# A typology of quantum algorithms

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#### Abstract

We draw the current landscape of quantum algorithms, by classifying about 130 quantum algorithms, according to the fundamental mathematical problems they solve, their real-world applications, the main subroutines they employ, and several other relevant criteria. The primary objectives include revealing trends of algorithms, identifying promising fields for implementations in the NISQ era, and identifying the key algorithmic primitives that power quantum advantage.

**Keywords:** Quantum Algorithms, Overview, Dependencies, NISQ, LSQ, Heuristics, Oracular, Promise, Sampling, Computational Model.

# Abbreviations (of single words)

Q. Quantum

Algo. Algorithm

# Acronyms

Algorithms: Families of algorithms:

QFT Q. Fourier Transform HS Hamiltonian Simulation

AA Amplitude Amplification QLSA Q. Linear-System Algo.

QA Q. Adiabatic VQA Variational Q. Algo.

**VQE** Variational Q. Eigensolver

BS Boson Sampling Others:

GBS Gaussian Boson Sampling NISQ Noisy Intermediate-Scale Quantum

QPE Q. Phase Estimation LSQ Large-Scale Quantum

QAE Q. Amplitude Estimation QCA Q. Cellular Automaton

**QSVT** Q. Singular-Value Transformation **QW** Q. Walk

QSP Q. Signal Processing HSP Hidden-Subgroup Problem

QAOA Q. Approximate Optimization Algo.

QUBO Quadratic Unconstrained Binary

Optimization

**QITE** Q. Imaginary-Time Evolution

**LCU** Linear Combination of Unitaries

**QSDP** Q. Semi-Definite Programming

**QSVD** Q. Singular-Value Decomposition

**HHL** Harrow-Hassidim-Lloyd

**Note 1:** The above **Abbreviations** will be used almost exclusively for names or descriptions of algorithms. The above **Acronyms** will be used almost systematically, except when we mention the concept for the first time in the main text (not if this is done in the figures), or when we want to insist on, or recall the full name of the concept. Both the **Abbreviations** and the **Acronyms** are used both in the body of the paper and in the classification table of the appendix.

Note 2: In this work, we choose to limit as much as possible the use of acronyms to names of algorithms, in order not to confuse the reader. As one can see above, there are however a few exceptions. But, in particular, we have not used acronyms for computational models (the acronyms QCA and QW above should hence not be understood as acronyms of computational models but rather of certain types or frameworks for certain algorithms), and similarly for the mathematical classes and the application domains that we define in our work.

# 1 Introduction

Quantum computing, an innovative and rapidly evolving field, holds the potential to reshape conventional paradigms in computation, by exploiting quantum-mechanical phenomena. One cornerstone of this transformation are quantum algorithms, whose design must enable a quantum computer to execute calculations more efficiently than classical computers. As the field thrives, the need for categorizing the multitude of quantum algorithms becomes vital. This paper sets out to meet this important need.

It is essential to first refer to previous overviews of quantum algorithms: Montanaro's [1], which provides a broad examination of quantum algorithms, emphasizing their main applications and their precise performance bounds; the "Quantum Algorithm Zoo" [2], which serves as a wide repository of quantum algorithms, including 435 references (on 6 April 2024); Mosca's [3], which focuses on quantum computational models and complexity; and several others [4–6]. The present work builds upon these overviews, with some important differences. We do not focus on complexity speedups over classical algorithms. Instead, we focus on the classification of the vast array of existing quantum algorithms, according to specific criteria, chosen for their ability to reveal different aspects of the algorithms, namely, the fundamental mathematical problems they solve, their real-world applications, the computational model they use, and several others; this constitutes the first core objective of this work, which we meet by the production of a large classification table, see Sec. 2 for the methodology followed and the description of the chosen classification criteria. We also establish a genealogy of quantum algorithms by analyzing their structural dependencies; this allows us to identify both the main, established algorithmic primitives, and some of the newer, emerging primitives; all this constitutes part of the second core objective of this work, second core objective which is the analysis of the classification table we have produced, analysis that we present in Sec. 3. Although we made our best attempt to analyze a broad-enough sample of representative, major quantum algorithms, our categorization cannot, of course, be fully exhaustive. We apologize for any major omission we may have done. The number of papers in this area of research is growing faster day by day. Still, our framework could be used over a broader set of quantum algorithms: we make our data and visualization codes available online for anyone willing to undertake such an extension, at https://github.com/Evi-Ksn/Typology-of-Quantum-Algorithms.

We believe that this work will be useful in two main ways: firstly, in providing a differential

understanding of how the algorithms can be grouped and what are the connections between these groups, and secondly, in providing a practical reference usable in different contexts. For example, industries may use this survey to identify quantum algorithms that are related to their business cases, and to evaluate whether these could be implementable in the Noisy Intermediate-Scale Quantum (NISQ) era. Researchers may find in our survey insights about the mathematical outcomes of different quantum algorithms or about the general trends in the field. They may use our dependency analysis as a roadmap to try and find their way back to the sources of quantum advantage. Lecturers may choose to focus the syllabuses of their courses on quantum algorithms down to the core set of primitives that we identify. Graduate and Ph.D. students may again rely on this roadmap to navigate quantum-algorithms learning routes whilst keeping their preferred applications in sight.

This paper is organized as follows. Sec. 2 describes (i) the methodology that we followed, especially in order to build our large classification table, and (ii) each of the classification criteria that we used. In Sec. 3, we present our analysis of this classification table, more precisely, our analysis of the correlations between the different classification criteria. We conclude in Sec. 4 and briefly mention possible extensions of this work. The appendix provides the classification table.

# 2 Methodology and criteria to build the classification table

To achieve the two core objectives of this work, namely, (i) the classification of the main existing quantum algorithms and (ii) their analysis, we followed various steps sequentially, which we now explain. To make a long story short: we listed a total of 134 quantum algorithms as rows in a large classification table given in Appendix A (according to the selection process described below in Subsection 2.1); we then listed as columns of this classification table our chosen key criteria, whose description and relevance is given below in Subsection 2.2; the filled classification table finally served as the raw data for our analysis of Section 3, whose aim was the discovery of relationships, dependencies or patterns among the algorithms and the criteria used, which was achieved by employing statistical and visualization methods.

#### 2.1 Algorithm selection process

The selection of the algorithms was by no means random, it was the result of a literature review-based approach, governed by the goal to create an all-inclusive overview that captures the overall landscape of quantum algorithms. The algorithm selection process of course prioritized those algorithms that are highly recognized within the quantum-computing community, ensuring the inclusion of foundational algorithms that have significantly contributed to the advancement of the field over the years. In addition to these most well-known algorithms, one of the several criteria for choosing new ones was their popularity, based on citation frequency. Subsequently, the classification table was updated to try and capture recent developments within current research trends. Digging into each of these topics, we then took the liberty to include further results that appeared to us as quite significant scientifically. Eventually, a last factor that was considered, was the consistency of the classification table itself, e.g., making sure that all main subroutines were present, all application domains inhabited, etc.. Again, although we tried to capture the main algorithms using objective and scientific criteria to minimize bias, it is inevitable that some degree of arbitrariness remains in the selection process.

#### 2.2 Classification criteria

We now outline the different classification criteria that were chosen in order to explore and enclose the various dimensions of quantum algorithms. Examples of algorithms will be given for each criterion.

#### 2.2.1 Main subroutine

In algorithm design, a subroutine is a building block that performs one (or several) specific task(s), packaged as a unit and typically used in larger, more complex algorithm. A subroutine can be called every time that this task needs to be performed, and furthermore, each subroutine can call another subroutine, creating a chain of calls. In the context of this work, the "Main subroutine" criterion, refers to the first layer of subroutines that an algorithm can have, i.e., not including subroutines of subroutines. In our classification table this criterion can take as "value" either some other algorithms that we have listed in our classification table, or the term "Primitive". In the latter case, this means that the algorithm is a fundamental building block, i.e., it is not using any other algorithm as subroutine. Furthermore, in a few cases the algorithm is classified as either a "variant of" or "inspired by" some other algorithm. In the former, the

algorithm is a modified or adapted version, while in the latter, it takes inspiration or concepts from another algorithm, using distinct features.

Example: Shor's algorithm uses as main subroutine the Quantum Phase Estimation (QPE) algorithm.

#### 2.2.2 Fundamental mathematical problem

This criterion refers to the specific mathematical problem that the algorithm is designed to solve. In Subsec. 3.2 of the analysis, we extracted wider "classes of mathematical problems", or "mathematical classes", to which these specific mathematical problems belong. The wider class is mentioned in the same criterion column, in each box, before the colon sign ":".

Example: In the case of Shor's algorithm again, the fundamental mathematical problems solved are prime-numbers factorization and computing discrete logarithms, which belong to the wider class of "Hidden-Subgroup Problems", see Subsec. 3.2 for the description of this class.

#### 2.2.3 Applications

This criterion refers to the particular problem-solving context(s) in which the algorithm can be useful. Just as for the "Fundamental mathematical problem" criterion, we first listed the possible applications of each algorithm. Then, in Subsec. 3.2, we extracted the wider "domains of application", or "application domains", that encompass them. The wider domain is mentioned in the same criterion column, in each box, before the colon sign ":".

Example: The quantum counterpart of the Principal-Component Analysis, has applications in the domain "Machine Learning & Data Science", see Subsec. 3.2 for the description of this application domain.

#### 2.2.4 Heuristic versus Proven algorithms

A heuristic algorithm, or heuristic, is a method of solving problems that uses practical and intuitive techniques to quickly find an approximate solution. Usually, this is because the problem is too difficult or impossible to solve exactly in an optimal way. Heuristics are often based on past experience or rules of thumb, and their solutions come without guarantee. The evaluation

of the performance of these algorithms is empirical and statistical.

Unlike exact, i.e., proven quantum algorithms such as Shor's or Grover's, which have been proven mathematically to provide complexity speedups over the best known classical algorithms for certain problems, heuristic quantum algorithms do not come with a formal proof of their performance. Instead, these algorithms are based on intuition and experimentation, and are often motivated by insights from classical algorithms or by physical principles. Indeed, even by these standards, the term "heuristic" is perhaps something of a misnomer, as in many cases there is a paucity of *empirical* evidence that the quantum algorithms in question will deliver useful quantum advantage at scale. Nevertheless, the "heuristics" that we included in our classification table constitute important algorithms because of their use in experiments probing the capabilities of early NISQ devices.

Example: The Quantum Approximate Optimization Algorithm (QAOA) [7] is at the heart of numerous quantum heuristics.

#### 2.2.5 NISQ versus LSQ algorithms

A Large-Scale Quantum (LSQ) computer is a theoretical type of quantum computer that can perform a wide range of tasks and applications beyond those that can be performed by current or near-term quantum computers. It is typically characterized by having a large number of qubits (of the order of millions or more) – notably, sufficiently many so that effective error correction is possible – and high-fidelity gate operations. The development of a large-scale general-purpose quantum computer is considered to be a major milestone in the field of quantum computing, as it is expected to enable significant advances in a wide range of fields, such as cryptanalysis, physics, chemistry, materials science, machine learning, data analysis, or optimization. However, building such a device remains a significant challenge due to the inherent fragility of quantum systems and the difficulty in scaling up quantum processors while maintaining the required level of coherence and control.

The quantum devices that are available commercially or in research laboratories today are more limited and belong, for now at least, to the so-called Noisy Intermediate-Scale Quantum (NISQ) regime. NISQ devices are a necessary step on the path towards LSQ computers, but in principle they could already enable quantum computational advantages, at least in specialized problems that have been explicitly designed to demonstrate a separation between classical

and quantum performances. The original definition of NISQ devices is that they contain 50 to 100 qubits and that they are sensitive to the noise induced by their environment, the so-called quantum decoherence [8]. They suffer from errors, and have insufficiently many qubits to perform effective error correction.

In the context of the classification table, the algorithms were hence separated as either of the NISQ type or of the LSQ type, according to their design, indicating whether they are intended for quantum processors in the NISQ or the LSQ eras. Whilst NISQ versus LSQ is a useful division for the purpose of algorithm classification, which is the intention of this paper, more generally it is worth appreciating that this is somewhat of a false dichotomy insofar as that the first generations of error-corrected quantum computers will still be heavily resource-constrained, and hence NISQ-like design principles (e.g., decomposing circuits to use as few qubits and gates as possible, handing off some computations to classical computers where possible, etc.) will still have to be employed to achieve effective computation for real-world problems.

Example: Both the QAOA, for approximate solutions to combinatorial optimization problems, and the Variational Quantum Eigensolver (VQE) for solving quantum chemistry problems, can be classified as NISQ-era algorithms.

## 2.2.6 Oracular (versus Non-oracular) algorithms

An oracular algorithm is a type of algorithm that makes use of a call to an unspecified generic function, also known as (a.k.a.) "oracle", to solve a specific computational problem. Quite often, the problem relates to a property of the given function.

Typically, in the context of quantum computing, the oracle function needs to be made unitary. Thus, the quantum circuit of an oracular quantum algorithm will contain a unitary gate, a.k.a. "black box", whose definition depends on the specific oracle that is used. Each use of the black box is referred to as a "query". In this context, algorithmic complexity is measured in terms of the number of these queries, hence the terminology "query complexity" [3]. The strength of oracular algorithms lies in their generality. Their weakness lies in the fact that query complexity ignores many details, including potential simplifications due to specific oracles.

Example: Grover's algorithm [9] for searching for a specific item in an unsorted database of N items, represents that database by means of a black book  $U_f$  that maps the basis state  $|x\rangle$ 

onto  $(-1)^{f(x)}|x\rangle$ , where f is the oracle Boolean function telling whether x is the desired item.

#### 2.2.7 Promise algorithms

An algorithm is of the "promise" type when the input of the algorithm needs, for it to work, to take as input one belonging to a subset of the natural set of all possible inputs. This subset is defined as that for which the input satisfies some strong conditions (or promises) that make the problem well-defined and simpler to solve (or even, simply, solvable with that algorithm).

Example: Simon's algorithm [10] solves Simon's problem, defined as follows: given a function  $f: \{0,1\}^n \to \{0,1\}^n$  with the promise that for some  $s \in \{0,1\}^n$ , it holds that f(x) = f(y) if and only if  $x \oplus y \in \{\{0\}^n, s\}$ , find the *n*-bit string s, by making as few queries to f(x) as possible.

## 2.2.8 Sampling algorithms

In the context of computation, sampling problems refer to the task of generating samples from a given probability distribution. The probability distribution may be defined in purely classical terms, or be defined as arising from some underlying quantum process. For example, suppose we prepare an n-qubit initial state  $|0\rangle^{\otimes n}$  and we apply a series of quantum gates, e.g., a quantum circuit implementing some unitary U, before we measure it all in the computational basis. This results in a final state, i.e., an n-bit string  $x \in \{0,1\}^n$  sampled with probability  $p_x = |\langle x|U|0\rangle^{\otimes n}|^2$  [11]. One sampling problem is to produce x with probability  $p_x$ .

Example: The Boson-Sampling (BS) problem, introduced by Aaronson and Arkipov [12], has the following setup: n non-interacting identical photons pass through a linear optical circuit, which is used as an interferometer. The task is to sample the probability distribution of their output positions. This model is easily simulated on a standard LSQ computer, whereas efficient classical BS would imply a polynomial hierarchy collapse.

#### 2.2.9 Partial dequantization

A recent [13], major realization has been that numerous quantum algorithms can be classically simulated efficiently in some limited regimes (e.g., low rank) by means of classical sampling mimicking the process of state preparation. We have identified which of the algorithms we listed have given rise to quantum-inspired classical algorithms using this methodology. We refer to

this criterion as *partial* dequantization, because typically the classical algorithm will serve as a *partial* replacement only (of its quantum counterpart), i.e., it will be applicable to a subset only of possible inputs.

#### 2.2.10 Computational model

Generally, the expression "model of computation", or "computational model", in computability an theory, refers to a formal mathematical definition of "what a computer is". It thus provides a theoretical framework about how some function may be computed, by specifying things such as: the presentation/encoding of inputs and outputs, the computational steps, the memory structure, and the communication methods. The computational complexity of an algorithm can then be defined in those terms [14].

In quantum computing, there exists a variety of computational models. We will list the most important ones, and mark them with the sign "(\*)" when they have been used in our typology, i.e., as a possible "value" taken by the criterion "Computational model" in the relevant column of our classification table. We choose not to abbreviate or use acronyms for the computational models in this work. Among these computational models we find, most relevantly, Quantum Turing Machines [15, 16] – often acronymized QTMs in the literature –, the Circuit Model (\*) [17], Quantum Cellular Automata (\*) [18] – often acronymized QCAs in the literature –, Adiabatic Quantum Computation (\*) [19] – often acronymized AQC in the literature –, Quantum Walks (\*) [20] – often acronymized QWs in the literature –, or Measurement-Based Quantum Computation (\*) [21] – often acronymized MBQC in the literature. All of these computational models can do universal quantum computation in the sense of approximating any arbitrary unitary operation  $U \in U(n)$ ,  $n \in \mathbb{N}$ , up to arbitrary precision [22–25].

Note that some of these models can be viewed as mathematical abstractions of certain physical implementations or experiment protocols: Quantum Annealing approximately implements Adiabatic Quantum Computation [26], Linear Optical Quantum Computation (\*) – often acronymized LOQC in the literature – is the preferred architecture to realize Measurement-Based Quantum Computation [27, 28], some architectures for the Circuit Model correspond to Quantum Random-Access Memories (\*) [29] – often acronymized QRAMs in the literature –, etc..

#### 2.2.11 Citations

The last criterion column of the classification table is that of the number of Google-Scholar citations. For each algorithm of the classification table, we picked as reference either the seminal paper explaining this algorithm, or a very relevant paper explaining this algorithm, and then consulted the number of citations of this reference paper, on Google Scholar. This check was done between October 2023 and June 2024. We have not used this number to weight our data visualization. We used it to determine (amongst other factors) whether some non-recent papers ought to be present in the classification table or not.

# 3 Analysis of the classification table

The diagrams presented in this part can be found at https://github.com/Evi-Ksn/Typolog y-of-Quantum-Algorithms for a better resolution.

## 3.1 Dependency and algorithmic-primitives analyses

Whilst the area of quantum algorithms was, two or three decades ago, dominated by a handful of seminal algorithms like Shor's and Grover's, it has now expanded into a rich landscape of numerous novel algorithms. However, a legitimate question is whether the new algorithms of this field are made from combinations of the old ones used as subroutines, or whether they are fundamentally novel altogether. Analyzing the "Main subroutine" criterion of the classification table, can give us an answer to this question.

#### 3.1.1 Dependency network

First of all, we used these data of the column "Main subroutine" to generate a network of dependencies (or dependency network) of quantum algorithms, that we present in Fig. 1. Let us first describe very generally the dependency network. It provides a detailed visualization of all the layers of subroutines that are used as "steps" in each algorithm. It offers an insight into the logical relationships between these algorithms, revealing also, to some extent, the genealogy of ideas underlying their design.

Let us now describe the dependency network in more detail. In this dependency network, arrows go from some subroutine to the algorithm making use of that subroutine, subroutine which may itself use another subroutine, and/or be used by another algorithm. An algorithm

sometimes uses several subroutines. Primitive algorithms, are thus shown as root nodes, i.e., without any arrow pointing towards them. When a primitive is not used by any other algorithm, it does not appear in our dependency network. When we write "a variant of" or "inspired by" some algorithm in the classification table, that latter algorithm is treated in the dependency network exactly as if it was a subroutine – i.e., there's an arrow from it towards the algorithm indicated as "a variant of" or as "inspired by". Notice that some dependencies vary with the version of the algorithm: for instance, whilst the original QPE relies on the Quantum Fourier Transform (QFT), there exists Bayesian QPE or Iterative QPE, which do not; similarly, different Quantum Amplitude Estimation (QAE) algorithms may rely on QPE or not. Notice also that several algorithms dependent on some early Quantum Linear-System Algorithm (QLSA), could in fact be made dependent on the Optimal QLSA [30].

Let us now start analyzing the dependency network. By examining it, we clearly observes four large clusters. At their respective centers lie four core primitives: Quantum Fourier Transform (QFT) – or the original QFT-based QPE, if we allow ourselves to speak about non-primitive subroutines –, Amplitude Amplification (AA), Quantum Adiabatic (QA), and Variational Quantum Eigensolver (VQE). It is interesting, and reassuring, that these four primitives having emerged as "cores" through the methodology followed by this study matches with what a practitioner in quantum algorithms would expect. One can also witness on this dependency network, to some extent, the consequences of a natural shift of interest in quantum algorithms (this will be clearer further down). Indeed, let us first notice that two important and distinct sources of quantum advantage that have been identified since the early days of quantum computing are the following: the original, QFT-based QPE, underpinning Shor's algorithm; and AA, underpinning Grover's algorithm. But, with the advent of real-world quantum hardware, attention has been drawn towards running algorithms on present-day devices seeking to implement VQE and, to some extent, QA; that is to say, since the proliferation of small-scale quantum hardware, the focus has switched from algorithms exhibiting asymptotic quantum advantage, to those that can run on such small-scale quantum hardware (even, on occasions, algorithms for which there is little evidence that there will be quantum advantage as the hardware scales).

#### 3.1.2 Algorithmic-primitives analysis

Going back to the first column "Main subroutine" of our classification table, and focusing on the Primitives, one immediately sees that, beyond 8 usual "suspects", many other primitives,

Popular Primitives	Popular Subroutines	Other Primitives
QFT [31]	QPE [32]	QITE [33]
AA [34]	QAE [35]	Q. Lanczos [36]
QA [37]	Grover [38]	Variational QSVD [39]
VQE [40]	QLSAs $[30, 41-44]$	Bayesian QPE [45]
HSs [46-51]	QSVT [52]	Hadamard Test [53]
Block Encoding [49]	QSP [54]	Hilbert-Schmidt Test [55]
BS [56]	Szegedy's QW [57]	Q. Hierarchical Clustering [58]
Swap Test [59]	SKW's QW Search [60]	Q. Distance Classifier [61]
	Adiabatic Search [62]	QEnhanced Markov-Chain Monte Carlo [63]
	D. & H. Minimization [64]	Simon's Algo. [65]
	Q. Counting [66]	Fermionic QFT [67]
	Q. Monte-Carlo Integral [68]	QCA for Q. Electrodynamics [69]
	Gaussian BS [70]	Q. Final State Shower [71]
	Ising-QUBO [72]	Free-QField-Theory Ground State [73]
	QAOA [74]	Q. Fock Space Dynamics [75]
	QSDP [ <b>76</b> ]	Dirac-Equation QW [77]
	Dihedral HSP Algo. [78]	
	Shor [79]	

Table 1: Table of primitives and non-primitive popular subroutines (none of the subroutines of the second column is, indeed, a primitive). Note that in order to be intuitive, all HS algorithms are clustered as "HSs", and all QLSA algorithms are clustered as "QLSAs". The same convention applies for Fig. 4, which uses the second column only, and for Fig. 5, which uses the first column only.

or non-primitive popular subroutines (not necessarily used by other algorithms) could be listed. Those findings are summarized in Table 1. In the first column of that table, we listed the 8 usual primitives mentioned above. In the second column, we listed other, non-primitive popular subroutines, either already old, or new ones, more or less half of which are already indeed used as subroutines of other algorithms. In the last column, we listed the remaining primitives.

Let us briefly speculate about which primitives or popular subroutines may be most dominant in the near or further future. It seems likely that the Quantum Singular-Value Transformation (QSVT) will play a prominent role in the future. Indeed, QSVT is famously a *unifying* algorithm, that subsumes most of the other prominent algorithms as special cases [80, 81]. Moreover, QSVT also plays the role of a *framework* that captures the "modern" approach to quantum-algorithm design, namely, using quantum computers to manipulate the eigenvalues or

singular values of exponentially large matrices – applied to some suitable input. This idea is extremely powerful, and the first hint of it actually came in the Harrow-Hassidim-Lloyd (HHL) algorithm, for which one of the primitives is QPE. Indeed, unlike most other algorithms that use QPE, rather than just extracting a particular eigenvalue phase, HHL manipulates the quantum state by accessing the several phases of eigenvalues in superposition in order to reach the desired result.

An interesting development is Tang's [13] methodology for extracting quantum-inspired classical algorithms from existing quantum algorithms, which are efficient for a certain range of parameters. This methodology is useful mainly for the HHL, QPE, and QLSA algorithms (a.k.a. qBLAS algorithms [82]), with impact on linear-algebra problems and machine-learning and datascience applications, amongst others. But the methodology can also be used for QSVT. Again, given the unifying nature of the latter, it is at this stage rather difficult to gauge the potential ramifications of dequantization that may ensue from a use of Tang's methodology on QSVT.

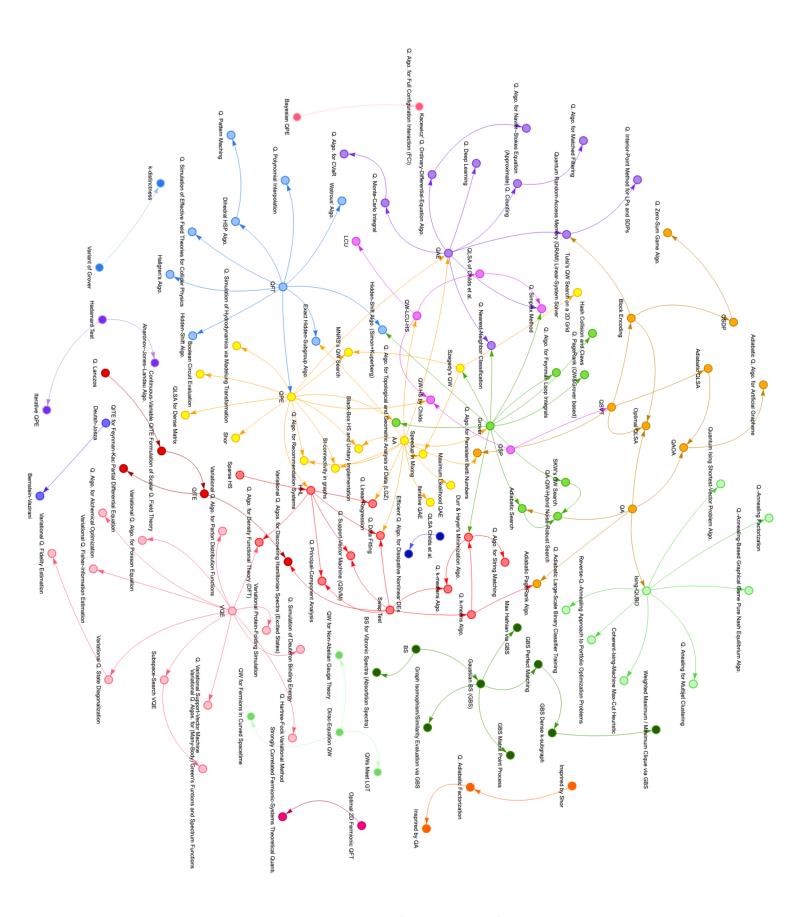


Figure 1: Dependency Network.

#### 3.2 Extracting mathematical classes and application domains

After thoroughly filling, for each algorithm, the specifics (i) for the "Fundamental mathematical problem" criterion/column of our classification table, and (ii) for the "Applications" criterion/column, we realized the need to take a step back to get to the bigger picture.

For the "Fundamental mathematical problem" criterion, we identified 6 classes, described below.

- Hidden-Subgroup Problems: This class of problems is about identifying a subgroup from a function that is constant "on" each coset of that subgroup and that takes distinct values on different cosets of this subgroup. Let us give a formal definition. First of all, let us define what a coset of a subgroup is. A left coset of some subgroup S of G is a set gS for some  $g \in G$ . The definition for right cosets is analogous. A fundamental result about cosets is that two left cosets or two right cosets are either the same or disjoint. Let us now define the standard Hidden-Subgroup Problem (HSP). Given a group G, a finite set X and a function  $f: G \to X$ , the goal is to identify a hidden subgroup  $H \subsetneq G$  of the group G such that f(x) is constant "on" each coset of H, which means that, for all  $g_1, g_2 \in G$ ,  $f(g_1) = f(g_2)$  if and only if  $g_1H = g_2H$  [83]. Shor's algorithm, which solves the problems of prime-numbers factorization and of computing discrete logarithms, can be generalized as an HSP for finite Abelian groups [84].
- Linear Algebra: Linear-algebra problems deal with linear equations, linear transformations and their representations using matrices. For example, the Quantum Singular-Value Decomposition (QSVD) algorithm [39] produces the singular-value decomposition of a bipartite pure state, which is indeed a linear-algebra problem.
- Dynamical Systems: A dynamical system is a system whose state changes over time. This could be either (i) in discrete time, via finite-difference equations in either only time, or both time and space, or (ii) in continuous time, with a partial derivative with respect to time at least, and, if we are also in continuous space, another partial derivative, with respect to space this time, leading to a partial differential equation (otherwise it is an ordinary differential equation with respect to time, with finite differences in space). Quantum algorithms that belong to this class seek to predict and analyze the system's time evolution, e.g., by preparing an initial state, encoding information therein, evolving it, or evaluating key features of this evolution. For instance, the Quantum Lanczos algorithm [36]

computes the ground state of some Hamiltonian, which is a key feature of the evolution of the system described by the Hamiltonian, so that this algorithm belongs to the class "Dynamical Systems".

- Stochastic Processes & Statistics: A stochastic process, also called a random process, is a mathematical concept used to model and describe the evolution of variables/quantities that evolve randomly over time (or some other parameter), using probabilistic rules. Statistics is a branch of mathematics that studies the collection, analysis, interpretation, and presentation of data, so as to make inferences or decisions based on them. Typically, the stochastic process is what generates the data that statistics analyze. In the context of this work, an example of an algorithm that solves a stochastic-processes problem is the Gaussian-Boson-Sampling (GBS) Matrix-Point Process [85] which, in addition, is a sampling algorithm, as defined above. An example of an algorithm that solves a statistics problem is the Quantum k-means algorithm [86].
- Optimization: "Optimization" refers to the class of problems that deal with finding the best solution amongst a given set of candidates. The "best" solution is determined by minimizing or maximizing some target function while satisfying a set of constraints. Optimization includes numerical optimization and combinatorial optimization problems. When an algorithm is of the combinatorial-optimization type, we assigned it both the "Optimization" class described here, and the "Combinatorics" class described below; otherwise (algorithm of the numerical-optimization type), we just assigned it the "Optimization" class. An example is the Quantum Linear Regression algorithm [87], whose fundamental mathematical problem is linear regression using least-squares optimization, which is convex optimization (which is a subclass of numerical optimization).
- Combinatorics: Combinatorial problems involve finding, for a discrete finite set of objects that satisfy some given conditions, either a grouping, or an ordering, or an assignment. They can be divided into tree basic types, that are, enumeration problems, existence problems, and optimization problems. An algorithm classified under "Combinatorics" in the classification table, focuses on solving problems like graph-theoretical problems, search tasks, counting tasks, etc.. For example, Grover's search algorithm has been extended to Quantum Counting [66], and both can be classified as combinatorial.

Regarding the "Applications" criterion, we extracted 6 application domains, described below.

- Cryptanalysis: Cryptanalysis refers to the study of cryptographic systems with the aim of understanding their strengths and weaknesses. Most works in this domain study the applicability of quantum algorithms to break classical cryptographic schemes. The obvious, early example is Shor's factorization algorithm, whose LSQ implementation would break the RSA public-key encryption system [88].
- Machine Learning & Data Science: This domain involves (i) performing tasks like classification, regression, clustering, etc., and (ii) applications of quantum algorithms in the fields of data analysis, statistical modeling, or predictive analytics for managing large datasets. For instance, the Quantum Algorithm for Recommendation Systems [89] has applications in both these areas.
- First-Principle Quantum Simulation: "First-Principle (also called "ab initio") Quantum Qimulation" refers to using quantum computing devices in order to simulate and predict physical properties of a system, based upon its fundamental quantum mechanical description without any detour via empirical models or parameter fittings. Indeed, although classical algorithms for matrix arithmetics have only polynomial complexity in the matrix dimension, the Hilbert-space dimension of multi-particle systems/many-body problems is exponential in the system's size. Thus, quantum algorithms are bound to quickly overcome their classical competitors [90]. In fact, using programmable quantum systems to simulate quantum physical systems was Feynman's original motivation for inventing the very concept of a quantum computer [91]. Typically, first-principle quantum simulations involve solving particle-structure problems, electronic structures [92], atomic-nucleus structures [93], or material properties. We distinguish this application domain from other scientific computing approaches, which we refer to as "classical" below.
- Classical Scientific Computing: This domain is used to accommodate scientific-computing applications that are not first-principle quantum simulations, including empirical-modeling applications and data/signal-processing applications. Quantum algorithms of this application domain may outperform their classical counterparts for specific tasks, including classical-mechanics simulations [94], empirical/semi-empirical numerical model prediction and optimization [95, 96], and experimental data processing/mining [97].

- Operational Research: "Operational Research" involves employing analytical methods, like modeling, statistics, optimization, in order to arrive at optimal or near-optimal solutions to decision-making problems, in complex scenarios like logistics, supply-chain management, resource allocation, etc.. Within the context of this work, mathematical programming, optimization, and game-theory algorithms are classified as "Operational Research". As an example, let us cite the Quantum Algorithm for CVaR [98], which has a specific application in finance, for risk management.
- Quantum Enablement: This domain covers the particular quantum algorithms, closer to the hardware part, that underlie the experimental implementation of other quantum algorithms, closer to the software part, or belong to the construction of the quantum technical stack itself. Typically, they have no direct applications by themselves, although some may be applied in quantum sensing or metrology [76, 99], or to demonstrate quantum advantage [100–102]. They often represent general algorithmic-design ideas on which many more specific quantum algorithms are based [31, 103], e.g., quantum test algorithms are classified in this domain [104–106].

# 3.3 Correlations and trends regarding the three principal criteria, "Main subroutine", "Fundamental mathematical problem" and "Applications"

Having coarse-grained fundamental mathematical problems into classes, and applications into domains, helps identifying correlations and trends.

# 3.3.1 Correlations between mathematical classes and application domains, related trends, and correlations with important subroutines

In Fig. 2, we plot the number of algorithms for each of the 6 application domains, over time. Some trends become apparent. Indeed, the application domains in which there is an increasing number of new algorithms in the last years (here we roughly merge the acceleration of this number and its absolute value when we say "increasing number", we do not disambiguate between both features at this stage), are, by decreasing increasing number, "First-Principle Quantum Simulation", "Machine Learning & Data Science", "Operational Research", and, to a lesser extent "Classical Scientific Computing" and "Quantum Enablement". The "Cryptanalysis" domain is the one that evolves slowest in terms of acceleration of the number of new algorithms.

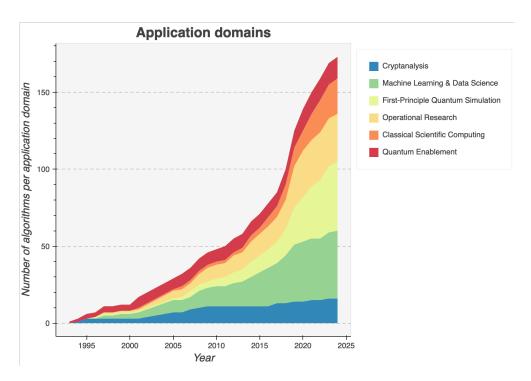


Figure 2: **Application-Domains Growth.** Evolution over time of the number of new algorithms per application domain.

A similar diagram, focusing this time on the mathematical classes, is presented in Fig. 3. It shows that an important proportion of new algorithms fall into the "Dynamical Systems" class. This trend aligns with the previously noted growth in "First-Principle Quantum Simulation" applications, as these applications often require solving dynamical-system problems. Similarly, an important proportion of new algorithms fall into the "Linear Algebra" and "Optimization" classes, which again aligns with the previously noted growth in "Machine Learning & Data Science" and "Operational Research" applications, since the latter require solving a combination of linear algebra, statistical, and optimization problems. The "Hidden-Subgroup Problems" class is stable, correlated with the stability of the "Cryptanalysis" application.

The previous trend analyses clearly suggest correlations between mathematical classes and their application domains, through the algorithms serving them. Let us evaluate this more precisely. We have computed these correlations and visualized [107] the result in Fig. 4. Notice how these findings were enabled by the dependency analysis of Subsec. 3.1, allowing us to focus on main subroutines for increased robustness: indeed, the x-axes of the two diagrams of Fig. 4, which are both the same, correspond to the list of Popular Subroutines identified thanks to the dependency network, and which appears in the second column of Table 1. Hopefully these two

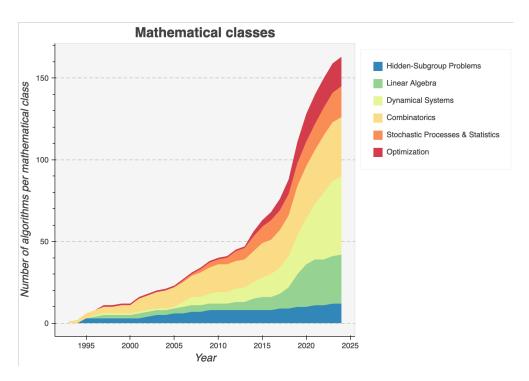


Figure 3: Mathematical-Classes Growth. Evolution over time of the number of new algorithms per mathematical class.

diagrams can help trace back the root of some quantum advantage, according to one's objective.

# 3.3.2 Correlations between algorithmic primitives, mathematical classes, and application domains

Let us dig in the correlations between mathematical classes and application domains, via the algorithmic primitives that serve them both. Those are represented in Fig. 5. Notice that the results are relatively elaborate and do not immediately follow from common sense. We are going to comment on them progressively as we advance in this subsubsection. We are going to start by making a number of observations, organized per primitive, and partly based on both Figs. 4 and 5, but also on general knowledge acquired during our investigation.

## • About the use of Grover's algorithm – based on the primitive AA:

Amplitude Amplification (AA) is a major primitive, on which Grover's algorithm [9] depends logically (not chronologically). Based on Grover's algorithm, which uses the quantum Circuit Model of computation, many quantum search algorithms have been developed with other computational models such as Adiabatic Quantum Computation [62] and Quantum Walks [60, 108]. The ability

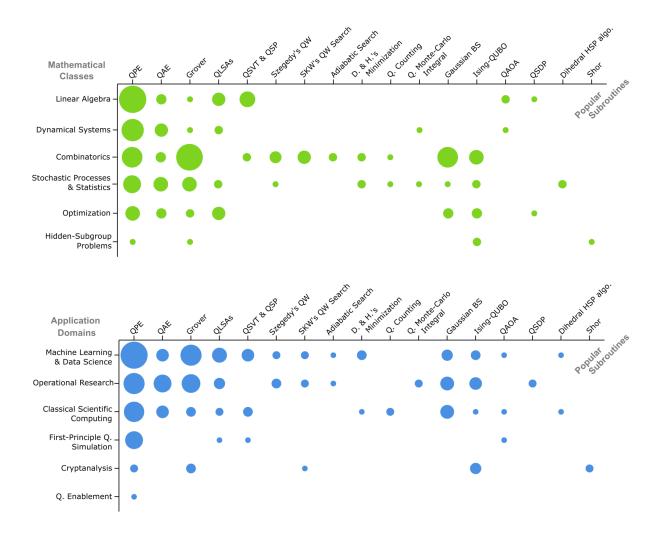


Figure 4: Correlations Array. Array of correlations between the Popular (non-primitive) Subroutines (of the second column of Table 1) and either (i) mathematical classes for the top array, or (ii) application domains for the bottom array.

to search faster leads to advantages in solving problems in the "Combinatorics" [57, 64, 109–116], "Hidden-Subgroup Problems" [117, 118] and "Stochastic Processes & statistics" [119–123] classes.

#### • About the use of QPE – based on the primitive QFT – and of the QLSAs:

As we can see on Fig. 4, QPE is one of the most popular subroutines. When aiming at solving problems in the "Dynamical Systems" class, QPE provides a fundamental method for estimating the ground-state energy. When aiming at solving problems in the "Linear Algebra" class, QPE served as a subroutine of early QLSAs (i.e., HHL).

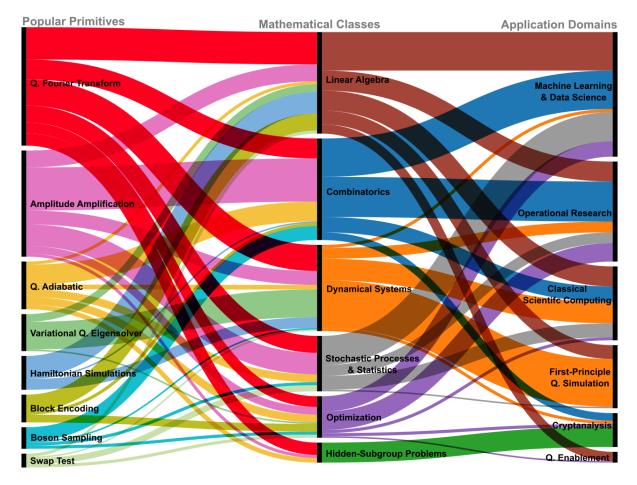


Figure 5: Sankey Diagram. Correlation between Popular Primitives (first column of Table 1), classes of the criterion "Fundamental mathematical problem", and domains of the criterion "Applications". In this diagram, the meaning of "Q. Adiabatic" in the column of Popular Primitives is "any algorithm of the QA type", i.e., we do not limit ourselves to the original QA algorithm. The same applies to "Hamiltonian Simulations". The positions of the "connections" which "arrive" on the column "Fundamental mathematical problems" from the left, have no relationship with the positions of the "connections" which "go out" of that column towards the right. All connections are always ordered very logically from top to bottom.

Further QLSAs escape QPE and make use of other primitives and subroutines instead, such as AA, QA, and QSVT [30, 42, 124, 125]. Still, whatever the QLSA that we use, and since many problems can be formulated in terms of systems of linear equations, QLSAs help to solve problems in the "Linear Algebra", "Optimization", "Stochastic Processes & Statistics", and "Dynamical Systems" classes.

#### • About QAE and, in general, the mixing of QPE and AA:

QPE and Grover's algorithm are often used in conjuction in quantum walk-based search algorithms for solving problems in the "Combinatorics" class.

QAE was also first obtained by combining QPE with AA [35]. QAE allows one to estimate the amplitude of a state within a superposition [35]. It acts as a fundamental primitive for those algorithms that use amplitude encoding and require retrieving the information encoded by the amplitude. The first use that comes to mind is in counting and matching, i.e., usually in the "Combinatorics" class. But QAE is also used for solving stochastic differential equations and some quantum machine-learning tasks. Moreover, further progress achieved QAE without QPE [126, 127]. This, in turn, is used by other algorithms to perform numerical calculations and solve problems of the "Linear Algebra" [128], "Optimization" [129, 130], "Stochastic Processes & Statistics" [68, 131], and "Dynamical Systems" [46, 47, 132–134] classes.

#### • About QSVT and QSP:

The eventual impact of the QSVT and QSP algorithms is expected to be greater than what Fig. 4 indicates, not only because many of the earlier subroutine usages of HHL can be directly replaced by advanced QLSAs that use QSVT as a subroutine, but also because using QSVT for Hamiltonian Simulation (HS) could become more popular, as it can sometimes provide optimal complexity, for example for the BQP-complete problem [135] of Ref. [30], in the LSQ sense. Moreover, powerful features such as eigenvalue transformation and filtering (of arbitrary Chebyshev polynomials) are likely to be found more uses as the exploration goes.

## • About HS:

Hamiltonian Simulation (HS), as a fundamental tool for encoding information and constructing the time evolution of the system under study, has bred a variety of approaches [47, 49, 136–139]. Besides the multi-purpose subroutines such as QSVT and QSP [80, 140], which are not confined exclusively to HS, and excluding the Suzuki-Trotter decomposition [141], which we have not considered as a quantum algorithm, in Fig. 5 we have put all the HS algorithms under the umbrella of "Hamiltonian Simulations" (exactly as we did in Table 1 above, which is mentioned in the caption). Among all of these algorithms, Block Encoding emerges as the most significant primitive in our dependency network, so that we have isolated it from other HS primitives in Fig. 5; we see in Fig. 5 that it is mostly used to solve problems in the "Linear Algebra" class. Generally speaking, Fig. 5 shows that HS algorithms, in addition to serving problems in the "Linear Algebra" class, also serve problems in the "Dynamical Systems" class, as we may expect, most often with applications in the "First-Principle Quantum Simulation" domain.

#### • About VQAs:

The Variational Quantum Eigensolver (VQE) [142] is an algorithm designed to find the eigenvalue of an operator and it is quite often used to find the ground-state energy of a quantum system. Many quantum algorithms use the VQE as their main subroutine to solve problems in the "Dynamical Systems" class, with applications in chemistry or materials science, e.g., for simulating the electronic structure of a molecule [143–146]. As a consequence, there is a large proportion of VQE uses in the "First-Principle Quantum Simulation" application domain, as we can see on Fig. 5. However, we can also notice a good proportion of VQE uses in the "Operational Research" application domain, due to the growth of variational quantum algorithms for problems in the "Combinatorics" and "Optimization" classes. It is noteworthy that following the development of the VQE, numerous Variational Quantum Algorithms (VQAs) based on the variational principle have been developed. These algorithms are capable of solving a broader range of problems in the (numerical) "Optimization" and "Linear Algebra" classes, beyond the computation of ground-state energies.

#### • More applications of the QFT (apart from the original QPE):

The Quantum Fourier Transform (QFT) is one of the major primitives among quantum algorithms. With the generalization of the Fourier sampling technique [147], algorithms could rely on QFT to solve "Hidden-Subgroup Problems" [148–150], which have many applications in the Cryptanalysis domain. QFT is also used as a primitive to solve problems in the "Dynamical Systems" class [151, 152], e.g., for Hamiltonian simulation [47, 50, 51]. Algorithms that solve problems in the "Linear Algebra" [41, 44, 52, 135, 153] and "Optimization" [154–159] classes also use QFT as subroutine, with applications in the "Machine Learning & Data Science" and "Classical Scientific Computing" domains.

#### • About QA and QUBO:

Again in the context of our dependency and correlation analyses, we have put *all* the Quantum Adiabatic (QA)-type algorithms under the same umbrella [37], especially in Fig. 5, and we will refer to them in the body of this paper as "QA algorithms". As for solving problems in the "Linear Algebra" and (numerical) "Optimization" classes, one of the crucial usages of QA algorithms is state preparation [30, 160], which is often used as a subroutine among QLSAs. Another branch

of usage of QA algorithms is (heuristically) solving problems in the "Combinatorics" class, where the Ising-QUBO formulation [72], as well as the the QAOA [74] (in the quantum Circuit Model context), give a uniform way of encoding solutions of many NP problems onto the ground state of an Ising-type spin Hamiltonian. With ingeniously designed adiabatic time scheduling, the minimum energy gap of the adiabatic Hamiltonian can be reasonably manipulated, which enables a time-complexity advantage [19]. Notice that, due to the capacities of QA algorithms in terms of combinatorial-problem encoding and time-evolution implementation, QA algorithms give rise to some NISQ experimental implementations (i.e., Quantum Annealing, or Coherent Ising Machines) [161–166], which claimed to have shown evidence of quantum advantage.

#### • About BS:

Again from the perspective of NISQ devices, Boson Sampling (BS)-based algorithms [100, 167], including Gaussian BS-based algorithms, also claimed to have shown evidence of quantum advantage in solving problems in the "Combinatorics" and "Stochastic Processes & Statistics" classes [70, 95, 168–172].

#### • About the variety of quantum test algorithms:

Finally, quantum test algorithms form an indispensable toolbox for quantum-algorithm research. The Swap Test is used to compare two states and can be used for fidelity checks [105]. This solves problems in the "Linear Algebra" class. The Hadamard Test is used to sample the expectation of acting with a given operator on some input state, which enables to tackle problems in the "Stochastic Processes & Statistics" class [104]. In addition, the Hadamard Test is used to calculate the distance between two operators, which is meaningful for evaluating some quantum algorithms that update gate parameters iteratively and solve problems in the "Linear Algebra" class.

#### • Discussing Fig. 5:

Overall, it is noteworthy that the quantum subroutines do not entirely dictate the relationship between application domains and mathematical classes. Instead, formulating application-domain problems into quantum-solvable mathematical problems by modeling and reduction is a crucial part of exploring practical applications of quantum computing. Moreover, a given, general field of application, actually often spans across several application domains: for instance, the task of drug design spans across application domains such as "Operational Research", "First-Principle Quantum Simulation", and "Machine Learning and Data Science"; indeed and hence – since we now merge (i) this large spectrum of applications with (ii) its influence on the precise mathematical problems to be solved, just to give an example of the fact that a strict distinction between both is not always that relevant –, depending on the scale the chemical properties of a drug can be modeled and analyzed either from first principles, semi-empirically, or entirely numerically, and the pharmacological and toxicological properties of the drug could also be optimized along these paths [173]. A counter example example is the Cryptanalysis application domain. The relevance of this application domain to mathematical problems is quite singular: quantum algorithms for solving hidden-subgroup problems are almost exclusively applied in this field, and they are based on QFT. All that being said, despite the complexity of Fig. 5, when faced with computationally intensive tasks, our typology could provide an effective perspective, offering potential quantum-computing users a path to find quantum speed-up solutions.

## 3.4 NISQ and secondary criteria

#### • NISQ/LSQ criterion with respect to the two main criteria

Our analysis of the "NISQ/LSQ" criterion resulted in two figures illustrating the percentage of NISQ/LSQ algorithms categorized by mathematical classes (Fig. 6) or application domains (Fig. 7). Judging by Fig. 7, it seems that the most promising application domain is "First-Principle Quantum Simulation", likely due to the variational algorithms that have been developed lately for this purpose, although full quantum simulation will still require LSQ devices. The "Classical Scientific Computing" application domain also seems promising, which is primarily due to the development of variational linear-algebra algorithms (also called QLSAs), as well as of other algorithms tackling dynamical-system problems. In contrast, there is a limited number of NISQ algorithms in the "Cryptanalysis" application domain, which is due to the fact that this domain prominently depends on the QFT. The large percentage of "Stochastic Processes & Statistics" algorithms falling into the NISQ category is due to the probabilistic nature of stochastic process problems, which are often tackled with sampling algorithms and seem more compatible with noisy devices, showing great promise in achieving quantum-computational advantages.

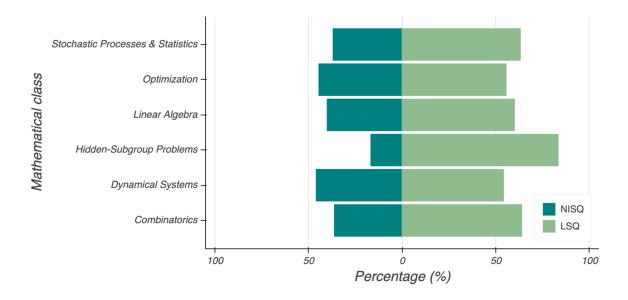


Figure 6: **NISQs per Mathematical Class.** Percentage of NISQ/LSQ algorithms per mathematical class.

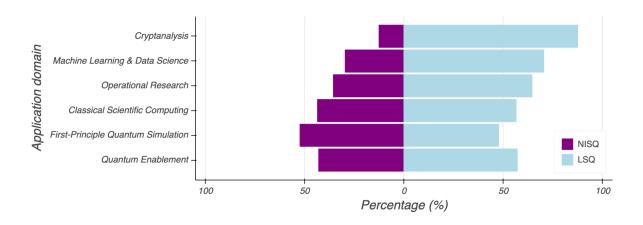


Figure 7: **NISQs per Application Domain.** Percentage of NISQ/LSQ algorithms per application domain.

#### • Other criteria

Let us close this analysis section by commenting about some correlations unraveled by the classification table, and involving secondary criteria such a being "Heuristic/Proven", "Oracular", "Promise", or the subject of "Partial dequantization".

We notice that NISQ is correlated with being a -heuristic algorithm. This is as one would expect. The general philosophy behind NISQ algorithm design is to obtain results in spite of errors and limited resources – which, in particular, unfortunately implies that any advantage

is going to be hard to prove in this context. Conversely, LSQ is correlated with Proven algorithms, showing that whenever researchers are working in this more idealized setting, they do strive for proofs of quantum advantage. Similarly, the correlation between being Oracular or Promise, which plant an even more idealized setting, with being Proven, comes to reinforce this understanding. Let us close this loop by noticing that there are almost no Oracular NISQ algorithms – again, because the NISQ rationale to come up with things that work right now on present-day hardware, is somewhat orthogonal to the mathematically inclined considerations of there existing some abstract oracle.

Within LSQ, the mathematical class "Combinatorics" is correlated with the algorithm being Oracular. This is likely explained by the fact that the AA algorithm (and search algorithms in general) are Oracular. The mathematical class "Dynamical Systems", on the other hand, is anti-correlated with being Oracular, likely reflecting the importance of the QPE and Non-oracular VQE algorithms in this field.

Partial dequantization mainly affects Proven LSQ algorithms, and mainly from the mathematical class "Linear Algebra" but not only (some "Optimization", "Combinatorics" and "Dynamical Systems" problems are impacted), and mainly from the "Machine Learning & Data Science" application domain but not only (some "Operational Research" and "First-Principle Quantum Simulation" applications are affected). This is mostly explained by the underlying use of the QPE and HHL algorithms.

The "Sampling" criterion does not show any significant correlation.

## 4 Conclusion

Quantum computing, driven by its exploitation of quantum-mechanical phenomena, stands poised to revolutionize computation. This paper has undertaken the task of mapping the ever-expanding landscape of quantum algorithms. To handle this rapidly moving field, we built a large sample of significant quantum algorithms based on a literature-review approach, seeking to capture both the most established algorithms and the most recent trends.

We analyzed the dependencies of these algorithms, drawing up their family tree and thereby making it apparent which are the true primitives and the most popular subroutines that are powering up quantum advantage.

For each of these algorithms, we listed the "Fundamental mathematical problem" addressed,

from which we then identified 6 larger classes. The same bottom-up approach was followed for the real-world "Applications" of these algorithms, which we first listed and then gathered into 6 application domains. Analyzing algorithmic trends over time, we observed shifts in the development of quantum algorithms, with certain mathematical classes and application domains gaining prominence. Notably, the domains of "Machine Learning & Data Science", of "First-Principle Quantum Simulation", and of "Operational Research", demonstrated substantial growth.

We took a closer look at the significance of the major primitives and/or subroutines in this unravelling story: Amplitude Amplification (AA), Quantum Phase Estimation (QPE), Quantum Linear-System Algorithms (QLSAs), Quantum Singular-Value Transformation (QSVT), Quantum Signal Processing (QSP), Hamiltonian Simulation (HS), Variational Quantum Eigensolver (VQE), Quantum Fourier Transform (QFT), Quantum Adiabatic (QA) algorithms, Ising-QUBO formulation, Boson Sampling (BS), Testing.

Other criteria ("Sampling", "Promise", "Heuristic/Proven", "NISQ/LSQ") were gathered and their correlations analyzed. We hope that all of these insights will provide guidance for students, researchers, companies and decision makers, navigating the NISQ era. For instance, the "First-Principle Quantum Simulation", "Classical Scientific Computing", and "Stochastic Processes & Statistics" application domains where identified as particularly promising in the short term.

Whilst we did our best to offer a good snapshot and analysis of the current landscape, this work will no doubt require periodic updates in order to accommodate continuing developments: our data and codes are made publicly available for anyone willing to undertake this task. During our research, we became aware of a similar effort presented as an arXiv preprint [174], which is more descriptive and topic-focused in nature, while try to focus on the bigger picture. Independent perspectives and methods are needed to illuminate such a vast and complex landscape, and constitute an essential ingredient of the scientific method. Future research could try and incorporate algorithmic complexity and performance bounds as a further criterion (of some classification table similar to ours, for example).

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## A Classification table

All the columns of the classification table we provide in this appendix have been explained in Sec. 2 – there are, apart from that which gives the name of the algorithm, 11 of them, plus a column providing the link towards the paper that dequantizes the algorithm whenever applicable –, and used in our analysis of Sec. 3, except for three extra columns, which have not been used in our analysis, and provide, respectively:

- 1. The link towards the arXiv version of the paper whenever available;
- 2. The link towards the peer-reviewed version of the paper whenever available;
- 3. The date of publication, which is the minimum date between the arXiv and the peer-reviewed versions of the paper.

Hidden-Shift Algo. (Simon+Kuperberg)	Hidden-Shift Algo.	Q. Polynomial Interpolation	Hash Collision and Claws	Hallgren's Algo.	Simon's Algo.	Bernstein-Vazirani	Deutsh-Josza	Grover	Q. Lanczos	QA	QITE	AA	VQm	푿	Shor	QPE	QFT	Quantum algorithm
QFT, Grover	QFT	OF T	Grover	QFT	Primitive	Deutsh-Josza	Primitive	AA	Primitive	Primitive	Primitive	Primitive	Primitive	QPE, Sparse HS	QPE	QFT	Primitive	m Main subroutine
Hidden-Subgroup Problems: Hidden- shift problem in non-Abelian groups	Hidden-Subgroup Problems: Hidden- shift problem	Combinatorics, Optimization: Finding a polynomial that passes through a given set of data points	Combinatorics: Collision problem	Hidden-Subgroup Problems: Principal ideal, Pell's equation, Determination of the class group	Hidden-Subgroup Problems: Period Finding. Combinatorics: Collision detection	Combinatorics: Boolean function classification (bitwise scalar product, or which string?)	Combinatorics: Boolean function classification (balanced or constant?)	Combinatorics: Binary classification on finite sets	Dynamical Systems: Finding the ground state of a Hamiltonian with improved accuracy	Combinatorics: Satisfiability problem	Dynamical Systems: Finding the ground state of a Hamiltonian without ansatz	Combinatorics: Binary classification on finite sets	Linear Algebra: Estimate the expectation value of a Hermitian operator. Dynamical Systems: Finding the ground state of a Hamiltonian	Linear Algebra: Solving systems for linear equations	Hidden-Subgroup Problems: Factorization, Discrete logarithms	Linear Algebra: Estimation of the eigenvalue of an eigenvector of a unitary operator. Dynamical Systems: Finding the spectrum of a Hamiltonian	Hidden-Subgroup Problems: Fourier transform of quantum states	Fundamental mathematical problem
Cryptanalysis: Quantum attacks, Symmetric Cryptography		Cryptanalysis: Shamir's secret- sharing scheme	Cyptanalysis: Hash-based cryptanalysis, Symmetric cryptography	Cryptanalysis: Buchman-Williams and (through reduction) RSA & Diffie-Hellman, Assymetric cryptography	Cryptanalysis: Quantum attacks, post-quantum cryptography, symmetric cryptography	Quantum Enablement, Cryptanalysis	Quantum Enablement	Machine Learning & Data Science: Data-base search. Operational Research	First-Principle Quantum Simulation	Quantum Enablement, Machine Learning & Data Science: Data-base search	First-Principle Quantum Simulation: Molecule electronic-structure simulation	Quantum Enablement	First-Principle Quantum Simulation	Machine Learning & Data Science, First-Principle Quantum Simulation, Operational Research	Cryptanalysis: Breaking RSA, Diffie-Hellman, Elliptic curve cryptography	First-Principle Quantum Simulation	Quantum Enablement	Applications
Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Heuristic	Proven	Heuristic	Proven	Proven	Proven	Proven	Heuristic/ Proven
Lso	LSQ	LSQ	Lsa	LSQ	Lso	Lso	LSQ	Lsa	LSQ	Lso	NISQ	LSQ	NISQ	Lso	LSQ	Lso	LSQ	NISQ
Yes	Yes	Yes	Yes	Z o	Yes	Yes	Yes	Yes	No	Z o	Z o	Yes	Z o	Z o	N <sub>o</sub>	Z o	N <sub>o</sub>	Oracular
S	8	Z <sub>o</sub>	N <sub>o</sub>	8	Yes	Yes	Yes	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	No	R	Yes	N <sub>o</sub>	8	No	Promise
N <sub>O</sub>	No	N <sub>O</sub>	No	No	No	N <sub>o</sub>	No	No	Yes	N <sub>o</sub>	No	N	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	N	Sampling
No.	8	S	No	8	S	S	No	N <sub>o</sub>	No	N <sub>o</sub>	N <sub>o</sub>	No	8	Yes	No	Yes	Discussed	Partial dequanti zation
														https://doi. org/10. 1145/3519		https://doi. org/10. 1145/3519 935. 3519991	https://doi. org/10. 1016/i	Dequan tization link
Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Adiabatic Quantum Computation	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Computational model
(D	233		22	364	204	2682	383	9536	467	1647	283	1625	3393	2700	13640	1150	485	Google- Scholar citations
57 None	https://arxiv.	https: //arxiv. 10 org/pdf/15	249 None	None	2046 None	None	3830 None	https://arxiv.	https: //arxiv. org/abs/19	https: //arxiv. org/pdf/qu	[1806.057 07] Variational	https: //arxiv.	https: //arxiv. org/abs/13 04.3061		4 to to to		https: //arxiv.	Link to arXiv s paper
https://link. springer.	https://epubs.	https: //drops. // dagstuhl.	https: //link. springer.	https://dl. acm. org/doi/ab s/10. 1145/500	https: //epubs. siam. org/doi/ab	https: //doi. org/10. 1145/167	https: //doi.	https://dl. acm. pu org/doi/10	https: //doi. 19 org/10.	None	iz https: //doi. lal org/10. 1103/Phv		https://doi. 13 0rg/10. 1038/nco			None		peer- review ed paper
02/12/2018	10 21/11/2002	01/05/2016	01/01/2006	ab 09/05/2006	ab 22/11/1994	01/06/1993	08/12/1992	10 29/05/1996	11/11/2019	28/01/2000	09/04/2018	15/05/2000	10/04/2013	19/11/2018	20/11/1994	20/11/1995	1995	o V

Q. Zero-Sum Game Algo.	Q. Algo. for CVaR	Q. Simplex Method	Q. Interior-Point Method for LPs and SDPs	QSDP	Quantum Random-Access Memory (QRAM) Linear- System Solver	Block Encoding	Optimal QLSA	QLSA for Dense Matrix	QLSA of Childs et al.	Variational Q. Singular-Value Decomposition	Variational Q. State Diagonalization	QSVT	Dihedral HSP Algo.	Exact Hidden-Subgroup Algo.	Watrous' Algo.	Quantum algorithm
QSDP	Q. Monte-Carlo Integral	Grover, QAE, QLSA of Childs et al.	Quantum Random- Access Memory (QRAM) Linear-System Solver	Block Encoding	QAE, Block Encoding	Primitive	QA, QSVT, Block Encoding	QPE	QW-LCU-HS	Primitive	VQE	QSP, Block Encoding	QFT	QFT, AA	QFT	Main subroutine
Optimization, Linear algebra: Solving linear programming	Dynamical Systems, Stochastic Processes & Statistics: Determine the Conditioned Value at Risk metric (CVaR) of a portfolio	Optimization, Linear Algebra: Solving linear programming	Optimization, Linear Algebra: Solving linear programming (LP) and semi-definite programming (SDP)	Optimization, Linear algebra: Solving semi-definite programming	Linear Algebra: Solving linear equation system, Weighted/generalized least squares. Combinatorics: Graph sparsification, Maximum flows	Linear Albebra: Encoding a matrix as a sub-block of a unitary matrix	Linear Algebra: Solving systems for linear equations	Linear Algebra: Solving systems for linear equations	Linear Algebra: Solving systems for linear equations	Linear Algebra: Production of singular value decomposition of a bipartite pure state	Linear Algebra: Diagonalization	Linear Algebra: Singular-value transformation	Hidden-Subgroup Problems: Dihedral group, Hidden-shift problem for finite Abelian group	Hidden-Subgroup Problems: Compute the structure of Abelian and solvable groups	Hidden-Subgroup Problems: Solvable groups	Fundamental mathematical problem
Operational Research: Game theory (solving zero-sum game NEQ)	Operational Research: Finance (risk management)	Operational Research	Operational Research	Quantum Enablement: Shadow tomography of quantum states, Quantum state discrimination. Operational Research	Operational Research: Estimating electrical network quantities	Machine Learning & Data Science: Encoding information into the system. First-Principle Quantum Simulation	Machine learning & Data science, Classical scientific computing: Computational mechanics	Machine Learning & Data Science, Classical Scientific Computing: Computational mechanics	Machine learning & Data science, Classical scientific computing: Computational mechanics		First-Principle quantum simulation, Machine learning & Data science	First-Principle Quantum Simulation Machine Learning & Data Science, Classical Scientific Computing	Cryptanalysis: Algebraic-structure analysis	Crypanalysis: Algebraic-structure analysis, Unique encoding of elements of a group, Exact version of the Swap Test	Crypanalysis: Algebraic-structure analysis, Unique encoding of elements of a group	Applications
Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Heuristic	Heuristic	Proven	Proven	Proven	Proven	Heuristic/ Proven
LSQ	LSQ	LSQ	LSQ	LsQ	Lso	LSQ	LSQ	Lso	LSQ	NISQ	NISQ	LSQ	LSQ	LSQ	LSQ	NISQ/
N <sub>o</sub>	Z o	N <sub>o</sub>	Zo	N <sub>O</sub>	Z <sub>o</sub>	Z o	Yes	Z <sub>o</sub>	Yes	N o	No	Yes/No	Yes	Yes	Yes	Oracular
Yes, sparsity	N <sub>O</sub>	Yes, condition number	Yes, require well-conditioned matrix	Yes, sparsity	Yes, require well-conditioned matrix	N <sub>o</sub>	So	No	No (Fourier Approach), Yes (Chebyshev approach)	8	No	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	8	Promise
S	S	Z	8	8	8	Z <sub>o</sub>	Z	No	Z <sub>o</sub>	Yes	Yes	Z <sub>o</sub>	Z <sub>o</sub>	S	N <sub>o</sub>	Sampling
Primitive is	N <sub>O</sub>	Z <sub>o</sub>	Yes	Yes	Yes	No	Primitive is	Yes	Yes	N <sub>o</sub>	S	Yes	N <sub>o</sub>	No.	N <sub>o</sub>	Partial Dequan dequanti tization link
			https://doi. org/10. 1145/3357 713.	https://doi. org/10. 1145/3357 713. 3384314	https://doi. org/10. 48550/arXi			https://doi. org/10. 22331/q- 2022-06-	https://doi. org/10. 48550/arXi v. 1811.0485			https://doi. org/10. 1145/3519 935. 3519991				Dequan tization link
Quantum Random- Access Memory	Circuit Model	Quantum Random- Access Memory	Quantum Random- Access Memory	Purely mathematical	Quantum Random- Access Memory	Quantum Random- Access Memory	Adiabatic Quantum Computation, Circuit Model	Quantum Random- Access Memory, Quantum Walks	Quantum Walks	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Computational model
ω	18	-	·	93	192	205		198	463	54	148	417	370		110	Google- Scholar citations
https: //arxiv. 30 org/abs/19	https: //arxiv. org/abs/18 06.06893	https: //arxiv. 12 org/abs/19 10 10649	https: //arxiv. org/abs/18 99 08.09266	https: //arxiv. org/abs/18 04.05058		https: //arxiv. org/pdf/18 04.01973.	https://arxiv. //arxiv. 64 org/abs/21	https: //arxiv. org/pdf/17 04.06174.	https://arxiv.org/pdf/15	https://arxiv.	https: //arxiv. org/pdf/18	https: //arxiv. org/abs/18 06.01838	https: //arxiv. org/abs/qu	https: //arxiv. org/abs/22 5 02.04047	https: //arxiv. org/abs/qu	Link to arXiv s paper
None	https: //doi. g org/10. 1038/s41	https: //doi. 9 org/10. 1287/opre	https: //doi. g org/10. hans	https://doi. //doi. org/10. 1109/FO		https: //drops. dagstuhl. de/entitie		2 IO IN IN IN						https://www. rintonpres s. com/iourn	https://dl. acm. org/doi/ab	peer- review ed
05/04/2019	18/06/2018	23/10/2019	28/08/2018	15/10/2017	05/04/2018	05/04/2018	11/11/2021	03/05/2017	29/09/2017	05/06/2020	26/06/2019	05/06/2018	20/12/2004	02/05/2022	26/01/2001	Date

Q. Algo. for Persistent Betti Numbers	Q. Algo. for Topological and Geometric Analysis of Data (LGZ)	Q. Algo. for Recommendation Systems	Q. Pattern Maching	Q. Linear Regression	Q. Principal-Component Analysis	Q. Deep Learning	Q. Data Fitting	Q. Distance Classifier (QDC)	Q. Variational Support- Vector Machine	Q. Support-Vector Machine (QSVM)	Q. Hierarchical Clustering Algo.	Q. Nearest-Neighbor Classification	Q. k-medians Algo.	Q. k-means Algo.	Quantum algorithm
AA, QSVT	QPE, Grover	QPE, AA	Dihedral HSP Algo.	HHL, AA	HHL, Swap Test	QAE	HHL, AA, Swap Test	Primitive	Inspired by VQE	HHL, Swap Test	Primitive	Grover, QAE	Swap Test, Durr & Høyer's Minimization Algo.	Swap Test, Durr & Høyer's Minimization Algo., Adiabatic PageRank Algo.	Main subroutine
Combinatorics: Algebraic topology (estimation of persistent Betti numbers)	Combinatorics: Estimate Betti numbers in persistent homology and finding eigen vectors and eigenvalues of combinatorial Lapacian	Stochastic Processes & Statistics: Matrix Sampling. Linear Algebra: Singular-value decomposition	Stochastic Processes & Statistics: Pattern matching	Optimization, Linear Algebra: Linear Regression using least-squares optimization	Optimization, Linear Algebra: Dimensionality reduction	Optimization: Gradient estimation	Stochastic Processes & Statistics: Determination of the quality of a least- squares fit over a data set: Least- squares fitting - Estimating Fit Quality	Optimization: Computation of distance between datapoints, Classification	Optimization, Linear Algebra: Linear discrimination problems (which is subcategory of pattern/linear classification)	Optimization, Linear Algebra: Linear discrimination problems (which is subcategory of pattern/linear classification)	Stochastic Processes & Statistics: Classification of data (divise clustering approach)	Stochastic Processes & Statistics: Binary classification (clustering), Calculating Euclidean distance between vectors or calculation of inner product	Stochastic Processes & Statistics: Classification of data (clustering)	Stochastic Processes & Statistics: Classification of data (clustering)	Fundamental mathematical problem
Machine Learning & Data Science	Machine Learning & Data Science: Topological data analysis (information retrieval)	Machine Learning & Data Science: Personalized recommendations to individual users	Machine learning & Data science, Classical Scientific Computing: Text and image processing, bioinformatics	Machine Learning & Data Science: Supervised learning, Pattern recognition, Prediction	Machine Learning & Data Science: Unsupervised machine learning, Data analysis	Machine learning & Data science: Classification, Inference, Training deep restricted & full Boltzmann machine	Machine Learning & Data Science: Fitting data to theoretical models, quality of a fit, structure analysis. Classical Scientific Computing: High-energy physics	Machine Learning & Data Science: Supervised learning, Pattern recognition	Machine Learning & Data Science: Supervised learning, Pattern recognition	Machine Learning & Data Science: Supervised learning, Pattern Recognition	Machine Learning & Data Science: Unsupervised learning	Machine Learning & Data Science: Handwriting recognition	Machine Learning & Data Science: Unsupervised learning, Data analysis	Machine Learning & Data Science: Unsupervised learning, Data analysis	Applications
Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Heuristic	Proven	Proven	Proven	Proven	Proven	Heuristic/ Proven
Lso	NISO	LSQ	Lsa	LSQ	LSQ	LSQ	Lsa	NISQ	NISO	LSQ	NSQ	LSQ	LSQ	LSQ	NISQ/ LSQ
Yes	N <sub>o</sub>	Z o	Yes	Z o	Z o	Yes	Yes	No	N <sub>o</sub>	Yes	N <sub>o</sub>	Yes	Yes	Yes	Oracular
S	no	No	No o	Yes	Yes	No	Yes	Yes	N <sub>O</sub>	Yes, low-rank matrices	No	Yes, requires training data not to be atypical of Haar-random unit vectors	No	No o	Promise
N <sub>o</sub>	N <sub>o</sub>	Yes	Z <sub>o</sub>	No	Yes	Yes	Algo 1. No Algo 2. Yes	S	Yes	N <sub>O</sub>	N	S	N <sub>O</sub>	Yes	Sampling
Yes	Yes	Yes	S	Yes	Yes	No	No	Z <sub>o</sub>	No	Yes	Z <sub>o</sub>	No o	Yes	Yes	Partial dequanti zation
https: //arxiv. org/pdf/220	https: //arxiv. org/pdf/220 9.13581. ndf	http://dx. doi.org/10. 1145/3313		https://doi. org/10. 1145/3357	https://doi. org/10. 1103/Phys RevLett					https://doi. org/10. 1145/3357 713. 3384314			https://doi. org/10. 1103/Phys	https://doi. org/10. 1103/Phys RevLett.	Dequan tization link
Circuit Model	Quantum Random-Access Memory	Circuit Model	Circuit Model	Circuit Model	Quantum Random- Access Memory	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Measurement-Based Quantum Computation	Circuit Model	Circuit Model	Adiabatic Quantum Computation	Computational model
<u> </u>	208	344	64	234	1098	240	496	219	1214	1407		224	102	678	Google- Scholar citations
https: //arxiv. 14 org/pdf/21	https: //arxiv. org/pdf/14 08.3106.	https: //arxiv. org/pdf/16		https: //arxiv. org/pdf/16	https: //arxiv. org/abs/13 07.0401	https: //arxiv. org/pdf/14 12.3489. odf	https: //arxiv. org/abs/12 6 04.5242	https: //arxiv. org/abs/17 9 03.10793	https: //arxiv. org/pdf/18 04.11326. 4 pdf		https: //arxiv. 2 org/pdf/23	https: //arxiv. org/pdf/14 01.2142.	None	https: //arxiv. org/abs/13 07.0411	Link to arXiv paper
	https: //www. 4 nature. com/articl	https://drops.6 dagstuhl.			https: //www. 3 nature. com/articl	144	https: //journals. aps. corg/prl/ab stract/10. 1103/Phv	https: //iopscien //ce.iop. org/article	https://www. 8 nature. com/articles/s4158		None	https://dl. acm. org/doi/10 4 5555/287 1393. 2871400	https://dl. acm. org/doi/10	None	peer- review ed
<u>um</u> al. 06/12/2022	13/08/2014	<u>os.</u> <u>hl.</u> 22/09/2017	26/08/2015	04/08/2016	16/09/2013	22/05/2015	ls. lb. lb. lb. lb. lb. lb. lb. lb. lb. lb	e 28/08/2017	05/06/2018	10/07/2014	06/03/2023	110 87 0 18/07/2014	10 20/06/2007	04/11/2013	N Date

Reverse-QAnnealing Approach to Portfolio Optimization Problems	QAOA	Adiabatic QLSA	Q. Anealing for Multijet Clustering	Q. Adiabatic Large-Scale Binary Classifier Training	QAnnealing Factorization	Q. Adiabatic Factorization	QAnnealing-Based Graphical Game Pure Nash Equilibrium Algo.	Adiabatic PageRank Algo.	QA-QW-Hybrid Noise- Robust Search	Adiabatic Search	Ising-QUBO	QW-LCU-HS	ГСП	QSP	QW-HS by Childs	Sparse HS	Black-Box HS and Unitary Implementation	Quantum algorithm
Ising-QUBO	Q <sub>A</sub>	QAOA, QSVT	Ising-QUBO	Ising-QUBO	Ising-QUBO	Insprired by QA, Insprired by Shor	Ising-QUBO	QA	Adiabatic Search, SKW's QW Search	QA, Inspired by Grover	Q <sub>A</sub>	QW-HS by Childs, LCU, AA	Primitive	QW-HS by Childs	QPE	Primitive	QPE, AA	Main subroutine
Combinatorics, Optimization: Quadratic unconstrained optimization problem	Dynamical Systems: Finding Hamiltonian ground state. Combinatorics: Max-cut, Search subset of the n-bit strings	Linear Algebra: Linear system of equations	Stochastic Processes & Statistics: Clustering	Combinatorics: Binary classification on finite sets	Hidden-Subgroup Problems: Integer factorization	Hidden-Subgroup Problems: Integer factorization. Dynamical Systems: Lowest energy state of nuclear spins	Combinatorics, Optimization: Nash equilibrium	Stochastic Processes & Statistics: Measure the relative importance of an element within a set	Combinatorics: Binary classification on finite sets	Combinatorics: Binary classification on finite sets	Combinatorics, Optimization: Quadratic unconstraint binary optimization	, Dynamical Systems: Hamiltonian simulation	Dynamical Systems: Hamiltonian simulation (time-independent sparse Hamiltonian simulation)	Dynamical Systems: Hamiltonian simulation (time-independent sparse Hamiltonian simulation)	Dynamical Systems: Hamiltonian simulation (sparse time-independent & non-sparse)	Dynamical Systems: Hamiltonian simulation	Dynamical Systems: Hamiltonian simulation. Linear Algebra: Implementation of unitary transformation	Fundamental mathematical problem
Operational Research: Finance:	Operational Research.	Machine Learning & Data Science, Classical Scientific Computing: Computational Mechanics	Classical Scientific Computing: Multijet clustering in high energy physics	Machine Learning & Data Science	Cyptanalysis	Cryptanalysis	Operational Research: Strategic decision making	Operational Research: Search engine optimization, Machine Learning & Data Science: Data mining, info retrieving	Machine Learning & Data Science: Data base search, Operational Research	Machine Learning & Data Science: Data base search, Operational Research	Operational Research. Classical Scientific Computing	First-Principle Quantum Simulation	First-Principle Quantum Simulation	First-Principle Quantum Simulation	First-Principle Quantum Simulation	First-Principle Quantum Simulation	First-Principle Quantum Simulation	Applications
Heuristic	Heuristic	Proven	Heuristic	Heuristic	Heuristic	Proven	Heuristic	Proven	Proven	Proven	Heuristic	Proven	Proven	Proven	Proven	Proven	Proven	Heuristic/ Proven
N S S S	N S Q	N S Q	NISQ	LSQ	NISQ	LSQ	NISQ	Lso	NISQ	LSQ	N S Q	LSQ	LSQ	LSQ	LSQ	LSQ	Lso	NISQ/
Z	Z <sub>o</sub>	Z o	Z <sub>o</sub>	Z <sub>o</sub>	Z <sub>o</sub>	Z <sub>o</sub>	Z <sub>o</sub>	N <sub>o</sub>	Yes	Yes	Z	Yes	Z o	Yes	Yes	Yes	Yes	Oracular
5	N <sub>o</sub>	Yes, same as	N <sub>O</sub>	No	N <sub>O</sub>	N	No	Yes, advantage only for top rank log(n) pages	Yes, currently only on hyper cube	No	8	Yes	Yes	Yes	No	Yes	No	Promise
Z o	S <sub>o</sub>	No.	No	N <sub>o</sub>	No	No	N <sub>O</sub>	Yes	No	No	S	No	No.	No	N <sub>o</sub>	N <sub>o</sub>	No	Sampling
No No	Z	R	No	N <sub>o</sub>	No o	N <sub>o</sub>	N <sub>o</sub>	No o	N <sub>o</sub>	N <sub>0</sub>	Yes	Yes 11	S	http: org/ Yes 114:	Primitive is	No	8	Partial dequanti zation
											ocal search	https://doi. org/10. 1145/3357		s://doi. 10. 5/3357				
Adiabatic Quantum Computation	Adiabatic Quantum Computation, Circuit Model	Adiabatic Quantum Computation, Circuit Model	Adiabatic Quantum Computation, Circuit Model	Adiabatic Quantum Computation	Adiabatic Quantum Computation, Circuit Model	Adiabatic Quantum Computation	Adiabatic Quantum Computation, Circuit Model	Adiabatic Quantum Computation	Adiabatic Quantum Computation, Quantum Walks	Adiabatic Quantum Computation	Adiabatic Quantum Local searcl Computation	Quantum Walks	Circuit Model	Quantum Walks	Quantum Walks	Circuit Model	Quantum Walks	Computational model
163	2433	71	17	88	91	185	N	104	38	718	1884	354	396	538	416	723	212	Google- Scholar citations
https: //arxiv. org/pdf/18	https://arxiv. org/abs/14 11.4028	https: //arxiv. org/abs/19	https://arxiv. org/abs/20	https://arxiv. org/abs/09	https: //arxiv. org/abs/16 04 05796	https: //arxiv. org/abs/08	[1903.064 54] A Quantum	https: //arxiv. org/abs/11 104 09.6546	https: //arxiv. org/abs/17	https: //arxiv. org/abs/qu	https: //arxiv. org/abs/13 02 5843	https: //arxiv. org/abs/15	https: //arxiv. org/abs/12 02 5822	https://arxiv. org/abs/16	https://arxiv.	https: //arxiv.	https: //arxiv. org/abs/09 10.4157	Link to arXiv paper
https: //link. springer.	None	https://dl. acm. org/doi/ful	https://www.	None	https: //doi. org/10.			https: //link.aps. org/doi/10	https: //doi. org/10.	https: //doi. org/10.	https://www. frontiersin	https://doi.	None			https: //doi.	None	Link to peer-review ed
19/10/2018	14/11/2014	04/03/2022	28/12/2020	04/12/2009	16/06/2016	14/08/2008	15/03/2019	29/09/2011	01/09/2017	03/07/2001	23/02/2013	07/12/2015	27/02/2012	20/12/2016	10/10/2009	07/02/2006	02/07/2011	Date

	Maximum Likelihood QAE AA	Iterative QAE AA	QAE QPE, AA	Q. Algo. for String Matching Minimization Algo.	Q. PageRank (QW&Grover Variant of Grover)	Speedup in Mixing Szegedy	Tulsi's QW Search on a 2D Inspired I	MNRS's QW Search Szegedy	Szegedy's QW Variant of Grover	SKW's QW Search Variant of Groven	(Approximate) Q. Counting QAE	Durr & Høyer's Minimization Algo. Grover	Q. Monte-Carlo Integral QAE	QEnhanced Markov-Chain  Monte Carlo  Primitive	Quantum Ising Shortest- Vector Problem Algo. Ising-QUBO	Coherent-Ising-Machine Max-Cut Heuristic Ising-QUBO	Quantum algorithm Main s
				øyer's tion Algo.	f Grover	Szegedy's QW, AA	Inspired by Szegedy's QW	Szegedy's QW, QPE	f Grover	f Grover					ВО	ВО	Main subroutine
	Linear Algebra: Estimate the amplitude of a substate of a superpositioned state	Linear Algebra: Estimate the amplitude of a substate of a superpositioned state	Linear Algebra: Estimate the amplitude of a substate of a superpositioned state	Combinatorics: String matching	Stochastic Processes & Statistics: Measure the relative importance of an element within a set	Stochastic Processes & Statistics: Sampling	Combinatorics: Searching elements on a 2D grid in O(sqrt(N) sqrt(log N)) rather than O(sqrt(N)) log N) as the previous algorithms (such as Szegedy's)	Combinatorics: Searching elements on a graph, more general than Tulsi's	Combinatorics: Searching elements in a set	Combinatorics: Search elements on a given graph	Combinatorics: Counting the number of elements that fulfill some specific requirements	Optimization: Searching the index with the minimum value	Stochastic Processes & Statistics: Approximate numerical integration	Stochastic Processes & Statistics: Boltzmann sampling	Hidden-Subgroup Problems, Combinatorics, Optimization: Shortest Vector Problem	Combinatorics: Max-cut	Fundamental mathematical problem
Cryptanalysis/Cryptography, Machine Learning & Data Science, Operational Research: Circuit	Quantum Enablement	Quantum Enablement	Quantum Enablement	Machine Learning & Data Science: Pattern matching, recommendation system. Classical Scientific Computing: Cosmology data processing	Operational Research: Search engine optimization. Machine Learning & Data Science: Data mining, information retrieving	Operational Research	Machine Learning & Data Science: Data base search, Operational Research	Machine Learning & Data Science: Data base search, Operational Research	Machine Learning & Data Science: Data base search, Operational Research	Machine Learning & Data Science: Data base search, Operational Research	Machine Learning & Data Science: Computer vision - histogram-based image processing	Machine Learning & Data Science	Machine Learning & Data Science. Classical Scientific Computing. Operational Research	Machine Learning & Data Science.	Cryptanalysis.	Machine Learning & Data Science: Binary classification. Operational Research.	Applications
DOVE	Proven	Proven	Proven	Proven	Heuristic	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Proven	Heuristic	Heuristic	Heuristic	Heuristic/ Proven
, 0	NISQ	LSQ	LSQ	LSQ	LSQ	LSQ	LSQ	LSQ	LSQ	LSQ	LSQ	LSQ	LSQ	NISQ	NISQ	NISQ	NISQ/ LSQ
<b>∀</b> es	Yes	Yes	Yes	Yes	N <sub>o</sub>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Z <sub>o</sub>	No	Z <sub>o</sub>	Z <sub>o</sub>	Oracular
Yes.	No	No	No	N <sub>O</sub>	No	No	No.	N <sub>o</sub>	No	Yes, only on hepercube and 2-d lattice	8	No	N <sub>o</sub>	No	N <sub>o</sub>	No	Promise
Z	N <sub>O</sub>	S	S	No.	No	Yes	N <sub>o</sub>	No	No o	No	No	N <sub>o</sub>	No	Yes	No	S	Sampling
5	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	Partial Dequan dequanti tization link
Ouantum Walks	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Quantum Walks	Quantum Walks	Quantum Walks	Quantum Walks	Quantum Walks	Quantum Walks	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Adiabatic Quantum Computation	Adiabatic Quantum Computation	an Computational
231 None	196 Or.	111 UI:	1690 ant-	010 010 010 010 010 010 010 010 010 010	[111 9] G in a 141 Qua	116 <u>//a</u>	111 / 112 /	463 OII	712 No	1292 <u>01</u>	https://arx org/. 553 ant-	517 <u>///a</u>	114 337 <u>91</u> 1	14 <u>//a</u>	22 <u>01</u>	112 013 312 013	Google- Li Scholar ar
		odf/19	iv. abs/qu	<u>s:</u> abs/qu 001104		https: h	https: // //arxiv. a org/abs/08 01.0497 b	https: h //arxiv. a org/abs/qu o	None L	https://arxiv.// org/abs/qu/a	abs/qu				https: h //arxiv. // org/abs/20 a	https: h	Link to rarXiv e
https: //doi. org/10. 1137/080	https: //link. springer.	https://www.	https://www.ams.		e. articl	https: //journals. ans	https://journals.aps.org/pra/abstract/10	https://dl. acm. org/doi/ab	https: //ieeexplo re.ieee.	https: //journals. aps.	https://link.springer.com/chap	None	https: //royalsoci etypublish	https: //www.	https://journals.	https: //journals. aps.	Link to peer- review ed
27/11/2007	23/04/2019	11/12/2019	15/05/2000	13/11/2000	09/12/2011	31/10/2008	09/07/2008	01/06/2007	17/10/2004	9/10/2002	27/05/1998	18/07/1996	08/09/2015	23/03/2022	24/06/2020	26/08/2013	Date

Strongly Correlated Fermionic-Systems Theoretical Quantum Simulation	Optimal 2D Fermionic QFT	Max Hafnian via GBS	BS for Vibronic Spectra (Absorbtion Spectra)	GBS Matrix Point Process	Graph Isomophism/Similarity Evaluation via GBS	Weighted Maximum / Maximum Clique via GBS	GBS Dense k-subgraph	GBS Perfect Matching	Gaussian BS (GBS)	BS	Variational Q. Fidelity Estimation	Variational Q. Fisher- Information Estimation	Swap Test	Hadamard Test	Hilbert-Schmidt Test	k-distinctness	St-connectivity in graphs	Quantum algorithm
Optimal 2D Fermionic	T Primitive	Gaussian BS (GBS)	88	Gaussian BS (GBS)		GBS Dense k-subgraph	GBS Perfect Matching	Gaussian BS (GBS)	Variant of BS	Primitive	Variational Q. State Diagonalization	VQE	Primitive	Primitive	Primitive	Variant of Grover	AA, QPE	n Main subroutine
Dynamical Systems; Fermionic- systems simulation for models that suffer from the sign problem such as the Fermi-Hubbard model, frustrated spin systems, the f-J model, or lattice gauge theories.	Dynamical Systems: Fourier transform	Combinatorics, Optimization: Enumerating max Hafnian of given input	Dynamical Systems: Computing Franck Condon profile of a mocule system (vibronic spectra)	Stochastic Processes & Statistics: Matrix point process	Combinatorics: Graph isomophism/similarity	Combinatorics, Optimization: Weighted maximum / maximum clique	Combinatorics: Dense k-subgraph identification	Combinatorics: Perfect matching	Stochastic Processes & Statistics: Boson sampling	Stochastic Processes & Statistics: Boson sampling	Linear Algebra: Estimating fidelity of two states	Dynamical Systems: Estimating quantum Cramér-Rao bound. Linear Algebra: Comparing two open-system quantum operation	Linear Algebra: Compare two states	Linear Algebra, Stochastic Processes & Statistics: Calculate the expectation value of a unitary projecting on a state.	Linear Algebra: Complex Unitary distance evaluation	Combinatorics: Element distinctness	Combinatorics: Graph connectivity (cycle detection / bipartitieness testing)	Fundamental mathematical problem
First-Principle Quantum Simulation	First-Principle Quantum Simulation	Operational Research: Graph optimization	First-Principle Quantum Simulation. Computing molecular vibronic spectra, including complicated effects such as Duschinsky rotations	Machine Learning & Data Science: Information retrieval	Classical Scientific Computing: Chemical identification. Machine Learning & Data Science: Data mining, Image recognition. Operational Research		Operational Research, Machine Learning & Data Science	Classical Scientific Computing, Operational Research	First-Principle Quantum Simulation	First-Principle Quantum Simulation, Quantum Enablement	Quantum Enablement	First-Principle Quantum Simulation: Simulating quantum sensors and evaluating its reliability	Quantum Enablement	Quantum Enablement	Quantum Enablement	Cryptanalysis, Machine Learning & Data Science: Data-base search. Operational Research	Operational Research: Network analysis, graph coloring, optimization. Classical Scientific Computing: Capacitance of a circuit.	Applications
Proven	Proven	Heuristic	Heuristic	Proven	Proven	Heuristic	Heuristic	Proven	Proven	Proven	Heuristic	Heuristic	Proven	Proven	Proven	Proven	Proven	Heuristic/ Proven
Lso	Lso	Z S O	N S Q	NSQ	N S Q	N S S	NSQ	NISQ	NISQ	NSQ	N S Q	NISQ	NISQ	NISQ	NISQ	LSQ	Lsa	NISQ
Z o	No	Z o	N <sub>o</sub>	No	Z <sub>o</sub>	N <sub>o</sub>	No	No	No	No	Z o	N <sub>o</sub>	No	N <sub>o</sub>	No	Z <sub>o</sub>	Yes	Oracular
Z <sub>0</sub>	N <sub>o</sub>	N <sub>O</sub>	Z	N <sub>o</sub>	N <sub>O</sub>	N <sub>o</sub>	N <sub>O</sub>	N <sub>o</sub>	N <sub>O</sub>	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	No o	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	Yes	Promise
Z	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	N <sub>o</sub>	N <sub>O</sub>	Sampling
S	No	No	Z	N <sub>O</sub>	Z o	S	S	8	S	N <sub>O</sub>	So	No o	8	N <sub>o</sub>	No	No	N <sub>o</sub>	Partial dequanti zation
																		Dequan tization link
Circuit Model	Circuit Model	Linear Optical Quantum Computing	Linear Optical Quantum Computing	Linear Optical Quantum Computing	Linear Optical Quantum Computing	Linear Optical Quantum Computing	Linear Optical Quantum Computing	Linear Optical Quantum Computing	Linear Optical Quantum Computing	Linear Optical Quantum Computing	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Quantum Walks	Quantum Walks	Computational
16	16									1485	<u></u>	(n	23	36	31	1052	N	Google- Scholar citations
[1711.053] 95] Quantum algorithms to simulate many-	162 951 162 951	https://arxiv. 47 org/abs/18	https: //arxiv. org/abs/14 319 12.8427	https: 34 //arxiv.	[1810.106 44] Graph isomorphis m and Gaussian	https: //arxiv. 99 org/abs/19	https://arxiv.	95 //arxiv.	https: 326 //arxiv.	https:	https: //arxiv. 115 org/pdf/19	https: //arxiv. org/pdf/20 10.10488.	https: 239 //arxiv.	https: //arxiv. org/abs/qu ant- ant- ph/051109	311 //arxiv.	https: //arxiv. 52 org/abs/qu	https: //arxiv. org/abs/16 26 10.00581	Link to arXiv s paper
100	https:	https: //journals 18 aps.	https: //www. nature. com/articl 14 es/nphoto	https: //journals.	bb https:  h //www.  his degruyter.  com/docu  n ment/doi/	https://www.	https: //journals. //sans	https: //journals.	https: //journals.			https: //journals. 20 aps. 8. org/prres	https: //epubs.	https://dl. acm. qu org/doi/ab s/10. 1145/113	https: //quantum	https://epubs.	(0)	Link to peer- peer- review ed namer
https: //doi. org/10. 1103/Phy sRevAppli ed.	15/11/2017	28/03/2018	id 29/12/2014	27/06/2019	er. SU 24/10/2018	01/02/2019	28/03/2018	19/12/2017	ls. 19/05/2013	14/11/2010	21/06/2019	S 20/10/2020	25/04/1996	ab 09/11/2005	02/07/2018	01/11/2003	es 03/10/2016	v o

Efficient Q. Algo. for Dissipative Nonlinear DEs	Q. Simulation of Hydrodynamics via Madelung Transformation	Variational Q. Algo. for Poisson Equation	Q. Algo. for Matched Filtering	Q. Many-Body Simulation on Fock Space	Q. Algo. for Generating the Ground State of a Quantum Field Theory	Variational Q. Algo. for Parton Distribution Functions	QITE for Feynman-Kac Partial Differential Equation	Continuous-Variable QITE Formulation of Scalar Q. Field Theory	Q. Algo. of Final State Shower	Q. Simulation of Deuteron Binding Energy	Q. Simulation of Effective Field Theories for Collider Physics	Q. Algo, for Feynman Loop Integrals	Aharonov-Jones-Landau Algo.	QCA for (3+1)D Q. Electrodynamics	Quantum algorithm
QLSA Childs et al.	QPE	VQE	(Approximate) Q. Counting	Primitive	Primitive	VQE	QITE	QITE, Q. Lanczos	Primitive	VQE	QFT	Grover	Hadamard Test	Primitive	Main subroutine
Dynamical Systems: Solving dissipative non-linear equations	Dynamical Systems: Solving hydrodynamical PDEs discretized as quantum lattice gas automata	Dynamical Systems: Solving Poisson equation	Stochastic Processes & Statistics, Combinatorics: Comparing templates to the data and evaluate the matching	Dynamical Systems: Mapping the many-body system onto an unconventional Anderson model on a complex Fock space network of many-	Dynamical Systems: Generating an approximation for the ground state of a quantum field theory	Dynamical Systems: Computing parton distribution functions in QCD	Dynamical Systems: Solving Feymann-Kac equation	Dynamical Systems: Computing Hamiltonian ground states and excited states	Dynamical Systems: Simulating parton showers of high-energy scattering processes	Dynamical Systems: Pionless effective field theory dynamics (deuteron binding-energy computation)	Dynamical Systems: Low-energy effective-field-theory dynamics (soft collinear effective theory)	Dynamical Systems, Combinatorics: Bootstrapping of the causal representation of multiloop Feynman integrals in the loop-tree duality formalism from the identification of all internal configurations that fulfill causality among the 2'n potential solutions. Combinatorics: Identifying directed acyclic graphs	Dynamical Systems: Obtaining an approximation of the Jones polynomial of any braid.	Dynamical Systems: Simulating Q. electrodynamics Hamiltonians, Hamiltonian simulation	Fundamental mathematical problem
Classical Scientific Computing: Computational fluid dynamics, (middle-scale) reaction simulation, epidemiology, numerical modeling	Classical Scientific Computing, First-Principle Quantum Simulation: Solvings hydrodynamical PDEs	Classical Scientific Computing: Variational quantum solver	Classical Scientific Computing: Data analysis (ex: gravitational- wave detection).	First-Principle Quantum Simulation: Simulation of a many-body system in real space.	First-Principle Quantum Simulation: Simulating a massive scalar bosonic QFT on a quantum computer	First-Principle Quantum Simulation: Quantum-chromodynamics computation	Classical Scientific Computing: Heat-physics simulation, quantum- system simulation. Operational Research: Finance (option pricing with stochastic volatility)	First-Principle Quantum Simulation: Scalar QFT simulation over 1D lattice	First-Principle Quantum Simulation: High-energy physics (high-energy scattering amplitudes)	First-Principle Quantum Simulation: Nuclear-physics simulation	First-Principle Quantum Simulation: Hadronic jet simulation	Classical Scientific Computing: High-energy physics, Perturbative QFT, Evaluation of quantum fluctuations at higher orders in the perturbative expansion by means of multiloop Feynman diagrams	First-Principle Quantum Simulation: Quantum simulation of topological quantum field theory	First-Principle Quantum Simulation: (3+1)D Q. electrodynamics simulation	Applications
Proven	Proven	Heuristic	Proven	Proven	Proven	Heuristic	Heuristic	Heuristic	Proven	Heuristic	Proven	Proven	Proven	Proven	Heuristic/ Proven
Lsa	N S S O	NISQ	LSQ	N S Q	LSQ	NISQ	NISQ	LSQ	NISQ	NISQ	NISQ	LSQ	LSQ	LSQ	NISQ/
Z <sub>o</sub>	No	No	Yes	Zo	Z o	N <sub>O</sub>	Z	N <sub>o</sub>	Z o	Z o	No	Z	N <sub>o</sub>	Z o	Oracular
Yes, nonlinearity ratio R<1	No o	N <sub>o</sub>	N <sub>o</sub>	No	Z <sub>0</sub>	Z	No	N <sub>O</sub>	N <sub>o</sub>	N <sub>o</sub>	R	Yes, only for complex topology whose number of states fulfilling causal-compatible conditions is much smaller than the total combinations of	No	N <sub>O</sub>	Promise
No	N <sub>O</sub>	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	Yes	Yes	N <sub>O</sub>	Yes	Yes	N <sub>O</sub>	No No	No	No	Sampling
Primitive is	Primitive is	No	No	No	No	No	8	No	No o	No o	N <sub>o</sub>	8	N <sub>o</sub>	N <sub>o</sub>	Partial Dequadequanti tizat
Ω	δÖ	Ω	Ω	Ω	≱ Q	Ω	Ω	₽ ⊑:	Ω	Ω	Ω	Ω	Ω	₽Q	ion
Circuit Model	Circuit Model, Quantum Walks	Circuit Model	Circuit Model	Circuit Model	Quantum Random- Access Memory	Circuit Model	Circuit Model	Linear Optical Quantum Computing	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Quantum Cellular Automata	Computational model
126		60			10	43	20	22	11	394	44	20	358		Google- Scholar citations
https: //arxiv. org/abs/20 6 11.03185	https: //arxiv. org/pdf/17 09.10417. pdf	https:	https: //arxiv. org/abs/21 9 09.01535	https://arxiv. //arxiv. org/pdf/22 4 11.05803.	https: //arxiv. org/abs/21 0 10.05708	https: //arxiv. org/pdf/20 11.13934.	https: //arxiv. org/abs/21 0 08.10846	https://arxiv.	https: //arxiv. org/abs/19 04.03196	https: //arxiv. org/pdf/18 01.03897.	https: //arxiv. 4 org/abs/21	https: //arxiv. org/abs/21 0 05.08703	https: //arxiv. 358 org/abs/qu	https://arxiv.	Link to arXiv paper
https: 0 //doi. 0 org/10.	https: //journals. // aps. org/pra/a bstract/10	https://doi.			https: 1 //doi. org/10.	https:  //doi. org/10.	https: //doi. org/10. 22331/q- 1 2022-06- 07-730	https:	l@	https: 3 //doi. org/10.	https:	https: //doi. org/10. 1007/JHE P05% 1 282022%	https: //link. u springer.		peer- review ed
06/11/2020	11/05/2023	13/12/2020	03/09/2021	10/11/2022	12/10/2021	27/11/2020	24/08/2021	02/07/2021	05/04/2019	11/01/2018	09/02/2021	18/05/2021	09/11/2005	06/05/2022	Date

QW for Fermions in Curved Spacetime	QW for Non-Abelian Gauge Theory	Dirac-Equation QW	Q. Algo. for Alchemical Optimization	Variational Protein-Folding Simulation	Adiabatic Q. Algo. for Artificial Graphene	Q. Algo. for Density Functional Theory (DFT)	Q. Algo. for Full Configuration Interaction (FCI)	Variational Q. Algos, for (Many-Body) Green's Funtions and Spectrum Functions	Variational Q. Algos. for Discovering Hamiltonian Spectra (Excited States)	Subspace-Search VQE	Bayesian QPE	Iterative QPE	Q. Hartree-Fock Variational Method	Q. Algo. for Navier-Stokes Equation	Kacewicz' Q. Ordinary- Differential-Equation Algo.	Quantum algorithm
Variant of Dirac- Equation QW	Variant of Dirac- equation QW	Primitive	VQE	VQE	QAOA	HHL, VQE	Bayesian QPE	Subspace-Search VQE	QITE, Swap Test	VQE	Primitive	Hadamard Test	VQE	Kacewicz' Q. Ordinary- Differential-Equation Algo.	QAE	Main subroutine
Dynamical Systems: Dynamical evolution of fermions in curved spacetime	Dynamical Systems: Dynamical evolution of a non-Abelian gauge theory	Dynamical Systems: Dynamical evolution of the Dirac and Weyl equations	Dynamical Systems: Finding the eigenspectrum of a quantum system's Hamiltonian	Dynamical Systems: Simulating a coarse-grained model for protein folding	Dynamical Systems: Fermi-Hubbard-model simulation	Dynamical Systems: Density functional theory	Dynamical Systems: Computes the ground-state energy of a many-body quantum eletronic system	Dynamical Systems: Computes the Green's function of a given Hamiltonian	Dynamical Systems: Computes excited states of a given Hamiltonian	Dynamical Systems: Computes excited states of a given Hamiltonian	Linear Algebra: Estimation of the eigenvalue of an eigenvector of a unitary operator. Dynamical Systems: Finding the spectrum of a Hamiltonian.	Dynamical Systems: Finding the eigenvalue of a unitary operator	Dynamical Systems: Solving the time- independent Schrödinger equation	Dynamical Systems: Solving Navier-Stokes equation	Dynamical Systems: Solving ordinary differential equation in an optimal way	Fundamental mathematical problem
First-Principle Quantum Simulation: Quantum simulating fermions in curved spacetime	First-Principle Quantum Simulation: Quantum simulating non-Abelian gauge theories	First-Principle Quantum Simulation: Quantum simulating the Dirac and the Weyl equations	First-Principle Quantum Simulation: Design of molecular systems that best fit in a given external potential. Material design, drug design.	First-Principle Quantum Simulation:	First-Principle Quantum Simulation: Material-structure simulation	First-Principle Quantum Simulation: Simulation of many-body fermions systems	First-Principle Quantum Simulation: Simulation of many-body fermions systems	First-Principle Quantum Simulation: Computes the Green's funtion and spectrum function of a quantum system	First-Principle Quantum Simulation: Finding the excited states of molecular systems	First-Principle Quantum Simulation: Computing excited states of a Hamiltonian	First-Principle Quantum Simulation	First-Principle Quantum Simulation: Quantum-chemistry simulation of particle spectrums	First-Principle Quantum Simulation: Quantum chemistry simulation using Hartree-Fock method.	Classical Scientific Computing: Computational mechanics	Classical Scientific Computing: Solving ODE	Applications
Proven	Proven	Proven	Heuristic	Heuristic	Proven	Heuristic	Heuristic	Heuristic	Heuristic	Heuristic	Heuristic	Proven	Heuristic	Proven	Proven	Heuristic/ Proven
LSQ	LSQ	LSQ	NISQ	NISO	LSQ	LSQ	NISQ	NISQ	NISQ	NISQ	N S S O	NISQ	N S Q	LSQ	LSQ	NISQ/
Z o	Z <sub>o</sub>	Z o	Z o	N <sub>O</sub>	N <sub>o</sub>	Z o	Z o	No	Z o	N <sub>o</sub>	N <sub>o</sub>	Z o	N <sub>o</sub>	Z o	Yes: driven function operator for f	Oracular
N <sub>O</sub>	N <sub>o</sub>	N <sub>o</sub>	No	Yes: only for branched heteropolymers comprised of N monomers on a tetrahedral (or "diamond")	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	Z	N <sub>o</sub>	N <sub>o</sub>	No	N <sub>o</sub>	No	N <sub>o</sub>	N <sub>O</sub>	Promise
N <sub>O</sub>	No	No	No.	8	No	N <sub>O</sub>	No	No	N <sub>O</sub>	No	No o	No	No	No	No	Sampling
N <sub>o</sub>	No	No	8	Ö	No	No	No	N <sub>o</sub>	No	No	No o	No	N <sub>o</sub>	No	No	Partial dequanti zation
8	No	No														Dequan tization link
Quantum Walks	Quantum Walks	Quantum Walks	Circuit Model	Circuit Model	Adiabatic Quantum Computation, Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model	Circuit Model, Quantum Walks	Purely mathematical	Computational model
40	69	262	22	72				7	220	250		179	527	31 (arXiv + peer-reviewed)	60	Google- Scholar citations
https: //arxiv. org/abs/16	https: //arxiv. 9 org/abs/16	https: //arxiv. 2 org/abs/he	https: //arxiv. org/abs/20 2 08.06449	https: //arxiv. org/abs/19 2 08.02163	[2204.030 13] 5 Adiabatic	https://arxiv.	No Arxiv 11 paper	[1909.122 50] Calculatio n of the		1810.0943 4.pdf 0 (arxiv.org)	2303.0151 7.pdf 0 (arxiv.org)		[2004.041 74] Hartree- Fock on a supercond		https://arxiv. o org/abs/qu	Link to arXiv paper
https://www.6 rintonpres	https://journals		https://pubs. //pubs. //sc. org/en/co ntent/artic	https: //www. nature. com/articl es/s4153 9 4-021- 00368-4		https://scipost.	https: //pubs. acs. org/doi/od	2 https: //journals. 2 aps. org/prres	Z https: //journals. al aps.	3 https: //journals.	Not published		1 https://doi. a org/10. d 1126/scie	https://www.		peer- review ed
01/08/2017	19/06/2016	19/04/1993	16/10/2020	06/08/2019	06/04/2022	04/04/2022	08/11/2021	S. 26/09/2019	14/06/2018	22/10/2018	oz/03/2023	25/10/2006	08/04/2020	05/03/2021	o6/10/2005	v Date

QWs Meet LGT	Quantum algorithm
Variant of Dirac- Equation QW	Rundame   Fundame   Fundame   Fundame   Problem
First-Principle Quantum Simulation Dynamical Systems: Unitary dynamical Quantum simulating fermions on evolution of fermions on lattices	Fundamental mathematical problem
First-Principle Quantum Simulation:  Quantum simulating fermions on lattices	Applications
Proven	Heuristic/ NISQ Proven LSQ
LSQ	NISQ/
N <sub>o</sub>	NISQ/ LSQ Oracular Promi
N <sub>O</sub>	Promise
N <sub>O</sub>	Sampling
8	Partial dequant zation
N <sub>o</sub>	Partial Dequan dequanti tization C
Quantum Walks	computationa nodel
_	Google- I Scholar citations
https://arxiv. org/abs/22	Link to arXiv paper
https://iopscien	Link to peer- review ed
28/12/2022	Date