

1.1.3. Points A,B,C are defined to be collinear if

$$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ A & B & C \end{pmatrix} = 2$$

Are the given points in (1.1) collinear?

Solution:

Given,

$$A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, B = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, C = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ A & B & C \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -4 & -3 \\ -1 & 6 & -5 \end{pmatrix}$$

Evaluating rank of the matrix using echelon form:

$$R_2 \rightarrow R_2 - R_1;$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -5 & -4 \\ -1 & 6 & -5 \end{pmatrix}$$

$$R_3 \rightarrow R_3 + R_2 - R_1;$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -5 & -4 \\ 0 & 0 & -10 \end{pmatrix}$$

As the no. of non-zero rows are "3", the rank of the matrix is "3".

Hence,

$$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ A & B & C \end{pmatrix} \neq 2$$

Therefore the given points are not collinear.