CHAPTER SEVEN

PROBABILITY DISTRIBUTION

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Binomial Distribution

A discrete random variable X is said to have binomial distribution if its probability function is as follows:

$$f(x) = n_{C_x}\ p^x\ q^{n-x}\ ;\ x = 0,1,2,\ldots\ldots\ldots\ldots\ldots\ldots\ldots,n$$

Where n and p are the parameters of the distribution and p + q = 1

Assumption of binomial distribution:

Four Assumptions or properties of a Binomial

• The sample consists of a fixed number of observations, n.

• Each observation is classified into one of two mutually exclusive and collectively exhaustive categories, called success and failure.

• The probability of an observation being classified as a success, p, or a failure, 1 - p, is constant over all observations.

• The outcome (success or failure) of any observation is independent of the outcome of any other observation.

Application problem: A family has five children. The probability of a child being a boy or a girl is equal. Find the probability that the family has (i) no boy; (ii) at most 2 boys and (iii) at least 3 boys.

Solution: Here, n = 5, p = q = 0.50 (since, The probability of a child being a boy or a girl is equal)

We Know, The probability function of a binomial distribution is

$$f(x) = n_{C_x}\ p^x\ q^{n-x}\ ;\ x = 0,1,2,\ldots\ldots\ldots\ldots\ldots\ldots\ldots,n$$

Therefore, $f(x) = 5_{C_x}$ $(0.50)^{x}$ $(0.50)^{5-x}$

(i) The probability that the family has no boy i.e,

$$P[X = 0] = f(0) = 5_{C_0} (0.50)^{0} (0.50)^{5-0} = 0.03125$$

(ii) The probability that at most 2 boys is

$$P[X \leqslant 2] = P[X = 0] + P[X = 1] + P[X = 2]$$
$$= 5_{C_0} (0.50)^{0} (0.50)^{5-0} + 5_{C_1} (0.50)^{1} (0.50)^{5-1} + 5_{C_2} (0.50)^{2} (0.50)^{5-2}$$
$$= 0.5$$

(iii) The probability that at least 3 boys is

$$P[X \geqslant 3] = P[X = 3] + P[X = 4] + P[X =]$$
$$= 5_{C_3} (0.50)^{3} (0.50)^{5-3} + 5_{C_4} (0.50)^{4} (0.50)^{5-4} + 5_{C_5} (0.50)^{5} (0.50)^{5-5}$$
$$= 0.5$$

Assignment problem: A fair coin is tossed 5 times. Find the probability of (i) exactly two heads, (ii) at least 3 heads, (iii0 no heads and (iv) at most 2 heads.

Poisson distribution

A discrete random variable X is said to have Poisson distribution if its probability function is as follows:

$$f(x) = \frac{e^{-m} m^{x}}{x!} \text{ ; x= 0,1,2,}\ldots\ldots\ldots\ldots,\infty$$

Where m is the parameter of the distribution and e = 2.718

Some practical application of Poisson distribution:

(i) The number of Suicides reported in a particular day.
(ii) The number of faulty blades in a packet of 100.
(iii) The number of printing mistakes per page of a book.
(iv) The number of letters lost in a mail per day.
(v) Number of deaths from a rare disease in a locality.

(vi) **What is normal distribution?**

Ans: A continuous random variable X is said to have normal distribution if its probability density function is as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ; \quad \begin{array}{l} \infty \leqslant x \leqslant \infty \\ \infty \leq \mu \leq \infty \\ \sigma^2 \geqslant 0 \end{array}$$

μ and σ^2 are the parameters of the distribution.

Properties of normal distribution:

1. Normal distributions are symmetric around their mean.
2. The mean, median, and mode of a normal distribution are equal.
3. The area under the normal curve is equal to 1.0.
4. Normal distributions are denser in the center and less dense in the tails.
5. Normal distributions are defined by two parameters, the mean (μ) and the standard deviation (σ).
6. 68% of the area of a normal distribution is within one standard deviation of the mean.
7. Approximately 95% of the area of a normal distribution is within two standard deviations of the mean.

STANDARD Normal Variate

Standard normal variate: A variate is called standard normal variate if its mean 0 and variance 1 respectively. It is denoted by Z where,

$$Z = \frac{X - \mu}{\sigma}$$

Mean: standard normal variate,

$$Z = \frac{X - \mu}{\sigma}$$

$$\textbf{Or, } E[Z] = E[\frac{X - \mu}{\sigma}]$$

$$\textbf{Or, } E[Z] = \frac{1}{\sigma} E[X - \mu]$$

$$\textbf{Or, } E[Z] = \frac{1}{\sigma}[E(x) - E(\mu)]$$

$$\textbf{Or, } E[Z] = \frac{1}{\sigma}[\mu - \mu]$$

$$\textbf{Or, } E[Z] = 0$$

Variance: $$Z = \frac{X - \mu}{\sigma}$$

$$\textbf{Or, V}[Z] = V[\frac{X - \mu}{\sigma}]$$

$$\textbf{Or, } V[Z] = \frac{1}{\sigma^2} V[X - \mu]$$

$$\textbf{Or, } V[Z] = \frac{1}{\sigma^2}[V(x) - V(\mu)]$$

$$\textbf{Or, } V[Z] = \frac{1}{\sigma^2}[V(X)]$$

$$\textbf{Or, } V[Z] = \frac{1}{\sigma^2}\sigma^2$$

$$\textbf{Or, } V[Z] = 1$$

Importance of normal distribution:

1. In practice under certain condition most of the probability and sampling distributions can be approximated by normal distribution.
2. According to central limit theorem, if mean and variance of a distribution exist, then the distribution converted to normal distribution.
3. Nomal distribution is the basis of all the sampling distribution.
4. Assumption of normaility is the basis of all the test of significance in applied statistics.
5. Normal distributions find its application in industrial statistics such as quality control.

Application problems: Suppose X is a normal variable with mean 8 and variance 4. Find (i) $P[0 < X < 8]$; (ii) $P[X<12]$ and (iii) $P[X > 12]$

Solution: From class lecture

Application problems: The heights of 1000 young boys follows normal distribution with mean 164 cm and standard deviation 6.4 cm. Estimate the number of boys who are (i) between 165 and 170 cm; (ii) Shorter than 162 cm and (iii) greater than 170 cm.