

Set

① What is Discrete mathematics?

⇒ Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous. [meaning of word "Discrete" → individually separate or distinct]

②

Discrete vs continuous

- | | |
|--|--|
| ① Can't be broken down into fractions or decimals | ① can be broken down into fractions or decimals. |
| ② Number of students purchasing tickets, text message etc. | ② Time, temperature, weight, distance etc. |
| ③ Discrete elements are countable | ③ Continuous objects are uncountable and measurable. |

⇒ G. Cantor (German mathematician) introduced the concept of sets.

Set: • A set is a collection of objects which are clearly identified.

• The objects in the set are called the members or elements of the set.

• Each member is said to belong to the set.

• Sets are denoted by capital letters:

A, B, C ---, X, Y, Z

• The elements of a set are represented by lower case letters:

a, b, c, ---, x, y, z

⇒ $\in \Rightarrow$ The symbol ' \in ' is used to denote belongs to or is an element of or is a member of set.

• $a \in A \Rightarrow$ "a is an element of set A"
"a is a member of set A"

• $a \notin A \Rightarrow$ "a is not an element of set A"

• Order of elements is meaningless.

- For example, the items you wear: shoes, socks, hat, shirt, pants, and so on. This is known as a set.
- The set includes index, middle, ring and pinky.

Notation of set:

- There is a fairly simple notation for sets. We simply list each element ("members") separated by a comma, and then put some curly brackets around the whole thing: $\{3, 6, 9, 12\}$
- $\{\}$ \Rightarrow Set brackets or braces
- The three dots ... are called an ellipsis, and mean "Continue on".

Numerical set (well defined)

\Rightarrow Vowels in the English alphabet

$$V = \{a, e, i, o, u\}$$

\Rightarrow Odd positive integers less than 10

$$O = \{1, 3, 5, 7, 9\}$$

\Rightarrow Set of even numbers: $\{-\dots, -4, -2, 0, 2, 4\}$

\Rightarrow Set of prime numbers: $\{2, 3, 5, 7, 11, 13, 17\}$

\Rightarrow Set of odd numbers: $\{-\dots, -3, -1, 1, 3, \dots\}$

→ Positive multiples of 3 that are less than 10:
 $\{3, 6, 9\}$

Numerical sets: (Not well-defined)

There can also be sets of numbers that have no common property, they are just defined that way.

- $\{2, 3, 6, 828, 3839\}$
- $\{4, 5, 6, 10, 21\}$
- $\{11, 8888\}$

Task:

- ① All the colors in the rainbow — well defined
- ② All the points that lie on a straight line — Not well defined
- ③ All the honest members in the family. — Not well defined
- ④ All the consonants of the English alphabet — well defined
- ⑤ All the tall boys of the school. — Not well defined
- ⑥ All the hardworking teachers in a school. — Not well defined
- ⑦ All the prime numbers less than 100. — Well defined
- ⑧ All the letters in the word geometry. — Well defined

Member of set:

To a member of

To not a member of

- | | |
|---------------------------|--------------------------|
| ① $3 \in \{1, 2, 3\}$ | ① $3 \notin \{4, 5, 6\}$ |
| ② $-1 \in \{-5, -1, -7\}$ | ② $a \notin \{b, c, d\}$ |

Element of set:

→ A set of odd numbers between 1 and 10.

$$A = \{3, 5, 7, 9\}$$

- Hence, 3, 5, 7, 9 are called elements of the set 'A'
- That is : $3, 5, 7, 9 \in A$
- Number of elements in set 'A' is 4.
- $n(A) = 4$

where, $n(A)$ mean the number of elements in set 'A'.

Representation of a set: Sets can be represented

in two ways:

- Roster or Tabular form

- Descriptive form

- Set builder Notation

Roster or Tabular form:

Elements শূলোভ ভাবে কর্তৃ দিয়ে — অর্থাৎ কর্তৃ — আবণল প্রেরণ

2nd bracket ফিল করে আবণল ভাবে Roster/ Tabular form বলে,

$$A = \{1, 2, 3, 5\}.$$

Descriptive form: Word দিয়ে Set বুকানো হয়।

$$A = \{1, 2, 3, 4, 5\} \rightarrow \text{Tabular form}$$

A = set of 5 natural numbers \rightarrow Descriptive form

Tabular to descriptive form:

$$C = \{1, 3, 5, 7, 9\}$$

C = set of positive odd integers

Descriptive কর্তৃ আবণল

বীজেট বুকানো হামাছ
অ,

X. $\{x : x \text{ is a root of } x^2 - 4x + 3 = 0\}$

$\{x : x^2 - 4x + 3 = 0\}$

X. $\{x : x \text{ is a natural number such that } 2x + 1 = 7\}$

Set builder form: ആറ്റര അവസ്ഥ ഉപാധാനമുള്ള ഫലം

common characteristic എന്ന് ആക്കി symbolic form എന്നിലും
എൻ,

$$A = \{x \in N \mid x \leq 5\}$$

$$B = \{x \in N \mid 0 < x \leq 50\}$$

$$C = \{x \in O \mid 0 < x\}$$

Task

(a) $A = \{2, 4, 6, 8\} \Rightarrow A = \{x \in \text{even numbers} \mid x \leq 8\}$

(b) $B = \{3, 9, 27, 81\} \Rightarrow B = \{x \mid x = 3^n; n \in N, n \leq 4\}$

(c) $C = \{1, 4, 9, 16, 25\} \Rightarrow C = \{x \mid x = n^2; n^2 \in N; n \leq N\}$

(d) $D = \{1, 3, 5, \dots\} \Rightarrow D = \{x \mid x \text{ is odd}\}$

(e) $E = \{a, e, i, o, u\} \Rightarrow \{x \mid x = \text{Vowels in English alphabets}\}$

Descriptive form to Roster or Tabular form:

(a) $A = \{x \mid x \in W, x \leq 5\}$
 $= \{0, 1, 2, 3, 4, 5\}$

(b) $B = \{\text{The set of all even numbers less than } 12\}$
 $= \{2, 4, 6, 8, 10\}$

(c) $C = \{x \mid x \text{ is divisible by } 12\}$
 $= \{12, 24, 36, \dots\}$

(d) $D = \{ \text{The set of first seven natural numbers} \}$
= $\{1, 2, 3, 4, 5, 6, 7\}$

(e) $E = \{ \text{The set of whole numbers less than } 5 \}$
= $\{0, 1, 2, 3, 4\}$.

Famous sets in Math:

N = Set of natural numbers

Z = Set of integers

Z^+ = Set (of) int positive integer : $A \in \{ \text{...}, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 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⑧ The symbol "... " is called an ellipsis. It is a short form "and so forth".

⑨ When a and b are real numbers with $a < b$, we write,

• $[a, b] = \{x | a \leq x \leq b\} \Rightarrow$ close interval from a to b

• $[a, b) = \{x | a \leq x < b\}$

• $(a, b] = \{x | a < x \leq b\} \Rightarrow$ half open interval from a to b

• $(a, b) = \{x | a < x < b\} \Rightarrow$ open interval from a to b

Finite sets: A finite set is one in which it is possible to list and count all the members of the set.

Example: ① $D = \{$ days of week $\} = \{\text{sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$

$D = \{\text{sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$

Hence, $n(D) = 7$ which is countable so it is finite set

② $A = \{1, 2, 3, 4, 5\}$

The set 'A' has 5 elements and so it is finite set because 'A' has a countable number.

Infinite set: An infinite set is one in which it is not possible to list and count all the members of the set.

Example: ① $E = \{ \text{even numbers greater than } 9 \}$

$$E = \{ 10, 12, 14, 16, \dots \}$$

Here $n(E)$ is infinite.

② $G = \{ \text{whole numbers greater than } 2000 \}$

$$G = \{ 2001, 2002, 2003, 2004, \dots \}$$

Here $n(G) = \infty$ because G has a uncountable number.

Task:

- $A = \{ x : x \in N \text{ and } x \text{ is even} \}$ — Infinite
- $B = \{ x : x \in N \text{ and } x \text{ is composite} \}$ — Infinite
- $C = \{ x : x \in N \text{ and } 3x - 2 = 0 \}$ — Finite
- $D = \{ x : x \in N \text{ and } x^2 = 9 \}$ — Finite
- $E = \{ \text{The set of numbers which are multiple of } 8 \}$ — Infinite
- $F = \{ \text{The set of letters in English alphabet} \}$ — Finite
- $G = \{ \text{The set of persons living in a house} \}$ — Finite
- $H = \{ x : x \in P, P \text{ is a number} \}$ — Infinite
- $I = \{ \text{The set of fractions with numerator } 3 \}$ — Infinite.

Empty set / Null set: An empty set is a set which has no members.

Example: $H = \{\text{the number of dinosaurs on earth}\}$

Hence H is an empty set.

$$H = \{\}$$

⇒ An empty set is denoted by the symbol $\{\}$ or \emptyset .

⇒ the subtlety in $\emptyset \neq \{\emptyset\}$

- The left hand side is the empty set

- The right hand side is a singleton set, and a set containing a set, have to be written like this

Singleton set or Unit set: A singleton is a set that contains exactly one element.

⇒ Sometimes it is known as unit set.

⇒ The singleton containing only the element a can be written as $\{a\}$.

⇒ \emptyset is empty set and $\{\emptyset\}$ is not empty set but it is a singleton set.

⇒ Singleton set or unit set contains only

⇒ one element. A singleton set is denoted

⇒ by $\{s\}$

$$S = \{x \mid x \in N, 7 < x < 9\}$$

Task:

- a. $A = \{x : x \in N, 1 < x < 2\}$ - Null
- b. $B = \{\text{point of intersection of two lines}\}$ - singleton
- c. $C = \{x : x \text{ is an even prime number greater than } 2\}$ - Null
- d. $\emptyset = \{x : x \text{ is an even prime number}\}$ - singleton
- e. $E = \{x : x^2 = 9, x \text{ is even}\}$ - Null
- f. $B = \{0\}$ - Singleton
- g. $D = \{\text{The set of largest 1 digit numbers}\}$ - singleton
- h. $F = \{\text{The set of triangles having 4 sides}\}$ - Null
- i. $H = \{\text{The set of even numbers not divisible by 2}\}$ - Null

Equal sets: Two sets are equal if they have the same number of elements.

Example: ① $A = \{1, 2, 3\}$

$$B = \{1, 2, 3\}$$

$\therefore A = B$, that is both sets are equal.

② $C = \{1, 2, 5\}$

$$D = \{5, 1, 2\}$$

$\therefore C = D$, that is both sets are equal.

Equivalent sets: Two sets are equivalent if they have the same number of elements.

Example: ① $F = \{2, 4, 6, 8, 10\}$

$$G = \{10, 20, 30, 40, 50\}$$

$\therefore n(F) = n(G)$, that is sets F and G are equivalent.

② $A = \{1, 2, 3\}$

$$B = \{a, b, c\}$$

$\therefore n(A) = n(B)$, that is, sets A and B are equivalent.

\Rightarrow Sets cannot be paired in a 1-1 correspondence are called non-equivalent sets.

Task:

- $A = \{x : x \in \mathbb{N}, x \leq 6\}$ $B = \{x : x \in \mathbb{N}, 1 \leq x \leq 6\}$ \Rightarrow Equal
- $P = \{\text{The set of letters in word "plane"}\}$ $Q = \{\text{The set of letters in word "plain"}\}$ \Rightarrow Equivalent
- $X = \{\text{The set of colors in the rainbow}\}$ $Y = \{\text{The set of days in a week}\}$ \Rightarrow Equivalent
- $M = \{4, 8, 12, 16\}$ $N = \{8, 12, 4, 16\}$ \Rightarrow Equal

e. $\{2, 3, 5, 7\}$, $\{2, 2, 3, 5, 3, 7\}$ — Equal

$\{2, 3, 5, 7\}$, $\{2, 3\}$ — Not equal

② Equal sets are always equivalent.

Equivalent sets are may not be equal.

Subsets:

$$B = \{3, 5, 6, 8, 9, 10, 11, 13\}$$

$$A = \{5, 11, 13\}$$

Hence, A is a subset of B $A \subseteq B$

- A set is a subset of a set B, if all the elements of A are contained in members of the larger set B.
- Set A is a subset of B if and only if every element of A is also an element of B.
- The empty set ($\{\} \text{ or } \emptyset$) is a subset of every set.
- $A \subseteq B$ (where \subseteq means 'is a subset of')
- A is a subset of B if and only if every element of A is in B.

Example: $F = \{1, 2, 3\}$

$G_1 = \{1, 2\}$

Hence, G_1 is a subset of F . (i.e. $G_1 \subseteq F$)

■ Not a subset

$A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4, 5\}$

$\therefore B$ is not a subset of A

\therefore That is $B \not\subseteq A$ (where $\not\subseteq$ means 'not is a subset of')

\Rightarrow Every set is a subset of itself. The empty set is a subset of every set.

Tarokro:

① $A = \{3, 9\}$

$B = \{5, 9, 1, 3\}$

$\therefore A \subseteq B = \text{Yes}$

② $A = \{3, 3, 3, 9\}$

$B = \{5, 9, 1, 3\}$

$A \subseteq B = \text{Yes}$

③ $A = \{1, 2, 3\}$

$B = \{2, 3, 4\}$

$\therefore A \subseteq B = \text{No}$

Number of subsets:

If $M = \{a, b, c\}$

Subsets of M are: $\{\text{a}\}, \{\text{b}\}, \{\text{c}\}, \{\text{a}, \text{b}\}, \{\text{a}, \text{c}\}, \{\text{b}, \text{c}\}, \{\text{a}, \text{b}, \text{c}\}, \{\}$

The number of subsets, $S = 8$

$$\text{Formula: } S = 2^n$$

where s is the number of sets and n is the number of elements of the set.

Example: $M = \{a, b, c\}$ To find the number of subsets of M .

$$S = 2^n$$

$$= 2^3$$

$$= 8.$$

Proper subset: A is a proper subset of B if and only if every element in A is also in B , and there exists at least one element in B that is not in A .

Example: ① $A = \{1, 2, 3\}$ is a subset of $B = \{1, 2, 3\}$ but is not a proper subset of $\{1, 2, 3\}$.

② $A = \{1, 2, 3\}$ is a proper subset of $B = \{1, 2, 3, 4\}$
because the element 4 is not in the set.
Hence $A \subset B$.

Subset and proper set:

Difference between subset and proper set:

$$A \subset B \iff B - A \neq \emptyset$$

\Rightarrow The set $\{2, 3, 5, 7\}$ is a subset of $\{2, 3, 5, 7\}$

\Rightarrow The set $\{2, 3, 5, 7\}$ is not a proper subset of $\{2, 3, 5, 7\}$

\Rightarrow The set $\{2, 3, 5\}$ is a proper subset of $\{2, 3, 5, 7\}$

Task:

- $x \in \{x\}$ — True (x is the member of the singleton $\{x\}$)
- $\{x\} \subseteq \{x\}$ — True
- $\{x\} \in \{x\}$ — False
- $\{x\} \in \{\{x\}\}$ — True
- $\emptyset \subseteq \{x\}$ — True
- $\emptyset \in \{x\}$ — False

Cardinality of set: The cardinality of set A , denoted $|A|$, is the number of elements in A . If the set has an infinite number of elements, then its cardinality is ∞ .

\Rightarrow The cardinality of a set A is denoted by $|A|$.

- if $A = \emptyset$, then $|A|=0$

If A has n elements, then $|A|=n$.
In addition, if A is a nonnegative number, then $|A|=n$.
• A is an infinite set, then $|A|=\infty$.

Example: ① Let A be the set of odd positive integers less than 10.

Then $|A|=5$

② Let S be the set of letters in the English alphabet.

Then $|S|=26$

③ Let P be the set of infinite numbers.

Then $|P|=\infty$

\Rightarrow The cardinality of the empty set is $|\emptyset|=0$

\Rightarrow The sets N, Z, Q, R are all infinite.

- $A = \{Mercedes, BMW, Porше\}$
 $|A| = 3$
- $B = \{x : x \text{ is an odd number divisible by } 2\}$
 $|B| = 0$
- $C = \{1, \{2, 3\}, \{4, 5\}, 6\}$
 $|C| = 4$
- $D = \{x : x \text{ is a counting number} < 10\}$
 $|D| = 9$
- $E = \emptyset$
 $|E| = 0$
- F: Let in the words BANANA
 $|F| = 3$

- $G = \{n \in N \mid n \leq 17000\}$
 $|G| = 7001$

Power set: A power set is a set of all the subsets of a set.

The power set of S is denoted by $P(S)$
 $P(S) = 2^n$

$$A = \{a, b\}, P(A) = 2^4 = 16$$

$$B = \{1, 2, 3\}, P(B) = 2^3 = 8$$

- $A = \{1, 2, 3\}$

Powerset of $A = 2^3 = 8$

$$P(A) = \{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$$

Disjoint set: Two sets are called disjoint if they have no elements in common.

Example: $S = \{2, 4, 6, 8\}$

$$T = \{1, 3, 5, 7\}$$

They are disjoint set.

- Two sets A and B are disjoint if $A \cap B = \emptyset$

Task:

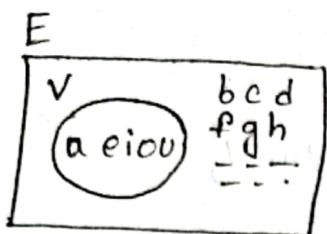
a. $A = \{\text{The set of boys in the school}\}$
 $B = \{\text{The set of girls in the school}\}$

b. $P = \{\text{The set of vowels letters in the English alphabet}\}$
 $Q = \{\text{The set of letters in the English alphabet}\}$

c. $X = \{x : x \text{ is an odd number, } x < 9\}$
 $Y = \{x : x \text{ is an even number, } x < 10\}$

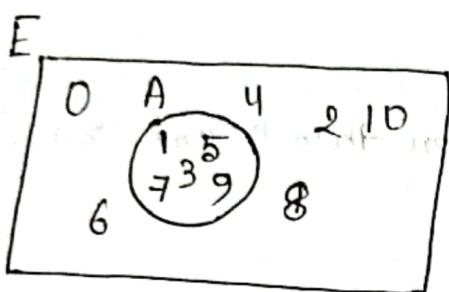
d. $E = \{9, 99, 999\}$
 $F = \{1, 10, 100\}$

Venn Diagrams: English mathematician John Venn. (1834-1923) began using diagrams to represent this.



Hence, $V = \{\text{vowels}\}$

and $E = \{\text{letters of the alphabet}\}$



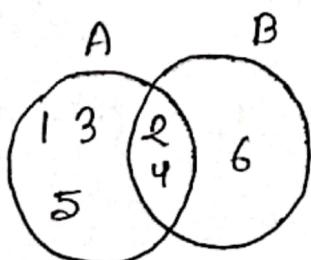
The region inside the circle represents the set A of odd whole numbers between 0 and 10.

Thus we place the numbers 1, 3, 5, 7 and 9 inside the circle. because $A = \{1, 3, 5, 7, 9\}$. outside the circle we place the other numbers 0, 2, 4, 6, 8 and 10 are in E not in A .

Union of sets; $A = \{1, 2, 3, 4, 5\}$

$B = \{2, 4, 6\}$

Union of these sets is $= A \cup B = \{1, 2, 3, 4, 5, 6\}$



$A \cup B$



Example: • $A = \{x | x \text{ is a number bigger than } 4 \text{ and smaller than } 8\}$
 $= \{5, 6, 7\}$

$B = \{x | x \text{ is a number smaller than } 7\}$
 $= \{1, 2, 3, 4, 5, 6\}$

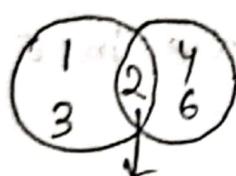
$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

$= \{x | x \text{ is a number bigger than } 0 \text{ and smaller than } 8\}$

Intersection of sets: $A = \{1, 2, 3\}$

$$B = \{2, 4, 6\}$$

$$A \cap B = \{2\}$$



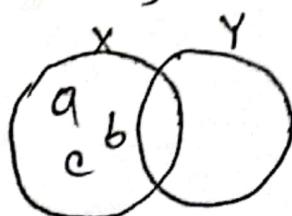
$$A \cap B = \{2\} \Rightarrow \text{common elements}$$

• $A = \{a, b, c, d\}, B = \{1, a, 2, b\}$

$$A \cap B = \{a, b\}$$

• $X = \{a, b, c\}, Y = \{\emptyset\}$

$$X \cap Y = \{\}$$



$$X \cap Y = \{\}$$



Non empty sets which have no members in common are called disjoint sets.

Task: ① $A = \{4, 6, 8, 10, 12\}$, $B = \{3, 6, 9, 12, 15, 18\}$

$C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

① $A \cap B = \{6, 12\}$

② $A \cap C = \{4, 6, 8, 10\}$

③ $B \cap C = \{3, 6, 9\}$

Difference of sets:

$A = \{0, 1, 2, 3\}$

$B = \{2, 3\}$

The difference set is $\{0, 1\}$

We can write it as $A - B$ or $A \setminus B$.

We say: 'A difference B' means all elements which are in A but not in B.

• $B - A$ or $B \setminus A = \{\}$

Task: ① $A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

$A - B = \{1, 2, 3\} - \{4, 5, 6\}$

= A

$B - A = \{4, 5, 6\} - \{1, 2, 3\}$

= B

② • $P = \{x : x \text{ is a natural number between } 10 \text{ and } 16\}$

• $Q = \{y : y \text{ is an even number between } 8 \text{ and } 20\}$

$R = \{7, 9, 11, 14, 18, 20\}$

$$\text{Solution: } P = \{11, 12, 13, 14, 15\}$$

$$Q = \{10, 12, 14, 16, 18\}$$

$$R = \{7, 9, 11, 14, 18, 20\}$$

$$(i) P-Q = \{11, 13, 15\}$$

$$(ii) Q-R = \{10, 12, 16\}$$

$$(iii) R-P = \{7, 9, 18, 20\}$$

$$(iv) Q-P = \{10, 16, 18\}$$

Universal set: The universal set is the set of all elements that are considered in a specific theory.

$$U = \{6, 7, 8, 9, 15, 16, 17, 18, 20, 21\}$$

$$M = \{x/x \text{ are the multiple of 3}\}$$

$$N = \{x/x \text{ are the multiple of 5}\}$$

$$\rightarrow M = \{6, 9, 15, 18, 21\}$$

$$\rightarrow N = \{15, 10\}$$

Complement of a set: $A' = U - A$ or $U \setminus A$

$$\therefore U = A \cup A' = \{5, 7, 19, 17, 19\} \cup \{3, 7, 9, 13, 15\}$$

$$U = A \cup A' = \{5, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

Task:

- ① $A = \{x/x \text{ is a number bigger than 4 and smaller than 8}\}$

$U = \{x | x \text{ is a positive number smaller than } 7\}$

$A = \{5, 6, 7\}$ and $U = \{1, 2, 3, 4, 5, 6\}$

$A' = \{1, 2, 3, 4\}$.

- $A' = \{x | x \text{ is a } \cancel{\text{smaller than}} \text{ bigger than } 1 \text{ and smaller than } 5\}$

$U = \{1, 2, 3, 4\}$

$A = \{\}$

$A' = \{1, 2, 3, 4\}$ or we simply say U

- A' is the complement of A
- $A \cap A' = \{\} \text{ or } \emptyset$

\Rightarrow The complement of a universal set is an empty set.

$U = \{1, 2, 3, 4\}$

$U' = \emptyset$

\Rightarrow The complement of an empty set is a universal set.

$U = \{1, 2, 3, 4\}$

$A = \{\}$

$A' = \{1, 2, 3, 4\}$