Highlights

A rich Vehicle Routing Problem with multiple Functionally-Diverse Nodes allowing Hierarchical and Multimodal Time-Dependant Transhipment of multiple Node-and-Vehicle-compatible-Cargo with Cascaded-Time-Minimization Objective for Disaster/Emergency Decision Support Systems

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- Multiple depots
- Traffic considered for the static Time-Network
- Research highlight 3
- Research highlight 4
- Research highlight 5

A rich Vehicle Routing Problem with multiple Functionally-Diverse Nodes allowing Hierarchical and Multimodal Time-Dependant Transhipment of multiple Node-and-Vehicle-compatible-Cargo with Cascaded-Time-Minimization Objective for Disaster/Emergency Decision Support Systems

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Abstract

Keywords:

1. Introduction

2. Previous Works and State of Art

3. Problem Description and Mathematical Formulation

A rich vehicle route planning problem is considered, having various types of vertices of a graph. These diverse vertex types correspond to different functionalities as mentioned in Table 2.

We initially allow all network linkages, i.e. all possible connection combinations between all vertices clustered within sets mentioned in Table 1 is allowed. It needs to be evident that Nodes (N) are the controlling elements of the formulation and a problem without any Nodes will not require any vehicle to ply. Keeping this in mind we remove linkages connecting Vehicle Depots, linkages connecting Warehouses to Vehicle Depots, as well as linkages connecting Vehicle Depot to Relief Centres; since these would never be used for our considered problem.

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We consider multiple types of loads being carried by compatible vehicles. Each vehicle can have multiple trips, which is formulated here using a layered approach such that each vehicle starts and ends at its respective vehicle depot while traveling through other vertices as required and performing necessary tasks of relief-and-rescue. Here we initially assign a huge number of levels (*i.e.*, vehicle layers) to each individual vehicle. The upper-limit of the number of layers which need to be provided is calculated as per equation (TBD).

Each individual vehicle (v) may only be accurately identified by its Vehicle Depot (VD), Vehicle Type (vT), and a unique number from among the number of vehicles of type vT available at VD.

Table 1: **Sets and Parameters**

Notation	Description
VD	Set of Vehicle Depots
W	Set of Warehouses
NM	Set of Nodes allowing only a single vehicle to perform simultaneous pickup and delivery
NP	Set of Nodes allowing split pickup and delivery
N	Set of all Nodes, *i.e.*, $NM \cup NP$
TP	Set of Transhipment Ports
RC	Set of Relief Centers
V_0	Vertex set consisting of $W \cup N \cup TP \cup RC$
V	Entire vertex set consisting of $VD \cup W \cup N \cup TP \cup RC$
E	Set of all Edges joining all two-vertex permutations from the vertex set V. However, a few permutations are not considered since they will never be used during the routing. All self-linkages alongwith the Edges $W \to VD$, $VD \to VD$ and $VD \to RC$ are hence removed from the network.
G	Network graph consisting of V vertices and E edges
v	refers to each vehicle which is defined as a combination of $\left\{ \substack{Vehicle\ Depot, \\ Vehicle\ Type\ at\ \ that\ Vehicle\ Depot, \\ Vehicle\ Numbers\ of\ those\ Types\ in\ that\ Depot} \right\}$. This refers to the set of specific vehicles of the corresponding $Vehicle\ Type$ in the respective $Vehicle\ Depot$ and the unique $Vehicle-Numbers$ which is within the maximum number of allowed vehicles of the same type in the same depot.
$vT(h)$	vT indicates the $Vehicle\ Types$ present at some specific $Vehicle\ Depot$ which is referenced through h, $h \subseteq VD$. Each VD has some specific types of vehicles which are available for performing tasks.
$vN(h,k)$	vN is referenced using the two parameters (h,k), which indicates the vehicles of type $k \in vT$ available at a specific vehicle depot h. It is the entire list of numbers starting from 1 to the maximum available value.
l_v	refers to all the levels l available to vehicle v. These integer numbers start from 1, the base level, till a maximum value l_v^{max} as per (Equation **TBD**)
DY	The set of all Delivery cargos
PU	The set of all Pickup cargos
$CC(k)$	Compatible Cargo: $CC(k)$ refers to the set of all load types that are allowed to be carried by a vehicle type k, where $k \in vT$
b_i^c	Intermediate continuous variable representing the amount of load transfer of type c at only the lowermost/first level/layer, for all vehicles, at Node i
q_i^c	Intermediate continuous variable representing the amount of load transfer of type c at layers apart from the lowermost, for all vehicles, at Node i
Q_i^c	Represents quantity of Cargo Type c at PickUp and/or Delivery Vertex i (availability of each DY in W, or, capacity for each PU at RC, or, requirement of each CC at N)
$CP(i)$	Compatibility at Ports: Refers to the set of compatible cargos which are allowed to be transhipped at the specific TP; *i.e.*, if $i \in TP$, then $CP(i)$ is the set of all delivery and pickup load types for which transhipment is allowed through i. In other words, loading or unloading of all load-types in the set $CP(i)$ is allowed at i. It is assumed that the rest of the incompatible cargo would not be taken out of the vehicles during transhipment at the concerned incompatible ports (as the formulation does not allow this).
$C_{i,j}^k$	Represents time taken to travel from vertex i to vertex j by vehicle type k
M	A very large positive real number
$U^{k,c}$	Represents the loading/unloading time of cargo type c into/from vehicle type k.
$CV(c)$	Compatible Vehicles: $CV(c)$ represents the set of all vehicle types that are compatible to carry the load type c. If c is a set of load types instead of an individual load type, *e.g.* as in section 3.4.1 and in Eq. 67r, then CV would represent the union of all the vehicle types that are compatible to carry any of the load types represented within c.
E_c	Volume of a unit of load-type c
E^k	Volume of a vehicle of type k
F_c	Weight of a unit of load-type c
F^k	Weight of a vehicle of type k
OT	Set of vehicle types which shall not return back to their starting depots

Table 2: **Multi-functionality of Graph Vertices**

Types of Vertices based on functionality	
Mathematical View Point	**Nomenclature Description**
Source / Origin only	Not considered in our Problem or Formulation, applicable only for open VRPs
Sink / Destination only	
Both Source & Sink *i.e.*, Vertex acts as both Origin & Destination	Vehicle Depot (VD) [these vertices are also the time-origin for corresponding vehicles]
PickUp only	Warehouses (W)
Delivery only	Relief Centers (RC)
PickUp & Delivery (Simultaneous)	Simultaneous Nodes (NM) [for single vehicle relief-rescue]
PickUp & Delivery (Split)	Split Nodes (NP) [allows multiple vehicles to perform predefined operations over the entire course of the mission]
Time-Dependent Cargo-Compatible Echelon Points	Transhipment Ports (TP) [also acts as Multimodal Junctions]
Route Split	For deploying Water Boats from a Land Vehicle for urban-flood relief [not considered in current formulation or problem]
Route Capture	For accepting boats with rescued victims back into the Mother Vehicle [not considered in our problem or formulation]
Allows both Route Splitting and Capture	Deploy Points [not considered in current formulation and suggested as a possible future extension]

Table 3: **Variable Definitions**

Notation	Description
$maxT$	Entire Emergency Operation Time
$x_{i,j}^{v,l}$	Binary decision variable referring to a vehicle v travelling from vertex i to vertex j on its level l
$x_{i}^{v,l,m}$	Binary decision variable referring to a vehicle v at vertex i travelling from level l to level m
$y_{i,j}^{v,l,c}$	Continuous-positive variable referring to the amount of cargo load of category c being taken by vehicle v traveling from vertex i to vertex j both on its level l, the carried cargo being compatible with the vehicle type
$y_{i}^{v,l,m,c}$	Continuous-positive variable referring to the amount of cargo load of category c being taken by vehicle v at vertex i traveling from level l to level m, the carried cargo being compatible with the vehicle type
$r_{i}^{v,l,c}$	Residue refers to the amount of actual transhipment operation of each cargo-type at each TP. It is a Real variable allowed to take negative or positive values depending on whether the compatible cargo-type was deposited or collected by the vehicle during any of its layer-wise visits. In this formulation, we calculate the Residue similar to deposit at TPs, and therefore the variable takes negative values whenever the resources are collected for subsequent transhipment.
$n_{i}^{v,l,c}$	This is an intermediate binary variable used during calculations of modulus of the Residue. This is constrained to take a value of 1 when the corresponding residue is positive, and 0 when the residue is negative.
$a_{i}^{v,l,c}$	This continuous-positive variable stores the modulus or absolute value of the Residues.
$at_{i}^{v,l}$	Continuous-positive variable, refers to the arrival time of the vehicle v at vertex i for a specific visit *i.e.*, at layer l
$dt_{i}^{v,l}$	Continuous-positive variable, refers to the departure time of the vehicle v at vertex i after a specific visit *i.e.*, at layer l
$tt_{i,j}^{v,l}$	Continuous-positive variable, refers to the travel time of the vehicle v when journeying from vertex i to j on it's layer l
$tt_{i}^{v,l,m}$	Continuous-positive variable, refers to the cascaded (or, continually added) travel time of the vehicle v at vertex i when journeying from layer l to layer m
$nt_{i}^{v,l}$	Continuous-positive variable, refers to the non-working or waiting time of the vehicle v at vertex i during a specific visit *i.e.*, at layer l. The waiting time would be non-zero only during collection at TPs', when the resource to be collected is yet to arrive at the TP as well as the cumulative addition of the working-durations (working durations refer to the loading/unloading done by the vehicle v at i during this visit l) is less than the required resource arrival time during since arrival of v.
$o_{(vv,ll,\$\$)}^{(v,l),i,c}$	The residues obtained needs to be populated over load-type and TP specific space-time matrices. This continuous variable o represents the cells in each of these matrices. For each $i \in TP$ and $c \in CP(i)$, matrices are created with the space axis varying over (v,l) and the time axis being varied over $(vv, ll, \$\$)$, where l refers to a layer of vehicle v, and ll refers to a layer of vehicle vv. Here $\$\$ \in \{A, D\}$, where A refers to arrival and D refers to departure. These space-time matrices are used to ensure that the temporal accumulation of the residues always remain positive.
$g_{(vv,ll,\$\$)}^{(v,l,\$),i}$	This intermediate binary variable compares the arrival or departure times of the subscripted vehicle vv *w.r.t.* the super-scripted vehicle v during their visits on layers ll and l respectively, only at the Transhipment Ports *i.e.*, $i \in TP$. Here $\$, \$\$ \in \{A, D\}$ representing arrival or departure for the vehicle v and vv respectively. The variable takes value 1 when: • for \$=A and \$\$=A: if $at_i^{vv,ll} \leq at_i^{v,l}$, else takes value 0 • for \$=A and \$\$=D: if $dt_i^{vv,ll} \leq at_i^{v,l}$, else takes value 0 • for \$=D and \$\$=A: if $at_i^{vv,ll} \geq dt_i^{v,l}$, else takes value 0 • for \$=D and \$\$=D: if $dt_i^{vv,ll} \geq dt_i^{v,l}$, else takes value 0

$$Minimize \quad maxT \tag{1}$$

subject to the following constraints:

3.1. Routing Constraints

Eq. 2 allows atmost a single path to start from the respective VD of each vehicle, to any of the joining vertices in the sets W, N, TP for the base level 1.

$$\sum_{j \in W,N,TP} x_{i,j}^{v,1} \leq 1, \quad \forall i \in VD, \forall v \in \left\{ \begin{matrix} i, \\ k \in vT(i), \\ vN(i,k) \end{matrix} \right\}, \tag{2}$$

Eq. 3 constrains the number of incoming paths/edges into the respective VD across all it's levels, to be limited to 1, *i.e.*, only when that vehicle is used.

$$\sum_{l \in l_v} \sum_{j \in N,TP,RC} x_{j,i}^{v,l} = \sum_{j \in W,N,TP} x_{i,j}^{v,1}, \quad \forall v \in \left\{ \begin{matrix} i \in VD, \\ k \in vT(i), \\ vN(i,k) \end{matrix} \right\}, \tag{3}$$

Eq. 4 conserves the route flows for each vertex for all layers constraining the sum of all incoming edges to be equal to the sum of all outgoing edges for that vertex; eq. 4a is specific for the first layer, while eq. 4c is specific for the last layer. Only the first layer has vertices W, TP, N being connected with edges (decision variables) from the Vehicle Depot. All layers have connections from vertices TP, N, RC to the Vehicle Depot.

$$x_i^{v,2,1} + \sum_{\substack{j \in V_0,h \\ i \neq j \\ if \ i \in RC \Rightarrow j \neq h}} x_{j,i}^{v,1} = x_i^{v,1,2} + \sum_{\substack{j \in V_0,h \\ i \neq j \\ if \ i \in W \Rightarrow j \neq h}} x_{i,j}^{v,1}, \quad \forall v \in \left\{ \begin{matrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{matrix} \right\}, \quad \forall i \in V_0, \tag{4a}$$

$$x_i^{v,l-1,l} + x_i^{v,l+1,l} + \sum_{\substack{j \in V_0 \\ i \neq j}} x_{j,i}^{v,l} = x_i^{v,l,l-1} + x_i^{v,l,l+1} + \sum_{\substack{j \in V_0,h \\ i \neq j \\ if \ i \in W \Rightarrow j \neq h}} x_{i,j}^{v,l}, \quad \forall v \in \left\{ \begin{matrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{matrix} \right\}, \forall i \in V_0, \forall l \in l_v \setminus \{1, l_v^{max}\}, \tag{4b}$$

$$x_i^{v,l_v^{max}-1,l_v^{max}} + \sum_{\substack{j \in V_0 \\ i \neq j}} x_{j,i}^{v,l_v^{max}} = x_i^{v,l_v^{max},l_v^{max}-1} + \sum_{\substack{j \in V_0,h \\ i \neq j \\ if\ \ i \in W \Rightarrow j \neq h}} x_{i,j}^{v,l_v^{max}},$$

$$\forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix} \right\}, \forall i \in V_0, if\ \ l_v^{max} > 1, \quad \text{(4c)}$$

Extra optional constraints, which may be relaxed, are presently considered, to allow a new level to be used only if the level below cannot be used. These are provided to progressively decrease the number of initial levels required of respective vehicles to get to the optimal solution:

$$\sum_{\substack{z \in V_0,h \\ i \neq z \\ if\ \ i \in W \Rightarrow z \neq h}} x_{i,z}^{v,1} + \sum_{\substack{z \in V_0,h \\ z \neq j \\ if\ \ j \in RC \Rightarrow z \neq h}} x_{z,j}^{v,1} \geq x_{i,j}^{v,2}, \quad \forall i,j \in V_0, (i \neq j), \forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix} \right\}, \quad \text{(5a)}$$

$$x_i^{v,l,l-1} + x_j^{v,l-1,l} + \sum_{\substack{z \in V_0,h \\ i \neq z \\ if\ \ i \in W \Rightarrow z \neq h}} x_{i,z}^{v,l} + \sum_{\substack{z \in V_0 \\ z \neq j}} x_{z,j}^{v,l} \geq x_{i,j}^{v,l+1}, \quad \forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix} \right\},$$

$$\forall i,j \in V_0, (i \neq j), \forall l \in l_v \smallsetminus \{1, l_v^{max}\}, \quad \text{(5b)}$$

$$\sum_{\substack{z \in V_0,h \\ i \neq z}} x_{i,z}^{v,1} \geq x_{i,h}^{v,2}, \quad \forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix} \right\}, \forall i \in N \cup TP \cup RC, \quad \text{(6a)}$$

$$x_i^{v,l,l-1} + \sum_{\substack{z \in V_0,h \\ i \neq z}} x_{i,z}^{v,l} \geq x_{i,h}^{v,l+1}, \quad \forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix} \right\}, \forall i \in N \cup TP \cup RC,$$

$$\forall l \in l_v \smallsetminus \{1, l_v^{max}\}, \quad \text{(6b)}$$

For Simultaneous Delivery and PickUp Nodes, only one visit in total is allowed by any of vehicles; which is constrained in eq. 7.

$$\sum_{v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix} \right\}} \sum_{\substack{j \in V_0,h \\ i \neq j}} \sum_{l \in l_v} x_{i,j}^{v,l} \leq 1, \quad \forall i \in NM, \quad \text{(7)}$$

3.2. Flow Constraints

We use single flow variables for each of the load-types and as per compatibility with the vehicle type.

3.2.1. For Vehicle Depots

All outflows from the VD's must be zero:

$$y_{h,j}^{v,1,c} \leq 0, \quad \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall c \in CC(k), \forall j \in W \cup TP \cup N, \tag{8}$$

All inflows into the VD's must be zero:

$$y_{i,h}^{v,l,c} \leq 0, \quad \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall l \in l_v, \forall c \in CC(k), \forall i \in TP \cup N \cup RC, \tag{9}$$

These nullifications will help us for the remainder of the flow formulations as we may not consider any of the flow variables into or out of any VD.

3.2.2. For Warehouses

For Warehouses, there will always be an increase from incoming links to outgoing links at each level of vehicle for each compatible delivery load type. (In case of non-existing variables during the equation framing, *e.g.* when l takes values of 1 or l_v^{max}, the variables must be ignored. Since the constraints differ slightly when ignoring these non-existent variables, constraints dealing with the inter-layer variables are bifurcated into 3 constraints, 1 for the lowest layer, 1 for the in-between layers, and 1 for the top-most layer, similar to the bifurcations in Eq. (4).)

$$y_i^{v,1,2,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{i,j}^{v,1,c} \geq y_i^{v,2,1,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{j,i}^{v,1,c}, \quad \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix},$$
$$\forall c \in DY \cap CC(k), \forall i \in W, \tag{10a}$$

$$y_i^{v,l,l+1,c} + y_i^{v,l,l-1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{i,j}^{v,l,c} \geq y_i^{v,l+1,l,c} + y_i^{v,l-1,l,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c},$$
$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall l \in l_v \setminus \{1, l_v^{max}\}, \forall c \in DY \cap CC(k), \forall i \in W, \tag{10b}$$

$$y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{i,j}^{v,l_v^{max},c} \geq y_i^{v,l_v^{max}-1,l_v^{max},c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{j,i}^{v,l_v^{max},c},$$

$$\forall v \in \left\{\substack{h\in VD,\\ k\in vT(h),\\ vN(h,k)}\right\}, \forall c \in DY \cap CC(k), \forall i \in W, \quad (10c)$$

There would be no change in the amounts of pickup loads at Warehouses:

$$y_i^{v,1,2,c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{i,j}^{v,1,c} = y_i^{v,2,1,c} + \sum_{\substack{j\in V_0,h\\ i\neq j}} y_{j,i}^{v,1,c}, \quad \forall v \in \left\{\substack{h\in VD,\\ k\in vT(h),\\ vN(h,k)}\right\},$$

$$\forall c \in PU \cap CC(k), \forall i \in W, \quad (11a)$$

$$y_i^{v,l,l+1,c} + y_i^{v,l,l-1,c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{i,j}^{v,l,c} = y_i^{v,l+1,l,c} + y_i^{v,l-1,l,c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{j,i}^{v,l,c},$$

$$\forall v \in \left\{\substack{h\in VD,\\ k\in vT(h),\\ vN(h,k)}\right\}, \forall l \in l_v \setminus \{1, l_v^{max}\}, \forall c \in PU \cap CC(k), \forall i \in W, \quad (11b)$$

$$y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{i,j}^{v,l_v^{max},c} = y_i^{v,l_v^{max}-1,l_v^{max},c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{j,i}^{v,l_v^{max},c},$$

$$\forall v \in \left\{\substack{h\in VD,\\ k\in vT(h),\\ vN(h,k)}\right\}, \forall c \in PU \cap CC(k), \forall i \in W, \quad (11c)$$

Warehouse capacity constraints:

$$\sum_{v\in\left\{\substack{h\in VD,\\ k\in vT(h)\cap CV(c),\\ vN(h,k)}\right\}} \sum_{l=1} \left(\sum_{\substack{j\in V_0\\ i\neq j}} y_{i,j}^{v,l,c} - \sum_{\substack{j\in V_0,h\\ i\neq j}} y_{j,i}^{v,l,c} \right) = b_i^c, \quad \forall i \in W,$$

$$\forall c \in DY \quad (12a)$$

$$\sum_{v \in \left\{ \substack{h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k)} \right\}} \sum_{l \in l_v \setminus \{1\}} \sum_{\substack{j \in V_0 \\ i \neq j}} \left(y_{i,j}^{v,l,c} - y_{j,i}^{v,l,c} \right) = q_i^c, \quad \forall i \in W,$$

$$\forall c \in DY \tag{12b}$$

$$b_i^c + q_i^c \leq Q_i^c, \quad \forall i \in W, \forall c \in DY \tag{12c}$$

3.2.3. For Relief Centres

There will always be a decrease of the incoming pickup values as the picked-up evacuees deboard at Relief Centres.

$$y_i^{v,1,2,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,1,c} \leq y_i^{v,2,1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,1,c}, \quad \forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h), \\ vN(h,k)} \right\},$$

$$\forall c \in PU \cap CC(k), \forall i \in RC, \tag{13a}$$

$$y_i^{v,l,l+1,c} + y_i^{v,l,l-1,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l,c} \leq y_i^{v,l+1,l,c} + y_i^{v,l-1,l,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c},$$

$$\forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h), \\ vN(h,k)} \right\}, \forall l \in l_v \setminus \{1, l_v^{max}\}, \forall c \in PU \cap CC(k), \forall i \in RC, \tag{13b}$$

$$y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l_v^{max},c} \leq y_i^{v,l_v^{max}-1,l_v^{max},c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l_v^{max},c},$$

$$\forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h), \\ vN(h,k)} \right\}, \forall c \in PU \cap CC(k), \forall i \in RC, \tag{13c}$$

No change in the delivery values would occur during any vehicle's visit to the Relief Centres.

$$y_i^{v,1,2,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,1,c} = y_i^{v,2,1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,1,c}, \quad \forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h), \\ vN(h,k)} \right\},$$

$$\forall c \in DY \cap CC(k), \forall i \in RC, \tag{14a}$$

$$y_i^{v,l,l+1,c} + y_i^{v,l,l-1,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l,c} = y_i^{v,l+1,l,c} + y_i^{v,l-1,l,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c},$$
$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall l \in l_v \setminus \{1, l_v^{max}\}, \forall c \in DY \cap CC(k),$$
$$\forall i \in RC, \quad (14b)$$

$$y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l_v^{max},c} = y_i^{v,l_v^{max}-1,l_v^{max},c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l_v^{max},c},$$
$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall c \in DY \cap CC(k), \forall i \in RC, \quad (14c)$$

Capacity constraints at each relief centre for each type of the pickup load type:

$$\sum_{v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{Bmatrix}} \sum_{l \in l_v} \left(\sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c} - \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l,c} \right) \leq Q_i^c,$$
$$\forall i \in RC, \forall c \in PU \quad (15)$$

3.2.4. Flow Constraints at Nodes

During any vehicle's visit at Nodes, outgoing pickup values at each Node should be greater than the incoming pickup values

$$y_i^{v,1,2,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,1,c} \geq y_i^{v,2,1,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{j,i}^{v,1,c},$$
$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall c \in PU \cap CC(k), \forall i \in N, \quad (16a)$$

$$y_i^{v,l,l+1,c} + y_i^{v,l,l-1,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l,c} \geq y_i^{v,l+1,l,c} + y_i^{v,l-1,l,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c},$$
$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall l \in l_v \setminus \{1, l_v^{max}\}, \forall c \in PU \cap CC(k), \forall i \in N, \quad (16b)$$

$$y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j\in V_0,h\\ i\neq j}} y_{i,j}^{v,l_v^{max},c} \geq y_i^{v,l_v^{max}-1,l_v^{max},c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{j,i}^{v,l_v^{max},c},$$

$$\forall v \in \left\{\substack{h\in VD,\\ k\in vT(h),\\ vN(h,k)}\right\}, \forall c \in PU \cap CC(k), \forall i \in N, \quad (16c)$$

For the delivery variables, sum of the outgoing values at each level at each Node should be greater than the sum of all incoming values, during any vehicle's visit at Nodes,

$$y_i^{v,1,2,c} + \sum_{\substack{j\in V_0,h\\ i\neq j}} y_{i,j}^{v,1,c} \leq y_i^{v,2,1,c} + \sum_{\substack{j\in V_0,h\\ i\neq j}} y_{j,i}^{v,1,c},$$

$$\forall v \in \left\{\substack{h\in VD,\\ k\in vT(h),\\ vN(h,k)}\right\}, \forall c \in DY \cap CC(k), \forall i \in N, \quad (17a)$$

$$y_i^{v,l,l+1,c} + y_i^{v,l,l-1,c} + \sum_{\substack{j\in V_0,h\\ i\neq j}} y_{i,j}^{v,l,c} \leq y_i^{v,l+1,l,c} + y_i^{v,l-1,l,c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{j,i}^{v,l,c},$$

$$\forall v \in \left\{\substack{h\in VD,\\ k\in vT(h),\\ vN(h,k)}\right\}, \forall l \in l_v \setminus \{1, l_v^{max}\}, \forall c \in DY \cap CC(k), \forall i \in N, \quad (17b)$$

$$y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j\in V_0,h\\ i\neq j}} y_{i,j}^{v,l_v^{max},c} \leq y_i^{v,l_v^{max}-1,l_v^{max},c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{j,i}^{v,l_v^{max},c},$$

$$\forall v \in \left\{\substack{h\in VD,\\ k\in vT(h),\\ vN(h,k)}\right\}, \forall c \in DY \cap CC(k), \forall i \in N, \quad (17c)$$

Minimal resource commitment at the Nodes:

$$\sum_{v\in\left\{\substack{h\in VD,\\ k\in vT(h)\cap CV(c),\\ vN(h,k)}\right\}} \sum_{l=1} \sum_{\substack{j\in V_0,h\\ i\neq j}} \left(y_{i,j}^{v,l,c} - y_{j,i}^{v,l,c}\right) = b_i^c, \quad \forall i \in N, \forall c \in PU, \quad (18a)$$

$$\sum_{v\in\left\{\substack{h\in VD,\\ k\in vT(h)\cap CV(c),\\ vN(h,k)}\right\}} \sum_{l\in l_v\setminus\{1\}} \left(\sum_{\substack{j\in V_0,h\\ i\neq j}} y_{i,j}^{v,l,c} - \sum_{\substack{j\in V_0\\ i\neq j}} y_{j,i}^{v,l,c} \right) = q_i^c, \quad \forall i\in N,$$

$$\forall c\in PU, \quad (18b)$$

$$b_i^c + q_i^c \geq Q_i^c, \quad \forall i\in N, \forall c\in PU, \quad (18c)$$

$$\sum_{v\in\left\{\substack{h\in VD,\\ k\in vT(h)\cap CV(c),\\ vN(h,k)}\right\}} \sum_{\substack{j\in V_0,h\\ i\neq j}} \left(y_{j,i}^{v,1,c} - y_{i,j}^{v,1,c} \right) = b_i^c, \quad \forall i\in N, \forall c\in DY, \quad (19a)$$

$$\sum_{v\in\left\{\substack{h\in VD,\\ k\in vT(h)\cap CV(c),\\ vN(h,k)}\right\}} \sum_{l\in l_v\setminus\{1\}} \left(\sum_{\substack{j\in V_0\\ i\neq j}} y_{j,i}^{v,l,c} - \sum_{\substack{j\in V_0,h\\ i\neq j}} y_{i,j}^{v,l,c} \right) = q_i^c, \quad \forall i\in N,$$

$$\forall c\in DY, \quad (19b)$$

$$b_i^c + q_i^c \geq Q_i^c, \quad \forall i\in N, \forall c\in DY, \quad (19c)$$

3.2.5. Flow Constraints at Transhipment Ports:

The residues of each type of TP-compatible load during each visit of any vehicle is calculated as below:

$$y_i^{v,2,1,c} - y_i^{v,1,2,c} + \sum_{\substack{j\in V_0,h\\ i\neq j}} \left(y_{j,i}^{v,1,c} - y_{i,j}^{v,1,c} \right) = r_i^{v,1,c}, \quad \forall v\in \left\{\substack{h\in VD,\\ k\in vT(h),\\ vN(h,k)}\right\},$$

$$\forall i\in TP, \forall c\in CC(k)\cap CP(i), \quad (20a)$$

$$y_i^{v,l+1,l,c} - y_i^{v,l,l+1,c} + y_i^{v,l-1,l,c} - y_i^{v,l,l-1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c} - \sum_{\substack{j \in V_0, h \\ i \neq j}} y_{i,j}^{v,l,c} = r_i^{v,l,c},$$

$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall i \in TP, \forall c \in CC(k) \cap CP(i), \forall l \in l_v \setminus \{1, l_v^{max}\}, \tag{20b}$$

$$y_i^{v,l_v^{max}-1,l_v^{max},c} - y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l_v^{max},c} - \sum_{\substack{j \in V_0, h \\ i \neq j}} y_{i,j}^{v,l_v^{max},c} = r_i^{v,l_v^{max},c},$$

$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall i \in TP, \forall c \in CC(k) \cap CP(i), \tag{20c}$$

For the loads which are not TP-compatible, no changes in the flow quantity is allowed at the respective TP as below:

$$y_i^{v,2,1,c} - y_i^{v,1,2,c} + \sum_{\substack{j \in V_0, h \\ i \neq j}} \left(y_{j,i}^{v,1,c} - y_{i,j}^{v,1,c} \right) = 0, \quad \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix},$$

$$\forall i \in TP, \forall c \in CC(k) \setminus CP(i), \tag{21a}$$

$$y_i^{v,l+1,l,c} - y_i^{v,l,l+1,c} + y_i^{v,l-1,l,c} - y_i^{v,l,l-1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c} - \sum_{\substack{j \in V_0, h \\ i \neq j}} y_{i,j}^{v,l,c} = 0,$$

$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall l \in l_v \setminus \{1, l_v^{max}\}, \forall i \in TP, \forall c \in CC(k) \setminus CP(i), \tag{21b}$$

$$y_i^{v,l_v^{max}-1,l_v^{max},c} - y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l_v^{max},c} - \sum_{\substack{j \in V_0, h \\ i \neq j}} y_{i,j}^{v,l_v^{max},c} = 0,$$

$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall i \in TP, \forall c \in CC(k) \setminus CP(i), \tag{21c}$$

3.3. Time constraints

We visualize the following process-flow during the operation at any typical TP during any vehicle visit *i.e.*, at any layer of any vehicle.

i. Vehicle reaches TP at an assumed time T_0

ii. Loading/Unloading of Vehicle-&-TP compatible cargo is done. For each of the compatible load categories, the time considered is the sum of the waiting time T_W loading/unloading time $T_{U/L}$.

   Here we assume this process to be done without any parallelization approach. As a future research area; or problem variant, this could be edited for task parallelization which could allow the transhipment operations to become organized. As a suggestion, simultaneous cargo-specific queues could also be pondered upon, depending on manpower available at the site and the number of extra vehicle-crew.

iii. Vehicle departs the TP during this same visit at a time T_z where,

$$T_Z = T_0 + \sum_{\substack{Compatible\\Cargos}} \left(T_W + T_{U/L}\right)$$

For a VD all departure times are considered 0, assuming that it takes no time to prepare the vehicles (fuelling, maintenance, etc.) or the crew (assigning the route instructions, providing a dry run of the tasks involved during their specific stops, etc.).

$$dt_i^{v,1} \leq 0, \quad \forall v \in \left\{\substack{i \in VD,\\ k \in vT(i),\\ vN(i,k)}\right\}, \tag{22}$$

3.3.1. Calculating the cascaded travel times

Calculating travel times on the same layer:

$$tt_{i,j}^{v,1} \leq M \cdot x_{i,j}^{v,1}, \quad \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h),\\ vN(h,k)}\right\}, \forall i,j \in V_0 \cup h, i \neq j,$$
$$(if\ \ i \in W \Rightarrow j \neq h), (if\ \ i = h \Rightarrow j \notin RC), \tag{23a}$$

$$tt_{i,j}^{v,l} \leq M \cdot x_{i,j}^{v,l}, \quad \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h),\\ vN(h,k)}\right\}, \forall l \in l_v \setminus \{1\}, \forall i \in V_0, \forall j \in V_0 \cup h,$$
$$i \neq j, (if\ \ i \in W \Rightarrow j \neq h), \tag{23b}$$

$$tt^{v,1}_{i,j} \geq -M \cdot \left(1 - x^{v,1}_{i,j}\right) + dt^{v,1}_{i} + C^{k}_{i,j} \cdot x^{v,1}_{i,j}, \quad \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix}\right\},$$
$$\forall i, j \in V_0 \cup h, i \neq j, (if \ \ i \in W \Rightarrow j \neq h), (if \ \ i = h \Rightarrow j \notin RC), \quad (24a)$$

$$tt^{v,l}_{i,j} \geq -M \cdot \left(1 - x^{v,l}_{i,j}\right) + dt^{v,l}_{i} + C^{k}_{i,j} \cdot x^{v,l}_{i,j}, \quad \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v \setminus \{1\},$$
$$\forall i \in V_0, \forall j \in V_0 \cup h, i \neq j, (if \ \ i \in W \Rightarrow j \neq h), \quad (24b)$$

$$tt^{v,1}_{i,j} \leq M \cdot \left(1 - x^{v,1}_{i,j}\right) + dt^{v,1}_{i} + C^{k}_{i,j} \cdot x^{v,1}_{i,j}, \quad \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix}\right\},$$
$$\forall i, j \in V_0 \cup h, i \neq j, (if \ \ i \in W \Rightarrow j \neq h), (if \ \ i = h \Rightarrow j \notin RC), \quad (25a)$$

$$tt^{v,l}_{i,j} \leq M \cdot \left(1 - x^{v,l}_{i,j}\right) + dt^{v,l}_{i} + C^{k}_{i,j} \cdot x^{v,l}_{i,j}, \quad \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v \setminus \{1\},$$
$$\forall i \in V_0, \forall j \in V_0 \cup h, i \neq j, (if \ \ i \in W \Rightarrow j \neq h), \quad (25b)$$

Calculating travel times between layers:

$$tt^{v,l,m}_{i} \leq M \cdot x^{v,l,m}_{i}, \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l, m \in l_v, |l - m| = 1, \forall i \in V_0, \quad (26)$$

$$tt^{v,l,m}_{i} \geq -M \cdot \left(1 - x^{v,l,m}_{i}\right) + dt^{v,l}_{i}, \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l, m \in l_v, |l - m| = 1, \forall i \in V_0, \quad (27)$$

$$tt^{v,l,m}_{i} \leq M \cdot \left(1 - x^{v,l,m}_{i}\right) + dt^{v,l}_{i}, \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l, m \in l_v, |l - m| = 1, \forall i \in V_0, \quad (28)$$

3.3.2. Calculating arrival times

$$at_i^{v,1} = tt_i^{v,2,1} + \sum_{\substack{j\in V_0,h \\ i\neq j \\ if\ \ i\in RC\Rightarrow j\neq h \\ if\ \ i=h\Rightarrow j\notin W}} tt_{j,i}^{v,1}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall i \in V_0, \tag{29a}$$

$$at_i^{v,l} = tt_i^{v,l+1,l} + tt_i^{v,l-1,l} + \sum_{\substack{j\in V_0 \\ i\neq j \\ if\ \ i=h\Rightarrow j\notin W}} tt_{j,i}^{v,l}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall i \in V_0, \\ \forall l \in l_v \setminus \{1, l_v^{max}\}, \tag{29b}$$

$$at_i^{v,l_v^{max}} = tt_i^{v,l_v^{max}-1,l_v^{max}} + \sum_{\substack{j\in V_0 \\ i\neq j \\ if\ \ i=h\Rightarrow j\notin W}} tt_{j,i}^{v,l_v^{max}}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall i \in V_0, \tag{29c}$$

$$at_h^{v,l} = \sum_{j\in N,TP,RC} tt_{j,h}^{v,l}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v, \tag{29d}$$

3.3.3. Calculating modulus of the residue

$$r_i^{v,l,c} \geq -M \cdot \left(1 - n_i^{v,l,c}\right), \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v, \forall i \in TP, \\ \forall c \in CP(i) \cap CC(k), \tag{30}$$

$$r_i^{v,l,c} \leq M \cdot n_i^{v,l,c}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v, \forall i \in TP, \\ \forall c \in CP(i) \cap CC(k), \tag{31}$$

$$a_i^{v,l,c} \geq r_i^{v,l,c}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v, \forall i \in TP, \forall c \in CP(i) \cap CC(k), \tag{32}$$

$$a_i^{v,l,c} \geq -r_i^{v,l,c}, \quad \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall l \in l_v, \forall i \in TP, \forall c \in CP(i) \cap CC(k), \tag{33}$$

$$a_i^{v,l,c} \leq r_i^{v,l,c} + M \cdot \left(1 - n_i^{v,l,c}\right), \quad \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall l \in l_v, \forall i \in TP,$$
$$\forall c \in CP(i) \cap CC(k), \tag{34}$$

$$a_i^{v,l,c} \leq -r_i^{v,l,c} + M \cdot n_i^{v,l,c}, \quad \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall l \in l_v, \forall i \in TP,$$
$$\forall c \in CP(i) \cap CC(k), \tag{35}$$

3.3.4. Calculating departure times

At Transhipment Ports:

$$dt_i^{v,l} = at_i^{v,l} + nt_i^{v,l} + \sum_{c \in CC(k) \cap CP(i)} \left(a_i^{v,l,c} \cdot U^{k,c}\right), \quad \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix},$$
$$\forall l \in l_v, \forall i \in TP, \tag{36}$$

At Nodes:

$$dt_i^{v,1} = at_i^{v,1} + \sum_{c \in CC(k) \cap PU} \left(U^{k,c} \cdot \left(y_i^{v,1,2,c} - y_i^{v,2,1,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} \left(y_{i,j}^{v,1,c} - y_{j,i}^{v,1,c}\right)\right)\right)$$
$$+ \sum_{c \in CC(k) \cap DY} \left(U^{k,c} \cdot \left(y_i^{v,2,1,c} - y_i^{v,1,2,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} \left(y_{j,i}^{v,1,c} - y_{i,j}^{v,1,c}\right)\right)\right),$$
$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{Bmatrix}, \forall i \in N, \tag{37a}$$

$$dt_i^{v,l} = at_i^{v,l} + \sum_{c \in CC(k) \cap PU} \Bigg(U^{k,c} \cdot \bigg(y_i^{v,l,l+1,c} + y_i^{v,l,l-1,c} - y_i^{v,l+1,l,c} - y_i^{v,l-1,l,c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l,c} - \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c} \bigg) \Bigg) + \sum_{c \in CC(k) \cap DY} \Bigg(U^{k,c} \cdot \bigg(y_i^{v,l+1,l,c} + y_i^{v,l-1,l,c} - y_i^{v,l,l+1,c} - y_i^{v,l,l-1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c} - \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l,c} \bigg) \Bigg), \quad \forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix} \right\}, \forall l \in l_v \setminus \{1, l_v^{max}\}, \forall i \in N, \quad (37b)$$

$$dt_i^{v,l_v^{max}} = at_i^{v,l_v^{max}} + \sum_{c \in CC(k) \cap PU} \Bigg(U^{k,c} \cdot \bigg(y_i^{v,l_v^{max},l_v^{max}-1,c} - y_i^{v,l_v^{max}-1,l_v^{max},c} + \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l_v^{max},c} - \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l_v^{max},c} \bigg) \Bigg) + \sum_{c \in CC(k) \cap DY} \Bigg(U^{k,c} \cdot \bigg(y_i^{v,l_v^{max}-1,l_v^{max},c} - y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l_v^{max},c} - \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{i,j}^{v,l_v^{max},c} \bigg) \Bigg), \quad \forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix} \right\}, \forall i \in N, \quad (37c)$$

At Warehouses:

$$dt_i^{v,1} = at_i^{v,1} + \sum_{c \in CC(k) \cap DY} \Bigg(U^{k,c} \cdot \bigg(y_i^{v,1,2,c} - y_i^{v,2,1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{i,j}^{v,1,c} - \sum_{\substack{j \in V_0,h \\ i \neq j}} y_{j,i}^{v,1,c} \bigg) \Bigg), \quad \forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{smallmatrix} \right\}, \forall i \in W, \quad (38a)$$

$$
dt_i^{v,l} = at_i^{v,l} + \sum_{c \in CC(k) \cap DY} \Bigg(U^{k,c} \cdot \Bigg(y_i^{v,l,l+1,c} + y_i^{v,l,l-1,c} - y_i^{v,l+1,l,c} - y_i^{v,l-1,l,c}
$$

$$
+ \sum_{\substack{j \in V_0 \\ i \neq j}} \left(y_{i,j}^{v,l,c} - y_{j,i}^{v,l,c} \right) \Bigg) \Bigg), \quad \forall v \in \left\{ \begin{matrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{matrix} \right\}, \forall l \in l_v \setminus \{1, l_v^{max}\},
$$

$$
\forall i \in W, \quad (38b)
$$

$$
dt_i^{v,l_v^{max}} = at_i^{v,l_v^{max}} + \sum_{c \in CC(k) \cap DY} \Bigg(U^{k,c} \cdot \Bigg(y_i^{v,l_v^{max},l_v^{max}-1,c} - y_i^{v,l_v^{max}-1,l_v^{max},c}
$$

$$
+ \sum_{\substack{j \in V_0 \\ i \neq j}} \left(y_{i,j}^{v,l_v^{max},c} - y_{j,i}^{v,l_v^{max},c} \right) \Bigg) \Bigg), \quad \forall v \in \left\{ \begin{matrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{matrix} \right\}, \forall i \in W, \quad (38c)
$$

At Relief Centres:

$$
dt_i^{v,1} = at_i^{v,1} + \sum_{c \in CC(k) \cap PU} \Bigg(U^{k,c} \cdot \Bigg(y_i^{v,2,1,c} - y_i^{v,1,2,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,1,c} - \sum_{\substack{j \in V_0, h \\ i \neq j}} y_{i,j}^{v,1,c} \Bigg) \Bigg),
$$

$$
\forall v \in \left\{ \begin{matrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{matrix} \right\}, \forall i \in RC, \quad (39a)
$$

$$
dt_i^{v,l} = at_i^{v,l} + \sum_{c \in CC(k) \cap PU} \Bigg(U^{k,c} \cdot \Bigg(y_i^{v,l+1,l,c} + y_i^{v,l-1,l,c} - y_i^{v,l,l+1,c}
$$

$$
- y_i^{v,l,l-1,c} + \sum_{\substack{j \in V_0 \\ i \neq j}} y_{j,i}^{v,l,c} - \sum_{\substack{j \in V_0, h \\ i \neq j}} y_{i,j}^{v,l,c} \Bigg) \Bigg), \quad \forall v \in \left\{ \begin{matrix} h \in VD, \\ k \in vT(h), \\ vN(h,k) \end{matrix} \right\},
$$

$$
\forall l \in l_v \setminus \{1, l_v^{max}\}, \forall i \in RC, \quad (39b)
$$

$$dt_i^{v,l_v^{max}} = at_i^{v,l_v^{max}} + \sum_{c\in CC(k)\cap PU}\Bigg(U^{k,c}\cdot\Bigg(y_i^{v,l_v^{max}-1,l_v^{max},c} - y_i^{v,l_v^{max},l_v^{max}-1,c} + \sum_{\substack{j\in V_0\\ i\neq j}} y_{j,i}^{v,l_v^{max},c} - \sum_{\substack{j\in V_0,h\\ i\neq j}} y_{i,j}^{v,l_v^{max},c}\Bigg)\Bigg), \quad \forall v \in \left\{\begin{matrix} h\in VD,\\ k\in vT(h),\\ vN(h,k)\end{matrix}\right\}, \forall i \in RC, \tag{39c}$$

3.4. Temporal Residue Constraints

The residues need to be arranged *w.r.t.* time so that we may ensure that at any point in time, the sum of residues of a specific load-type at a specific TP is never negative.

3.4.1. Comparing Arrival and Departure times of vehicles at Transhipment Ports

Populating the intermediate binary variable g when $\$ = D$ and $\$\$ = A$:

$$at_i^{vv,ll} - dt_i^{v,l} \geq -M\cdot\Big(1 - g_{(vv,ll,A)}^{(v,l,D),i}\Big), \quad \forall i \in TP, \forall v \in \left\{\begin{matrix} h\in VD,\\ k\in vT(h)\cap CV(CP(i)),\\ vN(h,k)\end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh\in VD,\\ kk\in vT(hh)\cap CV(CP(i)),\\ vN(hh,kk)\end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \tag{40a}$$

$$dt_i^{v,l} - at_i^{vv,ll} \geq -M\cdot g_{(vv,ll,A)}^{(v,l,D),i}, \quad \forall i \in TP, \forall v \in \left\{\begin{matrix} h\in VD,\\ k\in vT(h)\cap CV(CP(i)),\\ vN(h,k)\end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh\in VD,\\ kk\in vT(hh)\cap CV(CP(i)),\\ vN(hh,kk)\end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \tag{40b}$$

Populating the intermediate binary variable g when $\$ = D$ and $\$\$ = D$:

$$dt_i^{vv,ll} - dt_i^{v,l} \geq -M\cdot\Big(1 - g_{(vv,ll,D)}^{(v,l,D),i}\Big), \quad \forall i \in TP, \forall v \in \left\{\begin{matrix} h\in VD,\\ k\in vT(h)\cap CV(CP(i)),\\ vN(h,k)\end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh\in VD,\\ kk\in vT(hh)\cap CV(CP(i)),\\ vN(hh,kk)\end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \tag{41a}$$

$$dt_i^{v,l} - dt_i^{vv,ll} \geq -M\cdot g_{(vv,ll,D)}^{(v,l,D),i}, \quad \forall i \in TP, \forall v \in \left\{\begin{matrix} h\in VD,\\ k\in vT(h)\cap CV(CP(i)),\\ vN(h,k)\end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh\in VD,\\ kk\in vT(hh)\cap CV(CP(i)),\\ vN(hh,kk)\end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \tag{41b}$$

Populating the intermediate binary variable g when $\$ = A$ and $\$\$ = A$:

$$at_i^{vv,ll} - at_i^{v,l} \leq M \cdot \left(1 - g_{(vv,ll,A)}^{(v,l,A),i}\right), \quad \forall i \in TP, \forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h) \cap CV(CP(i)), \\ vN(h,k)} \right\},$$
$$\forall vv \in \left\{ \substack{hh \in VD, \\ kk \in vT(hh) \cap CV(CP(i)), \\ vN(hh,kk)} \right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (42a)$$

$$at_i^{v,l} - at_i^{vv,ll} \leq M \cdot g_{(vv,ll,A)}^{(v,l,A),i}, \quad \forall i \in TP, \forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h) \cap CV(CP(i)), \\ vN(h,k)} \right\},$$
$$\forall vv \in \left\{ \substack{hh \in VD, \\ kk \in vT(hh) \cap CV(CP(i)), \\ vN(hh,kk)} \right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (42b)$$

Populating the intermediate binary variable g when $\$ = A$ and $\$\$ = D$:

$$dt_i^{vv,ll} - at_i^{v,l} \leq M \cdot \left(1 - g_{(vv,ll,D)}^{(v,l,A),i}\right), \quad \forall i \in TP, \forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h) \cap CV(CP(i)), \\ vN(h,k)} \right\},$$
$$\forall vv \in \left\{ \substack{hh \in VD, \\ kk \in vT(hh) \cap CV(CP(i)), \\ vN(hh,kk)} \right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (43a)$$

$$at_i^{v,l} - dt_i^{vv,ll} \leq M \cdot g_{(vv,ll,D)}^{(v,l,A),i}, \quad \forall i \in TP, \forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h) \cap CV(CP(i)), \\ vN(h,k)} \right\},$$
$$\forall vv \in \left\{ \substack{hh \in VD, \\ kk \in vT(hh) \cap CV(CP(i)), \\ vN(hh,kk)} \right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (43b)$$

3.4.2. Constraining the cell values of the space-time matrices when the vehicles being compared don't overlap in spacetime

The cell values of the space-time matrices consist of the variables o which store some proportion of the load-component specific residue r available for transhipment during the arrival and departure time-stamps of all vehicles. This uses the comparison of a vehicle's visiting time with another at the same Transhipment Port for all its visiting possibilities individually, *i.e.*, across the various levels. Section 3.4.2 considers the cases when the visiting durations of the vehicles under comparison don't overlap, *i.e.*, they concerned vehicles don't meet during their corresponding layer-specific visit at the concerned TP. Due to this non-overlapping nature, it is easy to determine the available proportion of residue, the cell value o; which would either be the entire residue of the vehicle under comparison v *w.r.t.* a reference vehicle vv if v

has already departed the TP with some residue value r during its specific layer-wise visit l; otherwise it value would be 0 if the concerned vehicle v is yet to arrive in the TP *w.r.t.* the reference vehicle vv's visit duration.

Populating variable o as equal to the residue when $g = 1$, for $\$ = D$ and $\$\$ = A$:

$$o_{(vv,ll,A)}^{(v,l),i,c} \leq r_i^{v,l,c} + M \cdot \left(1 - g_{(vv,ll,A)}^{(v,l,D),i}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{\begin{matrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (44a)$$

$$o_{(vv,ll,A)}^{(v,l),i,c} \geq r_i^{v,l,c} - M \cdot \left(1 - g_{(vv,ll,A)}^{(v,l,D),i}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{\begin{matrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (44b)$$

Populating variable o as equal to the residue when $g = 1$, for $\$ = D$ and $\$\$ = D$:

$$o_{(vv,ll,D)}^{(v,l),i,c} \leq r_i^{v,l,c} + M \cdot \left(1 - g_{(vv,ll,D)}^{(v,l,D),i}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{\begin{matrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (45a)$$

$$o_{(vv,ll,D)}^{(v,l),i,c} \geq r_i^{v,l,c} - M \cdot \left(1 - g_{(vv,ll,D)}^{(v,l,D),i}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{\begin{matrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (45b)$$

Nullifying variable o when $g = 1$, for $\$ = A$ and $\$\$ = A$:

$$o_{(vv,ll,A)}^{(v,l),i,c} \leq M \cdot \left(1 - g_{(vv,ll,A)}^{(v,l,A),i}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{\begin{matrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (46a)$$

$$o_{(vv,ll,A)}^{(v,l),i,c} \geq -M \cdot \left(1 - g_{(vv,ll,A)}^{(v,l,A),i}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{\begin{matrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{matrix}\right\}, \forall vv \in \left\{\begin{matrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{matrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (46b)$$

Nullifying variable o when $g = 1$, for $\$ = A$ and $\$\$ = D$:

$$o^{(v,l),i,c}_{(vv,ll,D)} \le M \cdot \left(1 - g^{(v,l,A),i}_{(vv,ll,D)}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix} \right\}, \forall vv \in \left\{ \begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix} \right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (47a)$$

$$o^{(v,l),i,c}_{(vv,ll,D)} \ge -M \cdot \left(1 - g^{(v,l,A),i}_{(vv,ll,D)}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix} \right\}, \forall vv \in \left\{ \begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix} \right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (47b)$$

3.4.3. Constraining the cell values of the space-time matrices when the visits of the vehicles being compared overlap in spacetime, residues being positive

Constraints in section 3.4.2 allow an exact value of the variable o when $g = 1$. However, when $g = 0$ for both $\$ = A$ and $\$ = D$, *i.e.*, when the vehicles under comparison overlap during their visiting times at the same TP, those constraints become redundant and the below constraints are developed to populate the o variables of the space-time matrix within certain bounds. The o variable is allowed to take any value within these bounds, but as per the optimization criteria, it should take the upper bound due to minimization and when the residues are positive.

Constraining the cell values within their limits of 0 to the maximum value being the residue r itself (this is irrespective of whether the vehicles overlap in spacetime):

$$0 \le o^{(v,l),i,c}_{(vv,ll,A)} + M \cdot \left(1 - n^{v,l,c}_{i}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix} \right\}, \forall vv \in \left\{ \begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix} \right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (48)$$

$$o^{(v,l),i,c}_{(vv,ll,A)} \le o^{(v,l),i,c}_{(vv,ll,D)} + M \cdot \left(1 - n^{v,l,c}_{i}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix} \right\}, \forall vv \in \left\{ \begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix} \right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (49)$$

$$o^{(v,l),i,c}_{(vv,ll,D)} \le r^{v,l,c}_{i} + M \cdot \left(1 - n^{v,l,c}_{i}\right), \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \left\{ \begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix} \right\}, \forall vv \in \left\{ \begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix} \right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (50)$$

Upper bounds:

$$o_{(vv,ll,A)}^{(v,l),i,c} \leq \frac{at_i^{vv,ll} - at_i^{v,l}}{U^{k,c}} + M \cdot \left(g_{(vv,ll,A)}^{(v,l,A),i} + g_{(vv,ll,A)}^{(v,l,D),i}\right) + M \cdot \left(1 - n_i^{v,l,c}\right),$$
$$\forall i \in TP, \forall c \in CP(i), \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix}\right\}, \forall vv \in \left\{\begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix}\right\},$$
$$\forall l \in l_v, \forall ll \in l_{vv}, \quad (51)$$

$$o_{(vv,ll,D)}^{(v,l),i,c} \leq \frac{dt_i^{vv,ll} - at_i^{v,l}}{U^{k,c}} + M \cdot \left(g_{(vv,ll,D)}^{(v,l,A),i} + g_{(vv,ll,D)}^{(v,l,D),i}\right) + M \cdot \left(1 - n_i^{v,l,c}\right),$$
$$\forall i \in TP, \forall c \in CP(i), \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix}\right\}, \forall vv \in \left\{\begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix}\right\},$$
$$\forall l \in l_v, \forall ll \in l_{vv}, \quad (52)$$

Lower Bounds:

$$o_{(vv,ll,A)}^{(v,l),i,c} \geq r_i^{v,l,c} - \frac{dt_i^{v,l} - at_i^{vv,ll}}{U^{k,c}} + M \cdot \left(g_{(vv,ll,A)}^{(v,l,A),i} + g_{(vv,ll,A)}^{(v,l,D),i}\right) + M \cdot \left(1 - n_i^{v,l,c}\right),$$
$$\forall i \in TP, \forall c \in CP(i), \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix}\right\}, \forall vv \in \left\{\begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix}\right\},$$
$$\forall l \in l_v, \forall ll \in l_{vv}, \quad (53)$$

$$o_{(vv,ll,D)}^{(v,l),i,c} \geq r_i^{v,l,c} - \frac{dt_i^{v,l} - dt_i^{vv,ll}}{U^{k,c}} + M \cdot \left(g_{(vv,ll,D)}^{(v,l,A),i} + g_{(vv,ll,D)}^{(v,l,D),i}\right) + M \cdot \left(1 - n_i^{v,l,c}\right),$$
$$\forall i \in TP, \forall c \in CP(i), \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix}\right\}, \forall vv \in \left\{\begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix}\right\},$$
$$\forall l \in l_v, \forall ll \in l_{vv}, \quad (54)$$

3.4.4. Constraining the cell values of the space-time matrices when the visits of the vehicles being compared overlap in spacetime, residues being negative

Constraining the cell values within their limits of 0 to the minimum value being the residue r itself (this is irrespective of whether the vehicles overlap in spacetime):

$$0 \geq o_{(vv,ll,A)}^{(v,l),i,c} - M \cdot n_i^{v,l,c}, \quad \forall i \in TP, \forall c \in CP(i), \forall v \in \left\{\begin{smallmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{smallmatrix}\right\},$$
$$\forall vv \in \left\{\begin{smallmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{smallmatrix}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (55)$$

$$o_{(vv,ll,A)}^{(v,l),i,c} \geq o_{(vv,ll,D)}^{(v,l),i,c} - M \cdot n_i^{v,l,c}, \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{Bmatrix}, \forall vv \in \begin{Bmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{Bmatrix}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (56)$$

$$o_{(vv,ll,D)}^{(v,l),i,c} \geq r_i^{v,l,c} - M \cdot n_i^{v,l,c}, \quad \forall i \in TP, \forall c \in CP(i), \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{Bmatrix},$$
$$\forall vv \in \begin{Bmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{Bmatrix}, \forall l \in l_v, \forall ll \in l_{vv}, \quad (57)$$

Lower bounds:

$$o_{(vv,ll,A)}^{(v,l),i,c} \geq \frac{at_i^{vv,ll} - at_i^{v,l}}{-U^{k,c}} - M \cdot \left(g_{(vv,ll,A)}^{(v,l,A),i} + g_{(vv,ll,A)}^{(v,l,D),i} \right) - M \cdot n_i^{v,l,c},$$
$$\forall i \in TP, \forall c \in CP(i), \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{Bmatrix}, \forall vv \in \begin{Bmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{Bmatrix},$$
$$\forall l \in l_v, \forall ll \in l_{vv}, \quad (58)$$

$$o_{(vv,ll,D)}^{(v,l),i,c} \geq \frac{dt_i^{vv,ll} - at_i^{v,l}}{-U^{k,c}} - M \cdot \left(g_{(vv,ll,D)}^{(v,l,A),i} + g_{(vv,ll,D)}^{(v,l,D),i} \right) - M \cdot n_i^{v,l,c},$$
$$\forall i \in TP, \forall c \in CP(i), \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{Bmatrix}, \forall vv \in \begin{Bmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{Bmatrix},$$
$$\forall l \in l_v, \forall ll \in l_{vv}, \quad (59)$$

Upper Bounds:

$$o_{(vv,ll,A)}^{(v,l),i,c} \leq r_i^{v,l,c} + \frac{dt_i^{v,l} - at_i^{vv,ll}}{U^{k,c}} + M \cdot \left(g_{(vv,ll,A)}^{(v,l,A),i} + g_{(vv,ll,A)}^{(v,l,D),i} \right) + M \cdot n_i^{v,l,c},$$
$$\forall i \in TP, \forall c \in CP(i), \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{Bmatrix}, \forall vv \in \begin{Bmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{Bmatrix},$$
$$\forall l \in l_v, \forall ll \in l_{vv}, \quad (60)$$

$$o_{(vv,ll,D)}^{(v,l),i,c} \leq r_i^{v,l,c} + \frac{dt_i^{v,l} - dt_i^{vv,ll}}{U^{k,c}} + M \cdot \left(g_{(vv,ll,D)}^{(v,l,A),i} + g_{(vv,ll,D)}^{(v,l,D),i} \right) + M \cdot n_i^{v,l,c},$$
$$\forall i \in TP, \forall c \in CP(i), \forall v \in \begin{Bmatrix} h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k) \end{Bmatrix}, \forall vv \in \begin{Bmatrix} hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk) \end{Bmatrix},$$
$$\forall l \in l_v, \forall ll \in l_{vv}, \quad (61)$$

3.4.5. Constraining any collection to occur at transhipment ports only when the concerned material is available, i.e., previously deposited by some other vehicle at the same TP

The previous sections in 3.4 are used to develop the space-time matrix, for each load-type at each TP considering compatibilities for allowable transhipments, populated by respective o variables. The typical space-time matrix may be imagined as a matrix with rows as various space-stamps, each compatible vehicles' all levels in our case; and columns as various time-stamps. For each vehicle level in the rows, there are two time-stamps namely arrival and departure. Therefore all space-time matrices constructed by us are rectangular in nature with columns being twice that of rows, *i.e.*, the time-stamps correspond to the arrival and departure times of the compatible vehicles' levels. To ensure any collection happens only with available resources at TP's, we constrain the sum of the cells across each time axis (each column) for every space-time matrix to be non-negative. This ensures that at every time-stamp the total component-specific residue at any TP never becomes negative, *i.e.*, no collection of loads takes place before the load was deposited.

Constraining the cascaded-temporal sum of residues to be non-negative at each time-stamp, for every load-type at every TP:

$$\sum_{v \in \left\{ \substack{h \in VD, \\ k \in vT(h) \cap CV(c), \\ vN(h,k)} \right\}} \sum_{l \in l_v} o^{(v,l),i,c}_{(vv,ll,\$\$)} \geq 0, \quad \forall i \in TP, \forall c \in CP(i),$$
$$\forall vv \in \left\{ \substack{hh \in VD, \\ kk \in vT(hh) \cap CV(c), \\ vN(hh,kk)} \right\}, \forall ll \in l_{vv}, \forall \$\$ \in \{A, D\}, \quad (62)$$

3.5. Vehicle Capacity Constraints

Constraining all flow variables associated with an edge *w.r.t.* the total available vehicle-volume:

$$\sum_{c \in CC(k)} \left(E_c \cdot y^{v,1,c}_{i,j} \right) \leq E^k \cdot x^{v,1}_{i,j}, \quad \forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h), \\ vN(h,k)} \right\}, \forall i,j \in V_0 \cup h,$$
$$i \neq j, (if\ \ i \in W \Rightarrow j \neq h), (if\ \ i = h \Rightarrow j \notin RC), \quad (63a)$$

$$\sum_{c \in CC(k)} \left(E_c \cdot y^{v,l,c}_{i,j} \right) \leq E^k \cdot x^{v,l}_{i,j}, \quad \forall v \in \left\{ \substack{h \in VD, \\ k \in vT(h), \\ vN(h,k)} \right\}, \forall l \in l_v \setminus \{1\}, \forall i \in V_0,$$
$$\forall j \in V_0 \cup h, i \neq j, (if\ \ i \in W \Rightarrow j \neq h), \quad (63b)$$

$$\sum_{c\in CC(k)} \left(E_c \cdot y_i^{v,l,m,c}\right) \le E^k \cdot x_i^{v,l,m}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l,m \in l_v, |l-m| = 1,$$
$$\forall i \in V_0, \quad (63c)$$

Constraining all flow variables associated with an edge *w.r.t.* the total allowable vehicle-weight:

$$\sum_{c\in CC(k)} \left(F_c \cdot y_{i,j}^{v,1,c}\right) \le F^k \cdot x_{i,j}^{v,1}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall i,j \in V_0 \cup h,$$
$$i \ne j, (if\ \ i \in W \Rightarrow j \ne h), (if\ \ i = h \Rightarrow j \notin RC), \quad (64a)$$

$$\sum_{c\in CC(k)} \left(F_c \cdot y_{i,j}^{v,l,c}\right) \le F^k \cdot x_{i,j}^{v,l}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v \setminus \{1\}, \forall i \in V_0,$$
$$\forall j \in V_0 \cup h, i \ne j, (if\ \ i \in W \Rightarrow j \ne h), \quad (64b)$$

$$\sum_{c\in CC(k)} \left(F_c \cdot y_i^{v,l,m,c}\right) \le F^k \cdot x_i^{v,l,m}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l,m \in l_v, |l-m| = 1,$$
$$\forall i \in V_0, \quad (64c)$$

3.6. Assigning the objective variable to be the maximum among any vehicle's arrival time at its respective depot after the entire emergency operation

For the vehicle types that follow open-tour (*e.g.* trains, *etc.*), the last route back towards the depot is subtracted from the total time:

$$maxT \ge at_h^{v,l} - \sum_{i\in TP,N,RC} C_{i,h}^k \cdot x_{i,h}^{v,l}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h)\cap OT, \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v, \quad (65)$$

For the vehicle types that follow closed-tour, the entire tour length till arrival at the respective depot is considered:

$$maxT \ge at_h^{v,l}, \quad \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h)\setminus OT, \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v, \quad (66)$$

3.7. Variable definitions

$$x_i^{v,l,m} \in \{0,1\}, \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l, m \in l_v, |l-m| = 1, \forall i \in V_0, \tag{67a}$$

$$\begin{aligned} x_{i,j}^{v,1} \in \{0,1\}, \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall i, j \in V_0 \cup h, \\ i \neq j, (if\ \ i \in W \Rightarrow j \neq h), (if\ \ i = h \Rightarrow j \notin RC), \end{aligned} \tag{67b}$$

$$\begin{aligned} x_{i,j}^{v,l} \in \{0,1\}, \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v \setminus \{1\}, \forall i \in V_0, \forall j \in V_0 \cup h, \\ i \neq j, (if\ \ i \in W \Rightarrow j \neq h), \end{aligned} \tag{67c}$$

$$y_i^{v,l,m,c} \geq 0, \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l, m \in l_v, |l-m| = 1, \forall c \in CC(k), \forall i \in V_0, \tag{67d}$$

$$\begin{aligned} y_{i,j}^{v,1,c} \geq 0, \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall c \in CC(k), \forall i, j \in V_0 \cup h, \\ i \neq j, (if\ \ i \in W \Rightarrow j \neq h), (if\ \ i = h \Rightarrow j \notin RC), \end{aligned} \tag{67e}$$

$$\begin{aligned} y_{i,j}^{v,l,c} \geq 0, \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v \setminus \{1\}, \forall c \in CC(k), \forall i \in V_0, \\ \forall j \in V_0 \cup h, i \neq j, (if\ \ i \in W \Rightarrow j \neq h), \end{aligned} \tag{67f}$$

$$r_i^{v,l,c} \in \mathbb{R}, \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v, \forall i \in TP, \forall c \in CP(i) \cap CC(k), \tag{67g}$$

$$n_i^{v,l,c} \in \{0,1\}, \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v, \forall i \in TP, \forall c \in CP(i) \cap CC(k), \tag{67h}$$

$$a_i^{v,l,c} \geq 0, \forall v \in \left\{\begin{smallmatrix} h\in VD, \\ k\in vT(h), \\ vN(h,k) \end{smallmatrix}\right\}, \forall l \in l_v, \forall i \in TP, \forall c \in CP(i) \cap CC(k), \tag{67i}$$

$$at_i^{v,l} \geq 0, \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h),\\ vN(h,k)}\right\}, \forall l \in l_v, \forall i \in V_0 \cup h, \tag{67j}$$

$$dt_h^{v,1} \geq 0, \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h),\\ vN(h,k)}\right\}, \tag{67k}$$

$$dt_i^{v,l} \geq 0, \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h),\\ vN(h,k)}\right\}, \forall l \in l_v, \forall i \in V_0, \tag{67l}$$

$$tt_i^{v,l,m} \geq 0, \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h),\\ vN(h,k)}\right\}, \forall l, m \in l_v, |l - m| = 1, \forall i \in V_0, \tag{67m}$$

$$\begin{aligned} tt_{i,j}^{v,1} \geq 0, \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h),\\ vN(h,k)}\right\}, \forall i, j \in V_0 \cup h,& \\ i \neq j, (if\ i \in W \Rightarrow j \neq h), (if\ i = h \Rightarrow j \notin RC),& \end{aligned} \tag{67n}$$

$$\begin{aligned} tt_{i,j}^{v,l} \geq 0, \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h),\\ vN(h,k)}\right\}, \forall l \in l_v \setminus \{1\}, \forall i \in V_0, \forall j \in V_0 \cup h,& \\ i \neq j, (if\ i \in W \Rightarrow j \neq h),& \end{aligned} \tag{67o}$$

$$nt_i^{v,l} \geq 0, \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h),\\ vN(h,k)}\right\}, \forall l \in l_v, \forall i \in TP, \tag{67p}$$

$$\begin{aligned} o_{(vv,ll,\$\$)}^{(v,l),i,c} \in \mathbb{R}, \forall i \in TP, \forall c \in CP(i), \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h) \cap CV(c),\\ vN(h,k)}\right\},& \\ \forall vv \in \left\{\substack{hh \in VD,\\ kk \in vT(hh) \cap CV(c),\\ vN(hh,kk)}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \forall \$\$ \in \{A, D\},& \end{aligned} \tag{67q}$$

$$\begin{aligned} g_{(vv,ll,\$\$)}^{(v,l,\$),i} \in \{0,1\}, \forall i \in TP, \forall v \in \left\{\substack{h \in VD,\\ k \in vT(h) \cap CV(CP(i)),\\ vN(h,k)}\right\},& \\ \forall vv \in \left\{\substack{hh \in VD,\\ kk \in vT(hh) \cap CV(CP(i)),\\ vN(hh,kk)}\right\}, \forall l \in l_v, \forall ll \in l_{vv}, \forall \$, \$\$ \in \{A, D\},& \end{aligned} \tag{67r}$$

$$maxT \geq 0. \tag{67s}$$

$$b_i^c \geq 0, \forall i \in N, \forall c \in DY \cup PU \tag{67t}$$

$$b_i^c \geq 0, \forall i \in W, \forall c \in DY \tag{67u}$$

$$q_i^c \geq 0, \forall i \in N, \forall c \in DY \cup PU \tag{67v}$$

$$q_i^c \geq 0, \forall i \in W, \forall c \in DY \tag{67w}$$

The equations [references] get slightly altered while taking the extreme values of l_v, since when $l = 1$ incoming variables x or y connecting the bottom-most layer 1 from any below layer is non-existent; and similarly when $l = l_v^{max}$ outgoing variables from the top-most layer l_v^{max} to any other higher layer is non-existent. Further if $l_v^{max} = 1$, then both the above non-existent variables would further reduce such constraints. Therefore in the equations [references] if such non-existent variables are encountered while codifying the constraints, they are ignored. As an example, the alteration process of equation [1st eq. in the reference] is shown below:

Let's solve System (**??**):

$$x + y \geq 0 \tag{68a}$$
$$x - y \leq 4 \tag{68b}$$
$$x \times y \geq 1 \tag{??a}$$
$$x^y + y^x \geq 1 \tag{68c}$$

We should pay special attention to Equations (**??**a) and (68c) because they're tricky.

\$ = ✡
A = Δ
D = ∇

4. Heuristic Development

5. Discussion and Computational Study

6. Conclusion

References

Caution: The nodes in simultaneous must have each of their component-specific demand less than the maximum vehicle's capacity.