

# Graphs - IV



## Shortest Path in a Directed Acyclic Graph

Topological sorting

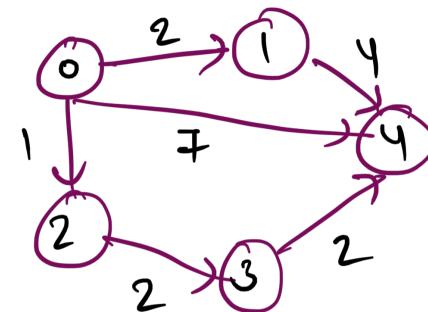


src = 0

dist[] = [ 0 2 1 3 5 ]

0	2	1	3	5
0	∞	1	3	5

u → v



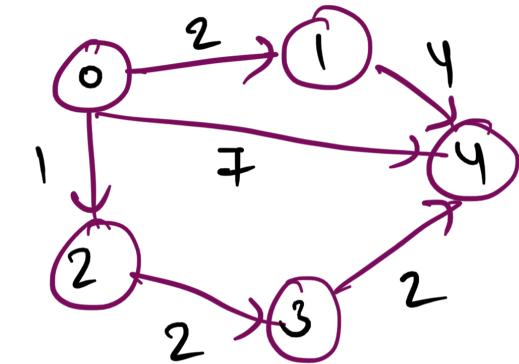
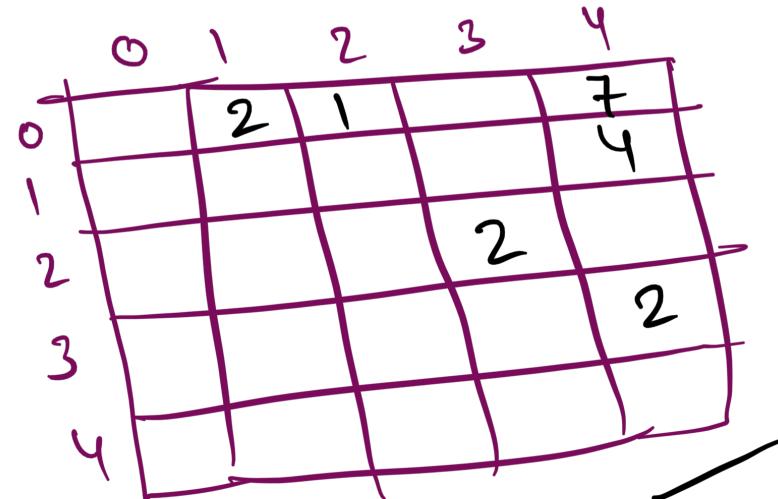
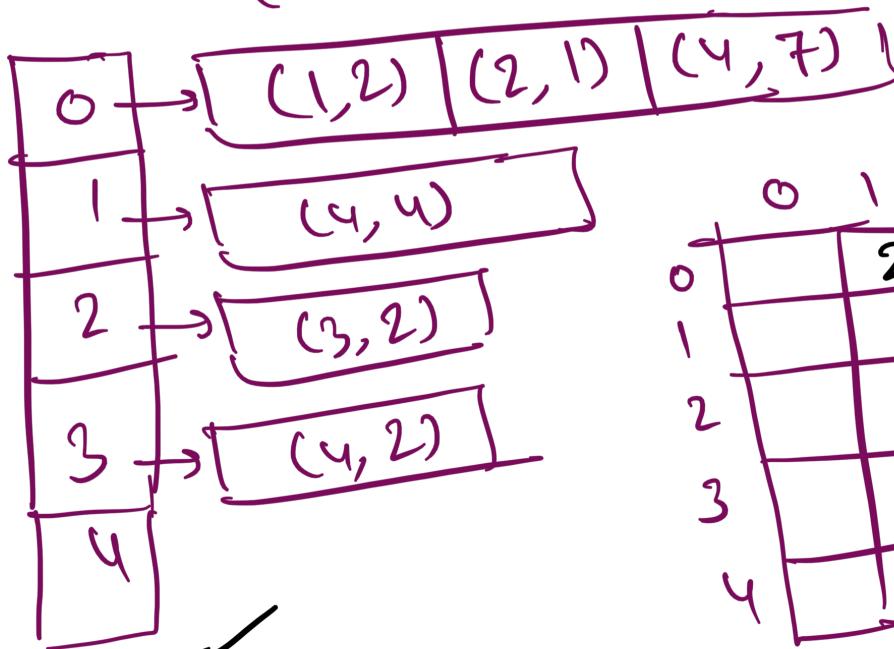
$$\begin{array}{l|l} \infty > 0 + 2 & \infty > 1 + 2 \\ \infty > 0 + 1 & 6 > 3 + 2 \\ \infty > 0 + 7 & \\ 7 > 2 + 4 & \end{array}$$

$O(V+E)$

for every neighbor ( $v$ ) of  $u$ :  
if ( $dist[v] > dist[u] + wt(u,v)$ )  
 $dist[v] = dist[u] + wt(u,v);$

}.

$(v, wt)$



class Pair {

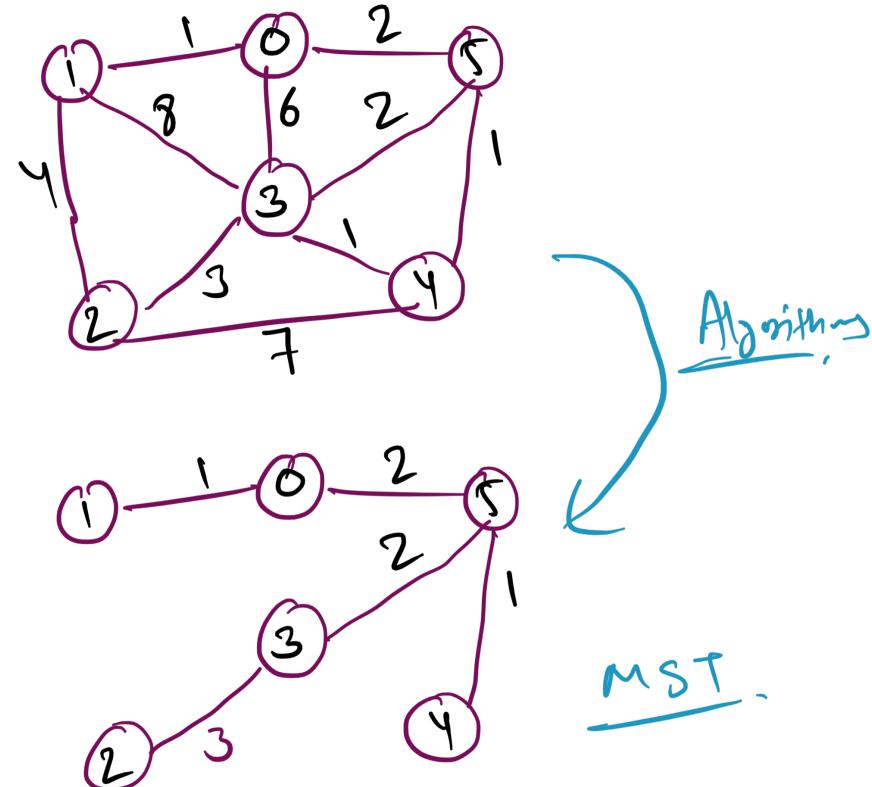
int v  
int wt

}

## Minimum Spanning Tree

# Minimum edge sum graph such that each vertex can be visited by another vertex.

V vertex  
MST  $\rightarrow$   $(V-1)$

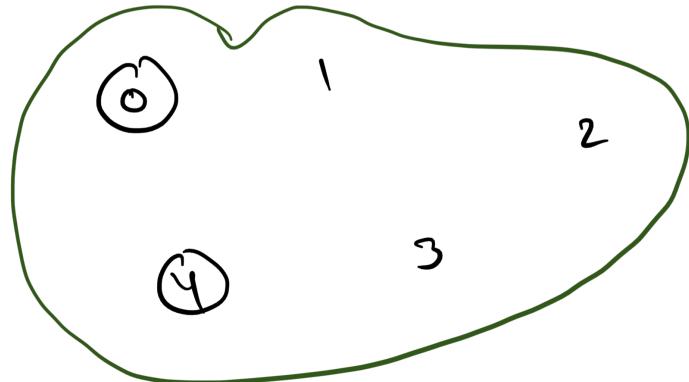


Kruskal

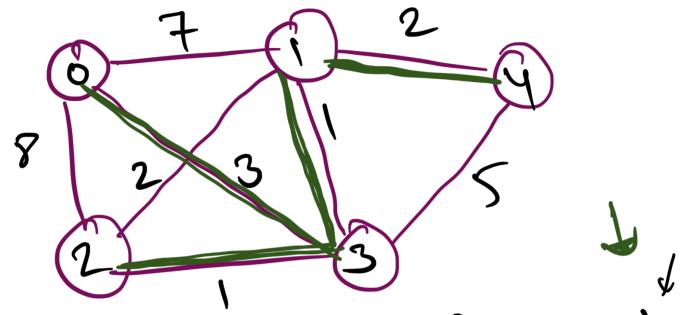
## Greedy Algorithm for MST

Greedy Algorithm

1. Sort the edges by their weights.



$$N = 5 \\ (V-1)$$



$\checkmark (1,3) \rightarrow 1$   
 $\checkmark (2,3) \rightarrow 1$   
 $\times (1,2) \rightarrow 2$   
 $\checkmark (1,4) \rightarrow 2$   
 $\checkmark (0,3) \rightarrow 3$   
 $(3,4) \rightarrow 5$   
 $(0,1) \rightarrow 7$   
 $(0,2) \rightarrow 8$

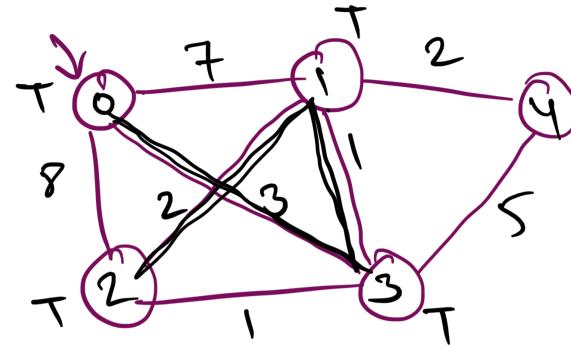
$$E \log E + E$$

## Prim's Algorithm for MST

MinHeap → Pair (wt).

Pq. →

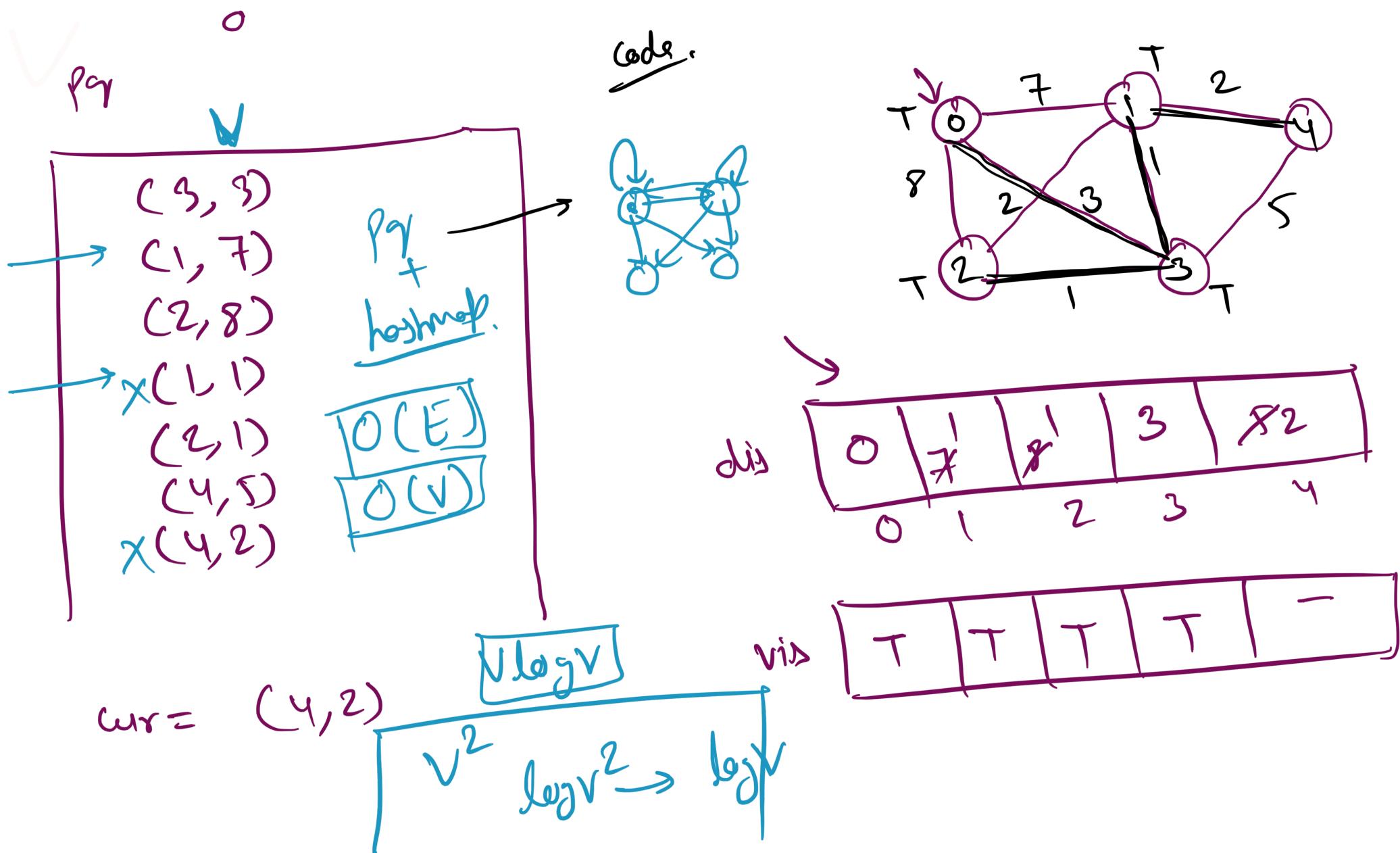
- (1, 7) X(2, 2)
- X(3, 3)
- (2, 8)
- X(1, 1)
- X(2, 1)
- (4, 5)
- (4, 2)



```
class Pair {  
    int v;  
    int wt;
```

y.

ans → (0, 3), (1, 1), (2, 1)

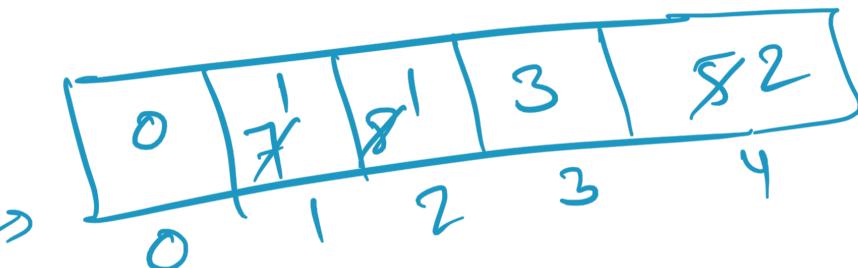


~~(1, 7)~~  
~~(2, 8)~~  
~~(3, 9)~~  
~~(1, 1)~~  
~~(2, 1)~~  
~~(4, 5)~~  
~~(4, 2)~~  
 E ↗

$$O(\boxed{E \log E}) = \boxed{E \log V}$$

ans = 0 + 3 + 1 + 1

+2  
7 -

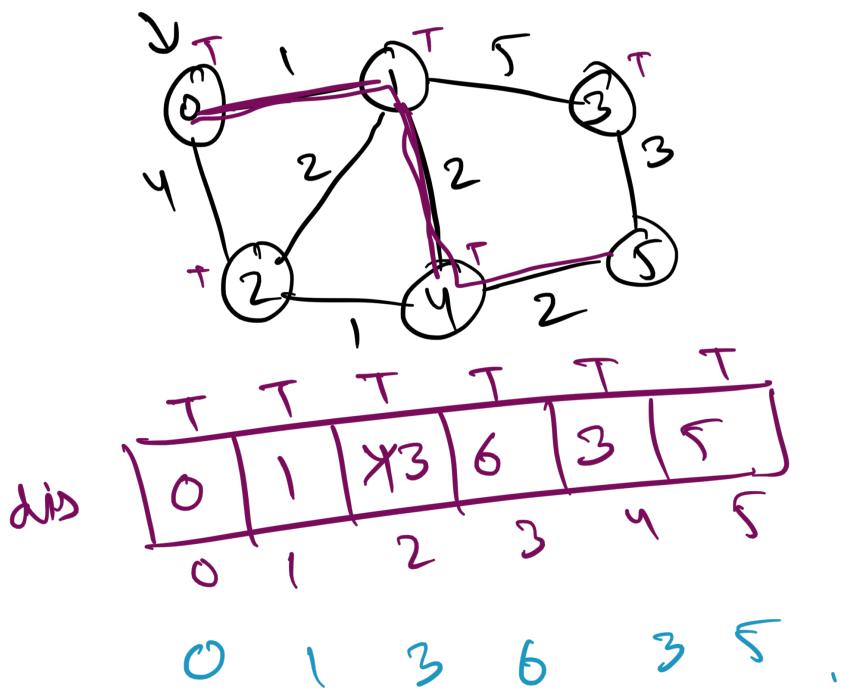


$E \ll V$   
 $\log E \rightarrow \log V$

## Dijkstra's Algorithm for Shortest Distance in a Weighted Graph

PQ

- ~~(1, 1)~~
- ~~(2, 4)~~
- ~~(2, 3)~~
- ~~(4, 3)~~
- ~~(3, 6)~~
- ~~(5, 5)~~



int cur = 5  
while (par[cur] != -1){  
 print (cur)  
 cur = par[cur]  
}

par

-1	0	1	1	1	1	4
0↑	1↑	2	3	4↑	5	



## **Bellman Ford Algorithm for Shortest Distance in a Weighted Graph**