Transformers

Transformer

A transformer is a static piece of equipment used either for raising or lowering the voltage of an AC supply with a corresponding decrease or increase in current.

Construction of a Transformer

A transformer consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in figure below.

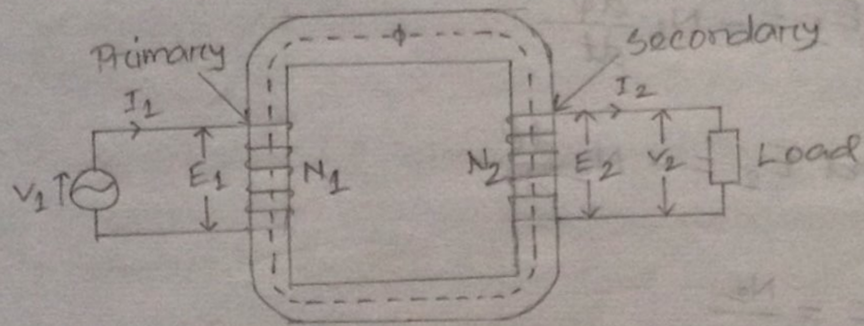

Figure : Transformer

The winding connected to the AC source is called primary winding and the one connected to load is called secondary winding. The alternating voltage V_1 whose magnitude is to be changed is applied to the primary. Depending upon the number of turns of primary (N_1) and secondary (N_2), an alternating emf E_2 is induced in the secondary. This causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load. If $V_2 > V_1$, it is called step up-transformer and if $V_2 < V_1$, it is called stepdown-transformer.

Working principle of a Transformer

When an altering voltage is applied to the primary, an alternating flux ϕ is set up in the core. This alternating flux links both the windings and induces emfs E_1 and E_2 in the windings according to Faraday's laws of electromagnetic induction. The emf E_1 is termed as primary emf and E_2 is termed as secondary emf.

Clearly, $E_1 = -N_1 \frac{d\phi}{dt}$

and $E_2 = -N_2 \frac{d\phi}{dt}$

$$\therefore \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

The magnitudes of E_2 and E_1 depend upon the number of turns on the secondary and primary respectively. If $N_2 > N_1$, then $E_2 > E_1$ (or, $V_2 > V_1$) and we get a step up-transformer. Inversely, we get a step down-transformer. If load is connected across the secondary winding, the secondary emf E_2 will cause a current I_2 to flow through the load. Thus, a transformer enables us to transfer AC power from one circuit to another with a change in voltage level.

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It is noted that there is no electrical connection between the primary and secondary. The AC power is transferred from primary to secondary through magnetic flux. There is no change in frequency i.e. output power has the same frequency as the input power.

⊞ E.M.F. Equation of a Transformer

Let an alternating voltage V_1 of frequency f be applied to the primary. The sinusoidal flux ϕ produced by the primary can be represented as

$$\phi = \phi_m \sin \omega t$$

The instantaneous emf e_1 induced in the primary is

$$e_1 = -N_1 \frac{d\phi}{dt}$$

$$= -N_1 \frac{d}{dt}(\phi_m \sin \omega t)$$

$$= -\omega N_1 \phi_m \cos \omega t$$

$$= -2\pi f N_1 \phi_m \cos \omega t$$

$$\Rightarrow e_1 = 2\pi f N_1 \phi_m \sin(\omega t - 90^\circ) \quad \text{——} \quad (1)$$

It is clear from the above equation that the maximum value of induced emf in the primary is

$$E_{m1} = 2\pi f N_1 \phi_m$$

The RMS value $E^\wedge$ of the primary emf is

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}}$$

$$\text{or, } E_1 = 4.44 f N_1 \phi_m$$

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Similarly $E_2 = 4.44 f N_2 \phi_m$.

In an ideal transformer, $E_1 = V_1$ and $E_2 = V_2$.

In common, we can write

$$E = \frac{2\pi}{\sqrt{2}} \phi_m f N \quad \text{or,} \quad E = 4.44 f N \phi_m .$$

⊞ What will happen if the primary of a transformer is connected to d.c. supply?

If DC supply is given to the primary of transfer, the primary will draw a stedy current due to which a constant flux is generated. Hence no emf will be produced. Because for production of emf in any winding, the current flowing through that must be sinusoidal since $e = L \frac{d}{dt}(I)$. Thus the primary of the transformer which is a low resistance side draws excessive current ultimately resulting in burning out of the terminals. To limit the effects of application of DC supply to transformer, a high resistance is connected in series to the primary of the transformer.

⊞ Define transformation ratio. According to this, classify transformer.

The transformation ratio is defined as the ratio of the secondary voltage to primary voltage. It is denoted by the letter K.

From the emf equations of a transformer, we can write

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

If $E_2 > E_1$ or $N_2 > N_1$ i.e. $K > 1$, then it is called step-up transformer.
If $E_2 < E_1$ or $N_2 < N_1$ i.e. $K < 1$, then it is called step-down transformer.

Q1 For an ideal transformation, derive the following relation : $E_1/E_2 = N_1/N_2 = I_2/I_1$

From emf equations of transformer, we know that

$$E_1 = 4.44 f N_1 \phi_m \quad \text{--- (1)}$$

and $$E_2 = 4.44 f N_2 \phi_m \quad \text{--- (2)}$$

Dividing (1) by (2), $$\frac{E_1}{E_2} = \frac{N_1}{N_2} \quad \text{--- (3)}$$

For an ideal transformer, $E_1 = V_1$ and $E_2 = V_2$ as there is no voltage drop in the windings.

$$\therefore \frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \text{--- (4)}$$

There are no losses. Therefore, volt-amperes input to primary are equal to output volt-amperes i.e.

$$V_1 I_1 = V_2 I_2$$

or, $$\frac{V_1}{V_2} = \frac{I_2}{I_1} \quad \text{--- (5)}$$

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From eq^n (4) and (5),

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}.$$

Hence, primary and secondary currents are inversely proportional to their respective turns.

Mathematical Problems

① Find the secondary current of a transformer having primary current of 20mA with 400 turns in primary and 200 turns in secondary.

Sol^n: Given,

Primary turns, $N_1 = 400$
Secondary turns, $N_2 = 200$
Primary current, $I_1 = 20mA$
Secondary current, $I_2 = ?$

We know that,

$$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$$\text{or, } I_2 = \frac{N_1}{N_2} \times I_1$$

$$= \frac{400}{200} \times 20$$

$$= 40 \text{ mA}.$$

(Ans)

② The maximum flux density in the core of a 250/3000 volts, 50 Hz single phase transformer is 1.2 Wb/m^2. If the emf per turn is 8 volt, determine

(i) The number of primary and secondary turns.

(ii) The area of the core

Soln: Given,

Maximum flux density, $B_{max} = 1.2 \text{ Wb/m}^2$

$E_1 = 250 \text{ V}$

$E_2 = 3000 \text{ V}$

emf per turn, $\frac{E_1}{N_1} = \frac{E_2}{N_2} = 8 \text{ V}$

$f = 50 \text{ Hz}$

(i) Turns in primary, $N_1 = \frac{E_1}{8} = \frac{250}{8} = 31.25$

and secondary, $N_2 = \frac{E_2}{8} = \frac{3000}{8} = 375$

(Ans)

(ii) We know that,

$$E_1 = 4.44 f \Phi_{max} N_1$$

hence, maximum flux, $\Phi_{max} = \frac{E_1}{N_1} \times \frac{1}{4.44 f}$

$$= 8 \times \frac{1}{4.44 \times 50}$$

$$\approx 0.036 \text{ Wb}$$

So, area of the core, $A = \frac{\Phi_{max}}{B_{max}} = \frac{0.036}{1.2}$

$$= 3 \times 10^{-3} \text{ m}^2 \text{ (Ans)}$$

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③ For a 400V/200V single phase transformer, determine the ratio of primary and secondary turns.

Solⁿ: Given,

$$E_1 = 400V$$

$$E_2 = 200V$$

∴ Ratio of primary to secondary turns,

$$\frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{400}{200} = 2$$

(Ans)

CIRCUIT THEOREMS

Thevenin's theorem

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

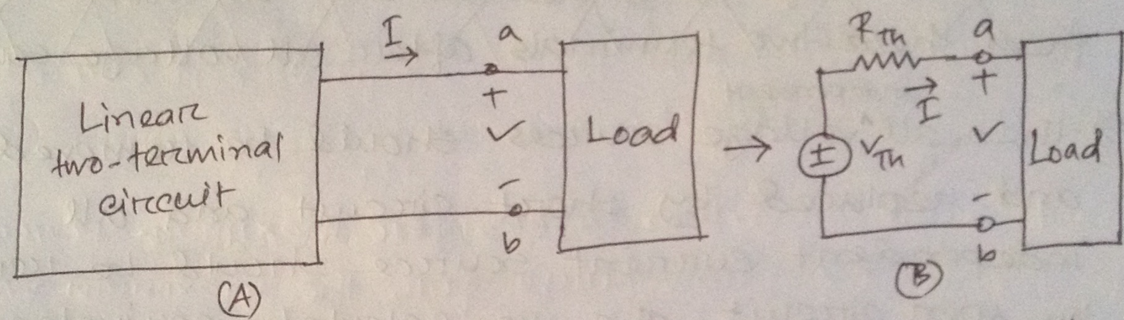

Figure: Replacing a linear two-terminal circuits by its Thevenin's equivalent :
(A) Original circuit
(B) Thevenin equivalent circuit

How do we Theveninize a given circuit?

The steps of theveninizing a given circuit are:

1) We have to remove the load resistance R_L temporarily.

2) Then we have to find the open circuit voltage (V_{oc}) which appears the two terminals from where R_L has been removed. This V_{oc} is called Thevenin voltage V_{TH}.

3) Then all independent voltage sources should be removed and replaced by short-circuit and all independent current source should be replaced by open circuit. The we calculate equivalent resistance from those two terminals which is called Thevenin resistance R_{TH}.

4) Finally, we have to replace the entire network by V_{TH} and R_{TH} in series connection. Load resistance must be back into its terminals from where it was previously disconnected.

Thus, we can find a Thevenin equivalent circuit for a given circuit.

Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

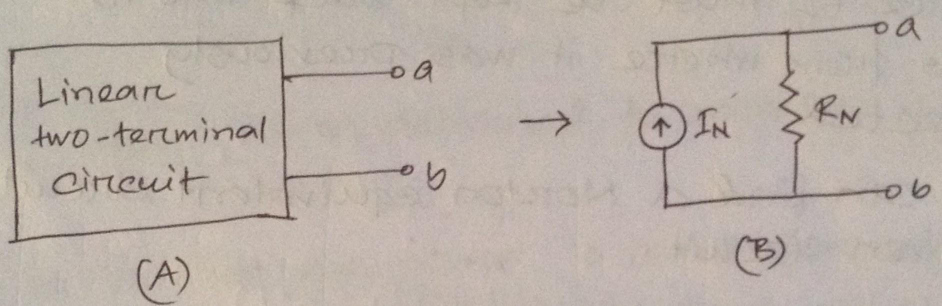

Figure: (A) Original circuit
(B) Norton equivalent circuit

How can you find Norton equivalent circuit from a given circuit?

The steps of Nortonizing a given circuit are:

1) We have to remove the load Resistance R_L across the two terminals and put a short circuit across them.

2) Then we have to compute short circuit current (I_{sc}) using nodal or mesh analysis. This I_{sc} is called Norton current I_N.

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3) Then all independent voltage sources should be removed and replaced by short circuit and all independent current sources should be replace by open circuit. Then we calculate equivalent resistance from those two terminals which is called Norton resistance R_N.

4) Finally, we have to replace the entire network by I_N and R_N in parallel connection. Load resistance R_L must be kept back into its terminals from where it was previously disconnected.

Thus, we can find a Norton equivalent circuit for a given circuit.

⊞ What is the condition to transfer maximum power to the load? Draw curve for the maximum power delivered to the load.

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load i.e. $R_L = R_{Th}$.

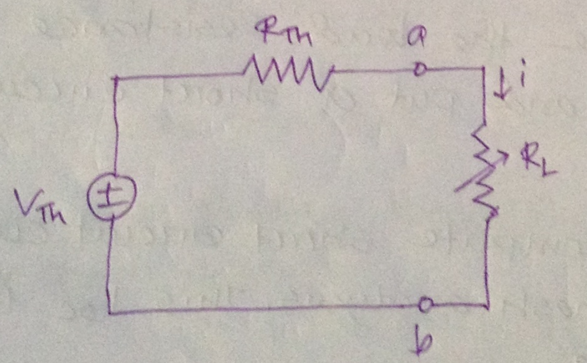

Figure: The circuit used for maximum power transfer.

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched below.

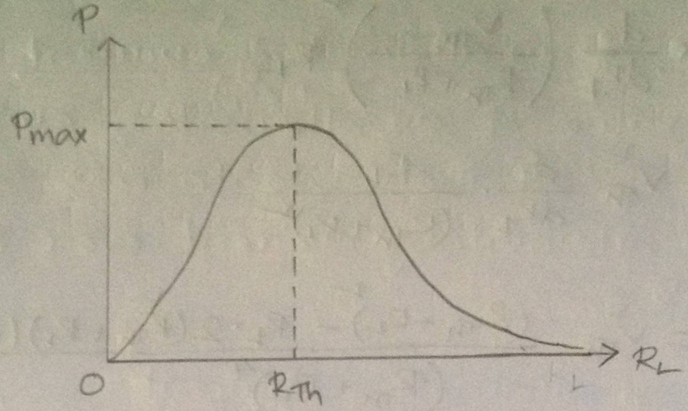

Figure: Power delivered to load as a function of R_L.

We notice that the power is small for small or large values of R_L but maximum for some value of R_L between 0 and α. The load acquires the maximum power when R_L equals the Thevenin resistance R_{Th}.

⊞ Prove the maximum power transfer theorem.
or, Prove that the maximum power transferred is
$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}.$$

We know that the power delivered to load is

$$P = I^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L \quad \text{———} \quad (1)$$

To prove the maximum power theorem, we differentiate eq^n (1) w.r.t. to R_L and set the result equal to zero. We obtain,

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

$$\Rightarrow 0 = V_{Th}^2 \frac{d}{dR_L} \cdot \frac{R_L}{(R_{Th}+R_L)^2}$$

$$= V_{Th}^2 \frac{(R_{Th}+R_L)^2 - R_L \cdot 2(R_{Th}+R_L)(0+1)}{(R_{Th}+R_L)^4}$$

$$= V_{Th}^2 \frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}$$

$$= V_{Th}^2 \frac{(R_{Th}+R_L)(R_{Th}+R_L-2R_L)}{(R_{Th}+R_L)^4}$$

$$= V_{Th}^2 \frac{R_{Th}-R_L}{(R_{Th}+R_L)^3}$$

$$\Rightarrow R_{Th}-R_L = 0$$

$$\Rightarrow R_L = R_{Th}$$

So, $$P_{max} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

$$= \frac{V_{Th}^2}{(R_{Th}+R_{Th})^2} \times R_{Th} \quad [\because R_{Th} = R_L]$$

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$$= \frac{V_{Th}^2}{4R_{Th}^2} \times R_{Th}$$

$$\therefore P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

(proved)

AC CIRCUIT

Definitions

1) Average value of an alternating current

The average value of an alternating current is the average of all the instantaneous values ~~during~~ of an alternating current over one complete cycle.

2) RMS value of an alternating current

The RMS value of an alternating current is the dc current that delivers the same average power to a resistor as the alternating current.

The RMS value of an alternating current,

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

3) Instantaneous value

The instantaneous value of an alternating voltage or current is the value of voltage or current at one particular instant.

4) Peak value

The maximum value attained by an alternating voltage or current during one complete cycle is called its peak value. It is also known as the maximum value or amplitude or crest value.

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5) Peak to Peak value

During each complete cycle of alternating voltage or current, there are always two maximum or peak values, one for the positive half-cycle and the other for the negative half-cycle. The differences between the peak positive value and the peak negative value is called the peak-to-peak value.

6) Frequency

The number of cycles per unit of time is called the frequency. It is denoted by 'f' or 'n'. The unit of frequency is cycle/s or Hz.

7) Phase

Phase is the initial angle of a sinusoidal function at its origin.

8) Apparent power

The apparent power (in VA) is the product of the rms values of voltage and current. It is measured in volt-amperes (VA) to distinguish it from the average or real power.

9) Power factor

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

If the voltage and current at the terminals of a circuit are,

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

or, in phasor form,

$$V = V_m \angle \theta_v$$
$$I = I_m \angle \theta_i$$

The average power, $P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$$= V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Here, the product $V_{rms} I_{rms}$ is known as apparent power and the factor $\cos(\theta_v - \theta_i)$ is called the power factor (pf).

10) Active and Reactive power

The portion of power that, averaged over a complete cycle of the AC waveform, results in net transfer of energy in one direction is known as active power (real power).

The portion of power due to stored energy, which returns to the source in each cycle, is known as reactive power.

11) Impedance

The impedance (Z) of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms (Ω).

$$Z = \frac{V}{I} = R + jX$$

where R = Re, Z is the resistance and X = Im.

12) Admittance

The admittance (Y) is the reciprocal of impedance (Z), measured in siemens (S).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

13) Reactance

Reactance is the oppsition of a circuit element to a change in current or voltage due to that element's inductance or capacitance.

Reactance of an inductor ~~is~~ is denoted by X_L.

$$X_L = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

Reactance of a capacitor is denoted by X_c.

$$X_c = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

14) Phasor

A phasor is a complex number that represents the amplitude and phase of a sinusoid.

It can be represented in three types of form.

→ Rectangular : $z = x + jy = r(\cos\theta + j\sin\theta)$

→ Polar : $z = r\angle\theta$

where,

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y/x)$$

→ Exponential : $z = re^{j\theta}$

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(ii) Derive that the Phasor relationship for capacitor is $V = \frac{I}{j\omega C}$ and show that the phasor current of a capacitor leads the voltage by 90 degree.

For a capacitor, let us assume that the voltage through it,

$$v = V_m \cos(\omega t + \theta)$$

The current through the capacitor is,

$$i = C\frac{dv}{dt}$$

$$= C\frac{d}{dt} V_m \cos(\omega t + \theta)$$

$$= -\omega C V_m \sin(\omega t + \theta)$$

$$= \omega C V_m \cos(\omega t + \theta + 90^\circ)$$

$$= Re(\omega C V_m e^{j(\omega t + \theta + 90^\circ)})$$

Tranforming into phasors,

$$I = \omega C V_m e^{j(\theta + 90^\circ)}$$

$$= \omega C V_m e^{j\theta} \cdot e^{j90^\circ}$$

$$= j\omega C V_m e^{j\theta} \quad [\because e^{j90^\circ} = j]$$

$$= j\omega C V \quad [\because V_m e^{j\theta} = V_m \angle \theta = V]$$

$$\therefore V = \frac{I}{j\omega C}$$

Showing that the current and voltage are out of phase and current leads voltage by 90°.

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