Improvement of Izhikevich's Neuronal and Neural Network Model

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***Abstract*—To enhance the veracity of Izhikevich's neuronal model and plausibility of Izhikevich's neural network model, we revised Izhikevich's neuronal model, deleted neurons' synaptic connections to themselves and established the description of short-term depression and time delay along axon in Izhikevich's neural network model. The simulation results with the improved neuronal model show it is more efficient and accurate for computation, the improved neural network model spikes with less synchronicity.**

Keywords- neuron; neural network; short-term depression; time delay; modeling

I. INTRODUCTION

In computational neuroscience, theory, mathematical model and physiological experiment are used to explore the working mechanism of brain. The modeling of neurons in cerebral cortex should firstly simple in computing, and then can reproduce all known types of spiking and bursting patterns of cortical neurons. For neural networks, the modification of synapses and time delay along axons should be described in the modeling. Among the neuronal models in common use, integrate-and-fire model is the most simple in computing, but it is too simple to reproduce all kind of spiking and bursting behaviors. H-H model is biophysically accurate but computationally prohibitive[1]. Using bifurcation methodologies to reduce H-H model, Izhikevich presented a new kind of neuronal model. It is as simple as integrate-and-fire model in computation and as accurate as H-H model in biophysics. He also built up a neural network model which may run in real time with the neuronal model to argue that it is computationally simple[2]. But Izhikevich's neuronal model is not accurate in mathematical expression as it cannot reproduce the 0.03 V apex values of spiking neurons, and his neural network model is short of realistic plausibility as there are not description of short-term depression and time delay along axon of spiking impulse. In this paper, improvement were made on the Izhikevich's models, the simulation show that the improved neuronal model can reproduce various behavior without staggered apex values and there is less rhythemicity in the improved neural network.

This work was supported by the Natural Science Foundation of Shandong Province (Y2008D09).

II. IZHIKEVICH'S NEURONAL AND NEURAL NETWORK MODEL

Izhikevich's neuronal model is a two-dimensional (2-D) system of ordinary differential equations of the form:

$$v' = 0.04v^2 + 5v + 140 - u + I \tag{1}$$

$$u' = a(bv - u) \tag{2}$$

with the auxiliary after-spike resetting

$$\text{If } v \geq 30 \text{ then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases} \tag{3}$$

Here, v and u are dimensionless variables, and a, b, c, and d are dimensionless parameters, and $'=d/dt$, where t is the time. The variable v stands for the membrane potential of the neuron and u corresponds to a membrane recovery variable that explains the activation of K^+ ionic current and inactivation of Na^+ ionic current and supplies negative feedback to v. As soon as the spike arrives its vertex, the membrane potential is reset in agreement with (3). Synaptic currents or injected dc-currents are expressed by means of variable I. The parameter a represents the time scale of the recovery variable u, larger values result in quicker recovery. A typical value is $a = 0.02$. The parameter b is the sensitivity of the recovery variable u to the subthreshold variations of the membrane voltage v. Smaller values couple v and u more weakly leading to possible subthreshold oscillations and low-threshold spiking dynamics. A typical value is $b = 0.2$. The parameter c is the after-spike reset value of the membrane voltage v induced by the swift high-threshold K^+ conductances. A typical value is $c=-65$. The parameter d represents after-spike reset of the recovery variable u produced by slow high-threshold Na^+ and K^+ conductances. A typical value is $d=2$. A typical Izhikevich's neuronal model can reproduce different spiking or bursting patterns of cortical neurons with various choices of parameters.

Izhikevich's neural network is an all-to-all network of 1000 spiking cortical neurons with about 1000000 synaptic

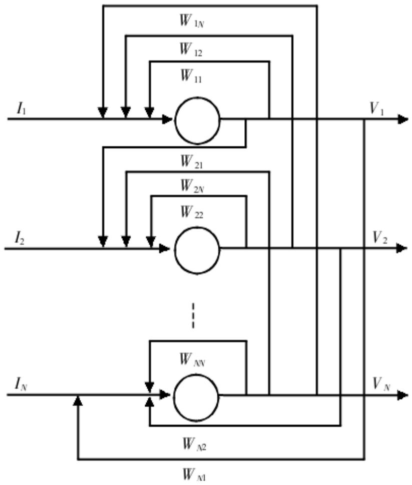

Figure.1 The structure of Izhikevich's neural network

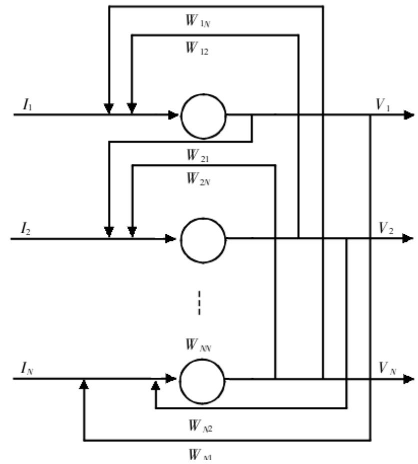

Figure. 2 The structure of improved Ichikevich's neural network

connections. In the neural network, the ratio of excitatory to inhibitory neurons is chosen as 4 to 1, and inhibitory synaptic connections are made stronger than excitatory synaptic connections. In principle, RS cells are used to model excitatory neurons and FS cells to model inhibitory ones. For achieving heterogeneity, each excitatory cell is assigned $(a_i,b_i)=(0.02,0.2)$ and $(c_i,d_i)=(-65,8)+(15,\ -6)r_i^2$, where r_i is a random variable uniformly distributed on the interval [0,1], and i is the neuron index. r_i^2 is used to bias the distribution toward RS cells. Similarly, every inhibitory cell is of $(a_i,b_i)=(0.02,0.25)+(0.08,\ -0.05)r_i$ and $(c_i,d_i)=(-65,2)$. This model is a kind of pulse-coupled neural networks, the synaptic connection weights among neurons set by the matrix $S=(s_{ij})$, the firing of the jth neuron are spontaneously adjusts variable v_i by s_{ij}. The structure of Izhikevich's neural network is shown in Fig.1.

III. Improvement of Ichikevich's Neuronal and Neural Network Model

Although Izhikevich's neuronal model is computationally simple, realistically plausible, and can reproduce all known types of spiking and bursting patterns of cortical neurons, it is not mathematically accurate. As it does not limit its max value in (3), the value of v is permitted to be greater than +0.03 V, while the model is simulated according to (1)—(3) directly, the apices of spiking or bursting in results will be staggered. For example, the simulation of RS neuron and FS neuron are shown in Fig. 3 (a), (b). The apex values may be staggered and greater than 0.03 V.

We revised the neuronal model as follow:

if $v<30$ and $v^{-}\neq 30$ then

$$v'=0.04v^2+5v+140-u+I \tag{4}$$

if $v\geq 30$ then v=30 (5)

if $v^{-}=30$ then $v=c$ (6)

if $v<30$ and $v^{-}\neq 30$ then

$$u'=a(bv-u) \tag{7}$$

if $v^{-}=30$ then $u=u^{-}+d$ (8)

Here v^{-} and u^{-} represent left neighborhood values of current values of u and v. In (5), the apices of spiking and bursting are limited to 30 (computed in mV), so peak values of v will not be staggered and greater than +0.03V. Simulation of spiking RS and FS neuron according to improved model is shown in Fig. 3 (c), (d).

Izhikevich's neural network model, which is described by MATLAB program, may run in real-time. As s_{ij} in Matrix $S=(s_{ij})$ expresses the synaptic connection weight between neurons i and j, so s_{ii} represents synaptic connection weight of neuron i to itself. Synaptic connections between different neurons in cortex are simulated in neural network, so s_{ii} should be 0. Secondly, no synaptic modification is set in the model. As the spiking of the jth neuron spontaneously affect the ith one, no time delay along axon is given in the model.

We revised Izhikevich's neural network model according to our and others' previous works[3-5] in three aspects:

1. let s_{ii}=0 delete the synaptic connection of each neuron to itself (do not consider the function of autonapses). The structure of improved Izhikevich's neural network model is shown in Fig.2.

2. introduce short-term depression of synapses, s_{ij} is multiplied by a quantity D, which obeys the dynamical equation:

$$\frac{\mathrm{d}D}{\mathrm{d}t}=-p_v D\sum_j \delta(t-t^{-})+(1-D)/\tau_D\ .$$

Here p_v is considered as the release probability per vesicle. τ_D is recovering time constant of available vesicles. As in our previous work, τ_D =0.5 sec and p_v=0.65.

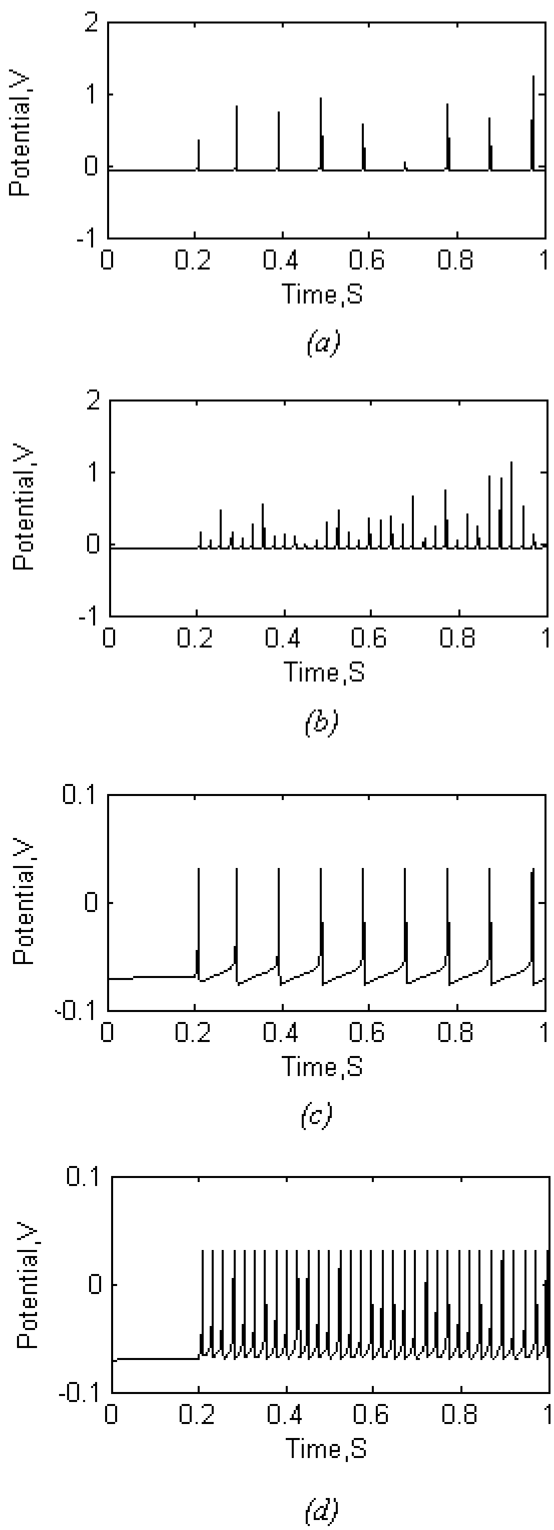

Figure.3 Simulation of Izhikevich's neuronal and improved neuronal model RS neuron and FS cell. (a) Izhikevich's model of RS neuron. (b) Izhikevich's model of FS cell. (c) Improved Izhikevich's model of RS neuron. (d) Improved Izhikevich's model of FS cell.

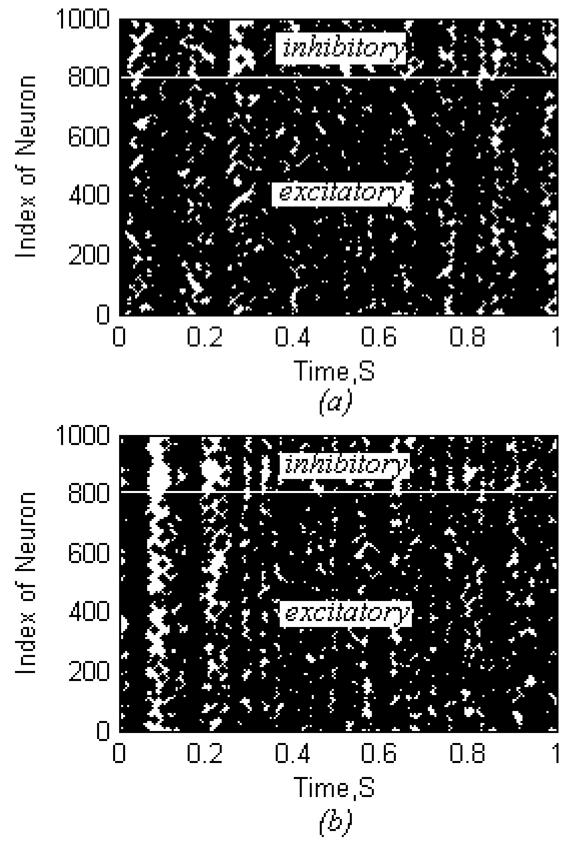

Figure.4 Simulation of a network of 1000 all-to-all coupled spiking neurons. (a) Result of Izhikevich's neural network model. (b) Result of improved neural network model.

3. present time delay along axon, which is chosen as 0.2mS with normal error 2.5%.[6]

IV. Simulation Results

A. *Comparision of Izhikevich's and Improved Models*

Now we use Izhikevich's neuronal model and neural network to simulate spiking and bursting behavior of neurons and the synchronicity of cortical areas. Simulation results of RS and FS cell of Izhikevich's model are shown in Fig. 3 (a), (b). Apex values are staggered and greater than 0.03V. The simulation results of improved Izhikevich's RS and FS neuron are shown in Fig.3 (c),(d). The peak values are uniform and equal accurately to 0.03 V.

The simulation results with Izhikevich's neural network and improved neural network model are shown in Fig. 4. Without connection from output end of every neuron to the input end of itself and with the realization of short-term depression and time delay of firing impulses along axons, the improved neural network model is of less synchronicity.

B. *Effect of Connection Strength and Input Noise*

The effect of connection strength among nurons on firing activity is simulated with changed connection strength, the results are shown in Fig.5. Firing frequency and synchronicity

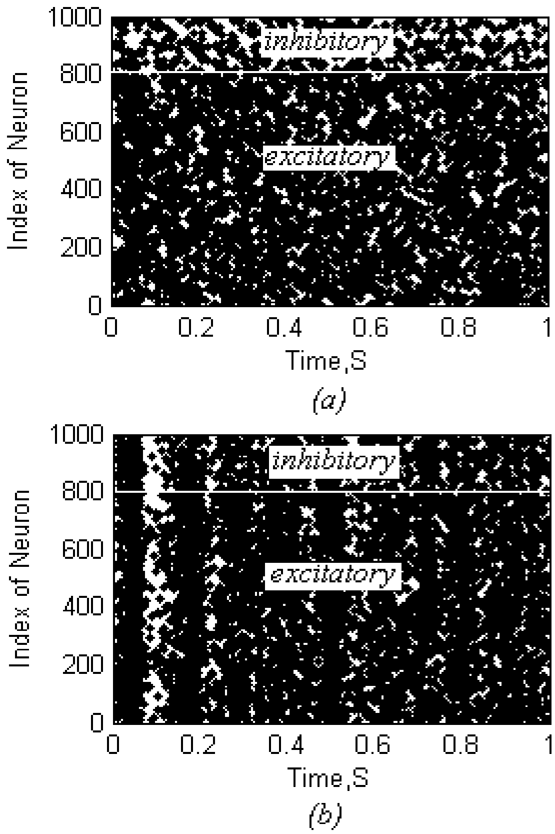

Figure.5 Comparision of firing times of inproved neural network model with different connection strengths. (a) Connction strength is 0.2. (b) Connction strength is 0.8.

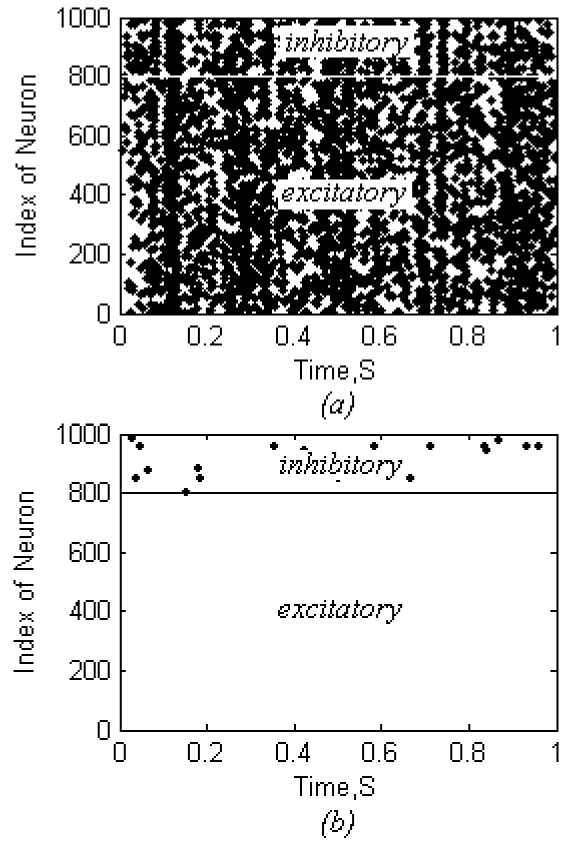

Figure.6 Comparision of firing times of inproved neural network model with different noise levels. (a) Noise strength is 0.8. (b) Noise strength is 0.4

are less with smaller connection strength.

The neurons in cortices are always affected by various noise in the physiological enviroment. In this study, we simulate the background noise in vital reality with Gaussian white noise. With changed noise strength, the firing of neurons in the network is shown in Fig.6. The firing frequency is greater with higher level of noise.

V. Discussion

Simulation results of improved Izhikevich's neuronal model show that spiking and bursting apices are all 0.03 V, so it may reproduce more biophysically realistic results than the original neuronal model does. As short-term depression and time delay along axon are introduced in improving of neural network model, the improved neural network model is realistically plausible. Simulation results with changed noise level and connection strength show firing fequency and synchronicity are greater with higher level noise and stronger connection for improved Izhikevich's neural network model.

We will describe coactivation of complex pre-synaptic and post-synaptic spikes with different rules to lay a foundation for modeling of large scale thalamocortical system used in the study of schizophrenia and Internet addiction.

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