International Islamic University Chittagong

Department of Computer Science and Engineering

B. Sc. in CSE Midterm Examination, Autumn 2023

Course Code: CSE 1223 Course Title: Discrete Mathematics

Total marks: ***30*** *Time:* ***1 hour 30 mins***

Figures in the right-hand margin indicate full marks.]

1.

a) Define Cardinality of set. Explain with example. **1 CLO1**

Ans: Cardinality of a set is the number of elements or members in the set. It represents the size or count of the set.
Example: If we have a set A = {1, 2, 3, 4, 5}, then the cardinality of set A is denoted as |A|, which equals 5.

b) Use set builder notation to give a description of each of the following set: i){2,4,6,8,10} ii){a,e,i,o,u} iii){1,4,9,16} iv){2,3,5,7} **2 CLO2**

Ans: i) {2, 4, 6, 8, 10} can be described as: {x | x is an even number between 1 and 10}
ii) {a, e, i, o, u} can be described as: {x | x is a vowel in the English alphabet}
iii) {1, 4, 9, 16} can be described as: {x | x is a perfect square between 1 and 16}
iv) {2, 3, 5, 7} can be described as: {x | x is a prime number between 1 and 10}

c) Prove that $(A \cap B)' = A' \cup B'$ using computer representation of set i.e. bit string. Consider, U={0,1,2,3,4,5,6,7,8,9}, A={2,5,9} and B={3,6,8,9} **3 CLO2**

Ans: Given U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 5, 9}, and B = {3, 6, 8, 9}.
Using bit strings, you can represent the sets as follows:
A = 0010010001
B = 0001001011
A ∩ B = 0000000001
A′ = 1101101110
B′ = 1110110100
(A ∩ B)′ = 1111111110
A′ ∪ B′ = = 1101101110 ∪ 1110110100 = 1111111110
(A ∩ B)′ = A′ ∪ B′, so the statement is proven.

or) Let U={ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8, 10} and C = {1, 2, 3, 4, 5} . What bit strings represent the following:
i) B⊕C ii) A-B

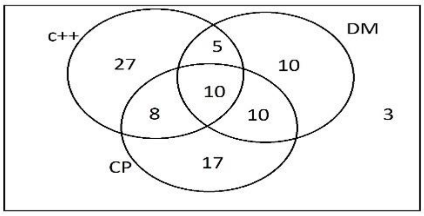
Ans: Given sets:
U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {1, 3, 5, 7, 9}
B = {2, 4, 6, 8, 10}
C = {1, 2, 3, 4, 5}
Let's represent each set using a bit string where each position in the bit string corresponds to an element in U, and the value at that position is 1 if the element is in the set and 0 if it's not.
Bit String Representation:
For sets A, B, and C:
A = {1, 3, 5, 7, 9} => Bit string for A: 1010101010
B = {2, 4, 6, 8, 10} => Bit string for B: 0101010101
C = {1, 2, 3, 4, 5} => Bit string for C: 1111100000

Now, let's perform the requested operations:
i) B⊕C (Symmetric Difference):
B⊕C is the set of elements that are in either B or C but not in both.
Bit string for B⊕C: (Bit string for B) XOR (Bit string for C)
Bit string for B⊕C: : 0101010101 XOR 1111100000 = 1010110101
ii) A-B (Set Difference):
A-B is the set of elements that are in A but not in B.
A-B= {1, 3, 5, 7, 9}-{2, 4, 6, 8, 10}
={{1, 3, 5, 7, 9}
Bit string for A-B: 1010101010
So, the bit string representations for the requested set operations are:
i) Bit string for B⊕C: 1010110101
ii) Bit string for A-B: 1010101010

d) In a survey there are 90 students, it was found that-50 had taken C++, 35 had taken Discrete mathematics and 45 had taken Competitive programming. 15 had taken C++ and Discrete mathematics, 20 had taken Discrete mathematics and Competitive programming, 18 had taken C++ and Competitive programming and 10 had taken all the three subjects. Find the number of students that had **4 CLO2**

i) read exactly two subjects
ii) read none of the subjects
iii) read exactly one subject

Ans:



Given:
Total number of students surveyed (universal set), n(S) = 90
Number of students who took C++, n(C++)= 50
Number of students who took Discrete mathematics, n(DM) = 35
Number of students who took Competitive programming, n(CP) = 45
Number of students who took C++ and Discrete mathematics,
n(C++ ∩ DM) = 15
Number of students who took Discrete mathematics and Competitive programming, n(DM ∩ CP) = 20
Number of students who took C++ and Competitive programming,
n(C++ ∩ CP) = 18
Number of students who took all three subjects, n(C++ ∩ DM ∩ CP) = 10
We can use the principle of inclusion-exclusion to find the number of students who fall into each category:

i) Number of students who read exactly two subjects:

Students who read exactly two subjects
= (n(C++ ∩ DM) + n(DM ∩ CP)+ n(C++ ∩ CP))-3* n(C++ ∩ DM ∩ CP)
=(15+20+18)-3*10
=53-30
=23

ii) Number of students who read none of the subjects:
Students who took at least one subject, n(C++ ∪ DM ∪ CP)
= n(C++) + n(DM) + n(CP) - n(C++ ∩ DM) - n(DM ∩ CP) - n(C++ ∩ CP)+ n(C++ ∩ DM ∩ CP)
= 50+35+45-15-20-18+10
=87
Students who read none of the subjects
= Total students - Students who took at least one subject
=n(S)- n(C++ ∪ DM ∪ CP)
= 90 – 87
= 3

iii) Number of students who read exactly one subject:
Students who took exactly one subject
=n (C++) + n(DM) + n(CP) – 2* n(C++ ∩ DM)- 2* n(DM ∩ CP)
– 2* n(C++ ∩ CP)+ 3* n(C++ ∩ DM ∩ CP)
= 50+35+45-2*15-2*20-2*18+3*10
= 130-30-40-36+30
= 160-106
=54

So, the answers are:
i) 23 students read exactly two subjects.
ii) 3 students read none of the subjects.
iii) 54 students read exactly one subject.

2.

a) Write down the importance of quantification with example. **2 CLO2**

Ans: Quantification is essential in mathematics and logic as it allows us to make general statements about groups of objects. For example, "For all integers x, x is greater than 0" is a universal quantification statement that asserts something about all integers.

b) Consider the following propositions: **2 CLO2**

- p: "The weather is sunny."
- q: "I will go for a walk."
- r: "I will take an umbrella."

Translate the following propositions into English statements using the provided propositions:
a) p ∧ q b) ¬r c) p → r d) q ↔ r

Ans: a) p ∧ q: "The weather is sunny, and I will go for a walk."
b) ¬r: "I will not take an umbrella."
c) p → r: "If the weather is sunny, then I will take an umbrella."
d) q ↔ r: "I will go for a walk if and only if I will take an umbrella."

c) Consider the following predicates: **3 CLO2**

L(x): "Student x loves mathematics.", S(x): "Student x studies regularly." , H(x): "Student x has high grades."
Given the domain of all students in your class, translate the following quantifier expressions into English statements using the provided predicates: a) (∀x) L(x) → S(x)
b) (∃x) H(x) ∧ ¬S(x)
c) (∃x) (∀y) (x ≠ y → L(x) → L(y))

Ans: a) (∀x) L(x) → S(x): "For all students, if a student loves mathematics, then they study regularly."
b) (∃x) H(x) ∧ ¬S(x): "There exists a student who has high grades and does not study regularly."

c) (∃x) (∀y) (x ≠ y → L(x) → L(y)): "There exists a student such that for any other student (not equal to the first one), if the first student loves mathematics, then the other student loves mathematics too."

or) Consider the following predicates:
A(x): "Animal x can fly.", B(x): "Animal x has feathers.", C(x): "Animalx is a carnivore."
Given the domain of all animals, translate the following English statements into quantifier expressions using the provided predicates:
a) "All animals that can fly have feathers."
b) "There exists an animal that is a carnivore and can fly."
c) "No animals that have feathers are carnivores."

Ans: a) "All animals that can fly have feathers."
Translation: (∀x) [A(x) → B(x)]
b) "There exists an animal that is a carnivore and can fly."
Translation: (∃x) [C(x) ∧ A(x)]
c) "No animals that have feathers are carnivores."
Translation: (∀x) [B(x) → ¬C(x)]

d) Show ¬(p → q) is logically equivalent to p ∧ ¬q. **3** **CLO2**

Ans:

p	**q**	**¬(p → q)**	**p � ¬q**
T	T	F	F
T	F	T	T
F	T	F	F
F	F	F	F

For all possible combinations of truth values for p and q, ¬(p → q) and p ∧ ¬q have the same truth values, which demonstrates their logical equivalence.

3.
a) Write down the difference between one-to-one and onto function with proper example. **2** **CLO2**

Ans: A one-to-one function (injective) is a function in which no two different elements in the domain map to the same element in the codomain.
Example: f(x) = 2x is one-to-one.
An onto function (surjective) is a function in which every element in the codomain is mapped to by at least one element in the domain. Example: f(x) = x^2 is not onto because it misses negative numbers in the codomain.

or) Given sets A={1,2,3,4} and B={2,4,6,8}, determine the zero-one matrices that represent the following relations $R1$ to $R4$:
1. $R1$={(1,2),(2,4),(3,6)} 2. $R2$={(1,1),(2,4),(3,9)}
3. $R3$={(2,3),(4,6),(3,9)} 4. $R4$={(1,2),(2,4),(2,6)}

Ans: *** Consider [ ] on each matrix
i) R1={(1,2),(2,4),(3,6)} is represented by:
1000
0100
0010
0000

ii) R2={(1,1),(2,4),(3,9)} is represented by:
0000
0100
0000
0000

Here,
(1,1) can't be mapped as B={2,4,6,8}
(3,9) can't me mapped as B={2,4,6,8}

iii) R3={(2,3),(4,6),(3,9)} is represented by:
0000
0000
0000
0000

Here, (3,9) can't me mapped as B={2,4,6,8}

iv) R4={(1,2),(2,4),(2,6)} is represented by:
1000
0110
0000
0000

b) Suppose that g is a function from A to B and f is a function from B to C. Prove each of these statements. **2 CLO2**

a) If f ∘ g is onto, then f must also be onto.
b) If f ∘ g is one-to-one, then g must also be one-to-one.

Ans: a) If f ∘ g is onto, then f must also be onto.
Proof: If f ∘ g is onto, it means that for every element in the codomain of f ∘ g, there exists an element in the domain of f ∘ g that maps to it.
Let y be an arbitrary element in the codomain of f.
Since f ∘ g is onto, there exists an element x in the domain of f ∘ g such that (f ∘ g)(x) = y.
But (f ∘ g)(x) = f(g(x)), so we have f(g(x)) = y.
This means that for every y in the codomain of f, there exists an x in the domain of f such that f(x) = y.
Therefore, f is also onto.

OR,

Proof:
Let's consider two functions:
Function g: {1, 2, 3} → {a, b, c} defined as g(1) = a, g(2) = b, g(3) = c.
Function f: {a, b, c} → {X, Y, Z} defined as f(a) = X, f(b) = Y, f(c) = Z.
Now, let's compute f ∘ g:

$(f \circ g)(1) = f(g(1)) = f(a) = X$
$(f \circ g)(2) = f(g(2)) = f(b) = Y$
$(f \circ g)(3) = f(g(3)) = f(c) = Z$
$f \circ g$ is onto because for every element in the codomain of $f \circ g$ ({X, Y, Z}), there exists an element in the domain of $f \circ g$ ({1, 2, 3}) that maps to it.
Now, let's prove that f is also onto:
For every element in the codomain of f ({X, Y, Z}), there exists an element in the domain of f ({a, b, c}) that maps to it.
$f(a) = X$
$f(b) = Y$
$f(c) = Z$
Therefore, f is onto as well.

b) If $f \circ g$ is one-to-one, then g must also be one-to-one.
Proof: If $f \circ g$ is one-to-one, it means that for every pair of distinct elements in the domain of $f \circ g$, their images under $f \circ g$ are also distinct.
Let x1 and x2 be two distinct elements in the domain of g.
Since $f \circ g$ is one-to-one, $(f \circ g)(x1) \neq (f \circ g)(x2)$.
But $(f \circ g)(x1) = f(g(x1))$ and $(f \circ g)(x2) = f(g(x2))$.
Therefore, $f(g(x1)) \neq f(g(x2))$, which implies that $g(x1) \neq g(x2)$.
So, for every pair of distinct elements x1 and x2 in the domain of g, their images under g are also distinct.
Hence, g is one-to-one.

OR,

Proof:
Let's consider two functions:
Function g: {1, 2, 3} → {a, b, c} defined as g(1) = a, g(2) = b, g(3) = c.
Function f: {a, b, c} → {X, Y, Z} defined as f(a) = X, f(b) = Y, f(c) = Z.
Now, let's compute $f \circ g$:
$(f \circ g)(1) = f(g(1)) = f(a) = X$
$(f \circ g)(2) = f(g(2)) = f(b) = Y$
$(f \circ g)(3) = f(g(3)) = f(c) = Z$
$f \circ g$ is one-to-one because for every pair of distinct elements in the domain of $f \circ g$ ({1, 2, 3}), their images under $f \circ g$ are also distinct.
Now, let's prove that g is also one-to-one:
For every pair of distinct elements in the domain of g ({1, 2, 3}), their images under g are also distinct.
$g(1) = a$
$g(2) = b$
$g(3) = c$
Therefore, g is one-to-one as well.

or) Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

a) the function that assigns to each nonnegative integer its last digit

b) the function that assigns the next largest integer to a positive integer

Ans: i) The function that assigns to each nonnegative integer its last digit.

Domain: The domain is the set of nonnegative integers, which is {0, 1, 2, 3, ...}.

Range: The range is the set of possible last digits, which is {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

ii) The function that assigns the next largest integer to a positive integer.

Domain: The domain is the set of positive integers, which is {1, 2, 3, 4, ...}.

Range: The range is the set of positive integers that are one greater than the input, which is {2, 3, 4, 5, ...}.

c) A person deposits \$1000 in an account that yields 9% interest compounded annually. Set up a recurrence relation for the amount in the account at the end of n years. 2 CLO2

Ans: To set up a recurrence relation for the amount in the account at the end of n years with 9% interest compounded annually, we can use the formula for compound interest:

A = P(1 + r/n)^(nt)

In this case, we have:

P = \$1000 (the principal amount)

r = 0.09 (9% annual interest rate in decimal form)

n = 1 (compounded annually)

t = n years, so t = n

We want to find A(n), which is the amount at the end of n years.

A(n) = P(1 + r/n)^(nt)

Substituting the given values:

A(n) = 1000(1 + 0.09/1)^(1*n)

Simplify this expression:

A(n) = 1000(1 + 0.09)^n

Now, the recurrence relation can be expressed as:

A(n) = 1000(1.09)^n

This recurrence relation represents the amount in the account at the end of n years with 9% interest compounded annually. We can use this relation to calculate the amount for any given n.

d) For each of these relations on the set {1, 2, 3, 4), decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive. 4 **CLO2**

i) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

ii) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4))

iii){(2, 4), (4, 2)

iv){(1, 1), (2, 2), (3, 3), (4, 4)}

Ans: i) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

- Not Reflexive: This relation is not reflexive because every element in the set {1, 2, 3, 4} needs to appear as (a, a) in the relation but (1,1) is absent.
- Not Symmetric: This relation is not symmetric because, for example, (2, 3) is present, but (3, 2) is not.
- Not Antisymmetric: This relation is also not antisymmetric because it contains both (2, 3) and (3, 2), and they are not equal.
- Transitive: This relation is transitive because whenever you have (a, b) and (b, c) in the relation, it implies (a, c). For example, (2, 3) and (3, 4) are present, so (2, 4) is also present.

ii) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}

- Reflexive: This relation is reflexive because every element in the set {1, 2, 3, 4} appears as (a, a) in the relation.
- Symmetric: This relation is symmetric because for every (a, b) present in the relation, (b, a) is also present.
- Not Antisymmetric: This relation is not antisymmetric because it contains both (1, 2) and (2, 1), and they are not equal.
- Transitive: This relation is transitive because, by its nature, it satisfies the transitive property. Whenever you have (a, b) and (b, c), it implies (a, c).

iii) {(2, 4), (4, 2)}

- Not Reflexive: This relation is not reflexive because it lacks pairs of the form (a, a).
- Symmetric: This relation is symmetric because for every (a, b) present in the relation, (b, a) is also present.
- Not Antisymmetric: This relation is not antisymmetric because it contains both (2, 4) and (4, 2), and they are not equal.
- Transitive: This relation is vacuously transitive because it has only two elements, and there are no combinations (a, b) and (b, c) to check for transitivity.

iv) {(1, 1), (2, 2), (3, 3), (4, 4)}

- Reflexive: This relation is reflexive because every element in the set {1, 2, 3, 4} appears as (a, a) in the relation.
- Symmetric: This relation is symmetric because for every (a, b) present in the relation, (b, a) is also present.
- Antisymmetric: This relation is antisymmetric because it contains only pairs of the form (a, a), and there are no (a, b) and (b, a) pairs where a ≠ b.
- Transitive: This relation is transitive because, by its nature, it satisfies the transitive property. Whenever you have (a, b) and (b, c), it implies (a, c).