To prove the maximum power theorem, we differentiate eqn (1) w.r.t. to R_L and set the result equal to zero. We obtain,

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

$$\Rightarrow 0 = V_{Th}^2 \frac{d}{dR_L} \cdot \frac{R_L}{(R_{Th}+R_L)^2}$$

$$= V_{Th}^2 \frac{(R_{Th}+R_L)^2 - R_L \cdot 2(R_{Th}+R_L)(0+1)}{(R_{Th}+R_L)^4}$$

$$= V_{Th}^2 \frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}$$

$$= V_{Th}^2 \frac{(R_{Th}+R_L)(R_{Th}+R_L-2R_L)}{(R_{Th}+R_L)^4}$$

$$= V_{Th}^2 \frac{R_{Th}-R_L}{(R_{Th}+R_L)^3}$$

$$\Rightarrow R_{Th} - R_L = 0$$

$$\Rightarrow R_L = R_{Th}$$

So, $P_{max} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$

$$= \frac{V_{Th}^2}{(R_{Th}+R_{Th})^2} \times R_{Th} \quad [\because R_{Th} = R_L]$$

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$$= \frac{V_{Th}^2}{4R_{Th}^2} \times R_{Th}$$

$$\therefore P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

(proved)

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AC CIRCUIT

Definitions

1) Average value of an alternating current

The average value of an alternating current is the average of all the instantaneous values ~~during~~ of an alternating current over one complete cycle.

2) RMS value of an alternating current

The RMS value of an alternating current is the dc current that delivers the same average power to a resistor as the alternating current.

The RMS value of an alternating current,

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

3) Instantaneous value

The instantaneous value of an alternating voltage or current is the value of voltage or current at one particular instant.

4) Peak value

The maximum value attained by an alternating voltage or current during one complete cycle is called its peak value. It is also known as the maximum value or amplitude or crest value.

5) Peak to Peak value

During each complete cycle of alternating voltage or current, there are always two maximum or peak values, one for the positive half-cycle and the other for the negative half-cycle. The differences between the peak positive value and the peak negative value is called the peak-to-peak value.

6) Frequency

The number of cycles per unit of time is called the frequency. It is denoted by 'f' or 'n'. The unit of frequency is cycle/s or Hz.

7) Phase

Phase is the initial angle of a sinusoidal function at its origin.

8) Apparent power

The apparent power (in VA) is the product of the rms values of voltage and current. It is measured in volt-amperes (VA) to distinguish it from the average or real power.

9) Power factor

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedence.

If the voltage and current at the terminals of a circuit are,

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

or, in phasor form,

$$V = V_m \angle\theta_v$$
$$I = I_m \angle\theta_i$$

The average power, $P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$$= V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Here, the product $V_{rms} I_{rms}$ is known as apparent power and the factor $\cos(\theta_v - \theta_i)$ is called the power factor (pf).

10) Active and Reactive power

The portion of power that, averaged over a complete cycle of the AC waveform, results in net transfer of energy in one direction is known as active power (real power).

The portion of power due to stored energy, which returns to the source in each cycle, is known as reactive power.

11) Impedance

The impedance (Z) of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms (Ω).

$$Z = \frac{V}{I} = R + jX$$

where $R = Re, Z$ is the resistance and $X = Im$.

12) Admittance

The admittance (Y) is the reciprocal of impedance (Z), measured in siemens (S).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

13) Reactance

Reactance is the oppsition of a circuit element to a change in current or voltage due to that element's inductance or capacitance.

Reactance of an inductor ~~is~~ is denoted by X_L.

$$X_L = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

Reactance of a capacitor is denoted by X_C.

$$X_C = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

14) Phasor

A phasor is a complex number that represents the amplitude and phase of a sinusoid.

It can be represented in three types of form.

→ Rectangular : $z = x + jy = r(\cos\theta + j\sin\theta)$

→ Polar : $z = r\angle\theta$

where,

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y/x)$$

→ Exponential : $z = re^{j\theta}$

Derive that the Phasor relationship for capacitor is $V = \frac{I}{j\omega C}$ and show that the phasor current of a capacitor leads the voltage by 90 degree.

For a capacitor, let us assume that the voltage through it,

$$v = V_m \cos(\omega t + \theta)$$

The current through the capacitor is,

$$i = C\frac{dv}{dt}$$

$$= C\frac{d}{dt} V_m \cos(\omega t + \theta)$$

$$= -\omega C V_m \sin(\omega t + \theta)$$

$$= \omega C V_m \cos(\omega t + \theta + 90^\circ)$$

$$= Re(\omega C V_m e^{j(\omega t + \theta + 90^\circ)})$$

Tranforming into phasors,

$$I = \omega C V_m e^{j(\theta + 90^\circ)}$$

$$= \omega C V_m e^{j\theta} \cdot e^{j90^\circ}$$

$$= j\omega C V_m e^{j\theta} \quad [\because e^{j90^\circ} = j]$$

$$= j\omega C V \quad [\because V_m e^{j\theta} = V_m \angle\theta = V]$$

$$\therefore V = \frac{I}{j\omega C}$$

Showing that the current and voltage are out of phase and current leads voltage by 90°.

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⊞ Show that $V = j\omega L I$.

For an inductor, let us assume that the current through it,

$$i = I_m \cos(\omega t + \phi)$$

The voltage accross it,

$$v = L\frac{di}{dt}$$

$$= L\frac{d}{dt} I_m \cos(\omega t + \phi)$$

$$= -\omega L I_m \sin(\omega t + \phi)$$

$$= \omega L I_m \cos(\omega t + \phi + 90^\circ)$$

$$= Re(\omega L I_m e^{j(\omega t + \phi + 90^\circ)})$$

Transforming into phasor,

$$V = \omega L I_m e^{j(\phi + 90^\circ)}$$

$$= \omega L I_m \cdot e^{j\phi} \cdot e^{j90^\circ}$$

$$= j\omega L I_m e^{j\phi} \quad [\because e^{j90^\circ} = j]$$

$$\Rightarrow V = j\omega L I \quad [\because I_m e^{j\phi} = I_m \angle\phi = I]$$

We can see that the current lags the voltage by 90°.

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▣ Draw the Phasor relationship for circuit elements: Capacitor and Inductor.

For a capacitor,

$$v = V_m \cos(\omega t + \phi)$$
$$i = I_m \cos(\omega t + \phi + 90^\circ)$$

So, current leads the voltage by 90°.

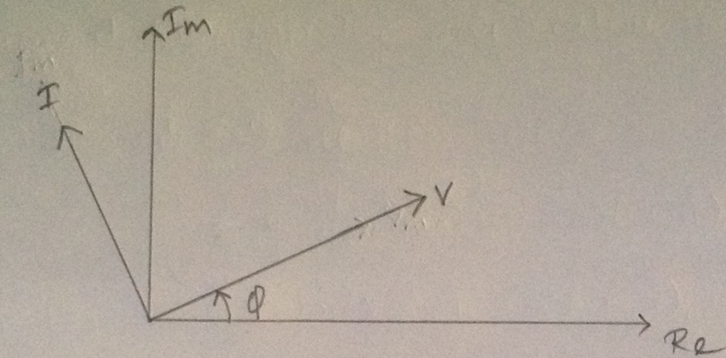

Figure: Phasor diagram for capacitor.

For an inductor,

$$i = I_m \cos(\omega t + \phi)$$
$$v = V_m \cos(\omega t + \phi + 90^\circ)$$

So, voltage leads the current by 90°

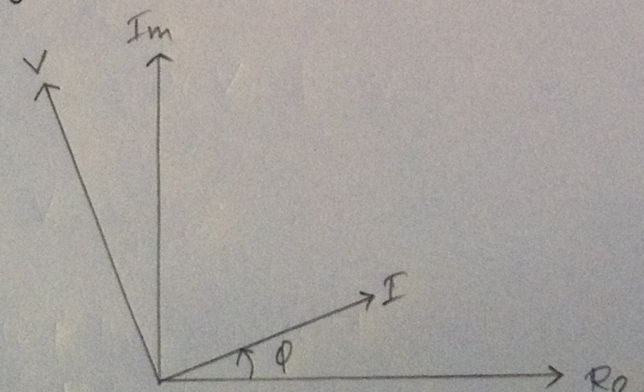

Figure: Phasor diagram for Inductor.

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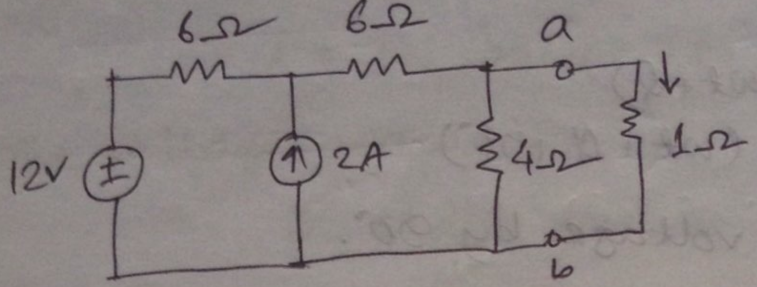
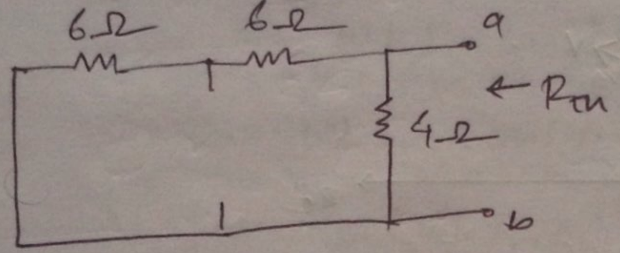
CIRCUIT PROBLEMS

Prac - 4.8

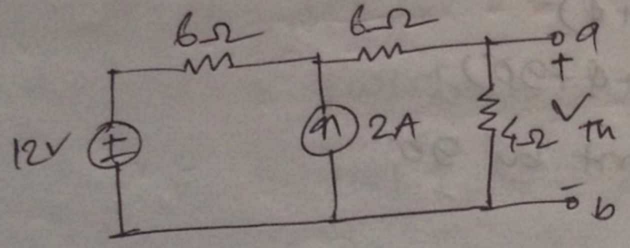


For obtaining R_{TH},

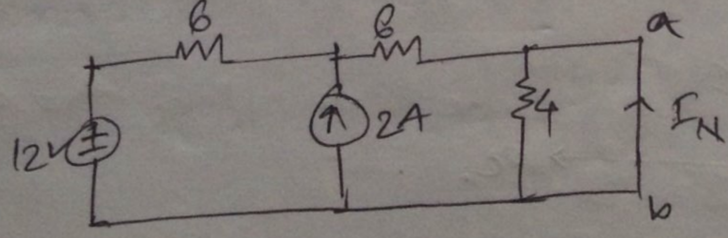


$$R_{TH} = (6+6) \parallel 4 \quad = 3\Omega$$

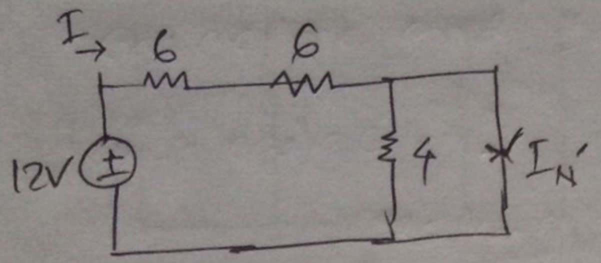
For obtaining V_{TH},



We will use Norton analysis



When 12V is active,



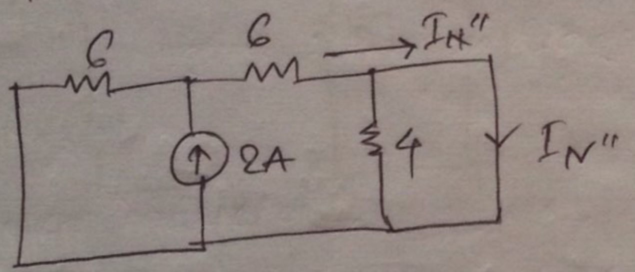
Current won't flow through 4Ω as there is a short circuit across it.

$$R' = 6+6 = 12$$

$$\therefore I = \frac{V}{R} = \frac{12}{12} = 1A$$

So, $\downarrow I_N' = I = 1A$

When 2A is active,



Current won't flow through 4Ω.

$$\therefore \downarrow I_N'' = \frac{6}{6+6} \times 2$$

$$= 1A$$

$$\therefore I_N = I_N' + I_N'' = 1A + 1A = 2A$$

$$\therefore V_{Th} = I_N R_{Th}$$

$$= 2 \times 3$$

$$= 6V$$

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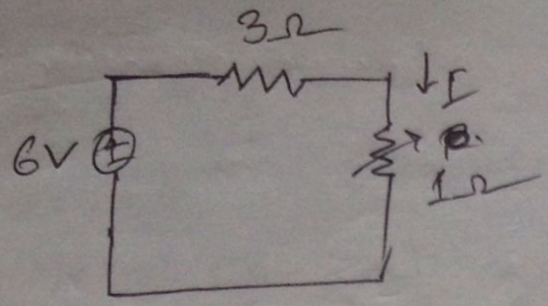
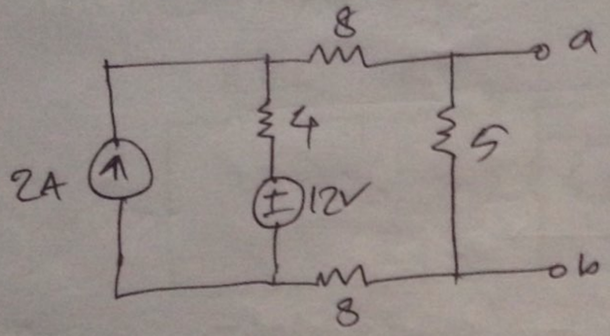
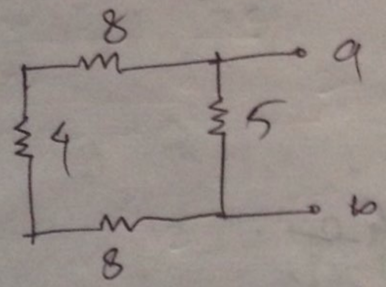


$$\therefore I = \frac{6}{3+1} = 1.5A$$

Prac-4.11



R_N



$$R_N = (8+4+8) \parallel 5$$
$$= 4\Omega$$

I_N

when 2A is active

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