Matrix

1. Definition of Matrix and determinant.

Matrix: Matrix is a set of numbers arranged in rows and columns that forms a rectangular array.

Determinant: The determinant is a scaler value that is a function of entries of a square matrix.

2. Distinguish between matrix and determinant.

Matrix	Determinant
1. Matrix can be denoted by [ ], ( ), ‖ ‖.	1. Determinant can be denoted by | |.
2. Number of rows and columns are not always equal.	2. Number of rows and columns are always equal.
3. It has no definite value.	3. It has a definite value.
4. Rows and columns cannot be interchanged.	4. Rows and columns can be interchanged.
5. If a matrix is multiplied by a number, every element of the entire matrix is multiplied by it.	5. If a determinant is multiplied by a number every element of any one row or any one column is multiplied by it.
6. A matrix cannot be resolved into two matrices.	6. A determinant can be resolved into two determinants by using only one row or column.
7. The product of two matrices may change the order.	7. The product of two determinants does not change the order.

Various types of Matrix.

1. Square Matrix: If the number of rows and columns are equal then the matrix is called a square matrix.

Exp: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2x2 order matrix.

2. Rectangular Matrix: If the number of rows and columns are not equal then the matrix is called a rectangular matrix.

Exp: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

3. Horizontal Matrix: If in a matrix the number of columns is more than the number of rows then it is called the horizontal matrix.

Exp: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

4. Vertical Matrix: If in a matrix the number of rows is more than the number of columns then it is called the vertical matrix.

Exp. $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

5. Row matrix: If in a matrix there is only one row, it is called a row matrix.

Exp: $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

6. Column Matrix: If in a matrix there is only one column then it is called a column matrix.

Exp: $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

7. Zero / Null matrix: If all elements of a matrix are zero then it is called a null or zero matrix.

Exp: $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = [0] = 0$

8. Unity / Identity matrix: A square matrix having unity for its elements in the leading diagonal and all other elements as zero is called a unit / identity matrix.

Exp: $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9. Diagonal matrix: A square matrix in which all elements are zero except those in the leading diagonal, is known as a diagonal matrix.

Exp: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

10. Sub Matrix: A matrix, which is obtained from a given matrix by deleting any number of rows and number of columns is called a sub matrix of the given matrix.

Exp: $B = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$ is the sub matrix of $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

11. Scaler Matrix: A scaler matrix is a diagonal matrix that has all elements in the diagonal equal to each other.

Exp: $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

12. Triangular matrix:

$$A = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}, B = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

13. Lower Triangular Matrix: Lower triangular matrix is a square matrix A whose elements $a_{ij} = 0$ for $j > i$.

Exp: $A = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$

14. upper Triangular matrix: upper triangular matrix is a square matrix A whose elements $a_{ij} = 0$ for $i > j$

Exp: $A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$

15. Singular Matrix: A matrix is singular if only its determinant is 0. $|A| = 0$.

16. Non-Singular matrix: A matrix is non-singular if only its determinant is not equal 0.

$|A| \neq 0$.

17. Binary matrix: Binary matrix is a matrix, each of whose elements is 0 or 1.

Exp: $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

18. Complex matrix: The matrix consists of complex numbers is called complex matrix.

Exp: $A = \begin{bmatrix} 2+3i & 2i+4 \\ 3 & 5 \end{bmatrix}$

19. Complex conjugate or conjugate matrix: A conjugate matrix of a complex matrix is obtained by replacing each term with its complex conjugate.

Exp: If $A = \begin{bmatrix} 2+3i & 2i+4 \\ 3 & 5 \end{bmatrix}$ then $\bar{A} = \begin{bmatrix} 2-3i & -2i+4 \\ 3 & 5 \end{bmatrix}$

20. Real matrix: A matrix A is called real if it satisfies the relation $A = \bar{A}$.

Exp: $A = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 9 & 4 \end{bmatrix}$

21. Imaginary matrix: A matrix A is called imaginary if it satisfies the relation $A = -\bar{A}$.

Exp: $A = \begin{bmatrix} 2i & 3i \\ 0 & 4i \end{bmatrix}$

22. Transpose of a matrix: The transpose of a matrix is created by converting its rows into columns. It is denoted by, A^t, AT, A'.

23. Symmetric matrix: A symmetric matrix is a square matrix that satisfies $AT = A$.

24. Skew symmetric matrix: A skew symmetric matrix is a square matrix that satisfies $A = -AT$.

25. Hermitan matrix: A square matrix is called hermitan if it satisfies $\overline{AT} = A$.

26. Unitary matrix: An unitary matrix is a square complex valued matrix that satisfies the relation $A^{-1} = A^{-T}$

27. Skew Hermitian matrix: A is said to be skew hermitian if it satisfies $\overline{A}^T = -A$.

28. Orthogonal matrix: A matrix A is orthogonal if $AA^T = A^TA = I$.

29. Inverse matrix: The inverse of a matrix A.

$$A = A^{-1} = \frac{\text{Adjoint of } A}{|A|}.$$

Addition and subtraction

1. Find A+B, A−B where, $A = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 5 & 9 \end{bmatrix}$

Solution: Givent that, $A = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 5 & 9 \end{bmatrix}$

$$\therefore A+B = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 4 \\ 6 & 5 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 3+2 & 4+4 \\ 6+6 & 7+5 & 5+9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & 8 \\ 12 & 12 & 14 \end{bmatrix}$$

$$\therefore A-B = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 4 \\ 6 & 5 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 3-2 & 4-4 \\ 6-6 & 7-5 & 5-9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -4 \end{bmatrix}$$

* Two matrix will be multiplicable if the numbers of columns of the first matrix is equal the number of rows of the second matrix.

② Find $A \times B$ when $A = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 9 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & 5 & 6 \\ 7 & 9 & 0 & 2 \\ 1 & 2 & 4 & 5 \end{bmatrix}$

Solution: $A \times B = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 9 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 4 & 5 & 6 \\ 7 & 9 & 0 & 2 \\ 1 & 2 & 4 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 1.3+3.7+5.1 & 1.4+3.9+5.2 & 1.5+3.0+5.4 & 1.6+3.2+5.5 \\ 7.3+9.7+0.1 & 7.4+9.9+0.2 & 7.5+9.0+0.4 & 7.6+9.2+0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 3+21+5 & 4+27+10 & 5+0+20 & 6+6+25 \\ 21+63+0 & 28+81+0 & 35+0+0 & 42+18+0 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 41 & 25 & 37 \\ 84 & 109 & 35 & 60 \end{bmatrix}$$

(Ans)

④ Test the following matrix is singular or not.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

Solution: Here, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix}$

$$= 1 \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2 \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3 \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$$

$$= 1(24-25) - 2(12-15) + 3(10-12)$$

$$= -1$$

$\therefore$ As, $|A| \neq 0$, hence the matrix is not singular.

☐ Prove that, $A = \begin{bmatrix} 3 & 2+i \\ 2-i & 1 \end{bmatrix}$ is a Hermitian matrix.

/OR, Test the following matrix Hermitian or not.

Solution: Given that,

$$A = \begin{bmatrix} 3 & 2+i \\ 2-i & 1 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 3 & 2-i \\ 2+i & 1 \end{bmatrix}$$

$$\therefore \overline{A^T} = \begin{bmatrix} 3 & 2+i \\ 2-i & 1 \end{bmatrix}$$

$$= A$$

$\therefore$ As $\overline{A^T} = A$ therefore A is a hermitian matrix.

[Proved]

☐ Prove that, the following matrix is skew hermitian.

$A = \begin{bmatrix} i & 2+i \\ -2+i & 3i \end{bmatrix}$ / OR, Test the following matrix skew hermitian or not

Solution: Given that, $A = \begin{bmatrix} i & 2+i \\ -2+i & 3i \end{bmatrix}$

$$\therefore A^T = \begin{bmatrix} i & -2+i \\ 2+i & 3i \end{bmatrix}$$

$$\therefore \overline{A^T} = \begin{bmatrix} -i & -2-i \\ 2-i & -3i \end{bmatrix}$$

Again, $-A = \begin{bmatrix} -i & -2-i \\ 2-i & -3i \end{bmatrix}$

As, $\overline{A^T} = -A$. Therefore, the given matrix is a skew Hermitian matrix.

[Proved].

☐ Prove that, the following matrix is a unitary matrix.

$$A = \begin{bmatrix} 2^{-1/2} & 2^{-1/2} & 0 \\ -2^{-1/2}i & 2^{-1/2}i & 0 \\ 0 & 0 & i \end{bmatrix}$$

<u>Solution:</u> Given that, $A = \begin{bmatrix} 2^{-1/2} & 2^{-1/2} & 0 \\ -2^{-1/2}i & 2^{-1/2}i & 0 \\ 0 & 0 & i \end{bmatrix}$

The given matrix A will be proved unitary if $A^{-1} = \bar{A}^T$

Now, $|A| = i \begin{vmatrix} 2^{-1/2} & 2^{-1/2} \\ -2^{-1/2}i & 2^{-1/2}i \end{vmatrix}$

$= i\left(\frac{1}{2}i + \frac{1}{2}i\right)$

$= i.i$

$= i^2$

$= (\sqrt{-1})^2$

$= 1$

Now, the cofactors are given by,

$C_{11} = \begin{vmatrix} 2^{-1/2}i & 0 \\ 0 & i \end{vmatrix} = -2^{-1/2}$

$C_{12} = -\begin{vmatrix} -2^{-1/2}i & 0 \\ 0 & i \end{vmatrix} = -2^{-1/2}$

$C_{13} = 0$

$$C_{21} = -\begin{vmatrix} -2^{-1/2} & 0 \\ 0 & i \end{vmatrix} = -2^{-1/2} i$$

$$\cancel{-2^{-1/2} i}$$

$$C_{22} = \begin{vmatrix} 2^{-1/2} & 0 \\ 0 & i \end{vmatrix} = 2^{-1/2} i$$

$$C_{23} = 0$$

$$C_{31} = 0$$

$$C_{32} = 0$$

$$C_{33} = \begin{vmatrix} 2^{-1/2} & 2^{-1/2} \\ -2^{-1/2} i & 2^{-1/2} i \end{vmatrix} = \frac{1}{2} i + \frac{1}{2} i = i$$

So the cofactor matrix is given by

$$A^c = \begin{bmatrix} -2^{-1/2} & -2^{-1/2} & 0 \\ -2^{-1/2} i & 2^{-1/2} i & 0 \\ 0 & 0 & i \end{bmatrix}$$

Now, the adjoint matrix is given by,

$$(A^c)^T = \begin{bmatrix} -2^{-1/2} & -2^{-1/2} i & 0 \\ -2^{-1/2} & 2^{-1/2} i & 0 \\ 0 & 0 & i \end{bmatrix}$$

Hence, the inverse matrix, $A^{-1} = \dfrac{\text{adjoint of } A}{|A|}$

$$= \frac{1}{-1} \cdot \begin{bmatrix} -2^{-1/2} & -2^{-1/2} i & 0 \\ -2^{-1/2} & 2^{-1/2} i & 0 \\ 0 & 0 & i \end{bmatrix}$$

$$= \begin{bmatrix} 2^{-1/2} & 2^{-1/2} i & 0 \\ 2^{-1/2} & -2^{-1/2} i & 0 \\ 0 & 0 & -i \end{bmatrix}$$

Now, $\bar{A} = \begin{bmatrix} 2^{-1/2} & 2^{-1/2} & 0 \\ 2^{-1/2}i & -2^{-1/2}i & 0 \\ 0 & 0 & -i \end{bmatrix}$

$\therefore \bar{A}^T = \begin{bmatrix} 2^{-1/2} & 2^{-1/2}i & 0 \\ 2^{-1/2} & -2^{-1/2}i & 0 \\ 0 & 0 & -i \end{bmatrix}$

As, $A^{-1} = \bar{A}^T$

Therefore, A is a unitary matrix. [Proved]

Example-4 (Pdf)

A manufacturer produces three products: Toothpaste, soap and Shampoo, which sell in the market. Annual sale volumes are indicat as follows:

Markets	Products		
	Toothpaste	Soap	Shampoo
New market	8000	10000	15000
Shopping complex	10000	2000	20000

(I) If unit (mini packet) sale prices of Toothpaste, soap, and shampo are tk 2.25, tk 3.50 and tk. 3.00 respectively. Find the total revenue in each market with the help of matrix.

(II) If the unit costs of the above three products are tk 1.60, tk 1.90 and tk 2.40 respectively. Find the gross profit with the help of matrices.

Solution: (I) Let,

unit sale price of three products be

$$A = \begin{bmatrix} 2.25 & 3.50 & 3.00 \end{bmatrix}$$

Annual sale of three products in two markets be

$$B = \begin{bmatrix} 8000 & 10000 \\ 10000 & 2000 \\ 15000 & 20000 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 2.25 & 3.50 & 3.00 \end{bmatrix} \times \begin{bmatrix} 8000 & 10000 \\ 10000 & 2000 \\ 15000 & 20000 \end{bmatrix}$$

$$= \begin{bmatrix} 2.25 \times 8000 + 3.50 \times 10000 + 3.00 \times 15000 & 2.25 \times 10000 + 3.50 \times 2000 + 3 \times 20000 \end{bmatrix}$$

$$= \begin{bmatrix} 98000 & 89500 \end{bmatrix}$$

Total revenue from the new market Tk. 98000/-

Total revenue from the shopping complex Tk. 89500/-

$\therefore$ Total revenue from two markets : Tk. 1, 87, 500/-

(Ans)

(II) Let, unit costs of three products be,

$$C = \begin{bmatrix} 1.60 & 1.90 & 2.40 \end{bmatrix}$$

∴ The costs of products in the market are:

$$C \times B = \begin{bmatrix} 1.60 & 1.90 & 2.40 \end{bmatrix} \times \begin{bmatrix} 8000 & 10000 \\ 10000 & 2000 \\ 15000 & 20000 \end{bmatrix}$$

$$= \begin{bmatrix} 1.60 \times 8000 + 1.90 \times 10000 + 2.40 \times 15000 & 1.60 \times 10000 + 1.90 \times 2000 + 2.40 \times 20000 \end{bmatrix}$$

$$= \begin{bmatrix} 67800 & 67800 \end{bmatrix}$$

Total cost of products for new market Tk. 67,800/-

Total cost of products for shopping complex - Tk. 67800/-

Total cost from two markets : Tk. 1,35,600

So, required gross profit = Total revenue - Total cost

$= 187500 - 135600$

$=$ Tk. 51,900/-

(Ans)

Example:06 (PdP)

Show that the following matrix $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$ is orthogonal.

Solution: Given that, $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

$$\therefore A^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Now,

$$AA^T = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & -\cos\alpha\sin\alpha + \cos\alpha\sin\alpha \\ -\sin\alpha\cos\alpha + \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I$$

$$A^TA = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I$$

As, $AA^T = A^TA = I$

Therefore, A is a orthogonal matrix.

Example-9

Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

Solution: Given that,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

Now, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix}$

$= 1(24-25) - 2(12-15) + 3(10-12)$

$= -1 + 6 - 6$

$= -1$

Now the cofactors are given by,

$C_{11} = \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix}$
$= 24-25$
$= -1$

$C_{12} = -\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix}$
$= -(12-15)$
$= 3$

$C_{13} = \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$
$= 10-12$
$= -2$

$C_{21} = -\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}$
$= -(12-15)$
$= 3$

$C_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix}$
$= 6-9$
$= -3$

$C_{23} = -\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix}$
$= -(5-6)$
$= 1$

$C_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$
$= 10-12$
$= -2$

$C_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix}$
$= -(5-6)$
$= 1$

$C_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$
$= (4-4)$
$= 0$

So, the cofactor matrix is given by

$$A^C = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

Now, the adjoint matrix of A is given by,

$$(A^C)^T = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

Hence, the inverse matrix of A is given by

$$A^{-1} = \frac{\text{Adjoint of } A}{|A|}$$

$$= \frac{1}{-1} \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

(Ans)

Perfection test:

$$AA^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

Characteristic equation of a Matrix

1. Characteristic matrix of A: $A - \lambda I$ (I = Identity matrix)
2. Characteristic polynomial of A: $|A - \lambda I|$ ~~= 0~~
3. Characteristic equation of A: $|A - \lambda I| = 0$
4. Characteristic ~~equation or~~ roots / values or latent roots or latent values or Eigen values or proper values of A: The value of λ. (The value of λ is called Eigen values)
5. Spectrum of A: $\lambda = \{1, 3\}$

   The set of all Eigen values of the matrix A is called the spectrum of A.

6. Eigen Value Problems: $\lambda = \{1, 2, 3, 4\}$

① Find the characteristic roots of the matrix, $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$

<u>Solution:</u> Given that, $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$

Here, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$$\therefore A - \lambda I = \begin{bmatrix} \cos\theta - \lambda & -\sin\theta \\ -\sin\theta & -\cos\theta - \lambda \end{bmatrix}$$

Hence, the characteristic equation,

$$|A - \lambda I| = 0$$

or, $$\begin{vmatrix} \cos\theta - \lambda & -\sin\theta \\ -\sin\theta & -\cos\theta - \lambda \end{vmatrix} = 0$$

or, $(\cos\theta - \lambda)(-\cos\theta - \lambda) - \sin^2\theta = 0$

or, $-\cos^2\theta - \lambda\cos\theta + \lambda\cos\theta + \lambda^2 - \sin^2\theta = 0$

or, $\lambda^2 - (\sin^2\theta + \cos^2\theta) = 0$

or, $\lambda^2 - 1 = 0$

or, $\lambda^2 = 1$

$\therefore \lambda = \pm 1$

$\therefore$ Characteristics roots of the given matrix are ± 1.

② Find the characteristic roots of the matrix,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

Solution: Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\therefore \lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\therefore A - \lambda I = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 2-\lambda \end{bmatrix}$$

Hence, the characteristic equation,

$$|A - \lambda I| = 0$$

or, $\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0$

or, $(1-\lambda)\begin{vmatrix} 2-\lambda & 3 \\ 0 & 2-\lambda \end{vmatrix} - 2\begin{vmatrix} 0 & 3 \\ 0 & 2-\lambda \end{vmatrix} + 3\begin{vmatrix} 0 & 2-\lambda \\ 0 & 0 \end{vmatrix} = 0$

or, $(1-\lambda)\{(2-\lambda)^2 - 0\} - 2(0) + 3(0) = 0$

or, $(1-\lambda)(2-\lambda)^2 = 0$

or, $(1-\lambda)(2-\lambda)(2-\lambda) = 0$

$\therefore \lambda = 1, 2, 2$

$\therefore$ Characteristic roots of the given matrix are 1, 2 and 2

(Ans)

③ Find the eigen values of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Solution: Given that, $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$\therefore A - \lambda I = \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$

Hence the characteristic equation,

$|A - \lambda I| = 0$

or, $\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$

or, $(8-\lambda)\begin{vmatrix} 7-\lambda & -4 \\ -4 & 3-\lambda \end{vmatrix} + 6\begin{vmatrix} -6 & -4 \\ 2 & 3-\lambda \end{vmatrix} + 2\begin{vmatrix} -6 & 7-\lambda \\ 2 & -4 \end{vmatrix} = 0$

or, $(8-\lambda)\{(7-\lambda)(3-\lambda)-16\}+6\{-6(3-\lambda)+8\}+2\{24-2(7-\lambda)\}=0$

or, $(8-\lambda)\{21-7\lambda-3\lambda+\lambda^2-16\}+6\{-18+6\lambda+8\}+2\{24-14+2\lambda\}=0$

or, $(8-\lambda)(\lambda^2-10\lambda+5)+6(6\lambda-10)+2(2\lambda+10)=0$

or, $8\lambda^2-80\lambda+40-\lambda^3+10\lambda^2-5\lambda+36\lambda-60+4\lambda+20=0$

or, $-\lambda^3+18\lambda^2-45\lambda=0$

~~or, $\lambda^3-3\lambda^2-15\lambda^2$~~

or, $\lambda^3-18\lambda^2+45\lambda=0$

or, $\lambda^3-3\lambda^2-15\lambda^2+45\lambda=0$

or, $\lambda^2(\lambda-3)-15\lambda(\lambda-3)=0$

or, $(\lambda-3)(\lambda^2-15\lambda)=0$

or, $\lambda(\lambda-3)(\lambda-15)=0$

$\therefore \lambda=0,3,15$

$\therefore$ The required eigen values of A are $0, 3, 15$.

(Ans)

Linear system of equations

Let a system of m linear equations in n unknowns $x_1, x_2 \dots x_n$ has the general form:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3n}x_n = b_3$$
$$\dots \dots \dots \dots \dots \dots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n = b_m$$

Then this system can be written in the matrix form as $AX = K$

where, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ x_n \end{bmatrix}$; $K = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \dots \\ b_m \end{bmatrix}$

We have, $AX = K$

Multiplying by A^{-1} on both sides

$$A^{-1}AX = A^{-1}K$$
$$\therefore X = A^{-1}K.$$

(iii) Solve the following equations by matrix method.

$$x + 2y + 3z = 14$$
$$3x + y + 2z = 11$$
$$2x + 3y + z = 11$$

<u>Solution:</u> Given that,
$$x + 2y + 3z = 14$$
$$3x + y + 2z = 11$$
$$2x + 3y + z = 11$$

Let, the system can be written as, $AX = k$ —— (1)

where, $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and $k = \begin{bmatrix} 14 \\ 11 \\ 11 \end{bmatrix}$

Again, from (1) we get, $A^{-1}AX = A^{-1}k$

or, $X = A^{-1}k$ —— (ii)

Now, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}$

$= 1(1-6) - 2(3-4) + 3(9-2)$

$= -5 + 2 + 21$

$= 18$

Now, the cofactor matrix are given by,

$C_{11} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ $\quad C_{12} = -\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$ $\quad C_{13} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix}$

$= (1-6)$ $\quad = -(3-4)$ $\quad = 9-2$

$= -5$ $\quad = 1$ $\quad = 7$

$$C_{21} = -\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = -(2-9) = 7$$

$$C_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1-6 = -5$$

$$C_{23} = -\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -(3-4) = 1$$

$$C_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = (4-3) = 1$$

$$C_{32} = -\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} = -(2-9) = 7$$

$$C_{33} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = (1-6) = 5$$

So, the cofactor matrix is given by,

$$AC = \begin{bmatrix} -5 & 1 & 7 \\ 7 & -5 & 1 \\ 1 & 7 & -5 \end{bmatrix}$$

Now, the Adjoint of A is given by,

$$(AC)^T = \begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix}$$

So, the inverse of A is given by,

$$A^{-1} = \frac{\text{Adjoint of } A}{|A|}$$

$$= \frac{1}{18} \begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix}$$

Now, from ② we have, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18} \begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix} \begin{bmatrix} 14 \\ 11 \\ 11 \end{bmatrix}$

$$= \frac{1}{18} \begin{bmatrix} -5\cdot 14 + 7\cdot 11 + 1\cdot 11 \\ 1\cdot 14 + (-5)\cdot 11 + 7\cdot 11 \\ 7\cdot 14 + 1\cdot 11 + (-5)\cdot 11 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore x = 1,\ y = 2,\ z = 3.$

(Ans)

Rank of a matrix

The number of non zero rows of a matrix is called its rank.

Ex-13 (PYQ) Find the rank of the matrix $A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$

Solution: Given that, $A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 2 & 1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{bmatrix} \quad [C_3' = C_3 - C_2]$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix} \quad [C_2' = C_2 - 2C_3]$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad [R_2' = R_2 - 2R_1]$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [R_3' = R_3 - 3R_1]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [C_1 \rightleftharpoons C_3]$$

Hence, the rank of the given matrix is one.

Normal form of a matrix:

Every non-zero matrix A of order $m \times n$ can be reduced by application of elementary row and column operations into equivalent matrix of one of the following form.

(I) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, (II) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$, (III) $\begin{bmatrix} I_r & 0 \end{bmatrix}$, (IV) $\begin{bmatrix} I_r \end{bmatrix}$

where I_r is $r \times r$ identity matrix and 0 is null matrix of any order.

These four forms are called Normal or canonical form.

(1) Reduce matrix A to its normal form and then find its rank, where, $A = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$.

Solution: Given that, $A = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 5 \\ 3 & 1 & 2 & 5 \end{bmatrix} \quad [C_2' = C_2 - C_1], [C_3' = C_3 - C_1], [C_4' = C_4 + C_1];$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 5 \\ 2 & 0 & 0 & 0 \end{bmatrix} \quad [R_3' = R_3 - R_2]$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad [R_3' = R_3 - 2R_1]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad [C_1' = C_1 - C_2]; \; [C_3' = C_3 - 2C_2]$$
$$[C_4' = C_4 - 5C_2]$$

$$= \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$ which is the normal form

Hence the the rank of A is 2.

(15/20 marks)

Linear system of Equations:-

Let us consider a system of m linear equations with n variables.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n = b_3$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_n$$

① Homogeneous system of linear equations: The system is said to be homogeneous if the constants $b_1, b_2, b_3, \ldots b_m$ are all zero. $[b_1, b_2 = 0]$.

② Non-homogeneous system of linear equation: The system is said to be non-homogeneous if the ~~elements~~ constants $b_1, b_2, b_3, \ldots, b_m$ are all non-zero.

③ Inconsistent system of linear equation: If the given system has no real solution then it is called inconsistent system of linear equation. (solution সম্ভব নয়)

④ Consistent system of linear equation: If the given system has a solution then the system is said to be consistent system of linear equation.

Solution of linear system of Equations:

1. No-solution
2. Unique solution
3. Zero solution
4. More than one solution

Formula: $L_i = -a_{i1} L_1 + a_{11} L_i$, where $i > 1$

Example-16 (Pdf)

Solve the linear system of equations, ~~2x+~~

$2x+y-2z=10$, $3x+2y+2z=1$, $5x+4y+3z=4$.

Solution: Given the linear system of equations,

$2x+y-2z=10 \rightarrow L_1$

$3x+2y+2z=1 \rightarrow L_2$

$5x+4y+3z=4 \rightarrow L_3$

We know that, $L_i = -a_{i1}L_1 + a_{11}L_i \quad (i>1)$ —— ①

Now putting $i=2$ we get,

$$
\begin{aligned}
L_2 &= -a_{21}L_1 + a_{11}L_2 \\
&= -3(2x+y-2z-10) + 2(3x+2y+2z-1) \\
&= -6x-3y+6z+6x+4y+4z+30-1 \\
&= y+10z+28
\end{aligned}
$$

Now, putting $i=3$ we get,

$$
\begin{aligned}
L_3 &= -a_{31}L_1 + a_{11}L_3 \\
&= -5(2x+y-2z-10) + 2(5x+4y+3z-4) \\
&= -10x-5y+10z+50+10x+8y+6z-8 \\
&= 3y+16z+42
\end{aligned}
$$

So, the given system

$2x+y-2z=10$

$y+10z=-28 \rightarrow L_1$

$3y+16z=-42 \rightarrow L_2$

Again, putting $i=2$ in eqn ① we get,

$$L_2 = -a_{21}L_1 + a_{11}L_2$$

$$= -3(y+10z+28) + 1(3y+16z+42)$$

$$= -3y-30z-84+3y+16z+42$$

$$= -14z - 42$$

Hence the given system becomes;

$$2x+y-2z=0$$

$$y+10z=-28$$

$$-14z=42$$

Now, $-14z=42$

$\Rightarrow z=-3$

Again, $y+10z=-28$

or, $y+10(-3)=-28$

or, $y=2$

Also, $2x+y-2z=10$

or, $2x+2-2(-3)=10$

or, $x=1$

$\therefore x=1,\ y=2$ and $z=-3$

(Ans)

Defitition of Eigen Values and Eigen Vectors:

(Must পারবে) An Eigen vector is a non-zero which satisfies the equation

$A\vec{v} = \lambda\vec{v}$ or, $A\vec{x} = \lambda\vec{x}$

where, A is the given square matrix.

λ is the scalar value which is called the Eigen value of the given matrix.

$\vec{v}$ or $\vec{x}$ is called the Eigen vector of the given matrix.

Example-23 (pdf)

Determine which of the following vectors are Eigen vectors for $A = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix}$ showing your analysis procedure. graphically.

(a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (f) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Solution: We know the relation between Eigen vectors and Eigen values of a matrix is given by, $A\vec{x} = \lambda\vec{x}$

where, A is the given matrix

λ is the Eigen value

x is the Eigen vector.

Now (a)

Mathematically test:

Given that, $\vec{A} = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Now, $AX = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 1\times1 + 2(-1) \\ -4\times1 + 7(-1) \end{bmatrix}$

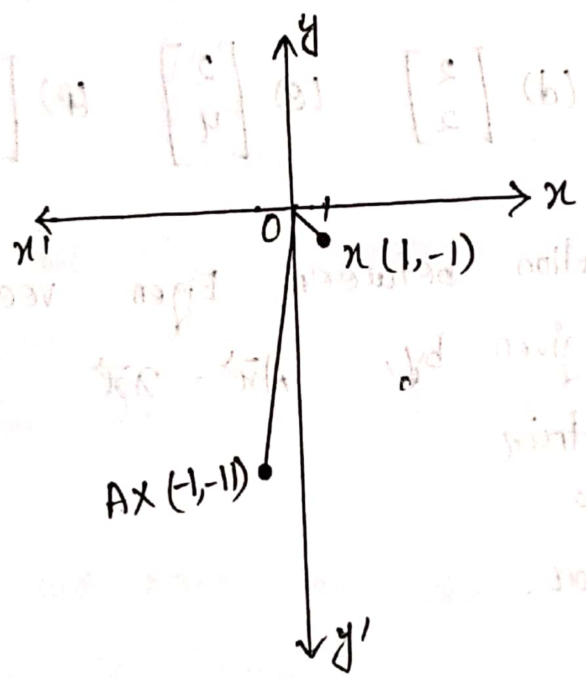
$= \begin{bmatrix} -1 \\ -11 \end{bmatrix}$

$= -1 \begin{bmatrix} 1 \\ 11 \end{bmatrix}$

As, $A\vec{x} = \lambda\vec{x}$ is not satisfied so the given vector is not Eigen vector. (Ans)

Graphical Analysis:-

Here, $Ax = \begin{bmatrix} -1 \\ -11 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$



As, the product vector Ax is not the same direction as the vector $\vec{x}$.

So, the given vector is not Eigen Vector.

(Ans)

Again (b)

Mathematically test:-

Given that, $\vec{A} = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Now, $Ax = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 2+6 \\ -8+21 \end{bmatrix}$

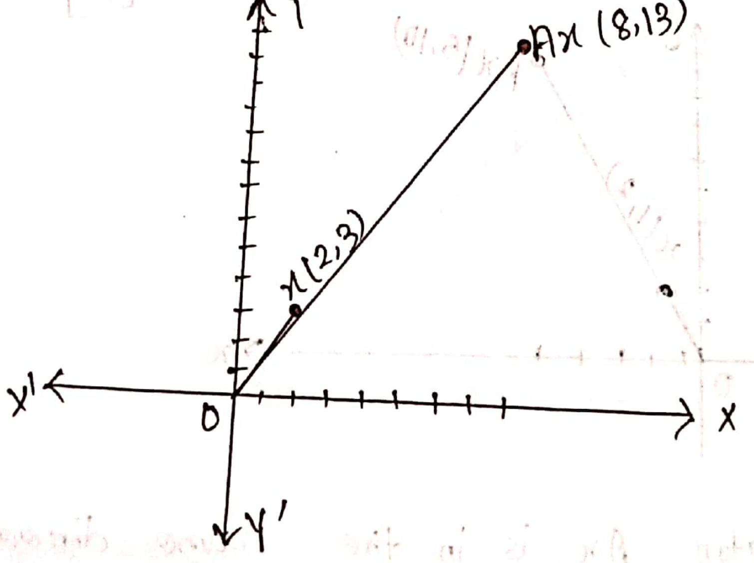
$= \begin{bmatrix} 8 \\ 13 \end{bmatrix}$

As, $A\vec{x} = \lambda\vec{x}$ is not satisfied.

So, the given vector is not Eigen vector. (Ans)

Graphical Analysis:- Here,

$Ax = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$, and $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$



As, the product vector Ax is not in the same direction as vector $\vec{x}$.

So, the given vector is not Eigen vector.

(Ans).

Again (c)

Mathematically test:

Given that, $\vec{A} = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Now, $Ax = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

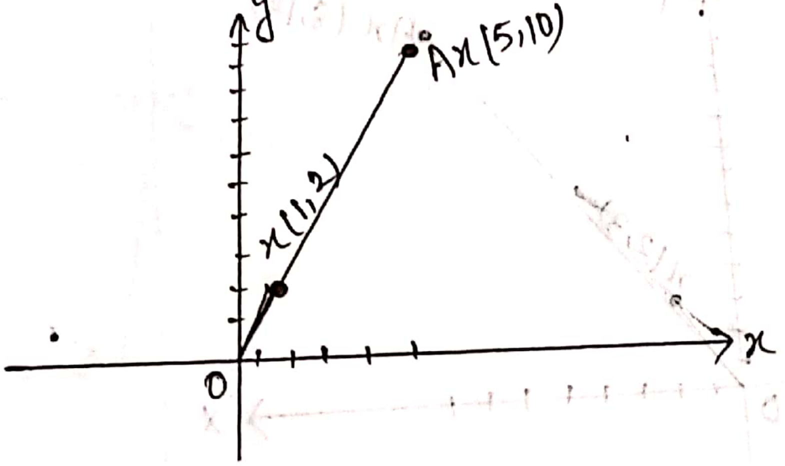
$= \begin{bmatrix} 1+4 \\ -4+14 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

$= 5 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

As, $A\vec{x} = \lambda \vec{x}$ is satisfied.

So, the given vector is an Eigen vector. (Ans)

Graphical Analysis: Here, $Ax = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$



As the product vector Ax is in the same direction as vector $\vec{x}$.

So, the given vector is an Eigen Vector.

(Ans)

Again (d)

Mathematically test:

Given that, $\vec{A} = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Now, $Ax = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 1\times2+2\times2 \\ -4\times2+7\times2 \end{bmatrix}$

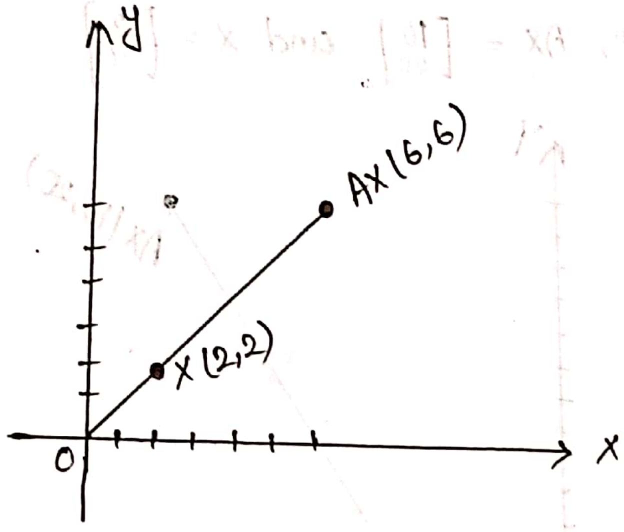
$= \begin{bmatrix} 6 \\ 6 \end{bmatrix}$

$= 3\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

As, $A\vec{x} = \lambda\vec{x}$ is satisfied.

So, the given vector is an Eigen Vector. (Ans)

Graphical Analysis: Here, $Ax = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$ and $x = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$



As the product vector Ax is in the same direction as vector $\vec{x}$.

So, the given vector is an Eigen Vector.

(Ans)

Again (c)

Mathematically Test:

Given that, $\vec{A} = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

Now, $AX = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} 1\times 2 + 2\times 4 \\ -4\times 2 + 7\times 4 \end{bmatrix}$

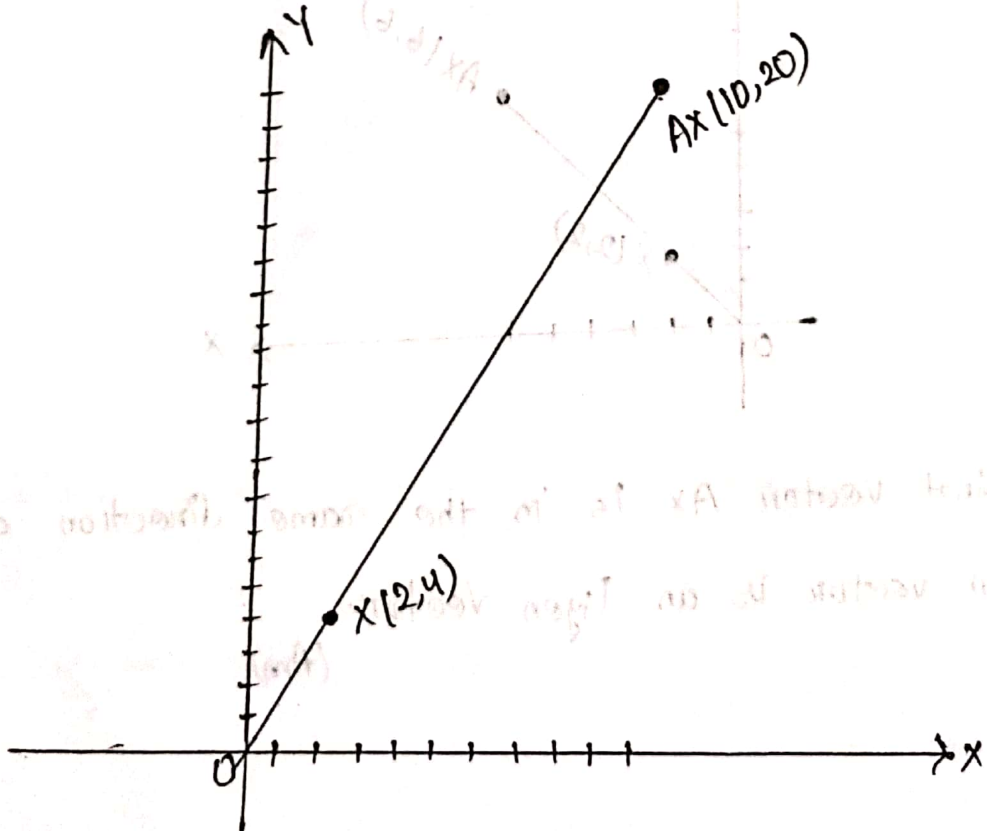
$= \begin{bmatrix} 10 \\ 20 \end{bmatrix}$

$= 5 \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

As, $A\vec{x} = \lambda\vec{x}$ is satisfied.

So, the given vector is an Eigen vector. (Ans)

Graphical Analysis: Here, $AX = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$ and $X = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$



As, the product vector AX is in the same direction as $\vec{X}$.

So, the given vector is an Eigen vector. (Ans)

Again, (P)

Mathematically Test:

Given that, $\vec{A} = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix}$

$$\vec{x} = \cancel{\begin{bmatrix} 2 \\ 4 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Now, $Ax = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

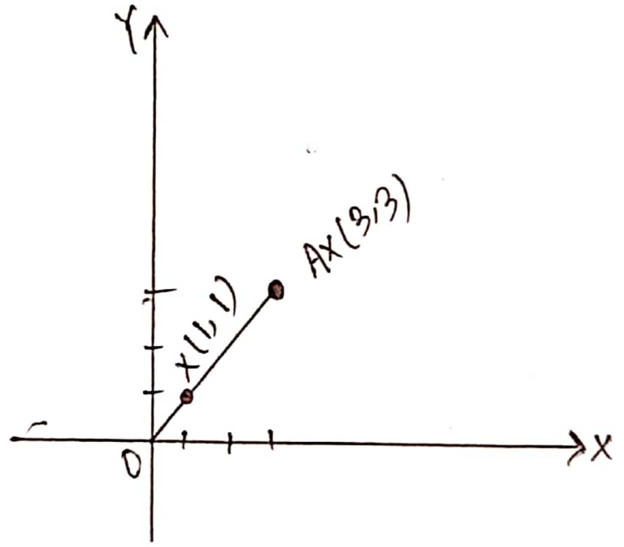
$$= \begin{bmatrix} 1\times1+2\times1 \\ -4\times1+7\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

As, $A\vec{x} = \lambda\vec{x}$ is satisfied.
So, the given vector is an Eigen vector. (Ans)

Graphical Analysis: Here, $Ax = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



As, the product vector Ax is in the same direction as $\vec{x}$.
So, the given vector is an Eigen vector. (Ans)