Lecture Sheet-2

GRAVITY AND GRAVITATION

Chapter Outline

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Definition of Gravity

The force of attraction between the earth and other bodies on the earth is known as gravity.

Or,

The force that attracts a body toward the center of the earth is called gravity.

Definition of Gravitation

The natural phenomenon of attraction between physical objects with mass is known as gravitation.

Or,

The force of attraction between any two bodies of the universe is called gravitation.

The gravitation depends on the mass of the bodies and the distance between them not other properties such as the medium.

Newton's Law of Gravitation

Every body in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

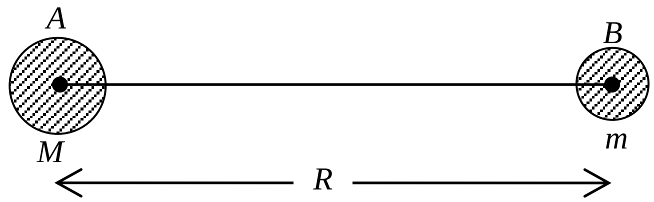

Fig. 2.1

Let us consider two bodies A and B of masses M and m respectively. The distance between their centers is R.

Then we get,

$$F \propto Mm \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$F \propto \frac{1}{R^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From Eqs. (1) & (2) we get,

$$F \propto \frac{Mm}{R^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

From which we get,

$$F = G\frac{Mm}{R^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Where G is proportionality constant which is the universal constant of gravitation.

$$G = 6.67 \times 10^{-11} Nm^2/kg^2$$

Compound Pendulum

Pendulum consisting of an actual object allowed to rotate freely around a horizontal axis is known as compound pendulum.

Or,

A pendulum consisting of any swinging rigid body which is free to rotate about a fixed horizontal axis is called a compound pendulum.

Or,

A compound pendulum is a rigid mass capable of oscillating about a horizontal axis passing through any point of the mass. This point is called the point of suspension.

Gravitational Field

The space around a body within which the gravitational force is acted is called gravitational field. The gravitational field or intensity at a point is the force experienced by a unit mass at that point. If the gravitational field at a point is *E*, the force acting on a mass *m* is

$$F = mE$$

$$E = \frac{F}{m}$$

Also the gravitational field is defined as the negative gradient of the gravitational potential.

$$\therefore\ E = -\frac{dV}{dx}$$

Gravitational Potential

The gravitational potential at a point is defined as the work done to bring a unit mass from the point to infinity against the gravitational force of attraction.

Let us consider a particle A of mass m. P is a point at a distance r from A. The gravitational intensity at P,

$$E = \frac{F}{m} = \frac{GM}{r^2}$$

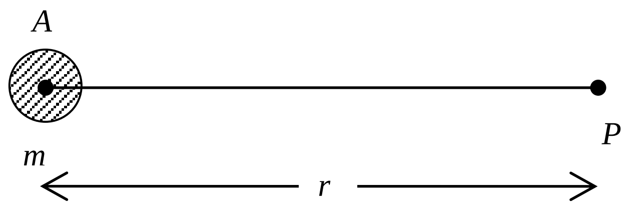

Fig. 2.2

Also we get,

$$E = -\frac{dV}{dr}$$

$$dV = -Edr$$

$$dV = -\frac{GM}{r^2}dr$$

Integrating above equation between the limits *r* to ∞ we get,

$$V = -\frac{GM}{r}$$

This is the equation of gravitational potential at a point due to a point mass. The gravitational potential is zero at infinity. At all other points its value is negative.

Gravitational potential due to spherical shell

(a) At a point outside the shell

Let us consider a uniform spherical shell of mass *M* and radius *a* (Fig. 2.3). Let ρ be the density per unit area of the shell. The planes AC and BD cut the shell vertically and the element between the two planes is a ring of radius, $AB = a.d\theta$.

Surface area of the element $= 2\pi(asin\theta)a.d\theta$

Mass of the element, $m = (2\pi a^2 sin\theta d\theta)\rho$

Also $AP = x$

Potential at *P* due to the element,

$$dV = -\frac{Gm}{x}$$

$$dV = -\frac{2G\pi a^2 \rho sin\theta d\theta}{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

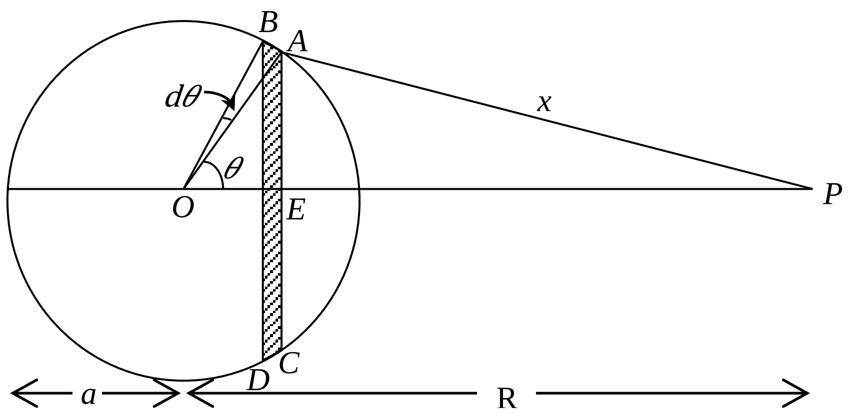

Fig. 2.3

In the ΔOAP

$$x^2 = a^2 + R^2 - 2aRcos\theta$$

Differentiating,

$$2xdx = 2aRsin\theta d\theta$$

$$x = \frac{aRsin\theta d\theta}{dx}$$

Putting this value of *x* in Eq.(1)

$$dV = -\frac{2G\pi a^2 \rho sin\theta d\theta dx}{aRsin\theta d\theta}$$

$$dV = -\left(\frac{2\pi Ga\rho}{R}\right) dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Integrating for the whole shell

$$V = -\int_{R-a}^{R+a} \left(\frac{2\pi Ga\rho}{R}\right) dx$$

$$V = -\frac{4\pi a^2 \rho G}{R}$$

But $M = 4\pi a^2 \rho$

$$\therefore V = -\frac{GM}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Thus for a point outside the shell, the shell behaves as if the whole of its mass concentrated at the centre of the shell.

Also we get from Eq.(3)

$$V \infty \frac{1}{R}$$

Gravitational Field

$$E = -\frac{dV}{dR}$$

$$E = -\frac{d}{dR}\left(-\frac{GM}{R}\right)$$

$$\therefore\ E = -\frac{GM}{R^2}$$

(b) At a point inside the inside a shell

Potential at a point inside the shell, due to the element (Fig. 2.4)

$$dV = -\left(\frac{2\pi G a\rho}{R}\right)dx$$

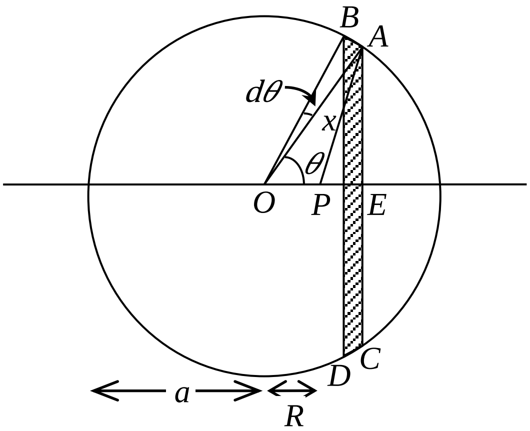

Fig. 2.4

For the whole shell integrating between the limits $(a - R)$ and $(a + R)$

$$V = -\int_{a-R}^{a+R}\left(\frac{2\pi G a\rho}{R}\right)dx$$

$$V = -4G\pi a\rho$$

But $M = 4\pi a^2\rho$

$$\therefore V = -\frac{GM}{a} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4)$$

Eq.(4) shows that V is independent of R. It is the same at all points inside the shell. This potential is equal to the potential on the surface of the shell and is constant.

Gravitational Field

$$E = -\frac{dV}{dR}$$

$$E = -\frac{d}{dR}\left(-\frac{GM}{a}\right)$$

$$\therefore\ E = 0$$

Therefore the gravitational field at any point inside a spherical shell is zero.

Escape velocity

Escape velocity is defined as the velocity with which a body has to be projected vertically upwards from the earth's surface so that it escapes the earth's gravitational field.

If *v* is the escape velocity, then the initial kinetic energy of the projection $\frac{1}{2}mv^2$ must be equal to the work done in moving the body from the surface of the earth to infinity. In such a case the body does not return to the earth's surface.

Let a body is at a distance *x* from the center of the earth. The force of gravity on the body

$$= \frac{GMm}{x^2}$$

Where,

$M \rightarrow$ Mass of the earth

$m \rightarrow$ Mass of the body

$G \rightarrow$ Gravitational constant

$R \rightarrow$ Radius of the earth

If the body is to be moved through a small distance *dx* away from the surface of the earth, the work done,

$$dW = \left(\frac{GMm}{x^2}\right)dx \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Work done in moving the body from the from the surface of the earth to infinity

$$\int dW = \int_R^\infty \left(\frac{GMm}{x^2}\right)dx$$

$$W = \frac{GMm}{R} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

We have the kinetic energy of the body,

$$K.E = \frac{1}{2}mv^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

For escape velocity we get,

$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$\therefore v = \sqrt{\frac{2GM}{R}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(4)$$

If the body is at the surface of the earth, then we get,

$$F = \frac{GMm}{R^2}$$

$$mg = \frac{GMm}{R^2}$$

$$GM = gR^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

From Eqs. (5) & (6) we get,

$$v = \sqrt{2gR} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6)$$

Example 2.1. Calculate the escape velocity of a body from the surface of the earth.
[Answer: $11.2\ kms^{-1}$]

Example 2.2. Calculate the escape velocity of a body from the surface of the moon. The radius of moon is $1.7 \times 10^6 m$ and acceleration due to gravity for moon is $1.63\ ms^{-2}$.
[Answer: $2.354\ kms^{-1}$]

Example 2.3. What will be the escape velocity on the surface of moon if the radius of moon were $1/4^{th}$ radius of earth and mass $1/80^{th}$ of the mass of earth?
[Answer: $2.504\ kms^{-1}$]

Example 2.4. What will be the acceleration due to gravity on the surface of the moon if its radius were $1/4^{th}$ radius of earth and mass $1/80^{th}$ of the mass of earth? What will be the escape velocity on the surface of the moon if it is $11.2\ kms^{-1}$ on the surface of the earth?
[Answer: $1.96\ ms^{-2}$ $2.504\ kms^{-1}$]

Kepler's Law of Planetary Motion

There are three laws of planetary motion:

First law (The law of orbit): Every planet moves in an elliptical orbit with the sun being at one of its foci.

Second law (The law of areas): The line joining any planet to the sun sweeps out equal areas in equal intervals of time. i.e., the area velocity is constant.

Third law (The law of period): The square of the time period of revolution of a planet around the sun is proportional to the cube of the mean distance from the planet to the sun.