Final note (Fourier Analysis)

09.09.2023

Q-1: Define signal, Stationary Signal, DC signal.

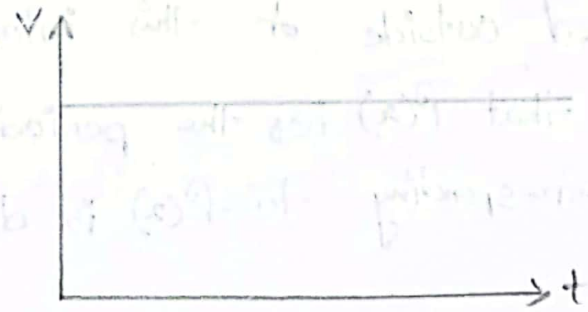
⇒ **Signal:** Signal is the physical quantity that is measurable. Signals are functions of time. System is a physical extity that exists. Signal is produced from a system.

Stationary Signal: Stationary signals are further divided into deterministic and random signals.

Random signals are unpredictable in their frequency content and their amplitude level, but they still have relatively uniform stastical characteristics over time.

DC Signal: In electronic circuits things happen. Voltage/time, V/t, graphs provide a useful method of describing the changes which take place.

The diagram below shows the V/t graph, which represents a DC signal.



Direct current (DC) is produced by sources such as batteries, thermcouples, Solar cells etc.

Q-2: What is periodic Signal? Write the characteristics of periodic signal.

⇒ A Signal which is repeating itself is a periodic signal.

Periodic Signal characteristics:-

i. Amplitude (A): Signal value, measured in volts.

ii. Frequency (f): repletion rate, cycles per second or Hertz.

iii. Period (T): Amount of time it takes for one repetition

iv. Phase (ϕ): Relative position in time, measured in degrees.

Q-3: Define fourier series in the interval $(-L, L)$

Soln: let $f(x)$ be defined in the interval $(-L, L)$ and determined outside of this interval by $f(x+2L) = f(x)$

i.e. assume that $f(x)$ has the period $2L$. The fourier expension corresponding to $f(x)$ is defined by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos \frac{n\pi x}{L} + b_n \cos \frac{n\pi x}{L}\right)$$

Where the fourier coefficients a_n and b_n are given by

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x) . \cos \frac{n\pi x}{2L}\, dx$$

and $$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin \frac{n\pi x}{2L}\, dx$$

Where $n = 0, 1, 2, ----$

Q-4: Math Example-12

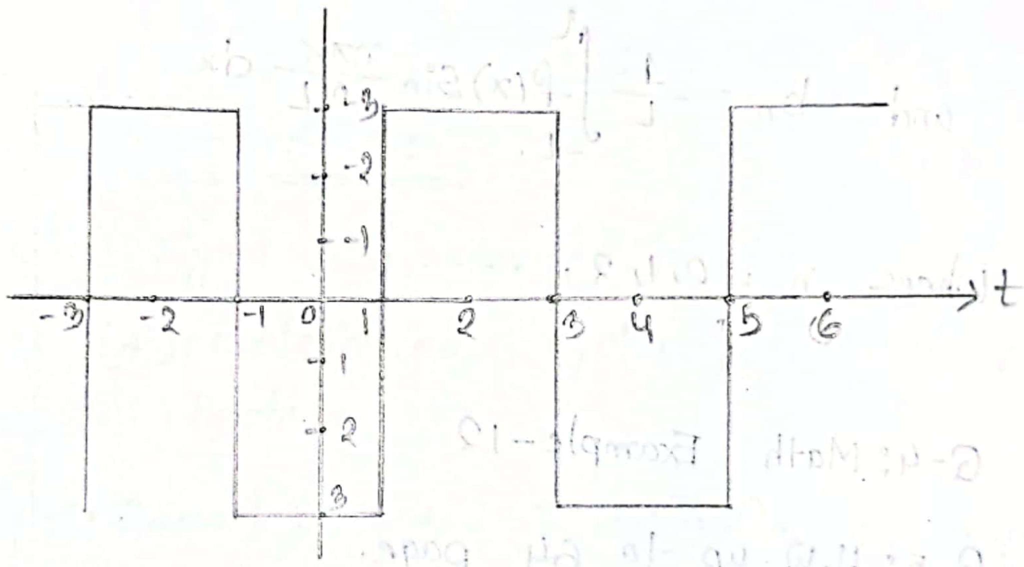
Q-5: H.W up to 64 page.

Example :- 6: $f(t) = -3 \; ; \; -1 \le t < 1$

$= 3 \; ; \; 1 \le t < 3$

$f(t) = f(t+4)$ [This period is 4]

$f(t)$



Period $= P = 2L = 4$

fig: 08

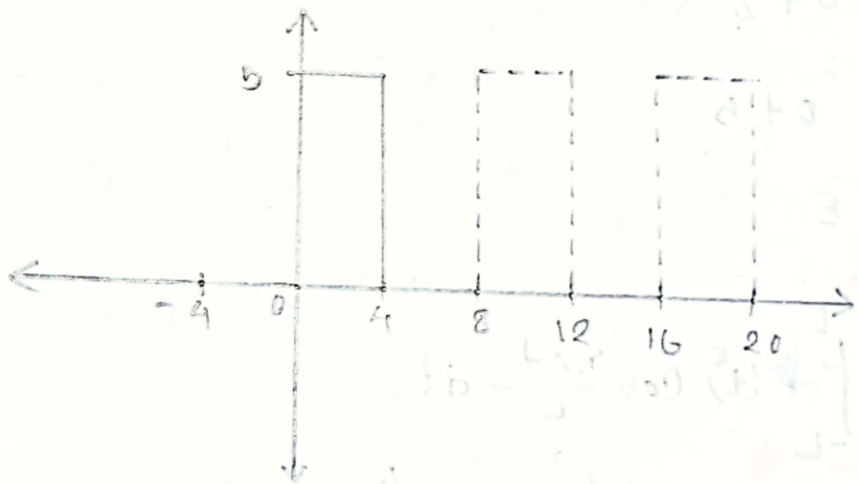
Example - 22: $y = f(t) = \begin{cases} 0 \; ; & -4 \le t \le 0 \\ 5 \; ; & 0 \le t \le 4 \end{cases}$

$f(t) = f(t+8)$ Here $T = 2L = 8 \quad \therefore L = 4$

a) Sketch the function for 3 cyles:

b) Find the fourier series for the function.

a) Soln:



b) Soln: We know the fourier series in the interval $(-L, L)$ for $f(t)$ is given by

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right) \text{ ------ (i)}$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt$$

$$= \frac{1}{4}\int_{-4}^{4} f(t)\,dt$$

$$= \frac{1}{4}\int_{-4}^{0} f(t)\,dt + \frac{1}{4}\int_{0}^{4} f(t)\,dt$$

$$= \frac{1}{4}\int_{-4}^{0} 0\,dt + \frac{1}{4}\int_{0}^{4} 5\,dt$$

$$= 0 + \frac{5}{4}(4-0)$$

$$= 0 + 5$$

$$= 5$$

Again,

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}\,dt$$

$$= \frac{1}{4}\int_{-4}^{0} 0.\cos\frac{n\pi t}{4}\,dt + \frac{1}{4}\int_{0}^{4} 5\cos\frac{n\pi t}{4}\,dt$$

$$= \frac{1}{4}.0 + \frac{5}{4}\cdot\frac{4}{n\pi}\left[\sin\frac{n\pi t}{4}\right]_0^4$$

$$= 0 + \frac{5}{n\pi}(\sin n\pi - \sin 0)$$

$$= 0$$

Again,

$$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{n\pi t}{L}\,dt$$

$$= \frac{1}{4}\int_{-4}^{0} 0\sin\frac{n\pi t}{4}\,dt + \frac{1}{4}\int_{0}^{4} 5\sin\frac{n\pi t}{4}\,dt$$

$$= 0 + \frac{5}{4}\cdot\frac{4}{n\pi}\left[-\cos\frac{n\pi t}{4}\right]_0^4$$

$$= \frac{5}{n\pi}\left[-\cos n\pi + \cos 0\right]$$

$$= \frac{-5}{n\pi}\left[\cos n\pi - 1\right]$$

The fourier series for above function is

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right)$$

$$= \frac{5}{2} + \sum_{n=1}^{\infty}\left(0 + \frac{-5}{n\pi}\left[\cos n\pi - 1\right]\sin\frac{n\pi x}{4}\right)$$

$$= \frac{5}{2} - \frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{5}\left(\left[\cos n\pi - 1\right]\sin\frac{n\pi x}{4}\right)$$

(Ans)

12.09.2023

Problem: 10 (Only - Formula)

$$a\cos\theta + b\sin\theta = R\sin(\theta+\alpha)$$

$a \rightarrow$ Amplitude of the cosine wave

$b \rightarrow$ Amplitude of the Sine wave

$R \rightarrow$ Amplitude of the new Signal

$\theta \rightarrow$ Phase Shift.

$$\sin(A+B) = \sin A\cos B + \cos A\sin B \text{ ——— (iii)}$$

$$R\sin(\theta+\alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$$

$$a\cos\theta + b\sin\theta = R\cos\theta\sin\alpha + R\sin\theta\cos\alpha$$

$$a = R\sin\alpha \text{ ——— (i)}$$

$$b = R\cos\alpha \text{ ——— (ii)}$$

From (i) & (ii)

$$\frac{a}{b} = \frac{R\sin\alpha}{R\cos\alpha}$$

or, $\frac{a}{b} = \tan\alpha$

or, $\tan\alpha = \frac{a}{b}$

or, $\alpha = \tan^{-1}(a/b)$

Again,

from (i) & (ii) Squaring and then adding,

$$a^2+b^2 = R^2\sin^2\alpha + R^2\cos^2\alpha$$

or, $a^2+b^2 = R^2(\sin^2\alpha + \cos^2\alpha)$

or, $a^2+b^2 = R^2.1$ $[\sin^2\alpha + \cos^2\alpha = 1]$

or, $a^2+b^2 = R^2$

or, $R^2 = a^2+b^2$

$\therefore R = \sqrt{a^2+b^2}$

Example - 32

• $x = \frac{1}{4}\cos 8t + \frac{1}{8}\sin 8t$. What is the amplitude of the resultant signal?

Soln: Given that,

$$x = \frac{1}{4}\cos 8t + \frac{1}{8}\sin 8t \text{ ------ (i)}$$

Now comparing (i) with

$$a\cos\theta + b\sin\theta = R\sin(\theta+\alpha)$$

We get,

$$a = \frac{1}{4} \quad \& \quad b = \frac{1}{8}$$

Now,

the Required new signal,

$$R = \sqrt{a^2+b^2}$$

$$\text{or, } R = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{8}\right)^2}$$

$$\text{or, } R = \sqrt{\frac{1}{16} + \frac{1}{64}}$$

$$\text{or, } R = \sqrt{\frac{4+1}{64}}$$

$$\text{or, } R = \sqrt{\frac{5}{64}}$$

$$\therefore R = \frac{\sqrt{5}}{8}$$

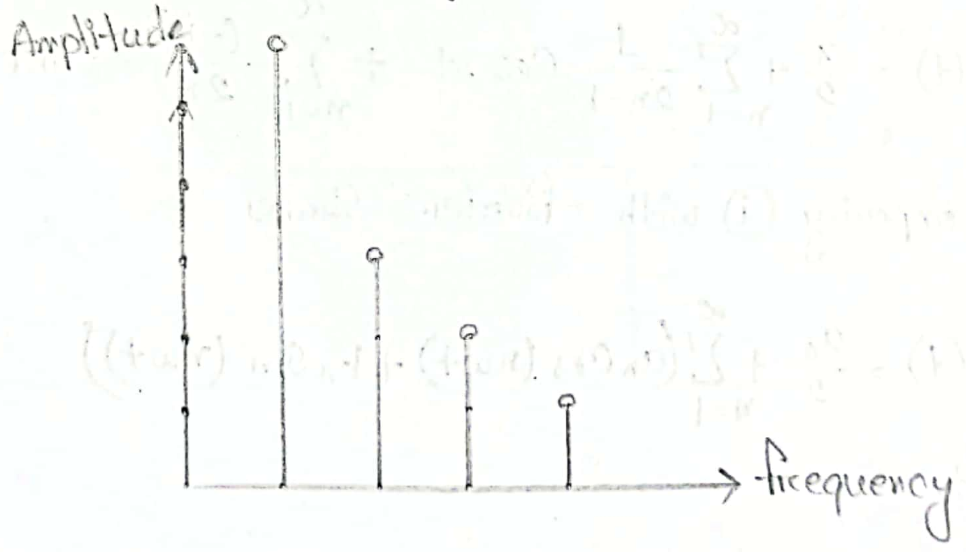
(Ans)

Problem- 12

Q: Define Line Specturm or amplitude specturm and Phase specturm (must আসবে).

Line Specturm or amplitude specturm: A plot of amplitude against angular frequency is called the line specturm or amplitude specturm.

Phase Specturm: While that of phase against angular frequency is called the phase specturm.



Ex-33 আসবে না কিন্তু explanation গুলো জানা দরকার

Example - 34

- Plot the line Specturem (discrete frequency spectra) for the fourier Series:

$$f(t) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{1}{2n-1} \cos nt + \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \sin t$$

Sol^n: Given that,

the fourier Series

$$f(t) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{1}{2n-1} \cos nt + \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \sin t \quad \text{----} (i)$$

Now comparing (i) with fourier Series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$

We get,

$$a_n = \frac{1}{2n-1}$$

and $$b_n = \frac{(-1)^n}{2n}$$

We know that,

$$R = \sqrt{a^2+b^2}$$

$$\therefore R_n = C_n = \sqrt{a_n^2+b_n^2}$$

$a_n = \frac{1}{2n-1}$	$b_n = \frac{(-1)^n}{2n}$	$C_n = \sqrt{a_n^2+b_n^2}$
$a_1 = 1$	$b_1 = \frac{1}{2}$	$C_1 = \sqrt{(1)^2+(1/2)^2}$ $= 1.118$
$a_2 = \frac{1}{3}$	$b_2 = \frac{1}{4}$	$C_2 = \sqrt{(\frac{1}{3}) + (1/4)^2}$ $= 0.4167$
$a_3 = \frac{1}{5}$	$b_3 = -\frac{1}{6}$	$C_3 = \sqrt{(1/5)^2+(-1/6)^2}$ $= 0.260$
$a_4 = \frac{1}{7}$	$b_4 = \frac{1}{8}$	$C_4 = \sqrt{(\frac{1}{7})^2+(1/8)^2}$ $= 0.190$

Again for frequency,

From eq^n (i) we have the frequency $= n\omega$

When $n=1$ then the 1^{st} frequency $\omega = 1$

" $n=2$ " " 2^{nd} " $2\omega = 2$

" $n=3$ " " 3^{rd} " $3\omega = 3$

" $n=4$ " " 4^{th} " $4\omega = 4$

Hence the required line Specturm is

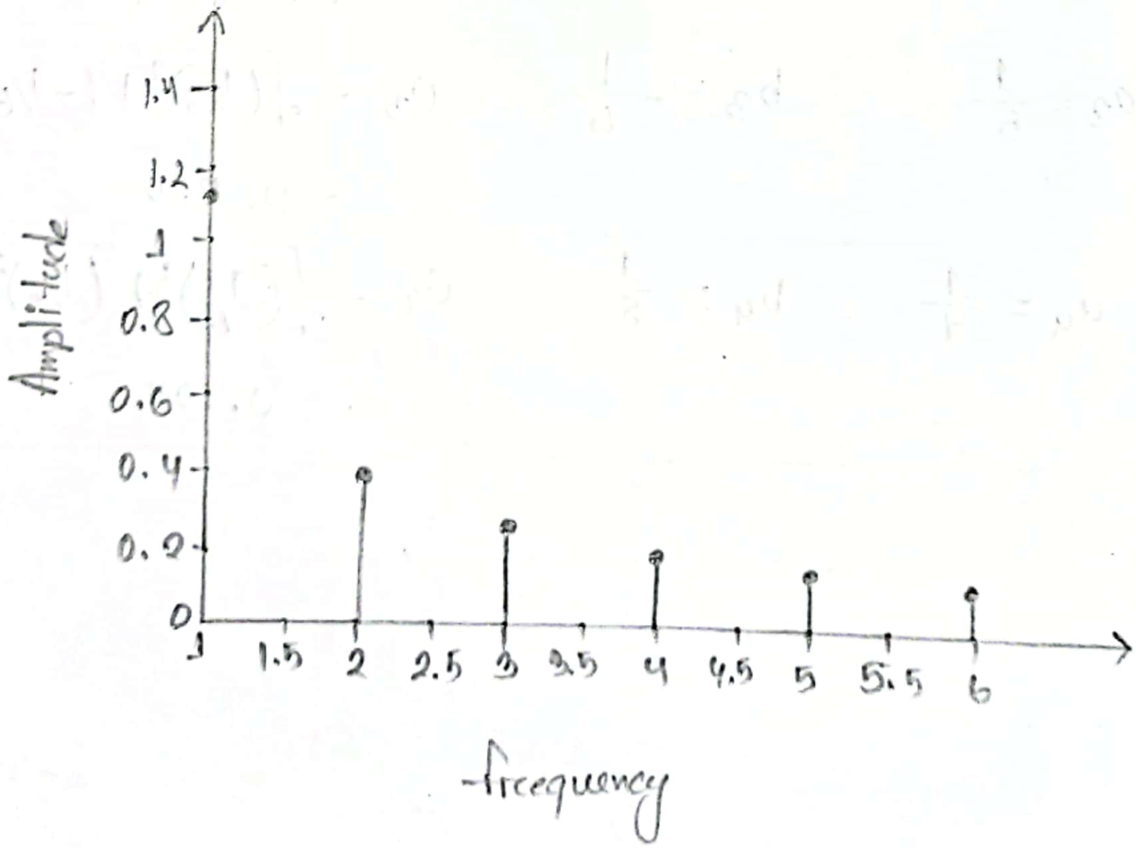


Fig: Line Specturm

Example - 35 (Home ~~task~~ work)

- Line Spectrum : Example

$$s(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos(2\pi f_0 t) - \frac{1}{3}\cos(6\pi f_0) + \frac{1}{5}\cos(10\pi f_0 t) - \ldots\right)$$

• Write the complex plot of fourier Series.