Spring 2024

Group A

Answer to the question No. 1

(a)

$A(1,4) = A(0, A(1,3))$

$A(1,3) = A(0, A(1,2))$

$A(1,2) = A(0, A(1,1))$

$A(1,1) = A(0, A(1,0))$

$A(1,0) = A(0,1)$

$A(0,1) = 1+1 = 2$

$A(1,1) = A(0,2)$

$A(0,2) = 2+1 = 3$

$A(1,2) = A(0,3)$

$A(0,3) = \cancel{A}\ 3+1 = 4$

$A(1,3) = A(0,4)$

$A(0,4) = \cancel{A}\ 4+1 = 5$

$A(1,4) = A(0,5)$

$A(0,5) = 5+1 = 6$

$\therefore A(1,4) = \underline{\underline{6}}$

(b)

Overflow condition:

The circular queue is ~~can~~ considered full when the next position of the rear index is equal to the front index.

Mathematically:

$front = rear + 1$

Example:

Suppose the queue has size $n=5$, and the indices are as follows:

* front = 1
* rear = 3

After inserting two more elements:

* rear moves to 4, then 0
* Now, if front = rear+1, the queue is full.

(c)

Tower of Hanoi

Rules:

a) Only one disk may be moved at a time. Specifically, only the top disk on any peg may be moved to any other peg.

b) At no time can a larger disk be placed on a smaller disk.

Complexity of Tower of Hanoi

Recursive Equation

$T(n) = 2T(n-1) + 1$ ——— eq 1

solving it by Backsubstitution:

$T(n-1) = 2T(n-2) + 1$ ——— 2

$T(n-2) = 2T(n-3) + 1$ ——— 3

putting the value of $T(n-2)$ in eq^n 2

$T(n-1) = 2(2T(n-3)+1) + 1$ ——— 4

$\cancel{= 4T(n-3)+1}$

putting the value of $T(n-1)$ in eq^n 1

$T(n) = 2(2(2T(n-3)+1)+1)+1$

$T(n) = 2^3 T(n-3) + 2^2 + 2^1 + 1$

$= 2^K T(n-K) + 2^{(K-1)} + 2^{K-2} + \ldots\ldots + 2^2 + 2^1 + 1$

$\therefore T(n) = 2^n - 1 \quad [\because T(1) = 1]$

Time complexity: $O(2^n)$

Space complexity: $O(n)$.

Answer to the question No. 3 (a)

Pass	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]	A[9]	A[10]	A[11]	A[12]	A[13]	A[14]
K=1, LOC=2	D	A	T	A	S	T	R	U	C	T	U	R	E	S
K=2, LOC=4	A	D	T	A	S	T	R	U	C	T	U	R	E	S
K=3, LOC=9	A	A	T	D	S	T	R	U	C	T	U	R	E	S
K=4, LOC=4	A	A	C	D	S	T	R	U	T	T	U	R	E	S
K=5, LOC=13	A	A	C	D	S	T	R	U	T	T	U	R	E	S
K=6, LOC=12	A	A	C	D	E	T	R	U	T	T	U	R	S	S
K=7, LOC=7	A	A	C	D	E	R	R	U	T	T	U	T	S	S
K=8, LOC=13	A	A	C	D	E	R	R	U	T	T	U	T	S	S
K=9, LOC=14	A	A	C	D	E	R	R	S	T	T	U	T	U	S
K=10, LOC=10	A	A	C	D	E	R	R	S	S	T	U	T	U	T
K=11, LOC=12	A	A	C	D	E	R	R	S	S	T	U	T	U	T
K=12, LOC=14	A	A	C	D	E	R	R	S	S	T	T	U	U	T
K=13, LOC=13	A	A	C	D	E	R	R	S	S	T	T	T	U	U
Sorted:	A	A	C	D	E	R	R	S	S	T	T	T	U	U

Answer to the question No. 3
(C)

INSERTION (A,N)

This algorithm sorts the array A with N elements

1. Set A[0] := $-\infty$ [Initializes sentinel element]
2. Repeat step 3 to 5 for K = 2, 3, ... N:
3. Set TEMP := A[K] and PTR := K−1
4. Repeat while TEMP < A[PTR]:
   (a) Set A[PTR+1] := A[PTR] [Moves element forward]
   (b) Set PTR := PTR − 1
   [End of loop]
5. Set A[PTR+1] := TEMP. [Inserts element in proper place.]
   [End of step 2 loop.]
6. Return.

Complexity of Insertion sort

$O(n^2)$

(d)

35, 58, 102, 79, 131, 46, 112, 177, 240 (ID = C233040)

Table size = 11

$H(K) = K \% \text{Table size}$

$H(35) = 35 \% 11 = 2$

$H(58) = 58 \% 11 = 3$

$H(102) = 102 \% 11 = 3$

using linear probing

$H(102) = 3+1 = 4$

$H(79) = 79 \% 11 = 2$

$H(79) = 2+1 = 3+1 = 4+1 = 5$

$H(131) = 131 \% 11 = 10$

$H(46) = 46 \% 11 = 2$

$H(46) = 2+1 = 3+1 = 4+1 = 5+1 = 6$

$H(112) = 112 \% 11 = 2$

$H(112) = 2+1 = 3+1 = 4+1 = 5+1 = 6+1 = 7$

$H(117) = 117 \% 11 = 7$

$H(117) = 7+1 = 8$

$H(240) = 240 \% 11 = 9$

H(

	1
35	2
58	3
102	4
79	5
46	6
112	7
117	8
240	9
131	10
	11

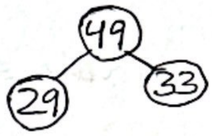
Answer to the question No.4

(a)

33, ~~49~~, 29, 49, 21, 57, 62, 73, 54

ITEM=33.

ITEM = 29



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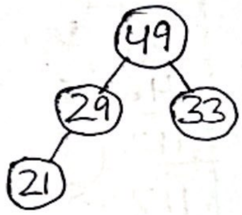
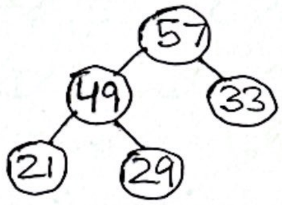
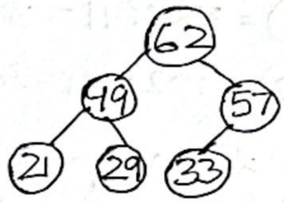


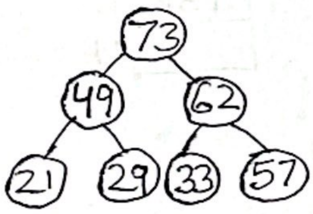
ITEM=21



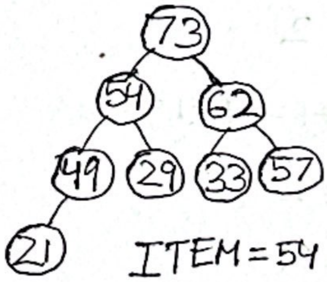
ITEM = 57



ITEM = 62



ITEM=73

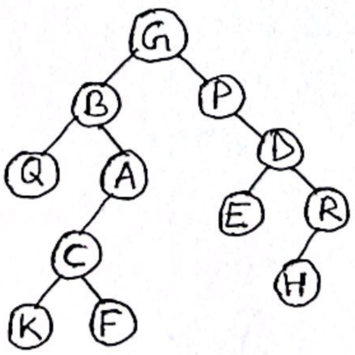


ITEM=54

(c)

Preorder: G, B, Q, A, C, K, F, P, D, E, R, H

Inorder: Q, B, K, C, F, A, G, P, E, D, H, R



Answer to the question No. 5

(a)

$$P_0 = \begin{array}{c} \\ X \\ Y \\ S \\ T \end{array} \begin{array}{c} \begin{array}{cccc} X & Y & S & T \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$C_1 = \{Y, T\}$ $R_1 = \{Y\}$

$C_1 \times R_1 = \{(Y, Y), (T, Y)\}$

$$P_1 = \begin{array}{c} \\ X \\ Y \\ S \\ T \end{array} \begin{array}{c} \begin{array}{cccc} X & Y & S & T \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$C_2 = \{X, Y, T\}$ $R_2 = \{X, Y, S\}$

$C_2 \times R_2 = \{(X,X), (X,Y), (X,S), (Y,X), (Y,Y), (Y,S), (T,X)$
$(T,Y), (T,S)\}$

$$P_2 = \begin{array}{c} \\ X \\ Y \\ S \\ T \end{array} \begin{array}{c} \begin{array}{cccc} X & Y & S & T \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$C_3 = \{X, Y, T\}$ $R_3 = \{T\}$

$C_3 \times R_3 = \{(X, T), (Y, T), (T, T)\}$

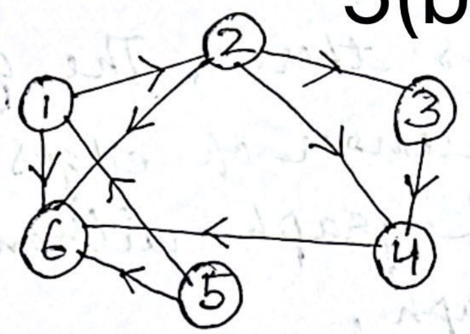
$$P_3 = \begin{array}{c|cccc} & X & Y & S & T \\ \hline X & 1 & 1 & 1 & 1 \\ Y & 1 & 1 & 1 & 1 \\ S & 0 & 0 & 0 & 1 \\ T & 1 & 1 & 1 & 1 \end{array}$$

$C_4 = \{X, Y, S, T\}$ $R_4 = \{X, Y, S, T\}$

$$P_4 = \begin{array}{c|cccc} & X & Y & S & T \\ \hline X & 1 & 1 & 1 & 1 \\ Y & 1 & 1 & 1 & 1 \\ S & 1 & 1 & 1 & 1 \\ T & 1 & 1 & 1 & 1 \end{array}$$

5(b).

i)



i) <u>Adjacency matrix :</u>

$$
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 0 & 1 & 0 & 0 & 0 & 1 \\
2 & 0 & 0 & 1 & 1 & 0 & 1 \\
3 & 0 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 0 & 1 \\
5 & 1 & 0 & 0 & 0 & 0 & 1 \\
6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

ii) <u>Adjacency list</u>

[1|] → [2|] → [6|X]

[2|] → [3|] → [4|] → [6|X]

[3|] → [4|X]

[4|] → [6|X]

[5|] → [1|] → [6|X]

[6|X]