**Double Integral**

Example: $\int_0^1\int_0^1 x^2 dxdy$

$$= \int_0^1 \left[ \int_0^1 x^2 dx \right] dy$$

$$= \int_0^1 \left[ \frac{x^3}{3} \right]_0^1 dy$$

$$= \int_0^1 \left[ \frac{1^3}{3} - \frac{0^3}{3} \right] dy$$

$$= \int_0^1 \left[ \frac{1}{3} - 0 \right] dy$$

$$= \int_0^1 \left[ \frac{1}{3} \right] dy$$

$$= \frac{1}{3}\int_0^1 dy$$

$$= \frac{1}{3}[y]_0^1$$

$$= \frac{1}{3}[1-0]$$

$$= \frac{1}{3}[1]$$

$$= \frac{1}{3}$$

Or

Example: $\int_0^1\int_0^1 x^2 dydx$

$$= \int_0^1 \left[ \int_0^1 x^2 dy \right] dx$$

$$= \int_0^1 \left[ x^2 \int_0^1 dy \right] dx$$

$$= \int_0^1 x^2 [y]_0^1 dx$$

$$= \int_0^1 x^2 [1-0] dx$$

$$= \int_0^1 x^2\, dx$$

$$=\left[\frac{x^3}{3}\right]_0^1$$

$$=\left[\frac{1^3}{3}-\frac{0^3}{3}\right]$$

$$=\left[\frac{1}{3}-0\right]$$

$$=\frac{1}{3}$$

**Triple integral**

Evaluate

$$\int_0^1\int_0^{1-x}\int_0^{1-y^2} z\,dz\,dy\,dx$$

$$=\int_0^1\int_0^{1-x}\left[\int_0^{1-y^2} z\,dz\right]dy\,dx$$

$$=\int_0^1\int_0^{1-x}\left[\frac{z^2}{2}\right]_0^{1-y^2}dy\,dx \qquad [\int x^n dx=\frac{x^{n+1}}{n+1}]$$

$$=\int_0^1\int_0^{1-x}\left[\frac{(1-y^2)^2}{2}-\frac{0^2}{2}\right]dy\,dx$$

$$=\int_0^1\int_0^{1-x}\left[\frac{(1-2y^2+y^4)}{2}-0\right]dy\,dx$$

$$=\int_0^1\int_0^{1-x}\left[\frac{(1-2y^2+y^4)}{2}\right]dy\,dx$$

$$=\frac{1}{2}\int_0^1\int_0^{1-x}\left[(1-2y^2+y^4)\right]dy\,dx$$

$$=\frac{1}{2}\int_0^1\left[\int_0^{1-x}\left[(1-2y^2+y^4)\right]dy\right]dx$$

$$=\frac{1}{2}\int_0^1\left[\int_0^{1-x}1\,dy+\int_0^{1-x}(-2y^2)\,dy+\int_0^{1-x}y^4dy\right]dx$$

$$=\frac{1}{2}\int_0^1\left[[y]_0^{1-x}-2\left[\frac{y^{2+1}}{3}\right]_0^{1-x}+\left[\frac{y^{4+1}}{5}\right]_0^{1-x}\right]dx \qquad [\int dy=y]$$

$$=\frac{1}{2}\int_0^1\left[[y]_0^{1-x}-2\left[\frac{y^3}{3}\right]_0^{1-x}+\left[\frac{y^5}{5}\right]_0^{1-x}\right]dx$$

$$=\frac{1}{2}\int_0^1\left[[(1-x)-0]-2\left[\frac{(1-x)^3}{3}-\frac{0^3}{3}\right]+\left[\frac{(1-x)^5}{5}-\frac{0^5}{5}\right]\right]dx$$

$$=\frac{1}{2}\int_0^1\left[[(1-x)]-2\left[\frac{(1-x)^3}{3}\right]+\left[\frac{(1-x)^5}{5}\right]\right]dx$$

$$=\frac{1}{2}\left[\int_0^1[(1-x)]dx-\frac{2}{3}\int_0^1(1-x)^3\,dx+\frac{1}{5}\int_0^1(1-x)^5\,dx\right]$$

$$=\frac{1}{2}\left[\int_0^1 1\,dx-\int_0^1 x\,dx-\frac{2}{3}\int_0^1(1-x)^3\,dx+\frac{1}{5}\int_0^1(1-x)^5\,dx\right]$$

$$=\frac{1}{2}\left[[x]_0^1-\left[\frac{x^2}{2}\right]_0^1-\frac{2}{3}\left[\frac{(1-x)^{3+1}}{3+1}(-1)\right]_0^1+\frac{1}{5}\left[\frac{(1-x)^{5+1}}{5+1}(-1)\right]_0^1\right] \qquad \left[\int dx=x\right]$$

$$=\frac{1}{2}\left[[1-0]-\left[\frac{1^2}{2}-\frac{0^2}{2}\right]+\frac{2}{3}\left[\frac{(1-x)^4}{4}\right]_0^1-\frac{1}{5}\left[\frac{(1-x)^6}{6}\right]_0^1\right]$$

$$=\frac{1}{2}\left[[1-0]-\left[\frac{1^2}{2}-\frac{0^2}{2}\right]+\frac{2}{3}\left[\frac{(1-1)^4}{4}-\frac{(1-0)^4}{4}\right]-\frac{1}{5}\left[\frac{(1-1)^6}{6}-\frac{(1-0)^6}{6}\right]\right]$$

$$=\frac{1}{2}\left[[1]-\left[\frac{1}{2}\right]+\frac{2}{3}\left[0-\frac{1}{4}\right]-\frac{1}{5}\left[0-\frac{1}{6}\right]\right]$$

$$=\frac{1}{2}\left[1-\left[\frac{1}{2}\right]+\frac{2}{3}\left[-\frac{1}{4}\right]-\frac{1}{5}\left[-\frac{1}{6}\right]\right]$$

$$=\frac{1}{2}\left[1-\frac{1}{2}-\frac{2}{12}+\frac{1}{30}\right]$$

$$=\frac{1}{2}\left[\frac{60-30-10+2}{60}\right]$$

$$=\frac{1}{2}\left[\frac{22}{60}\right]$$

$$=\frac{11}{60}$$