Course Code: MATH-1207

Course Title: Mathematics -II

Formulas

Change of Axes.

1. If the origin is shifted to another point (α, β) where the direction of axes remains unaltered then putting, $x = x+\alpha$ and $y = y+\beta$.
2. If the axes rotated through at an angle θ where the origin of coordinates remains the same then putting, $x = x\cos\theta - y\sin\theta$.

   and $y = x\sin\theta + y\cos\theta$.

3. If the origin is shifted to another point (α, β) and the direction of axes rotated through at an angle θ, then putting:

   $x = \alpha + x\cos\theta - y\sin\theta$,

   and $y = \beta + x\sin\theta + y\cos\theta$.

4. In order to remove the xy term from the expression $ax^2+2hxy+by^2$ putting

$$\tan 2\theta = \frac{2h}{a-b}$$

Pair of Straight lines.

1. Homogeneous Quadratic/Second degree equation,

$$ax^2+2hxy+by^2=0.$$

always represents a pair of straight lines real or imaginary through the origin.

2. If $ax^2+2hxy+by^2=0$ represented by the lines are $y-m_1x=0$ and $y-m_2x=0$

then, $m_1+m_2 = -\frac{2h}{b}$

and $m_1m_2 = \frac{a}{b}$.

3. Angle between the lines represented by the equation $ax^2+2hxy+by^2=0$ is $\theta = \tan^{-1}\left(\frac{2\sqrt{h^2-ab}}{a+b}\right)$.

4. The lines which are represented by $ax^2+2hxy+by^2=0$ will be.

i) real and different if, $h^2>ab$

ii) real and coincident or parallel if, $h^2=ab$

iii) perpendicular if, $a+b=0$.

iv) imaginary if, $h^2<ab$.

5. The bisectors of the angles between the lines repesented by $ax^2+2hxy+by^2=0$ is $\frac{x^2-y^2}{a-b}=\frac{xy}{h}$

6. General equation of second degree, $ax^2+2hxy+by^2+2gx+2fy+c=0$. may represent a pair of straight lines if,

$$\Delta \equiv \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

or, $\Delta \equiv abc+2fgh-af^2-bg^2-ch^2=0$.

7. The conic is called a parabola and Ellipse or a Hyperbola according as the eccentricity $e=1$, $e<1$ or $e>1$ respectively.

8. General equation of the second degree, $ax^2+2hxy+by^2+2gx+2fy+c=0$ will represent,

i) a pair of straight lines if $\Delta \equiv \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

or, $\Delta \equiv abc+2fgh-af^2-bg^2-ch^2=0$.

ii) two parallel lines if, $\Delta=0$ and $h^2=ab$.

iii) two perpendicular lines, if $\Delta=0$ and $a+b=0$.

iv) a circle if, $\Delta \neq 0$, $a=b$ and $h=0$

v) a parabola if, $\Delta \neq 0$ and $h^2-ab=0$

vi) an ellipse if, $\Delta \neq 0$ and $h^2-ab<0$

vii) a hyperbola if, $\Delta \neq 0$ and $h^2-ab>0$

viii) a rectangular hyperbola if, $a+b=0$, $h^2-ab>0$ and $\Delta \neq 0$.

Rectangular Co-ordinates

1. If the three numbers x, y, z are called the co-ordinates of any point P then the point represented by $P(x,y,z)$.

2. Distance between the points $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ is $PQ=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$.

3. Distance between the ~~origin~~ origin $O(0,0,0)$ and $P(x_1, y_1, z_1)$ is $OP = \sqrt{x_1^2 + y_1^2 + z_1^2}$.

4. Co-ordinates of the point which divides the straight line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $m_1 : m_2$ then,

Internal section ratio $= \cancel{\left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, m_1y_1\right)}$

$$= \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}, \frac{m_1z_2 + m_2z_1}{m_1+m_2}\right)$$

and external section ratio,

$$= \left(\frac{m_1x_2 + m_2x_1}{m_1-m_2}, \frac{m_1y_2 - m_2y_1}{m_1-m_2}, \frac{m_1z_2 - m_2z_1}{m_1-m_2}\right)$$

5. Centre of gravity of a triangle

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

6. Direction cosine (d'cs) are denoted by l, m, n where, $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$ also $l^2+m^2+n^2 = 1$

7. Direction ratio (d'rs) are denoted by a, b, c

8. Relation between dc's and dr's,

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

9. The direction cosines of the line joining the two points (x_1, y_1, z_1) and (x_2, y_2, z_2) are projectional to $x_2 - x_1$; $y_2 - y_1$; $z_2 - z_1$

10. Angle between two lines,

According to direction cosine, $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

According to direction ratio, $\cos\theta = \dfrac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$

11. Condition for perpendicularity of two lines,

According to direction cosine, $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

According to directin ratio, $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

12. Condition for parallelism of two lines,

According to direction cosine, $l_1 = l_2$; $m_1 = m_2$; $n_1 = n_2$

According to direction ratio, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

The Plane

1. General equation of a plane, $ax + by + cz + d = 0$
2. Equation of a plane through the origin, $ax + by + cz = 0$
3. Equation of a plane through the point (x_1, y_1, z_1)

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

4. Standards forms of the equatin of a plane
   Intercept form, $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$

   Normal form, $lx+my+nz=p.$

5. Angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is $\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}$

6. Condition for perpendicularity of two planes,
   $a_1a_2+b_1b_2+c_1c_2=0$

7. Condition of parallelism of two planes,
   $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}.$

8. Plane parallel to the plane $ax+by+cz+d=0$ is $ax+by+cz+k=0.$

9. Equation of x-plane or yz plane, $x=0$
   Equation of y-plane or xz plane, $y=0$
   Equation of z-plane or xy plane, $z=0.$

10. plane through the line of intersection of a given plane is $a_1x+b_1y+c_1z+d_1+K(a_2x+b_2y+c_2z+d_2)=0$

11. Perpendicular distance from the point (x_1, y_1, z_1) to the plane $ax+by+cz+d=0$ is ~~a_1x+b_1y+~~ given by,

$$D = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

12. If a tetrahedron form by the four planes

$$a_1x+b_1y+c_1z+d_1=0$$
$$a_2x+b_2y+c_2z+d_2=0$$
$$a_3x+b_3y+c_3z+d_3=0$$
$$a_4x+b_4y+c_4z+d_4=0$$

then the volume of tetrahedron, $V = \dfrac{\Delta^3}{6D_1D_2D_3D_4}$

Where, $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix}$, $D_1 = -\begin{vmatrix} a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_4 & b_4 & c_4 \end{vmatrix}$

$D_2 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \\ a_4 & b_4 & c_4 \end{vmatrix}$, $D_3 = -\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_4 & b_4 & c_4 \end{vmatrix}$, $D_4 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

The Straight lines.

1. General equation of a straight line,
$$a_1x+b_1y+c_1z+d_1=0=a_2x+b_2y+c_2z+d_2$$

2. Symmetric form of a straight line

According to dc's, $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$

According to dr's, $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

3. Equation of straight line through the points (x_1,y_1,z_1) and (x_2,y_2,z_2) is $\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$.

4. Equation of straight line through the origin,
$$a_1x+b_1y+c_1z=0=a_2x+b_2y+c_2z$$

5. Angle between the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ and the plane $ax+by+cz+d=0$ is,
$$\sin\theta=\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{l^2+m^2+n^2}}$$

6. Condition of parallelism of a line and a plane
$$al+bm+cn=0.$$

7. Condition of perpendicularity of a line and a plane
$$l/a=m/b=n/c.$$

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8. The straight line lie on the plane if, $al+bm+cn=0$ and $ax_1+by_1+cz_1+d=0$.

9. The straight lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ & $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ will be coplanar if $\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$

and the equation of the plane in which the above lines lie or containing the above lines is,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

10. Shortest Distance between two lines,

$$S.D = l(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1).$$

Where l, m, n are the dc's of the line S.D and (x_1, y_1, z_1) & (x_2, y_2, z_2) are two points on the lines respectively.

The Sphere

1. General equation of a sphere,
$x^2+y^2+z^2+2ux+2vy+2wz+d=0$, whose centre is at $(-u,-v,-w)$ and radius is $\sqrt{u^2+v^2+w^2-d}$

2. Equation of the sphere through the origin,
$$x^2+y^2+z^2+2ux+2vy+2wz=0$$
whose centre is at $(-u,-v,-w)$ and radius $\sqrt{u^2+v^2+w^2}$

3. Equation of a sphere whose centre is at (a,b,c) and radius ~~r is for~~ r is,
$$(x-a)^2+(y-b)^2+(z-c)^2=r^2.$$

4. Equation of a sphere whose centre is at $(0,0,0)$ i.e. origin and radius r is, $x^2+y^2+z^2=r^2$.

5. The equation of a sphere through the two end points of a diameter is
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0.$$

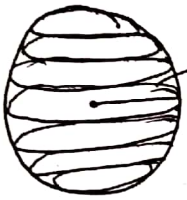
6. The equation of the tangent plane to a sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ at the point (x_1,y_1,z_1) is, $xx_1+yy_1+zz_1+u(x+x_1)+v(y+y_1)+w(z+z_1)+d=0$.

7. The equation of a circle,

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0=ax+by+cz+d$$

or, $\left.\begin{array}{l} x^2+y^2+z^2+2ux+2vy+2wz+d=0 \\ ax+by+cz+d=0 \end{array}\right\}$

8. The equation of the sphere through the above circle, $x^2+y^2+z^2+2ux+2vy+2wz+k(ax+by+cz+d)=0$

9. 

→ Great circle with respect to the sphere.

Centre of the great circle = Centre of the sphere.

Radius of the great circle = Radius of the sphere.