International Islamic University Chittagong

Department of Electrical and Electronic Engineering

B. Sc. Engineering in EEE

Final Exam, Spring 2022

Course Code: **MATH 1107** Course Title: **Mathematics I**

Time: 2 hours 30 minutes Full Marks: 50

(i) The figures in the right-hand margin indicate full marks

(ii) Course Outcomes and Bloom's Levels are mentioned in additional Columns

Course Outcomes (COs) of the Questions	
CO1	For complex Engineering problems, it is essential to get Knowledge of the limit, continuity, and differentiability, power series, Rolle's Theorem, Mean value theorem, Taylor, and McLaurin series. Also, the need concept of the partial derivative and Integration.
CO2	By using the above mentioned foundational mathematical information; One can implement it to solve the mathematical problems, which is expressing engineering principles.

Bloom's Levels of the Questions						
Letter Symbols	R	U	App	An	E	C
Meaning	Remember	Understand	Apply	Analyze	Evaluate	Create

Part A

[Answer the questions from the followings]

1.	a)	If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, Prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$ and verify Euler's Theorem and prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$.	CO1	U	5
1.	b)	Find maximum minimum points and value of the function $f(x,y) = x^3 + y^3 - 13(x+y) + 10xy$.	CO1	Ap	5
2.	a)	Evaluate: $\int \left(\frac{2x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} \right) dx$.	CO2	E	4
2.	b)	Evaluate: (i). $\int \frac{x^2}{(x+1)(x+2)(x-3)}\, dx$, (ii). $\int \frac{1-\sin x}{x+\cos x}\, dx$, and (iii). $\int \frac{x^{n-1}}{1+x^n}\, dx$.	CO2	E	2+2+2

Or,

2.	a)	A cylinder closed on all sides of fixed volume is to be made of tinned sheet. Prove that the sheet used will be a minimum if the height is equal to the diameter.	CO2	E	4
2.	b)	(i)If $u = \frac{x}{r^3}$ and $r^2 = x^2 + y^2 + z^2$, Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.	CO2	E	3+3

(ii)Prove that the necessary conditions for the existence of maximum value and

Part B

[Answer the questions from the followings]

3.	a)	Evaluate the integral $\int_a^b 2x\,dx$ by the definition of the limit of sum.	CO1	Ap	6
3.	b)	Express $\int_0^1 x^m(1-x^n)^q\,dx$ in terms of Beta function then Evaluate $\int_0^1 x^7(1-x^5)^{11}dx$.	CO2	E	4
4.	a)	Prove that $\int_0^{\frac{\pi}{2}} d\theta \int_0^{a\,sin\theta} dr \int_0^{a-\frac{r^2}{a}} r\,dz = \frac{5a^3\pi}{64}$.	CO1	Un	5
4.	b)	Find the length of the arc of the parabola $y^2 = 8x$ from the vertex to an extremity of the latus rectum.	CO2	E	5
5	a)	Find the area of the region bounded by the parabolas $y^2 = 4ax$ and$x^2 = 4ay$	CO1	U	4
5.	b)	(i) Determine $\int_a^b xdx$ as the limit of a sum (ii) Evaluate $\lim_{n\to\infty}\left(\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}\ldots\ldots\ldots\ldots+\frac{1}{8n}\right)$	CO1	Ap	6

Or,

5.	a)	Write the physical interpretation of definite integral.	CO1	U	4
5.	b)	Find the volume of the solid having its top in the plane $x + z = 2$. Its base in the plane $z = 0$, and its sides in the plane $x = 0, y = 0$ and $x + y = 2$.	CO2	E	6