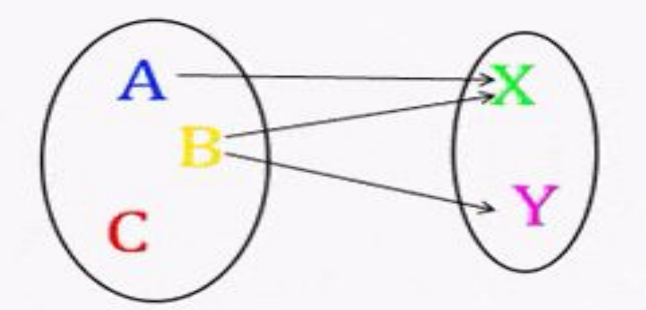
Discrete mathematics



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1.2 Propositional Equivalence

Introduction

What is Propositional Equivalence?

- Two logical expressions are said to be equivalent if they have the same truth value in all cases.

- Sometimes this fact helps in proving a mathematical result by replacing one expression with another equivalent expression, without changing the truth value of the original compound proposition.

- To prove two Compound Propositions logically equivalent we use the symbol " $\equiv$ "

Classification

Classification

Compound Proposition

- Tautology
- Contradiction
- Contingency

Tautology, Contradiction & Contingency (Example)

a) $p \vee \neg p$

b) $q \wedge \neg q$

c) $(p \rightarrow q)$

Tautology

Contingency

Solution :

p	q	!p	!q			
F	F	T	T	T	F	T
F	T	T	F	T	F	T
T	F	F	T	T	F	F
T	T	F	F	T	F	T

Contradiction

DEFINITIONS

- Tautology: A compound proposition that is always true, no matter what the truth values of the propositions that occur in it, is called a “Tautology”.
- Contradiction: A compound proposition that is always false, no matter what the truth values of the propositions that occur in it, is called a “Contradiction”.
- Contingency: A compound proposition that is neither tautology nor contradiction is called “Contingency”.

Tautology using Truth Table

Example-1

- Show that, $(p \to q) \land (q \to r) \to (p \to r)$ is a tautology using the truth table.

Solution :

p	q	r					
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	F	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

This compound proposition is a tautology

Example-2

Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow p$ is a tautology.

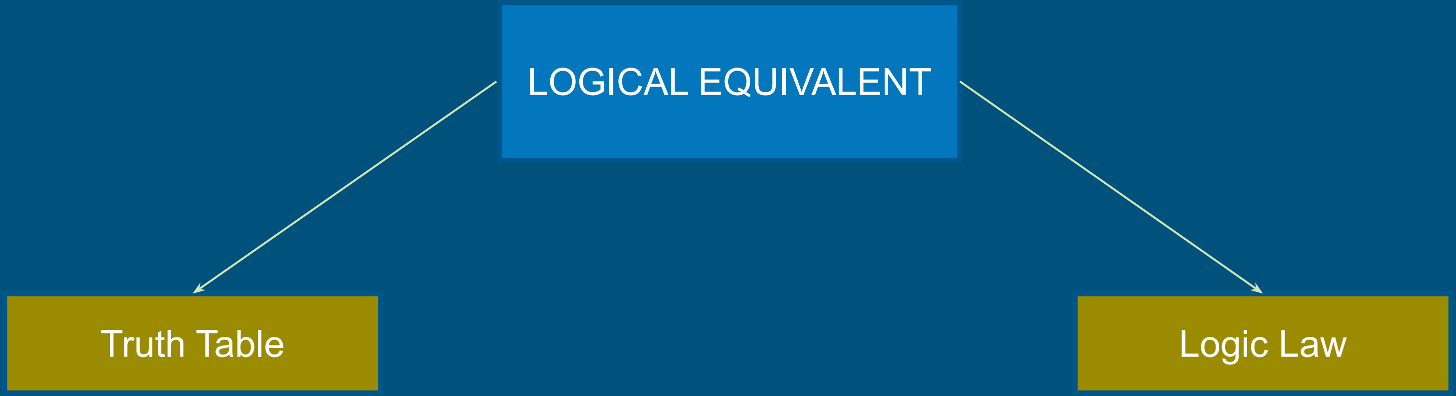
Solution :

p	q	r					
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	F	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

This compound proposition is a tautology

Logical Equivalence

Ways to prove logical equivalence



Example-1

Show that, $p \to q$ and $(\neg p \vee q)$ are logically equivalent.

Solution :

p	q			
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

This two compound proposition are logically equivalent as they have the same truth values.

$$\therefore (p \to q) \equiv (\neg p \vee q)$$

Example-2

Show that, $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

Solution :

p	q	r					
F	F	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	T	T	T	T	T	F	T
T	F	F	F	T	T	F	T
T	F	T	T	T	T	F	T
T	T	F	F	F	F	T	F
T	T	T	T	T	T	T	T

$$\therefore (p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

Exercises

I. Show that, $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are logically equivalent.

II. Show that, $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are logically equivalent.

Logic Law

DEFINITIONS

- <u>**Logic law:**</u>

Logic laws are such formula which are used for compound and complex propositions to simply them

Logic Laws

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

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Logic Laws(DE Morgan’s Law)

DE Morgan's Law

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De Morgan's Law:

$\neg(p \land q) \equiv \neg p \lor \neg q$

$\neg(p \lor q) \equiv \neg p \land \neg q$

F	F	T	T	F	T	T
F	T	T	F	F	T	T
T	F	F	T	F	T	T
T	T	F	F	T	F	F

F	F	T	T	F	T	T
F	T	T	F	T	F	T
T	F	F	T	T	F	F
T	T	F	F	T	F	F

Proving Tautology using Logic Law

Examples

Example-1

(a) $(p \wedge q) \rightarrow p$

Solution:

$(p \wedge q) \rightarrow p$

$\equiv \neg(p \wedge q) \vee p$ [Conditional Law]

$\equiv \neg p \vee \neg q \vee p$ [De Morgan's Law]

$\equiv (\neg p \vee p) \vee \neg q$ [Associative law]

$\equiv T \vee \neg q$ [Negation Law]

$\equiv T$ [Domination Law] [Proved]

Example-2

(b) $(p \wedge q) \rightarrow (p \vee q)$

Solution:

$(p \wedge q) \rightarrow (p \vee q)$

$\equiv \neg(p \wedge q) \vee (p \vee q)$ [Conditional law]

$\equiv \neg p \vee \neg q \vee (p \vee q)$ [De Morgan's law]

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$ [Associative law]

$\equiv T \vee T$ [Negation law]

$\equiv T$ [Proved]

Proving Logical Equivalence using Logic Law

Examples

Example-1

(a) $\neg(p \rightarrow q) \equiv p \wedge \neg q$

Solution:

L.H.S. = $\neg(p \rightarrow q)$

$\equiv \neg(\neg p \vee q)$ [Conditional Law]

$\equiv \neg(\neg p) \wedge \neg q$ [De Morgan's Law]

$\equiv p \wedge \neg q$ [Double Negation Law]

= R.H.S

Example-2

(b) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Solution:

L.H.S.= $\neg(p \vee (\neg p \wedge q))$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$ [De Morgan's law]

$\equiv \neg p \wedge \neg(\neg p) \vee \neg q)$ [De Morgans law]

$\equiv \neg p \wedge (p \vee \neg q)$ [Double Negation Law]

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ [distributive law]

$\equiv F \vee (\neg p \wedge \neg q)$ [negation law]

Examples

Example-3

Simply the expression: $\neg(R \wedge S \wedge T) \wedge \neg(R \vee S \vee T)$

DE MORGANS LAW:

$\therefore \neg(R \wedge S \wedge T) \equiv \neg R \vee \neg S \vee \neg T$

$\therefore \neg(R \vee S \vee T) \equiv \neg R \wedge \neg S \wedge \neg T$

Solution:

$\neg(R \wedge S \wedge T) \wedge \neg(R \vee S \vee T)$

$\equiv (\neg R \vee \neg S \vee \neg T) \wedge \neg(R \vee S \vee T)$ [∴DE Morgan's Law]

$\equiv(\bar{R} + \bar{S} + \bar{T}) \cdot (\bar{R} \cdot \bar{S} \cdot \bar{T})$ [changing into algebraic form]

$\equiv \bar{R} \cdot \bar{R} \cdot \bar{S} \cdot \bar{T} + \bar{R} \cdot \bar{S} \cdot \bar{T} \cdot \bar{S} + \bar{R} \cdot \bar{S} \cdot \bar{T} \cdot \bar{T}$

$\equiv \bar{R} \cdot \bar{S} \cdot \bar{T} + \bar{R} \cdot \bar{S} \cdot \bar{T} + \bar{R} \cdot \bar{S} \cdot \bar{T}$ [∴Idempotent Law]

$\equiv \bar{R} \cdot \bar{S} \cdot \bar{T}$

$\equiv \neg R \wedge \neg S \wedge \neg T$

Negation Using De Morgan's Law

Exercises

- Use **De Morgan's Law** to find the negation of each of the following statements:

a) **John is rich and happy.**

Solution: **John is rich and happy.**

p ∧ q

$$\Rightarrow \neg(p \wedge q) = \neg p \vee \neg q$$

$\Rightarrow$ *John is not rich or John is not happy.*

b) Carlos will bicycle or run tomorrow.

Solution: Carlos will bicycle or run tomorrow.

p ∨ q

$$\Rightarrow \neg(p \vee q) = \neg p \wedge \neg q$$

$\Rightarrow$ *Carlos will not bicycle and not run tomorrow.*

THANK YOU!!!!