International Islamic University Chittagong
Department of Computer Science & Engineering
B.Sc. in CSE, Mid Term Examination, Autumn-2018
Course Code: MATH-3501 Course Title: Mathematics-V

Marks: 30

AUTUMN 18 ANS

a) Let the function $f:\Re^+ \rightarrow \Re^+$ be defined by $f(x)=x^2+x-2$ Find $f^{-1}(4)$.

b)

ANS 1a

Ans 1 (a)

1.a. Here

$f(x) = \cancel{x^2+x-2} \,/\, x^2+x-2$.

So,

$f^{-1}(4) = \{x \in R^{\#} ; x^2 + x - 2 = 4\}$

$= \{x \in R^{\#} ; x^2 + x - 6 = 0\}$

ax^2+bx+c so that used Quadratic eqn.

$= \{x \in R^{\#} ; x = \frac{-1 \pm \sqrt{1+24}}{2}\}$

$= \{x \in R^{\#} ; x = \frac{-1 \pm \sqrt{25}}{2}\}$

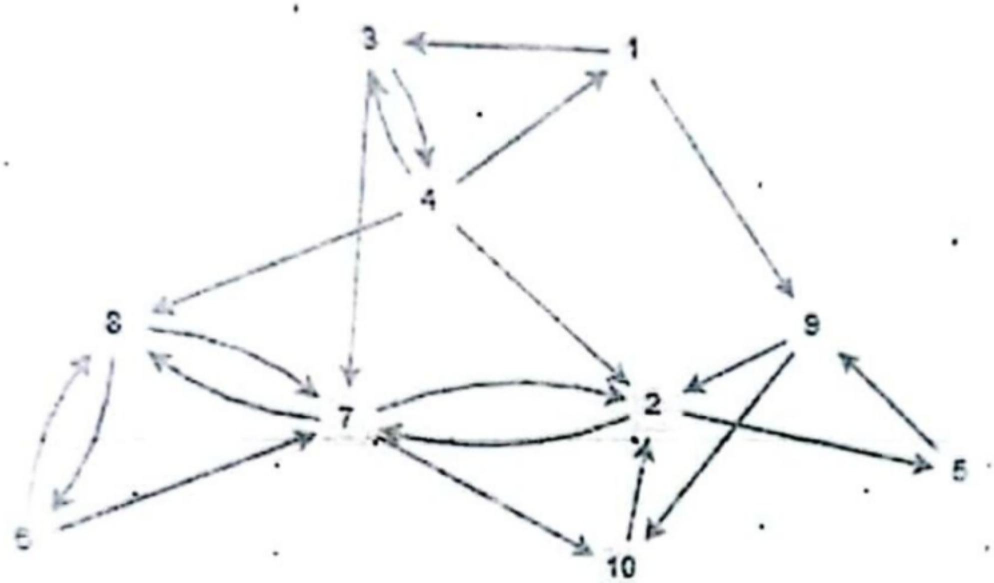
$= \{x \in R^{\#} ; x = \frac{-1 \pm 5}{2}\}$

$= x \in R^{\#} ; x = \frac{-6}{2}, x \frac{4}{2}\}$

$= -3, 2$

used eqn. $\Rightarrow \boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

b)



ANS 1B

2(a)

In this relation,

Let Relation is "R"

Relation is not reflexive, because.

$\{a,a\} \notin R$, Example - $\{1,1\}$, $\{3,3\} \notin R$.

Relation is symmetric, because.

$(a,b) \in R$ implies $(b,a) \in R$, Ex $(3,4) \in R$, $(4,3) \in R$.

Relation is not anti-symmetric, because.

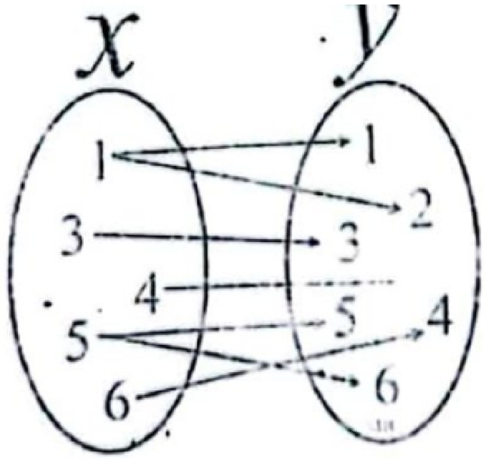
$(a,b) \in R$ and $(b,a) \in$ but $a=b \notin R$ so that it is not anti-symmetric. Ex - $(3,4) \in R$, $(4,3) \in R$ but $(3,3)$ are not.

Relation is ~~not~~ transitive, because.

$(a,b) \in R$ and $(b,c) \in R$ ~~and~~ implies $(a,c) \in R$.

Ex - $(3,4) \in R$ and $\cancel{(4,3)}$ $(4,1) \in R$ implies $(1,3) \in R$.

c)



Whether the top graph is a function or not? Show an explanation for your answer.

AND 1C

Ans. 1 (c)

(c) the graph is not function.

because

'x' set 1 element assign with ~~...~~

'y' set 2 element.

So that this is not function.

[redacted] 2. A circle in the z-plane has its centre at $z = 3$ and a radius of 2 units. Determine its image in the w-plane when transformation by $w = \dfrac{1}{z}$. Show your analsis with necessary justification

ANS 2A

Example 12

A circle in the z-plane has its centre at $\mathbf{z = 3}$ and a radius of 2 units. Determine its image in the w-plane when transformation by $\mathbf{w = \frac{1}{z}}$

Where c is the circle $|z-3| = 2$

We have,

$z = x + jy$

$z - 3 = x + jy - 3$

$z - 3 = x - 3 + jy$

$\therefore |z-3| = \sqrt{(x-3)^2 + y^2}$

Given,

$|z-3| = 2$

$\therefore |z-3| = \sqrt{(x-3)^2 + y^2} = 2$

$\therefore \sqrt{(x-3)^2 + y^2} = 2$

$\therefore (x-3)^2 + y^2 = 2^2$

$\therefore (x-3)^2 + (y-0)^2 = 2^2$

This is the equation of the circle whose centre (3,0) and radius 2

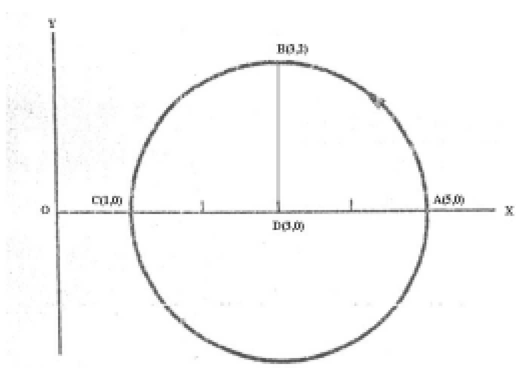


Figure 30

Solution:

Given,

$$\mathbf{z = 3}$$
$$\Rightarrow \mathbf{x + jy = 3}$$
$$\Rightarrow x + jy = 3 + 0.j \text{ ------------------------(i)}$$

Equating the coefficient of real and imaginary part, we get,

$$\mathbf{x = 3, \quad y = 0}$$

Hence, we can write,

$$(x, y) = (3,0)$$

and given, radius =2

So, Equation of the circle

$(x-3)^2 + (y-0)^2 = 2^2$ ----------------------(ii)

$$[\because (x-a)^2 + (y-b)^2 = r^2]$$

That is centre of the circle is (3,0) and radius 2

$(x-3)^2 + y^2 = 4$

$\Rightarrow x^2 - 6x + 9 + y^2 = 4$

$\Rightarrow x^2 + y^2 - 6x + 5 = 0$ ------------(iii)

Again, Given,

$w = \frac{1}{z}$

$w = \frac{1}{x + jy}$ $[z = x + jy]$

$w = \frac{x - jy}{(x + jy)(x - jy)}$ [Multiplying by $x - jy$]

$w = \frac{x - jy}{x^2 - jxy + jxy - j^2 y^2}$

$w = \frac{x - jy}{x^2 + y^2}$ $[\because j^2 = -1]$

$u + jv = \frac{x - jy}{x^2 + y^2}$ $[w = u + jv]$

$u + jv = \frac{x}{x^2 + y^2} - j\frac{y}{x^2 + y^2}$ ------------------------(iv)

Equating the coefficient of real and imaginary part, we get,

$u = \frac{x}{x^2 + y^2}$ ----------(v)

$v = \frac{-y}{x^2 + y^2}$ ---------(vi)

Again, Given

$w = \frac{1}{z}$

$\therefore z = \frac{1}{w}$

$z = \frac{1}{u + jv}$ $[w = u + jv]$

$$z=\frac{u-jv}{(u+jv)(u-jv)}$$

$$z=\frac{u-jv}{u^2-(jv)^2}$$

$$z=\frac{u-jv}{u^2+v^2} \qquad [\because j^2=-1]$$

$$x+jy=\frac{u-jv}{u^2+v^2} \qquad [z=x+jy]$$

i.e. $x+jy=\frac{u}{u^2+v^2}-j\frac{v}{u^2+v^2}$ ---------(vii)

Equating the coefficient of real and imaginary part, we get,

$$x=\frac{u}{u^2+v^2} \;;\quad y=\frac{-v}{u^2+v^2} \quad \text{----------(viii)}$$

Substituting the values of x and y in (iii),

$$x^2+y^2-6x+5=0$$

$$\Rightarrow\left(\frac{u}{u^2+v^2}\right)^2+\left(\frac{-v}{u^2+v^2}\right)^2-6\left(\frac{u}{u^2+v^2}\right)+5=0$$

$$\Rightarrow\frac{u^2}{(u^2+v^2)^2}+\frac{v^2}{(u^2+v^2)^2}-\frac{6u}{u^2+v^2}+5=0$$

$$\Rightarrow\frac{u^2+v^2}{(u^2+v^2)^2}-\frac{6u}{u^2+v^2}+5=0$$

$$\Rightarrow\frac{1}{u^2+v^2}-\frac{6u}{u^2+v^2}+5=0$$

$$\Rightarrow\frac{1-6u+5(u^2+v^2)}{u^2+v^2}=0$$

$$\Rightarrow 5(u^2+v^2)-6u+1=0$$

$$\Rightarrow u^2+v^2-\frac{6}{5}u+\frac{1}{5}=0 \quad \text{[dividing by 5]}$$

$$\Rightarrow u^2+v^2-2\cdot\frac{3}{5}\cdot u+2\cdot 0\cdot v+\frac{1}{5}=0$$

$$\Rightarrow u^2+v^2+2\cdot(-\frac{3}{5})\cdot u+2\cdot 0\cdot v+\frac{1}{5}=0 \text{-----------(ix)}$$

We know the general equation of the circle is

$$x^2+y^2+2gx+2fy+c=0$$

Whose centre is $(-g,-f)$ and radius is $\sqrt{g^2+f^2-c}$

Hence from (ix)

Here $\mathbf{g = -\frac{3}{5}}$, $\mathbf{f = 0}$ and $\mathbf{c = \frac{1}{5}}$

The centre of the new circle of (ix) is $\mathbf{(-g,-f) = (-(-\frac{3}{5}),-0) = (\frac{3}{5},0)}$

That is, centre of new circle in the w-plane is, $D(\frac{3}{5},0)$ [From figure 32]

Radius is $\sqrt{g^2 + f^2 - c} = \sqrt{(-\frac{3}{5})^2 + 0^2 - \frac{1}{5}}$

$$= \sqrt{(-\frac{3}{5})^2 + 0^2 - \frac{1}{5}} = \sqrt{\frac{9}{25} + 0 - \frac{1}{5}}$$

$$= \sqrt{\frac{9}{25} - \frac{1}{5}}$$

$$= \sqrt{\frac{9-5}{25}}$$

$$= \sqrt{\frac{4}{25}}$$

$$= \frac{2}{5}$$

That is, radius of new circle in the w-plane is, $\frac{2}{5}$

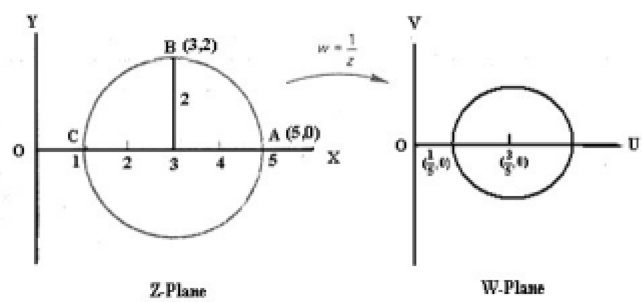


Figure 31

Taking three sample points A, B, C as shown, that is:

$\mathbf{A(5,0)}$, $\mathbf{B(3,2)}$, $\mathbf{C(1,0)}$

Putting the values of $\mathbf{A(5,0)}$, $\mathbf{B(3,2)}$ $\mathbf{C(1,0)}$ in (v) and (vi)

We have,

$$u = \frac{x}{x^2 + y^2} \qquad v = \frac{-y}{x^2 + y^2}$$

For $A(5,0)$; $u = \frac{x}{x^2 + y^2} = \frac{5}{5^2 + 0^2} = \frac{5}{25 + 0} = \frac{5}{25} = \frac{1}{5}$

For $A(5,0)$; $v = \frac{-y}{x^2 + y^2} = \frac{-0}{5^2 + 0^2} = \frac{0}{25 + 0} = 0$

$\therefore$ For $A(5,0)$; $w = u + jv = \frac{1}{5} + j.0$

The image of A is $A'(w = u + jv = \frac{1}{5} + j.0) = \frac{1}{5} + j.0$

That is $A'(\frac{1}{5}, 0)$ --------------------(x)

For $B(3,2)$; $u = \frac{x}{x^2 + y^2} = \frac{3}{3^2 + 2^2} = \frac{3}{9 + 4} = \frac{3}{13}$

For $B(3,2)$; $v = \frac{-y}{x^2 + y^2} = \frac{-2}{3^2 + 2^2} = \frac{-2}{9 + 4} = \frac{-2}{13}$

$\therefore$ For $B(3,2)$; $w = u + jv = \frac{3}{13} + j.(\frac{-2}{13})$

The image of B is $B'(w = u + jv = \frac{3}{13} + j.(-\frac{2}{13})) = \frac{3}{13} - j.\frac{2}{13}$

That is $B'(\frac{3}{13}, -\frac{2}{13})$ -----------------------(xi)

For $C(1,0)$; $u = \frac{x}{x^2 + y^2} = \frac{1}{1^2 + 0^2} = \frac{1}{1} = 1$

For $C(1,0)$; $v = \frac{-y}{x^2 + y^2} = \frac{-0}{1^2 + 0^2} = \frac{-0}{1} = 0$

$\therefore$ For $C(1,0)$; $w = u + jv = 1 + j.(0)$

The image of C is $C'(w = u + jv = 1 + j.0 = 1 + j.0$

That is $C'(1,0)$ --------------------------(xii)

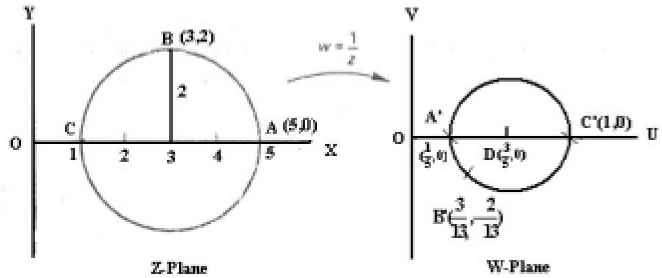


Figure 32

Justification:

We have $A'(\frac{1}{5},0), B'(\frac{3}{13},-\frac{2}{13}), C'(1,0), D(\frac{3}{5},0)$

Radius $= \frac{2}{5}$

We know the length of a line between two points (x_1, y_1) and (x_2, y_2) is:

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Here, in the w-plane

$\therefore A'D = \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2}$

$\therefore A'D = \sqrt{(\frac{1}{5} - \frac{3}{5})^2 + (0-0)^2}$

$\therefore A'D = \sqrt{(-\frac{2}{5})^2}$

$\therefore A'D = \sqrt{\frac{4}{25}}$

$\therefore A'D = \frac{2}{5}$ (Proved)

Again,

$\therefore B'D = \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2}$

$$B'D = \sqrt{(\frac{3}{13} - \frac{3}{5})^2 + (\frac{-2}{13} - 0)^2}$$

$$B'D = \sqrt{(\frac{15-39}{65})^2 + (\frac{4}{169})}$$

$$B'D = \sqrt{(\frac{-24}{65})^2 + \frac{4}{169}}$$

$$B'D = \sqrt{\frac{576}{4225} + \frac{4}{169}}$$

$$B'D = \sqrt{\frac{114244}{714025}}$$

$$B'D = \frac{2}{5} \qquad \text{(Proved)}$$

$$\therefore C'D = \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2}$$

$$C'D = \sqrt{(1 - \frac{3}{5})^2 + (0 - 0)^2}$$

$$C'D = \sqrt{(\frac{5-3}{5})^2}$$

$$C'D = \sqrt{(\frac{2}{5})^2}$$

$$C'D = \frac{2}{5} \qquad \text{(Proved)}$$

a) Determine whether $f(z) = (x^3 - 3xy^2 - 2x) + i(3x^2y - y^3 - 2y)$ is analytic.

ANS 3A

3 (a)

$f(z) = f(x+iy) = (x^3 - 3xy^2 - 2x) + i(3x^2y - y^3 - 2y)$

$u + iv = (x^3 - 3xy^2 - 2x) + i(3x^2 - y^3 - 2y)$

equating real and imaginary part

$u = x^3 - 3xy^2 - 2x$ — (i)

$v = 3x^2 - y^3 - 2y$ — (ii)

from (i) $\Rightarrow$

$$\frac{\delta u}{\delta x} = 3x^2 - 3y^2 - 2$$

$$\frac{\delta u}{\delta y} = -6xy$$

from (ii) $\Rightarrow$

$$\frac{\delta v}{\delta x} = 6xy$$

$$\frac{\delta v}{\delta y} = 3x^2 - 3y^2 - 2.$$

Now cauchy Riemann equation.

$$\frac{\delta u}{\delta x} = \frac{\delta v}{\delta y} \quad \Big| \quad \frac{\delta v}{\delta x} = -\frac{\delta u}{\delta y}$$

$$\therefore 3x^2 - 3y^2 - 2 = 3x^2 - 3y^2 - 2 \quad \Big| \quad \Rightarrow 6xy = -(-6xy)$$

$$\therefore 6xy = 6xy$$

$$[L.H.S] = [R.H.S]$$

Since the cauchy-Riemann equations are satisfied. So that

the function is analytic.

b) Is $f(x, y, z) = x^2 - y^2 - 2z^2$ harmonic? What about $f(x, y, z) = x^2 - y^2 + z^2$?

ANS 3B

Harmonic function:

Let us Consider any given function $U = f(x, y, z)$ Of three Variable Under the domain of R^3 Then we can say that the given function U is Said to be Harmonic if and only if it Satisfies the Laplace Equation.

3(b) must Satisfies $f_{xx} + f_{yy} + f_{zz} = 0$

Here in this Case, the given function is

$f(x, y, z) = x^2 + y^2 - 2z^2$

Now by taking Partial derivatives, we get.

$$f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 + y^2 - 2z^2)$$

$$= 2x$$

$$f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 + y^2 - 2z^2)$$

$$= 2y$$

$$f_z = \frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(x^2 + y^2 - 2z^2)$$

$$= -4z.$$

Similarly again applying Partial deri...

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}(2x) = 2$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}(2y) = 2$$

$$f_{zz} = \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z}\right) = \frac{\partial}{\partial z}(-2z^2) = -4$$

Hence, Obviously we can say that

$$f_{xx} + f_{yy} + f_{zz} = 2+2-4 = 0.$$

* $f(x,y,z) = x^2 - y^2 + z^2$

Now taking Partial derivative

$$f_x = \frac{df}{dx} = \frac{\partial}{\partial x}(x^2 - y^2 + z^2)$$

$$\Rightarrow 2x$$

$$f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 - y^2 + z^2)$$

$$\Rightarrow -2y$$

$$f_z = \frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(x^2 - y^2 + z^2)$$

$$= 2z$$

Similarly again applying (c) Partial derivatives

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}(2x) = 2$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}(-2y) = -2$$

$$f_{zz} = \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z}\right) = \frac{\partial}{\partial z}(2z) = 2$$

$$f_{xx} + f_{yy} + f_{zz} = \cancel{2} - \cancel{2} + 2$$

$$= 2$$

So not harmonic.

a) Let $f(z) = \dfrac{2 + z + z^2}{(z-2)(z-3)(z-4)(z-5)}$. Show all the poles and compute their residues 4

ANS 4a

Let $f(z) = \frac{2+z+z^2}{(z-2)(z-3)(z-4)(z-5)}$

Pole $\Rightarrow (z-2);(z-3);(z-4);(z-5)=0$

$z = 2, z=3, z=4, z=5$

Residue $z=2$ $f(z) = \lim_{z\to 2}(z-2)\frac{2+z+z^2}{(z-2)(z-3)(z-4)(z-5)}$

$$\Rightarrow \lim_{z\to 2}\frac{2+z+z^2}{(z-3)(z-4)(z-5)}$$

4(a)

$$\Rightarrow \frac{2+2+4}{(2-3)(2-4)(2-5)}$$

$$= \frac{8}{(-1)(-2)(-3)}$$

$$= \frac{\cancel{8}\,4}{-\cancel{6}\,3}$$

$$= \frac{4}{-3}$$

Residue $z=3$ $f(z) = \lim_{z \to 3} (z-3) \frac{2+z+z^2}{(z-2)(z-3)(z-4)(z-5)}$

$$\Rightarrow \lim_{z \to 3} \frac{2+z+z^2}{(z-2)(z-4)(z-5)}$$

$$\Rightarrow \frac{2+3+3^2}{(3-2)(3-4)(3-5)}$$

$$= \frac{14}{1 \cdot (-1) \cdot (-2)}$$

$$\Rightarrow \frac{\cancel{14}\ 7}{\cancel{2}} = 7$$

Residue $z=4$ $f(z) = \lim_{z \to 4} (z-4) \frac{2+z+z^2}{(z-2)(z-3)(z-4)(z-5)}$

$$= \lim_{z \to 4} \cancel{(z-4)} \frac{2+z+z^2}{(z-2)(z-3)(z-5)}$$

$$\Rightarrow \frac{2+4+16}{(4-2)(4-3)(4-5)}$$

$$\Rightarrow \frac{22}{2 \cdot 1 \cdot (-1)}$$

$$\Rightarrow \frac{22}{-2} = -11$$

Residue $z=5$

$$\Rightarrow \lim_{z \to 5} \frac{2+z+z^2}{(z-2)(z-3)(z-4)}$$

$$\Rightarrow \frac{2+5+25}{(5-2)(5-3)(5-4)}$$

$$= \frac{32}{3 \cdot 2 \cdot 1} = \frac{32}{6}$$

b) Evaluate Evaluate $\int_c \frac{2z+1}{z^2+z}dz$ by Cauchy's Integral Formula; where c is the circle $|z| = \frac{1}{2}$

ANS 4B

Q-3: Evaluate $\int_c \frac{2z+1}{z^2+z}dz$

Where c is the circle $|z| = \frac{1}{2}$

$\sqrt{x^2 + y^2} = \frac{1}{2}$

$\therefore x^2 + y^2 = \frac{1}{4}$

$(x-0)^2 + (y-0)^2 = (\frac{1}{2})^2$ ----------------(i)

[We have, $(x-a)^2 + (y-b)^2 = r^2$]

Which is the equation of a circle whose Center (0, 0), Radius $= \frac{1}{2}$

Poles: $z^2 + z = 0$

$z(z+1) = 0$

That is $z = 0, z = -1$

There is only one pole at $z = 0$ inside the given circle.

$\int_c \frac{2z+1}{z^2+z}dz$

$$\int_c \frac{2z+1}{z(z+1)}dz$$

$$= \int_c \frac{\frac{2z+1}{z+1}}{z}dz$$

Here, $f(z) = \frac{2z+1}{z+1}$

$\therefore f(0) = \frac{1}{1} = 1$

Hence, from Cauchy's Integral Formula:

$$\int_c \frac{f(z)}{z-a}dz = 2\pi i \times f(a)$$

$$\int_c \frac{f(z)}{z-0}dz = 2\pi i \times f(0) \qquad [a=0]$$

$$\int_c \frac{f(z)}{z}dz = 2\pi i \times f(0)$$

$$\int_c \frac{f(z)}{z}dz = \int_c \frac{\frac{2z+1}{z+1}}{z}dz = 2\pi i \times f(0)$$

$$\int_c \frac{f(z)}{z}dz = \int_c \frac{\frac{2z+1}{z+1}}{z}dz = 2\pi i \times 1$$

$$\int_c \frac{f(z)}{z}dz = \int_c \frac{\frac{2z+1}{z+1}}{z}dz = 2\pi i$$

c) Evaluate $\int_c (x+y)dz$ from $z=0$ to $z = 3+2j$ along the curve c.

ANS 4C

C

Evaluate ∫ (x+y)dz from z=0 to z=3+2j along the curve c

To evaluate the given integral, we need to know the curve c along which we are integrating. Without this information, we cannot provide a specific answer.

**** ANS 4C**

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