Set

① What is Discrete mathmatics?

⇒ Discrete mathmatics is the study of mathmetical structures that are fundamentally discrete rather than continuous. [Meaning of word "Discrete" → individually separate or distinct]

* Discrete vs continuous

Discrete	Continuous
① Can't be broken down into fractions or decimals	① can be broken down into fractions or decimals.
② Number of students Purchasing tickets, text message etc.	② Time, temperature, weight, distance etc.
③ Discrete elements are countable	③ Continuous objects are measurable.

⇒ G. Cantor (German mathmatician) introduced the concept of sets.

Set: • A set is a collection of objects that are clearly identified.

- The objects in the set are called the members or elements of the set.
- Each member is said to belong to the set.
- Sets are denoted by capital letters:

  A, B, C ---, X, Y, Z

- The elements of a set are represented by lower case letters:

  a, b, c, ------, x, y, z

$\in$ ⇒ The symbol '$\in$' is used to denote belongs to or is an element of or is a member of set.

- $a \in A$ ⇒ "a is an element of set A"
  "a is a member of set A"
- $a \notin A$ ⇒ "a is not an element of set A"
- Order of elements is meaningless.

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- For example, the items you wear: shoes, socks, hat, shirt, pants, and so on. This is known as a set.
- The set includes index, middle, ring and pinky.

Notation of set:

- There is a fairly simple notation for sets. We simply list each element (or "member") separated by a comma, and then put some curly brackets around the whole thing: $\{3, 6, 9, 12\}$
- $\{\}$ ⇒ Set brackets or braces
- The three dots ... are called an ellipsis, and mean "Continue on".

Numerical set (well defined)

⇒ Vowels in the english alphabet

$V = \{a, e, i, o, u\}$

⇒ Odd positive integer less than 10

$O = \{1, 3, 5, 7, 9\}$

⇒ Set of even numbers: $\{\ldots, -4, -2, 0, 2, 4\}$

⇒ Set of prime numbers: $\{2, 3, 5, 7, 11, 13, 17\}$

⇒ Set of odd numbers: $\{\ldots, -3, -1, 1, 3, \ldots\}$

⇒ Positive multiple of 3 that are less than 10:

{3, 6, 9}

Numerical sets: (Not well-defined)

There can also be sets of numbers that have no common property, they are just defined that way.

- {2, 3, 6, 828, 3839}
- {4, 5, 6, 10, 21}
- {111, 8888}

Task:

① All the colors in the rainbow – well defined

② All the points that lie on a straight line – Not well defined

③ All the honest members in the family. – Not well defined

④ All the ~~consto~~onants of the English alphabet – well defined

⑤ All the tall boys of the school. – Not well defined

⑥ All the hardworking teachers in a school. – Not well defined

⑦ All the prime numbers less than 100. – Well defined

⑧ All the letters in the word Geometry. – Well defined

Member of set:

Is a member of	Is not a member of
① $3 \in \{1, 2, 3\}$	① $3 \notin \{4, 5, 6\}$
② $-1 \in \{-5, -1, -7\}$	② $a \notin \{b, c, d\}$

Element of set:

→ A set of odd numbers between 1 and 10.

$A = \{3, 5, 7, 9\}$

- Here, 3, 5, 7, 9 are called elements of the set 'A'
    - That is : $3, 5, 7, 9 \in A$
    - Number of elements in set 'A' is 4.
    - $n(A) = 4$

where, n(A) mean the number of elements in set 'A'.

Representation of a set: Sets can be represented in two ways:

- Roster or Tabular form
- Descriptive form
- Set builder Notation

Roster or Tabular form:

Element গুলোর মধ্যে কমা দিয়ে আলাদা করা থাকলে এবং 2nd bracket দিয়ে close থাকলে তাকে Roster / Tabular form বলে,

$A = \{1, 2, 3, 5\}$.

Descriptive form: Word দিয়ে set বুঝানো হয়,

$A = \{1, 2, 3, 4, 5\} \rightarrow$ Tabular form

$A = \{$ set of 5 natural number $\rightarrow$ Descriptive form

Describe করা থাকবে কী সেট বুঝানো হয়েছে তা,

Tabular to descriptive form:

$C = \{1, 3, 5, 7, 9\}$

$C =$ set of positive odd integers

Set builder form: সেটের সকল উপাদানগুলোকে কিছু common characteristic এর সাথে symbolic form এ লিখা হয়।

$A = \{x \in N \mid x <= 5\}$

$B = \{x \in N \mid 0 < x <= 50\}$

$C = \{x \in Q \mid 0 < x\}$

Task

(a) $A = \{2, 4, 6, 8\} \Rightarrow A = \{x \in \text{even and } x \le 8\}$

(b) $B = \{3, 9, 27, 81\} \Rightarrow B = \{x ; x\ 3n ; n \in N,\ n \le 4\}$

(c) $C = \{1, 4, 9, 16, 25\} \Rightarrow C = \{x : x \in n^2 ; n^2 \in N : n \le N\}$

(d) $D = \{1, 3, 5, ---\} \Rightarrow D = \{x : x \text{ is odd}\}$

(e) $E = \{a, e, i, o, u\} \Rightarrow \{x : x = \text{Vowels in English alphabets}\}$

Descriptive form to Roster or Tabular form:

(a) $A = \{x : x \in W,\ x \le 5\}$

$= \{0, 1, 2, 3, 4, 5\}$

(b) $B = \{\text{The set all even numbers less than } 12\}$

$= \{2, 4, 6, 8, 10\}$

(c) $C = \{x : x \text{ is divisible by } 12\}$

$= \{12, 24, 36, ----\}$

(d) D = {The set of first seven natural numbers}

= {1, 2, 3, 4, 5, 6, 7}

(e) E = {The set of whole numbers less than 5}

= {0, 1, 2, 3, 4}.

Famous sets in Math:

N = Set of natural numbers

Z = Set of integers

Z+ = set of ~~int~~ positive integer

Q = $\{p/q \mid p \in Z, q \in Z, \text{ and } q \neq 0\}$, set of rational numbers.

Q+ = The set of positive rational number.

R = The set of real numbers.

R+ = The set of positive real numbers.

C = The set of complex numbers.

"Standard" sets:

- Natural numbers N = {0, 1, 2, 3, ---}
- Integers Z = {...., −2, −1, 0, 1, 2}
- Positive integer Z^+ = {1, 2, 3, 4}
- Real numbers R = {47.3, −12, π, ---}
- Rational numbers Q = {1.5, 2.6, −3.8, 15, --}

(x) The symbol "..." is called an ellipsis. It is a short for "and so forth".

[ii] When a and b are real numbers with $a<b$, we write,

- $[a,b] = \{x \mid a \le x \le b\}$ ⇒ close interval from a to b
- $[a, b) = \{x \mid a \le x < b\}$
- $(a, b] = \{x \mid a < x \le b\}$ ⇒ half open interval from a to b
- $(a,b) = \{x \mid a < x < b\}$ ⇒ open interval from a to b

Finite set: A finite set is one in which it is possible to list and count all the members of the set.

Example: ① D = {days of week}

D = {sunday, Monday, Tuesday, Wednesday, Thursday, Friday, saturday}

Here, $n(D) = 7$ which is countable so it is finite set

② $A = \{1, 2, 3, 4, 5\}$

The set 'A' has 5 elements and so it is finite set because 'A' has a countable number.

Infinite set: An infinite set is one in which it is not possible to list and count all the members of the set.

Example: ① $E = \{$even numbers greater than 9$\}$

$E = \{10, 12, 14, 16, -----\}$

Here $n(E)$ is infinite.

② $G - \{$whole numbers greater than 2000$\}$

$G = \{2001, 2002, 2003, 2004, ----\}$

Here $n(G)$ = infinite because G has a uncountable number.

Task:

a. $A = \{x : x \in N$ and x is even$\}$ — Infinite

b. $B = \{x : x \in N$ and x is composite$\}$ — Infinite

c. $C = \{x : x \in N$ and $3x - 2 = 0\}$ — Finite

d. $D = \{x : x \in N$ and $x^2 = 9\}$ — Finite

e. $E = \{$ The set of numbers which are multiple of 9$\}$ — Infinite

f. $F = \{$The set of letters in English alphabets$\}$ — Finite

g. $G = \{$ The set of persons living in a house$\}$ — Finite

h. $H = \{x : x \in P, P$ is a number$\}$ — Infinite

i. $I = \{$ The set of fractions with numerator 3$\}$ — Infinite.

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Empty set / Null set: An empty is a set which has no members.

Example: $H = \{$the number of dinosaurs on earth$\}$

Here H is an empty set.

$H = \{\}$

⇒ An empty set is denoted by the symbol $\{\}$ or ϕ.

⇒ the subtlety in $\phi \neq \{\phi\}$

- The left hand side is the empty set
- The right hand - side is a singleton set, and a set containing a set.

Singleton set or unit set: A singleton is a set that contains exactly one element.

⇒ Sometimes it is known as unit set.

⇒ The singleton containing only the element a can be written $\{a\}$.

⇒ ϕ is empty set and $\{\phi\}$ is not empty set but it is a singleton set.

⇒ singleton set or unit set contains only

⇒ one element. A singleton set is denoted

⇒ by $\{s\}$

$S = \{x \mid \in N, 7 < x < 9\}$

Task:

a. $A = \{x : x \in N, 1 < x < 2\}$ — Null

b. B = {point of intersection of two lines} — singleton

c. C = {x : x is an even prime number greater than 2} — Null

d. Q = {x : x is an even prime number} — singleton

e. $E = \{x : x^2 = 9, x \text{ is even}\}$ — Null

f. $B = \{0\}$ — Singleton

g. D = {The set of largest 1 digit number} — singleton

h. F = {The set of triangles having 4 sides} — Null

i. H = {The set of even numbers not divisible by 2} — Null

Equal sets: Two sets are equal if they have the same number of elements.

Example: ① $A = \{1, 2, 3\}$

$B = \{1, 2, 3\}$

$\therefore A = B$, that is both sets are equal.

② $C = \{1, 2, 5\}$

$D = \{5, 1, 2\}$

$\therefore C = D$, that is both sets are equal.

Equivalent sets: Two sets are equivalent if they have the same number of elements.

Example: ① $F = \{2, 4, 6, 8, 10\}$
$G = \{10, 20, 30, 40, 50\}$

$\therefore n(F) = n(G)$, that is sets F and G are equivalent.

② $A = \{1, 2, 3\}$
$B = \{a, b, c\}$

$\therefore n(A) = n(B)$, that is, sets A and B are equivalent.

⇒ Sets cannot be paired in a 1-1 correspondence are called non-equivalents sets.

Task:

a. $A = \{x : x \in N, x \le 6\}$
$B = \{x : x \in W, 1 \le x \le 6\}$ ⇒ Equal

b. P = {The set of letters in word "plane"}
Q = {The set of letters in word "plain"} → Equivalent

c. X = {The set of color in the rainbow}
Y = {The set of days in a week} ⇒ Equivalent

d. $M = \{4, 8, 12, 16\}$
$N = \{8, 12, 4, 16\}$ ⇒ Equal

e. $\{2,3,5,7\}$, $\{2,2,3,5,3,7\}$ — Equal

$\{2,3,5,7\}$, $\{2,3\}$ — Not equal

(*) Equal sets are always equivalent.

Equivalent sets are may not be equal.

Subsets: $B = \{3,5,6,8,9,10,11,13\}$

$A = \{5,11,13\}$

Here, A is a subset of B $A \subseteq B$

- A set is a subset of a set B, if all the elements of A are contained in members of the larger set B.

- Set A is a subset of B if and only if every ~~mem~~ element of A is also an element of B

- The empty set ($\{\}$ or $\emptyset$) is a subset of every set.

- $A \subseteq B$ (where $\subseteq$ means 'is a subset of')

- A is a subset of B if and only if every element of A is in B.

Example: $F = \{1, 2, 3\}$
$G = \{1, 2\}$

Here, G is a subset of F.

Not a subsets

$A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4, 5\}$

$\therefore$ B is not a subset of A

$\therefore$ That is $B \not\subseteq A$ (where $\not\subseteq$ means 'not is a subset of')

$\Rightarrow$ Every set is a subset of itself. The empty set is a subset of every set.

Tasks:

① $A = \{3, 9\}$

$B = \{5, 9, 1, 3\}$

$\therefore A \subseteq B$ = Yes

② $A = \{3, 3, 3, 9\}$

$B = \{5, 9, 1, 3\}$

$A \subseteq B$ = Yes

③ $A = \{1, 2, 3\}$

$B = \{2, 3, 4\}$

$\therefore A \subseteq B$ = No

Number of subsets:

If $M = \{a, b, c\}$

Subsets of M are: $\{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}$
$\{b,c\}, \{a,b,c\}, \{\}$

The number of subsets, $S = 8$

Formula: $S = 2^n$

where s is the number of sets and n is the number of elements of the set.

Example: $M = \{a, b, c\}$

$S = 2^n$
$= 2^3$
$= 8.$

Proper subset: A is a proper subset of B if and only if every element in A is also in B, and there exists at least one element in B that is not in A.

Example: ① $A = \{1,2,3\}$ is a subset of $B = \{1,2,3\}$ but is not a proper subset of $\{1,2,3\}$.

② $A = \{1,2,3\}$ is a proper subset of $B = \{1,2,3,4\}$ because the element 4 is not in the 1st set.

Hence $A \subset B$.

Subset and proper set:

Difference between subset and proper set:

⇒ The set $\{2,3,5,7\}$ is a subset of $\{2,3,5,7\}$

⇒ The set $\{2,3,5,7\}$ is not a proper subset of $\{2,3,5,7\}$

⇒ The set $\{2,3,5\}$ is a proper subset of $\{2,3,5,7\}$

Task:

- $x \in \{x\}$ — True (x is the member of the singleton $\{x\}$)
- $\{x\} \subseteq \{x\}$ — True
- $\{x\} \in \{x\}$ — False
- $\{x\} \in \{\{x\}\}$ — True
- $\emptyset \subseteq \{x\}$ — True
- $\emptyset \in \{x\}$ — False

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Cardinality of set: The cardinality of set S, denoted |S|, is the number of elements in S. If the set has an infinite number of elements in S then its cardinality is ∞.

⇒ The cardinality of a set A is denoted by |A|

- if $A = \phi$, then $|A| = 0$
- If A has an elements, then $|A| = n$
- n is a nonnegative number.
- A is an infinite set, then $|A| = \infty$.

Example: ① Let A be the set of odd positive integers less than 10.

Then $|A| = 5$

② Let S be the set of letters in the English alphabet.

Then $|S| = 26$

③ Let P be the set of infinite numbers.

Then $|P| = \infty$

⇒ The cardinality of the empty set is $|\emptyset| = 0$

→ The sets N, Z, Q, R are all infinite.

- $A = \{Mercedes, BMW, Porsche\}$

  $|A| = 3$

- $B = \{x : x$ is an odd number divisible by $2\}$,

  $|B| = 0$

- $C = \{1, \{2,3\}, \{4,5\}, 6\}$

  $|C| = 4$

- $D = \{x : x$ is a counting number $< 10\}$

  $|D| = 9$

- $E = \emptyset$

  $|E| = 0$

- $F = \{$ Let in the words BANANA $\}$

  $|F| = 3$

- $G = \{x \in N \mid x \leq 7000\}$

  $|G| = 7001$.

Power sets: A power set is a set all the subsets of a set.

The power set of S is denoted by P(S)

$P|S| = 2^n$

$A = \{a, b\}$, $P(A) = 2^4 = 16$

$B = \{1,2,3\}$, $P(B) = 2^3 = 8$

- $A = \{1, 2, 3\}$

Powers set of $A = 2^3 = 8$

$P(A) = \{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

Disjoint set: Two sets are called disjoint if they have no elements in common.

Example: $S = \{2, 4, 6, 8\}$

$T = \{1, 3, 5, 7\}$

They are disjoint set.

- Two sets A and B are disjoint if $A \cap B = \{\}$

Task:

a. A = {The set of boys in the school}
B = {The set of girls in the school} — Disjoint sets

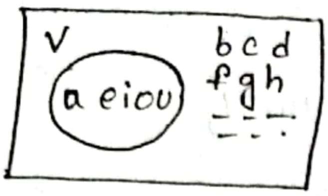
b. P = {The set of vowels letters in the English alphabet}
Q = {The set of letters in the English alphabets} — overlapping sets

c. $X = \{x : x$ is an odd number, $x < 9\}$
$Y = \{x : x$ is an even number, $x < 10\}$ — Disjoint sets

d. $E = \{9, 99, 999\}$
$F = \{1, 10, 100\}$ — Disjoint sets

Venn Diagrams: English mathmatician John Venn. (1834 – 1923) began using diagrams to represent this.

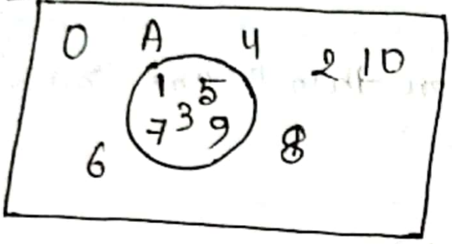
E



Here, V = {vowels}

E = {letters of the alphabet}

E



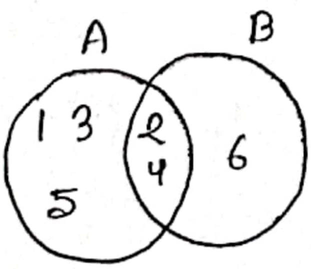
The region inside the circle represents the set A of odd whole numbers between 0 and 10.

Thus we place the numbers 1, 3, 5, 7 and 9 inside the circle. because A = {1, 3, 5, 7, 9}. Outside the circle we place the other numbers 0, 2, 4, 6, 8 and 10 are in E not in A.

Unions of sets: A = {1, 2, 3, 4, 5}

B = {2, 4, 6}

Union of these sets is A∪B = {1, 2, 3, 4, 5, 6}.



A∪B.

Example: • $A = \{x \mid x$ is a number bigger than 4 and smaller than 8$\}$

$= \{5, 6, 7\}$

$B = \{x \mid x$ is a number smaller than 7$\}$

$= \{1, 2, 3, 4, 5, 6\}$

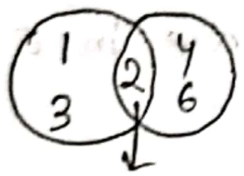
$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$= \{x \mid x$ is a number bigger than 0 and smaller than 8$\}$

Intersection of set: $A = \{1, 2, 3\}$

$B = \{2, 4, 6\}$

$A \cap B = \{2\}$



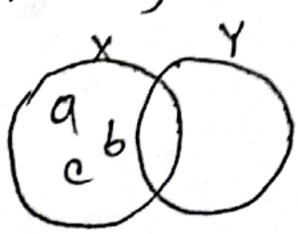
$A \cap B = \{2\} \Rightarrow$ common elements

• $A = \{a, b, c, d\}$, $B = \{1, a, 2, b\}$

$A \cap B = \{a, b\}$

• $X = \{a, b, c\}$, $Y = \{0\}$

$X \cap Y = \{\}$



$X \cap Y = \{\}$

Non empty sets which have no members in common are called disjoint sets.

Task: (1) $A = \{4, 6, 8, 10, 12\}$, $B = \{3, 6, 9, 12, 15, 18\}$

$C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(1) $A \cap B = \{6, 12\}$

(2) $A \cap C = \{4, 6, 8, 10\}$

(3) $B \cap C = \{3, 6, 9\}$

Difference of sets:

$A = \{0, 1, 2, 3\}$

$B = \{2, 3\}$

The difference set is $\{0, 1\}$

We can write it as $A-B$ or $A \setminus B$.

We say: 'A difference B'

- $B-A$ or $B \setminus A = \{\}$

Task: (1) $A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

$A-B = \{1, 2, 3\} - \{4, 5, 6\}$

$= A$

$B-A = \{4, 5, 6\} - \{1, 2, 3\}$

$= B$

(2) • $P = \{x : x \text{ is a natural number between } 10 \text{ and } 16\}$

$Q = \{y : y \text{ is a even number between } 8 \text{ and } 20\}$

$R = \{7, 9, 11, 14, 18, 20\}$

Solution: $P = \{11, 12, 13, 14, 15\}$

$Q = \{10, 12, 14, 16, 18\}$

$R = \{7, 9, 11, 14, 18, 20\}$

(i) $P - Q = \{11, 13, 15\}$

(ii) $Q - R = \{10, 12, 16\}$

(iii) $R - P = \{7, 9, 18, 20\}$

(iv) $Q - P = \{10, 16, 18\}$

Universal set: The universal set is the set of all elements that are considered in a specific theory.

$U = \{6, 7, 8, 9, 15, 16, 17, 18, 20, 21\}$

$M = \{x \mid x$ are the multiple of $3\}$

$N = \{x \mid x$ are the multiple of $5\}$

$M = \{6, 9, 15, 18, 21\}$

$N = \{15, 10\}$

Complement of a set: $A' = U - A$ or $U \setminus A$

- $U = A \cup A' = \{5, 7, 11, 17, 19\} \cup \{3, 7, 9, 13, 15\}$

  $U = A \cup A' = \{5, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

Task:

① $A = \{x \mid x$ is a number bigger than 4 and smaller than $8\}$

$U = \{x | x$ is a positive number smaller than $7\}$

$A = \{5, 6, 7\}$ and $U = \{1, 2, 3, 4, 5, 6\}$

$A' = \{1, 2, 3, 4\}$.

- $A' = \{x | x$ is a ~~smaller tha~~ bigger than 1 and smaller than $5\}$

$U = \{1, 2, 3, 4\}$

$A = \{\}$

$A' = \{1, 2, 3, 4\}$ or we simply say U

- A' is the complement of A
- $A \cap A' = \{\}$ or ϕ

⇒ The complement of a universal set is an empty set.

$U = \{1, 2, 3, 4\}$

$U' = \phi$

⇒ The complement of an empty set is a universal set.

$U = \{1, 2, 3, 4\}$

$A = \{\}$

$A' = \{1, 2, 3, 4\}$