

Finite Automata

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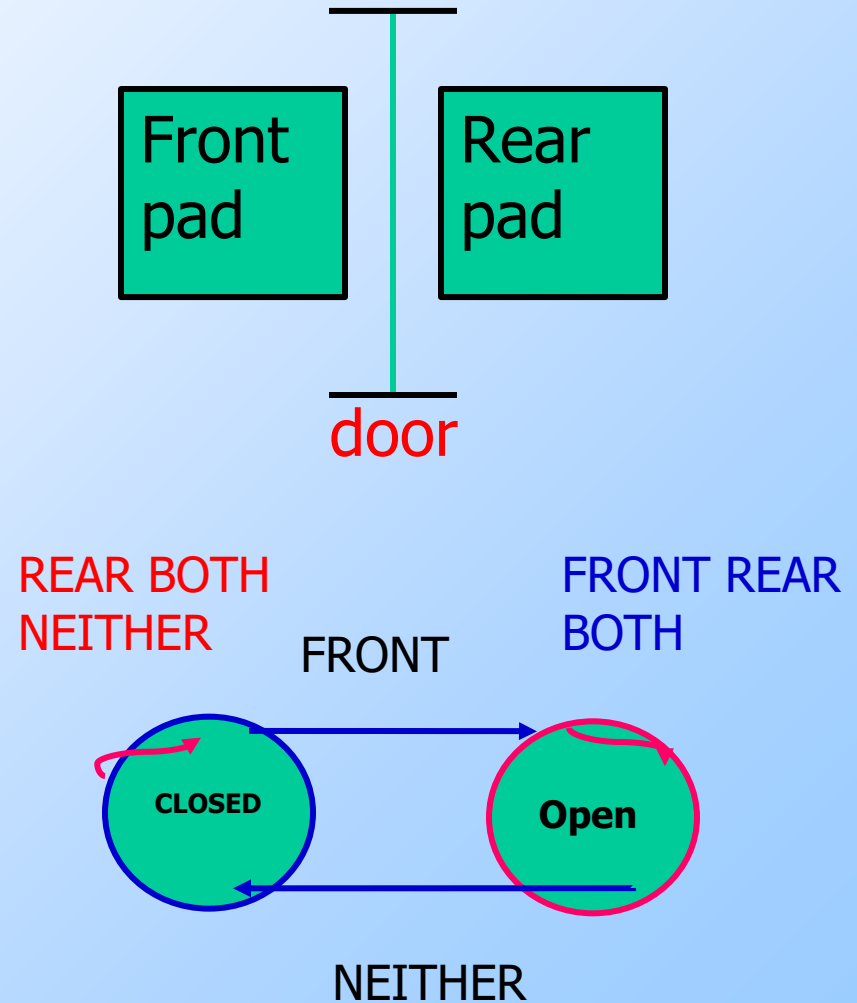
- A finite automaton has a set of states and it control moves from state to state in response to external inputs
- Deterministic: automation cannot be in more than one state at any time
- Nondeterministic: automation can be several states at once

Informal Picture of Finite Automata

- Finite automata are good models for computers with an extremely limited amount of memory
 - electromechanical devices
- The controller for an automatic door is one example of such a device
- Often found in supermarket entrances/exits, automatic doors swing open when sensing that a person is approaching

Informal Picture of Finite Automata

- Two states: OPEN or CLOSED
- 04 input conditions:
 1. **FRONT** (a person is standing on the pad in front of the doorway)
 2. **REAR** (a person is standing on the pad to the rear of the doorway)
 3. **BOTH** (people are standing on both pads)
 4. **NEITHER** (no one is standing on either pad)



State table

Input signal				
	NEITHER	FRONT	REAR	BOTH
CLOSED	CLOSED	OPEN	CLOSED	CLOSED
OPEN	CLOSED	OPEN	OPEN	OPEN

- Controller moves from state to state, depending on input
- when CLOSED state & input NEITHER/REAR, it remains in CLOSED state
- If the input BOTH, it stays CLOSED because opening the door Risks knocking someone over on the rear pad
- But if input FRONT, it moves to the OPEN state
- In the OPEN state, if input FRONT/REAR/BOTH, it remains OPEN

Other Examples

- This controller is a computer that has just a single bit of memory, capable of recording which is two states the controller is in
- Others: elevator controller, dishwashers, thermostat, digital watches, calculators etc

Deterministic Finite Automata (DFA)

- **Deterministic**: on each input there is one state to which the automaton can transition from its current state
- In terms of 5 tuples:

$$A = (Q, \Sigma, \delta, q_0, F)$$

- **A**: DFA, **Q**: set of states, **Σ** : input symbols, **δ** : transition function, **q_0** : start state, **F**: set of accepting state

DFA

- A formalism for defining languages, consisting of:
 1. A finite set of *states* (Q , typically).
 2. An *input alphabet* (Σ , typically).
 3. A *transition function* (δ , typically).
 4. A *start state* (q_0 , in Q , typically).
 5. A set of *final states* ($F \subseteq Q$, typically).
 - “Final” and “accepting” are synonyms denoted by **Double Loops**.

The Transition Function

- Takes two arguments: a state & an input symbol; returns a state
- δ represented by an arc between states and the labels on the arcs.
- $\delta(q, a)$ = the state that the DFA goes to when it is in state q & input a is received.
- q state (current) to p state (next): an arc labeled with a

How a DFA Process Strings

- How the DFA decides whether or not to 'accept' a sequence of input symbols
- The language of the DFA is the set of all strings that the DFA accepts.
- Sequence of input symbols: a_1, a_2, \dots, a_n
- initial state: q_0
- Transition function, $\delta(q_0, a_1) = q_1$ to find state that the DFA enters after processing the first input symbol a_1

How a DFA Process Strings

- Process next input symbol, a_2 : $\delta(q_1, a_2) = q_2$
- continue in this way, finding states $q_3, q_4, \dots q_n$, such that $\delta(q_{i-1}, a_i) = q_i$ for each i
- If q_n is a member of F , then the input $a_1 a_2 \dots a_n$ is **accepted**,
- if not then it is **rejected**.

Example

- Specify a DFA that accepts all and only the strings of 0's and 1's that have the sequence 01 somewhere in the string.
- L as:
 $\{w \mid w \text{ is of the form } x01y \text{ for some strings } x \text{ \& } y \text{ consisting of 0's \& 1's only}\}$
- $\{x01y \mid x \text{ \& } y \text{ are any strings of 0's \& 1's}\}$

Example

- Strings: 01, 11010, 100011 exists in L
- ϵ , 0, & 111000 not exist in L
- $\Sigma = \{0, 1\}$
- $\delta(q_0, 1) = q_0$
- $\delta(q_0, 0) = q_2$
- $\delta(q_2, 0) = q_2$
- $\delta(q_2, 1) = q_1$

Example

- $\delta(q_1, 0) = \delta(q_1, 1) = q_1$
- $Q = \{q_0, q_1, q_2\}, F = \{q_1\}$
- The complete specification of the automation A that accepts the L of strings that have a 01 substring:

$$A = (\{q_0, q_1, q_2\}, (0, 1), \delta, q_0, \{q_1\})$$

Notations for DFA's

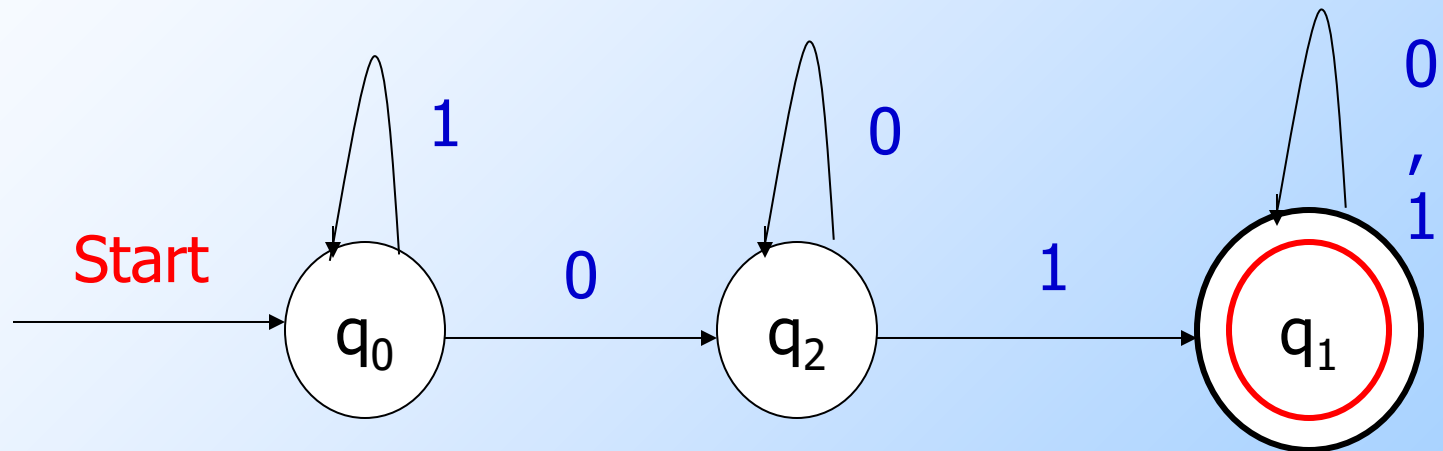
- Specifying a DFA as 5tuple with a detailed description of the δ transition function: tedious & hard to read
- Two notations:
 1. Transition diagram
 2. Transition table

Transition Diagram

- Nodes = states.
- Arcs represent transition function.
 - Arc from state p to state q labeled by all those input symbols that have transitions from p to q .
- Arrow labeled "Start" to the start state.
- Final states indicated by double circles.

Example

- Draw the Transition Diagram for the DFA accepting all string with a **substring 01**.

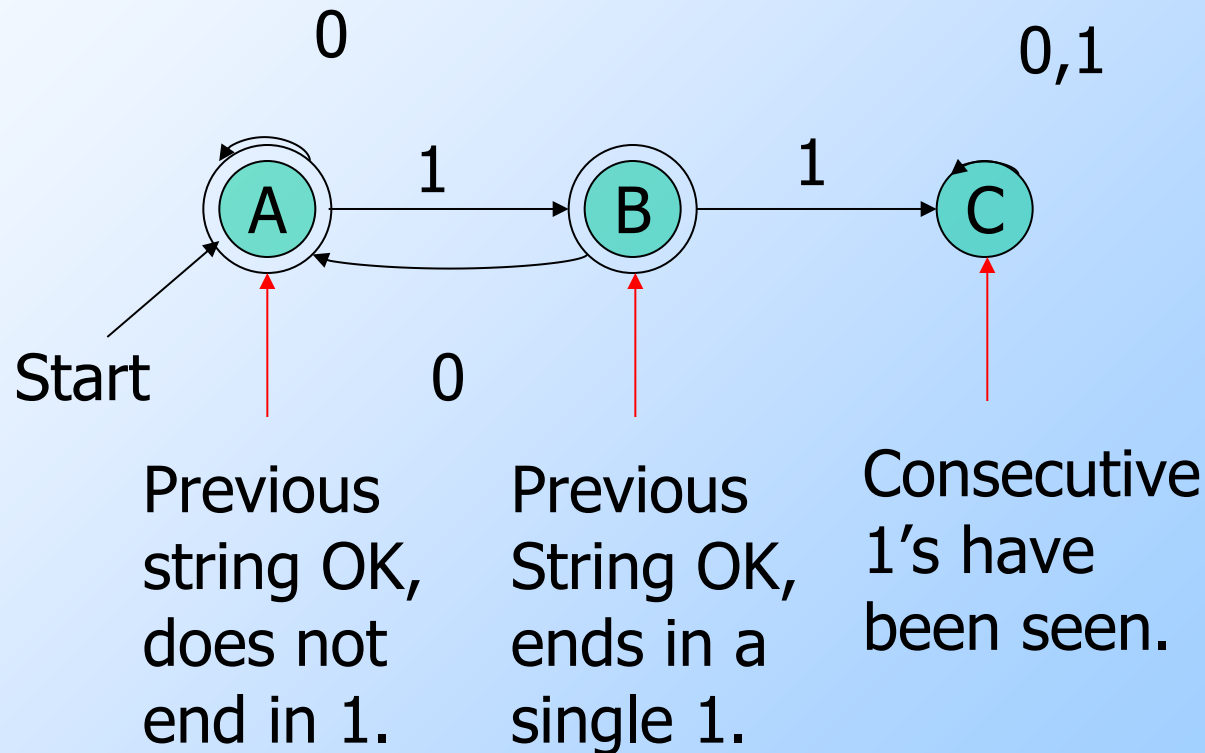


$$A = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_1\})$$

Check with the string 01, 11010, 100011,
0111, 110101, 11101101, **111000**

Another Example

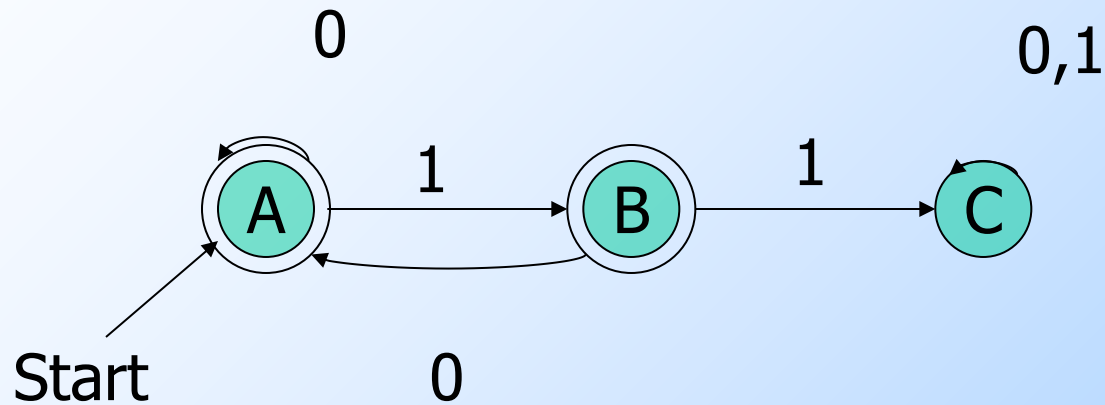
Accepts all strings without two consecutive 1's.



Transition Table

- a conventional, tabular representation of a function like δ that takes two arguments & return a value.
- **Rows:** states; corresponding to state q
- **Columns:** inputs; corresponding to input a is the state $\delta(q,a)$
- The start state is marked with an **arrow** and accepting states are marked with a **star**.

Example



Final states
starred

Columns =
input symbols

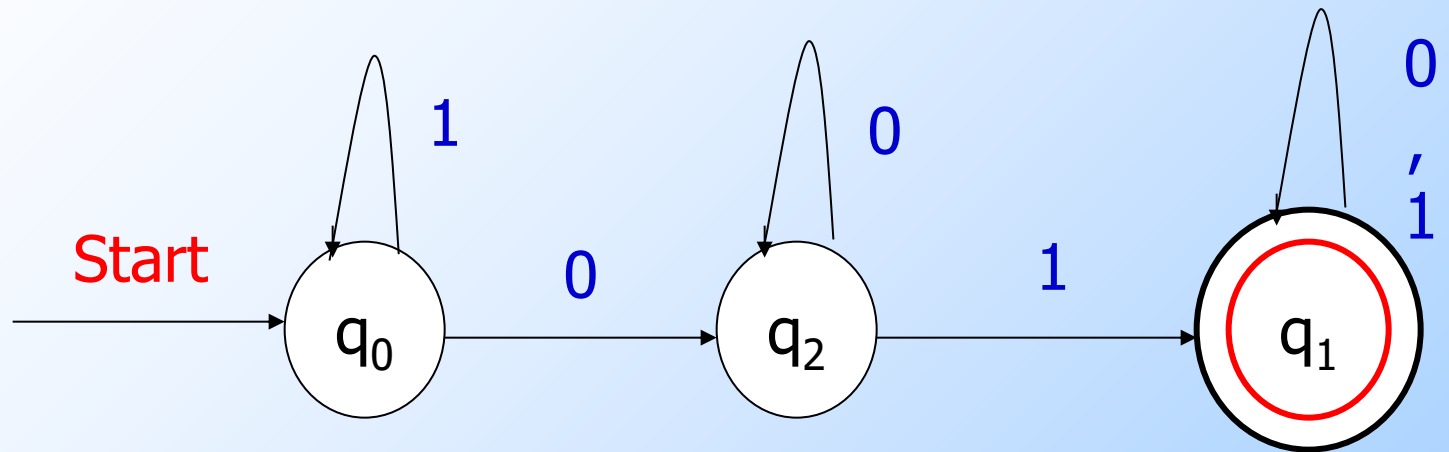
Arrow for
start state

→ * A
* B
C
↑

Rows = states

	0	1
A	A	B
A	A	C
C	C	C

Example



- $(q_0, 0) = q_2$
- $(q_0, 1) = q_0$
- $(q_1, 0) = q_1$
- $(q_1, 1) = q_1$
- $(q_2, 0) = q_2$
- $(q_2, 1) = q_1$

	0	1
→ q_0	q_2	q_0
* q_1	q_1	q_1
q_2	q_2	q_1

Extended Transition Function

- An Extended Transition Function that describes what happens when we start in any state and follow any sequence of inputs.

δ □ Transition Function

$\hat{\delta}$ □ Extended Transition Function

- a function that takes a state q & a string w & returns a state p -the state that the automaton reaches when starting in state q & processing the sequence of inputs w

Extended Transition Function

- We describe the effect of a string of inputs on a DFA by extending δ to a state and a string.
- Induction on length of string.
- **Basis:** $\hat{\delta}(q, \varepsilon) = q$
- **Induction:** $\hat{\delta}(q, wa) = \delta(\hat{\delta}(q, w), a)$
 - w is a string; a is an input symbol.

Induction

$$2. \quad \hat{\delta}(q, w) = \delta(\hat{\delta}(q, x), a) = \delta(p, a)$$

$w \sqsubseteq xa$

$a \sqsubseteq \text{last symbol} = 1$

$w \sqsubseteq \text{string} = 1101$

$x \sqsubseteq \text{string consisting of all but the last symbol} = 110$

Extended δ : Intuition

- **Convention:**
 - ... w, x, y, x are strings.
 - a, b, c, \dots are single symbols.
- Extended δ is computed for state q and inputs $a_1a_2\dots a_n$ by following a path in the transition graph, starting at q and selecting the arcs with labels a_1, a_2, \dots, a_n in turn.

Example: Extended Delta

	0	1
A	A	B
B	A	C
C	C	C

$$\delta(B, 011) = \delta(\delta(B, 01), 1) = \delta(\delta(\delta(B, 0), 1), 1) =$$
$$\delta(\delta(A, 1), 1) = \delta(B, 1) = C$$

Delta-hat

- In book, the extended δ has a “hat” to distinguish it from δ itself.
- Not needed, because both agree when the string is a single symbol.
- $\delta(q, a) = \delta(\delta(q, \varepsilon), a) = \delta(q, a)$

Extended deltas



Example

- Let us design a DFA to accept the language

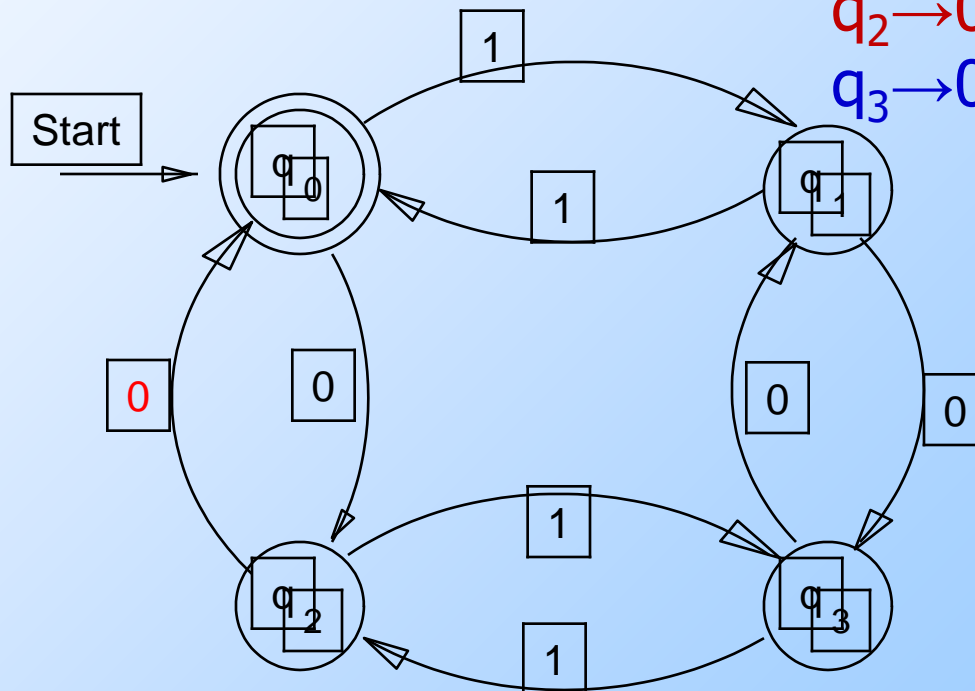
$L = \{w \mid w \text{ has both an even number of 0's and/or even number of 1's}\}$

$q_0 \rightarrow 0(\text{even}) \ 1(\text{even})$

$q_1 \rightarrow 0(\text{even}) \ 1(\text{odd})$

$q_2 \rightarrow 0(\text{odd}) \ 1(\text{even})$

$q_3 \rightarrow 0(\text{odd}) \ 1(\text{odd})$



	0	1
$\rightarrow^* q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_3	q_1

Example with Transition Table

	0	1
$\rightarrow^* q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

• **w = 110101**

• **x = 11010**

• **a = 1**

$\hat{\delta}(q_0, 110101) = q_0$

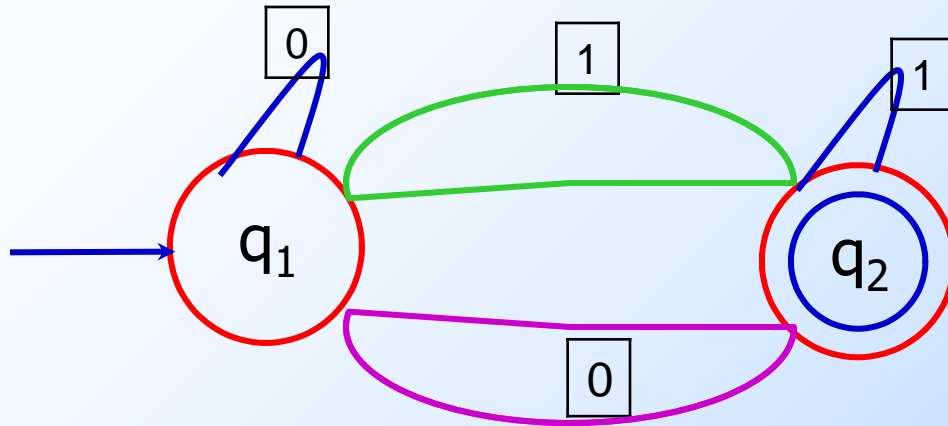
- $\hat{\delta}(q_0, \epsilon) = q_0$
- $\hat{\delta}(q_0, 1) = \delta(\hat{\delta}(q_0, \epsilon), 1) = \delta(q_0, 1) = q_1$
- $\hat{\delta}(q_0, 11) = \delta(\delta(\hat{q}_0, 1), 1) = \delta(q_1, 1) = q_0$
- $\hat{\delta}(q_0, 110) = \delta(\delta(\hat{q}_0, 11), 0) = \delta(q_0, 0) = q_2$
- $\hat{\delta}(q_0, 1101) = \delta(\delta(\hat{q}_0, 110), 1) = \delta(q_2, 1) = q_3$
- $\hat{\delta}(q_0, 11010) = \delta(\delta(\hat{q}_0, 1101), 0) = \delta(q_3, 0) = q_1$
- $\hat{\delta}(q_0, 110101) = \delta(\hat{\delta}(q_0, 11010), 1) = \delta(q_1, 1) = q_0$

Accepted

Example: Try Yourself

- $A = \{w \mid w \text{ contains at least one } 1 \text{ and an even number of } 0\text{s follow the last } 1\}$
- **Hints:** $A_1 = (Q, \Sigma, \delta, q_1, F)$
 1. $Q = \{q_1, q_2, q_3\}$
 2. $\Sigma = \{0, 1\}$
 3. δ try yourself
 4. Start state: q_1
 5. Final state: $\{q_2\}$

Example



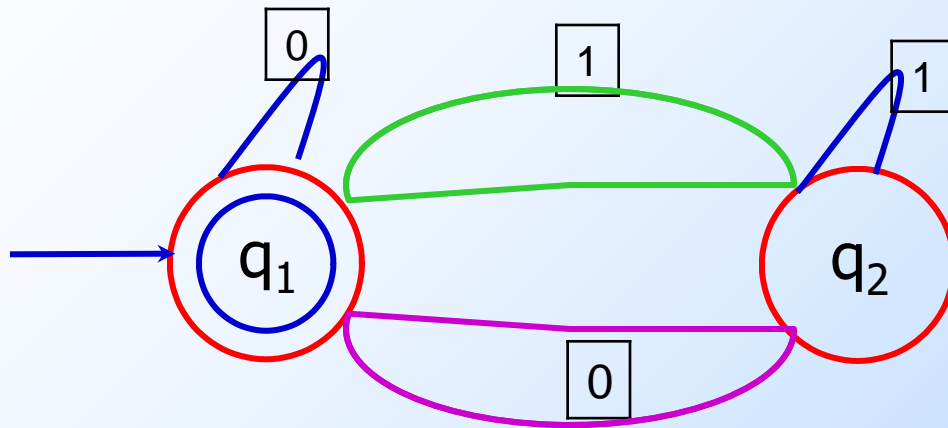
- $A_2 = (\{q_1, q_2\}, (0,1), \delta, q_1, \{q_2\})$
- Transition function, δ

Try: 1101, 11010, 0011010

$L(A_2) = \{w \mid w \text{ ends in a } 1\}$

	0	1
$\rightarrow q_1$	q_1	q_2
$*q_2$	q_1	q_2

Example



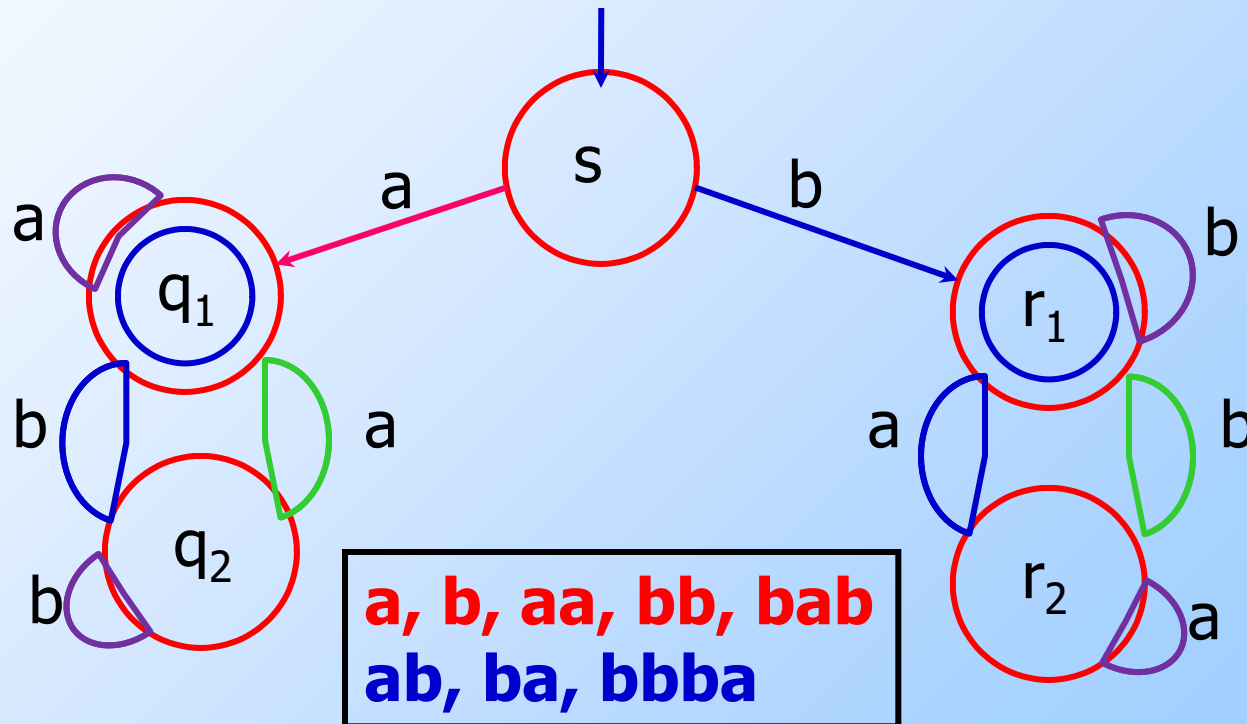
- $A_3 = (\{q_1, q_2\}, (0,1), \delta, q_1, \{q_1\})$
- Transition function, δ

	0	1
$\rightarrow^* q_1$	q_1	q_2
q_2	q_1	q_2

Try: 1101, 11010, 0011010
 $L(A_3) = \{w \mid w \text{ is } \varepsilon \text{ or ends in a } 0\}$

Example

- A_4 accepts all strings that start and end with a or that start and end with b
- A_4 accepts strings, that start and end with the same symbol

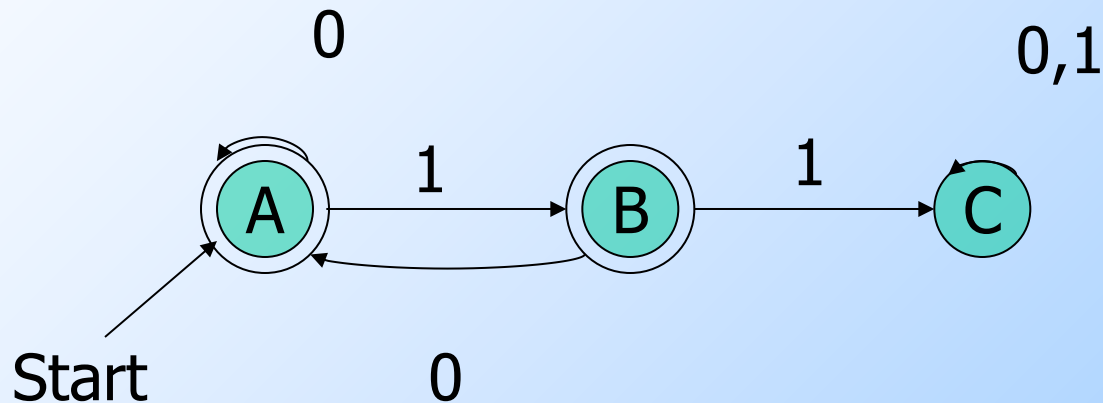


Language of a DFA

- Automata of all kinds define languages.
- If A is an automaton, $L(A)$ is its language.
- For a DFA A , $L(A)$ is the set of strings labeling paths from the start state to a final state.
- Formally: $L(A)$ = the set of strings w such that $\delta(q_0, w)$ is in F .

Example: String in a Language

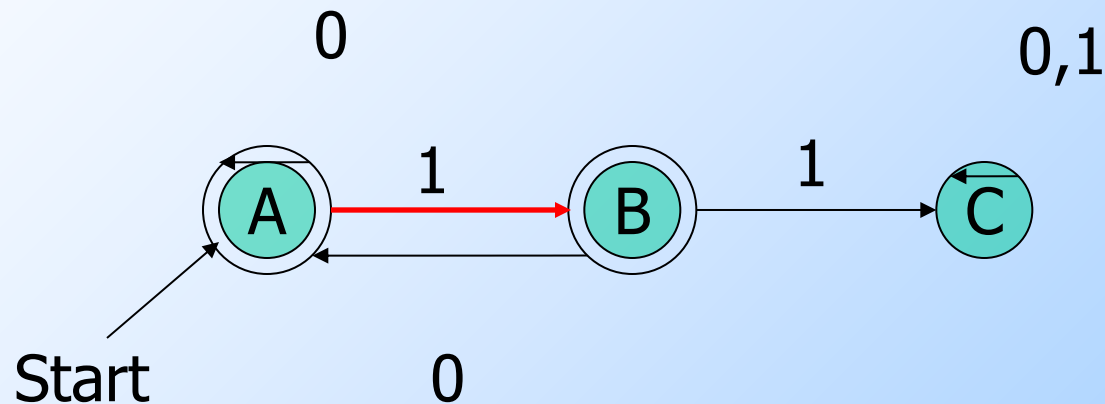
String 101 is in the language of the DFA below.
Start at A.



Example: String in a Language

String 101 is in the language of the DFA below.

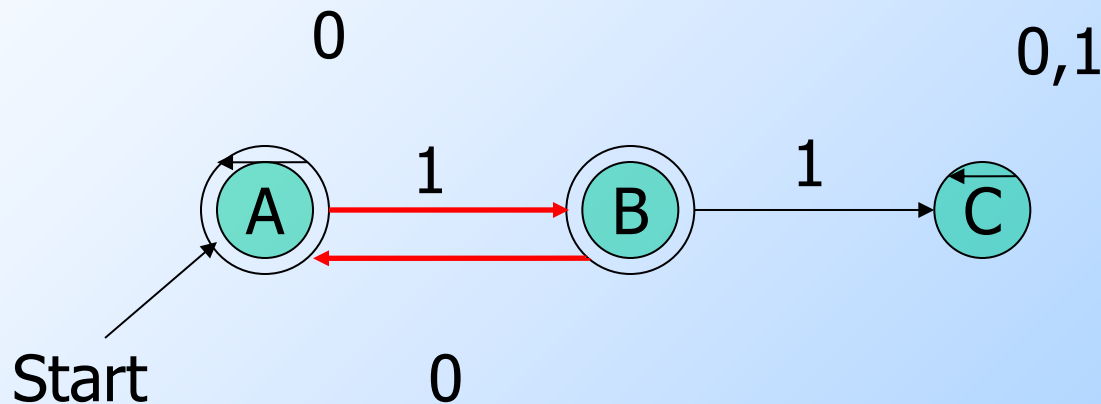
Follow arc labeled 1.



Example: String in a Language

String 101 is in the language of the DFA below.

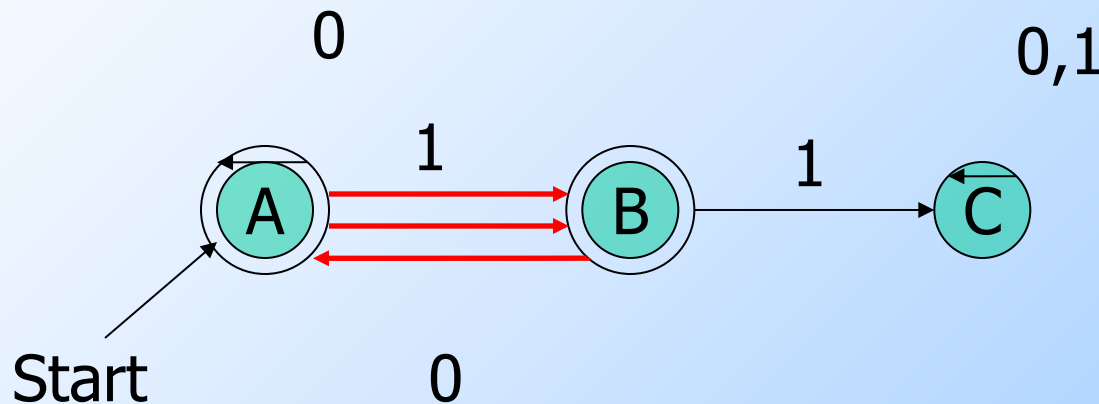
Then arc labeled 0 from current state B.



Example: String in a Language

String 101 is in the language of the DFA below.

Finally arc labeled 1 from current state A. Result is an accepting state, so 101 is in the language.



Example – Concluded

- The language of our example DFA is:
 $\{w \mid w \text{ is in } \{0,1\}^* \text{ and } w \text{ does not have two consecutive 1's}\}$

Such that...

These conditions
about w are true.

Read a *set former* as
"The set of strings w ..."

Try Yourself

- Give DFA's accepting the following languages over the alphabet $\{0,1\}$
 - a) The set of all strings ending in 00
 - b) The set of all strings with three consecutive 0's (not necessarily at the end)
 - c) The set of strings with 011 as a substring

Try Yourself

- Give DFA's accepting the following languages over the alphabets $\{0,1\}$
 - a) The set of all strings such that each block of five consecutive symbols contains at least two 0's
 - b) The set of all strings whose tenth symbol from right end is a 1
 - c) The set of strings such that the number of 0's is divisible by five, and the number of 1's is divisible by 3