

$$= \frac{\sqrt{V_{TH}}}{4R_{TH}^2} \times P_m$$

$$\therefore P_{max} = \frac{\sqrt{V_m}}{4R_m}$$

(proved)

## AC CIRCUIT

### ► Definitions

#### 1) Average value of an alternating current

The average value of an alternating current is the average of all the instantaneous values during of an alternating current over one complete cycle.

#### 2) RMS value of an alternating current

The RMS value of an alternating current is the dc current that delivers the same average power to a resistor as the alternating current.

The RMS value of an alternating current,

$$I_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$$

#### 3) Instantaneous value

The instantaneous value of an alternating voltage or current is the value of voltage or current at one particular instant.

#### 4) Peak value

The maximum value attained by an alternating voltage or current during one complete cycle is called its peak value. It is also known as the maximum value or amplitude or crest value.

### 5) Peak to Peak value

During each complete cycle of alternating voltage or current, there are always two maximum or peak values, one for the positive half-cycle and the other for the negative half-cycle. The difference between the peak positive value and the peak negative value is called the peak-to-peak value.

### 6) Frequency

The number of cycles per unit of time is called the frequency. It is denoted by 'f' or 'n'. The unit of frequency is cycles/s or Hz.

### 7) Phase

Phase is the initial angle of a sinusoidal function at its origin.

### 8) Apparent power

The apparent power (in VA) is the product of the rms values of voltage and current. It is measured in volt-amperes (VA) to distinguish it from the average or real power.

### 9) Power factor

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

If the voltage and current at the terminals of a circuit are,

$$v(t) = V_m \cos(\omega t + \varphi_v)$$

$$i(t) = I_m \cos(\omega t + \varphi_i)$$

or, in phasor form,

$$V = V_m \angle \varphi_v$$

$$I = I_m \angle \varphi_i$$

$$\text{The average Power, } P_{av} = \frac{1}{2} V_m I_m \cos(\varphi_v - \varphi_i)$$

$$= V_{rms} I_{rms} \cos(\varphi_v - \varphi_i)$$

Hence, the product  $V_{rms} I_{rms}$  is known as apparent power and the factor  $\cos(\varphi_v - \varphi_i)$  is called the power factor (PF).

#### 10) Active and Reactive power

The portion of power that, averaged over a complete cycle of the AC waveform, results in net transfer of energy in one direction is known as active power (real power).

The portion of power due to stored energy, which returns to the source in each cycle, is known as reactive power.

### 11) Impedance

The impedance ( $Z$ ) of a circuit is the ratio of the phasor voltage  $V$  to the phasor current  $I$ , measured in ohms ( $\Omega$ ).

$$Z = \frac{V}{I} = R + jX$$

where  $R = R_L$ ,  $Z$  is the resistance and  $X = I_m$ .

### 12) Admittance

The admittance ( $Y$ ) is the reciprocal of impedance ( $Z$ ), measured in siemens ( $S$ ).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

### 13) Reactance

Reactance is the opposition of a circuit element to a change in current or voltage due to that element's inductance or capacitance.

Reactance of an inductor is denoted by  $X_L$ .

$$X_L = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

Reactance of a capacitor is denoted by  $X_C$ .

$$X_C = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

### 14) Phasor

A phasor is a complex number that represents the amplitude and phase of a sinusoid.

It can be represented in three types of form.

$$\rightarrow \text{Rectangular} : z = x + iy = r(\cos\theta + i\sin\theta)$$

$$\rightarrow \text{Polar} : z = r\angle\theta$$

where,

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$\rightarrow \text{Exponential} : z = re^{i\theta}$$

Derive that the Phasor relationship for capacitor is  $V = \frac{I}{j\omega C}$

and show that the phasor current of a capacitor leads the voltage by 90 degree.

For a capacitor, let us assume that the voltage through it,

$$v = V_m \cos(\omega t + \phi)$$

The current through the capacitor is,

$$\begin{aligned} i &= C \frac{dv}{dt} \\ &= C \frac{d}{dt} V_m \cos(\omega t + \phi) \\ &= -\omega C V_m \sin(\omega t + \phi) \\ &= \omega C V_m \cos(\omega t + \phi + 90^\circ) \\ &= R_c (\omega C V_m e^{j(\omega t + \phi + 90^\circ)}) \end{aligned}$$

Transforming into phasors,

$$\begin{aligned} I &= \omega C V_m e^{j(\phi + 90^\circ)} \\ &= \omega C V_m e^{j\phi} \cdot e^{j90^\circ} \\ &= j\omega C V_m e^{j\phi} \quad [ \because e^{j90^\circ} = j ] \\ &= j\omega C V \quad [ \because V_m e^{j\phi} = V_m \angle \phi = V ] \end{aligned}$$

$$\therefore V = \frac{I}{j\omega C}$$

Showing that the current and voltage are out of phase and current leads voltage by 90°.

By show that  $V = j\omega L I$ .

For an inductor, let us assume that the current through it,

$$i = I_m \cos(\omega t + \phi)$$

The voltage across it,

$$\begin{aligned} V &= L \frac{di}{dt} \\ &= L \frac{d}{dt} I_m \cos(\omega t + \phi) \\ &= -\omega L I_m \sin(\omega t + \phi) \\ &= \omega L I_m \cos(\omega t + \phi + 90^\circ) \\ &= \operatorname{Re}(\omega L I_m e^{j(\omega t + \phi + 90^\circ)}) \end{aligned}$$

Transforming into phasor,

$$\begin{aligned} V &= \omega L I_m e^{j(\phi + 90^\circ)} \\ &= \omega L I_m \cdot e^{j\phi} \cdot e^{j90^\circ} \\ &= j\omega L I_m e^{j\phi} [\because e^{j90^\circ} = j] \\ \Rightarrow V &= j\omega L I \quad [\because I_m e^{j\phi} = I_m \angle \phi = I] \end{aligned}$$

We can see that the current lags the voltage by  $90^\circ$ .

Draw the Phasor relationship for circuit elements :  
capacitor and Inductor.

For a capacitor,

$$V = V_m \cos(\omega t + \phi)$$

$$i = I_m \cos(\omega t + \phi + 90^\circ)$$

So, current leads the voltage by  $90^\circ$ .

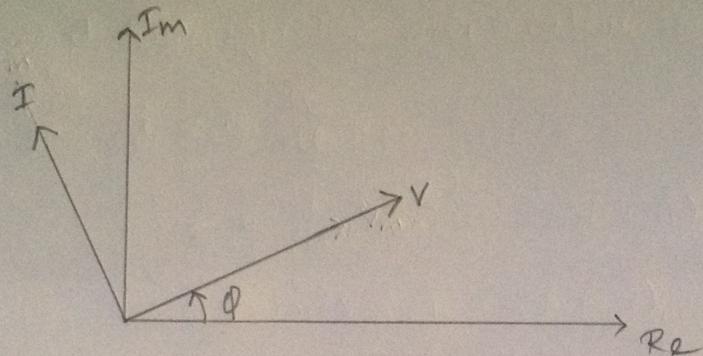


Figure : Phasor diagram for Capacitor .

For an inductor,

$$i = I_m \cos(\omega t + \phi)$$

$$V = V_m \cos(\omega t + \phi - 90^\circ)$$

so, voltage leads the current by  $90^\circ$

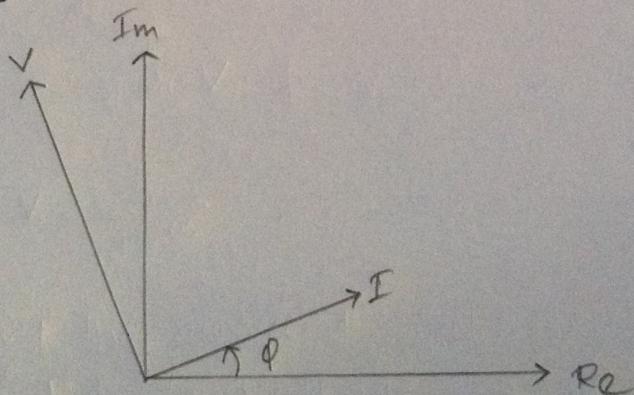
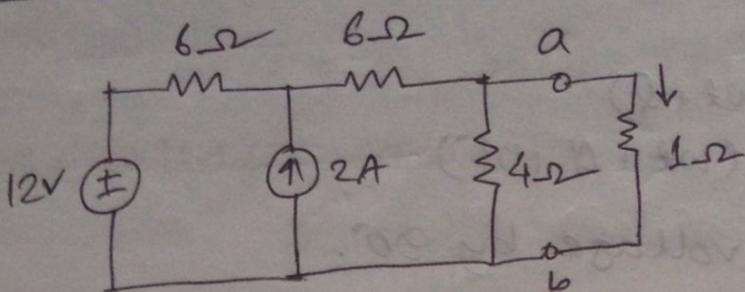


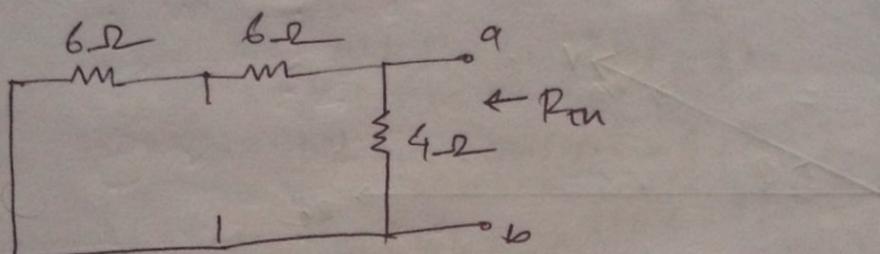
Figure : Phasor diagram for Inductor .  
Name : Mahmood / 1AM / C161026

CIRCUIT PROBLEMS

Prac - 4.8

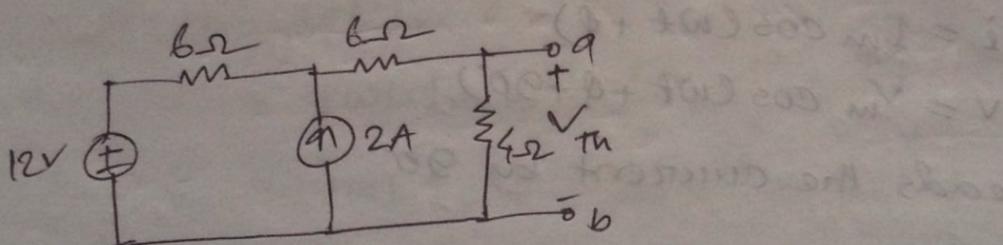


For obtaining  $R_{Th}$ ,

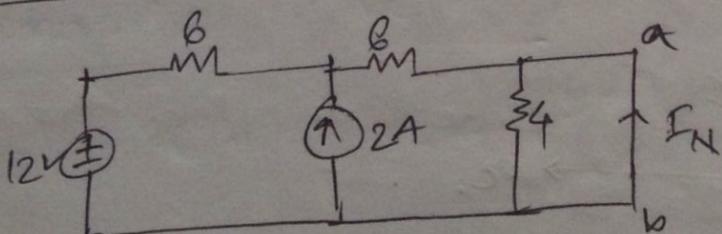


$$R_{Th} = (6+6) \parallel 4 = 3\Omega$$

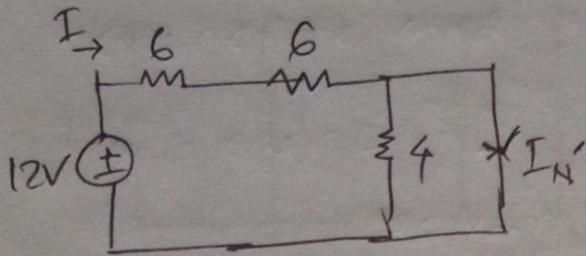
For obtaining  $V_{Th}$ ,



We will use Norton analysis



When 12V is active,



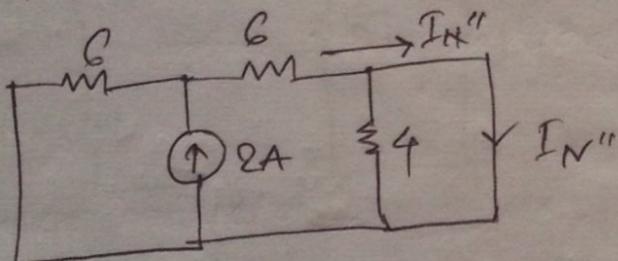
Current won't flow through  $4\Omega$  as there is a short circuit across it.

$$R' = 6 + 6 = 12$$

$$\therefore I = \frac{V}{R} = \frac{12}{12} = 1A$$

$$\text{So, } \downarrow I_{N'} = I = 1A$$

When 2A is active,



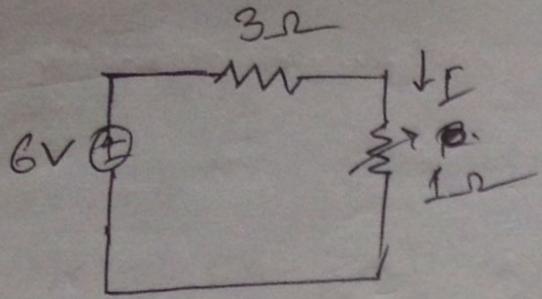
Current won't flow through  $4\Omega$ .

$$\therefore \downarrow I_{N''} = \frac{6}{6+6} \times 2$$

$$= 1A$$

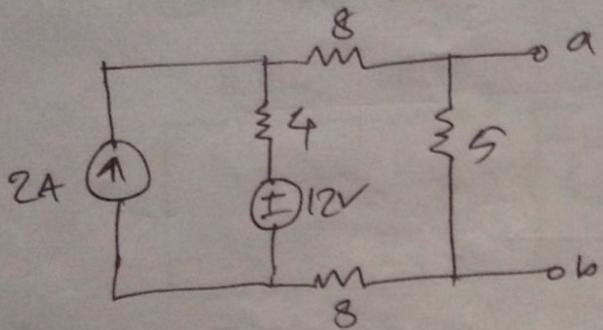
$$\therefore I_H = I_{N'} + I_{N''} = 1A + 1A = 2A$$

$$\begin{aligned}\therefore V_{Th} &= I_H R_{Th} \\ &= 2 \times 3 \\ &= 6V\end{aligned}$$

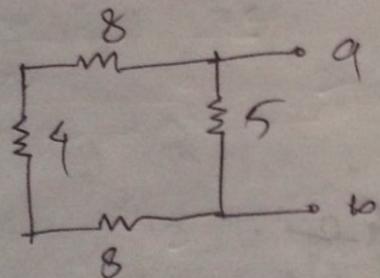


$$\therefore I = \frac{6}{3+1} = 1.5A$$

Prob 4.11

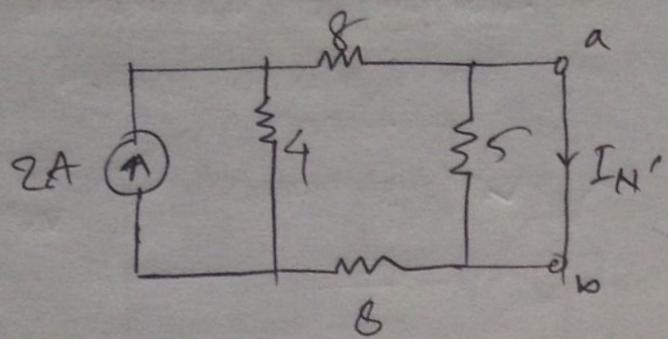


R\_N



$$R_N = (8+4+8) \parallel 5 \\ = 4\Omega$$

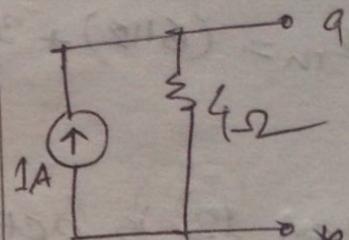
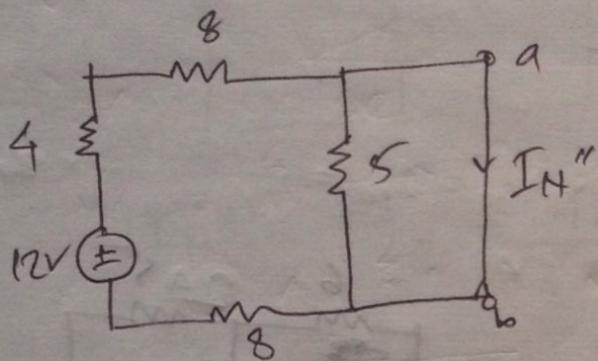
I\_N  
when 2A is active



Current won't flow through  $S_{22}$ .

$$\downarrow I_{N'} = \frac{4 \times 2}{8+8+4} = 0.4 \text{ A}$$

when 12V is active



Current won't flow through  $S_{22}$

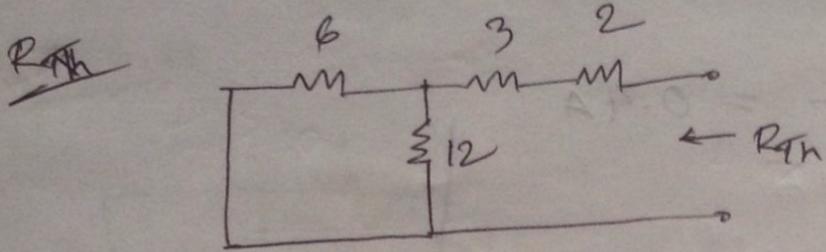
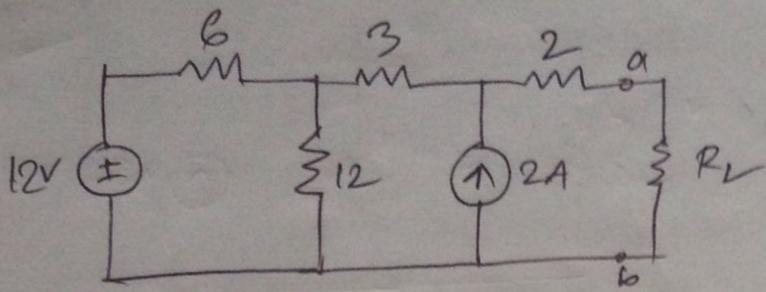
$$R_{eq} = 4 + 8 + 8 = 20 \Omega$$

$$\therefore I = \frac{V}{R_{eq}} = \frac{12}{20} = 0.6 \text{ A}$$

$$\therefore \downarrow I_{N''} = I = 0.6 \text{ A}$$

$$\begin{aligned} \therefore I_H &= \downarrow I_{H'} + \downarrow I_{H''} \\ &= 0.4 + 0.6 = 1 \text{ A} \end{aligned}$$

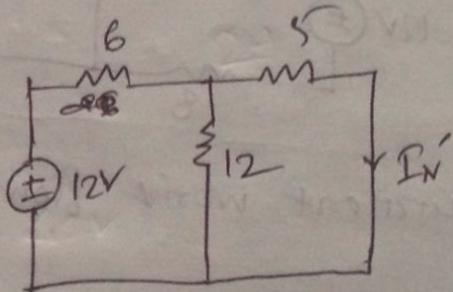
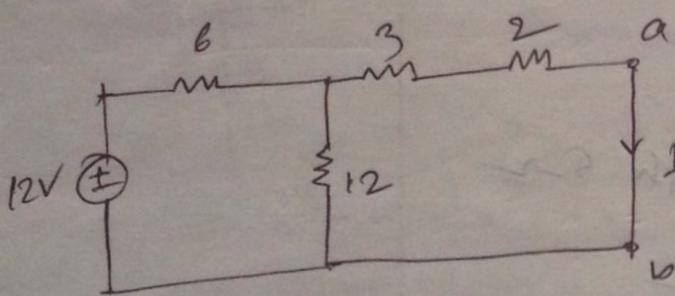
Ex-4.13



$$R_{Th} = (6 \parallel 12) + 3 + 2 = 9 \Omega$$

$V_{Th}$

when 12 is active.



$$\therefore I = \frac{V}{R_{eq}} = \frac{12}{9.53} \\ = 1.26 A$$

$$R_{eq} = (5 \parallel 12) + 6 \\ = 9.53 \Omega$$

$$\therefore I_{N'} = I_{5\Omega} = \frac{12 \times 1.26}{12+5} = 0.89 A$$

$$\left(\frac{8}{9}\right)$$