Two-factor ANOVA

Data Layout:

Row	Column 1	2	... j	... n	mean
1	x_{11}	x_{12}	... x_{1j}	... x_{1n}	$\bar{x}_{1.}$
2	x_{21}	x_{22}	... x_{2j}	... x_{2n}	$\bar{x}_{2.}$
i	x_{i1}	x_{i2}	... x_{ij}	... x_{in}	$\bar{x}_{i.}$
m	x_{m1}	x_{m2}	... x_{mj}	... x_{mn}	$\bar{x}_{m.}$
mean	$\bar{x}_{.1}$	$\bar{x}_{.2}$	... $\bar{x}_{.j}$	... $\bar{x}_{.n}$	$\bar{x}_{..}$

$$\text{Total SS} = \sum_{i}^{m}\sum_{j}^{n}(x_{ij} - \bar{x}_{..})^2$$

$$= \sum_{i}\sum_{j}\left[(x_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{x}_{..}) + (\bar{x}_{i.} - \bar{x}_{..}) + (\bar{x}_{.j} - \bar{x}_{..})\right]^2$$

$$= \sum_{i}\sum_{j}(x_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{x}_{..})^2 + n\sum_{i}(\bar{x}_{i.} - \bar{x}_{..})^2 + m\sum_{j}(\bar{x}_{.j} - \bar{x})^2$$

$$+ \text{product term} (=0)$$

$$= \text{Error SS} + \text{Row SS} + \text{Column SS}$$

where $\bar{x}_{i.} = \sum_{j=1}^{n} x_{ij}/n$, $\bar{x}_{.j} = \sum_{i=1}^{m} x_{ij}/m$

Grand mean, $\bar{x}_{..} = \sum_{i}\sum_{j} x_{ij}/nm$

ANOVA Table

Sources of Variance	df	SS	MS	f
Row	$m-1$	$n \sum_{i=1}^{m} (\bar{x}_{i\cdot} - \bar{x})^2$	Row SS/$m-1$ = RMS	RMS/MSE
Column	$n-1$	$m \sum_{j=1}^{n} (\bar{x}_{\cdot j} - \bar{x})^2$	Column SS/$n-1$ = CMS	CMS/MSE
Error	$(m-1)(n-1)$	$\sum_i \sum_j (x_{ij} - \bar{x}_{i\cdot} - \bar{x}_{\cdot j} + \bar{x})^2$	Error SS/$(m-1)(n-1)$ = MSE	
Total	$mn-1$			

$$TSS = \sum \sum x_{ij}^2 - mn \, \bar{x}_{\cdot\cdot}^2$$