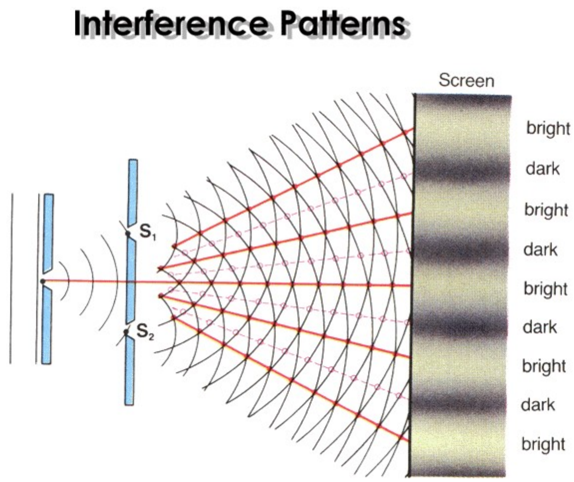
Interference of Light:

Due to superposition of two light waves emitted from coherent sources intensity of light increases at some points and decreases at other points. As a result alternate bright and dark state is produced on a plane. The alternate variation of intensity of light from point to point on a plane is called the interference of light.



Coherent Sources:

When two sources emit light waves of the same colour (wavelength), of the same frequency, nearly the same amplitude and always in phase with each other, then the two sources are called coherent sources.

It is not possible to have two independent sources which are coherent. But for experimental purpose, two virtual sources formed from a single source can act as coherent sources.

Condition for interference of light:

1. The waves from two sources must be of the same frequency.
2. The two light waves must be coherent.
3. The path difference between the overlapping waves must be less than the coherence length of the waves
4. The wavelengths of two interfering waves must be same and their amplitudes are either equal or nearly equal to each other.
5. The interfering waves must be produced in the same direction.
6. The light waves should be reach at the interfering points at the same time.
7. The sources must be monochromatic.
8. The two coherent sources must lie to each other in order to discern the fringe pattern.
9. The distance of the screen from the two sources must be large.

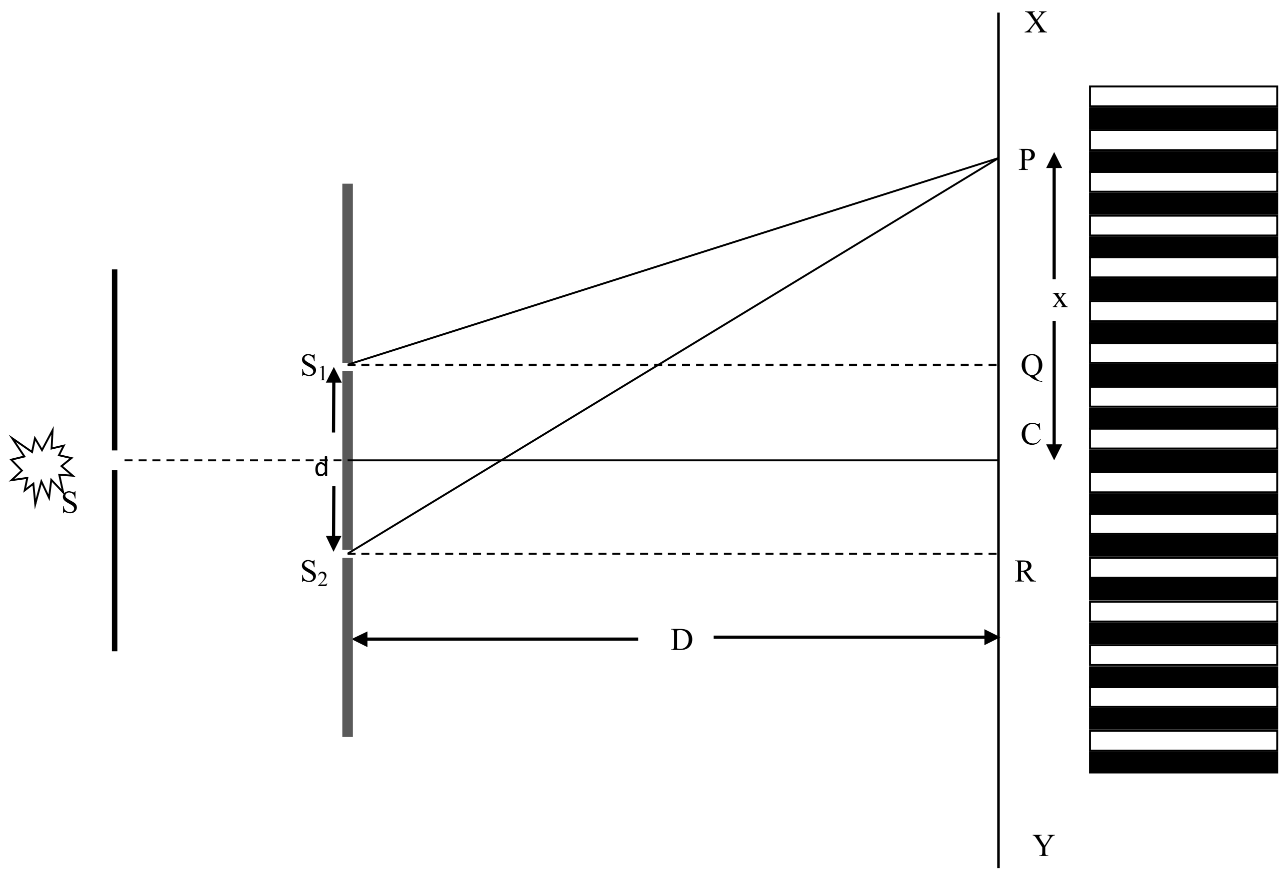
Young's Double Slit Experiment:

Consider a narrow monochromatic source of light S and two pinholes S_1 and S_2, equidistance from S.

S_1 and S_2 acts as two coherent sources separated by a distance d.

Let a screen XY be placed at a distance D from the coherent sources. The point C is the centre point of XY which is equidistance from the coherent sources. So the path difference between waves at C is Zero. Thus the point C has maximum intensity.

Consider a point P at a distance x from C on the screen XY. The waves reach at the point P from S_1 and S_2.



Now, from figure,

PC = x

S_1S_2 = QR = d

$QR = CR = d/2$

Hence,

$PQ = PC – QC = x – d/2$ …………………………..(1)

$PR = PC + CR = x + d/2$ …………………………..(2)

Now, From $\Delta\ S_1PQ$,

$S_1P^2 = S_1Q^2 + PQ^2$

$= D^2 + (x – d/2)^2$ …………………………….(3)

From $\Delta\ S_2PR$,

$S_2P^2 = S_2R^2 + PR^2$

$= D^2 + (x + d/2)^2$ …………………………….(4)

Subtracting equation (3) from equation (4),

$S_2P^2 - S_1P^2 = D^2 + (x + d/2)^2 - D^2 - (x – d/2)^2$

or, $S_2P - S_1P = 2xd / (S_2P + S_1P)$

But $S_2P = S_1P = D$ (approximately)

So,

$S_2P - S_1P = 2xd/2D = xd/D$

Where, $S_2P - S_1P$ is not equal to zero.

Therefore, Path difference,

$x' = xd/D$ ……………(5)

So, The phase difference,

$\delta = 2\pi x' / \lambda = 2\pi xd / \lambda D$……………………(6)

Case 1:

For Bright Frings,

If the path difference is a whole number multiple of wavelength λ, then the point P is bright.

Hence, For bright,

$x' = n\lambda$ [where, n = 0,1,2,3,…….]

or, $x_n\, d/D = n\lambda$

or, $x_n = n\lambda D/d$

When , n = o, $x_0 = 0$

n = 1, $x_1 = \lambda D/d$

n = 2, $x_2 = 2\lambda D/d$

n = 3, $x_3 = 3\lambda D/d$

…………………………………………………..

…………………………………………………

Now,

$x_1 - x_0 = \lambda D/d$

$x_2 - x_1 = \lambda D/d$

$x_3 - x_2 = \lambda D/d$

…………….

……………

$x_n - x_{n-1} = \lambda D/d$

Therefore, The distance between any two consecutive bright frings is $\lambda D/d$.

Case –II:

If the path difference is an odd number multiple of half wavelength then the point P is dark.

So, for dark fringe,

$x' = (2n + 1)\, \lambda/2$ [where, n = 0,1,2,3,……………]

or, , $x_n\, d/D = (2n + 1)\lambda/2$

or, , $x_n = (2n + 1)\, \lambda D /2d$

When , n = o, $x_0 = \lambda D/2d$

n = 1, $x_1 = 3\lambda D/2d$

$n = 2, \qquad x_2 = 5\lambda D/2d$

$n = 3, \qquad x_3 = 7\lambda D/2d$

…………………………………………………..

…………………………………………………

Now,

$x_1 - x_0 = \lambda D/d$

$x_2 - x_1 = \lambda D/d$

$x_3 - x_2 = \lambda D/d$

…………….

……………

$x_n - x_{n-1} = \lambda D/d$

Therefore, the distance between any two consecutive dark frings is $\lambda D/d$.

Fringe width:

The distance between any two consecutive bright or dark fringes is known as fringe width.

If the width of the fringe is represented by β,

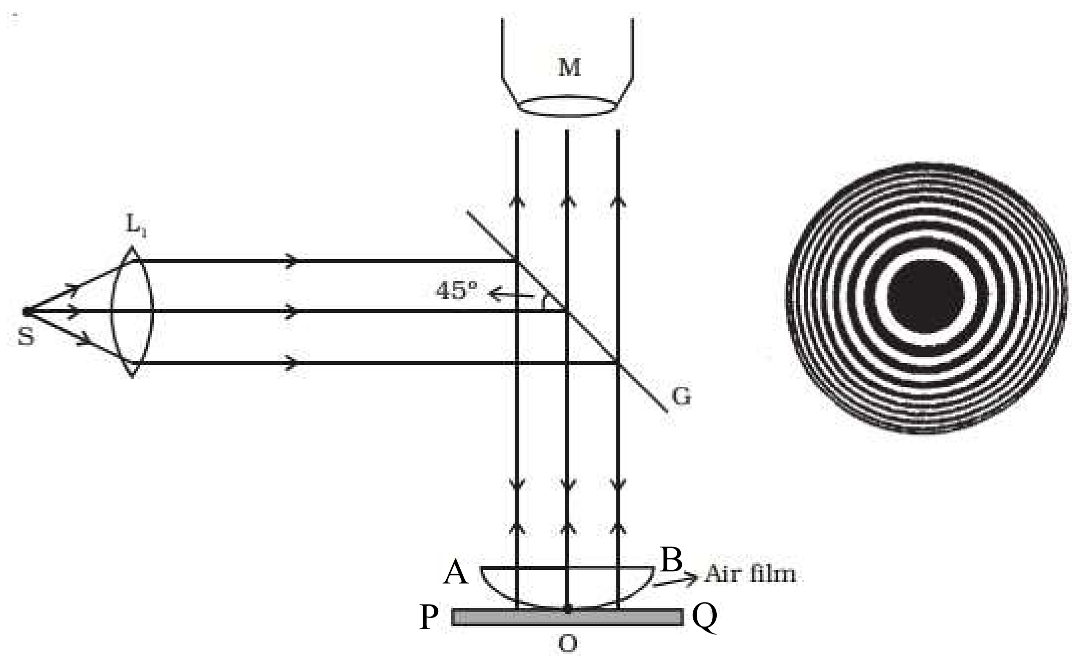
Then, $\beta = \lambda D /d$

Which means that, all the fringes are equal in width and are independent of the order of the fringe.

Newton's Rings:

If we place a plano-convex lens on a glass plate a thin air film is formed between the curved surface of lens and plane glass plate. If we allow monochromatic light to fall on the curved surface of the lens, then the frings are product in the air film and contour lens will be circular. The ring shaped frings thus produced wave analyze by Newton and is hence known as Newton's ring. When viewed with white light, the fringes are coloured.

Construction:



Let, AOB be a plano-convex lens on the glass plate POQ. A thin air film is formed between the AOB and POQ. The thickness of the air film is zero at the point of contact O and increases as one moves away from the point of contact.

If we allow a monochromatic source of light S to fall on the air film. To do this a glass plate G is placed at 45 with the incident light. This placed plate G reflects the light down on the plate.

After reflection from this surface POQ, it is transmitted through G and observed rings by a microscope M.

For near normal incidence the optical path difference between two waves is nearly equal to 2μt (where μ is the refractive index and t is the thickness).

Thus, whenever, the thickness of the air film satisfies the condition,

$2\mu t = (2n + 1)\, \lambda/2$ …………………….. (1), where, n =0,1,2,3,………………………

we will have the maxima.

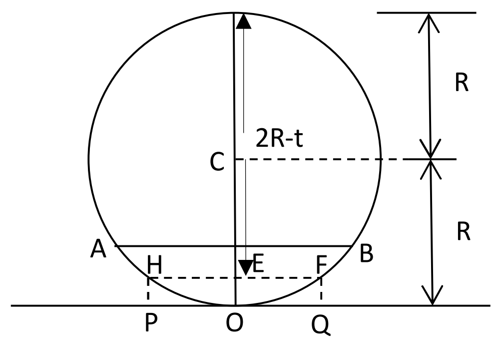
And similarly the condition,

$2\mu t = n\lambda$ ……………………………(2)

will correspond to minima.

Radius of the nth dark ring:

Suppose the radius of curvature of the lens is R and thickness of the air film is t at a distance of OQ = r, from the point of contact O.



Here, the interference is due to reflected light.

Let, the radius of nth ring is r_n.

For bright ring,

$2\mu t\cos\theta = (2n + 1)\, \lambda/2$

Here, θ is small. So, cosθ = 1

Then, $2\mu t = (2n + 1)\, \lambda/2$ ………………………………………(1)

For dark rings,

$2\mu t \cos\theta = n\lambda$

But, θ is very small. So, cosθ = 1

Then, $2\mu t = n\lambda$ …………………… (2)

From figure,

EF × HE = OE (2R –OE) …………..(3)

Here, EF = HE = r and OE = FQ = t

So, $r^2 = t\,(2R - t)$ ………………(4)

Generally, R>>t; So, we may neglect t in comparism to 2R.

So $r^2 = t\,(2R) = 2Rt$

or, $2t = r^2 / R$ ……………….(5)

From equation (1), for bright rings,

$\mu r^2 / R = (2n + 1)\,\lambda /2$

or, $r = \sqrt{\frac{(2n+1)\lambda R}{2\mu}}$

□ $r = \sqrt{\frac{(2n+1)\lambda R}{2}}$ [for air, $\mu = 1$]

From equation (2), for dark rings,

$\mu r^2 / R = n\,\lambda$

or, $r = \sqrt{\frac{n\lambda r}{\mu}}$

□ $r = \sqrt{n\lambda R}$, [for air, $\mu = 1$]

For nth ring,

$r_n = \sqrt{\frac{n\lambda R}{\mu}}$ ………………………….(6)

Equation (6) implies that the radii of the rings vary at square root of natural numbers. Thus, the rings will become close to each other as the radius increases.

Dark Centre:

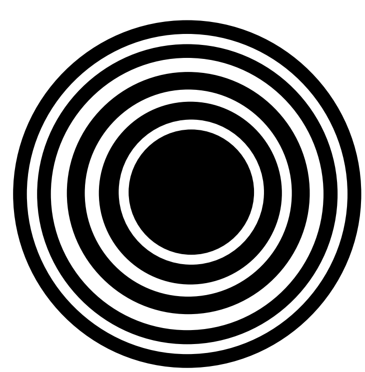
When, $n = 0$ the radius of the dark ring is zero and the radius of the bright ring is $\sqrt{\frac{\lambda R}{2}}$. So the centre is dark. Alternately dark and bright fringe are produced.

Wavelength:

If the diameter of the nth dark ring is D_n and radius r_n, then

$r_n^2 = n\lambda R$

□ $D_n^2 = 4n\lambda R$ …………………….. (7)



And also the diameter of the (n + m)th ring is,

$D_{n+m}^2 = 4(n+m)\lambda R$……………….(8)

Subtracting equation (7) from (8) we get,

$D_{n+m}^2 - D_n^2 = 4(n+m-n)\lambda R$

□ $\lambda = (D_{n+m}^2 - D_n^2) / 4\,mR$ …………………. (9)

Where, m is the difference between the higher and lower order of the ring.

Now, by knowing the value of radius of curvature of the lens with the help of a spherometer the wavelength of a given monochromatic source of light can be determined.

Refractive Index:

If this arrangement of figure is kept in a metal container C. The liquid is poured in the container C without disturbing the arrangement. The air film between the lower surface of the lens and upper surface of the plate is replaced by the liquid. Then for the liquid the nth order of the dark rings from equation (6) is given by,

$D'^2_n = 4n\lambda R/\mu$

Similarly for (n+m)th dark ring,

$D'^2_{n+m} = 4(n+m)\lambda R/\mu$

Therefore,

$D'^2_{n+m} - D'^2_n = 4m\lambda R/\mu$

or, $\mu = 4m\lambda R / (D'^2_{n+m} - D'^2_n)$ [for known wavelength]

□ $\mu = (D_{n+m}^2 - D_n^2) / (D'^2_{n+m} - D'^2_n)$ [for unknown wavelength]

Interferometers:

Interferometry is a family of techniques in which waves, usually electromagnetic waves, are superimposed causing the phenomenon of interference in order to extract information. Interferometry is an important investigative technique in the fields of astronomy, fiber optics, engineering metrology, optical metrology, oceanography, seismology, spectroscopy (and its applications to chemistry), quantum mechanics, nuclear and particle physics, plasma physics, remote sensing, biomolecular interactions, surface profiling, microfluidics, mechanical stress/strain measurement, velocimetry, and optometry.

Interferometers are widely used in science and industry for the measurement of small displacements, refractive index changes and surface irregularities. In an interferometer, light from a single source is split into two beams that travel different optical paths, then combined again to produce interference. The resulting interference fringes give information about the difference in optical path length. In analytical science, interferometers are used to measure lengths and the shape of optical components with nanometer precision; they are the highest precision length measuring instruments existing. In Fourier transform spectroscopy they are used to analyze light containing features of absorption or emission associated with a substance or mixture. An astronomical interferometer consists of two or more separate telescopes that combine their signals, offering a resolution equivalent to that of a telescope of diameter equal to the largest separation between its individual elements.

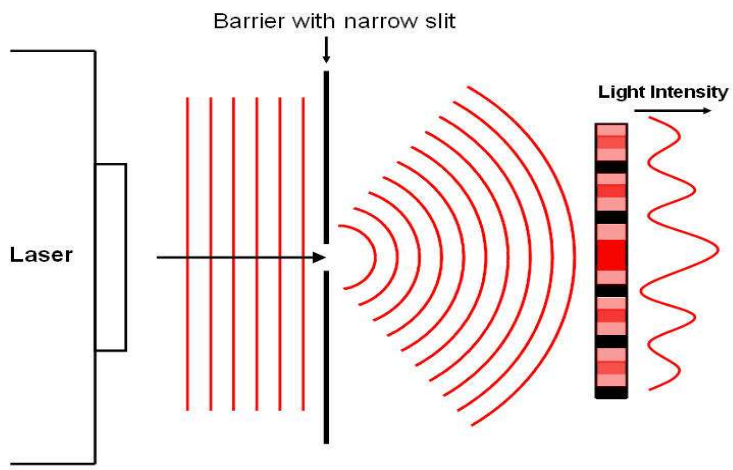
Types of interferometers:

- Michelson Interferometer
- Twyman-Green Interferometer
- Fabry-Perot Etalon
- Scanning Fabry-Perot Interferometer
- Mach-Zehnder Interferometer etc.

Diffraction of light:

When a beam of light passes through a narrow slit of aperture to a screen, then the geometrical shadow of the obstacle is not distinct uniform and a completely dark. This phenomenon is called diffraction of light.

In other word, when a beam of light passes through a narrow slit, it spreads out to a certain extend into a region of geometrical shadow of an object is known as diffraction.



Classification of diffraction of light:

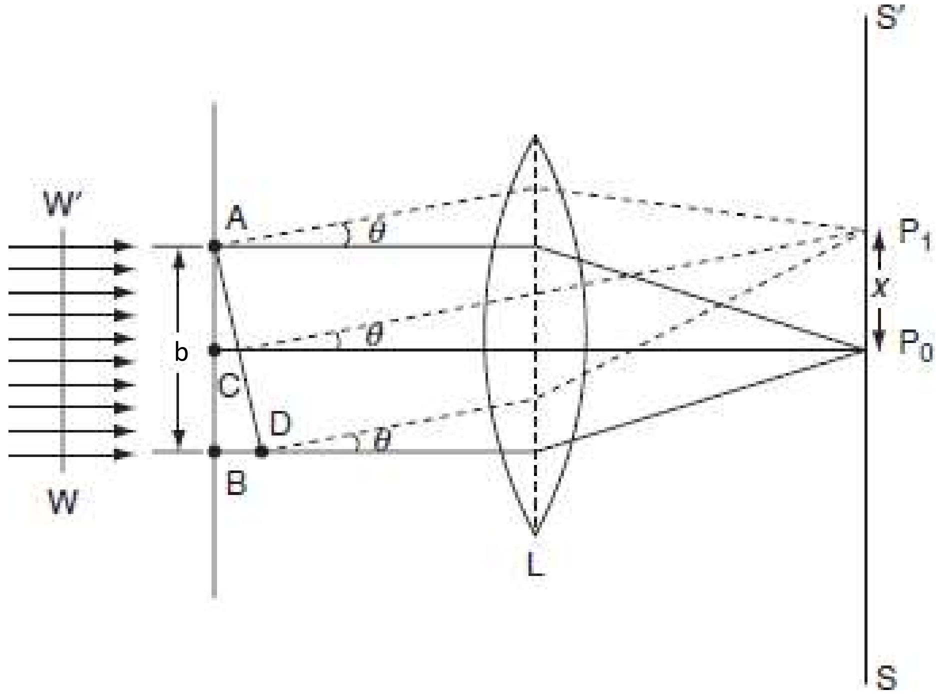
Diffraction phenomena are conveniently divided into two general classes-

1) Fresnel Diffraction
2) Fraunhofer Diffraction

The difference between Fresnel and Fraunhofer diffraction are as follows –

Fresnel diffraction	Fraunhofer diffraction
1) Either source of light or screen or both are effectively at finite distance from the aperture causing the diffraction of light.	1) Source and screen on which the diffraction phenomenon observed are at infinite distance from the aperture.
2) No lenses are required to occur the diffraction phenomenon.	2) More than one lens is required.
3) Incident wave front will be spherical or cylindrical.	3) Incident wave front will always plane.
4) It is difficult to treat theoretically.	4) It is easy to treat theoretically.
5) It is easy to treat experimentally.	5) It is difficult to treat experimentally.

Diffraction by a Single slit:



Let us consider a section of a slit of width b, illuminated parallel light from the left. Let ds be an element of the wave front in the plane of the slit. The parts of each secondary wave which travel normal to the plane will be focused at P_o and through an angle θ will reach at P.

Consider the wavelet emitted by the element ds situated at origin whose amplitude will be directly proportional to the ds and inversely proportional to the distance x.

The displacement of spherical wave at P be expressed as,

$$dy_0 = \frac{a\,ds}{x}\sin(\omega t - kx)$$

If ds is above the origin,

$$dy_{+s} = \frac{a\,ds}{x}\sin(\omega t - kx - ks\sin\theta)$$

If ds is below the origin,

$$dy_{-s} = \frac{a\,ds}{x}\sin(\omega t - kx + ks\sin\theta)$$

So, the resultant displacement is given by,

$$dy = dy_{+s} + dy_{-s} = \frac{a\,ds}{x}\sin(\omega t - kx - ks\sin\theta) + \frac{a\,ds}{x}\sin(\omega t - kx + ks\sin\theta)$$

$$= \frac{2a\,ds}{x}\{\sin(\omega t - kx)\cos(ks\sin\theta)\}$$

Integrating s = 0 to b/2 we get,

$$y = \int_0^{\frac{b}{2}} \frac{2ads}{x}\{\sin(\omega t - kx)\cos(ks\sin\theta)\}$$

$$= \frac{2a}{x}\sin(\omega t - kx)\int_0^{\frac{b}{2}} \cos(ks\sin\theta)ds$$

$$= \frac{2a}{x}\sin(\omega t - kx) \left[\frac{\sin(ks\sin\theta)}{k\sin\theta}\right]$$

$$= \frac{2a}{x}\sin(\omega t - kx) \left[\frac{\sin(k\frac{b}{2}\sin\theta)}{k\sin\theta}\right]$$

$$= \frac{ab}{x}\sin(\omega t - kx) \left[\frac{\sin(k\frac{b}{2}\sin\theta)}{k\frac{b}{2}\sin\theta}\right]$$

$$\therefore y = A\sin(\omega t - kx)$$

Where,

$A = \frac{ab}{x} \left[\frac{\sin(k\frac{b}{2}\sin\theta)}{k\frac{b}{2}\sin\theta}\right]$ is the resultant amplitude.

Let, $A_0 = \frac{ab}{x}$ and $\beta = \left[\frac{\sin(k\frac{b}{2}\sin\theta)}{k\frac{b}{2}\sin\theta}\right]$

So, the amplitude, $A = A_0 \frac{\sin\beta}{\beta}$

Thus the intensity of diffraction pattern on screen is given by,

$I = A^2 = A_0^2 (\sin^2\beta / \beta^2)$

Which is the intensity of diffraction pattern by single slit.

Condition for maximum and minimum intensity:

Since intensity depends on amplitude. So here,

$A = A_0 \frac{\sin\beta}{\beta}$ will define the maxima and minima.

For central maxima, $\beta \rightarrow 0$

□ $\lim_{\beta \to 0} \frac{\sin\beta}{\beta} = 1$

So, $I = A^2 = A_0^2 = I_0$

Maximum intensity falls to zero at $\beta = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= m\pi$

Condition for minima:

Intensity will be minimum if amplitude is minimum

□ $I_{min} = 0$

When $\sin\beta = 0$

or , $\sin\beta = \sin n\pi$, [where, n= 0,1,2,…..]

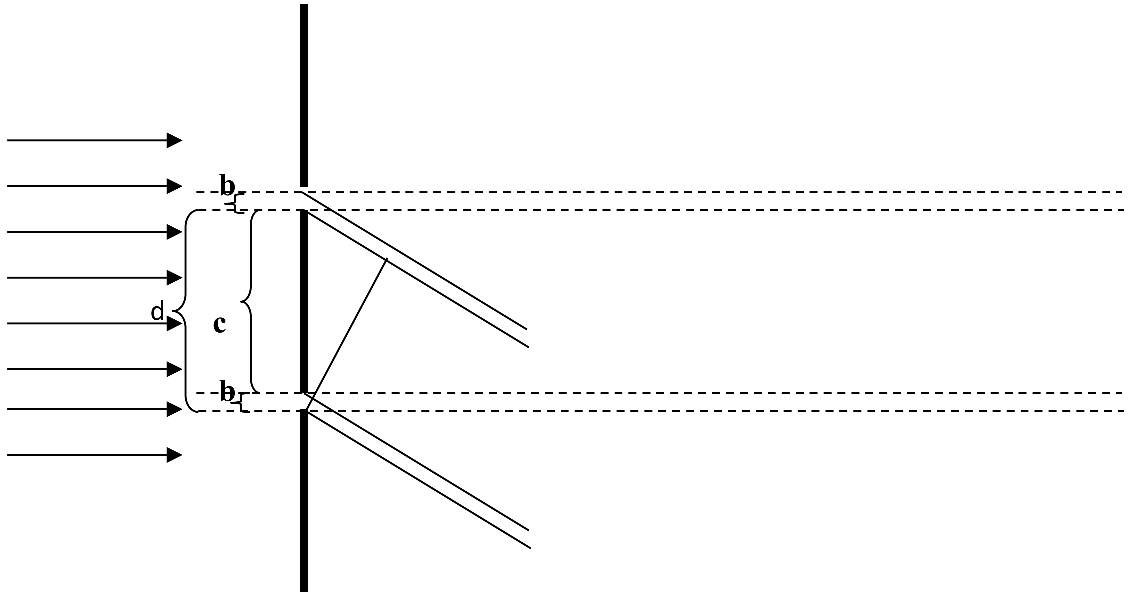
□$\beta = n\pi$

Hence, $(kb/2)\sin\theta = n\pi$

or, $(b/2)(2\pi/\lambda)\sin\theta = n\pi$ [where, $k = (2\pi/\lambda)$]

□ $b\sin\theta = n\lambda$

Diffraction due to a double slit :



Let us consider two equal slits of width b, separated by an opaque space of width c, the origin may be choosen at the center of c. So, the separation of the slits is d = b+c.

The parallel rays are reflected at an angle θ to the normal of the slit. Consider for small element ds due to the emitted wavelet by the element, the infinitesimal displacement may written as,

$$dy_0 = \frac{a\,ds}{x}\sin(\omega t - kx)$$

If ds is above the origin,

$$dy_{+s} = \frac{a\,ds}{x}\sin(\omega t - kx - ks\sin\theta)$$

If ds is below the origin,

$$dy_{-s} = \frac{a\,ds}{x}\sin(\omega t - kx + ks\sin\theta)$$

So, the resultant displacement is given by,

$$dy = dy_{+s} + dy_{-s} = \frac{a\,ds}{x}\sin(\omega t - kx - ks\sin\theta) + \frac{a\,ds}{x}\sin(\omega t - kx + ks\sin\theta)$$

$$= \frac{2a\,ds}{x}\{\sin(\omega t - kx)\cos(ks\sin\theta)\}$$

For total integrating by taking limit,

s = (d-b) /2 to (d+b)/2

we get,

$$y = \int_{\frac{d-b}{2}}^{\frac{d+b}{2}} \frac{2ads}{x} \{\sin(\omega t - kx)\cos(ks\sin\theta)\}$$

$$= \frac{2a}{x}\sin(\omega t - kx)\int_{\frac{d-b}{2}}^{\frac{d+b}{2}} \cos(ks\sin\theta)ds$$

$$= \frac{2a}{x}\sin(\omega t - kx) \quad \left[\sin\{\frac{k(d+b)\sin\theta}{2}\} - \sin\{\frac{k(d-b)\sin\theta}{2}\}\right]$$

$$= \frac{2a\sin(\square t - kx)}{kx\sin\theta} \quad \left[\sin(\frac{k\,d\sin\theta}{2} + \frac{k\,b\sin\theta}{2}) - \sin(\frac{k\,d\sin\theta}{2} - \frac{k\,b\sin\theta}{2})\right]$$

$$= \frac{2a\sin(\square t - kx)}{kx\sin\theta}\; 2\cos(\frac{k\,d\sin\theta}{2})\sin(\frac{k\,b\sin\theta}{2})$$

Let,

$$\beta = \frac{k\,b\sin\theta}{2} = \frac{2\pi}{\lambda}\frac{b\sin\theta}{2} = \frac{\pi}{\lambda}b\sin\theta$$

$$\gamma = \frac{k\,d\sin\theta}{2} = \frac{2\pi}{\lambda}\frac{d\sin\theta}{2} = \frac{\pi}{\lambda}d\sin\theta$$

Therefore,

$$y = \frac{2a\sin(\square t - kx)}{kx\sin\theta}\; 2\cos\gamma\sin\beta$$

$$= \frac{2ab}{x}\frac{\sin\beta}{\frac{kb\sin\theta}{2}}\cos\gamma\sin(\square t - kx)$$

$$= \frac{2ab}{x}\frac{\sin\beta}{\beta}\cos\gamma\sin(\square t - kx)$$

$$= 2A_0\frac{\sin\beta}{\beta}\cos\gamma\sin(\square t - kx) \quad [\text{where, } A_0 = \frac{ab}{x}]$$

$$= A\sin(\square t - kx)$$

Where, Amplitude, $A = 2A_0\frac{\sin\beta}{\beta}\cos\gamma$

 Intensity,

$$I = A^2 = (2A_0 \frac{\sin\beta}{\beta} \cos\gamma)^2 = 4\, A_0^2\, (\frac{\sin\beta}{\beta} \cos\gamma)^2$$

Condition for minima:

The resultant intensity will zero when either $(\sin^2\beta)/\beta^2$ or $\cos^2\gamma$ is zero.

That is,

$\beta = \pi, 2\pi, 3\pi, \ldots\ldots\ldots\ldots\ldots, m\pi$

 $(kb/2) \sin\theta = m\pi$,

So, $b\sin\theta = m\lambda$

This is the condition no 1 for minima

or,

$\gamma = \pi/2, 3\pi/2, 5\pi/2, \ldots\ldots\ldots\ldots\ldots, (2m+1)\pi/2$

so, $d\sin\theta = (2m+1)\lambda/2$

this is the condition no 2 for minima.

Condition for maxima:

For maximum,

$\gamma = 0, \pi, 2\pi, 3\pi, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. n\pi$

 $d\sin\theta = m\lambda$

This is the condition for maima.

Resolving power of optical instrument:

The ability of an optical instrument expressed in numerical measure, to resolve the images of two nearby points is termed as its resolving power.

Diffraction grating:

An arrangement which is consists of a large number of parallel equidistance slits of the same width is called diffraction grating. Corresponding diffraction pattern is known as grating spectra.

The sum of the width of each slit and width of the opaque space is called grating constant.

Let, b = breadth of each slits

c = width of opaque space

d = (b+c) = grating constant/ grating element

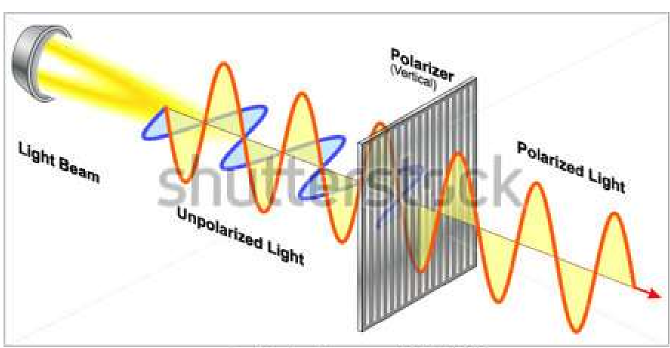
therefore, for a grating with N lines per inch,

d = (2.54 / N) cm

Polarization of light:

Ordinary light consists of transverse vibrations that is the vibrations are at right angles to the direction of propagation of the wave. An ordinary beam of light consist million of such waves, each with its own plane of vibration. If by some means the vibrations constituting the beam of light are confined to one plane, the light is said to be plane polarized.

The process by which light vibrations are confined to one particular direction is known as polarization.



Polarization by reflection or Brewster's Law:

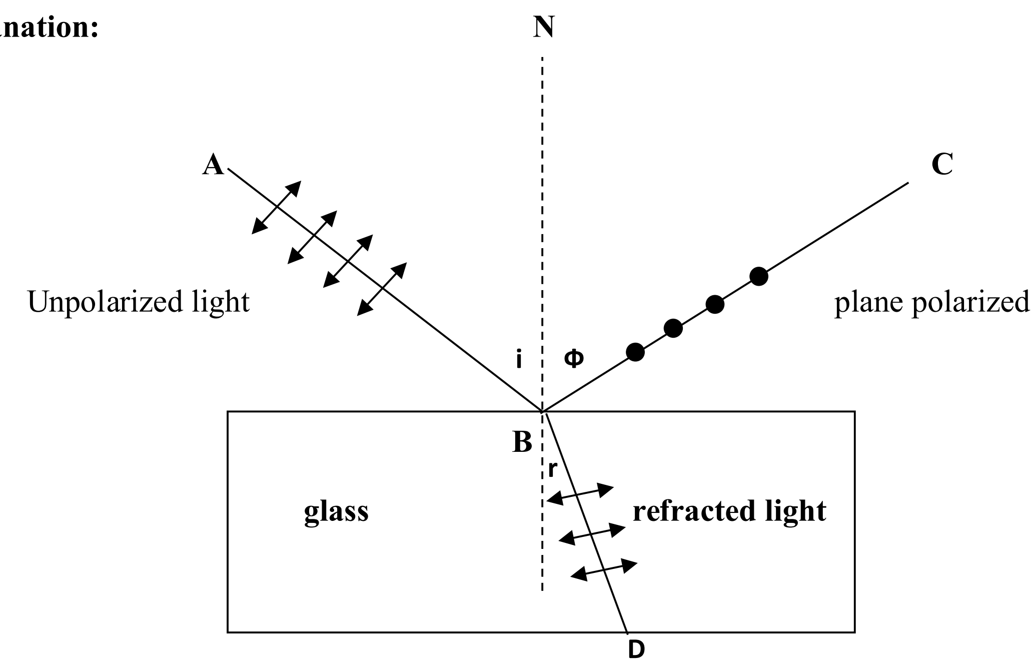
Brewster's law states that,

"*When the angle of incidence is equal to the angle of polarization for a transparent substance, the tangent of the angle of polarization will be equal to the refractive index of the substance. Moreover, the reflected and the refracted rays are perpendicular to each other.*"

If φ is the polarizing angle and μ is the refractive index of the medium, then according to the Brewster's law,

$$\mu = \tan\varphi$$

Explanation:



Let an uonpolarized light AB is incident at an angle equal to the polarizing angle on the glass surface. It is reflected along BC and refracted along BD.

From Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

and from Brewster's law,

$\mu = \tan\varphi = \frac{sin\varphi}{cos\varphi}$

By comparing, we get,

$\frac{\sin i}{\sin r} = \frac{sin\varphi}{cos\varphi}$

or, $\frac{\sin \varphi}{\sin r} = \frac{sin\varphi}{cos\varphi}$ [since, i = φ]

or, $\cos\varphi = \sin r$

or, $\cos\varphi = \cos(\frac{\pi}{2} - r)$

or, $\varphi = \frac{\pi}{2} - r$

 $\varphi + r = \frac{\pi}{2}$

Therefore, The refracted and the reflected (polarized) rays are perpendicular to each other.

Problem 1:

An unpolarized light is incident at an angle equal to the polarizing angle on glass surface. For a refractive index 1.54, what is the value of polarizing angle?

Solution:

Here,

Refractive index, $\mu = 1.54$

Polarizing angle, φ =?

We know that,

$\mu = \tan\varphi$

or, $\varphi = \tan^{-1}(1.54)$

□ $\varphi = 57$

Problem 2:

The path difference between two points of a wave is 4λ. What is the phase difference between these two points?

Solution:

Here,

Path difference, $x = 4\lambda$

Phase difference, $\delta = ?$

We know that,

$$\delta = (2\pi / \lambda)\ x$$

$$= (2\pi / \lambda) \times 4\ \lambda$$

$$= 8\pi \text{ radian}$$

Problem -3: The straight and narrow parallel slits of 1mm apart are illuminated by monochromatic light. Fringes formed on the screen held at a distance of 1m from the slits are 0.50mm apart. Calculate the wavelength of light used.

Solution:

We are given,

Distance between two slits, $d = 1\text{mm}$

Distance of the screen from source, $D = 1\text{m} = 1000\text{mm}$

Fringe width, $\beta = 0.50\text{mm}$

Wavelength, $\lambda = ?$

We know that,

$$\beta = \frac{\lambda D}{d}$$

$$\text{or, } \lambda = \frac{\beta d}{D} = 5 \times 10^{-4} \text{ mm} = 5 \times 10^{-4} \times 10^{6} \text{ Å} = 500\text{Å}$$

<u>Problem -4:</u> A plano-convex lens of radii 3 m is placed on an optically flat glass plate and illuminated by monochromatic light. The diameter of the 8th bright ring in the transmitted system is 0.72cm. Calculate the wavelength of light used.

Solution:

Here,

Radius of plano-convex lens, R = 3m = 300cm

No of dark ring, n = 8

Diameter of the nth ring, D = 0.72 cm

 radius of the nth ring, r = 0.36 cm

Wavelength, λ =?

We know that,

$$r^2 = \frac{(2n+1)\lambda R}{2}$$

$$\text{or, } \lambda = \frac{2r}{(2n+1)R} = 5760 \text{ Å}$$

<u>Problem – 5:</u> In Newton's ring experiment, the diameter of the 5th ring was 0.336cm and that of the 15th ring was 0.590cm. Find the radius of curvature of the plano-convex lens if the wavelength of light used was 5890 Å.

Solution:

Here,

Diameter of 5th ring, D_5 = 0.336cm

Diameter of 15th ring, D_{15} = 0.590cm

Wavelength, λ= 5890 Å = 5890 × 10^{-8}cm

n = 15 – 5 =10

Radius of curvature, R =?

We know that,

$$R = \{(D_{n+m})^2 - (D_n)^2\} / (4n\lambda)$$

$$= 99.83\text{cm}$$