

Disjoint Sets



Algorithms by
Robert Sedgewick

Introduction to Disjoint Sets

makeTeam(a, b)

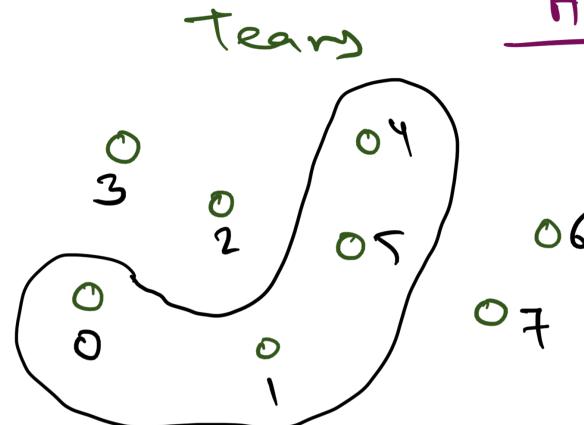
makeTeam(0, 1) \rightarrow void

areInTeam(2, 6) \rightarrow boolean

makeTeam(4, 5)

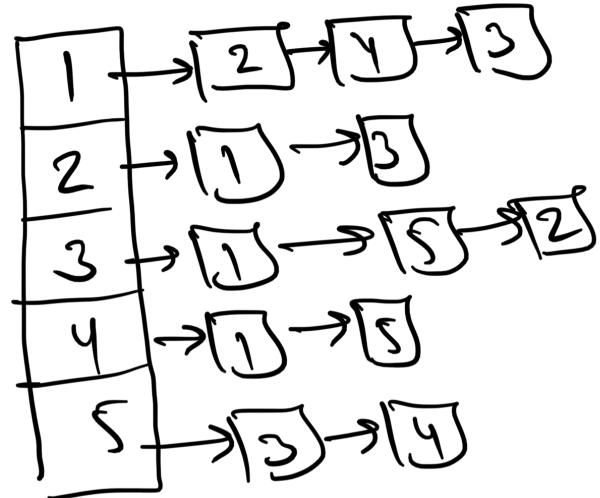
makeTeam(0, 5)

areInTeam(1, 4) \rightarrow True

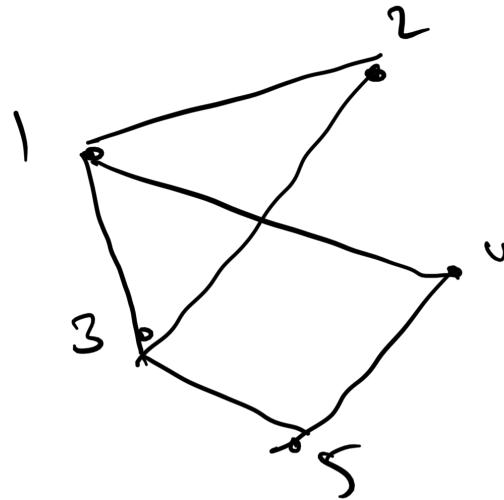


[union
find]

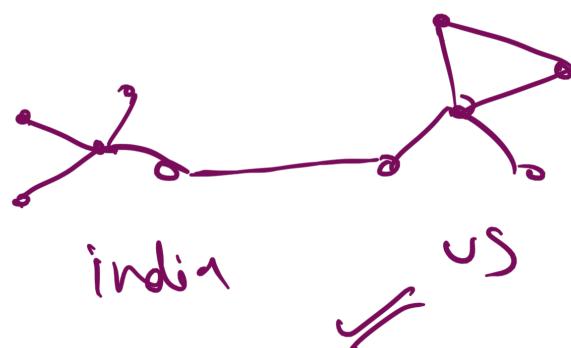
Adjacency List



Array of linked lists



facebook



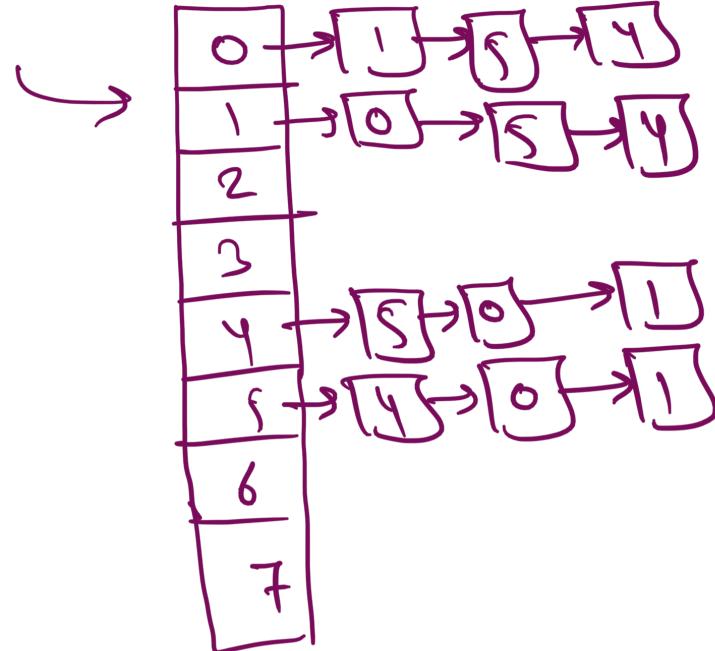
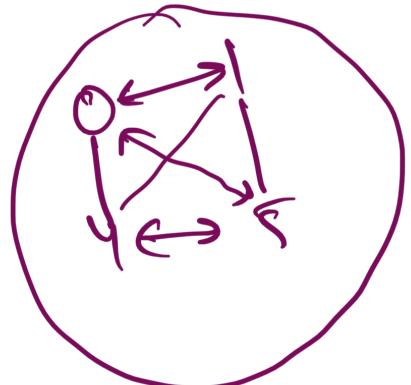
✓ makeTeam(0, 1) → void

areInTeam(2, 6) →

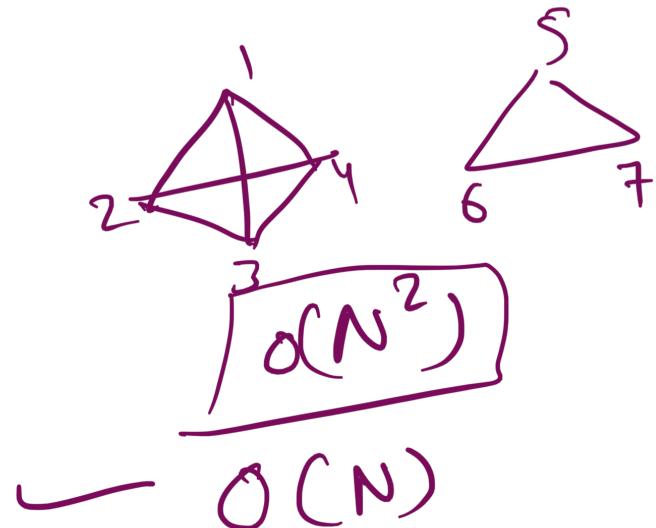
makeTeam(4, 5)

makeTeam(0, 5)

areInTeam(1, 4) → True



Space
Time.



$mT \rightarrow \text{makeTeam}$
 $aIT \rightarrow \text{areIn Team}$

Implementation of Union - Find

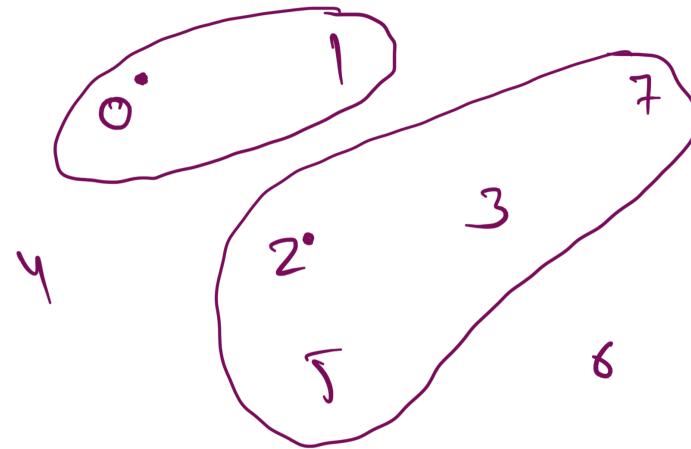
$mT(2, 5)$

$mT(3, 7)$

$mT(5, 7)$

$\rightarrow aIT(3, 5) \rightarrow$
 $\downarrow \quad \downarrow$ True
 2 2

$mT(0, 1)$



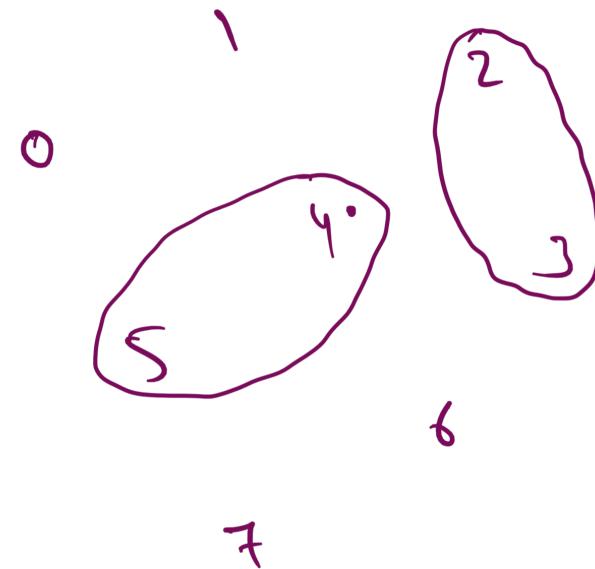
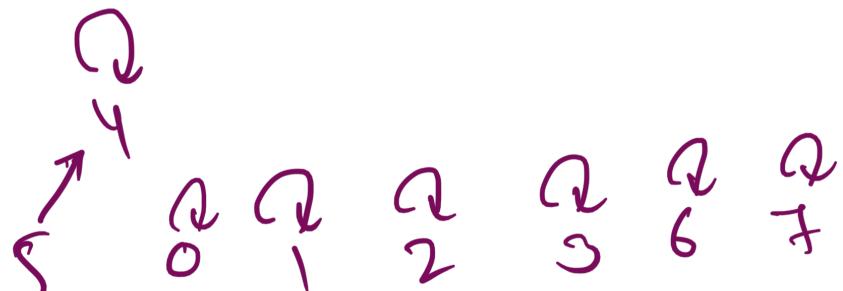
$aIT(1, 7) \rightarrow \text{false}$
 $\downarrow \quad \downarrow$
 0 2

✓

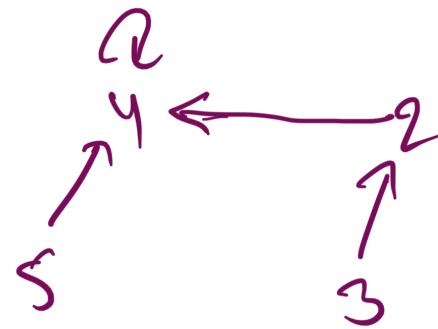
$\text{parent}[] =$	0	1	4	2	4	4	6	7
	0	1	2	3	4	5	6	7

$mT(4, 5)$

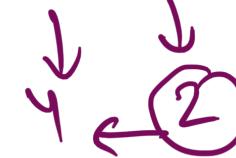
$\text{parent}[5] = 4$



$mT(2, 3)$



$mT(5, 2)$



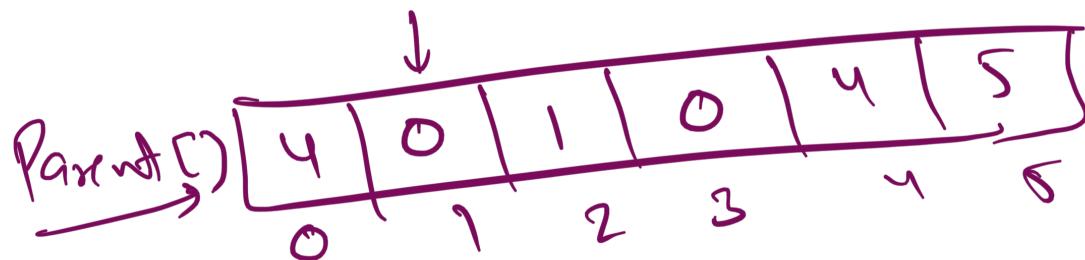
Union - find Operations



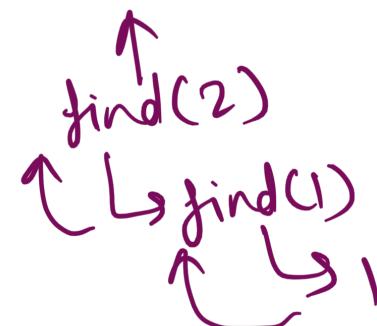
To make a team with two elements



To find the captain / root element of disjoint set tree.



$$n = 6$$



union(0, 3)

0 ← 3

union(1, 2)

1 ← 2

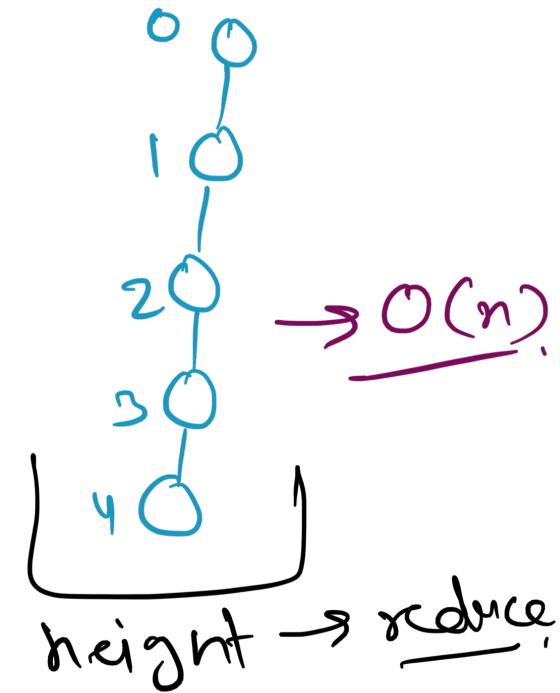
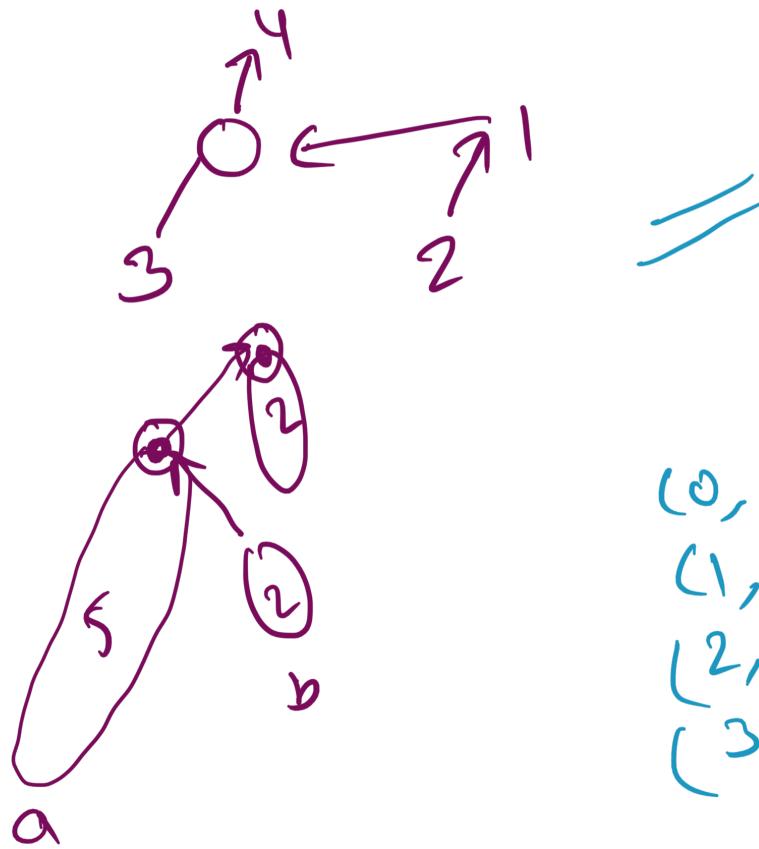
union(0, 2)

0 ← 1

$\text{union}(4, 2)$

$4 \leftarrow 0$

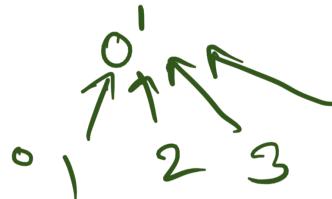
$\text{find}(2)$
↳ $\text{find}(1)$
↳ $\text{find}(0)$
↳ 0



Union - Find with Rank

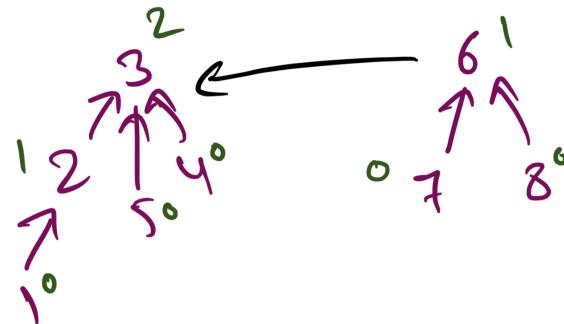


[
 (0, 1)
 (1, 2)
 (2, 3)
 (3, 4)



(3, 4)

Rank.



1

\downarrow	\downarrow
2	0
1	0
2	0
3	0
4	0
5	

$\frac{\text{rank}}{2}$

$(0, 1)$ $(2, 3)$

$0 \leftarrow 1$ $2 \leftarrow 3$

parent \rightarrow

0	0	0	2	2	5
0	1	2	3	4	5

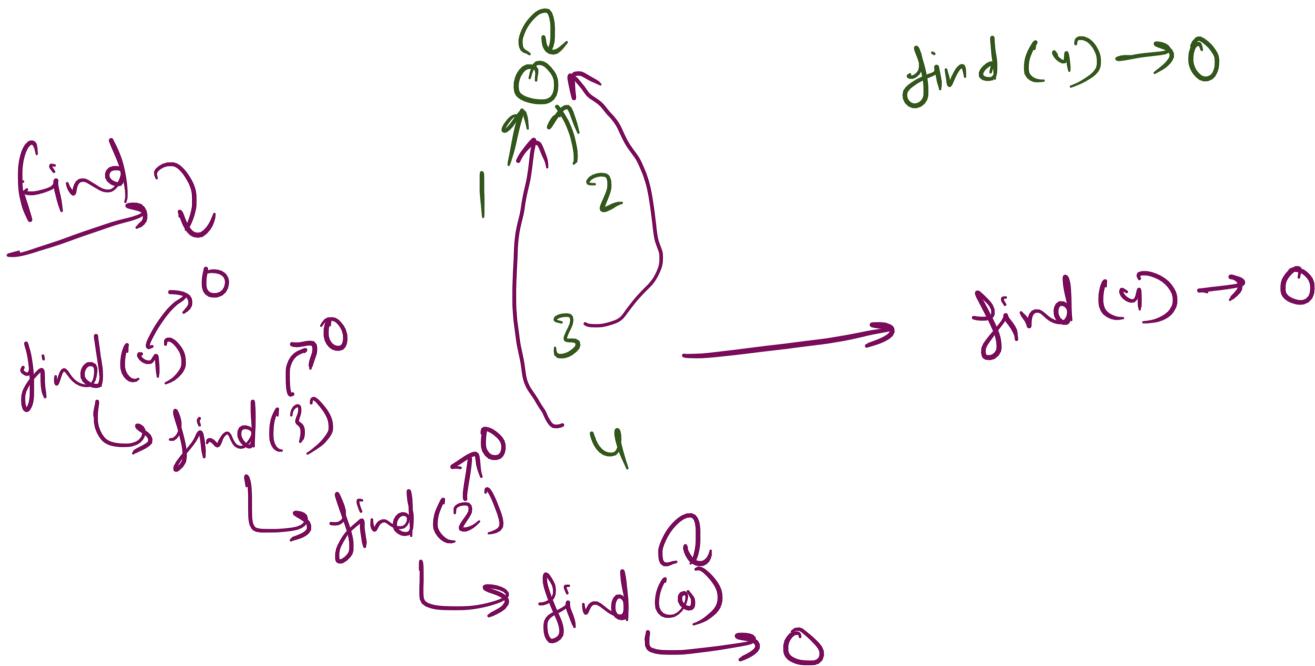
$^2 0 \leftarrow ^1 2$
 $^0 1 \leftarrow ^0 3$
 $^4 0 \leftarrow ^2 4$

$(3, 4)$
 $2 \leftarrow 4$

$(0, 4)$
 $0 \leftarrow 2$

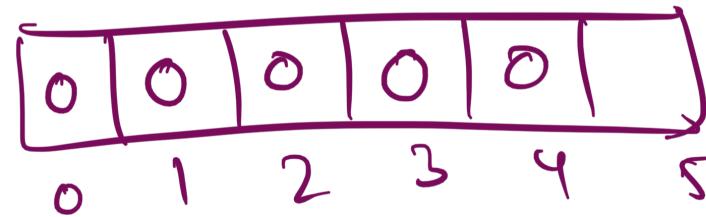
\uparrow

Union - Find with Path Compression





$O \curvearrowleft \text{find}(4)$
 $\hookrightarrow \text{parent}[4] = \text{find}(3)$
 $O \curvearrowleft \text{parent}[3] = \text{find}(2)$
 $O \uparrow \curvearrowleft \text{parent}[2] = \text{find}(0)$
 $\downarrow \uparrow O$



$m \rightarrow$ operations
union | find

$$O(m \times n) \rightsquigarrow O(m \times 4)$$