Case II. Let either A or B be zero ; then eq. (3) takes the form if $A = 0$

or, $By^2 + 2Gx + 2Fy + C = 0$

or, $B(y + F/B)^2 = -2Gx - C + F^2/B = -2G\left(x + \frac{BC - F^2}{BG}\right)$ ... ... (7)

(i) If $G = 0$, then eq. (7) will represent a pair of parallel straight lines.

(ii) If $G = 0$ and $F^2/B - C = 0$, (7) will represent a pair of coincident lines.

(iii) $G \neq 0$, shift the origin to $\left(\frac{BC - F^2}{BG}, \frac{-F}{B}\right)$;

then eq. (7) takes the form $y^2 = -\frac{2G}{4}x$ ... ... (8)

which represents a parabola.

Since $AB = ab - h^2$ from (4)

or, $ab - h^2 = 0$, as A is zero

Hence when $ab - h^2 = 0$, conic (1) will represent a parabola.

Case III. When $A = B$ in eq. (3) i. e. when $a = b$ and $h = 0$ from (1), the eq. (1) will represent a circle.

Hence Eq. (1) represents an ellipse, parabola or hyperbola according as $ab - h^2 >, = \text{ or } < 0$

Art. 48. From the above discussions the general equation of the second degree.

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ will represent

(1) a pair of straight lines if the determinant

$$\Delta \equiv \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

(i) **two parallel lines** if $\Delta = 0, ab = h^2$

(ii) **two perpendicular lines** if $\Delta = 0, a + b = 0$

(2) **a circle** if $a = b, h = 0,$

(3) **a parabola** if $ab = h^2, \Delta \neq 0$

(4) **an ellipse** if $ab - h^2 > 0, \Delta \neq 0$

(5) **a hyperbola** if $ab - h^2 < 0, \Delta \neq 0$

(6) **a rectangular hyperbola** if $a + b = 0, ab - h^2 < 0, \Delta \neq 0$

Note : $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 =$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

Art. 49. Centre of a conic.

Definition : The centre of a conic is the point in the plane of the conic such that every chord of the conic passing through it is bisected at it.

Let the equation to the conic be

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ... ... (2)

and let $\frac{x - x_1}{l} = \frac{y - y_1}{m} = r$ ... ... (3)

be the equation of any line through (x_1, y_1)

If the live (2) meets the conic (1) at a distance r from (x_1, y_1) then $(x = x_1 + lr, y = y_1 + mr)$ is a point of intersection of (1) and (2) The from (1)

$a(x_1 + lr)^2 + 2h(x_1 + lr)(y_1 + mr) + b(y_1 + mr)^2 + 2g(x_1 + lr) + 2f(y_1 + mr) + c = 0$

or, $(al^2 + 2hlm + bm^2) r^2 + 2r \{(ax_1 + hy_1 + g) l + (hx_1 + by_1 + f) m\} + (ax_1^2 + 2hx_1y_1 + by_1^2 + 2gx_1 + 2fy_1 + c) = 0$ ... ... (4)

From Eq. (4) we see that r has two roots say r_1 and r_2 which are the distances from (x_1, y_1) i.e. the line (2) inersects the conic (1) at two points.

If (x_1, y_1) be the middle point of the chord, then r_1 and r_2 have equal but opposite sign i. e. $r_1 = -r_2$, or $r_1 + r_2 = 0$

The sum of the roots of r in (3) = 0

or $(ax_1 + by_1 + g) l + (hx_1 + by_1 + f) m = 0$ ... ... (5)

If (x_1, y_1) is the centre of the conic, then the relation (4) holds good for all values of l and m

$\therefore \quad ax_1 + hy_1 + g = 0$ ... ... (6)

$\quad hx_1 + by_1 + f = 0$ ... ... (7)

The above two equations given the centre of the conic (1). Solve (6) and (7) for x_1 and y_1

$\therefore \quad x_1 = \frac{hf - bg}{ab - h^2}, \; y_1 = \frac{gh - af}{ab - h^2}$

Hence the co-ordinates of centre (1) is $\left(\frac{hf - bg}{ab - h^2}\right), \left(\frac{gh - af}{ab - h^2}\right)$

Cor. 1. When $ab - h^2 = 0$, the co-ordinates of the centre are at infinity and the curve is a parabola which has no centre.

Cor. 2. If $hf - bg = 0$ and $ab - h^2 = 0$

i. e. $a/h = h/b = g/f$

The equations (3) and (4) represent the same line. Is this case any point on the straight line (3) or (4) may be regarded as the centre, for every such point satisfies the definition of the centre.

The locus in this case is a pair of parallel straight lines.

***Note* : The equations (3) and (4) can be obtained by, diferentiating (1) partially with respect to x and y respectively.**

If $F(x, y) \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Then, $\frac{\delta F}{\delta x} \equiv 2(ax + hy + g) = 0$, $\frac{\delta F}{\delta y} \equiv 2(hx + by + f) = 0$,

Solve $\frac{\delta F}{\delta x} = 0$ and $\frac{\delta F}{\delta y} = 0$ for x and y. Thus the co-ordinates for the centre (x, y) are obtained.

Art. 50. Equation referred to centre of the conic. (Rectangular axex)

Let the equation to the conic be

$$F(x, y) \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \ldots \quad (1)$$

$$\frac{\delta F}{\delta x} \equiv 2(ax + hy + g) = 0, \; \frac{\delta F}{\delta y} \equiv 2(hx + by + f) = 0 \quad \ldots \quad (2)$$

If (x_1, y_1) be the co-ordinates of the centre of the conic, then these co-odinates are obtained by solving these equations,

$$ax_1 + hy_1 + g = 0 \text{ and } hx_1 + by_1 + f = 0 \quad \ldots \quad (3)$$

If the origin is transferred to (x_1, y_1) the Eq. (1) becomes $ax^2 + 2hxy + by^2 + c_1 = 0$ ... (4)

where $c_1 = ax_1{}^2 + 2hx_1y_1 + by_1{}^2 + 2gx_1 + 2y_1 + c$ ... (5)

by (2) Art. 49

$= x_1(ax_1 + hy_1 + g) + y_1(hx_1 + by_1 + f) + gx_1 + fy_1 + c$

$= gx_1 + fy_1 + c$ by (3) ... (6)

Put the values of x_1, and y_1 from (5), Art. 49. in (6)

$$\text{then } c_1 = \frac{abc + 2fgh - af^2 - bg^2 - ch^2}{ab - h^2} \times \frac{\Delta}{ab - h^2} \quad \ldots \quad (7)$$

Where Δ is called the discriminant of the general equation (1) The equation referred to the new axes through the centre is

$$ax^2 + 2hxy + by^2 + \frac{\Delta}{ab - h^2} = 0 \quad \ldots \quad (8)$$

Cor. If $\Delta = 0$, then $c_1 = 0$ then Eq. (8) will represent two straight lines

Working Rule : For the reduction of the equation of a conic referred to centre as origin.

Let the equation of the conic be.

$$F(x, y) \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \ldots \quad (1)$$

$$\text{then write } \frac{\delta F}{\delta x} \equiv (ax + hy + g) = 0 \text{ or } ax + hy + g = 0 \quad \ldots \quad (2)$$

$$\text{and } \frac{\delta F}{\delta y} = 2(hx + by + f) = 0 \text{ or, } hx + by + f = 0 \quad \ldots \quad (3)$$

The equations (2) and (3) *i. e.* $\frac{\delta F}{\delta x} = 0$, and $\frac{\delta F}{\delta y} = 0$

are the same as these giving the co-ordinates of the centre.

Now the constant $= \frac{\Delta}{ab - h^2} = gx + fy + c$ which is represented by another constant c_1.

$$c_1 = gx + fy + c \quad \ldots \quad (4)$$

where x and y are found from the equations $\frac{\delta F}{\delta x} = 0$, and $\frac{\delta F}{\delta y} = 0$

and g, f and c are found from the given equation (1)

Hence the equation reduces to $ax^2 + 2hxy + by^2 + c_1 = 0$

In other words "**Same terms of second degree as in the given equation + new constant** = 0. which we can always reduce to the FORM

$Ax^2 + 2H\,xy + By^2 = 1$ ... ... (5)

If the axes of the equation (5) are rotated about its new (centre) origin in such a way that xy-term vanishes, the new equation of the conic becomes $a_1x^2 + b_1y^2 + c_1 = 0$ ... (6)

which is called the **Standard Form** of the equation of the conic. Then by Invariants, (See Art. 35) we have from (5) and (6)

$a_1 + b_1 = a + b$

$a_1b_1 = ab - h^2$

where a_1 and b_1 can be easily found.

Thus the equation (1) reduces to the standard form

$a_1x^2 + b_1y^2 + c_1 = 0$

Ex. Reduce the equation $32x^2 + 52xy - 7y^2 - 64x - 52y - 148 = 0$ to the standard form.

Let $f(x, y) \equiv 32x^2 + 52xy - 7y^2 - 64x - 52y - 148 = 0$

$\delta f/\delta x = 0$ or, $64x + 52y - 64 = 0$ ... ... (1)

and $\delta f/\delta y = 0$ or, $52x - 14y - 52 = 0$ ... ... (2)

Solve (1) and (2) for x and y $\therefore$ $x = 1, y = 0$ Centre is at $(1, 0)$

New constant, $c_1 = gx + fy + c = -32\,(1) - 26\,(0) - 148 = -180$

Therefore, the equation of the conic referred as origin is

$32x^2 + 52xy - 7y^2 - 180 = 0$ *by* (5) working Rule (xi)

When the xy — term is removed *by* rotation of the axes, let the reduced equation be

$a_1x^2 + b_1y^2 = 180$ ... ... (3) *by* (6) (cor. Art 50)

Then $a_1 + b_1 = 32 - 7 = 25$.

$\therefore$ here $a = 32$, $b = -7$, $h = 26$

and $a_1b_1 = -32.7 - (26)^2 = -900$

Solve for a_1 and b_1 $\therefore$ $a_1 = 45$, $b_1 = -20$

Therefore, the equation (3) is

$45x^2 - 20y^2 = 180$ or, $x^2/_4 - y^2/\frac{9}{2} = 1$

The given equation represents a hyperbola.

T-(x) **Art 51. Axes : To find the lengths and position of the axes of the conic.** [C. U. 1986]

$ax^2 + 2hxy + by^2 = 1$ ... ... (1)

Let the axes be rectangular.

Let the conic be cut by a concentric circle of radius r. The diameters through the points of intersection will be equally inclined to the axes of the conic, and will be coincident if the radius of the circle be equal to either of the semi-axes of the conic.

Now suppose that equation of the circle as $\dfrac{x^2 + y^2}{r^2} = 1$ ... (2)

Making (1) homogeneous with the help of (2), we have

$$\left(a - \frac{1}{r^2}\right)x^2 + 2hxy + \left(b - \frac{1}{r^2}\right)y^2 = 0 \quad \ldots \quad (3)$$

which represents a pair of straight lines passing through the origin and the points of intersection of (1) and (2). When these lines (diameters) coincide they become axes of conic (1).

They will be coincident (by condition of equal roots)

$$(a - 1/r^2)(b - 1/r^2) = h^2 \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

or $$\boxed{\frac{1}{r^4} - (a+b)\frac{1}{r^2} + ab - h^2 = 0} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

This is a quadratic equation in r^2. Hence the square of lengths of the semi-axes of the conic. (4) are the roots of (5)

Let r_1^2 and r_2^2 be the two roots. The semi-axes of the conic are of lengths r_1 and r_2.

Now multiply the equation (3) by $(a - 1/r^2)$ then it becomes

$$(a - 1/r^2)^2 x^2 + 2h(a - 1/r^2)\, xy + (a - 1/r^2)(b - 1/r^2)\, y^2 = 0$$

or, $(a - 1/r^2)^2 x^2 + 2h(a - 1/r^2)\, xy + h^2y^2 = 0$ by (4)

or, $\{(a - 1/r^2)x + hy\}^2 = 0$ or, $(a - 1/r^2)\, x + hy = 0$

The equations of the axes are therefore

$$\boxed{\left(a - \frac{1}{r_1^2}\right)x + hy = 0 \text{ and } \left(a - \frac{1}{r_2^2}\right)x + hy = 0} \quad \ldots \quad \ldots \quad (6)$$

and length of the corres ponding semi-axes are r_1 and r_2 where r_1^2 and r_2^2 are the roots of (5)

Cor. For the general equation.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

which reduces to the equation

$ax^2 + 2hxy + by^2 = k$ by Eq 8 Art. 50.

where $k = -c_1$ from (7) Art. 50

Making the equation homogeneous by (2) we have

$$ax^2 + 2hxy + by^2 = k\,\frac{x^2 + y^2}{r^2}$$

$$\text{or, } \left(a - \frac{k}{r^2}\right)x^2 + 2hxy + \left(b - \frac{k}{r^2}\right)y^2 = 0 \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

In this case the square of the Semi-axes are given by (sec relation 4)

$$(a - k/r^2)(b - k/r^2) = h^2 \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

and the equations of the axes are given by

$$(a - k/r_1^2)\, x + hy = 0 \text{ and } (a - k/r_2^2)\ \ x + hy = 0 \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

Note **:** As the minor or conjugate axis is perpendicular to the major or transverse axis, the equation of minor or conjugate axis is easily obtained if equation of the major or transverse axis is known. say if $5x + 3y + 6 = 0$ is the equation of major axis the equation of the minor axis is $3x - 5y + k = 0$, Since it pass through the centre, then k is known.

Sometimes the minor or conjugate axis obtained from the fromula $(b - 1/r_2^2)\, x + hy = 0$ seems to be not perpendicular to the major or transverse axis. For example see worked out Example No. 10.

Lat us rectum, is parallel to minor or conjugate axis and passes through the focus S or S'

Directrices are parallel to minor or conjugate axis and passes through Z or Z'

Tangents at the vertices A and A' are parallel to minor or conjugate axis and pass through A or A'. So their equations can be easily determined.

Notes : Slope of the axis as given by ta- From it we get sin θ and cos θ. If tan θ is negative, then θ is obtuse. In this case we keep perpendicular positive and base negative. Thus sin θ is positive and cos θ is negative in this case.

Let us explain the above method in the following Example.

Ex. ***Reduce the equation***

$f(x, y) \equiv 8x^2 + 4xy + 5y^2 - 24x - 24y = 0$ ... ... (1)

to the standard form and find its all properties.

In the Eq. (i) $a = 8, h = 2, b = 5, g = -12, f = -12, c = 0$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = -$ ve and $ab - h^2 = 36$.

Therefore, $\Delta \neq 0, ab - h^2 > 0$

(i) The equation (i) will represent an ellipse.

$$\frac{\delta f(x, y)}{\delta x} \equiv 16x + 4y - 24 = 0, \text{ and } \frac{\delta f(x, y)}{\delta y} \equiv 10y + 4x - 24 = 0$$

or, $4x + y - 6 = 0$ and $2x + 5y - 12 = 0$, Centre of the conic is the intersection of these two lines. Solve $x_1 = 1, y_1 = 2$.

(ii) **Centre (1, 2)**

$C_1 = gx_1 + fy_1 + c = -12 \cdot 1 - 12 \cdot 2 + 0 = -36$

The eq. (1) referred to the centre (1, 2) is

$8x^2 + 4xy + 5y^2 - 36 = 0$, or, $\frac{2}{9}x^2 + \frac{1}{9}xy + \frac{5}{36}y^2 = 1$ ... ... (2)

Here $A = 2/9, H = 1/18, B = 5/36$ ... ... (3)

∴ The lengths of the semi-axes are given by

$\frac{1}{r^4} - (A + B)\frac{1}{r^2} + AB - H^2 = 0$ by 5, Art. 51.

Put the values of A, B and H from (3) then

$r^4 - 13r^2 + 36 = 0$ or, $r^2 = 9$ or, 4 i. e.; $r_1 = 3, r_2 = 2$ ... ... (4)

(iii) The lengths of the axes are 6 and 4.

The equations of the major and minor axes are

$(A - 1/r_1^2)/x + Hy = 0$ and $(A - 1/r_2^2)x + Hy = 0$

or, $2x + y = 0$ and $x - 2y = 0$ by (3) and (4), referred to the centre (1, 2),

or, $2(x - 1) + (y - 2) = 0$ and $(x - 1) - 2(y - 2) = 0$; referred to the old orgin:

or, $2x + y - 4 = 0$ and $x - 2y + 3 = 0$

(iv) Equation of the major axis is $2x + y - 4 = 0$ ... ... (5)

(v) Equation of the minor axis is $x - 5y + 3 = 0$ ... ... (6)

(Eccentricity. $e^2 = 1 - r_2^2/r_1^2 = 1 - 4/9 \therefore e = \sqrt{5}/3$ ... ... (7)

Now d for foci S and S′ $= \pm ae = \pm r_1 e = \pm \sqrt{5}$ by (4) and (7)

(vi) Solpe of major axis, eq. (5)

$\tan\theta = -2$ and $\sin\theta = 2/\sqrt{5}, \cos\theta = -1/\sqrt{5}$ ... ... (8)

(h, k) *i. e.* : (1, 2) are the centre of the conic (1) by (II) of

Art. 51. (a) $d = \sqrt{(1^2 + 2^2)} = \sqrt{5}$

(vii) Focus S ; $(h + d\cos\theta, k + d\sin\theta) = (1 + \sqrt{5}. -1/\sqrt{5}, 2 + \sqrt{5}. 2/\sqrt{5} = (0, 4)$,

Focus S′, $(1 - \sqrt{5}. -1/\sqrt{5}, 2 - \sqrt{5}.2/\sqrt{5}) = (2, 0)$

d for vertices A and A′, $= \pm a = \pm r_1 = \pm 3$

d for feet of directrices Z and Z′ $= \pm a/e = \pm r_1/e = \pm 9/\sqrt{5}$.

(viii) Vertex A, $(1 + 3. -1/\sqrt{5}, 2 + 3.2/\sqrt{5}) = (1 - 3/\sqrt{5}, 2 + 6/\sqrt{5})$

Vertex A′ $(1 - 3. -1/\sqrt{5}, 2 - 3.2/\sqrt{5}) = (1 + 3/\sqrt{5}, 2 - 6/\sqrt{5})$

(ix) Point Z $(1 + 9/\sqrt{5}. -1/\sqrt{5}, 2 + 9/\sqrt{5}. 2 - \sqrt{5}) = (-44/5, 28/5)$

Point Z′ $(1 - 9/\sqrt{5}. -1\sqrt{5}, 2 - 9/\sqrt{5}.2 - \sqrt{5}) = (14/5, -8/5)$

(x) d for the points B and B′ $= \pm b = \pm r^2 = \pm 2$

Slope of minor axis : $\tan\theta = \frac{1}{2}$, $\sin\theta = 1/\sqrt{5}.$, $\cos = 2/\sqrt{5}$

(h, k) is the centre (1, 2)

Points B and B′ $= (h \pm d\cos\theta, k \pm d\sin\theta)$

$= (1 \pm 2.2/\sqrt{5}, 2 \pm 2.1/1/\sqrt{5}) = (1 \pm 4 - \sqrt{5}, 2 \pm 2\sqrt{5})$

(xi) Length of the latus rectum $= \frac{2b^2}{a} = 2r_2^2/r_1 = 8/3$

(xii) Distance between S and L or, L′ is given by

$d = \pm b^2/a = \pm r_2^2/r_1 = \pm 4/3$

and (h, k) stands for S (0, 4) or, S′ (2, 0) by (vii)

Solpe of latus rectum is the same as the solpe of minor axis

Points : L and L′ $(0 \pm 4/3.2\sqrt{5}, 4 \pm 4/3.1/\sqrt{5}) = (+ 8/3\sqrt{5}, 4 + 4./3\sqrt{5})$

Similarly another pair of points with respect to S′(2, 0) can be determined.

(xiii) Equation of the latus rectum which is parallel to minor axis $x - 2y + 3 = 0$ is $x - 2y + k = 0$. Since it passes through the foci S (0, 4) or S′ (2, 0), hence $k = 8, -2$, Hence equations of the latar recta are $x - 2y + 8 = 0$ and $x - 2y - 2 = 0$

(xiv) Directrix through Z $(-4/5, 28/5)$ or, Z′ $(14/5, -18/5)$ is also parallel to minor axis $x - 2y + 3 = 0$. Therefore the equation is $x - 2y + 3 = 0$, where $\lambda = 12, -10$.

Hence the equations of the directrices are $x - 2y + 12 = 0$ and $x - 2y - 10 = 0$

Cor. Eccentricity of the conic, e is given by $e^2 = 1 - \frac{r_1^2}{r_2^2}$

If r_1^2 and r_2^2 are positive, the conic is an ellipse. If r_2^2 is negative the conic is a hyperbola. If $r_1^2 = r_2^2$ and r_2^2 is negative, the conic is a rectangular hyperbola.

Art 52. Tangents : Def : Let P be a given point on a curve and Q be any other point on it. If Q tends to P, the straight line PQ tends, in general, to a definite straight line to which the chord PQ of a curve tends as Q tends to P. This straight line is called the tangent to the curve at P.

Popularly tangent is defined as **'Tangent is the limiting position of a secant'.**

Art. 52. (a) To find the equation of the tangent to the conic

$F(x, y) \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ... ... (1)

Let (x_1, y_1) be any point on the conic

Let a line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = r$ ... ... (2)

Pass through the point (x_1, y_1). If the line (5) meets the conic $F(x, y) = 0$, the general co-ordinates $(x_1 + lr, y_1 + mr)$ will satisfy $F(x, y) = 0$,

$a(lr + x_1)^2 + 2h(lr + x_1)(mr + y_1) + b(mr + y_1)^2 + 2g(lr + x_1) + 2f(mr + y_1) + c = 0.$

or, $r^2(a^2l^2 + 2hlm + bm^2) + 2r\{(ax_1 + hy_1 + g)l + (hx_1 + by_1 + f)m\} + (ax_1^2 + 2hx_1y_1 + 2gx_1 + 2fy_1 + c) = 0.$

or, $r^2(a^2l^2 + 2hlm + bm^2) + 2r(ax_1 + hy_1 + g)l$
$+ (hx_1 + by_1 + f)m\} + F(x_1, y_1) = 0$ ... ... (3)

If the point (x_1, y_1) is on the conic $F(x_1, y_1) = 0$ then

$ax_1^2 + 2hx_1y_1 + by_1^2 + 2gx_1 + 2fy_1 + c = 0.$ ... ... (4)

or, $F(x_1, y_1) = 0$

Then from (3). one value of r is zero. If the line (2) is an tangent to the conic, then the other value of r is also zero from (3)

$(ax_1 + hy_1 + g)l + (hx_1 + by_1 + f)m = 0$ ... ... (5)

Since the tangent can be drawn in any direction and hence its equation is obtained by elimination l, m between (2) and (5)

Therefore, the equation of the tangent at (x_1, y_1) is

$(ax_1 + hy_1 + g)(x - x_1) + (hx_1 + by_1 + f)(y - y_1) = 0$

or, $x(ax_1 + hy_1 + g) + y(hx_1 + by_1 + f) = ax_1^2 + 2hx_1y_1 + by_1^2 + gx_1 + fy_1$

or, $x(ax_1 + hy_1 + g) + y(hx_1 + by_1 + f) = ax_1^2 + 2hx_1y_1 + by_1^2 + 2gx_1 + 2fy_1 + c - (gx_1 + fy_1 + c)$

or, $2x(ax_1 + hy_1 + g) + 2y(hx_1 + by_1 + f) + 2(gx_1 + fy_1 + c) = 0$ by (4)

or, $x\frac{\delta F}{\delta x_1} + y\frac{\delta F}{\delta y_1} + 2c_1 = 0$, where $c_1 = gx_1 + fy_1 + c$

or $axx_1 + h(xy_1 + x_1y) + byy_1 + g(x + x_1) + f(y + y_1) + c = 0$ ... ... (6)