⊞ Show that $V = j\omega L I$.

For an inductor, let us assume that the current through it,

$$i = I_m \cos(\omega t + \phi)$$

The voltage accross it,

$$v = L \frac{di}{dt}$$

$$= L \frac{d}{dt} I_m \cos(\omega t + \phi)$$

$$= -\omega L I_m \sin(\omega t + \phi)$$

$$= \omega L I_m \cos(\omega t + \phi + 90^\circ)$$

$$= Re\left(\omega L I_m e^{j(\omega t + \phi + 90^\circ)}\right)$$

Transforming into phasor,

$$V = \omega L I_m e^{j(\phi + 90^\circ)}$$

$$= \omega L I_m \cdot e^{j\phi} \cdot e^{j90^\circ}$$

$$= j\omega L I_m e^{j\phi} \quad [\because e^{j90^\circ} = j]$$

$$\Rightarrow V = j\omega L I \quad [\because I_m e^{j\phi} = I_m \angle \phi = I]$$

We can see that the current lags the voltage by 90°.

Nayeem Mahmood / 1AM / C161026

Draw the Phasor relationship for circuit elements: Capacitor and Inductor.

For a capacitor,

$$v = V_m \cos(\omega t + \phi)$$
$$i = I_m \cos(\omega t + \phi + 90^\circ)$$

So, current leads the voltage by 90°.

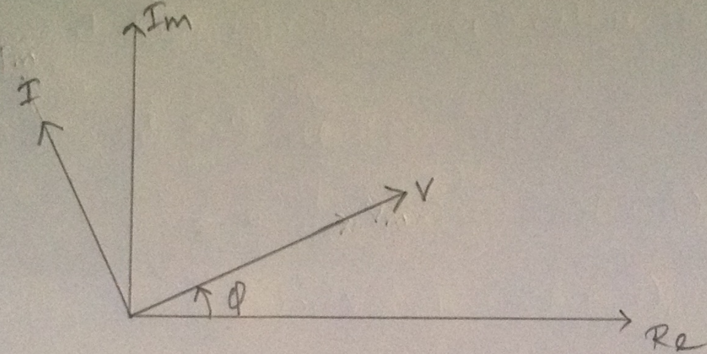


Figure : Phasor diagram for capacitor.

For an inductor,

$$i = I_m \cos(\omega t + \phi)$$
$$v = V_m \cos(\omega t + \phi + 90^\circ)$$

So, voltage leads the current by 90°

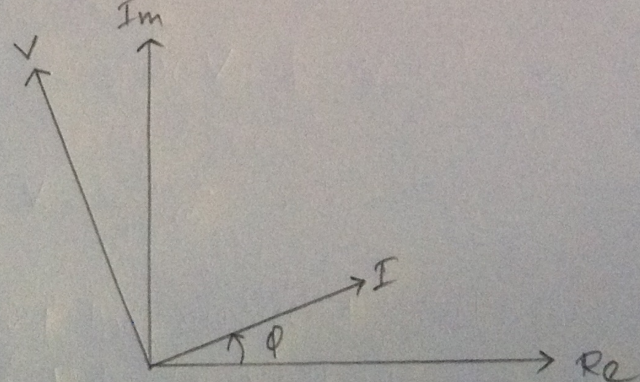
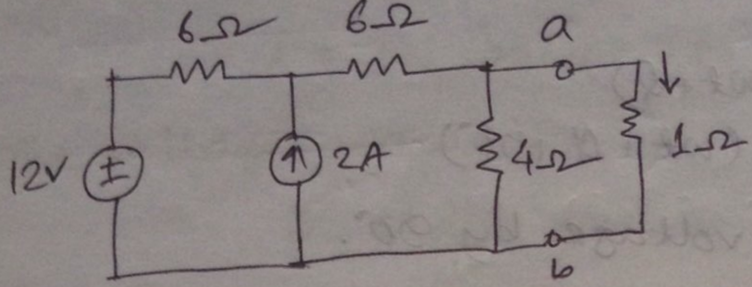


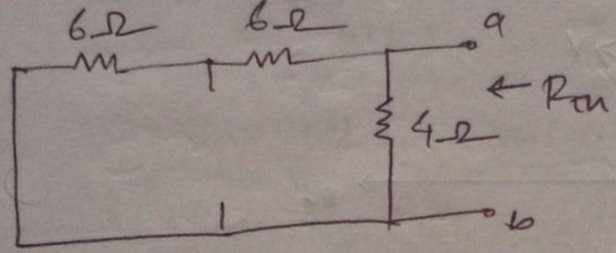
Figure : Phasor diagram for Inductor.

CIRCUIT PROBLEMS

Prac - 4.8

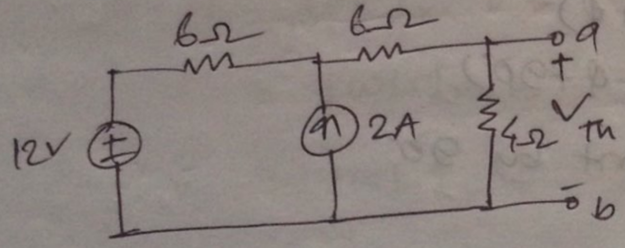


For obtaining R_{TH},

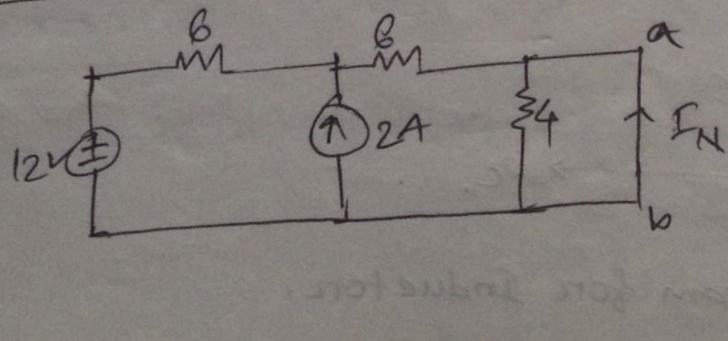


$$R_{TH} = (6+6) \parallel 4 = 3\Omega$$

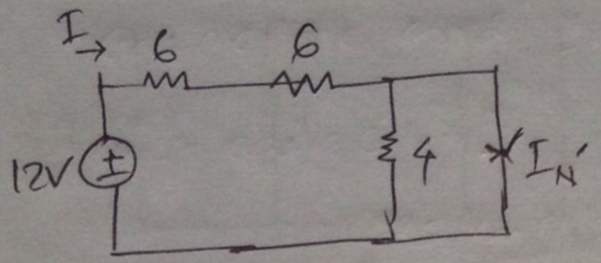
For obtainging V_{TH},



We will use Norton analysis



When 12V is active,



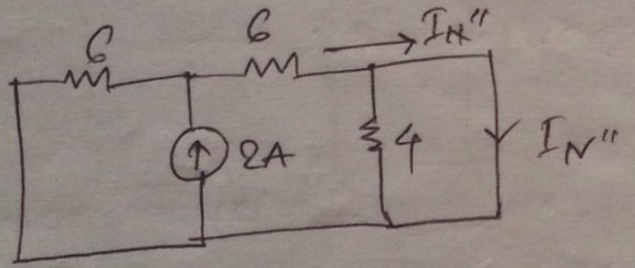
Current won't flow through 4Ω as there is a short circuit across it.

$$R' = 6+6 = 12$$

$$\therefore I = \frac{V}{R} = \frac{12}{12} = 1A$$

So, $\downarrow I_N' = I = 1A$

When 2A is active,



Current won't flow through 4Ω.

$$\therefore \downarrow I_N'' = \frac{6}{6+6} \times 2$$

$$= 1A$$

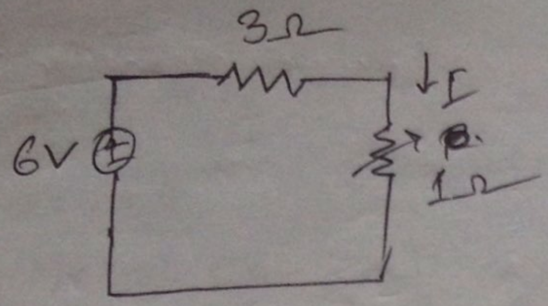
$$\therefore I_N = I_N' + I_N'' = 1A + 1A = 2A$$

$$\therefore V_{Th} = I_N R_{Th}$$

$$= 2 \times 3$$

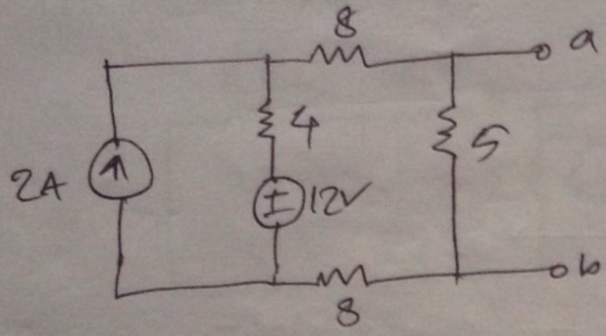
$$= 6V$$

Nayeem Mahmood / 1AM / C161026

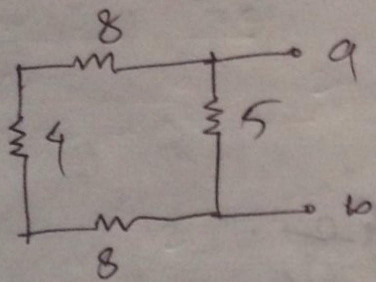


$$\therefore I = \frac{6}{3+1} = 1.5A$$

Prac-4.11



R_N

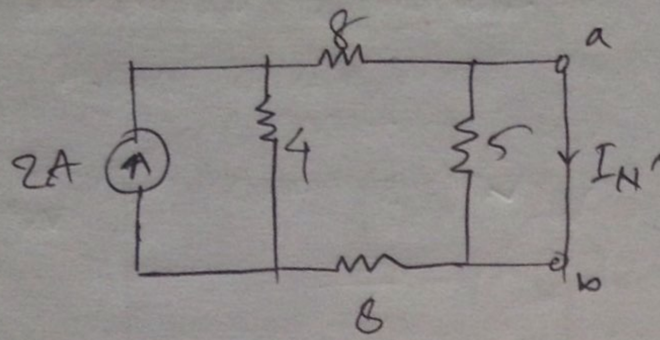


$$R_N = (8+4+8) \parallel 5$$
$$= 4\Omega$$

I_N

When 2A is active

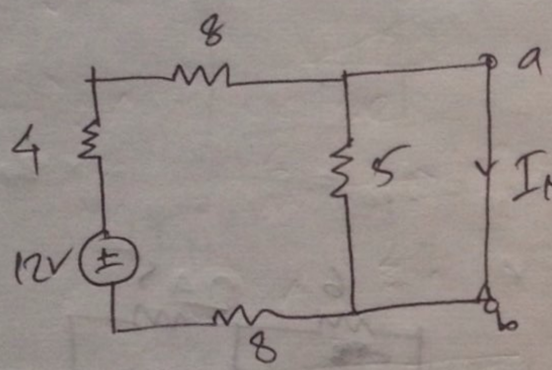
Nayeem Mahmood / 1AM / C161026

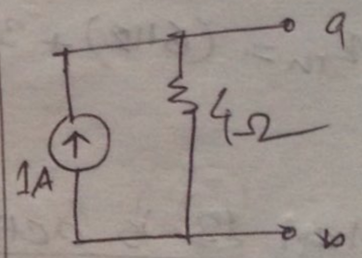


Current won't flow through 5Ω.

$$\downarrow I_N' = \frac{4\times 2}{8+8+4} = 0.4A$$

when 12V is active





Current won't flow through 5Ω

$$R_{eq} = 4+8+8 = 20\Omega$$

$$\therefore I = \frac{V}{R_{eq}} = \frac{12}{20} = 0.6A$$

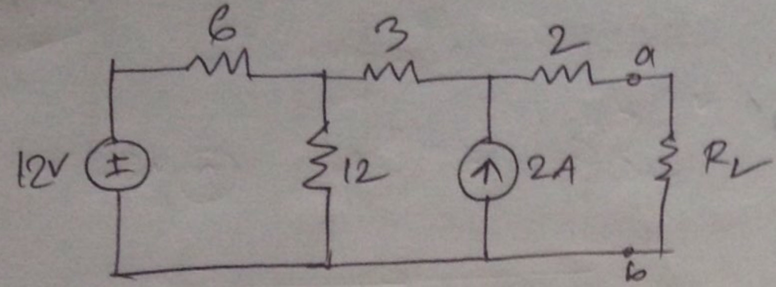
$$\therefore \downarrow I_N'' = I = 0.6A$$

$$\therefore I_N = \downarrow I_N' + \downarrow I_N''$$

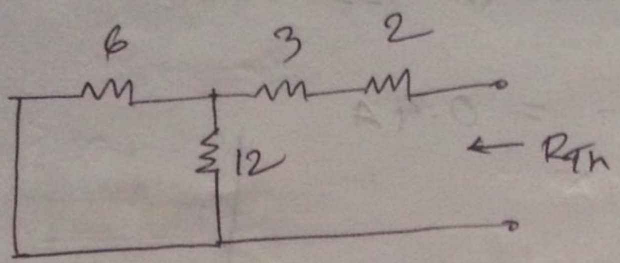
$$= 0.4 + 0.6 = 1A$$

Nayeem Mahmood / 1AM / C161026

Ex-4.13



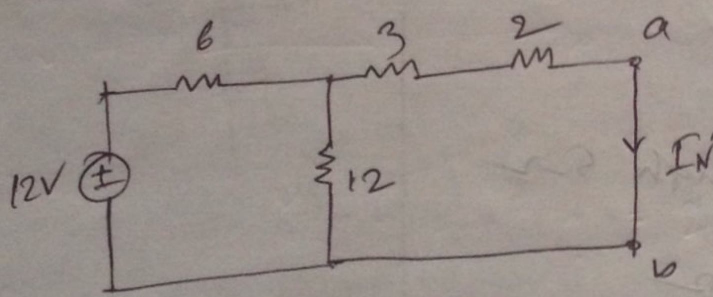
R_{Th}



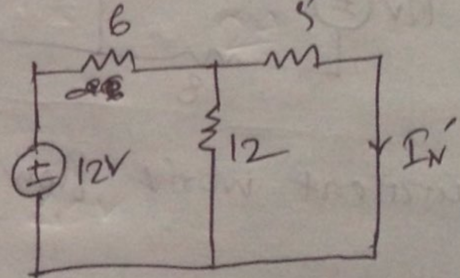
$$R_{Th} = (6\|12) + 3 + 2 = 9\,\Omega$$

V_{Th}

when 12 is active.



$\Rightarrow$



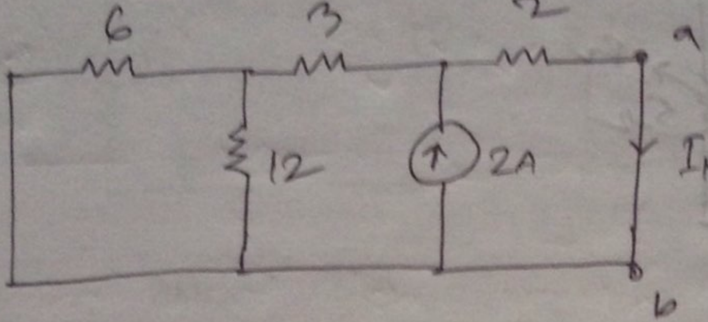
$$R_{eq} = (5\|12) + 6 = 9.53\,\Omega \quad \frac{162}{17}$$

$$\therefore I = \frac{V}{R_{eq}} = \frac{12}{9.53} = 1.26\,A \quad \frac{34}{27}$$

$$\therefore \downarrow I_N' = I_{5\Omega} = \frac{12 \times 1.26}{12+5} = 0.89\,A \quad \frac{8}{9}$$

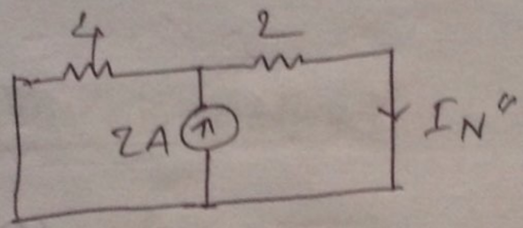
Nayeem Mahmood / 1AM / C161026

When 2A is active



$(12 \parallel 6) + 3 = 4\,\Omega$

The circuit can be ~~de~~ reduced to -



$\therefore \downarrow I_N'' = I_{2\Omega} = \frac{4\times 2}{4+2} = 1.33A \quad \left(\frac{4}{3}\right)$

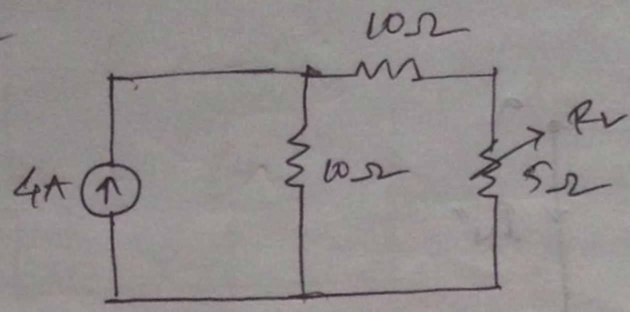
$\therefore I_N = \downarrow I_N' + \downarrow I_N'' = 0.89 + 1.33 = 2.22A \quad \left(\frac{20}{9}\right)$

$\therefore V_{Th} = I_N R_{Th} = 2.22 \times 9 = 19.98V \approx 20V$

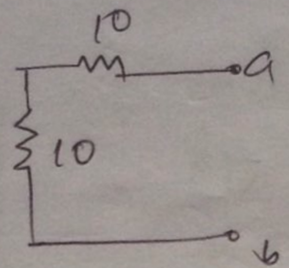
Nayeem Mahmood / 1AM / C161026

Exercise

4.33

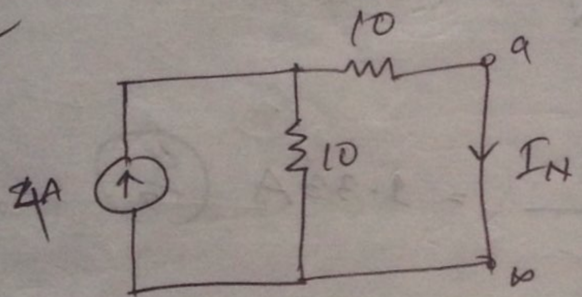


R_{TH}



$$R_{TH} = 10 + 10 = 20\Omega$$

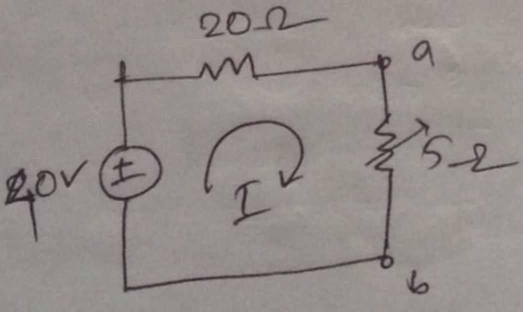
V_{TH}



$$I_N = I_{10} = \frac{10}{10+10} \times 4$$

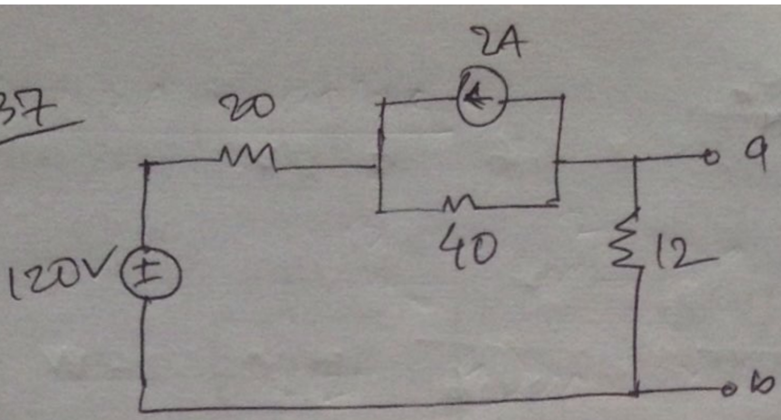
$$= \underline{2A}$$

$$\therefore V_{TH} = I_N R_{TH} = 2 \times 20 = 40V$$

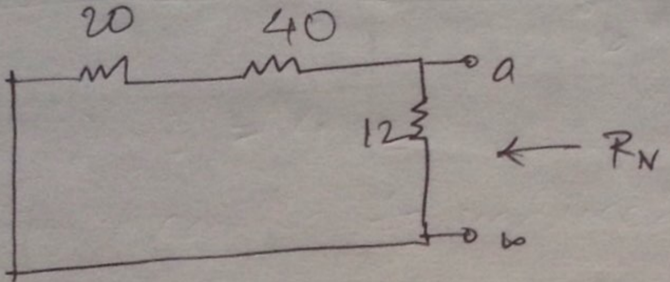


$$I_{5\Omega} = I = \frac{40}{20+5} = \cancel{0.8A}\ 1.6A$$

4.37



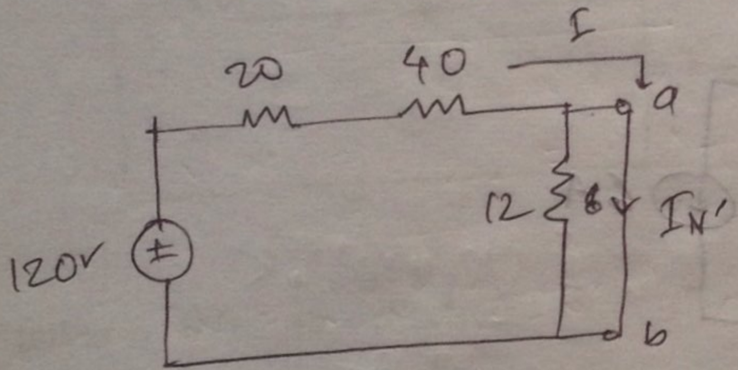
R_N



$R_N = (20+40) \parallel 12$

$= 10\Omega$

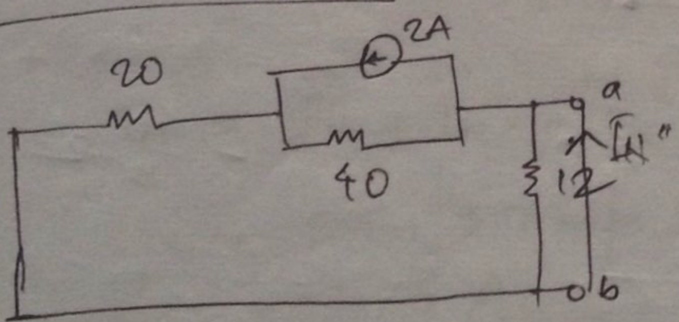
I_N when 120V is active



current won't flow through 12Ω

$$\downarrow I_N' = I = \frac{120}{20+40} = 2A$$

when 2A is active



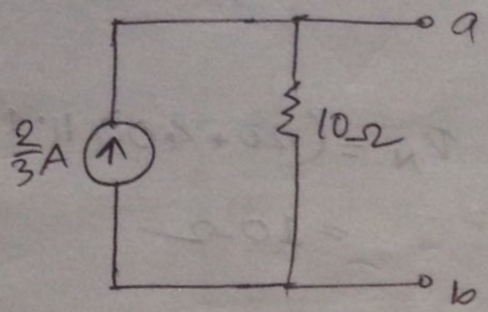
current won't flow through 12Ω.

$$\uparrow I_N'' = I_{20\Omega} = \frac{40\times 2}{40+20}$$

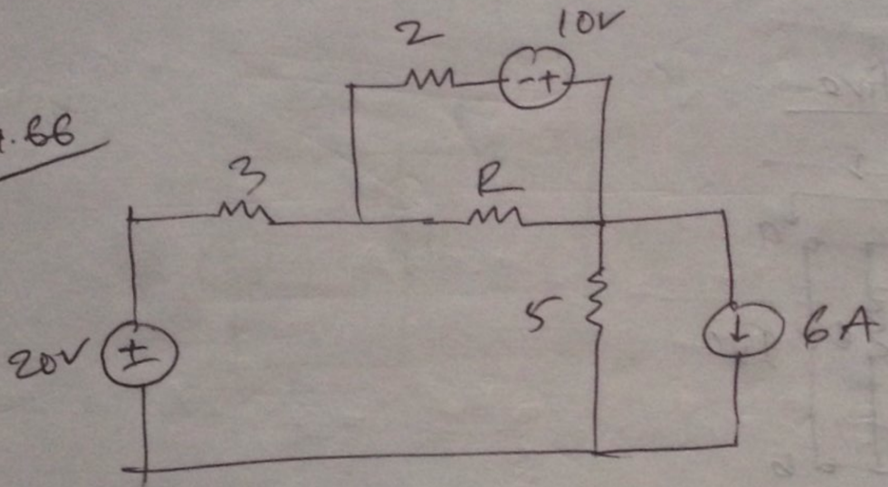
$$= \frac{4}{3} A$$

Nayeem Mahmood / 1AM / C161026

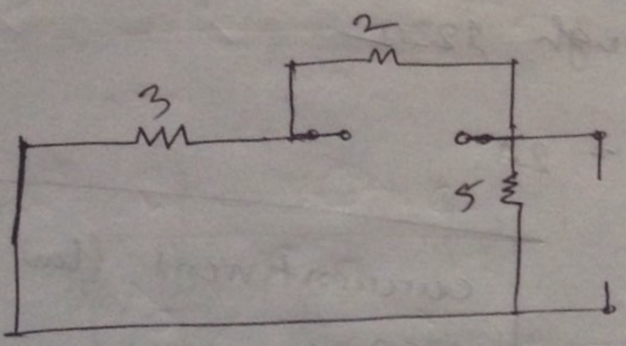
$$\therefore \downarrow I_N = \downarrow I_N' - \uparrow I_N''$$
$$= 2 - \frac{4}{3}$$
$$= \frac{2}{3} A$$



4.66



R_{Th}

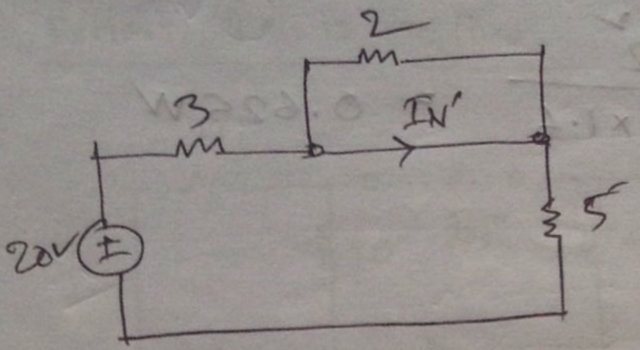


$$R_{Th} = (5+3) \parallel 2$$
$$= 1.6\ \Omega$$

Nayeem Mahmood / 1AM / C161026

I_N

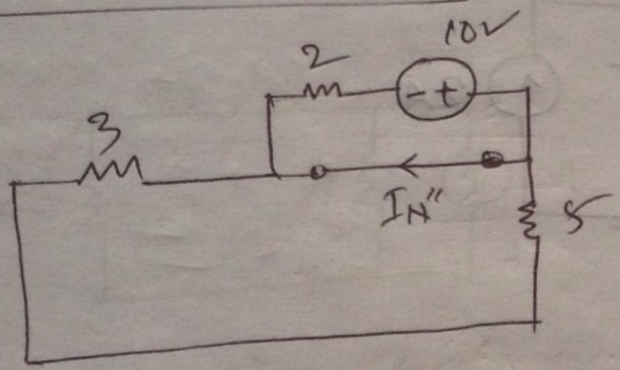
When 20V is active



Current won't flow through 2Ω.

$$\therefore \vec{I_N'} = I = \frac{20}{3+5} = 2.5A$$

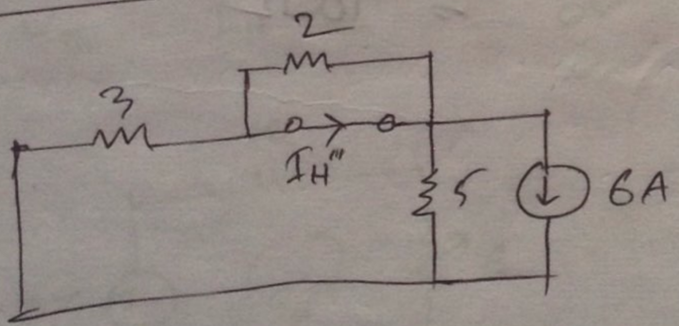
When 10V is active



Current won't flow through 5Ω and 3Ω.

$$\therefore \overleftarrow{I_N''} = \frac{10}{2} = 5A$$

When 6A is active



Current won't flow through 2Ω.

$$\therefore \vec{I_N'''} = I_{3\Omega} = \frac{5 \times 6}{3+5}$$

$$= 3.75A$$

$$\therefore I_N = \vec{I_{N'}} + \vec{I_N'''} - \overleftarrow{I_N''}$$

$$= 2.5 + 3.75 - 5$$

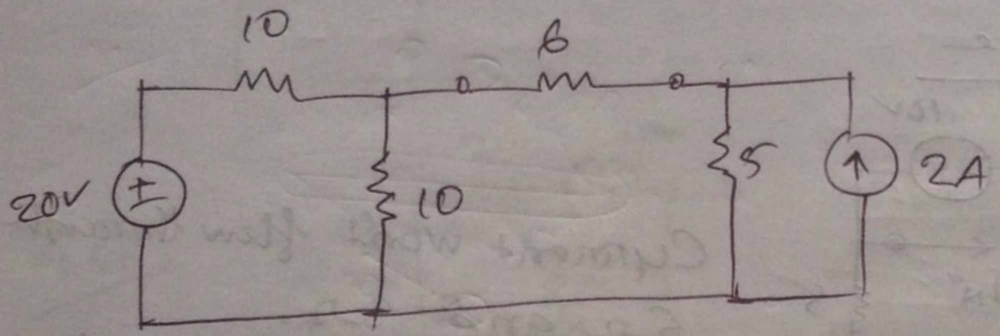
$$\therefore \vec{I_N} = 1.25 A$$

Nayeem Mahmood / 1AM / C161026

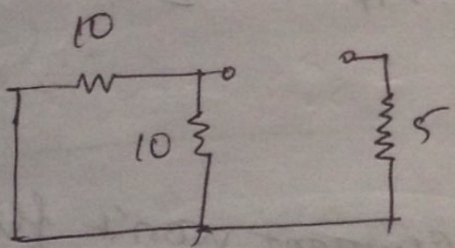
$\therefore V_{Th} = I_N R_{Th} = 1.25 \times 1.6 = 2V$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{2^2}{4 \times 1.6} = 0.625\,W$$

4.43



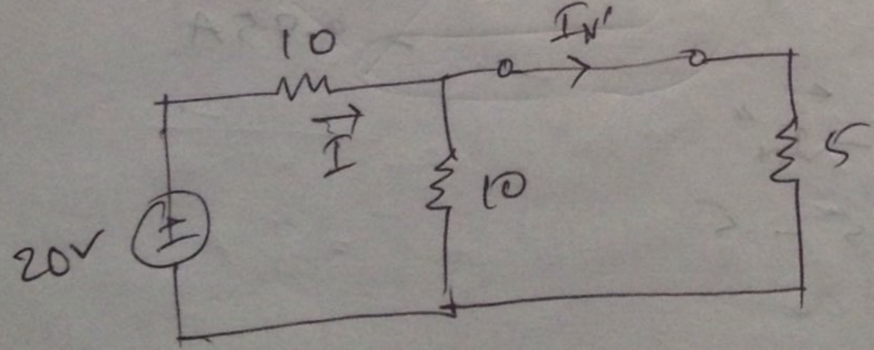
R_{Th}



$$R_{Th} = (10 \parallel 10) + 5 = 10\,\Omega$$

V_{Th}

When 20V is active



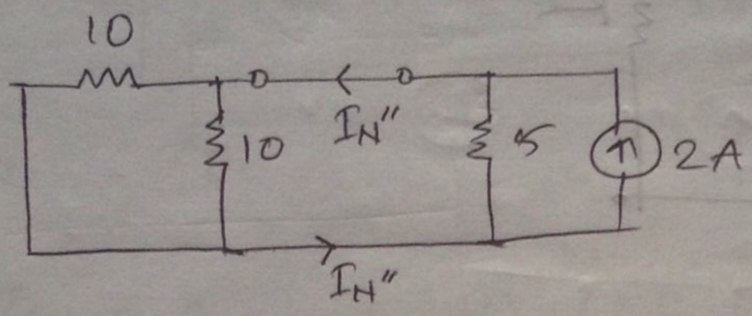
$$R_{eq} = (10 \parallel 5) + 10 = \frac{40}{3}\,\Omega$$

$$\therefore I = \frac{20}{\frac{40}{3}} = 1.5A$$

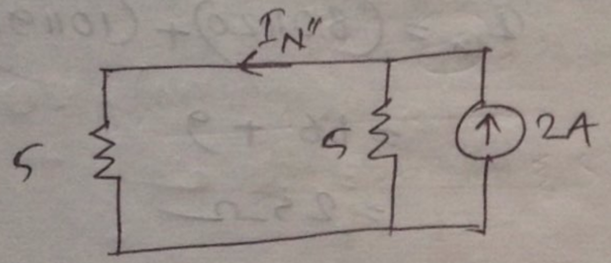
Nayeem Mahmood / 1AM / C161026

$$\therefore \vec{I_H'} = I_{5\Omega} = \frac{10\times1.5}{10+5} = 1A$$

<u>When 2A is active</u>



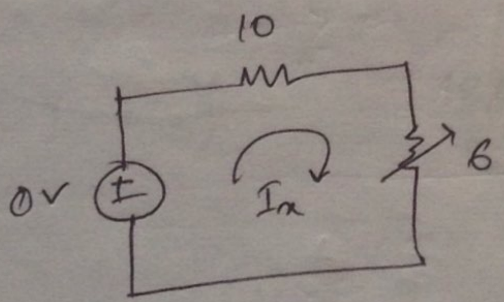
$$10 \parallel 10 = 5\Omega$$



$$I_H'' = \frac{5\times2}{5+5} = 1A$$

$$\therefore I_H = \vec{I_H'} - \overleftarrow{I_H''} = 1-1 = 0A$$

$$\therefore V_{Th} = I_H R_{Th} = 0V$$

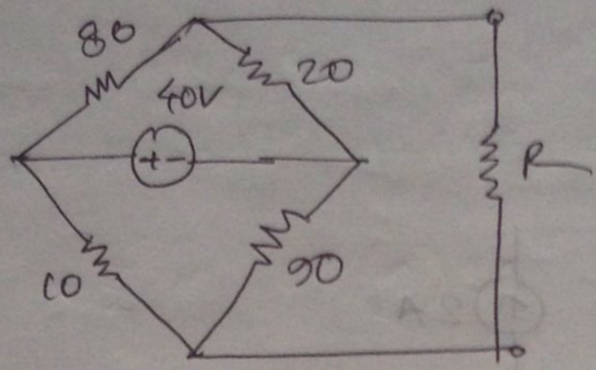


$$I_x = \frac{0}{10+6} = 0A$$

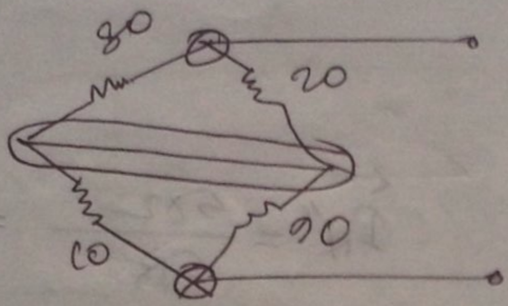
Nayeem Mahmood / 1AM / C161026

AUT-15

3(b)



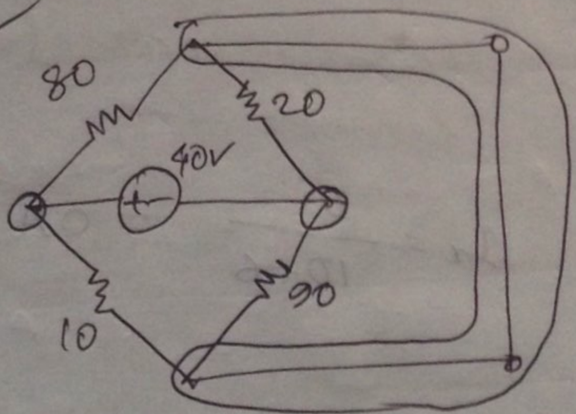
R_{Th}



$$R_{Th} = (80 \parallel 20) + (10 \parallel 90)$$
$$= 16 + 9$$
$$= 25\,\Omega$$

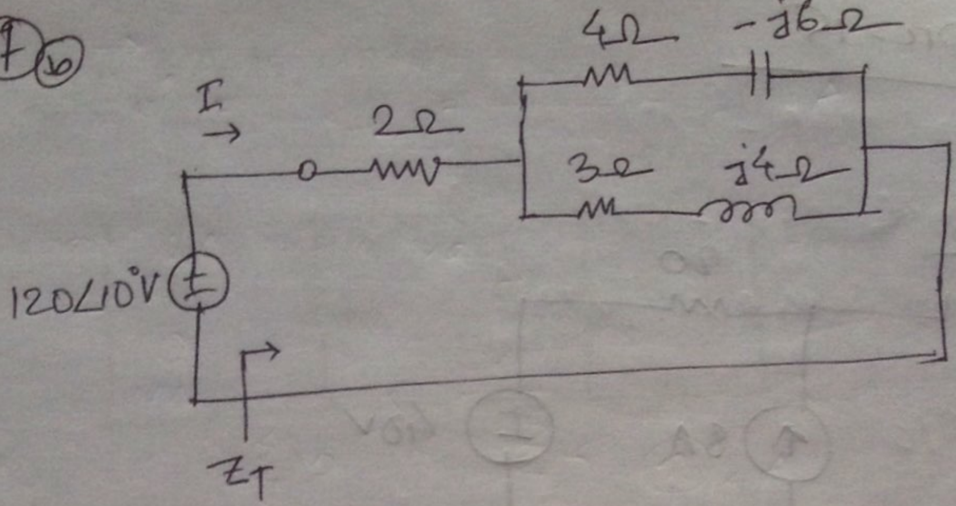
R must be $25\,\Omega$ for maximum power.

V_{Th}



Nayeem Mahmood / 1AM / C161026

7 (b)



$$Z_T = 2 + \{(4-j6) \parallel (3+j4)\}$$

$$= 2 + \frac{(4-j6)(3+j4)}{4-j6+3+j4}$$

$$= 2 + \frac{36-2j}{7-j2}$$

$$= 2 + 4.83 + j1.09$$

$$\therefore Z_T = 6.83 + j1.09$$

$$= 6.92 \angle 9.1^\circ$$

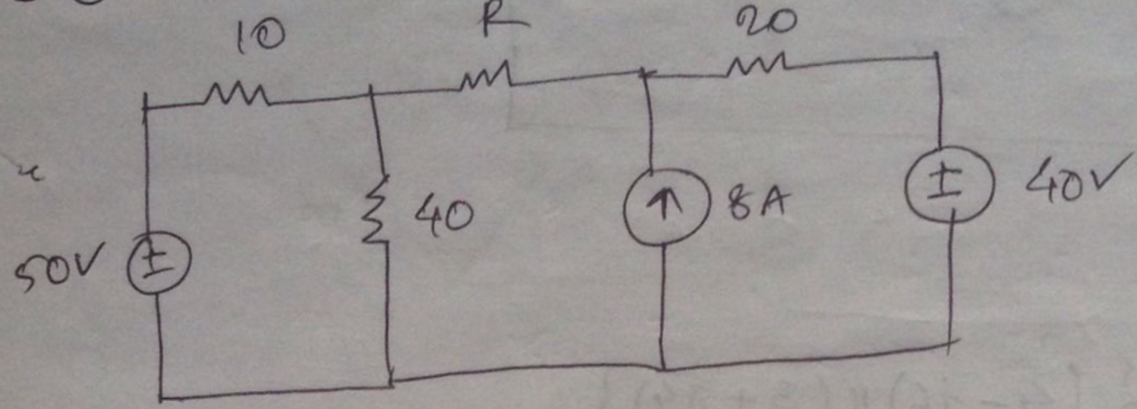
$$\therefore I = \frac{V}{Z_T} = \frac{120\angle 10^\circ}{6.92\angle 9.1^\circ} = 17.35 \angle 0.9^\circ$$

(Ans)

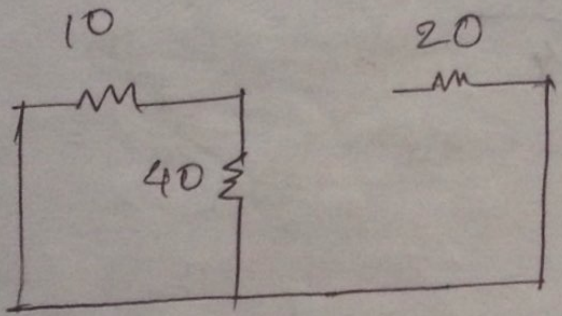
Nayeem Mahmood / 1AM / C161026

SPR-15

3 (b)



R_{Th}

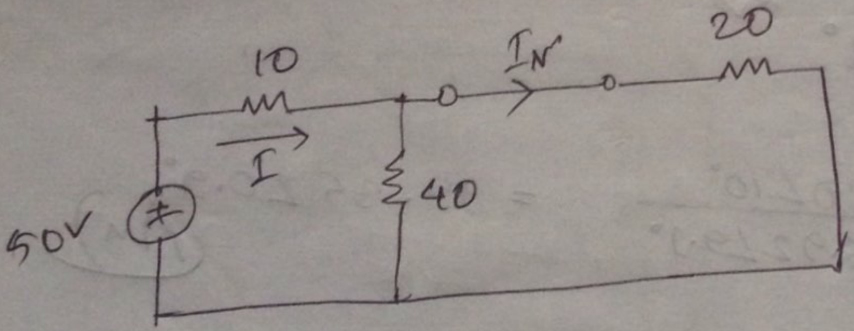


$R_{Th} = (10 \parallel 40) + 20$

$= 28\,\Omega$

V_{Th}

When 50V is active



$R_{eq} = (20 \parallel 40) + 10$

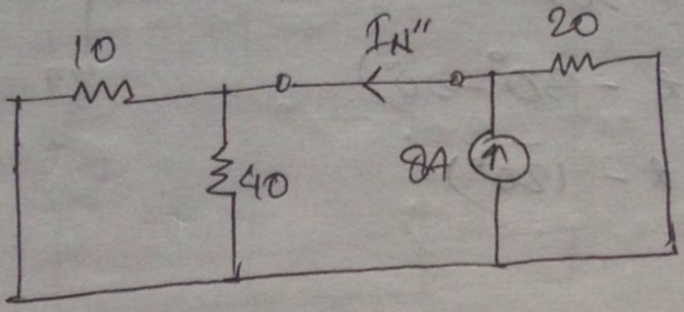
$= \frac{70}{3}$

$\therefore I = \frac{50}{70/3} = \frac{15}{7}$ A

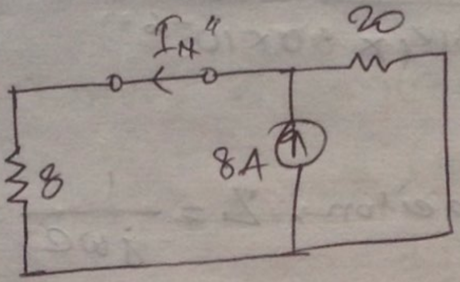
$\therefore \vec{I_N'} = I_{20\Omega} = \frac{40 \times \frac{15}{7}}{40+20} = \frac{10}{7}$ A

Nayeem Mahmood / 1AM / C161026

When 8A is active

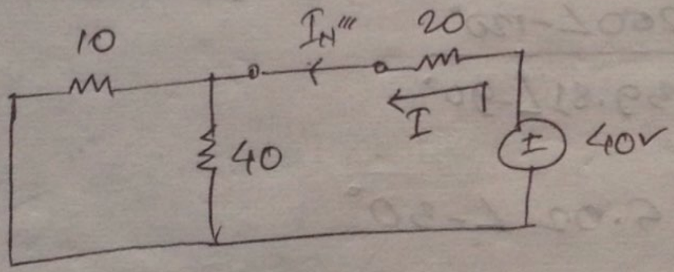


$10 \parallel 40 = 8\Omega$



$$\therefore \overleftarrow{I_H''} = \frac{20\times 8}{20+8} = \frac{40}{7}A$$

When 40V is active



$$R_{eq} = (10 \parallel 40) + 20$$
$$= 28\Omega$$

$$\therefore I = \frac{40}{28} = \frac{10}{7}A$$

$$\therefore \overleftarrow{I_H'''} = I = \frac{10}{7}A.$$

$$\therefore I_H = \overleftarrow{I_H''} + \overleftarrow{I_H'''} - \overrightarrow{I_H'}$$
$$= \frac{10}{7} + \frac{40}{7} - \frac{10}{7}$$

$$\therefore \overleftarrow{I_N} = \frac{40}{7}A$$

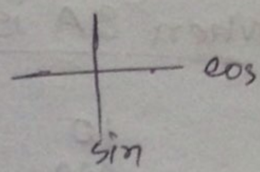
$$\therefore V_{Th} = I_H \times R_{Th}$$
$$= \frac{40}{7} \times 28$$
$$= 160V$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = 228.57W$$

Ans.

Nayeem Mahmood / 1AM / C161026

7 (a)



$V = 200 \sin(314t - 30^\circ)$

$= 200 \cos(314t - 30^\circ - 90^\circ)$

$= 200 \cos(314t - 120^\circ)$

$\omega = 314$ rad/s

Reactance $= \dfrac{1}{\omega C} = \dfrac{1}{314 \times 80 \times 10^{-6}} = 39.81\ \Omega$

Impedance of $80\mu F$ Capacitor, $Z = \dfrac{1}{j\omega C}$

$= -j39.81\ \Omega$

$= 39.81\angle -90^\circ$

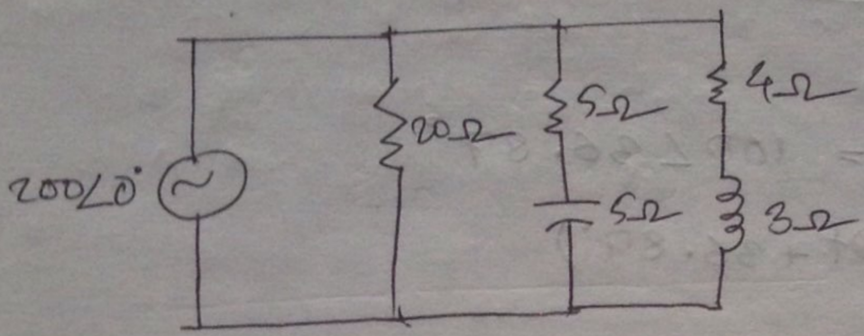
$\therefore I = \dfrac{V}{Z} = \dfrac{200\angle -120^\circ}{39.81\angle -90^\circ}$

$= 5.02\angle -30^\circ$

$= 5.02 \cos(314t - 30^\circ)$

(Ans)

(b)



~~(b)~~

Total Impedance, $Z_T = \{(4+3) \parallel (5+5)\} \parallel 20$

$= (7 \parallel 10) \parallel 20$

$= \frac{140}{41}\ \Omega \quad = \frac{140}{41}\angle 0°$

Total Admittance, $Y = \frac{1}{Z_T} = \frac{41}{140}$ S

$\therefore$ Total current $= \frac{200\angle 0°}{\frac{140}{41}\angle 0°} = 58.57\angle 0°$

Power supplied by source,

$P = \frac{V^2}{Z_T} = \frac{(200\angle 0°)^2}{\frac{140}{41}\angle 0°} = 11.71$ kW

(Ans)

Nayeem Mahmood / 1AM / C161026

Aut-14

⑦ ⓒ $V = 80 + j60^\circ = 100\angle 36.87^\circ$

$= 100\cos(\omega t + 36.87^\circ)$

$I = -4 + j10^\circ = 2\sqrt{29}\angle 111.80^\circ$

$= 2\sqrt{29}\cos(\omega t + 111.80^\circ)$

Impedance, $Z_T = \frac{V}{I} = 9.28\angle -74.93^\circ$

$P = \frac{V^2}{Z_T} = 1077\angle 148.67$ W

Nayeem Mahmood / 1AM / C161026