Spring 2023

Group - A

$\boxed{1}$

(a) Describe the components of context-free grammar. Is any of the components have a similarity with any of the components of regular language or finite automata?

Ans.

Here the components of context-free grammar :

i) V is a finite set called the variables. Also called N i.e. non-terminals.

ii) Σ is a finite set, disjoint from V, also called T i.e. terminals.

iii) R or P is a finite set of rules or productions.

iv) S is the start variable, $S \in V$.

There are few components that have similarity with components of regular language or finite automata :

i] Terminals are similar with the alphabet.

ii] Productions or rules are similar with the transition function.

iii] Start variable is similar in concept to the stat state.

OR

Define and differentiate between the following.

[a] Derivation and Parse Tree.

[b] Leftmost and Rightmost derivation.

Ans.

[a]

Derivation : The sequence of substitution to obtain a string is called derivation.

Parse Tree : A parse tree is a step-by-step diagram that shows how a string is made by a given grammar.

Difference between derivation and parse tree :

Derivation are represented as a linear

sequence of production rules.

On the other hand,
Parse Tree is a graphical structure represent the hierarchical relationship between symbols.

$\boxed{b}$

Leftmost Derivation: A leftmost derivation is a process of generating a string by always expanding the leftmost non-terminal first in each step.

Rightmost Derivation: A rightmost derivation is a process of generating a string by always expanding the rightmost non-terminal first in each step.

Difference between leftmost and rightmost derivation:

Both derivation result in the same string but they differ in the order in which non-terminals are expanded during the derivation process.

(b) What is ambiguity? Determine whether the following grammar is ambiguous.

$$S \rightarrow AB$$
$$A \rightarrow aA \mid abA \mid \varepsilon$$
$$B \rightarrow bB \mid abB \mid \varepsilon$$

Ans.

In context free grammar ambiguity refers to a situation where a grammar generates the same string in several different ways.

Given that,

$$S \rightarrow AB$$
$$A \rightarrow aA \mid abA \mid \varepsilon$$
$$B \rightarrow bB \mid abB \mid \varepsilon$$

$$\begin{aligned}
S &\rightarrow AB \\
&\rightarrow abAB \quad [A \rightarrow abA] \\
&\rightarrow abAabB \quad [B \rightarrow abB] \\
&\rightarrow ababB \quad [A \rightarrow \varepsilon] \\
&\rightarrow abab \quad [B \rightarrow \varepsilon]
\end{aligned}$$

$S \rightarrow AB$

$\rightarrow aAB \quad [A \rightarrow aA]$

$\rightarrow aB \quad [A \rightarrow \varepsilon]$

$\rightarrow abB \quad [B \rightarrow bB]$

$\rightarrow ababB \quad [B \rightarrow abB]$

$\rightarrow abab \quad [B \rightarrow \varepsilon]$

The given grammar is ambiguous because the string "abab" derived ambiguously in that grammar.

(C) Consider the following grammar

$S \rightarrow SSx \mid SSy \mid SSz \mid a \mid b \mid c$

Show how to derive the string cabxycz using this grammar using a left-most derivation. Draw the parse tree for the string.

Ans.

Given grammar,

$$S \rightarrow SSx \mid SSy \mid SSz \mid a \mid b \mid c$$

Leftmost derivation :

$S \rightarrow SSz \quad [S \rightarrow SSz]$

$\rightarrow SSySz \quad [S \rightarrow SSy]$

$\rightarrow cSySz \quad [S \rightarrow c]$

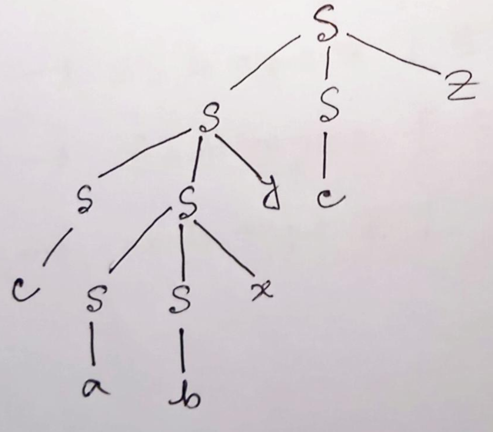
$\rightarrow cSSxySz \quad [S \rightarrow SSx]$

$\rightarrow caSxySz \quad [S \rightarrow a]$

$\rightarrow cabxySz \quad [S \rightarrow b]$

$\rightarrow cabxycz \quad [S \rightarrow c]$

Parse tree for the string :



OR Consider the following grammar

$S \rightarrow SSx \mid SSy \mid SSz \mid a \mid b \mid c$

Show how to derive the string cabxycz using this grammar using a right-most derivation. Draw the parse tree for the string.

Ans.

Given grammar,

$S \rightarrow SSx \mid SSy \mid SSz \mid a \mid b \mid c$

Right-most derivation:

$S \rightarrow SSz \quad [S \rightarrow SSz]$

$\rightarrow Scz \quad [S \rightarrow c]$

$\rightarrow SSycz \quad [S \rightarrow SSy]$

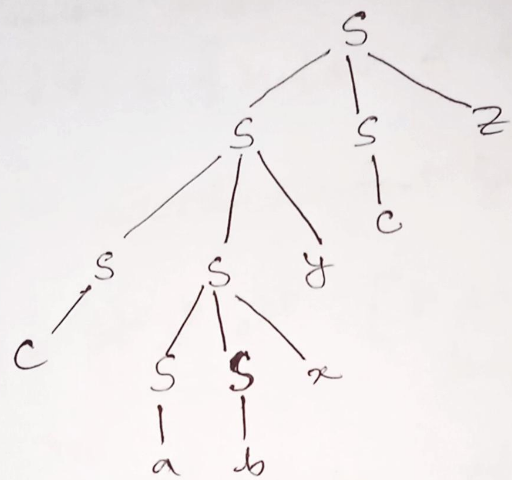
$\rightarrow SSSxycz \quad [S \rightarrow SSx]$

$\rightarrow SSbxycz \quad [S \rightarrow b]$

$\rightarrow Sabxycz \quad [S \rightarrow a]$

$\rightarrow cabxycz \quad [S \rightarrow c]$

Parse tree for the string:



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(a) Show using the pumping lemma which of the following languages are context-free.

i. $L1 = \{w \mid w \in a^n b^n c^{2n} \mid n \geq 0\}$

ii. $L2 = \{w \mid w \in a^n b^n c^n \mid n \geq 0\}$

Ans.

i] Given that,

$$L1 = \{w \mid w \in a^n b^n c^{2n} \mid n \geq 0\}$$

Assume L1 is context-free.

Now, $z = a^p b^p c^{2p}$ [p is the pumping length.]

In L1, the number of a's equals the number of b's, the number of c's is double.

Hence, L1 is not context-free.

ii] Given that,

$$L2 = \{W \mid W \in a^n b^n c^n \mid n \geq 0\}$$

Assume L2 is context-free.

The number of a's, b's and c's are equal.

Hence, L2 is not context-free.

OR Describe the following languages using a context-free grammar.

a. $0^* 1^*$

b. $1(01)^*$

c. $(11 \cup 0)^*$

Ans.

(a) $0^* 1^*$

CFG: $S \rightarrow 0S1 \mid \varepsilon$

(b) $1(01)^*$

CFG:

$S \rightarrow 1A$

$A \rightarrow 01A \mid \varepsilon$

(c) $(11 \cup 0)^*$

CFG:

$S \rightarrow 11S \mid 0S \mid \varepsilon$

(b) Give a CFG for each of the following languages over the alphabet $\Sigma = \{a, b\}$:-

i. $L = \{a^{2n} b^m c^n \mid n \geq 0\}$

ii. $L = \{a^m b^n c^{m+n} \mid n \geq 0\}$

Ans.

i) $L = \{a^{2n} b^m c^n \mid n \geq 0\}$

$A \to aaAc \mid B \mid \varepsilon$

$B \to bB \mid \varepsilon$

ii) $L = \{a^m b^n c^{m+n} \mid n \geq 0\}$

$A \to aAc \mid B \mid \varepsilon$

$B \to bBc \mid \varepsilon$

(c) Convert the following CFG into an equivalent CFG in Chomsky normal form.

$R \rightarrow aSa \mid bRb \mid S$

$S \rightarrow aTb \mid bTa \mid aS$

$T \rightarrow XTX \mid X \mid \varepsilon$

$X \rightarrow a \mid b$

Ans.

Step-1: Make a new start variable

$R_0 \rightarrow R$
$R \rightarrow aSa \mid bRb \mid S$
$S \rightarrow aTb \mid bTa \mid aS$
$T \rightarrow XTX \mid X \mid \varepsilon$
$X \rightarrow a \mid b$

Step-2: Remove null production.

$R_0 \rightarrow R$
$R \rightarrow aSa \mid bRb \mid S$
$S \rightarrow aTb \mid bTa \mid aS \mid ab \mid ba$
$T \rightarrow XTX \mid X \mid XX$
$X \rightarrow a \mid b$

Step-3: Remove unit rules.

$R_0 \rightarrow aSa \mid bRb \mid aTb \mid bTa \mid aS \mid ab \mid ba$

$R \rightarrow aSa \mid bRb \mid aTb \mid bTa \mid aS \mid ab \mid ba$

$S \rightarrow aTb \mid bTa \mid aS \mid ab \mid ba$

$T \rightarrow XTX \mid a \mid b \mid XX$

$X \rightarrow a \mid b$

Step-4:

$R_0 \rightarrow XSX \mid XRX \mid XTX \mid XTX \mid XS \mid XX \mid XX$

$R \rightarrow XSX \mid XRX \mid XTX \mid XTX \mid XS \mid XX \mid XX$

$S \rightarrow XTX \mid XTX \mid XS \mid XX \mid XX$

$T \rightarrow XTX \mid a \mid b \mid XX$

$X \rightarrow a \mid b$

Step-5:

$R_0 \rightarrow XSX \mid XRX \mid XTX \mid XS \mid XX$

$R \rightarrow XSX \mid XRX \mid XTX \mid XS \mid XX$

$S \rightarrow XTX \mid XS \mid XX$

$T \rightarrow XTX \mid a \mid b \mid XX$

$X \rightarrow a \mid b$

Step- 6:

$R_0 \rightarrow XU_1 \mid XU_2 \mid XU_3 \mid XS \mid XX$

$R \rightarrow XU_1 \mid XU_2 \mid XU_3 \mid XS \mid XX$

$S \rightarrow XU_3 \mid XS \mid XX$

$T \rightarrow XU_3 \mid a \mid b \mid XX$

$U_1 \rightarrow SX$

$U_2 \rightarrow RX$

$U_3 \rightarrow TX$

$X \rightarrow a \mid b$

This is the required chomsky normal form.