Modern physics

Outline:

① Bohr's atomic model (Def^n)

② Radius and total energy of Hydrogen atom (Proof)

③ Atomic Nucleus and properties (Def^n)

④ Mass defect, Binding energy (Explain)

⑤ photoelectric effect and einstein photo electric effect (Proof)

⑥ Compton effect

⑦ De-Brogle effect

⑧ X-Ray and properties

⑨ Zeeman effect

$\left.\begin{array}{l} ⑥ \\ ⑦ \\ ⑧ \\ ⑨ \end{array}\right\}$ (Def^n)

Bohr's atomic model :

~~Boh~~ Bhor assumed basically Rutherford Nuclear model of the atom and tried to overcome the defects of the model.

① An electron, while orbiting in a permanent orbit it's total angular momentum will be an integral multiple of $h/2\pi$, Here h is the plancks constant.

It's angular momentum, $mvr = \frac{nh}{2\pi}$,

Here, n is a integer, m = mass, V = velocity,

r = Permanent orbit of radius. n is called principle quantum number.

② Electron is an atom cannot revolve round the nucleus is all probable orbits, rather they rotate in certain fixed prescribed circular orbits. These orbits are called Permanent and non radiating orbits. When the electron revolves into a permanent orbits it does not emit electromagnetic radiation as predicted by electromagnetic theory of light.

③ Whenever an electron jumps from a convenient orbit to another convenient orbit, then radiation or absorption of energy takes place. An atom emitted energy when electron jumps from higher energy to lower energy level and absorbed when electron jumps from lower energy level higher energy level,

Let upper ~~energy~~ level energy E_2 and lower level energy E_1

$$E = E_2 - E = h\nu \text{ (radiation)}$$

$$E = E_1 - E_2 \text{ (absorption)}$$

Bohr-formula: Based on these postulates, Bohr derived the formula for (i) the radii of the stationary orbits and (ii) the total energy of the electron in the orbit.

Radius and total energy of hydrogen atom:

$\boxed{\text{Radius:}}$ Let Z number of proton in the nucleus of an atom and the electron whose charge is e and mass is m be orbitting round the nucleus with velocity V_n. If the radius of nth orbit be r_n, then the centripetal force, $F = \frac{mV_n^2}{r_n}$ —(1)

Again columbs attractive force between the positively charged nucleus and the negatively charged electron,

$$F = \frac{1}{4\pi\epsilon_0} \frac{Ze.e}{r_n^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n^2} \quad \text{—(2)}$$

From eq^n (1) and eq^n (2) we get,

$$\frac{mV_n^2}{r_n} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n^2}$$

$$\Rightarrow mV_n^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n} \quad \text{—(3)}$$

Now, moment of Inertia, $I = mr_n^2$

Angular velocity, $\omega = \frac{V_n}{r_n}$

According to Bohr 1st postulates,

Angular momentum, $L = I\omega$

$$= mr_n^2 \times \frac{V_n}{r_n}$$

$$= mV_n r_n$$

$$\therefore \; m v_n r_n = \frac{nh}{2\pi}$$

$$\Rightarrow \; v_n = \frac{nh}{2\pi m r_n}$$

Putting the value of v_n in eq^n (3).

$$m\left(\frac{nh}{2\pi m r_n}\right)^2 = \frac{Ze^2}{4\pi \epsilon_0 r_n}$$

$$\Rightarrow \; \frac{m n^2 h^2}{4\pi^2 m^2 r_n^2} = \frac{Ze^2}{4\pi \epsilon_0 r_n}$$

$$\Rightarrow \; \frac{n^2 h^2}{\pi m r_n} = \frac{Ze^2}{\epsilon_0}$$

$$\Rightarrow \; r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}$$

For, hydrogen atom, $Z = 1$, $n = 1$

$$r_n = \frac{\epsilon_0 h^2}{\pi m e^2}$$

$$= \frac{(8.854 \times 10^{-12}) \times (6.673 \times 10^{-34})^2}{\pi \times (9.1 \times 10^{-31}) \times (1.6 \times 10^{-19})^2}$$

$$= 5.29 \times 10^{-11}\ m.$$

$\epsilon_0 = 8.854 \times 10^{-12}\ C^2 N^{-1} m^2$

$h = 6.673 \times 10^{-34}\ JS$

$e = 1.6 \times 10^{-19}\ C$

$m = 9.1 \times 10^{-31}\ kg$

Total energy:

The total energy of an electron is the stable orbit consists of two parts. k.E and P.E.

i.e, the total energy, $E = E_K + E_P$

kinetic energy ~~of~~ for electron, $E_K = \frac{1}{2}mv_n^2$

$$= \frac{1}{2}m\left(\frac{ze^2}{4\pi\epsilon_0 r_n m}\right) \quad \left[v_n^2 = \frac{ze^2}{4\pi\epsilon_0 r_n m}\right]$$

$$= \frac{ze^2}{8\pi\epsilon_0 r_n}$$

As the electron is in the electric field of the positively charged nucleus. so its potential energy $= V\times(-e)$.

V is the potential at a distance r_n from the nucleus.

We know, $V = \frac{ze}{4\pi\epsilon_0 r_n}$

Potential energy, $E_P = -Ve = -\frac{ze^2}{4\pi\epsilon_0 r_n}$

Then total energy, $E_n = \frac{ze^2}{8\pi\epsilon_0 r_n} - \frac{ze^2}{4\pi\epsilon_0 r_n}$

$$= -\frac{ze^2}{8\pi\epsilon_0 r_n} \quad \text{——— (1)}$$

Putting the value of r_n to eqⁿ (1) we get

$$E_n = -\frac{1}{8\pi\epsilon_0} \times \frac{ze^2\pi m z e^2}{n^2h^2\epsilon_0}$$

$$= -\frac{mz^2e^4}{8\epsilon_0^2 n^2 h^2} \quad \text{——— (2)}$$

Eq^n ② represents the energy of electron of the nth orbit.

For hydrogen atom, $z=1$, $n=1$

$$E_1 = -\frac{me^4}{8\epsilon_0^2 h^2}$$

E_1 is called the lowest energy state or energy of the ground state of the atom.

$$E_1 = -13.6\ eV$$

Atomic Nucleus and properties:

Atomic Nucleus: The nucleus is the center of an atom. Atomic nucleus is made up of elementary Particle called protons and neutrons.

A proton has a positive charge of the same magnitude as that of an electron. A neutron is electrically neutral.

The proton and the neutron are considered to be two different charge states of the same Particle which is called a necleon.

$_Z X^A$ Z = The atomic number indicates the number of proton

A = the mass number (indicates the total number of Proton and neutrons)

(N = A - Z).

Properties of a Nucleus:

(i) Nuclear mass: Nucleus consist of Protons and Neutrons.

$$\text{Nuclear mass} = Zm_p + Nm_n$$

where, m_p and m_n are the respective proton and neutron masses and N is the neutron number.

Exp: Uranium nucleus U^{238} has a mass of 238 a.m.u

(ii) Nuclear size: The mean radius of an atomic nucleus is of the order of 10^{-14} to 10^{-15} m while that of the atom is about 10^{-10} m.

$$R = r_0 A^{1/3}$$

A = mass number

$r_0 = 1.3 \times 10^{-15}$ m = 1.3 fm

Exp: ${}_{6}C^{12}$, $R \approx (1.3)(12)^{1/3}$

$= 3$ fm

(iii) Nuclear Density: It is defined as the nuclear mass per unit volume. Since the nuclear mass is the mass of proton and neutron of an atom

$$\text{Nuclear density}, \rho_N = \frac{\text{Nuclear mass}}{\text{Nuclear Volume}}$$

$$\rho_N = \frac{Zm_p + Nm_n}{\frac{4}{3}\pi R^3}$$

Exp: ${}_6C^{12}$, $\rho_N = \frac{12 \times (1.66 \times 10^{-27} \text{ kg})}{\frac{4}{3} \times 3.1416 \times (3 \times 10^{-15})^3}$

$$= 1.8 \times \frac{10^{17} \text{ kg}}{m^3}$$

(iv) Nuclear charge: Each proton has a positive charge of 1.6×10^{-19} C. The nuclear charge Ze where Z is the atomic number of the nucleus.

Mass defect: The difference between the measured mass M and the mass number A of a nuclide is called the mass defect.

$\therefore$ Mass defect, $\Delta m = M - A$

Binding energy: When the 'Z' Protons and 'N' neutrons combine to make a nucleus, some of the mass (Δm) disappears because it is converted into an amount of energy $\Delta E = (\Delta m)c^2$. This energy is called the binding energy of the nucleus.

$\therefore B.E = \{(Zm_p + Zm_n) - A\}c^2$

B.E of the deutron : ($_1H^2$)

Mass of proton = 1.007276 amu

Mass of neutron = 1.008665 amu

$\therefore$ Mass in free station = (Proton+ neutron)

= 2.015941 amu

Mass of deutron nucleus = 2.013553 amu

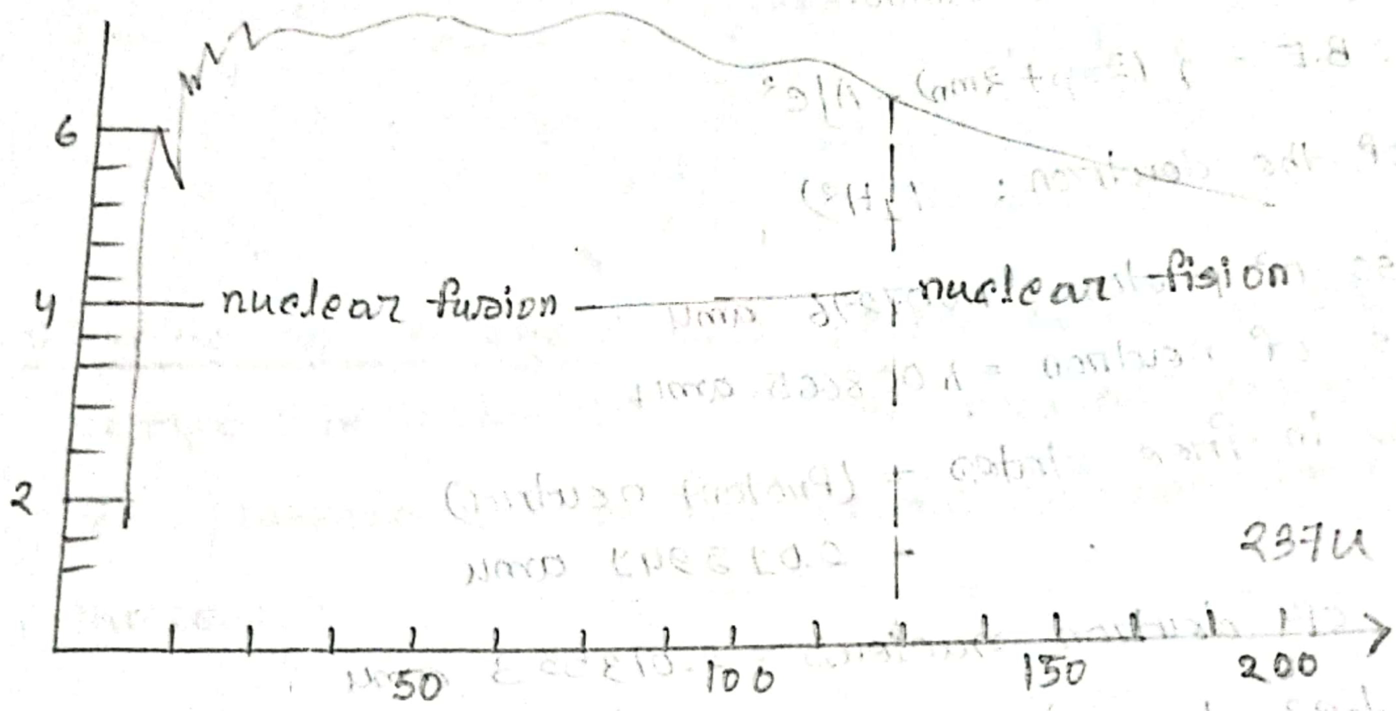
Mass deffect = Δm = 0.002388 amu

B.E = 0. 002388 x931 [1 amu = 931 MeV]

= 2.23 MeV

Nuclear Binding energy curve:

Binding energy per nuclear = $\dfrac{\text{Total B.E of a nucleus}}{\text{The number of nucleus it contains}}$



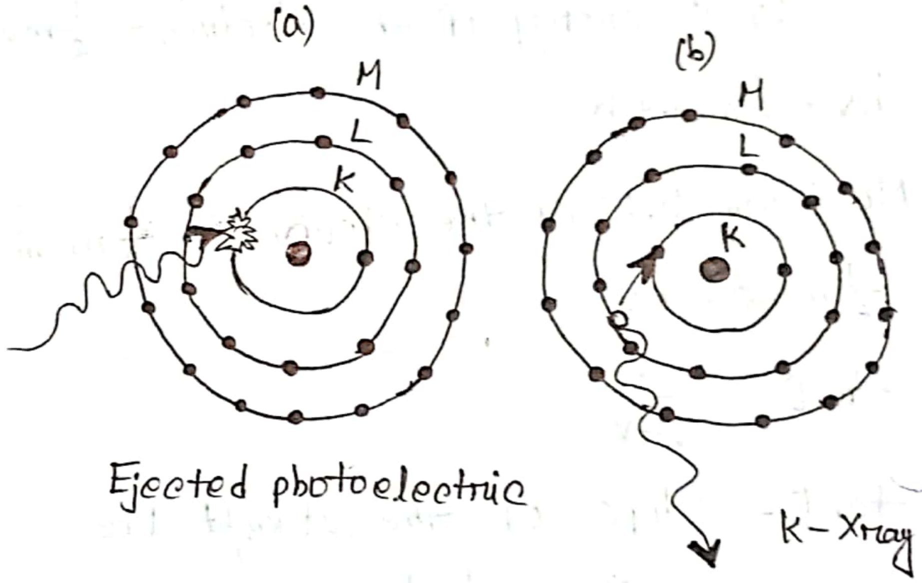
Photoelectric effect:

Hertz in 1887 ~~an~~ discovered the features of photo-electric effect. The electron ejected out of the metal under the action of light (photon) are known as photo electrons and this phenomenon is known as photo electric effect.

$$E_K = E - W_i$$

$$E = W_K - W_L$$

$$E_{aug} = (W_K - W_L) - W_i$$



Ejected photoelectric

Einstein's equation for photo-electric effect:

In 1905 A. Einstein made use of quantum theory of radiation to explain the basic facts and laws of photo-electric emission. According to quantum theory,

(i) A light of frequency ν consist of quanta of energy or photon. Each photon has an energy $h\nu$ and travels with the velocity of light.

(ii) In photon-electric effect, one photon is completely absorbed by one electron in the photo cathode.

Kinetic energy of the electron $= \frac{1}{2}mv^2$

$$h\nu = \frac{1}{2}mv^2 + W_0$$

Maximum K.E of the electron $= \frac{1}{2}mV_m^2$

$$\frac{1}{2}mV_m^2 = eV_0$$

$$\tan\theta = \frac{eV_0}{V-V_0}$$

$\tan\theta$ = slope of the straight line

= h, a constant

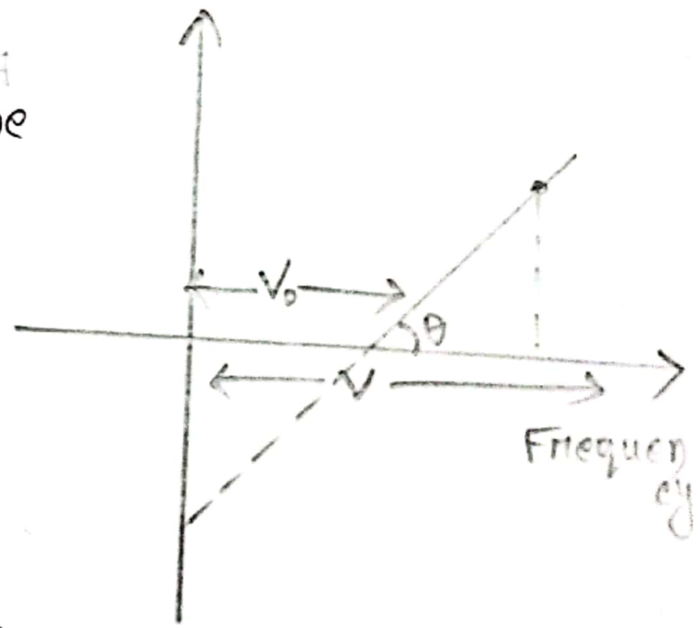
$$h = \frac{eV_0}{V-V_0}$$

$$\Rightarrow eV_0 = h(V-V_0)$$

$$\therefore \frac{1}{2}mV_m^2 = h(V-V_0)$$

$$\Rightarrow \frac{1}{2}mV_m^2 = hV - hV_0$$

$$\Rightarrow \frac{1}{2}mV_m^2 = hV - W_0 \quad \text{--- (1)}$$



where, $h\nu_0 = W_0 =$ photo- electric work function.

$\therefore$ Equation ① is the Einstein's equation for photo- electric effect and $h\nu_0 = \phi$ is the relation between work function and threshold frequency.

<u>Compton effect:</u> A reduction is the energy of x-rays or gamma ray photon when they are Sattered by free electrons. The phenomenon occurs when the photon energy is transferred to the electron and consequently the Photon loses energy $h(\nu-\nu')$ where is plank constant and ν and ν' are the frequencies before ~~the~~ and after collision. The scattering of a photon by an electron is called the compton effect.

The energy E_K of the compton electron is

$$E_K = 2\alpha h\nu \cos^2\phi \,/\, \{(1+\alpha)^2 - \alpha^2\cos^2\phi\}$$

De-Broglie effect:

In 1924 a young physicist, de Broglie, speculated that nature did not single out light as being the only matter which exhibits a wave particle duality. He proposed that ordinary 'Particles' such as electrons, protons or bowling balls could also exhibit wave characteristics in certain circumstances

photon of energy $E = h\nu$

momentum $p = \frac{h\nu}{c} = \frac{h}{\lambda}$

where λ is the wavelength,

De Broglie suggested that whenever there are Particles of momentum p, their motion is associated with a wavelength.

This hypothesis is called De Broglie.

Zeeman effect: A spectral line emitted by the excited atoms is split up ~~the~~ into a doublet or a triplet when the emitting atoms are placed in a magnetic field. This effect of the splitting of a spectral line under the action of a magnete field is known as normal zeeman effect.

X-ray: X-ray are high energy photons (1-100kev) or electromagnetic radiation having a very short wave-length of the order of 1Å.

Properties of X-rays:

① They are propagated in straight lines with the velocity of light. i.e 3×10^8 m/s.

② They are not deflected by magnetic or electric fields. and therefore they do not possess any charge.

③ They produce fluorescence in barium-platinocyanide, zinc sulphide and cadmium tungstate.

④ They have destructive effect on living tissues.
~~Long exposure of the skin to X-rays produce~~
X-ray are used for destroying and burning the ulcers and tumours in the body.

Uses of X-rays:

* X-rays are especially useful in the detection of pathology of the skeletal system, but are also useful for detecting some disease processes in soft tissue.
* chest X-ray which can be used to identify lung diseases such as pneumonia, lung cancer or pulmonary edema. ~~and the abnormal X-ray~~

* The abdominal X-ray, which can detect intestinal obstruction, free air and free fluid.

* X-ray maybe used to detect pathology.

* Traditional plain X-rays are less useful in the imaging of soft tissuses such as the brain or muscle.

* Airport security luggage scanners use X-rays for inspecting the interior of trucks for at country borders.

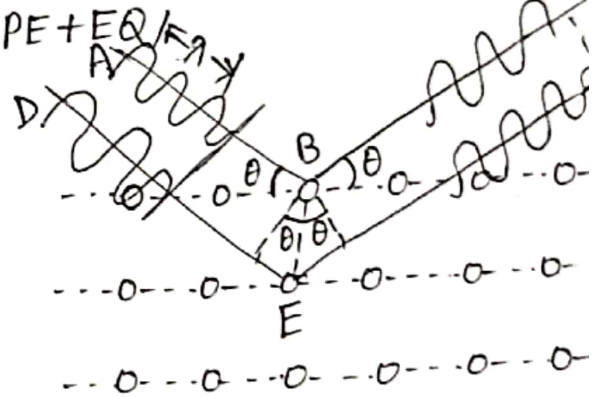
* Border security truck scanners use X-rays for inspecting the interior of trucks for at country borders.

Bragg's law:

Statement: The intensity of the reflected beam at certain angles will be maximum, where the two reflected wave from two different planes have a path difference equal to an integral multiple of the wave length of X-rays, while at some other angles, the intensity will be minimum.

Derivation: Path difference between the reflected waves along BC and EF

$= PE + EQ$



In the ΔPBE, $\sin\theta = \frac{PE}{BE}$

$\therefore PE = BE \sin\theta = d\sin\theta$

Similarly, in the ΔQBE,

$EQ = BE\sin\theta = d\sin\theta$

$\therefore$ Path difference, $PE + EQ = d\sin\theta + d\sin\theta$
$= 2d\sin\theta$

If the path difference is an integral multiple of lave length λ,

$$2d\sin\theta = n\lambda$$

where, $n = 1, 2, 3 \ldots$

This equation is known as Bragg's equation and represents Bragg's law for x-ray diffraction.

Example 11.21 A particle is moving with a speed of 0.5c. Calculate the ratio of its rest mass and the mass while in motion.

Sole.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

or, $$\frac{m_0}{m} = \sqrt{1-\frac{v^2}{c^2}}$$

Here $v = 0.5c$

$$\therefore \quad \frac{m_0}{m} = \sqrt{1-\frac{(0.5c)^2}{c^2}}$$

$$= \sqrt{1-(0.5c)^2}$$

$$= 0.866.$$

Example 11.22 Calculate the velocity that one atomic mass unit will have if it has a kinetic energy equal to twice the rest mass energy.

Soln.

We have

$$E = mc^2 = m_0c^2 + T$$

Here $T = 2m_0c^2$

$\therefore\ mc^2 = m_0c^2 + 2m_0c^2$

or, $3m_0c^2 = mc^2$

or, $m = 3m_0$

But $\quad m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$\therefore \quad 3m_0 = \frac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

or, $\sqrt{1-\dfrac{v^2}{c^2}} = \dfrac{m_0}{3m_0} = \dfrac{1}{3}$

or, $1-\dfrac{v^2}{c^2} = \dfrac{1}{9}$

or, $\dfrac{v^2}{c^2} = 1-\dfrac{1}{9} = \dfrac{8}{9}$

or, $\quad v^2 = \left(\dfrac{8}{9}\right)c^2$

or, $\quad v = \sqrt{\dfrac{8}{9}}\cdot c$

$$= 0.941\ c.$$

Example 11.23 For what value of v/c (=β) will be relativistic mass of a particle exceed its rest mass by a given fraction f?

Soln.

From eqn. $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$

we have

$$f = \frac{m - m_0}{m_0} = \frac{m}{m_0} - 1 = \frac{1}{\sqrt{1-\beta^2}} - 1$$

or, $$\frac{1}{\sqrt{1-\beta^2}} = 1 + f$$

Squaring

$$\frac{1}{1-\beta^2} = (1+f)^2$$

or, $$1 - \beta^2 = \frac{1}{(1+f)^2}$$

or, $$\beta^2 = 1 - \frac{1}{(1+f)^2} = \frac{1 + 2f + f^2 - 1}{(1+f)^2} = \frac{2f + f^2}{(1+f)^2} = \frac{f(2+f)}{(1+f)^2}$$

or, $$\beta = \frac{\sqrt{f(2+f)}}{(1+f)}$$

The table below shows some computed values, which hold for all particles regardless of their rest mass.

f	β
0.001 (0.1 percent)	0.014
0.01	0.14
0.1	0.42
1 (100 percent)	0.87
10	0.994
100	0.999

Example 22.24 What is the length of a metre stick moving parallel to its length when its mass is 3/2 of its rest mass?

Soln.

We have $m = \dfrac{m_0}{\sqrt{1-v^2/c^2}}$ .... (i)

or, $\dfrac{m}{m_0} = \dfrac{1}{\sqrt{1-v^2/c^2}}$; or $\dfrac{m_0}{m} = \sqrt{1-v^2/c^2}$

and $L = L_0\sqrt{1-v^2/c^2}$ .... (ii)

Dividing (ii) by (i)

$$\frac{L}{m} = \frac{L_0}{m_0}\left(1-v^2/c^2\right)$$

$$\therefore\ L = \frac{m}{m_0} \cdot L_0\,(1 - v^2/c^2) = \left(\frac{m}{m_0}\right) L_0 \left(\frac{m_0}{m}\right)^2 = \frac{m_0}{m} \cdot L_0$$

Here $\dfrac{m_0}{m} = \dfrac{2}{3}$ and $L = 1$ metre

$$\therefore\ L = \frac{2}{3} \times 1 = 0.667\text{m}.$$

***Example 11.25** A particle of mass 10^{-24} kg is moving with a speed of 1.8×10^8 m/s. Calculate its mass when it is motion.*

Soln.

$$m = \frac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Here $m_0 = 10^{-24}$ kg

$v = 1.8 \times 10^8$ m/sec

$c = 3 \times 10^8$ m/sec

$\dfrac{v}{c} = 0.6$

$$\therefore\quad m = \frac{10^{-24}}{\sqrt{1-(0.6)^2}}$$

$$= 1.25 \times 10^{-24}\text{ kg}.$$

Example 11.26 *4.18×10^{-3} kg of a substance is fully converted to heat energy. Calculate the amount of heat produced.*

Soln.

Here $m = 4.18 \times 10^{-3}$ kg

$C = 3 \times 10^{8}$ m/sec.

$$E = mc^2$$

$$= 4.18 \times 10^{-3} \times (3 \times 10^{8})^2 \text{ J}$$

$$\therefore \text{ Heat produced} = \frac{E}{4.18} \text{ calories}$$

$$= \frac{4.18 \times 10^{-3} \times (3 \times 10^{8})^2}{4.18}$$

$$= 9 \times 10^{13} \text{ calories.}$$

Example 11.28 *Electrons are accelerated upto a kinetic energy of 10^9 eV. Find (i) the ratio of their mass to the rest mass (ii) the ratio of their velocity to the velocity of light and (iii) the ratio of their energy to the rest mass energy.*

Soln.

$$U = 10^9 \text{ eV} = 10^9 \times 1.6 \times 10^{-19} \text{ J}$$

Also $U = mc^2$

$$m = \frac{U}{c^2} = \frac{10^9 \times 1.6 \times 10^{-19} \text{ J}}{(3 \times 10^8)^2}$$

$$= 1.77 \times 10^{-27} \text{ kg.}$$

Rest mass of electron,

$$m_0 = 9 \times 10^{-31} \text{ kg.}$$

(i) $$\frac{m}{m_0} = \frac{1.77 \times 10^{-27}}{9 \times 10^{-31}} = 1.95 \times 10^3$$

(ii) $$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or, $$\frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1.95 \times 10^3 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \left[\frac{1}{1.95\times10^3}\right]^2$$

$$\frac{v^2}{c^2} = 1 - \left[\frac{1}{1.95\times10^3}\right]^2$$

$$\frac{v}{c} = \left[1-\left\{\frac{1}{1.95\times10^3}\right\}^2\right]^{1/2}$$

$$= \left[1-\frac{1}{2\times(1.95)^2\times10^6}\right]^2$$

$$= \left[1-1.315\times10^{-7}\right]$$

(iii) Rest mass energy,

$$U_0 = m_0c^2 = 9 \times 10^{-31} \times (3 \times 10^8)^2$$

$$= 8.1 \times 10^{-14}\ J$$

$$\frac{U}{U_0} = \frac{10^9\times1.6\times10^{-19}}{8.1\times10^{-14}} = 1.975 \times 10^3.$$

Example 11.30 An electron has a total energy of 2 MeV. Calculate the effective mass of the electron in kg and also its speed. Assume rest mass of the electron to be 0.511 MeV.

Soln.

Here $U = 2\text{ MeV} = 2 \times 10^6 \times 1.6 \times 10^{-19}\text{ J}$

$= 3.2 \times 10^{-13}\text{ J}$

$U = mc^2$

$$\therefore\ m = \frac{U}{c^2} = \frac{3.2\times10^{-13}}{(3\times10^8)^2} = 35.6 \times 10^{-31}\text{ kg.}$$

(ii) rest mass, $m_0 = \dfrac{U}{c^2}$

$$= \frac{0.511\times10^6\times1.6\times10^{-19}}{(3\times10^8)^2}$$

$= 9.1 \times 10^{-31}\text{ kg.}$

(iii) $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

or, $$\frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

or, $$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0}{m} = \frac{9.1\times10^{-31}}{35.6\times10^{-31}} = 0.2556$$

$$1-\frac{v^2}{c^2} = (0.2556)^2 = 0.06533$$

or, $$\frac{v^2}{c^2} = 1 - 0.06533 = 0.93467$$

or, $v^2 = 0.93467c^2$

or, $v = 0.96\ c.$

Example 11.32 Calculate the kinetic energy of an electron with a velocity of 0.98 c times the velocity of light in the laboratory system.

Soln.

When the electron moves with a velocity 0.98 c, its mass becomes

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$ where m_0 is the rest mass of the electron.

$$= \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = 5.02\ m_0$$

Relativistic formula for kinetic energy

$$T = (m - m_0)\ c^2 = (5.02\ m_0 - m_0)\ c^2$$

$$= 4.02\ m_0 \times c^2 = 4.02 \times 9.11 \times 10^{-31} \times (3 \times 10^8)^2\ J$$

$$= 3.396 \times 10^{-13}\ J.$$

***Example 11.34** Calculate the mass of the electron when it is moving with a K.E. of 10 MeV.*

Soln.

$$K.E. = (m - m_0)\,c^2 = 10\text{ MeV} = 10 \times 1.6 \times 10^{-13}\text{ J}$$

$$\therefore \quad m = m_0 + \frac{10 \times 1.6 \times 10^{-13}\text{ J}}{\left(3 \times 10^8\right)^2}$$

$$= 9.1 \times 10^{-31} + 176 \times 10^{-31}$$

$$= 185.1 \times 10^{-31}\text{ Kg.}$$

Example 11.12 The length of a spaceship is measured to be exactly half its actual length. Calculate (i) the speed of the spaceship and (ii) the time dilation corresponding to one second on the spaceship.

Soln.

(i) $L = L_O\sqrt{1-\frac{v^2}{c^2}}$

$\frac{L}{L_O} = \sqrt{1-\frac{v^2}{c^2}}$ Here $\frac{L}{L_O} = 0.5$

$\therefore \quad 0.5 = \sqrt{1-\frac{v^2}{c^2}}$ $c = 3 \times 10^8$ m/sec; v = ?

or, $\frac{v^2}{c^2} = 0.75$

$\therefore \quad v = \sqrt{0.75}\,.\,c$

$= 0.866\ c$

$= 0.866 \times 3 \times 10^8$ m/sec

$= 2.598 \times 10^8$ m/sec.

(ii) The time t as observed from the stationary frame corresponding to the time $t_0 = 1$ second on the spaceship is given by

$t_0 = t\sqrt{1-\frac{v^2}{c^2}}$

or, $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ Here, $t_0 = 1$ second

$\frac{v^2}{c^2} = 0.75$

$\therefore \quad = \frac{1}{\sqrt{1-0.75}}$ sec

$= 2$ sec.

Example 11.13 The proper length of a rod is 5 metres. What would be its length for an observer if it be moving relative to him in a direction parallel to its own length (i) with velocity 0.8c, (ii) moving with velocity c? What would be its length for an observer who is himself moving along with it?

Soln.

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$ where l_0 is the proper length of the rod.

(i) when $v = 0.8\ c$

$$l = l_0\sqrt{1-\frac{(0.8c)^2}{c^2}}$$

$$= 5\sqrt{1-0.64}$$

$$= 5\sqrt{0.36}$$

$$= 5 \times 0.6 = 3\text{m}.$$

(ii) when $v = c$

$$l = l_0\sqrt{1-\frac{v^2}{c^2}} = 5 \times \sqrt{1-1} = 5 \times 0 = 0\ \text{m}.$$

when the observer moves with the rod, $v = 0$

$$\therefore \quad l = l_0\sqrt{1-\frac{v^2}{c^2}} = l_0\sqrt{1-0} = 5 \times 1 = 5\ \text{m}.$$

Example 11.14 How fast would a rocket ship have to go relative to an observer for its length to be contracted to 99% of its length when at rest?

Soln.

Let l_0 = proper length

then $\quad l = 0.99\, l_0$

Therefore $\frac{l}{l_0} = \frac{0.99\, l_0}{l_0} = \frac{99}{100}$

From, $l = l_0 \sqrt{1 - v^2/c^2}$, we have

$$\frac{l}{l_0} = \frac{99}{100} = \sqrt{1 - \frac{v^2}{c^2}}$$

or, $1 - \frac{v^2}{c^2} = \left(\frac{99}{100}\right)^2$

or, $\frac{v^2}{c^2} = 1 - \left(\frac{99}{100}\right)^2 = 0.0199$

or, $v^2 = 0.0199\, c^2$

or, $v = 0.1416\, c$

$= 0.1416 \times 3 \times 10^8$ m/sec

$= 4.245 \times 10^7$ m/sec.

***Example 11.15** Obtain the volume of a cube, the proper length of each edge of which is l_0, when it is moving with a velocity v along one of its edges.*

Soln.

Obviously, the only change in length, (*i.e.*, a contraction) will occur in the particular edge of the cube along which it is moving. The lengths of the other edges, being perpendicular to the direction of motion, will remain unaffected.

If l_0 be the proper length of each edge of the cube, the length of the edge along which it is moving will become $l_0\sqrt{1 - v^2/c^2}$.

Since the length of the other two edges remain unaffected, the length of those edges will remain same as before, *i.e.*, l_0.

Therefore, the volume of the moving cube.

$$= l_0\sqrt{1 - v^2/c^2} \cdot l_0 \cdot l_0.$$