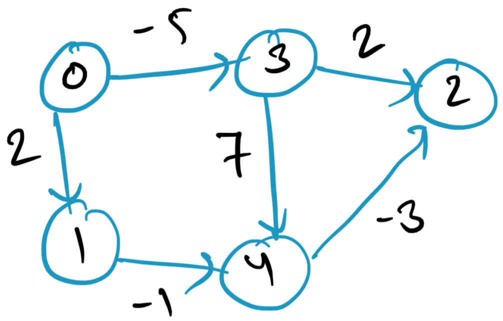
Graphs - V &
Greedy Algorithm - I

Negative weight cycle in a Graph



Shortest path Algorithms

Dijkstra's

-ve weight ✗

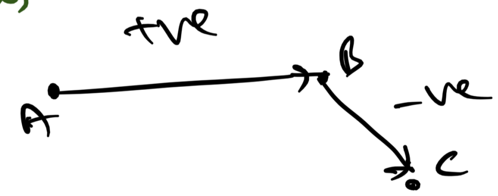
O(E log V)

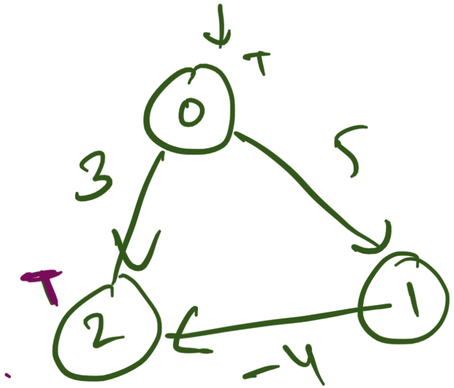
Bellman ford

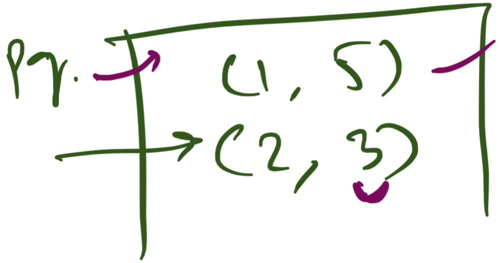
-ve weight edges ✓

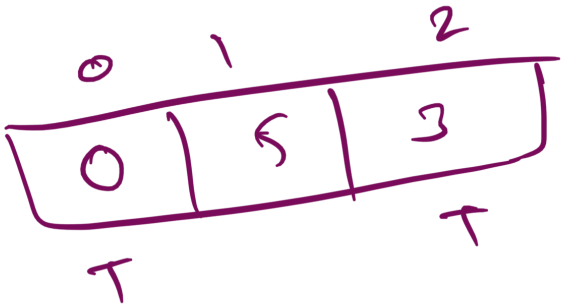
→ O(E.V)

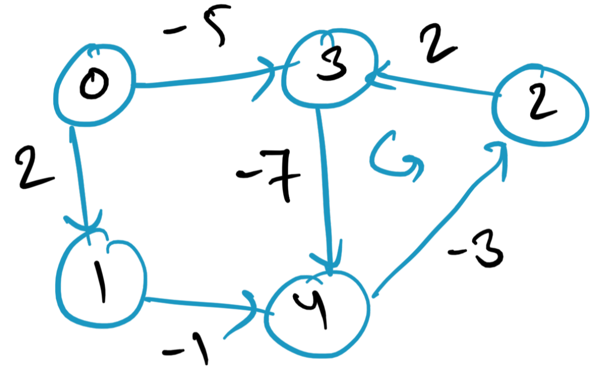
-ve cycles ✗











$7 + 2 + (-3)$

$= 6$ <u>+ve</u>

$-7 + 2 + -3$

$= -8$ <u>-ve</u>

Bellman Ford Algorithm

V vertices

V → Repeat (V-1) times: {

E [for every edge ($u \xrightarrow{wt} v$): {

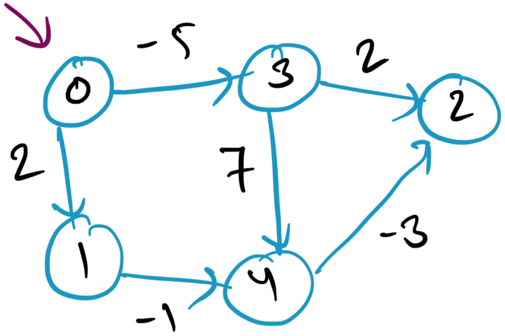
if (dis[u] + wt < dis[v]) {

dis[v] = dis[u] + wt

}

}

}



dis

0	1	2	3	4
0	∞	∞	∞	∞

$V = 5$

	0	1	2	3	4
dis[] =	0	∞	∞	∞	∞

$(V-1)$

1st →	0	2	-3	-5	-13
2nd →	0	2	-16	-5	-13
3rd	0	2	-16	-5	-13
4th →	0	2	-16	-5	-13

Shortest distance ?

$-3 > -13 - 3$

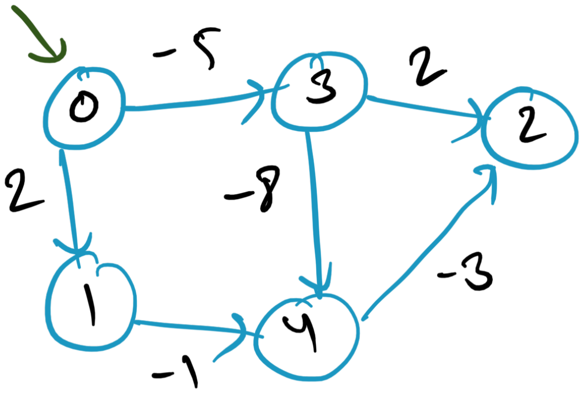
$-5 > 0 + -5$

$-16 > -5 + 2$

$-13 > -5 + (-8)$

$-13 > 2 + (-1)$

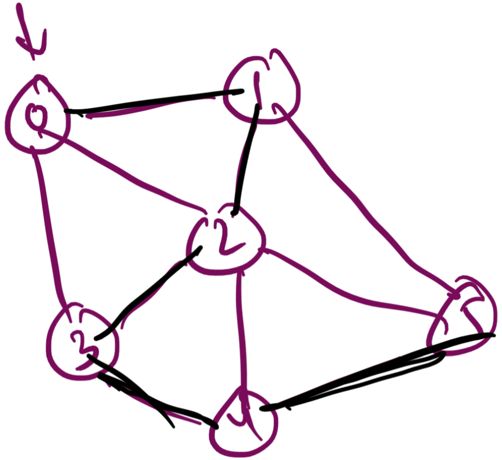
$2 > 0 + 2$

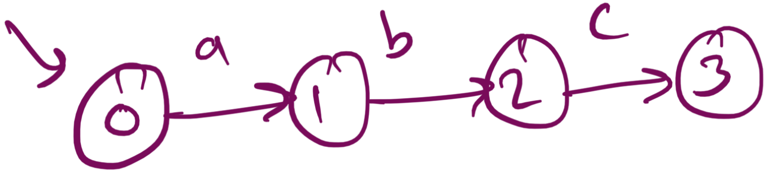


Edges:

4 → 2		-3
0 → 3		-5
3 → 2		2
→ 3 → 4		-8
1 → 4		-1
0 → 1		2

Proof: why $(v-1)$ iterations required?





$2 \to 3$

$1 \to 2$

$0 \to 1$

0	1	2	3
0	∞	∞	∞

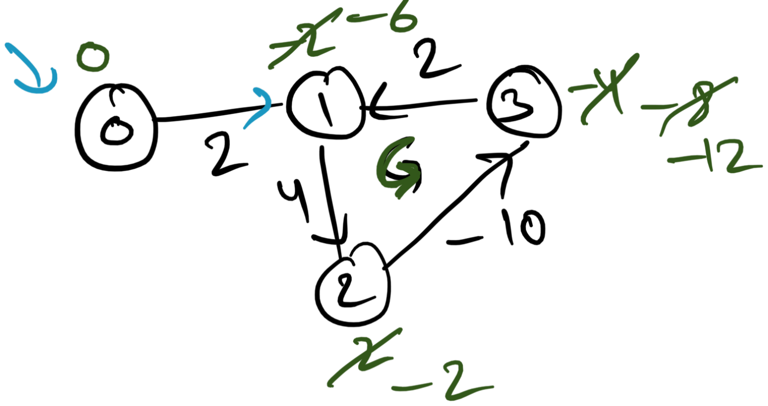
1st	0	a	∞	∞
2nd	0	a	$a+b$	∞
3rd	0	a	$a+b$	$a+b+c$

$\infty > \infty + b$

$\infty > 0 + a$

	0	1	2	3
1	0	2	6	∞
2	0	2	6	-4
3	0	-2	2	-4
+1				
4	0			-8

$-4 \quad 2 + (-10)$



$3 \to 1$

$2 \to 3$

$0 \to 1$

$1 \to 2$

Introduction to Greedy Algorithm

10 5 2

Coin change problem

(10, 8, 1)

16

6

1 + 6 → 7

10 × 2		20
5 × 1		5
2 × 1		2
4		27

greedy

D.P.

Fractional Knapsack Problem

Remaining

20

0

Value → 500

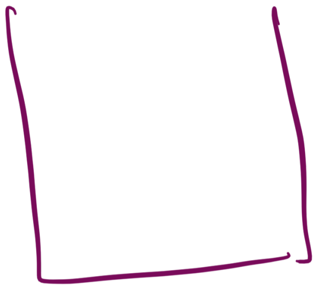
200

700

$\frac{500}{50} * W$

$\left(\frac{\text{Value}}{\text{wt}} \text{ ratio} \right)$

Maximum value



Bag.

Capacity → W = 30

	0	1	2	3
wt.	10	5	20	50
value	500	40	10	500
$\frac{\text{value}}{\text{wt}}$	50	8	0.5	10

$$\frac{(i_2 \cdot v)}{(i_2 \cdot wt)} - \frac{(i_1 \cdot v)}{(i_1 \cdot wt)} \geq 0$$

$$(i_1 \cdot wt) * (i_2 \cdot v) - \underline{(i_2 \cdot wt) * (i_1 \cdot v)} \geq 0$$

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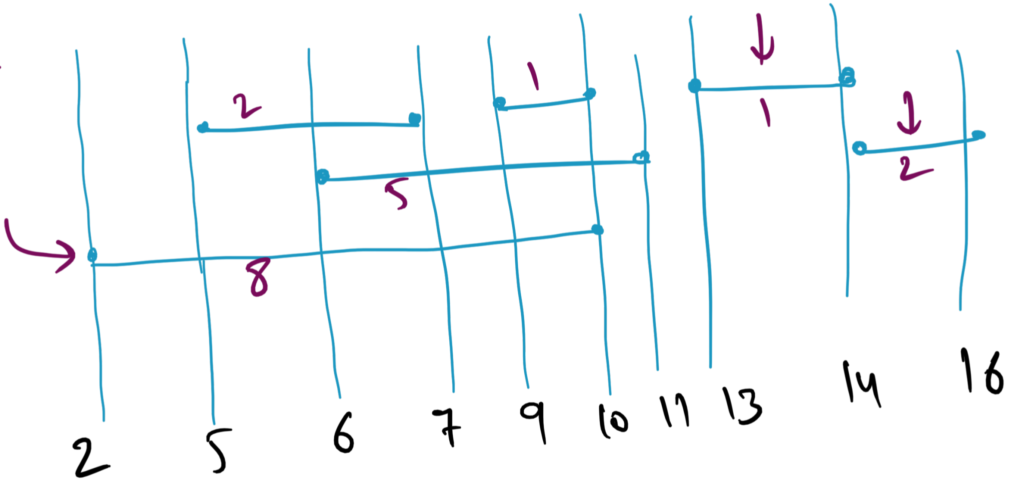
Activity Selection Problem

$O(N^2)$



(9, 10)
(13, 14)
(14, 16)
(2, 10)
(5, 7)
(6, 11)

Machine
↳ one at a time.

Pick maximum number of activities such that no two overlap.

(9, 10)
(13, 14)
(14, 16)
(2, 10)
(5, 7)
(6, 11)

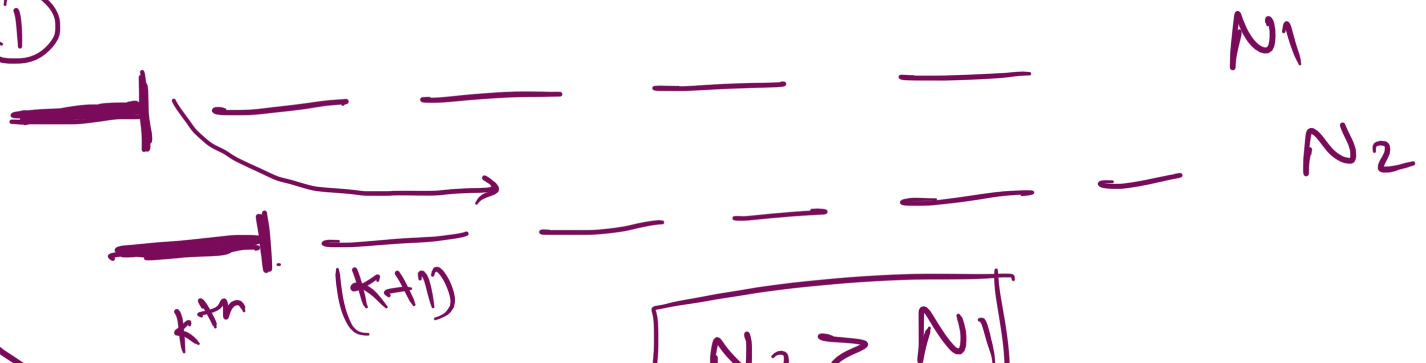
((5, 7))
(9, 10)
(2, 10)
(6, 11)
(13, 14)
(14, 16)

(5, 7)
(9, 10)
(13, 14)
(14, 16)

cur

Proof

1



$$\boxed{N_2 > N_1}$$

$$N_2 - 1 + 1 = N_2$$

Practice Problems

1. Wine selling Problem
2. Minimum product subset of an array
3. Maximum sum of absolute difference of any permutation