Answer to the question no : 01

Given that,

$$f(x) = \frac{x}{2}; \ 0 < x < 2.$$

i) $P[x \leq 1.25]$

$$= \int_0^{1.25} f(x)\,dx$$

$$= \int_0^{1.25} \frac{x}{2}\,dx.$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_0^{1.25}$$

$= 0.39$ (Ans)

ii) $E[x]$.

We know,

$$E[x] = \int_{-\infty}^{\infty} x f(x)\,dx.$$

$$= \int_0^2 x \cdot \frac{x}{2}\,dx.$$

$$= \frac{1}{2}\int_0^2 x^2\,dx.$$

$$= \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{8}{6} = 1.33 \text{ (Ans)}$$

iii) $V[X]$

We know,

$$V[X] = E[X^2] - [E(X)]^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left(\frac{4}{3}\right)^2 \quad [\text{From (ii)}]$$

$$= \int_0^2 x^2 \cdot \frac{x}{2}\,dx - \frac{16}{9}$$

$$= \frac{1}{2}\left[\frac{x^4}{4}\right]_0^2 - \frac{16}{9}$$

$$= \frac{2^4}{8} - \frac{16}{9}$$

$$= 2 - \frac{16}{9} = 0.22 \text{ (Ans)}$$

iv) $P[0.75 \leq x \leq 1.5]$.

$$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx.$$

$$\therefore \int_{0.75}^{1.5} \frac{x}{2}\,dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_{0.75}^{1.5}$$

$$= \frac{1}{4}\left(1.5^2 - 0.75^2\right) = 0.42 \text{ (Ans)}$$

v) $E[5x+7]$

We know,

$$E[ax+b] = aE[x] + b.$$

Therefore,

$$E[5x+7] = 5E[X] + 7$$

$$= 5 \times \frac{4}{3} + 7 \quad \left[\text{From (i)} \; E[X] = \frac{4}{3}\right]$$

$$= 13.67 \text{ (Ans)}$$

vi) $V[5x+7]$

We know,

$$V[ax+b] = a^2 V[X]$$

Therefore,

$$V[5x+7] = 5^2 V(X)$$

$$= 25 \times 0.22 \quad \left[\text{From (iii)}, \; V[X] = 0.22\right]$$

$$= 5.5 \text{ (Ans)}$$

Answer to the question no:02

Given that,

$$f(x) = kx^2 \cdot 0 \le x \le 1.$$

i) We know,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

or, $\int_0^1 kx^2\,dx = 1$

$\Rightarrow k\left[\frac{x^3}{3}\right]_0^1 = 1.$

$\Rightarrow \frac{k}{3} = 1.$

$\therefore k = 3$ (Ans)

ii) $P[0.20 < x < 0.50]$

$= \int_{0.20}^{0.50} kx^2\,dx.$

$= k\left[\frac{x^3}{3}\right]_{0.20}^{0.50}$

$= \frac{3}{3}(0.50)^3 - (0.20)^3$

$= 0.117.$ (Ans)

iii) $P[X<0.30]$

$$= \int_0^{0.30} kx^2\,dx$$

$$= k\cdot\left[\frac{x^3}{3}\right]_0^{0.30}$$

$$= \frac{3}{3}\{(0.30)^3 - 0^3\}$$

$$= 0.027 \quad \text{(Ans)}$$

iv) $P[X>0.75]$

$$= \int_{0.75}^{1} kx^2\,dx.$$

$$= k\left[\frac{x^3}{3}\right]_{0.75}^{1}$$

$$= \frac{3}{3}(1^3 - 0.75^3)$$

$$= 0.578. \quad \text{(Ans)}$$

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Answer to the question no: 03

Given that,

$$f(x) = k(2x - x^2) \; ; \; 0 < x < 2$$

i) We know,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

or, $$\int_{0}^{2} k(2x - x^2)\,dx = 1$$

$$\Rightarrow k\int_{0}^{2} 2x\,dx - k\int_{0}^{2} x^2\,dx = 1.$$

$$\Rightarrow 2k\left[\frac{x^2}{2}\right]_0^2 - k\left[\frac{x^3}{3}\right]_0^2 = 1$$

$$\Rightarrow 4k - \frac{8}{3}k = 1$$

$$\Rightarrow k\left(4 - \frac{8}{3}\right) = 1$$

$$\therefore k = \frac{3}{4} = 0.75 \quad \text{(Ans)}$$

ii) $$P[X > 1] = \int_{1}^{2} k(2x - x^2)\,dx$$

$$= k\int_{1}^{2} 2x\,dx - k\int_{1}^{2} x^2\,dx.$$

$$= 2k\left[\frac{x^2}{2}\right]_1^2 - k\left[\frac{x^3}{3}\right]_1^2$$

$$= 0.75(4-1) - \frac{0.75}{3}(8-1)$$

$$= \cancel{\tfrac{3}{4}\times 3}\ 2.25 - 1.75$$

$$= 0.5. \quad \text{(Ans)}$$

iii) $P[1.5 < x < 2.25]$

$$= \int_{1.5}^{2.25} k(2x-x^2)\,dx$$

$$= 2k\left[\frac{x^2}{2}\right]_{1.5}^{2.25} - k\left[\frac{x^3}{3}\right]_{1.5}^{2.025}$$

$$= 0.75(2.25^2-1.5^2) - \frac{0.75}{3}(2.25^3-1.5^3)$$

$$= 2.11 - 2.00$$

$$= 0.11 \quad \text{Ans}$$

iv) $E[3x+6]$

We know,

$$\Rightarrow E[x] = \int_{-\infty}^{\infty} x f(x)\,dx.$$

$$= \int_0^2 x k(2x-x^2)\,dx.$$

$$= 2k\int_0^2 x^2\,dx - k\int_0^2 x^2\,dx.$$

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$$= 2k\left[\frac{x^3}{3}\right]_0^2 - k\left[\frac{x^4}{4}\right]_0^2$$

$$= \frac{2\times0.75}{3}\times(8) - \frac{0.75}{4}\times 16$$

$$= 4 - 3$$

$$= 1$$

$$\therefore E[3x+6] = 3E[x]+6$$

$$= 3\times1+6$$

$$= 9. \quad \text{(Ans)}$$

v) $V[4x+9]$

We know,

$$\therefore V[x] = E[x^2] - [E(x)]^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - 1^2 \quad [\text{from (iv)}, E[x]=1]$$

$$= \int_0^2 x^2 k(2x - x^2)\,dx - 1$$

$$= 2k\int_0^2 x^3\,dx - k\int_0^2 x^4\,dx - 1$$

$$= 2\times0.75\left[\frac{x^4}{4}\right]_0^2 - k\left[\frac{x^5}{5}\right]_0^2 - 1$$

$$= \frac{2\times0.75}{4}\times(2^4) - \frac{0.75}{5}\times2^5 - 1$$

$$= 0.2$$

$\therefore V[4x+9] = 4^2 V[x]$

$= 16 \times (0.2)$

$= 3.2.$ Ans)

Answer to the question no: 04

Given that,

$f(x) = k(x-1)$, $2 \le x \le 6$.

i) We know,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

$$\Rightarrow \int_{2}^{6} k(x-1)\,dx = 1$$

$$\Rightarrow k\int_{2}^{6} x\,dx - k\int_{2}^{6} 1\,dx = 1$$

$$\Rightarrow k\left[\frac{x^2}{2}\right]_2^6 - k[x]_2^6 = 1$$

$$\Rightarrow \frac{k}{2}(36-4) - k(6-2) = 1$$

$$\Rightarrow 16k - 4k = 1$$

$$\therefore k = \frac{1}{12}.$$ Ans)

ii) $P[X>3]$

$= \int_3^6 k(x-1)\,dx.$

$= k\int_3^6 x\,dx - k\int_3^6 1\,dx.$

$\Rightarrow k\left[\frac{x^2}{2}\right]_3^6 - k[x]_3^6$

$= \frac{1}{12}\times\frac{1}{2}\times(36-9) - \frac{1}{12}\times 3$

$= \frac{27}{24} - \frac{3}{12}$

$= \frac{7}{8}.$ (Ans)

iii) $P[3<x<4]$

$= \int_3^4 k(x-1)\,dx.$

$= k\int_3^4 x\,dx - k\int_3^4 dx.$

$= k\left[\frac{x^2}{2}\right]_3^4 - k[x]_3^4.$

$= \frac{1}{12\times 2}(4^2-3^2) - \frac{1}{12}(4-3)$

$= \frac{7}{24} - \frac{1}{12}$

$= \frac{5}{24}$ (Ans)

iv) $E[2x+7]$

$\therefore$ We know,

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx.$$

$$= \int_{2}^{6} x k(x-1)\, dx.$$

$$= k\int_{2}^{6} x^2\, dx - k\int_{2}^{6} x\, dx.$$

$$= \frac{1}{12}\left[\frac{x^3}{3}\right]_2^6 - \frac{1}{12}\left[\frac{x^2}{2}\right]_2^6$$

$$= \frac{1}{12\times 3}(6^3-2^3) - \frac{1}{12\times 2}(6^2-2^2)$$

$$= \frac{52}{9} - \frac{4}{3}$$

$$= \frac{40}{9} = 4.44$$

$$\therefore E[2x+7] = 2E[X]+7$$

$$= 2\times\frac{40}{9}+7$$

$$= 15.89 \text{ (Ans)}$$

v) $V[9x+7]$.

We know,

$$V[X] = E[X^2] - [E(X)]^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - (4.44)^2$$

[From iv, $E[X] = 4.44$]

$$= \int_2^6 x^2 k(x-1)\, dx - \cancel{252.492}\ 19.71$$

$$= k\int_2^6 x^3\, dx - k\int_2^6 x^2 dx - \cancel{252.492}\ 19.71$$

$$= k\left[\frac{x^4}{4}\right]_2^6 - k\left[\frac{x^3}{3}\right]_2^6 - \cancel{252.492}\ 19.71$$

$$= \frac{1}{12\times 4}(6^4-2^4) - \frac{1}{12\times 3}(6^3-2^3) - \cancel{252.492}\ 19.71$$

$$= \cancel{\frac{178}{9}}\ 1.179$$

$$\therefore V[9x+7] = 9^2 V[x]$$

$$= 81 \times \cancel{\frac{178}{9}}\ 1.179$$

$$\cancel{= 9\times 178}\ 1.17 \quad 95.5 \text{ Ans}$$

$$\cancel{= 1611}$$

Answer to the question no: 06)

Given that,

$$f(x) = k(x+1); \quad 2 < x < 5$$

i) We know,

$$\int_{-\infty}^{\infty} f(x)\, dx = 1.$$

$$\Rightarrow \int_{2}^{5} k(x+1)\,dx = 1$$

$$\Rightarrow k\int_{2}^{5} x\,dx + k\int_{2}^{5} 1\,dx = 1$$

$$\Rightarrow k\left[\frac{x^2}{2}\right]_2^5 + k\,[x]_2^5 = 1$$

$$\Rightarrow \frac{k}{2}(5^2-2^2) + k(5-2) = 1.$$

$$\Rightarrow \frac{21}{2}k + 3k = 1$$

$$\Rightarrow \frac{21k+6k}{2} = 1$$

$$\Rightarrow 27k = 2$$

$$\therefore k = \frac{2}{27} = 0.074$$

Ans

ii) $P[x>3]$

$$= \int_{3}^{5} k(x+1)\,dx$$

$$= k\int_{3}^{5} x\,dx + k\int_{3}^{5} dx$$

$$= k\left[\frac{x^2}{2}\right]_3^5 + k\,[x]_3^5$$

$$= \frac{1}{27}(5^2-3^2) + \frac{2}{27}(5-3)$$

$= \frac{16}{27} + \frac{4}{27} = \frac{20}{27} = 0.740$ (Ans)

~~$= \frac{20}{27} = 0.74$ (Ans)~~

iii) $P(X=4)$

$= \int_0^4 k(x+1)\,dx$

$= k\int_0^4 x\,dx + k\int_0^4 dx$

$= k\left[\frac{x^2}{2}\right]_0^4 + k[x]_0^4$

$= \frac{2}{27\times 2}\times(4^2) + \frac{2}{27}\times 4.$

$= \frac{16}{27} + \frac{8}{27}$

$= \frac{8}{9} = 0.89$ (Ans)

iv) $P[3<X<4]$

$= \int_3^4 k(x+1)\,dx.$

$= k\int_3^4 x\,dx + k\int_3^4 dx.$

$$= \frac{2}{27}\left[\frac{x^2}{2}\right]_3^4 + k[x]_3^4.$$

$$= \frac{1}{27}(4^2-3^2) + \frac{2}{27}(4-3)$$

$$= \frac{7}{27} + \frac{2}{27}$$

$$= \frac{1}{3} = 0.33 \text{ Ans}$$

v) $E[x+7]$

we know,

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_2^5 x\, k(x+1)\,dx.$$

$$= k\int_2^5 x^2\,dx + k\int_2^5 x\,dx.$$

$$= k\left[\frac{x^3}{3}\right]_2^5 + k\left[\frac{x^2}{2}\right]_2^5$$

$$= \frac{2}{27\times 3}(5^3-2^3) + \frac{2}{27\times 2}(5^2-2^2).$$

$$= \frac{26}{9} + \frac{7}{9}$$

$$= \frac{11}{3} = 3.67$$

$$\therefore E[x+7] = E[x]+7$$
$$= 3.67+7$$
$$= 10.67 \quad \text{Ans}$$

vi) $V[x+7]$

We know,

$$V[x] = E[x^2] - [E(x)]^2$$
$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - (3.67)^2$$
$$= \int_2^5 x^2 k(x+1)\,dx - 13.47.$$
$$= k\int_2^5 x^3\,dx + k\int_2^5 x^2\,dx - 13.47$$
$$= \frac{2}{27}\left[\frac{x^4}{4}\right]_2^5 + \frac{2}{27}\left[\frac{x^3}{3}\right]_2^5 - 13.47.$$
$$= \frac{2}{27\times4}(5^4-2^4) + \frac{2}{27\times3}(5^3-2^3) - 13.47$$
$$= \frac{203}{18} + \frac{26}{9} - 13.47$$
$$= 0.7.$$

$$\therefore V[x+7] = 1^2 V[x]$$
$$= 0.7 \quad \text{Ans}$$

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Answer to the question no: 6

Given that,

$$f(x) = \frac{3}{12}(6x - 3x^2) \; ; \; 0 < x < 2.$$

i) Probability not more than 1.5 inches. $P(x < 1.5)$

$$= \int_0^{1.5} \frac{3}{12}(6x - 3x^2)\,dx.$$

$$= \frac{3}{12}\int_0^{1.5} 6x\,dx - \frac{3}{12}\int_0^{1.5} 3x^2\,dx.$$

$$= \frac{3}{12} \times 6 \left[\frac{x^2}{2}\right]_0^{1.5} - \frac{3}{12} \times 3 \left[\frac{x^3}{3}\right]_0^{1.5}$$

$$= \frac{3 \times 6}{12 \times 2} \times (1.5)^2 - \frac{9}{12 \times 3} \times (1.5)^3.$$

$$= 0.84. \text{ (Ans)}$$

ii) Probability between 0.5 and 1.5 inches, ~~P[0.5<x<~~

$$P[0.5 < x < 1.5].$$

$$= \int_{0.5}^{1.5} \frac{3}{12}(6x - 3x^2)\,dx.$$

$$= \frac{3 \times 6}{12}\int_{0.5}^{1.5} x\,dx - \frac{3 \times 3}{12}\int_{0.5}^{1.5} x^2\,dx$$

$$= \frac{3\times6}{12}\left[\frac{x^2}{2}\right]_{0.5}^{1.5} - \frac{3\times3}{12}\left[\frac{x^3}{3}\right]_{0.5}^{1.5}$$

$$= \frac{3\times6}{12\times2}(1.5^2-0.5^2) - \frac{3\times3}{12\times3}(1.5^3-0.5^3)$$

$$= 1.5 - 0.8125$$

$$= 0.69 \quad \text{(Ans)}$$

$$\text{Mean, } E[X] = \int_{-\infty}^{\infty} xf(x)\,dx.$$

$$= \int_0^2 x.\frac{3}{12}(6x-3x^2)\,dx.$$

$$= \frac{3\times6}{12}\left[\frac{x^3}{3}\right]_0^2 - \frac{3\times3}{12}\left[\frac{x^4}{4}\right]_0^2$$

$$= \frac{1}{2}(2^3-0^3) - \frac{3}{16}(2^4)$$

$$= \frac{1}{2}\times8 - \frac{3}{16}\times16$$

$$= 4-3$$

$$= 1 \quad \text{(Ans)}$$

We know,

Variance, $V(X) = E(X^2) - [E(X)]^2$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - 1^2$$

$$= \int_0^2 x^2 \left\{\frac{3}{12} \times (6x - 3x^2)\right\} dx - 1$$

$$= \frac{6\times 3}{12}\int_0^2 x^3\,dx - \frac{3\times 3}{12}\int_0^2 x^4\,dx - 1$$

$$= \frac{18}{12}\left[\frac{x^4}{4}\right]_0^2 - \frac{9}{12}\left[\frac{x^5}{5}\right]_0^2 - 1$$

$$= \frac{18\times 16}{12\times 4} - \frac{9\times 32}{12\times 5} - 1$$

$$= 6 - \frac{24}{5} - 1$$

$$= \frac{1}{5} = 0.2 \text{ (Ans)}$$