities of the active and passive system respectively to evaluate.

Appendix A reports all the results obtained in the simulation considering the vehicle velocity of 72 km/h, and all the performance indices are reported in Table I for the LQR controller and in Table II for the PID controller.

D. Time domain analysis

Figure 15 illustrates the time response of the suspension travel, which is related to vehicle handling. Regarding the PID controller, both the peak and the oscillations have been reduced by only some percentage points, 2.63% and 1.62% respectively, and also the settling time has been reduced. Regarding the LQR controller, we had a worse performance regarding the peak, it increases its magnitude when the suspension travel is negative, of about 21.32%, while the oscillations and the settling time have been reduced.

Figure 16 illustrates the time response of the sprung mass acceleration, which is related to ride comfort. Regarding the PID controller, there is a reduction in the peak of about 2.26%, while there is a slight worsening of the oscillations of about 1.82%. Regarding the LQR controller, both the peak and the oscillations have been strongly reduced by about 20.68% and 13.71 % respectively. For both active suspension controllers, the settling time has been reduced.

Figure 17 illustrates the time response of the sprung mass motion, which is related to body motion. Regarding the PID controller, both the peak and the oscillations have been reduced by about 10.99% and 6.26 % respectively. Regarding the LQR controller, both the peak and the oscillations have been strongly reduced by about 35.77% and 29.96% respectively. For both active suspension controllers, the settling time has been reduced.

Parameters	Max Passive	Max LQR	KPI_{rms}	KPI_{max}
$\ddot{z_s}$	55.10	47.55	20.68%	13.71%
z_s	0.06	0.04	35.77%	29.96%
$z_s - z_{us}$	0.04	0.047	1.31%	-21.32%

TABLE I
KEY PERFORMANCE INDICES (KPI) FOR LQR CONTROLLER

Parameters	Max Passive	Max PID	KPI_{rms}	KPI_{max}
$\ddot{z_s}$	55.10	56.10	2.26%	-1.82%
z_s	0.06	0.05	10.99%	6.26%
$z_s - z_{us}$	0.04	0.038	2.63%	1.62%

TABLE II
KEY PERFORMANCE INDICES (KPI) FOR PID CONTROLLER

E. Frequency domain analysis

To better evaluate the performances of the active suspension system a frequency domain analysis has been performed. In Figure 7, 8 and 9 the bode plot of the transfer functions for the suspension travel, sprung mass acceleration and sprung mass motion with respect to the derivative of the road input respectively are represented.

The bode plot of the suspension travel (Figure 7) shows a reduction in the peak for both the controller used. The LQR controller additionally shows a worsening in the low-frequency range.

The bode plot of the mass acceleration (Figure 8) shows a reduction in the peak for both the controller used, in particular

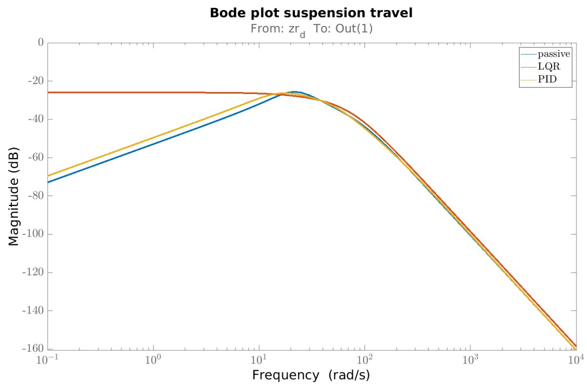

Fig. 7. Bode diagram of suspension travel

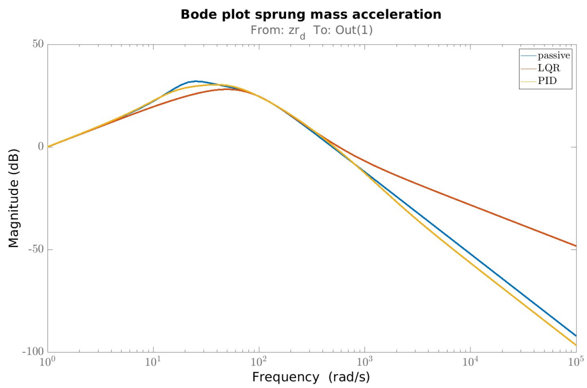

Fig. 8. Bode diagram of sprung mass acceleration

for the LQR controller. The LQR controller additionally shows a worsening in the high-frequency range. The PID controller shows an overall improvement along all the frequency-domain contrary to the LQR controller.

For the bode plot of the mass motion (Figure 9), hold the same considerations done for the bode plot of the mass acceleration.

The bode plots of the three transfer functions don't show a strong reduction but an overall improvement, especially in the peak, due to the fact that parameter tuning has been done in order to improve all the suspension performance.

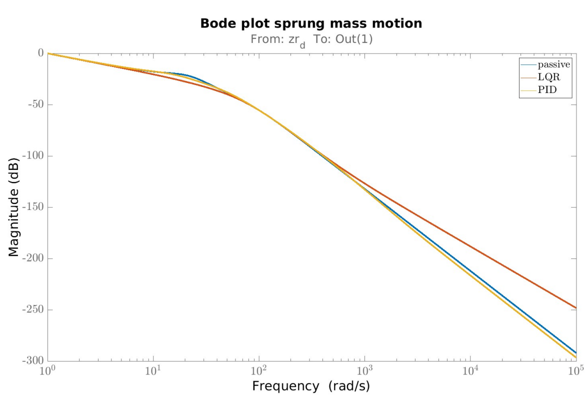

Fig. 9. Bode diagram of sprung mass motion

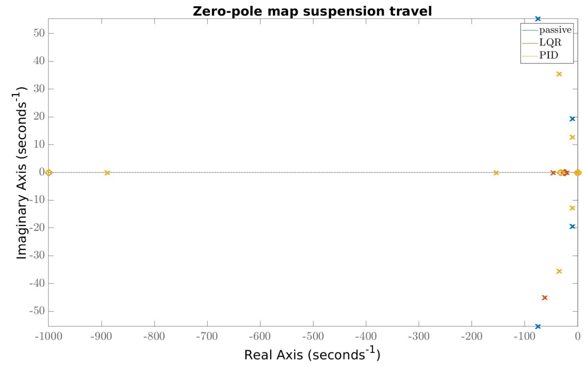

Fig. 10. Zero-pole map of suspension travel

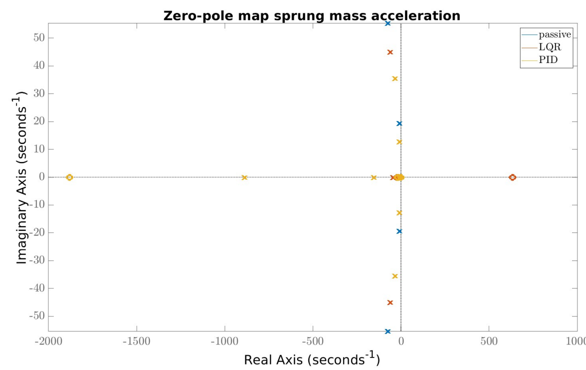

Fig. 11. Zero-pole map of sprung mass acceleration

F. Stability analysis

The stability for a continuous time linear system is given if all the poles of the transfer function have negative real part.

In Figure 10, 11 and 12 are reported the zero-pole map of the transfer functions related to the physical quantities associated with the suspension performances.

As we can see from the three zero-pole maps all three transfer functions have poles with negative real part, and thus can be concluded that are stable.

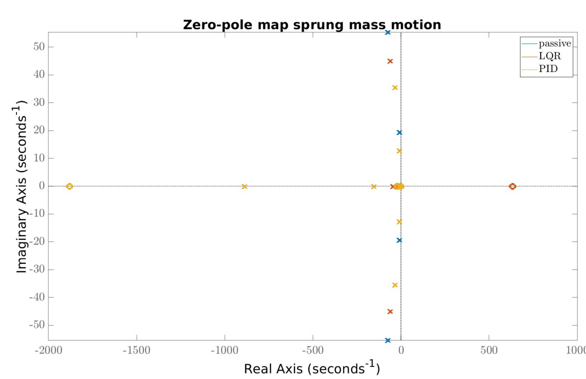

Fig. 12. Zero-pole map of sprung mass motion

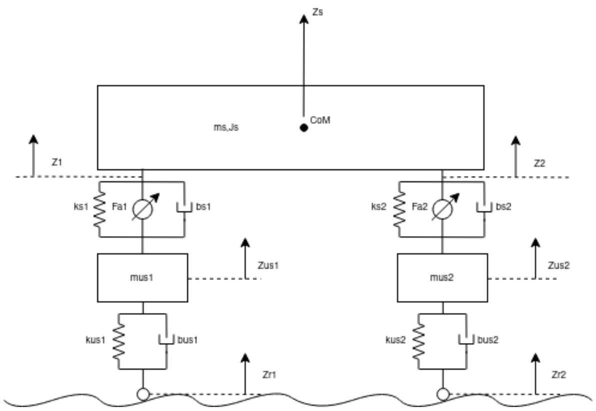

Fig. 13. Active Suspension Half car model

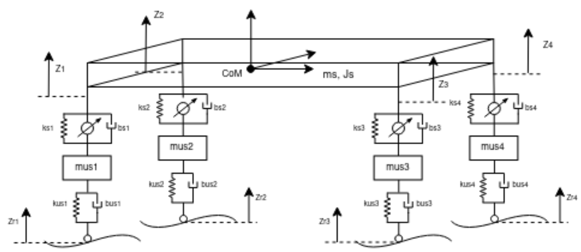

Fig. 14. Active Suspension Full car model

VI. Extension to Half car and full car model

Up to now, we only considered the quarter-car vehicle model (Figure 2). This simple model, however, does not fully represent the rigid-body motions of a real car. To better study the dynamics of the suspension system the model can be expanded to the Half-dynamic model, or the Full-dynamic model which takes into account both the roll and pitch motion of the car.

The Half-Dynamic Model of a vehicle, described in Figure 13. It can be roll model or pitch model. This model considers the influence of the vehicle's body roll or pitch angles depending on which half of the model is being studied. This model split the unsprung masses into two masses and the suspension-tyre system into two equivalents, given by the parallel of two suspension-tyre systems.

The Full-Dynamic Model of a vehicle, described in Figure 14. Overall, this model is quite complex. However, this model provides all the necessary elements to evaluate the vehicle's oscillation.

VII. Conclusion

The simulation results show that an active suspension system reduces the oscillations, as shown in Table I and II, giving better performance than the passive suspension. The peak performance instead depends on the type of controller used. In conclusion, from the simulation results, active suspension with an LQR or PID controller can be considered one of the valid solutions.

Appendix A
Simulation Plots

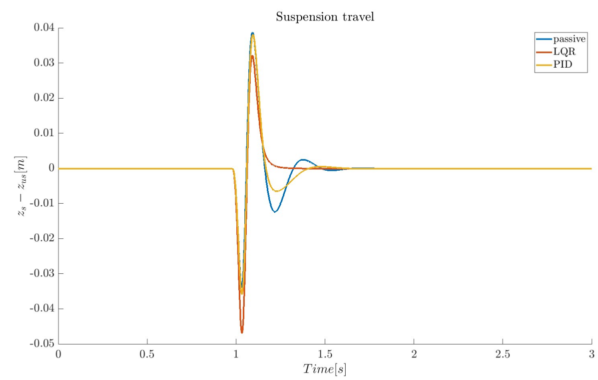

Fig. 15. Suspension travel with bump road

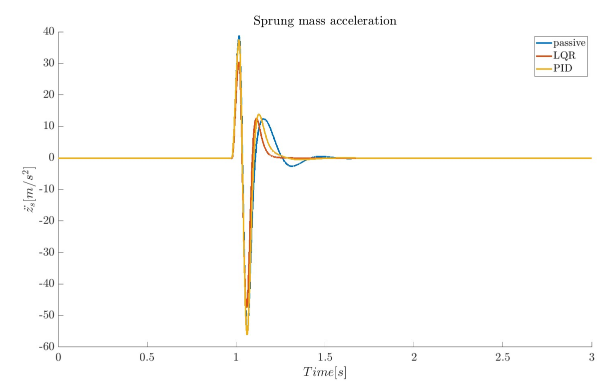

Fig. 16. Sprung mass acceleration with bump road

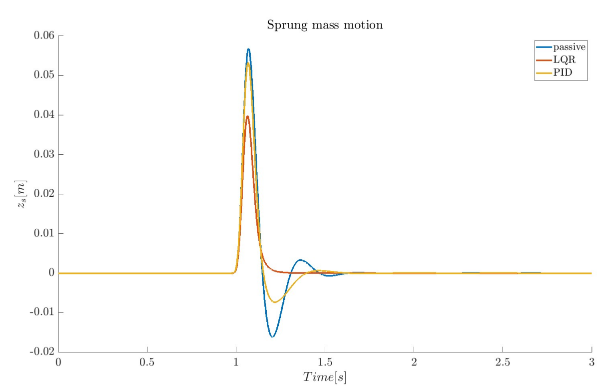

Fig. 17. Sprung mass motion with bump road

APPENDIX B
SIMULINK SCHEME

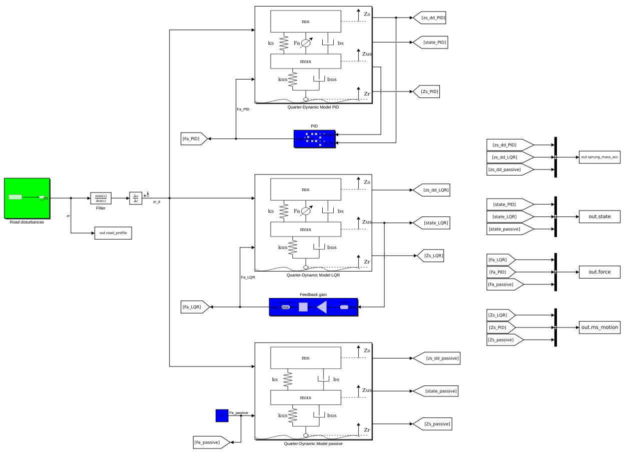

Fig. 18. SIMULINK scheme simulation

REFERENCES

[1] A. Kanjanavapastit and A. Thitinaruemit, "Estimation of a speed hump profile using quarter car model," *Procedia - Social and Behavioral Sciences*, vol. 88, pp. 265–273, 10 2013.