Autumn 2021

Group A

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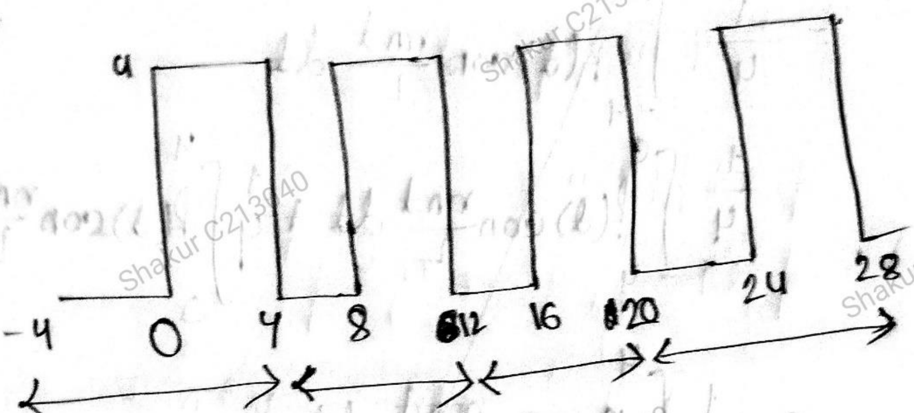


$$T = 2L = 8$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt$$

$$= \frac{1}{4}\int_{-4}^{4} f(t)\,dt$$

$$= \frac{1}{4}\int_{-4}^{0} f(t)\,dt + \frac{1}{4}\int_{0}^{4} f(t)\,dt$$

$$= \frac{1}{4}\int_{-4}^{0} 0\,dt + \frac{1}{4}\int_{0}^{4} 4\,dt$$

$$= \frac{1}{4}\left[4t\right]_0^4$$

Example 36: Find Harmonic Analysis for the given Fourier series

$$f(t) = 2.5 - \frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\cos n\pi - 1)\sin\frac{n\pi t}{4}$$ 2(a)

We have, from Example 22, (Page no 33)

$$f(t) = 2.5 - \frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\cos n\pi - 1)\sin\frac{n\pi t}{4}$$ [Answer of Example 22, Page no 33]

$$\underbrace{f(t)}_{\text{Complex wave}} = \underbrace{2.5}_{\text{DC value}} \underbrace{- \frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\cos n\pi - 1)\sin\frac{n\pi}{4}t}_{\text{AC value}} \text{-----------------------------(i)}$$

$$\underbrace{f(t)}_{\text{Complex wave}} = \underbrace{2.5}_{\text{DC value}} - \underbrace{\frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\cos n\pi - 1)}_{\text{Amplitude}} \sin \underbrace{\frac{n\pi}{4}}_{\text{Frequency}} t \text{-------------------(ii)}$$

We have the Fourier series is $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos(n\omega t) + b_n\sin(n\omega t))$

Here, $n\omega = \frac{n\pi}{4}$

For $n = 1$; **Fundamental Frequency = 1st Harmonic =** $\omega = \frac{n\pi}{4} == \frac{1.\pi}{4} = \frac{\pi}{4}$

For $n = 2$; **2nd Harmonic** $= 2\omega = \frac{n\pi}{4} == \frac{2.\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

For $n = 3$; **3rd Harmonic** $= 3\omega = \frac{n\pi}{4} == \frac{3.\pi}{4} = \frac{3\pi}{4}$

For $n = 4$; **4th Harmonic** $= 4\omega = \frac{n\pi}{4} == \frac{4.\pi}{4} = \frac{4\pi}{4} = \pi$

2(a) or

$$f(t) = 5 + 2\sum_{n=1}^{\infty} \frac{1}{n} \sin nt$$

we have fourier series is

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=}^{\infty} b_n \cos(n\omega t)$$

$a_n = 0$ and $b_n = \frac{2}{n}$

$R_n = C_n = \sqrt{a_n^2 + b_n^2}$

$a_n = 0$	$b_n = \frac{2}{n}$	$C_n = \sqrt{a_n^2 + b_n^2}$
$a_1 = 0$	$b_1 = 2$	$C_1 = \sqrt{0^2 + 2^2} = 2$
$a_2 = 0$	$b_2 = 1$	$C_2 = \sqrt{0^2 + 1^2} = 1$
$a_3 = 0$	$b_3 = \frac{2}{3}$	$C_3 = \sqrt{0^2 + \frac{2^2}{3^2}} = 0.66$
$a_4 = 0$	$b_4 = \frac{2}{4}$	$C_4 = \sqrt{0^2 + \left(\frac{1}{2}\right)^2} = 0.5$
$a_5 = 0$	$b_5 = \frac{2}{5}$	$C_5 = \sqrt{0^2 + (2/5)^2} = 0.4$
$a_6 = 0$	$b_6 = \frac{2}{6}$	$C_6 = \sqrt{0^2 + (2/6)^2} = 0.33$

Here $n\omega = n$

$n = 1$ = 1st Harmonic $= \omega = 1$

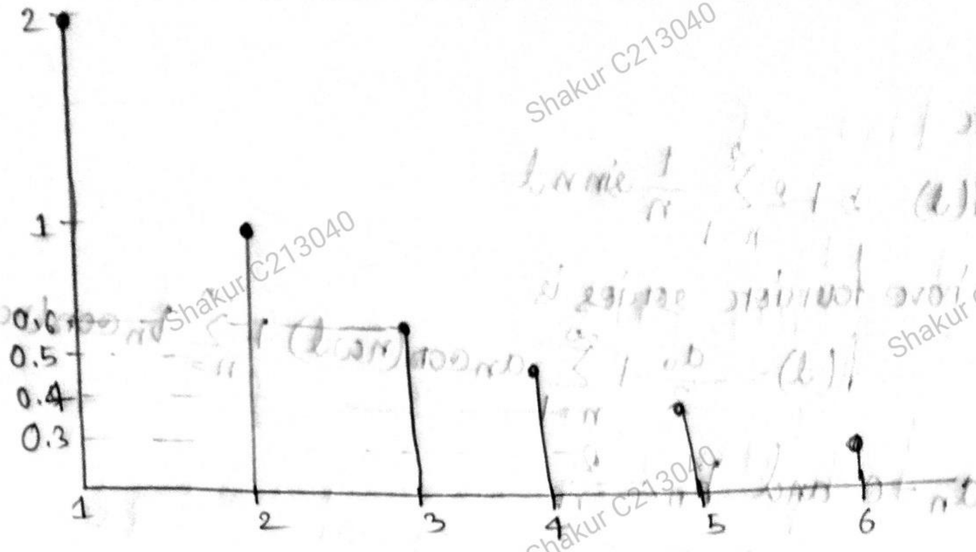
$n = 2$ = 2nd " $= 2\omega = 2$

$n = 3$ = 3rd " $= 3\omega = 3$

$n = 4$ = 4th " $= 4\omega = 4$

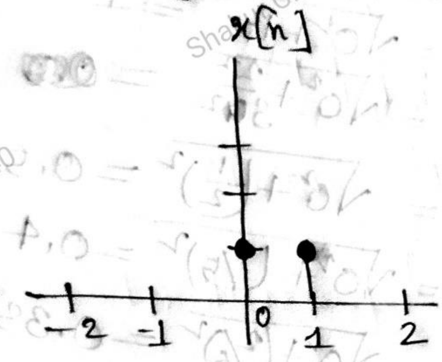
$n = 5$ = 5th " $= 5\omega = 5$

$n = 6$ = 6th " $= 6\omega = 6$



2(b)

$$y = x[n] * h[n]$$

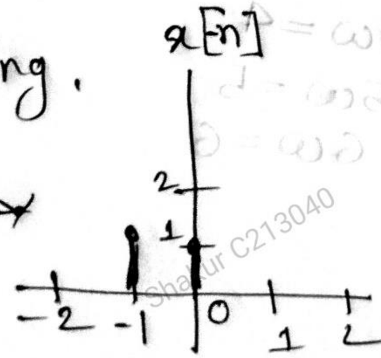


1st time

① Folding

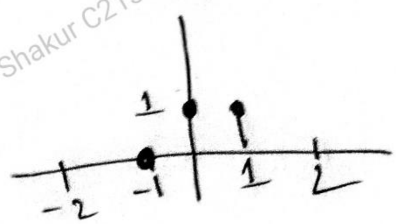
$$x[n] = x[-n]$$
$$x[0] = x[0]$$
$$x[1] = x[-1]$$

folding.



(ii) multiplication.

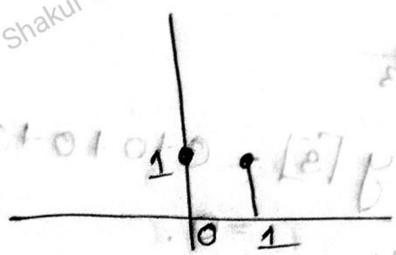
$$x[-n] * h[n]$$



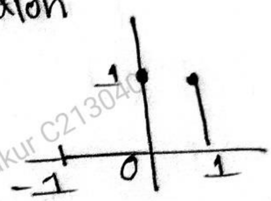
(iii) Summation $y[0] = 1+1 = 2$

2nd time

(i) Shifting



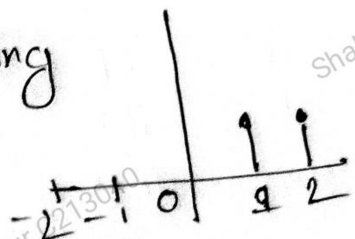
(ii) multiplication



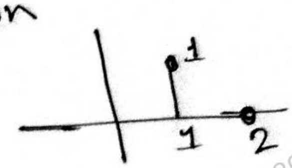
(iii) Summation $y[1] = 1+1 = 2$

3rd time

(i) Shifting



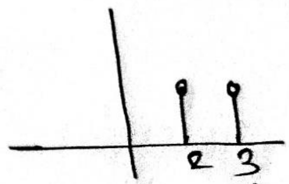
(ii) multiplication



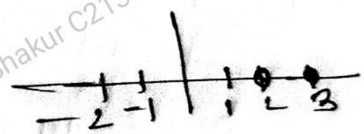
(iii) Summation $y[2] = 1+0 = 1$

4th time

(i) Shifting



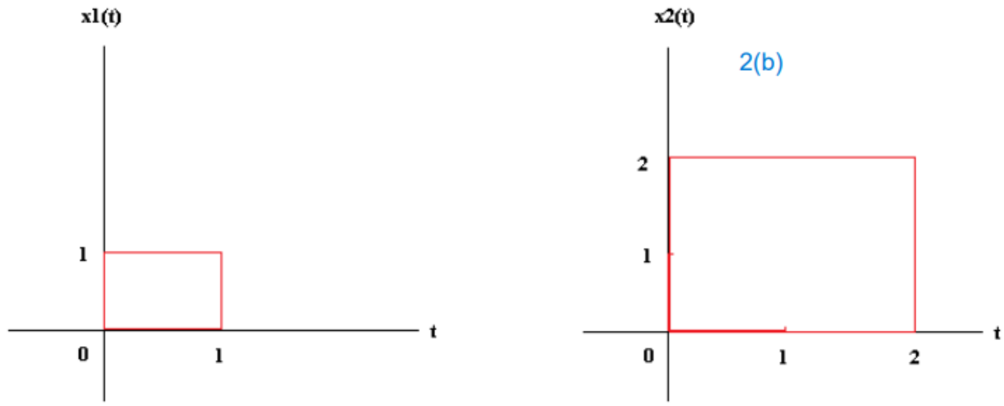
(ii) multiplication



(iii) Summation $y[3] = 0+0+0+0$
$= 0.$

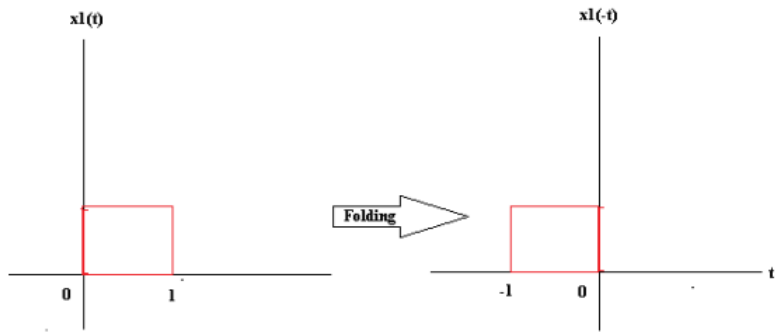
$\therefore$ Finally we got.

$y[0] = 2$
$y[1] = 2$
$y[2] = 1$
$y[3] = 0.$

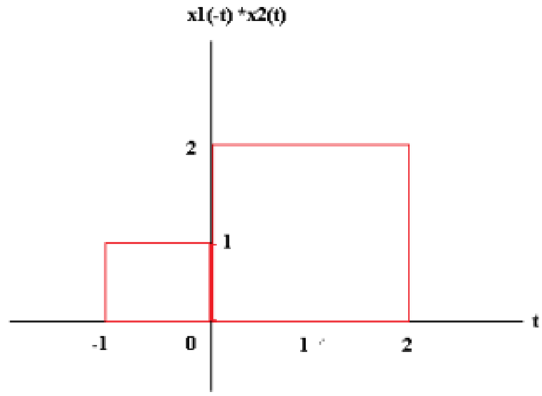


1[st] time:

i. Folding



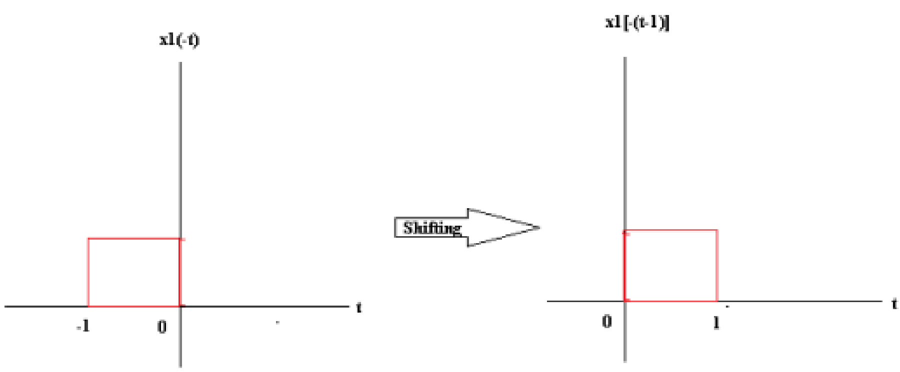
ii. Finding Area



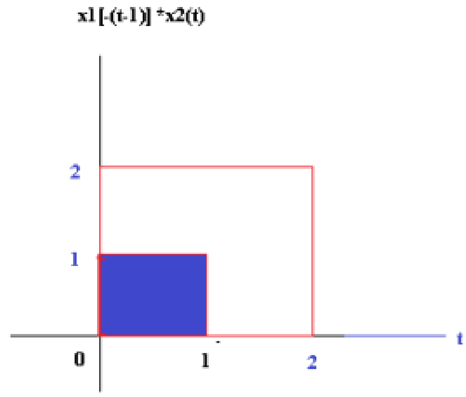
Finding Area: No overlapped area; x[0] =0

2nd time:

i. Shifting

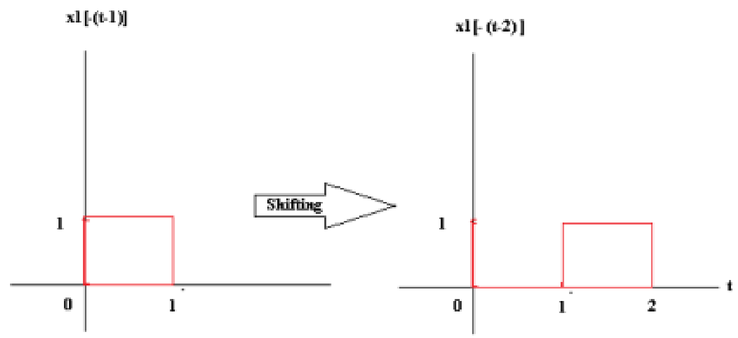


ii. Finding Area:

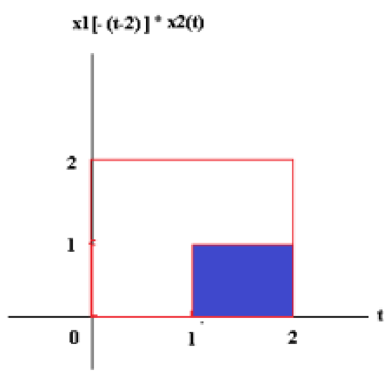


Area: x[1] $= 1 \times 1 = 1$

iii. Shifting

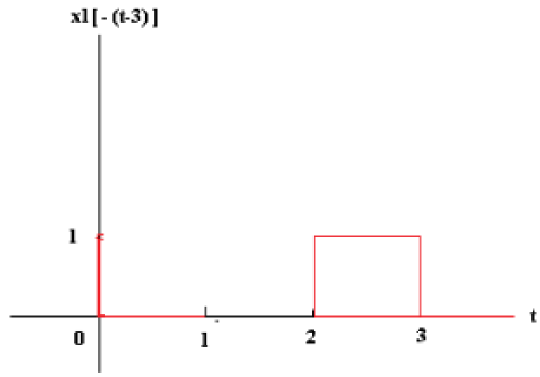


iv. Finding area

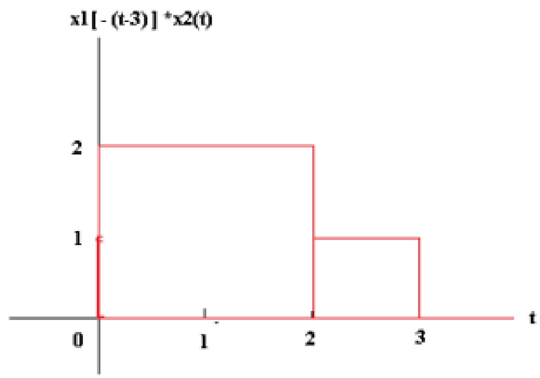


Area: $x[2] = 1 \times 1 = 1$

v. Shifting:

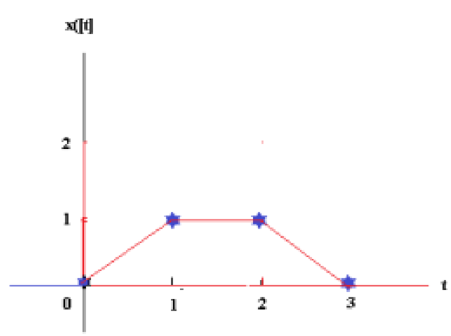


vi. Finding Area:



No overlapped: x [3] =0

Hence the convolution integral is



Example 41: Find Fourier Transform of

3(a)

$$f(t)=1 \quad ;0\le t<1$$
$$=-1 \quad ;-1\le t<0$$
$$=0 \quad ;|t|>1$$

We have $g(\omega)=\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$

$$g(\omega)=\int_{-\infty}^{-1} f(t)e^{-i\omega t}dt+\int_{-1}^{0} f(t)e^{-i\omega t}dt+\int_{0}^{1} f(t)e^{-i\omega t}dt+\int_{1}^{\infty} f(t)e^{-i\omega t}dt$$

$$g(\omega)=\int_{-\infty}^{-1} 0.e^{-i\omega t}dt+\int_{-1}^{0}(-1)e^{-i\omega t}dt+\int_{0}^{1}1.e^{-i\omega t}dt+\int_{1}^{\infty}0.e^{-i\omega t}dt$$

$$g(\omega)=-\int_{-1}^{0} e^{-i\omega t}dt+\int_{0}^{1} e^{-i\omega t}dt$$

$$g(\omega)=-\left[\frac{e^{-i\omega t}}{-i\omega}\right]_{-1}^{0}+\left[\frac{e^{-i\omega t}}{-i\omega}\right]_{0}^{1} \qquad [\because \int e^{-mx}dx=\frac{e^{-mx}}{-m}]$$

$$g(\omega)=-\left[\frac{e^{-i\omega.0}}{-i\omega}-\frac{e^{-i\omega(-1)}}{-i\omega}\right]+\left[\frac{e^{-i\omega.1}}{-i\omega}-\frac{e^{-i\omega.0}}{-i\omega}\right]$$

$$g(\omega)=-\left[\frac{e^{-0}}{-i\omega}-\frac{e^{i\omega}}{-i\omega}\right]+\left[\frac{e^{-i\omega}}{-i\omega}-\frac{e^{-0}}{-i\omega}\right]$$

$$g(\omega)=-\left[\frac{\frac{1}{e^{0}}}{-i\omega}-\frac{e^{i\omega}}{-i\omega}\right]+\left[\frac{e^{-i\omega}}{-i\omega}-\frac{\frac{1}{e^{0}}}{-i\omega}\right]$$

$$g(\omega)=-\left[\frac{\frac{1}{1}}{-i\omega}-\frac{e^{i\omega}}{-i\omega}\right]+\left[\frac{e^{-i\omega}}{-i\omega}-\frac{\frac{1}{1}}{-i\omega}\right]$$

$$g(\omega)=-\left[\frac{1}{-i\omega}-\frac{e^{i\omega}}{-i\omega}\right]+\left[\frac{e^{-i\omega}}{-i\omega}-\frac{1}{-i\omega}\right]$$

$$g(\omega)=\left[\frac{1}{i\omega}-\frac{e^{i\omega}}{i\omega}\right]+\left[-\frac{e^{-i\omega}}{i\omega}+\frac{1}{i\omega}\right]$$

$$g(\omega)=\frac{1}{i\omega}+\frac{1}{i\omega}-\frac{e^{i\omega}}{i\omega}-\frac{e^{-i\omega}}{i\omega}$$

$$g(\omega)=\left[\frac{e^{0}}{(1-i\omega)}-\frac{e^{-\infty}}{(1-i\omega)}\right]+\left[\frac{e^{-\infty}}{-(1+i\omega)}-\frac{e^{-0}}{-(1+i\omega)}\right]$$

$$g(\omega)=\left[\frac{1}{(1-i\omega)}-\frac{1}{e^{\infty}(1-i\omega)}\right]+\left[\frac{e^{-\infty}}{-(1+i\omega)}-\frac{e^{-0}}{-(1+i\omega)}\right]$$

$$g(\omega)=\frac{1}{(1-i\omega)}\left[1-\frac{1}{e^{\infty}}\right]+\frac{-1}{1+i\omega}\left[\frac{1}{e^{\infty}}-\frac{1}{e^{0}}\right]$$

$$g(\omega)=\frac{1}{(1-i\omega)}\left[1-\frac{1}{\infty}\right]+\frac{-1}{1+i\omega}\left[\frac{1}{\infty}-\frac{1}{1}\right]$$

$$g(\omega)=\frac{1}{(1-i\omega)}[1-0]+\frac{-1}{1+i\omega}\left[\frac{1}{\infty}-\frac{1}{1}\right]$$

$$g(\omega)=\frac{1}{(1-i\omega)}+\frac{-1}{1+i\omega}[0-1]$$

$$g(\omega)=\frac{1}{(1-i\omega)}+\frac{1}{1+i\omega}$$

$$g(\omega)=\frac{1+i\omega+1-i\omega}{(1-i\omega)(1+i\omega)}$$

$$g(\omega)=\frac{2}{(1-i\omega)(1+i\omega)}$$

$$g(\omega)=\frac{2}{(1-i^{2}\omega^{2})}$$

$$g(\omega)=\frac{2}{1+\omega^{2}} \qquad [\,i^{2}=-1\,]$$

Answer

Example 43: Find Fourier Transform for the given functions

$f(t)=e^{-2t}$;$t\geq 0$

$f(t)=0$;$t<0$

Answer:

Given

$f(t)=e^{-2t}$;$t\geq 0$

$f(t)=0$;$t<0$ --------------------------(i)

We have,

$$g(\omega)=\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

$$g(\omega)=\int_{-\infty}^{0} f(t)e^{-i\omega t}dt+\int_{0}^{\infty} f(t)e^{-i\omega t}dt$$

$$g(\omega)=\frac{2}{i\omega}-\frac{1}{i\omega}(e^{i\omega}+e^{-i\omega})$$

$$g(\omega)=\frac{2}{i\omega}-\frac{1}{i\omega}\frac{2}{2}(e^{i\omega}+e^{-i\omega})$$

$$g(\omega)=\frac{2}{i\omega}-\frac{2}{i\omega}\frac{1}{2}(e^{i\omega}+e^{-i\omega})$$

$$g(\omega)=\frac{2}{i\omega}-\frac{2}{i\omega}\cos\omega \qquad [\because \cos x=\frac{1}{2}(e^{ix}+e^{-ix})]$$

$$g(\omega)=\frac{2}{i\omega}(1-\cos\omega)$$ *Answer*

Example 42: Find Fourier Transform of $f(t)=e^{-|t|}$

Or

Find Fourier Transform of

$$f(t)=e^{-t} \qquad ;t>0$$
$$\quad =e^{t} \qquad ;t<0$$

Answer:

Given

$$f(t)=e^{-t} \qquad ;t>0$$
$$\quad =e^{t} \qquad ;t<0 \qquad \text{--------------------------(i)}$$

We have,

$$g(\omega)=\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

$$g(\omega)=\int_{-\infty}^{0} f(t)e^{-i\omega t}dt+\int_{0}^{\infty} f(t)e^{-i\omega t}dt$$

$$g(\omega)=\int_{-\infty}^{0} e^{t}\,e^{-i\omega t}dt+\int_{0}^{\infty} e^{-t}\,e^{-i\omega t}dt \qquad \text{[Given equation no (i)]}$$

$$g(\omega)=\int_{-\infty}^{0} e^{t}\,e^{-i\omega t}dt+\int_{0}^{\infty} e^{-t}\,e^{-i\omega t}dt$$

$$g(\omega)=\int_{-\infty}^{0} e^{t-i\omega t}dt+\int_{0}^{\infty} e^{-t-i\omega t}dt$$

$$g(\omega)=\int_{-\infty}^{0} e^{(1-i\omega)t}dt+\int_{0}^{\infty} e^{-(1+i\omega)t}dt$$

$$g(\omega)=\left[\frac{e^{(1-i\omega)t}}{(1-i\omega)}\right]_{-\infty}^{0}+\left[\frac{e^{-(1+i\omega)t}}{-(1+i\omega)}\right]_{0}^{\infty}$$

$$g(\omega)=\left[\frac{e^{(1-i\omega).0}}{(1-i\omega)}-\frac{e^{(1-i\omega)(-\infty)}}{(1-i\omega)}\right]+\left[\frac{e^{-(1+i\omega)\infty}}{-(1+i\omega)}-\frac{e^{-(1+i\omega).0}}{-(1+i\omega)}\right]$$

Q-105: Solve the following Initial Value Problem (IVP) by Laplace Transform $Y'' - 3Y' + 2Y = 4e^{2t}$

$$Y(0) = -3 \qquad Y'(0) = 5$$

Solution

$Y = f(t)$

4(a)

Given,

$Y'' - 3Y' + 2Y = 4e^{2t}$

Taking the Laplace transform of both sides of the differential equation and using the given conditions, we have

$Y'' - 3Y' + 2Y = 4e^{2t}$

$L\{Y''\} - 3L\{Y'\} + 2L\{Y\} = 4L\{e^{2t}\}$

We have,

$L(f''(t)) = s^2 L\{f(t)\} - s\,f(0) - f'(0)$

And

$\therefore L(f'(t)) = sL\{f(t)\} - f(0)$

$\therefore L\{Y''\} - 3L\{Y'\} + 2L\{Y\} = 4L\{e^{2t}\}$

$$s^2L\{f(t)\} - sf(0) - f'(0) - 3[sL\{f(t)\} - f(0)] + 2y = 4\frac{1}{s-2} \quad [\text{let}, L\{Y\} = y \ \& \ L(e^{at}) = \frac{1}{s-a}]$$

$$s^2L\{Y\} - sf(0) - f'(0) - 3[sL\{Y\} - f(0)] + 2y = 4\frac{1}{s-2} \quad [Y = f(t)]$$

$$s^2y - sf(0) - f'(0) - 3[sy - f(0)] + 2y = 4\frac{1}{s-2} \quad [\text{let}, L\{Y\} = y]$$

$$s^2y - sf(0) - f'(0) - 3sy + 3f(0) + 2y = 4\frac{1}{s-2}$$

$$s^2y - s\{-3\} - 5 - 3sy + 3(-3) + 2y = 4\frac{1}{s-2}$$

$$s^2y + 3s - 5 - 3sy - 9 + 2y = 4\frac{1}{s-2}$$

$$s^2y + 3s - 3sy - 14 + 2y = 4\frac{1}{s-2}$$

$$s^2y - 3sy + 2y + 3s - 14 = 4\frac{1}{s-2}$$

$$s^2y - 3sy + 2y = -3s + 14 + 4\frac{1}{s-2}$$

$$y(s^2 - 3s + 2) = -3s + 14 + 4\frac{1}{s-2}$$

$$y(s^2 - 2s - s + 2) = -3s + 14 + 4\frac{1}{s-2}$$

$$y\{s(s-2) - 1(s-2) = -3s + 14 + 4\frac{1}{s-2}$$

$$y(s-1)(s-2) = -3s + 14 + 4\frac{1}{s-2}$$

$$y = -3\frac{s}{(s-1)(s-2)} + 14\frac{1}{(s-1)(s-2)} + 4\frac{1}{(s-2)}\frac{1}{(s-1)(s-2)}$$

$$y = \frac{-3s}{(s-1)(s-2)} + \frac{14}{(s-1)(s-2)} + \frac{4}{(s-1)(s-2)^2}$$

$$y = \frac{-3s+14}{(s-1)(s-2)} + \frac{4}{(s-1)(s-2)^2}$$

$$y = \frac{(-3s+14)(s-2)+4}{(s-1)(s-2)^2}$$

$$y = \frac{-3s^2+6s+14s-28+4}{(s-1)(s-2)^2}$$

$$y = \frac{-3s^2+6s+14s-24}{(s-1)(s-2)^2}$$

Applying partial fraction

Let,

$$y = \frac{-3s^2+6s+14s-24}{(s-1)(s-2)^2} = \frac{A}{(s-1)} + \frac{B}{(s-2)} + \frac{C}{(s-2)^2}$$

....

....

$$y = \frac{-3s^2+6s+14s-24}{(s-1)(s-2)^2} = \frac{-7}{(s-1)} + \frac{4}{(s-2)} + \frac{4}{(s-2)^2}$$

$$\therefore L\{Y\} = y = \frac{-3s^2+6s+14s-24}{(s-1)(s-2)^2} = \frac{-7}{(s-1)} + \frac{4}{(s-2)} + \frac{4}{(s-2)^2}$$

$$\therefore Y = L^{-1}(y) = -7\frac{1}{2}L^{-1}[\frac{1}{(s-1)}] + 4L^{-1}[\frac{1}{(s-2)}] + 4L^{-1}[\frac{4}{(s-2)^2}]$$

$$\therefore Y = L^{-1}(y) = -7e^{t} + 4e^{2t} + 4te^{2t}$$

Example 89: **Given that,** $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

Answer:

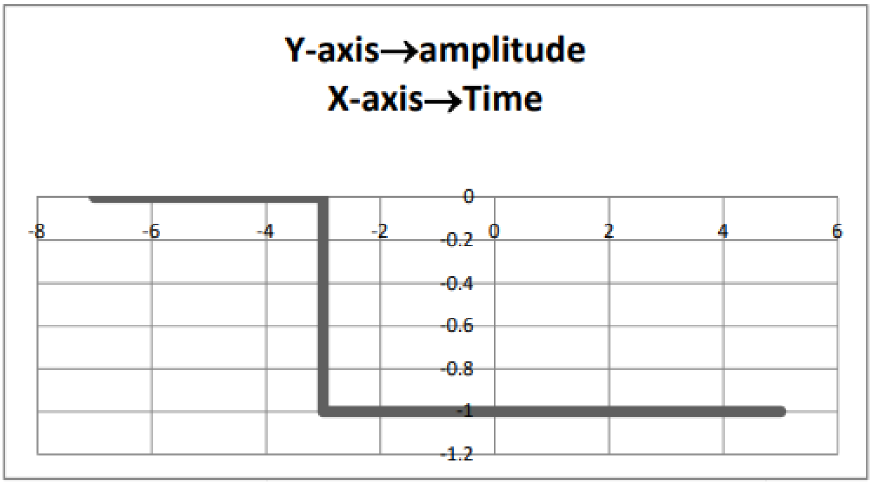
01. $-u(t+3)$ =>

4(b)

So,

$-u(t+3) = -1\ ;\ t \geq -3;$ $\qquad$ $here, t+3 = 0$

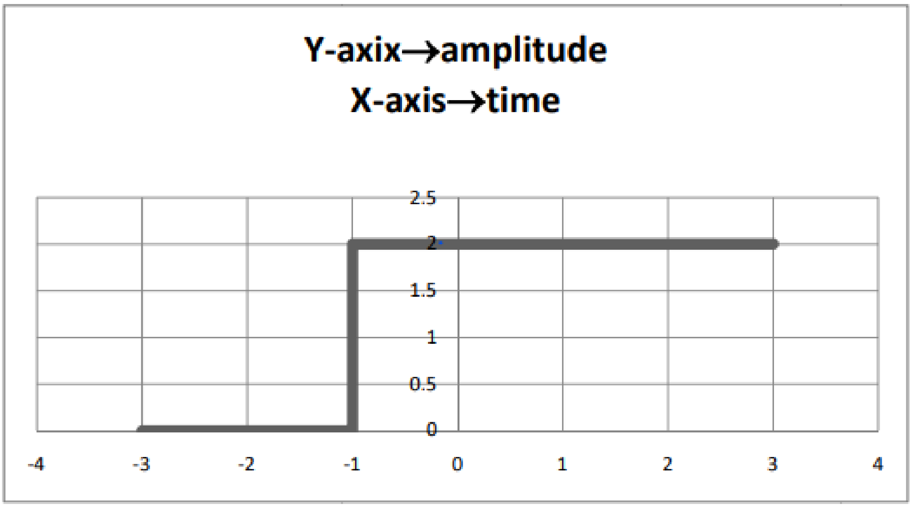
$\qquad = 0;\ \ t < -3$ $\qquad$ $\therefore t = -3$



02. $2u(t+1)$

$2u(t+1) = 2; t \geq -1$ $\qquad$ $here, t+1 = 0$

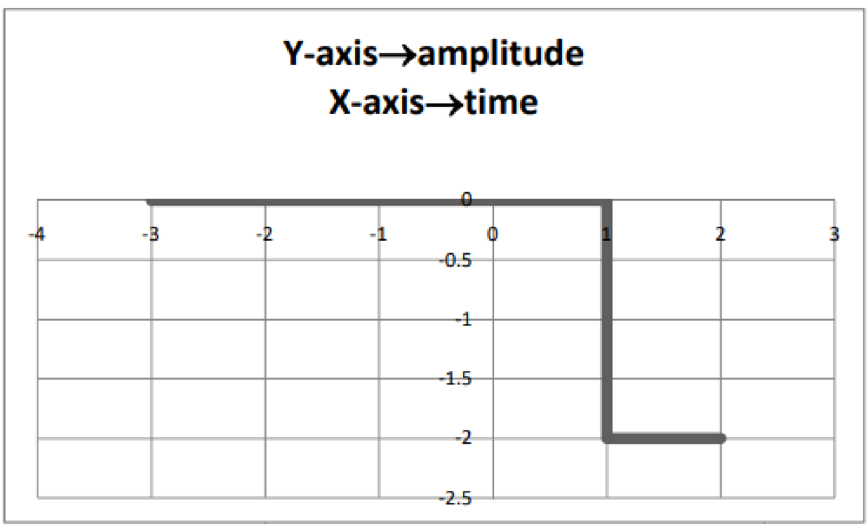
$= 0; t < -1$ $\qquad$ $t = -1$



03. $-2u\,(t-1)$

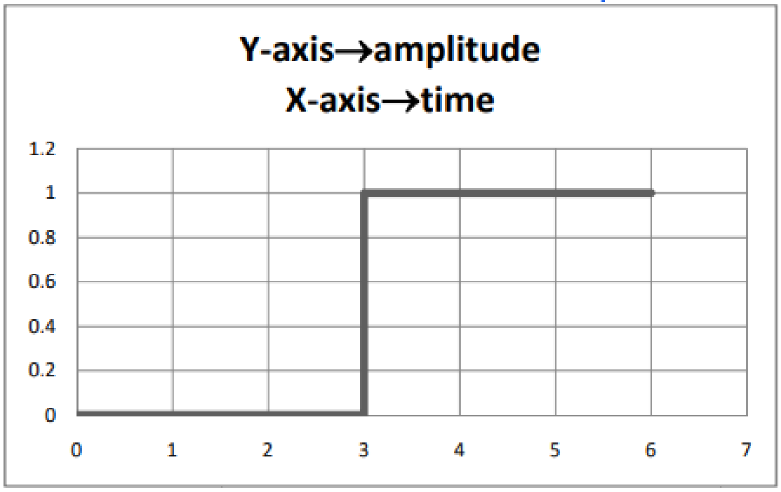
$\therefore\ -2u(t-1) = -2; t \geq 1$ $\qquad$ $here, t-1 = 0$

$= 0;\ t < 1$ $\qquad$ $t = 1$

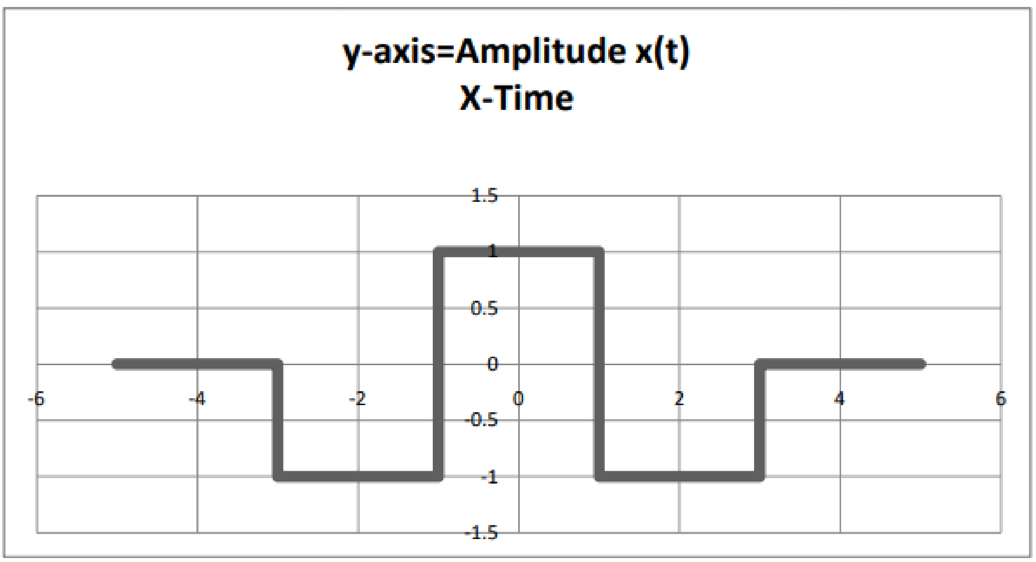


04. $\boldsymbol{u(t-3)} =>$ $here, t-3=0$

$t=3$



$\boldsymbol{x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)}$



Au-21

5 a)

$$f(t) = 2.5 + \left[-\frac{5}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (\cos n\pi - 1) \sin \frac{n\pi t}{4} \right]$$

we have fourier series $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega$

Here,

$a_n = 0$, $b_n = \frac{5}{\pi n} (\cos n\pi - 1)$

$R_n = \sqrt{a_n^2 + b_n^2}$

$n\omega = \frac{n\pi}{4}$

দরকার নাই লেখার

```
x = 1 : 6
W = x.*pi/4
V = ((-5/pi)*1./x).* (cos(x*pi)-1)
steam(ω,V)
```

b

```
function [result] = four1(n)
t = -4:.0001:20;
y = pi;
sum = y';
for (i=1:1:n)
sum = sum - ((2/i)*sin(i*pi*t))
end
```

c

$a = [1, 2]$

$b = [-1\ \ -2]$

$y = \text{conv}(x, h)$

Ans: