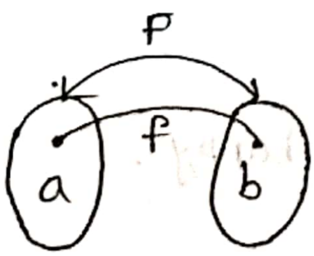
Functions

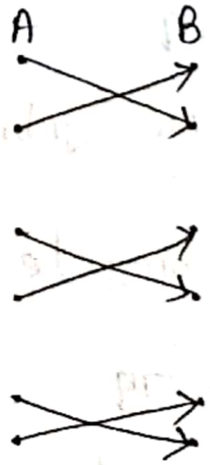
Function: A function is a rule that assigns each input exactly one output. We call the output the image of the input. The set of all inputs for a function is called the domain. The set of all allowable outputs is called the codomain. But the range is the set of values which actually comes out.

⇒ A সেটের একটি মানের জন্য B সেটের যদি একটি unique Value পাওয়া যায় তাকে function বলে।

⇒ Let A and B be non empty sets. A function f from A to B is an assignment of exactly one element of B to each element

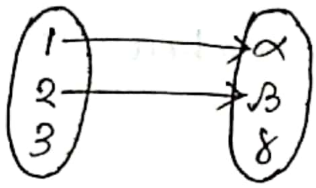


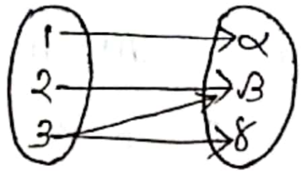
Venn diagram

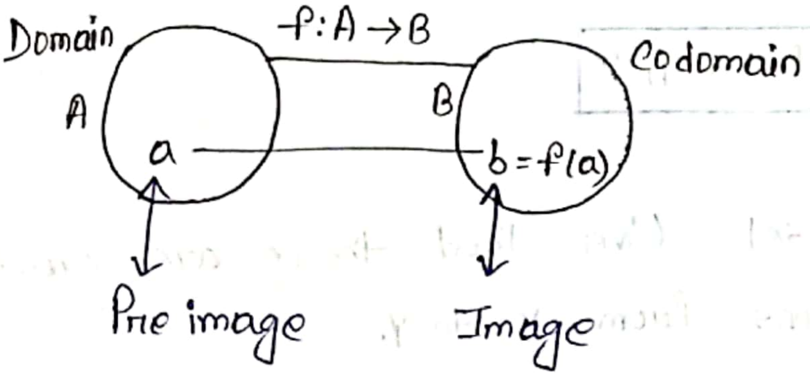


Graph

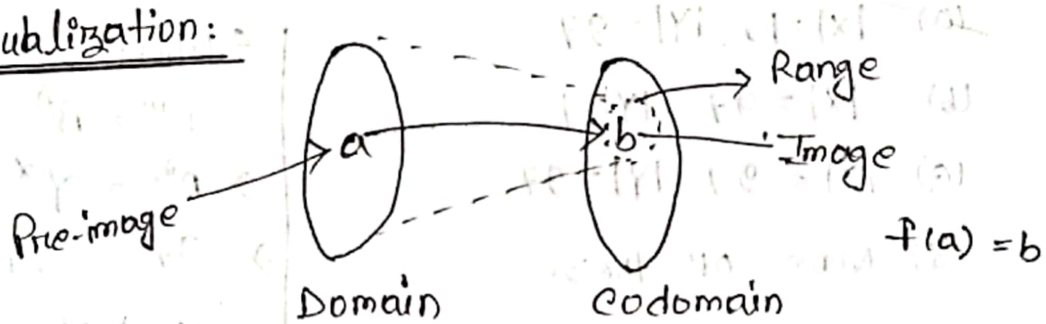
Not-function:







Function Visublization:



A function $f: A \to B$

Range vs Codomain Example:

- f is a function mapping students in the class to the sets of grade {A, B, C, D, E}. Find domain, co domain and range of f.

Solution: Domain: students of class

Co-domain: {A, B, C, D, E}

Range: Unknown

If all the students grades turn out A' or B' then Range = {A, B}

codomain: {A, B, D, E}

- $|A| = m$, $|A| = n$ then number of functions possible from A to B?

$2 \times 2 \times 2 = 8$

$$\boxed{\text{Function possible} = n^m}$$

Exercise: ① X and Y set. Given that there are exactly 97 functions from X to Y.

~~(a)~~ $|X| = 1$, $|Y| = 97$

(b) $|X| = 97$ $|Y| = 1$

(c) $|X| = 97$ $|Y| = 97$

(d) None of these

Solution:

$n^m = B^A$

$\Rightarrow n^m = Y^X$

$\Rightarrow 97 = 97^1$

$\therefore X = 1$ $Y = 97$

② $W = X \times Y$. Let E be the set of all subsets of W. The number from Z to E is,

~~(a)~~ Z^{2xy}

(b) $Z \times 2^{xy}$

(c) 2^Z

(d) 2^{xyz}

Solution:

$W = X \times Y = xy$

Now, $E = P(W)$

$= 2^{xy}$

$Z \rightarrow E$

Composition of function: The composition of two functions $g: A \to B$ and $f: B \to C$, denoted by $f \circ g$ is denoted by

$$(f \circ g)(a) = f(g(a))$$

It means that first function g is applied to element $a \in A$, mapping in onto an element of B, Then function f is applied to this element of B, mapping in onto an element of C.

Therefore the composite function maps from A to C.

$(f o g)(x)$

Inside function 1st

Outside function last.

Example: ① $f o g(x) = f(g(x))$

$f(x) = 2x+3, \quad g(x) = 3x+2$

$g(1) = 3 \cdot 1 + 2 = 5$

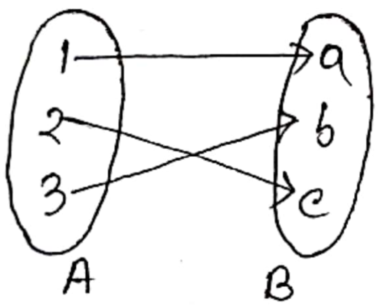
$f(5) = 2 \cdot 5 + 3 = 13$

Composition of a function and its inverse

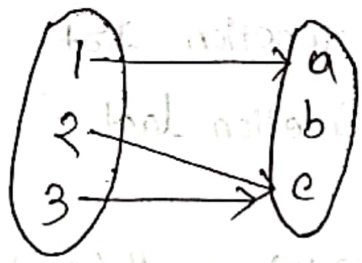
$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$$

The composition of a function and its inverse is the identity function

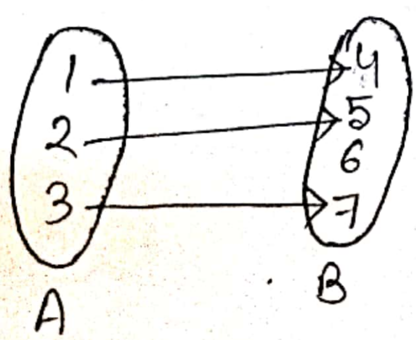
One to one function / Injection function: A function f is said to be one-to-one or injective, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f. A function is said to be an injection if it is one to one.



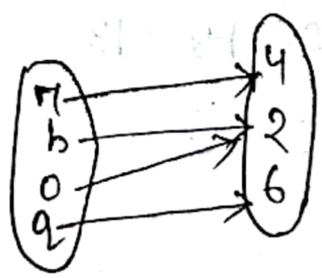
One to one function / Injective function



Not one to one function



One one function

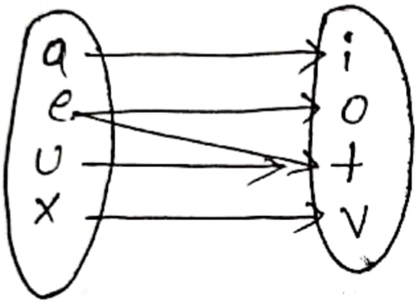


Not one one function

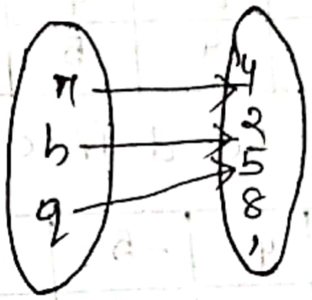
Onto function / surjections: A function f from A to B is called onto or surjective, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$. A function f is called surjective if it is onto.

$f: A \to B$ is surjective (onto) if the image of f = range

$f(a) = b$

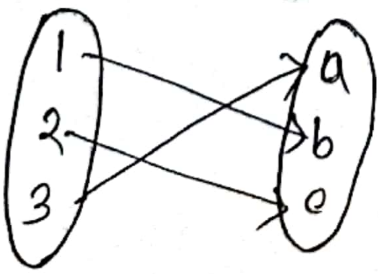


Onto functions

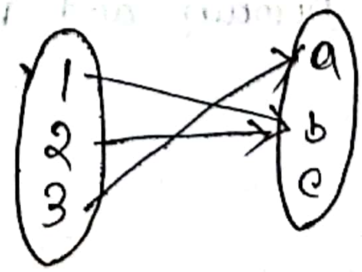


Not onto

Bijection functions: A function f is a one to one correspondence, or a bijection, if it is both one to one and onto.



Bijective



Not bijective

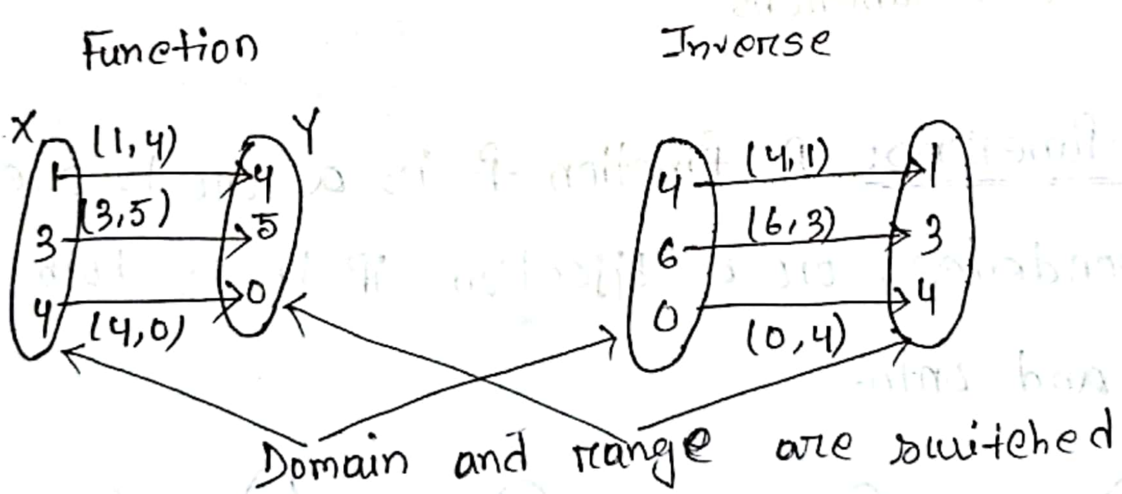
Inverse function: Let f be a bijection from A to B. Then the inverse of f, denoted f^{-1}. is the function from B to A defined as $f^{-1}(y) = x$. Iff $f(x) = y$. No inverse exists unless f is a bijection.

Let f be defined

x	-2	0	4	7
y	0	4	-5	10

f^{-1} can be defined

x	0	4	-5	10
y	-2	0	4	7



- Find Inverse function:

① $y = \frac{x}{5x-3}$

$\Rightarrow x = \frac{y}{5y-3}$

$\Rightarrow x(5y-3) = y$

$\Rightarrow 5xy - 3x = y$

$\Rightarrow 5yx - y = 3x$

$\Rightarrow y(5x-1) = 3x$

$\Rightarrow y = \frac{3x}{5x-1}$

$\Rightarrow f^{-1}(x) = \frac{3x}{5x-1}$

Exercise: • Why f is not a function from R to R if

(a) $f(x) = \frac{1}{x}$?

$\Rightarrow$ When x is 0, $1/x$ is not defined so $x=0$ does not map to a real number.

(b) $f(x) = \sqrt{x}$?

$\Rightarrow$ This function gives two values for each x, hence on its graph vertical line touches on two points.

(c) $f(x) = \pm\sqrt{(x^2+1)}$?

$\Rightarrow$ $f(x)$ has two possible answers. $\sqrt{(x^2+1)}$, $-\sqrt{x^2+1}$ এখানে x এর value বসালে each x এর জন্য unique value পাওয়া যাবে না।

- Find D_f, R_f

(a) The function that assigns to each non negative integer its last digit.

Domain = set of non negative

Range = $\{0, 1, \ldots 9\}$

(b) The function that assigns to next large number (i) to positive (i)

Domain = All positive (i)

Range = $\{~~1~~ 2, 3, 4, \ldots\}$

Constant function: A constant function is a function whose value is the same for every input value.

Dual function: If f and g are functions denoted on the same domain D and if $f(a) = g(a)$ for every $a \in D$ then the functions f and g are equal and we write $f = g$.

Floor function: $f(x) = \lfloor x \rfloor$, the floor function is the longest integer less than or equal to x.

Ceiling function: $f(x) = \lceil x \rceil$, the ceiling function is the smallest integer greater than or equal to x.

Remainder function: Let k be any integer and M be a positive integer. Then k (Mod M) will ~~delete~~ denote the integer reminder when k is divided by M

Integer value function: —

~~Int~~ INT (3.14) = 3, INT(-8.5) = -8

Absolute value function:

ABS (-15) = 15, ABS (15) = 15, ABS (0) = 0

Exponential function:

Let m be a positive integer,

Then, $a^m = a * a * a \ldots * a$ (m-times)

- $a^0 = 1$
- $a^{-m} = 1/a^m$
- $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $b^{x+y} = b^x . b^y$ and $(b^x)^y = b^{xy}$

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Logarithmic function: Let b be a positive number. The logarithmic of any positive number x to be the base b, written $\log_b x$ represents the exponent to which b must be raised to obtain x.

$y = \log_b x$ and $b^y = x$ are equivalent statement.

Recursively defined function: A function is said to be recursively defined if the function definition refers to itself. In order for the definition not to be circular, the function definition must have the following properties.

Factorial function: $f : N \to z^+$, denoted by $f(n) = n!$ is the product of the first n positive integers when n is a nonnegative integer.

$f(n) = 1 \cdot 2 \text{ --- } (n-1) n$

Sequences and Summation:

Sequences: A sequences is a function from a subset of the integers or to a set S.
Sequences are ordered lists of elements.

Arithmetic progression: A arithmetic progression is a sequence of the form,: $a, a+d, a+2d, \ldots, a+nd \ldots$
where the initial number a and the common difference d are real numbers.

① Let $a = -1$ and $d = 4$

$\{s_n\} = \{s_0, s_1, s_2, s_3, s_4 \ldots\}$
$= \{-1, 3, 7, 11, 15, \ldots\}$

Geometrical progression: It is a sequence of the form

$$a, ar, ar^2, ar^3, \ldots ar^n, \ldots$$

Here, a is the initial term and "b" is the ratio both of term are real numbers.

Strings: A string is a finite sequence of characters from a finite set.

→ Empty string is represented by λ.

→ The string abcde has length 5.

Fibonacci sequence: Define the fibonacci sequence $f_0, f_1, f_2 \ldots$ by:

Initial conditions: $f_0 = 0, \; f_1 = 1$

Recurrence Relation: $f_n = f_{n-1} + f_{n-2}$

Example: Find f_2, f_3, f_4, f_5 and f_6

$$f_2 = f_1 + f_0 = 1 + 0 = 1$$

$$f_3 = f_2 + f_1 = 1 + 1 = 2$$

$$f_4 = f_3 + f_2 = 2 + 1 = 3$$

$$f_5 = f_4 + f_3 = 3 + 2 = 5$$

$$f_6 = f_5 + f_4 = 5 + 3 = 8$$

Useful sequences:

nth term	First 10 term
n^2	1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ----
n^3	1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, ----
n^4	1, 16, 81, 256, 625, 1296, 2401, 4096, 6561, 10000, ---
2^n	2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, -----
3^n	3, 9, 27, 81, 243, 729, 2187, 6561, 19683, 59049, ----
$n!$	1, 2, 6, 24, 120, 720, 5040, 40320, 36288, 3628800, --
f_n	1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89. ----

Summations: Sum of terms from sequence.

Sum notation:

$$\sum_{j=m}^{n} a_j \quad \sum_{j=m}^{n} a_j \quad \sum_{m \leq j \leq n} a_j$$

represents,

$$a_m + a_{m+1} + \cdots + a_n$$

The variable j is called the index of summation. It runs through all the integers starting with its lower limit m and ending with its upper limit n.

Product notation:

$$\prod_{j=m}^{m} a_j \quad \prod_{j=m}^{n} a_j \quad \prod_{m \leq j \leq n} a_j$$

represents, $a_m \times a_{m+1} \times \cdots \times a_n$

Some useful summation formula:

Sum	closed form
$\sum_{k=0}^{n} ar^k \; (r \neq 0)$	$\frac{ar^{n-1} - a}{r-1}, \; r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{\infty} x^k, \; \lvert x \rvert < 1$	$\frac{1}{1-x}$
$\sum_{k=1}^{\infty} k \cdot x^{k-1}, \; \lvert x \rvert < 1$	$\frac{1}{(1-x)^2}$

Algorithms: An algorithm is a finite set of Precise instructions for performing a computation or for solving a problem.

The complexity of Algorithms:

when we use analyze the time the algorithm ~~sus~~ use to solve the problem given input of a Particular size, we are studying the time complexity of the algorithm.

The growth of a function: Growth function are used to estimate the number of steps an algorith uses as its input grows.

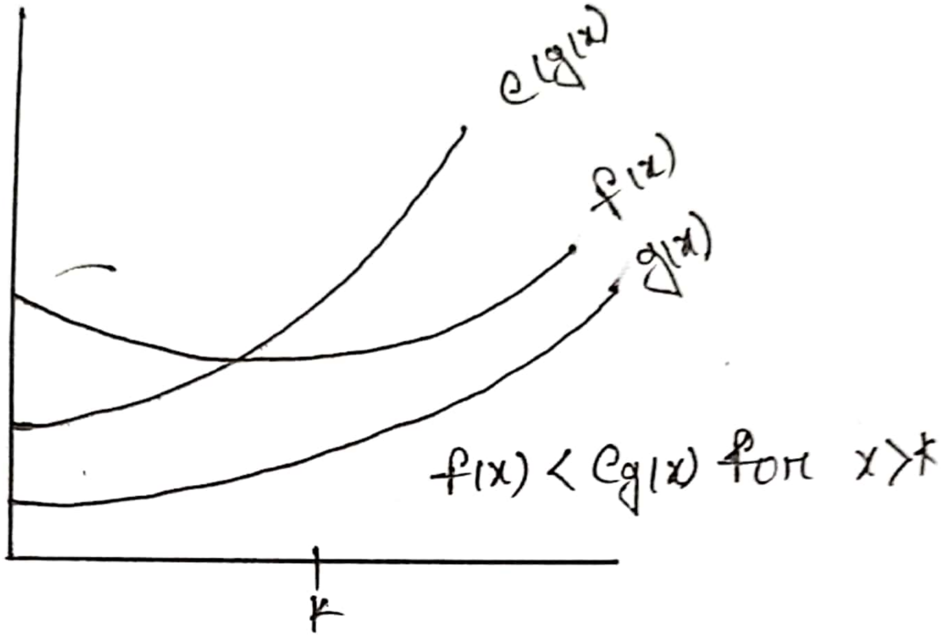
Big-O-Notation: $|f(x)| \leq c\,|g(x)|$

say: $f(x)$ is Big-O of $g(x)$

or: "g asymptotically dominates f".

Illustration of Big-O- Notation:

f(x) is O(g(x))- -



The part of the graph of f(x) that satisfies f(x) < cg(x) is shown in color.