

GATE Assignment

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Three unbiased coins were tossed. Provided that at least two outcomes are tails, the probability of having all three outcomes as tails is

Solution:

Parameter	value	description
X_i	1	first coin
	2	second coin
	3	third coin
n	3	number of coins
p, q	$\frac{1}{2}$	toss result in heads/tails
Y	$\sum_{i=0}^3 X_i$	three coins

TABLE 1: Definition of Y and parameters.

$$\Pr(Y = 3 | Y \geq 2) = \frac{\Pr(Y \geq 2, Y = 3)}{\Pr(Y \geq 2)} \quad (1)$$

$$= \frac{\Pr(Y = 3)}{\Pr(Y \geq 2)} \quad (2)$$

$$= \frac{p_Y(3)}{1 - F_Y(1)} \quad (3)$$

$$p_Y(k) = {}^nC_k p^k q^{n-k} \quad (4)$$

$$= {}^3C_k \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{3-k} \quad (5)$$

$$\Rightarrow p_Y(k) = \begin{cases} \frac{{}^3C_k}{8}; k = \{0, 1, 2, 3\} \\ 0; \text{otherwise} \end{cases} \quad (6)$$

$$F_Y(k) = \Pr(Y \leq k) \quad (7)$$

$$= \sum_{k=0}^k p_Y(k) \quad (8)$$

$$\Rightarrow F_Y(k) = \begin{cases} 0; k < 0 \\ \sum_{k=0}^k \frac{{}^3C_k}{8}; k = \{0, 1, 2, 3\} \\ 1; k > 3 \end{cases} \quad (9)$$

$$\Rightarrow \Pr(Y = 3 | Y \geq 2) = \frac{\left(\frac{1}{8}\right)}{\left(\frac{1}{2}\right)} \quad (10)$$

$$= \frac{1}{4} \quad (11)$$

\therefore The probability of having all three outcomes as tails is 0.25.