Radioactivity

The phenomenon of spontaneous emission of powerful radiations exhibited by heavy elements is called radioactivity.

Or:*Unstable atomic nuclei are spontaneously decomposes to form nuclei i.e. a higher stability. The decomposition process is called radioactivity.* The energy and particle which are released during the decomposition process are called radiation.

Or:*The property of substances giving out radioactive rays (e.g. α-rays, β-rays, γ-rays) is called radioactivity.*The substance that possesses this phenomenon is called radioactive substance.*e.g.* Radium (*Ra*), Uranium (*U*), Actinium (*Ac*).

Radioactive particles

The radioactive radiations emitted by these elements are found to consist of the following:

(i) Alpha (α) rays orα-particle
(ii) Beta (β) rays orβ-particle
(iii) Gamma (γ) rays orγ-particle.

(i) Alpha (α) rays or α-particle: Theα-particles are doubly-ionized helium atoms*i.e.* helium atoms with both of their electrons removed. Obviously, anα-particle has double the charge of a hydrogen nucleus (*or proton*) and a mass number or atomic weight four times as great. It is represented by the symbol $_2He^4$.

(ii) Beta (β) rays or β-particle:Theβ-particles consist of ordinary electrons with a mass equal to 1/1836 of the mass of the proton. Since it carries a unit negative charge and its mass is negligible, symbol for electron is $_{-1}e^0$.

(iii) Gamma (γ) rays or γ-particle:γ-rays are electromagnetic radiations having high frequency. They have no charge.

Properties of α-particle

- α-particles are doubly-ionized helium atoms *i.e.* helium atoms with both of their electrons removed.
- α-particle has double the charge of a hydrogen nucleus (*or proton*) and a mass number or atomic weight four times as great.
- It is represented by the symbol $_2He^4$.

Properties of β-particle

- Theβ-particles consist of ordinary electrons with a mass equal to 1/1836 of the mass of the proton
- β-particle has negative charge and a mass number equal to that of an electron.
- It is represented by the symbol$_{-1}^{0}e$.

Properties of γ-ray

- Gamma rays are not charged particles like α and β particles, having no mass.

- γ-rays are electromagnetic radiation with high frequency.
- γ-rays move with the velocity of light.

Activity of Radioactive Substance

The activity of a sample of any radioactive nuclide is the rate at whichthe nuclei of the constituent atoms decay. If N is the number of nuclei present in the sample at a certain time, its activity $R$is given by

$$R = \frac{-dN}{dt}$$

The minus sign is used to make R a positive quantity since$dN/dt$is, of course, negative.

Unit of Radioactivity

In radioactivity,the intensity is determined in terms of the rate of disintegration or number of particles emitted per second.

Curie (Ci) is a unit of radioactivity and

$$1\,Curie = 3.7 \times 10^{10}\,disintegrations/second$$

There is another unit of radioactivity, which is known as Rutherford (*Rd).*

$$1\,Rutherford = 1 \times 10^{6}\,disintegrations/second$$

The SI unit of radioactivity is Becquerel (Bq).

$$1\,Becquerel = 1\,disintegrations/second$$

Thus, Curie (Ci) is defined as that quantity of radioactive substance that gives $3.7 \times 10^{10}\,disintegrations/second$. Rutherford (*Rd)* is defined as that quantity of radioactive substance that gives$1 \times 10^{6}\,disintegrations/second$.Becquerel ($Bq$) is defined as that quantity of radioactive substance that gives$1\,disintegrations/second$.

Types of Radioactivity

There are two types of radioactivity:

(i) Natural radioactivity
(ii) Artificial or induced radioactivity

(i) Natural radioactivity:Natural radioactivity is that *which is exhibited by elements as found in nature*. When *unstable nuclei decompose in nature the process is referred to as natural radioactivity*. It is always found in heavier elements in the periodic table.

(ii) Artificial or induced radioactivity:Artificial transmutation of elements has made it possible to produce radioactivity in many other elements much lighter than those occur in nature. Such type radioactivity is known as *artificialor induced radioactivity*.

Radioactive transformation/Radioactive decay

There are three types of natural radioactivity. These are:

(a) Alpha (α) decay
(b) Beta (β) decay
(c) Gamma (γ) decay.

Beta (β) decay can be classified into three types. These are:

(i) Electron emission
(ii) Positron emission
(iii) Electron capture.

These are described below:

(a) ***Alpha (α) decay:*** When an atom of a substance emits an α-particle its mass number is decreased by four units and its atomic number is decreased by two units. As there is decrease in the atomic number the atom is transformed into a new atom of a new element whose atomic number is less by two units than the atomic number of the original atom.*i.e.*

$$ {}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He $$

Where, X = Original atom, Y = New atom, A = Mass number, and Z = Atomic number.

For example, Uranium with Z = 92 and A = 238 emits α-particles. Therefore, the new substance formed is Thorium with Z = 90 and$A$ = 234.

$$ {}_{92}^{238}U \rightarrow {}_{90}^{234}Th + {}_{2}^{4}He $$

Thus, it is evident that the emission of α-particle by the nuclei of the atom of an element transforms it into the atoms of the lower element.

(b) ***Beta (β) decay:***The spontaneous emission of electron (e^-), positron (e^+) from the nucleus of a radioactive substance is called beta decay. Again the capture of electron from the atomic shell by the nucleus is called beta decay.

There are three modes of beta decay. These are:

(i) ***Electron emission:***When an atom of a substance emits electron its mass numberremains the same and its atomic number is increased by one unit. As there is increase in the atomic number the atom is transformed into a new atom of a new element whose atomic number is more than one unit of the atomic number of the original atom. *i.e.*

$$ {}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e $$

For example, Carbon with A = 14 and Z = 6 emits β-particles. Therefore, the new substance formed is Nitrogen with A = 14 and Z = 7.

$$ {}_{6}^{14}C \rightarrow {}_{7}^{14}N + {}_{-1}^{0}e $$

Thus, it is evident that the emission of electronby the nuclei of the atom of an element transforms it into the atoms of the higher element.

(ii) ***Positron emission(e^{+},β^{+}):*** Emission of positron (anti electron/antimatter) by proton decreasesatomic number by one unit and mass number remains the same. As there is decrease in the atomic number the atom is transformed into a new atom of a new element whose atomic number is less than one unit of the atomic number of the original atom. *i.e.*

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}e$$

For example, Copper with Z = 29 and$A$ = 64 emits positron. Therefore, the new substance formed is Nickel with A = 64 and Z =90.

$$^{64}_{29}Cu \rightarrow {}^{64}_{28}¿ + {}^{0}_{1}e$$

Thus, it is evident that the emission of positron by proton of the atom of an element changes its proton to a neutron.

(iii) ***Electron capture:***Capture of electron by the proton of the atom decreasesatomic number by one unit and mass number remains the same. As there is decrease in the atomic number the atom is transformed into a new atom of a new element whose atomic number is less than one unit of the atomic number of the original atom. *i.e.*

$$^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}Y$$

For example, Copper with Z = 29 and$A$ = 64 capture electron. Therefore, the new substance formed is Nickel with Z = 90 and$A$ = 234.

$$^{64}_{29}Cu + {}^{0}_{-1}e \rightarrow {}^{64}_{28}¿$$

Thus, it is evident that the capture of electron by the proton of the atom of an element changes its proton to a neutron.

(c) ***Gamma (γ) decay:***The emission of gamma radiation results from an energy change within the atomic nucleus. Gamma emission changes neither the atomic number nor the mass number. *i.e.*

$$^{A}_{Z}X^{¿} \rightarrow {}^{A}_{Z}X + \gamma$$

Where, X^* = Original unstable atom, X = Stable atom.

For example, Strontium (*excited state*) with Z = 87 and$A$ = 38 emits energy in the form gamma radiation and unstable nucleus becomes stable.

$$^{87}_{38}Sr^{¿} \rightarrow {}^{87}_{38}Sr + \gamma$$

<u>***Rutherford and Soddy theory of radioactive decay***</u>

In 1902 E. Rutherford and F. Soddy came up with the original formula of radioactive decay as, *"Rate of decay of radioactive atoms is proportional to the number of atoms available for decay at that time"*.

Suppose at the beginning of disintegration, *i.e.* at $t=0$, the number of radioactive atoms present in a sample is N_0. As time passes, the number of the original atoms decreases due to continuous disintegration. At any time t, let the number of radioactive atom available is N. Let the number of atoms dN disintegrates in time dt.

Therefore according to this law we can write

$$\frac{-dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N \quad (1)$$

Where λ is a constant called disintegration constant or decay constant and depends only on the nature of radioactive substance. Since the number of radioactive atoms decreases with time the rate of disintegration has been integrated by negative sign.

$\left|\frac{dN}{dt}\right| = \lambda N$ defines the activity of radioactive substance.

From this we can write,

$$\lambda = \frac{\left|\frac{dN}{dt}\right|}{N}$$

Thus, *the decay constant is the ratio of radioactive decay per second to the total number of atom.*

Equation (1) can be written as

$$\frac{dN}{N} = -\lambda dt$$

Integrating both sides of the above equation we get,

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

Where, $N = N_0 \, at \, t = 0 \wedge N = N \, at \, t = t$. Thus we can write

$$[\ln N]_{N_0}^{N} = -\lambda t$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln\frac{N}{N_0}=-\lambda t$$

$$\frac{N}{N_0}=e^{-\lambda t}$$

$$\therefore N=N_0 e^{-\lambda t}(2)$$

This equation is called the radioactive decay equation.This equation shows that the number of radioactive atoms decay exponentially with time. This is shown in fig. 1.

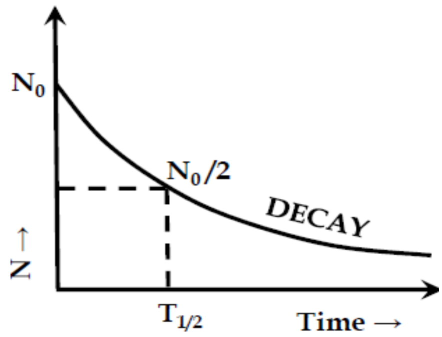

Fig. 1.

Half-life

The half-life of a radioactive substance is defined as the time required for one-half of the radioactive substance to disintegrate.

The half-life period is different for different substances and depends on radioactive constant of the substance.

We know from the equation of radioactive decay

$$N=N_0 e^{-\lambda t}$$

If $t=T_{1/2}$ then$N=\frac{N_0}{2}$, thus we can write

$$N=\frac{N_0}{2}$$

$$\frac{N_0}{2}=N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2}=e^{-\lambda T_{1/2}}$$

$$e^{\lambda T_{1/2}}=2$$

$$\lambda T_{1/2}=\ln 2$$

$$T_{1/2}=\frac{\ln 2}{\lambda}$$

$$\therefore T_{1/2} = \frac{0.693}{\lambda}$$

This is the expression for half-life.

Average life/mean life

The *average life or mean life* is defined as the ratio of the total life time of all the radioactive atoms to the total numbers of such atoms in it. It is denoted by τ.

$$\tau = \frac{Total\,life\,of\,all\,atoms}{Total\,number\,of\,atoms}$$

Suppose,

The number of atoms at the beginning is N_0,
At time t, the number of atoms is N,
and dN atoms disintegrate between t and $t+dt$.

Hence $(-dN)$ atoms have a life of t seconds.

$$Total\,life\,of\,(-dN)\,atoms = -dN\,.\,t$$

Since all the atoms disintegrate in time from zero to infinity, the sum of the life periods of all the atoms

¿ $$\int_0^{\infty} -dN\,.\,t$$

Thus the average life

$$\tau = \frac{1}{N_0}\int_0^{\infty} -dN\,.\,t \quad (1)$$

The radioactive decay law is

$$N = N_0 e^{-\lambda t}$$

We can write

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$$

$$\therefore dN = -\lambda N_0 e^{-\lambda t}\,dt$$

Substituting the value of dN in equation (1) we get

$$\tau = \frac{1}{N_0}\int_0^{\infty} \left(\lambda N_0 e^{-\lambda t}\,dt\right) t$$

$$\tau = \lambda \int_0^{\infty} t\,e^{-\lambda t}\,dt$$

Integrating by parts we get

$$\tau = \lambda \left[t\int_0^{\infty} e^{-\lambda t}\,dt - \int_0^{\infty} \left\{ \frac{d}{dt} t \int_0^{\infty} e^{-\lambda t}\,dt \right\} dt \right]_0^{\infty}$$

$$= \lambda \left[t\frac{e^{-\lambda t}}{-\lambda} - \int_0^{\infty} \frac{e^{-\lambda t}}{-\lambda}\,dt \right]_0^{\infty}$$

$$=\lambda\left[t\frac{e^{-\lambda t}}{-\lambda}-\frac{e^{-\lambda t}}{\lambda^2}\right]_0^\infty$$

$$=\lambda\left[\frac{\left(-\lambda te^{-\lambda t}\right)-\left(e^{-\lambda t}\right)}{\lambda^2}\right]_0^\infty$$

$$=-\frac{1}{\lambda}(0-1)$$

$$\boldsymbol{\tau=\frac{1}{\lambda}}$$

This is the expression for average life.

<u>Relation between half-life and average life/mean life</u>

We know, half-life

$$T_{1/2}=\frac{0.693}{\lambda}(1)$$

Mean life

$$\tau=\frac{1}{\lambda}(2)$$

From the above two equations we can write

$$T_{1/2}=0.693\,\tau$$

$$Half-life=0.693\times Mean\,life$$

$$Half-life=69.3\%\,of\ Mean\,life$$

These are the Relation between half-life and mean life.

Mathematical Problems

Problem-1: *The disintegration constant λ of a radioactive element is 0.00231 per day. Calculate its half-life and average life.*

Solution: We know, Half-life

$$T_{1/2}=\frac{0.693}{\lambda}$$

$$T_{1/2}=\frac{0.693}{0.00231}$$

$$\therefore T_{1/2}=300\,days\,(Ans)$$

Mean-life

$$\tau=\frac{1}{\lambda}=\frac{1}{0.0231}=432.9\,days\,(Ans)$$

Here,

$\lambda=0.00231\ per\ day$

$T_{1/2}=?$

$\tau=?$

Problem-2: *How long does it take for 60.0 percent of a sample of radon to decay?*

Solution: We know, Half-life

$$T_{1/2}=\frac{0.693}{\lambda}$$

$$\lambda=\frac{0.693}{T_{1/2}}$$

$$\lambda=\frac{0.693}{3.82}$$

$$\therefore \lambda=0.1814\ per\ day$$

We know,

$$N=N_0 e^{-\lambda t}$$

$$\frac{N}{N_0}=e^{-\lambda t}$$

$$\frac{N_0}{N}=e^{\lambda t}$$

Here,

$$N=\left(1-\frac{60}{100}\right)N_0$$

$$N=(1-0.6)N_0$$

$$N=0.4\,N_0$$

$$\frac{N_0}{N}=\frac{1}{0.4}=2.5$$

$T_{1/2}=3.82\,days$

$t=?$

$$\ln\frac{N_0}{N}=\lambda t$$

$$t=\frac{1}{\lambda}\ln\frac{N_0}{N}$$

$$t=\frac{1}{0.1814}\ln 2.5$$

$$\therefore t=5.05\, days\,(Ans)$$

Problem-3: *How long will it take for a sample radium D to decrease to 10%, if its half-life is 22 years?*

Solution: We know, Half-life

$$T_{1/2}=\frac{0.693}{\lambda}$$

$$\lambda=\frac{0.693}{T_{1/2}}$$

$$\lambda=\frac{0.693}{22}$$

$$\therefore \lambda=0.0315\ per\ year$$

Here,

$$N=\frac{10}{100}N_0$$

$$\frac{N_0}{N}=10$$

$$T_{1/2}=22\ years$$

$$t=?$$

We know,

$$N=N_0 e^{-\lambda t}$$

$$\frac{N}{N_0}=e^{-\lambda t}$$

$$\frac{N_0}{N}=e^{\lambda t}$$

$$\ln\frac{N_0}{N}=\lambda t$$

$$t=\frac{1}{\lambda}\ln\frac{N_0}{N}$$

$$t=\frac{1}{0.0315}\ln 10$$

$$t=73.11\ years\,(Ans)$$

Problem-4: *Calculate the time in which the activity of a sample of thorium reduces to 90% of its original value. Assume the half-life of thorium to be* 1.4×10^{10} *years.*

Solution: We know, Half-life

$$T_{1/2}=\frac{0.693}{\lambda}$$

$$\lambda=\frac{0.693}{T_{1/2}}$$

$$\lambda=\frac{0.693}{1.4\times10^{10}}$$

$$\therefore \lambda=4.95\times10^{-11}\ per\ year$$

We know,

$$N=N_0e^{-\lambda t}$$

$$\frac{N}{N_0}=e^{-\lambda t}$$

$$\frac{N_0}{N}=e^{\lambda t}$$

$$\ln\frac{N_0}{N}=\lambda t$$

$$t=\frac{1}{\lambda}\ln\frac{N_0}{N}$$

$$t=\frac{1}{4.95\times10^{-11}}\ln\left(\frac{10}{9}\right)$$

$$\therefore t=2.10\times10^{9}\ years\,(Ans)$$

Here,

$$N=\frac{90}{100}N_0$$

$$\frac{N_0}{N}=\frac{10}{9}$$

$$T_{1/2}=1.4\times10^{10}\ years$$

$$t=?$$

Problem-5: *The half-life of radon is 3.82 days. After how many days will only 5% of radon left over?*

Solution: We know, Half-life

$$T_{1/2}=\frac{0.693}{\lambda}$$

$$\lambda=\frac{0.693}{T_{1/2}}$$

Here,

$$N=\frac{5}{100}N_0$$

$$\frac{N_0}{N}=\frac{100}{5}=20$$

$$T_{1/2}=3.82\,days$$

$$t=?$$

$$\lambda = \frac{0.693}{3.82}$$

$$\therefore \lambda = 0.1814 \text{ per day}$$

We know,

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\frac{N_0}{N} = e^{\lambda t}$$

$$\ln \frac{N_0}{N} = \lambda t$$

$$t = \frac{1}{\lambda} \ln \frac{N_0}{N}$$

$$t = \frac{1}{0.1814} \ln 20$$

$$\therefore t = 16.51 \text{ days} (Ans)$$

Problem-6: *The disintegration constant of a radioactive element is 0.0693 per month. Calculate the required for 75% at the element to decay.*

Solution: We know,

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\frac{N_0}{N} = e^{\lambda t}$$

$$\ln \frac{N_0}{N} = \lambda t$$

$$t = \frac{1}{\lambda} \ln \frac{N_0}{N}$$

$$t = \frac{1}{0.0693} \ln 4$$

$$\therefore t = 20 \text{ months} (Ans)$$

Here,

As 75% element decays, 25% the element left behind

$$N = \frac{25}{100} N_0$$

$$\frac{N_0}{N} = \frac{100}{25} = 4$$

$$\lambda = 0.0693 \text{ per month}$$

$$t = ?$$

Problem-7: *Find the activity of 1.00 mg of radon, ^{222}Rn, whose atomic mass is 222 U*

Solution: We know, Half-life

$$T_{1/2}=\frac{0.693}{\lambda}$$

$$\lambda=\frac{0.693}{T_{1/2}}$$

$$\lambda=\frac{0.693}{3.30\times10^{5}}$$

$$\therefore \lambda=2.1\times10^{-6}\,sec^{-1}$$

Here,

$T_{1/2}=3.82\,days$

¿$(3.82\times24\times3600)\,sec$

¿$3.30\times10^{5}\,sec$

The number of N atoms in 1.00 mg of ^{222}Rn is

$$N=\frac{1\times10^{-3}\,gm}{222\,(gm/molecule)}\times6.023\times10^{23}\,(atom/molecule)$$

$$N=2.71\times10^{18}\,atoms$$

$$Activity=\lambda N=2.1\times10^{-6}\times2.71\times10^{18}$$

¿$5.69\times10^{12}\,decays/sec$

¿$5.69\times10^{12}\,Bq\,(Ans)$

Problem-8: *A radioactive substance has a half-life period of 30 days. calculate*

(i) The radioactive disintegration constant.

(ii) The average life

(iii) The time taken for 3/4 of the original number of atoms to disintegrate

(iv) The time for 1/8 of original number of atoms to remain unchanged.

Solution: (i) We know,

$$T_{1/2}=\frac{0.693}{\lambda}$$

$$\lambda=\frac{0.693}{T_{1/2}}$$

$$\lambda=\frac{0.693}{30}$$

$$\therefore \lambda=0.0231\,per\,day\,(Ans)$$

(ii) Average life

$$\tau=\frac{1}{\lambda}=\frac{1}{0.0231}=43.29\,days\,(Ans)$$

Here,

$T_{1/2}=30\,days$

For the 1st case

$$N=\left(1-\frac{3}{4}\right)N_0$$

$$N=\frac{1}{4}N_0$$

$$\frac{N_0}{N}=4$$

$$t_1=?$$

For the 2nd case

$$N=\frac{1}{8}N_0$$

$$\frac{N_0}{N}=8$$

$$t_2=?$$

(iii) We know,

$N=N_0e^{-\lambda t_1}$

$\frac{N_0}{N}=e^{\lambda t_1}$

$\ln\frac{N_0}{N}=\lambda t_1$

$t_1=\frac{1}{\lambda}\ln\frac{N_0}{N}$

$t_1=\frac{1}{0.0231}\ln 4$

$\therefore t_1=60\,days\,(Ans)$

(iv) We know,

$N=N_0e^{-\lambda t_2}$

$\frac{N_0}{N}=e^{\lambda t_2}$

$\ln\frac{N_0}{N}=\lambda t_2$

$t_2=\frac{1}{\lambda}\ln\frac{N_0}{N}$

$t_2=\frac{1}{0.0231}\ln 8$

$\therefore t_2=90\,da\;ys\,(Ans)$

Problem-9: *1 gm of radium is reduced by 2.1 mg in 5 years by alpha decay. Calculate the half-life period of radium.*

Solution: We know,

$N=N_0e^{-\lambda t}$

$\frac{N}{N_0}=e^{-\lambda t}$

$\frac{N_0}{N}=e^{\lambda t}$

$\ln\frac{N_0}{N}=\lambda t$

$\lambda=\frac{1}{t}\ln\frac{N_0}{N}$

Here,

$N_0=1\,gm$

$N=\left(1-2.1\times10^{-3}\right)gm$

$N=0.9979\,gm$

$\frac{N_0}{N}=\frac{1}{0.9979}$

$t=5\,years$

$T_{1/2}=?$

$$\lambda = \frac{1}{5}\ln\left(\frac{1}{0.9979}\right)$$

$$\therefore \lambda = 4.20 \times 10^{-4}\ per\ year$$

We know, Half-life

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$T_{1/2} = \frac{0.693}{4.20 \times 10^{-4}}$$

$$\therefore T_{1/2} = 1650\ year\ (Ans)$$

Problem-10: *A radioactive sample has its half-life equal to 60 days. Calculate the time required for 2/3 of the original number of atoms to disintegrate.*

Solution: We know, Half-life

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{T_{1/2}}$$

$$\lambda = \frac{0.693}{60}$$

$$\therefore \lambda = 0.01155\ per\ day$$

We know,

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\frac{N_0}{N} = e^{\lambda t}$$

$$\ln\frac{N_0}{N} = \lambda t$$

$$t = \frac{1}{\lambda}\ln\frac{N_0}{N}$$

$$t = \frac{1}{0.01155}\ln 3$$

$$\therefore t = 95.11\ days\ (Ans)$$

Here,

$$N = \left(1 - \frac{2}{3}\right)N_0$$

$$N = \frac{1}{3}N_0$$

$$\frac{N_0}{N} = 3$$

$$T_{1/2} = 60\ days$$

$$t = ?$$