Radioactivity

Outline:

① Definition of radioactivity] Definition
② Properties of radioactivity]
③ Radioactive Disintegration law
$N = N_0 e^{-\lambda t}$ (expression)] Proof
④ Half life and expression
⑤ Mean life and expression
⑥ Properties of α, β, γ ray / α, β, γ decay] Definition.
⑦ Nucleus fission and fusion
⑧ Nuclear cross section

Definition of radioactivity: The phenomenon of spontaneous emission of powerful radiation exhibited by heavy elements is called radioactivity.

Radioactivity is essentially a nuclear phenomenon.

Those elements which activity are called radioactive elements. Such as uranium, polonium, radium, radon, etc.

Unit: SI unit of radioactivity is Becquerel (Bq)

One radioactive decay per second is called Becquerel.

$$1\ Bq = 1\ decay\,s^{-1}$$

$$1\ Ci = 3.7 \times 10^{10}\ Bq$$

The radioactive radiations emitted by these elements are found to consist of the following.

(i) α - particles

(ii) β - particles

(iii) γ - Particles

Properties of radioactivity:

① These radiations are highly penetrating, could effect Photographic plates.

② As radiations are given out, new elements are formed in an irreversible process the new elements themeselves being usually radioactive

(3) The emission of radiations is spontaneous and is not affected by external agents.

(4) The emission is not instantaneous but is prolonged i.e. it is extended over a period of time. Otherwise it would not have been discovered at all.

(5) Except for radioactivity, there is nothing abnormal about the radioactive elements as regards their physical and chemical properties.

Radioactive distintegration law: ($N = N_0 e^{-\lambda t}$)

Radioactive disintegration is found to obey the following laws:

(1) Atoms of all radioactive elements undergo spontaneous distintegration to from fresh radioactive products with the emission of α-, β- and γ-rays.

(2) The number of disintegrations per second is not affected by environmental factors (like temperature, pressure, and chemical combination etc) but depends on the number of the atoms of the original kind present at time.

Here,

N_0 = Number of radioactive atoms present in a sample at the beginning of distintegration i.e. $t=0$

N = Number of radioactive atoms present in a sample at any time t.

dN - Number of radioactive atoms present in a sample at a time dt.

$$\cancel{dN \propto -N}$$

$$\Rightarrow \cancel{\frac{dN}{dt}} \quad \frac{dN}{dt} \propto -N$$

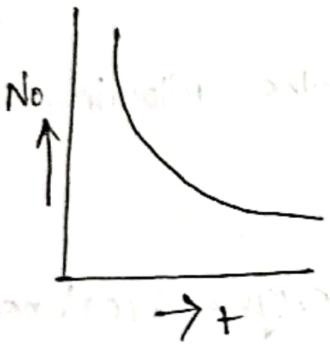
$$\Rightarrow \frac{dN}{dt} = -\lambda N \quad \text{—— (1)}$$

$$\Rightarrow \frac{dN}{N} = -\lambda dt$$

$$\Rightarrow \int \frac{dN}{N} = -\lambda \int dt$$

$$\Rightarrow \ln N = -\lambda t + c$$

where, λ is the constant of Proportionality and is known as the disintegration constant.



where c is constant.

when $t=0$, $N = N_0$,

$$\therefore \ln N_0 = c$$

Substituting this value of c in eq^n (1)

$$\ln N = -\lambda t + \ln N_0$$

$$\Rightarrow \ln \frac{N}{N_0} = -\lambda t$$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore N = N_0 e^{-\lambda t}$$

$\therefore$ This equation represents the laws of radioactive disintegration.

Half ~~time~~ life and expression:

The half period of a radioactive substance is defined as the time required for one-half of the radioactive substance to disintegrate.
The half life period is different for different substance and depends upon the radioactive constant of the substance.

Expression: We know, $N = N_0 e^{-\lambda t}$ —— (1)

Here, N_0 = The number of radioactive atoms present in a sample at the beginning of disintegration, $t = 0$.

N = The number of radioactive atoms present in a sample at any time t.

λ = Disintegration constant.

half-life period $= T_{1/2}$

Now, if $t = T_{1/2}$, then, $N = \frac{N_0}{2}$

From eqn (1)

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow \ln\left(\frac{1}{2}\right) = -\lambda T_{1/2}$$

$$\Rightarrow \ln 1 - \ln 2 = -\lambda T_{1/2}$$

$$\Rightarrow \ln 2 = \lambda T_{1/2}$$

$$\Rightarrow T_{1/2} = \frac{\ln 2}{\lambda}$$

$$\Rightarrow T_{1/2} = \frac{0.693}{\lambda}$$

The above equation represents the expression for half-life period.

As seen $T_{1/2}$ is inversely proportional to the radioactive constant.

Mean life and expression: The mean life of a radioactive element defined as the ratio of the total life time of all the radiactive atoms to the total member of such atoms in it.

The mean life of a radioactive element,

$$= \frac{\text{Sum of the lives of all atoms.}}{\text{Total number of atoms}}$$

Total life of dN atoms = (dN)t

The possible life of any of the total Number N_0 radioactive atoms ~~avi~~ varies from 0 to ∞

$\therefore$ Total life time of all N_0 atoms = $\int_0^\infty t\,dN$

Now, mean life = $\bar{\tau} = \frac{\text{total life-time}}{\text{total number of atoms}}$

$$= \frac{\int_0^\infty t\,dN}{N_0}$$

Now, $N = N_0 e^{-\lambda t}$

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$$

$$\Rightarrow dN = -\lambda N_0 e^{-\lambda t} dt$$

$$\Rightarrow dN = \lambda N_0 e^{-\lambda t} dt$$

Hence, $$\bar{\tau} = \frac{\int_0^\infty \lambda N_0 e^{-\lambda t} dt}{N_0}$$

$$= \lambda \int_0^\infty t e^{-\lambda t} dt$$

Integrating by parts, $$\bar{\tau} = \left[\frac{te^{-\lambda t}}{-\lambda} - \int \frac{e^{-\lambda t} dt}{-\lambda} \right]$$

$$\Rightarrow \bar{\tau} = \lambda \left[\frac{te^{-\lambda t}}{-\lambda} - \frac{e^{-\lambda t}}{-\lambda^2} \right]$$

$$= \frac{1}{\lambda}$$

$$\therefore \bar{\tau} = \frac{1}{\lambda}$$

Alpha radioactivity /particle / Rays / Decay:

A positively charged particle that consists of two Protons and two neutrons bound together. It is emitted by an atomic nucleus undergoing radioactive decay and is identical to the nucleus of a helium atom.

→ They can be stopped by a piece of paper.

→ As its range is less than a tenth of a milimeter its not suitable for radiation therapy.

→ It has great destructive power in short range.

Gammaradioactivity / Gamma rays / Decay / Particle:

A stream of high-energy electromagnetic radiation given off by an atomic nucleus undergoing radioactive decay.

→ As the wavelength of gamma rays are shorter than x-rays, it has greater energy and penetrating Power than x-rays.

→ It is most useful types of radiation for medical purpose.

→ It is most dangerous because of its ability to Penetrate large thickness of material.

Beta radioactivity / rays / decay / particle:

Beta particle are just electrons from the nucleus - the term beta particle being an historical term used in the early description of radioactivity.

⇒ The high energy electrons have greater range of Penetration than alpha particles, but still much less than gamma rays.

⇒ The radiation hazard from beta is greatest if they are ingested.

Nuclear fission: The process of breaking up of the nucleus of a heavy atom into two, more or equal fragments with release of a large amount of energy is known as fission.

The schematic equation for the fission process is

$$_{92}U^{235} + {_0n^1} \longrightarrow {_{92}U^{236*}} \longrightarrow X + Y + \text{neutrons}$$

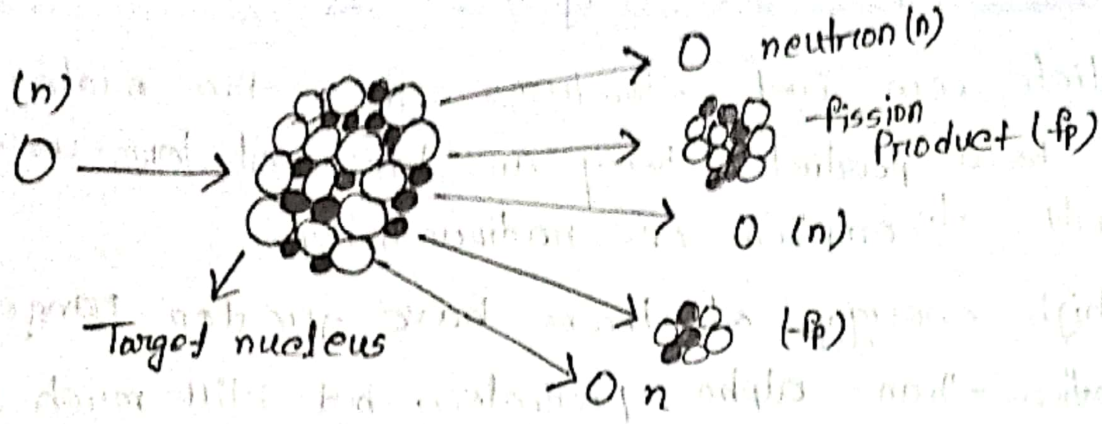
Here, $_{92}U^{236*}$ is highly unstable isotope.

Typical fission reactions are,

$$_{92}U^{235} + {_0n^1} \longrightarrow {_{92}U^{236*}} \longrightarrow {_{56}Ba^{141}} + {_{36}Kr^{92}} + 3\,{_0n^1} + Q$$

$$_{92}U^{235} + {_0n^1} \longrightarrow {_{92}U^{236*}} \longrightarrow {_{56}Xe^{140}} + {_{38}Sr^{94}} + 2\,{_0n^1} + Q$$

where Q is the energy released in the reaction.



Nuclear fusion: The process in which two or more light nucleus are combined into a single nucleus with the release of tremendous of energy is called as nuclear fusion.

- The sum of masses before the fusion is more than the sum of masses after the fusion and this difference appears as the fusion energy.

The most typical fusion Reaction is the fusion of two deutrium nuclei into helium.

$$_1H^2 + _1H^2 \longrightarrow _2He^4 + 21.6 \text{ MeV}.$$

Nuclear Cross section:

A measure of the probability for a reaction to occur between a nucleus and a particle, it is an area such that the number of reactions which occur in a sample exposed to a beam of particles equals the product of the number of nuclei in the sample and the number of incident particles which would pass through this area if their velocities were perpendicular to it.

Example 16.1 The half-life of a radioactive substance is 30 days. Calculate (i) the radioactive decay constant, (ii) the mean life (iii) the time taken for ¾ of the original number of atoms to disintegrate and (iv) the time for 1/8 of the original number of atoms to remain unchanged.

Soln:

(i) $T_{1/2} = 30$ days

$$\therefore \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{30\,d} = 0.0231 \text{ per day}$$

(ii) Mean life, $\tau = \frac{1}{\lambda} = \frac{1}{0.0231 d^{-1}} = 43.29$ days

(iii) From $N = N_0 e^{-\lambda t}$, we have

$$\frac{1}{4}N_0 = N_0 e^{-\lambda t} \quad \text{where} \quad N = N_0 - \frac{3N_0}{4} = \frac{1}{4}N_0$$

or $$\frac{\frac{1}{4}N_0}{N_0} = e^{-\lambda t}\ ; \quad \text{or} \quad e^{-\lambda t} = \frac{1}{4}$$

or $\quad e^{\lambda t} = 4 \;;\quad \lambda t = \log_e 4$

$$\therefore \quad t = \frac{\log_e 4}{\lambda} = \frac{}{0.0231} = 60 \text{ days}$$

(iv) The number of atoms left, $N = \frac{1}{8} N_0$

$$\therefore \quad \frac{N}{N_0} = \frac{1}{8} = e^{-\lambda t}$$

or $\quad e^{\lambda t} = 8$

$$\therefore \quad \lambda t = \log_e 8 \;;\text{ or }\; t = \frac{\log_e 8}{0.0231} = \frac{}{0.0231} = 90 \text{ days}$$

Example 16.2 The half-life of radium is 1620 years. In how many years will one gram of pure element (i) lose one centigram and (ii) be reduced to one centigram?

Soln.

The decay constant of radium is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1620 \text{ years}} = 4.28 \times 10^{-4}\, y^{-1}$$

(i) Let t be the time during which one centigram of radium is lost due to disintegration. The amount remaining is $(1 - \frac{1}{100}) = 0.99$ gm

From $N = N_0 e^{-\lambda t}$, we have

$$\frac{N}{N_0} = e^{-\lambda t};\text{ or } \frac{0.99\, N_0}{N_0} = e^{-\lambda t}$$

or $\quad e^{-\lambda t} = 0.99 = \frac{99}{100}$

$$e^{\lambda t} = \frac{100}{99} \;;\text{ or }\; \lambda t = \log_e\left(\frac{100}{99}\right)$$

Example 16.6 *A counter rate meter is used to measure the activity of a radioactive sample. At a certain instant, the count rate was recorded as4750 counts per minute. Five minutes later, the count rate recorded was 2700 counts per minute. Compute (i) the decay constant and (ii) the half-life of the sample.*

Soln.

(a) $N = N_0 e^{-\lambda t}$

or $\dfrac{N}{N_0} = e^{-\lambda t}$; $e^{\lambda t} = \dfrac{N_0}{N}$

Now $N_0 = dN_1/dt = 4750$

and $N = dN_2/dt = 2700$

$$\therefore \quad e^{5\lambda} = \frac{4750}{2700}$$

or $$5\lambda = \log e \left(\frac{4750}{2700}\right)$$

or $$\lambda = \frac{\log_e(1.76)}{5} = 0.113 \text{ per minute.}$$

(b) $$T_{1/2} = \frac{0.693}{0.113} = 6.1 \text{ minutes.}$$

Example 16.7 Find the activity of 1mg (10^{-3}gm) of radon (R_n^{222}). The half-life of radon is 3.8 days.

Soln.

$$\lambda = \frac{0.693}{3.8\times24\times3600} = 2.1\times10^{-6}\ s^{-1}$$

Number of atoms in 10^{-3}gm,

$$N = \frac{10^{-3}\times6.02\times10^{23}}{222}$$

So activity, $A = \frac{dN}{dt}$ (ignoring the minus sign)

$$= \lambda N$$

$$= \frac{2.1\times10^{-6}\times10^{-3}\times6.02\times10^{23}}{222}$$

$= 5.7 \times 10^{12}$ disintegration per second

$$= \frac{5.7\times10^{12}}{3.7\times10^{10}} = 153 \text{ Ci}$$

$$= \frac{5.7\times10^{12}}{10^{6}} = 5.7 \times 10^{6} \text{ rd}$$

$= 5.7 \times 10^{3}$ GBq.

Example 16.8 Some amount of a radio-active substance of half-life 30 days is spread inside a room. Consequently the level of radiation inside the room became 50 times the permissible level for normal occupancy of the room. After how many days the room would be safe for occupation?

Soln.

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = \frac{1}{50}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\lambda = \frac{0.693}{30} d^{-1} = 0.0231 d^{-1}$$

$$e^{\lambda t} = \frac{N_0}{N} = 50$$

$$\lambda t = \ln(50) = 3.912$$

$$t = \frac{3.912}{0.0231} = 169.35 \text{ days}$$

Example 16.10 A piece of an ancient wood boat shows an activity of C^{14} of 3.9 disintegrations per minute per gram of carbon. Estimate the age of the boat if the half-life of C^{14} is 5568 years. Assume that the activity of fresh C^{14} is 15.6 disintegrations per minute per gram.

Soln.

Let the age of the boat be t years.

From $N = N_0 e^{-\lambda t}$, we have

$$\frac{N}{N_0} = e^{-\lambda t}$$

or $$e^{\lambda t} = \frac{N_0}{N} = \frac{15.6}{3.9}$$

or $$\lambda t = \ln\left(\frac{15.6}{3.9}\right)$$

or $$t = \frac{\ln\left(\frac{15.6}{3.9}\right)}{\lambda}$$

$$= \frac{\ln\left(\frac{15.6}{3.9}\right)}{1.24\times10^{-4}\,y^{-1}} = 1.118 \times 10^4 \text{ yrs.}$$

Here

activity $\lambda N = 3.9$

and $\lambda N_0 = 15.6$

$$\therefore \quad \frac{\lambda N_0}{\lambda N} = \frac{N_0}{N} = \frac{15.6}{3.9}$$

$$\lambda = \frac{0.693}{5568} y^{-1} = 1.24 \times 10^{-4} y^{-1}.$$

$$\therefore \quad t = \frac{\log_e(100/99)}{\lambda} = \frac{}{4.28\times10^{-4}\ \text{years}} = 23.68 \text{ years}$$

(ii) Now N = 0.01 gm

$$\therefore \quad \frac{N}{N_0} = 0.01 = \frac{1}{100} = e^{-\lambda t}$$

or $\quad e^{\lambda t} = 100 \qquad \therefore \lambda t = \log_e 100$

$$\therefore \quad t = \frac{\log_e 100}{\lambda} = \frac{}{4.28\times10^{-4}\ \text{years}} = 10{,}760 \text{ years}$$

Example 16.3 1gram of radium is reduced by 2.1 mg in 5 years by α-decay. Calculate the half-life of radium.

Soln.

The amount of radium left at the end of 5 years is, $N = 1 - 2.1\times10^{-3}$ $= 1 - 0.0021 = 0.9979$ gm

From $\dfrac{N}{N_0} = e^{-\lambda t}$ we have

$$\frac{0.9979}{1.0} = e^{-\lambda t} = 0.9979$$

Now t = 5 years

$$e^{-5\lambda} = 0.9979 \ ; \ \text{ or } \ e^{5\lambda} = \frac{1}{0.9979}$$

or $\quad 5\lambda = \log_e\left(\dfrac{1}{0.9979}\right)$

$$\therefore \quad \lambda = \frac{\log_e(1/0.9979)}{5\ y} = \frac{}{5\ y} = 41.4468\times10^{-5}\ y$$

$$\therefore \quad T_{1/2} = \frac{0.6931}{41.4468\times10^{-5}\ y} = 1672 \ \text{ years.}$$