Name : ..............................................

Roll No. : ..............................................

Invigilator's Signature : ..............................................

**CS/B.TECH (NEW)/SEM-1/M-101/2013-14**

**2013**

# MATHEMATICS – I

*Time Allotted* : 3 Hours  Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*



## GROUP – A

### ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) The value of the determinant

$$\begin{vmatrix} 100 & 101 & 102 \\ 105 & 106 & 107 \\ 110 & 111 & 112 \end{vmatrix} \text{ is}$$

a) 0  b) 10

c) 100  d) 1000.

ii) The equation $x + y + z = 0$ has

a) infinite number of solutions

b) no solution

c) unique solution

d) two solutions.

iii) The value of $\int_1^0 \int_0^1 (x+y)\,dx\,dy =$

a) 2 b) 3

c) 1 d) 0.

iv) $f(x, y) = \dfrac{\sqrt{y} + \sqrt{x}}{y + x}$ is a homogeneous function of degree

a) $\dfrac{1}{2}$ b) $\dfrac{-1}{2}$

c) 1 d) 2.

v) In the MVT $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$, if $f(x) = \dfrac{1}{1+x}$ and $h = 3$, then the value of $\theta$ is

a) 1 b) $\dfrac{1}{3}$

c) $\dfrac{1}{\sqrt{2}}$ d) none of these.

vi) If $y = e^{ax+b}$ then $(y_5)_0 =$

a) $ae^b$ b) $a^5 e^b$

c) $a^b e^{ax}$ d) none of these.





vii) The series $\sum \dfrac{1}{(2n+1)^n}$ is

a) convergent b) divergent

c) oscillatory d) none of these.

viii) $\int_0^{\frac{\pi}{2}} \cos^6 x\,dx$ is equal to

a) $\dfrac{7\pi}{12}$ b) $\dfrac{5\pi}{32}$

c) $\dfrac{\pi}{32}$ d) $\dfrac{3\pi}{16}$.

ix) If $[\vec{a}\ \vec{b}\ \vec{c}] = 0$ then the vectors $\vec{a}\ \vec{b}\ \vec{c}$ are

a) colinear b) coplanar

c) orthogonal d) none of these.

x) If $u(x, y) = \tan^{-1}\left(\dfrac{y}{x}\right)$, then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is

a) 0 b) $2u(x, y)$

c) $u(x, y)$ d) none of these.

xi) The centre of the sphere given by the equation

$a\left(x^2+y^2+z^2\right)+2bx+2cy+2dz+e=0$ is

a) $\left(\frac{-b}{a}, \frac{-c}{a}, \frac{-d}{a}\right)$

b) $(-b, -c, -d)$

c) $\left(\frac{-b}{2a}, \frac{-c}{2a}, \frac{-d}{2a}\right)$

d) $\left(\frac{b}{2a}, \frac{c}{2a}, \frac{d}{2a}\right)$.

## GROUP – B

**( Short Answer Type Questions )**

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that every square matrix can be expressed as the sum of a symmetric matrix and a skew symmetric matrix.

3. Show that

$$\vec{f} = \left(6xy + z^2\right)\hat{i} + \left(3x^2 - z\right)\hat{j} + \left(3xz^2 - y\right)\hat{k}$$

is irrotational. Hence find a scalar function $\varphi$ such that

$$\vec{f} = \vec{\nabla}\varphi .$$

4. Using Mean Value Theorem prove that

$$x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}, \quad 0 < x < 1.$$

5. Show that the area bounded by a simple closed curve $C$ is given by $\frac{1}{2}\oint_C (x\,dy - y\,dx)$.





6. Prove that the function

$$f(x, y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$$

has neither maxima for minima at the origin.

## GROUP – C

**( Long Answer Type Questions )**

Answer any *three* of the following. $3 \times 15 = 45$

7. a) If $f = |\vec{r}|$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, prove that $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$.

b) Prove that

$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2 .$$

c) If $y = \cos\left(m \sin^{-1} x\right)$ then prove that

$$\left(1-x^2\right)y_{n+2} - (2n+1)\,xy_{n+1} + \left(m^2 - n^2\right)y_n = 0.$$

5 + 5 + 5

8. a) If the vector function $\vec{F}$ and $\vec{G}$ are irrotational, prove that $\vec{F} \times \vec{G}$ is solenoidal.

b) If $f(x, y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, verify that $f_{xy} = f_{yx}$.

c) Find the maxima and minima of the function $x^3 + y^3 - 3x + 12y + 20$. Also find the saddle point.

5 + 5 + 5

9. a) Evaluate $\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}$ by Laplace expansion method.

b) Verify Green's theorem for

$\oint_C \left[ \left( 3x - 8y^2 \right) dx + \left( 4y - 6x \right) dy \right]$ where $C$ is region bounded by $x = 0$, $y = 0$ and $x + y = 1$.

c) For what values of $\lambda$ and $\mu$ the system of equations

$x + y + z = 6$

$x + 2y + 3z = 10$, has (i) Unique solution, (ii) No solution, (iii) Infinite solutions. $x + 2y + \lambda z = \mu$.

5 + 5 + 5

10. a) If $u_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$, then prove that

$n\left( u_{n+1} + u_{n-1} \right) = 1$.

b) Prove that

( if $0 < a < b$ ), $\frac{(b-a)}{\left(1+b^2\right)} < \tan^{-1} b - \tan^{-1} a < \frac{(b-a)}{\left(1+a^2\right)}$.

Hence show that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

c) Test the convergence of the series

$\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \ldots$ 5 + 5 + 5





11. a) State Leibnitz's theorem for convergence of a series. Hence test the convergence of the following series :

$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots\ldots$

b) If $z = f(x, y)$ where $x = e^u \cos v$ and $y = e^v \sin v$, show that $y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}$.

c) Evaluate

$\int_a^0 \int_0^x \int_0^{x+y} e^{x+y+z} \, dx \, dy \, dz$. 5 + 5 + 5