INTERNATIONAL ISLAMIC UNIVERSITY CHITTAGONG

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

B. Sc. in CSE Midterm Examination, Autumn 2017

Course Code: CSE 3609 Course Title: Theory of Computing

Total Marks: 30 Time: 1 hours 30 minutes

*Answer any **three** of the following questions*

Figures in the right hand margin indicate full marks

1.

a) Give the formal definition of ***finite automata.*** 2

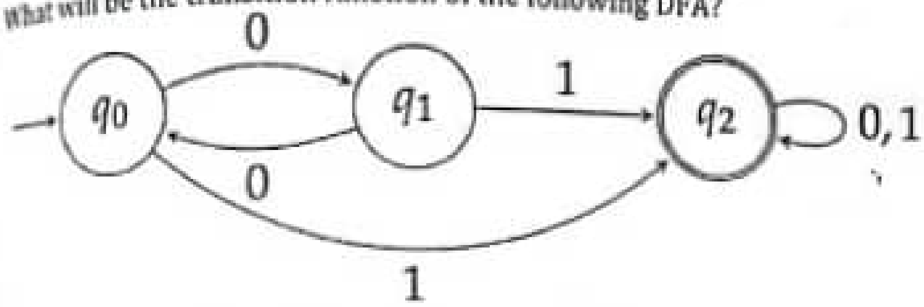
b) Construct DFA's for the following languages over the alphabet {a,b}: 4.5

i. {*w* | w has an odd number of a's and ends with a b}

ii. {*w*| w does not contain the substring 'baba'}

iii. {*w*| *w* has at exactly two a's and at least two b's}

c) What will be the transition function of the following DFA? 2.5



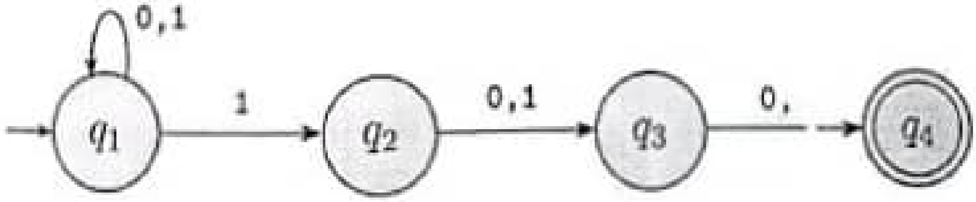
Write down 1 string that will be accepted by this DFA and 1 other string that will not be accepted. What is the language of this DFA?

d) Define *string* and ***language.*** 1

2.

a) What is a ***nondeterministic machine***? Explain. 2

b) What will be the language of the following NFA? 3



Show the computation of this NFA on input 1001101110.

c) Convert the NFA given above in an equivalent DFA. Show only the reachable states. 5

3.

a) Write down the regular expressions for the following languages where the alphabet is {a, b}. 3

i. {*w* | *w* begins with a b and every other b is preceded by an a}.

ii. {*w* | *w* contains at most two b's }.

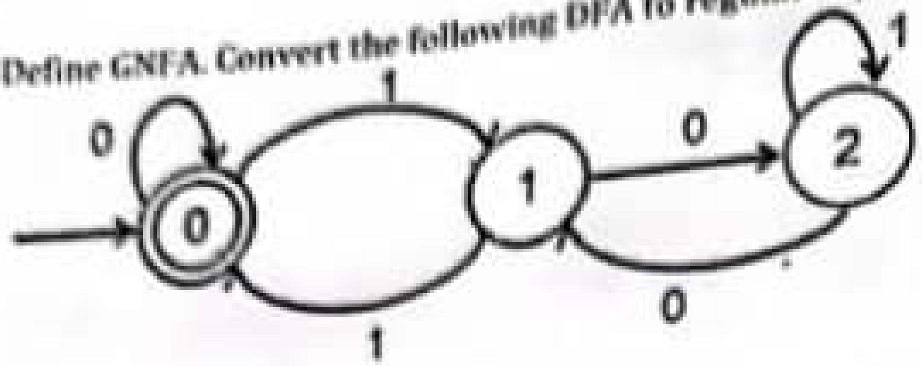
iii. {*w* | length of *w* is multiple of 3}.

b) Convert the following regular expressions to NFA: 3

i. ab* U abb

ii. (a U b)*aba

c) Define GNFA. Convert the following DFA to regular expression:



4.

a) Give the formal definition of context free grammar.

b) Given the grammar:

```
E → I | E+E | E*E | (E)
I → a | b | Ia | Ib | I0 | I1
```

Compute a leftmost derivation and a corresponding parse tree for the input string a*(a+b00).

c) Define Chomsky Normal Form. Convert the following grammar in Chomsky Normal Form:

$S \rightarrow aXbX$

$X \rightarrow aY|bY|\varepsilon$

$Y \rightarrow X|c$

d) Prove that the class of regular languages is closed under concatenation opération.

TOC

Mid-Term Autumn - 2017

1 a) Give the formal definition of finite automata.

Ans:- A finite automation is collection of 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where:

Q = finite set of states

Σ = finite set of input symbol

q_0 = initial state

F = final state

δ = Transition function

~~1b) Ex~~

1b) Construct DFA's for the following languages over the alphabet $\{a, b\}$:

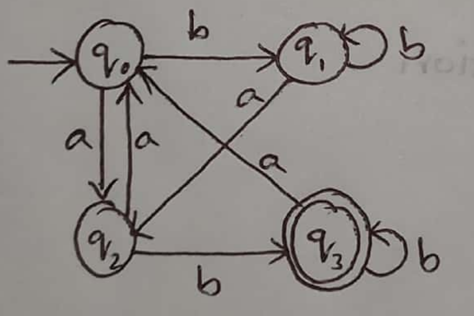
i. $\{ w \mid w$ has an odd number of a's and ends with a b $\}$

Ans :- $\Sigma = \{a, b\}$

$L(M) = \{ab, bab, aaab, abaab \ldots\}$

↓

Possible strings

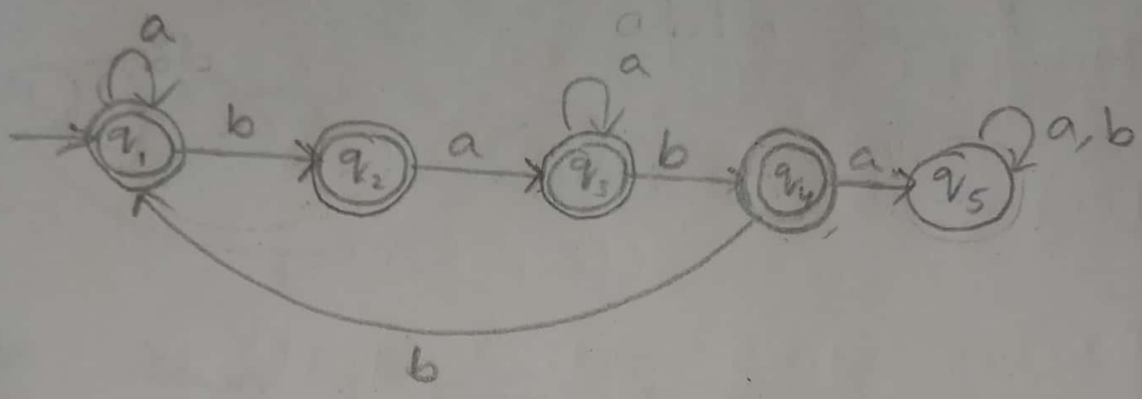


Explanation video:

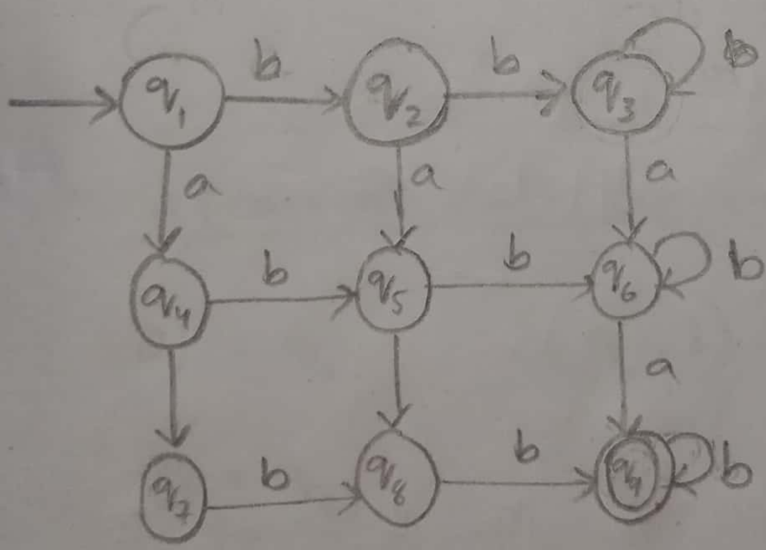
Channel : Sticky Notes

Construct DFA that has an odd number of a's and ends with 'b'.

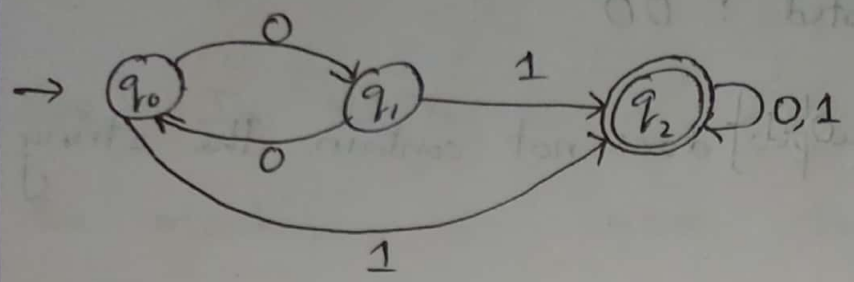
ii. $\{w \mid w$ does not contain the substring 'baba'$\}$



iii) $\{w \mid w$ has at exactly two a's and at least two b's$\}$



c) What will be the transition function of the following DFA ?



Write down 1 string that will be accepted by this DFA and 1 other string that will not be accepted. What is the language of this DFA ?

Ans :- Transition function, $\delta : Q \times \Sigma \rightarrow Q$

$\downarrow$

Cartisian Product

Formula: δ (current state, current input) = next state

$\delta(q_0, 0) = q_1$ $\quad$ $\delta(q_0, 1) = q_2$

$\delta(q_1, 1) = q_2$

$\delta(q_2, 0) = q_2$

$\delta(q_2, 1) = q_2$

String accepted : 011

String not accepted : 00

Language : $L = \{w \mid w$ does not contain the string '00'$\}$

d) Define string and language.

Ans:- String - It is a finite sequence of symbols taken from Σ.

Eg:- 'cabad' is a valid string on the ~~aph~~ alphabet set $\Sigma = \{a, b, c, d\}$

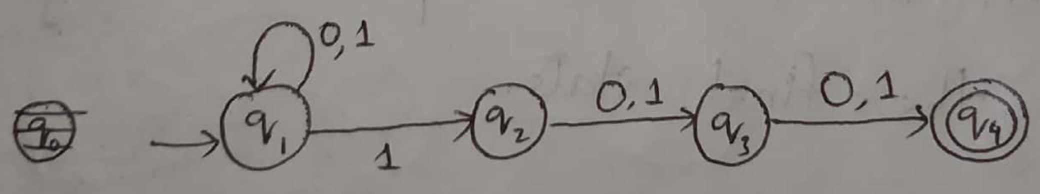
Language - It is a subset of Σ^* for some alphabet Σ. It can be finite or infinite.

Eg:- If the language takes all possible strings of length 2 over $\Sigma = \{a, b\}$, then $L = \{ab, bb, ba, aa\}$

2a) What is a nondeterministic machine? Explain.

Ans :- In NDFA, for a particular input symbol, the machine can move to any combination of the states in the machine. In other words, the exact state to which the machine moves cannot be determined. Hence, it is called non-deterministic automation. It has finite number of states.

2b) What will be the language of the following NFA?



Show the computation of this NFA on input 1001101110.

Language of the NFA

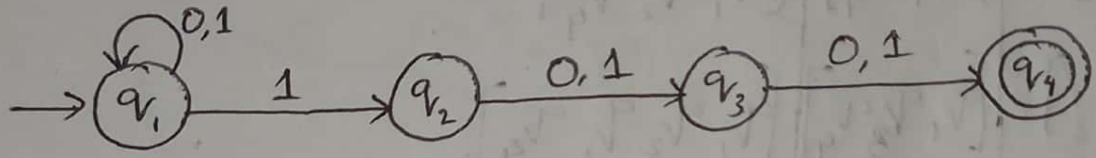
is = {w | w contains all strings over 0 and 1 containing a 1 in the third position from the end }

The computation of this NFA stays on the start state q_1 unit it "guesses" that it is three places from the end. For the input 100110111 0, the start state q_1 is looped till the part '1001101'. For the next input 1, it goes to state q_2 then the next 1, it goes to q_3 and then for the next input 0, it goes to q_4, the final state.

2c) Convert the NFA given above in an equivalent DFA. Show only the reachable states.

Ans:-

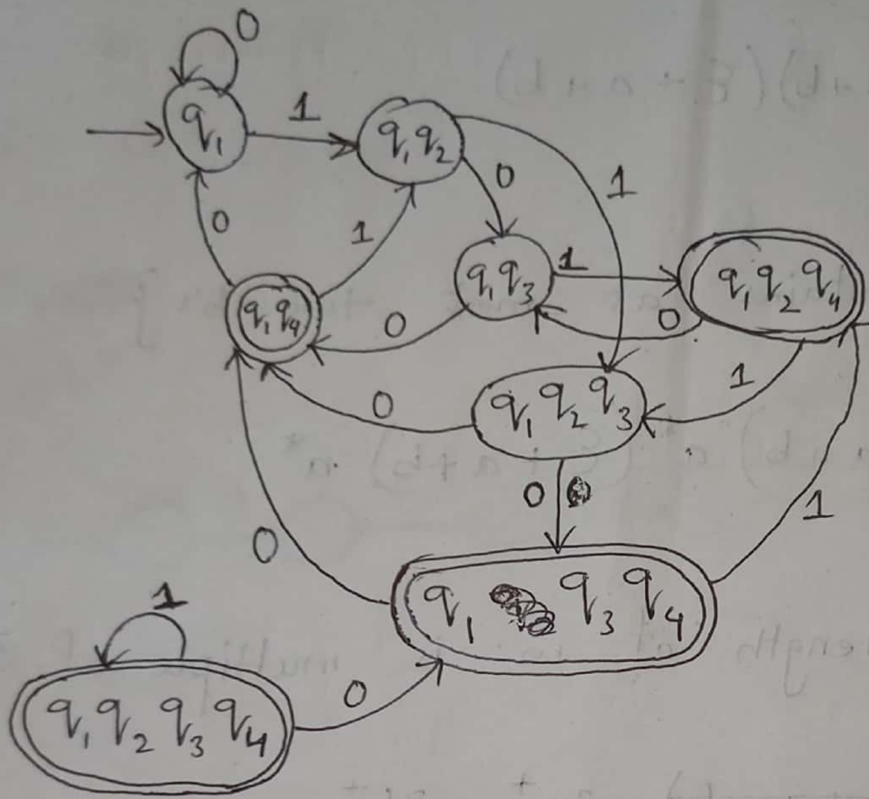
NFA



	0	1
$\rightarrow q_1$	$\{q_1\}$	$\{q_1, q_2\}$
q_2	$\{q_3\}$	$\{q_3\}$
q_3	$\{q_4$	$\{q_4\}$
q_4	—	—

DFA

	0	1
$\rightarrow [q_1]$	$[q_1]$	$[q_1 q_2]$
$[q_1 q_2]$	$[q_1 q_3]$	$[q_1 q_2 q_3]$
$[q_1 q_3]$	$[q_1 q_4]$	$[q_1 q_2 q_4]$
$[q_1 q_2 q_3]$	$[q_1 q_3 q_4]$	$[q_1 q_2 q_3 q_4]$
$[q_1 q_4]$	$[q_1]$	$[q_1 q_2]$
$[q_1 q_2 q_4]$	$[q_1 q_3]$	$[q_1 q_2 q_3]$
$[q_1 q_3 q_4]$	$[q_1 q_4]$	$[q_1 q_2 q_4]$
$[q_1 q_2 q_3 q_4]$	$[q_1 q_3 q_4]$	$[q_1 q_2 q_3 q_4]$



3a) Write down the regular expressions for the following languages where the alphabet is $\{a,b\}$.

i. $\{w \mid w$ begins with a 'b' and every other b is preceded by an a$\}$.

Ans :- ~~b o ab*~~ $b \circ (ab)^*$

$a^* = 0$ or more
$= \{\varepsilon, a, aa, \ldots\}$

$a^+ = 1$ or more
$= \{a, aa, aaa, \ldots\}$

ii. $\{w \mid w$ contains at most two b's $\}$

Ans:- $(\varepsilon + a + b)\, a^* (\varepsilon + a + b)\, a^*$

iii. $\{w \mid$ ~~w~~ length of w is multiple of $3\}$

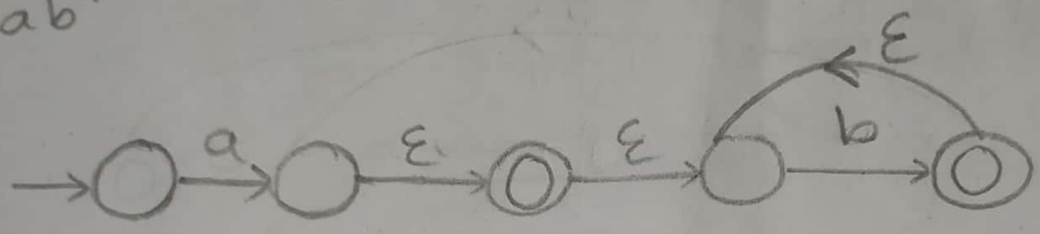
Ans:- $\cancel{3(\varepsilon + a + b)}\ 3a^+ + 3b^+$

3b) Convert the following regular expressions to NFA:

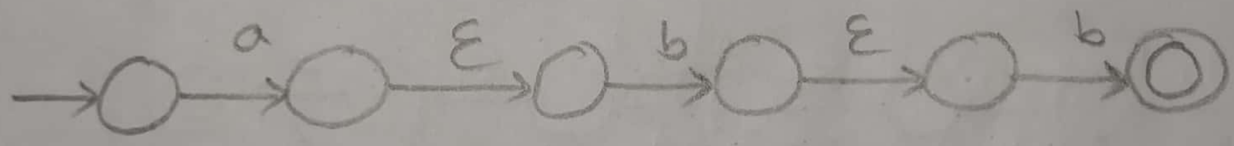
i. $ab^* \cup abb$

Ans:-

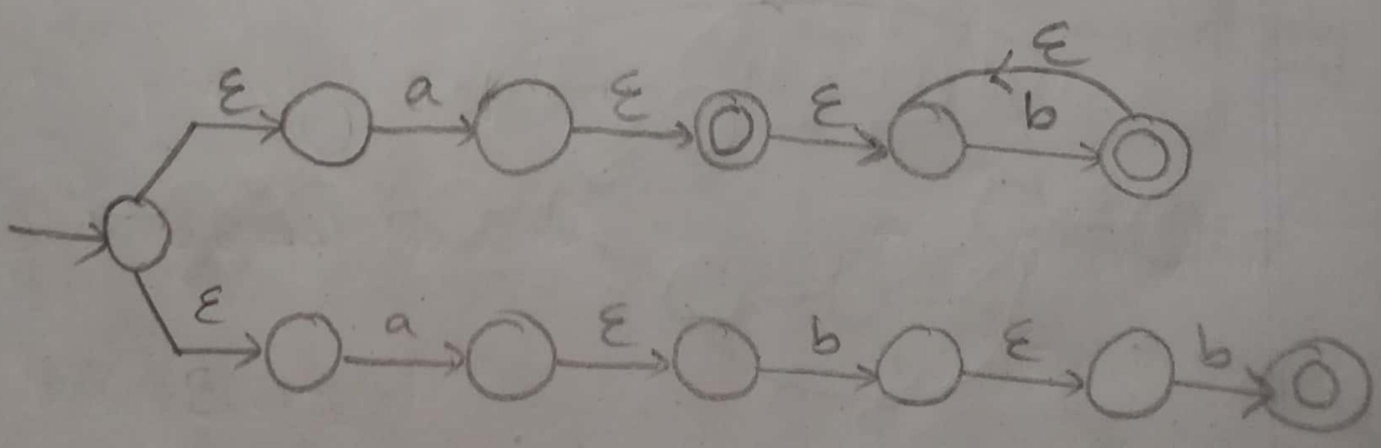
ab^*



abb

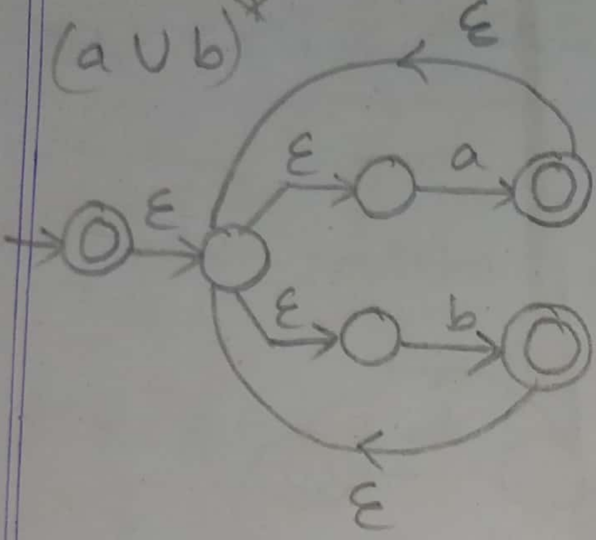


$ab^* \cup abb$

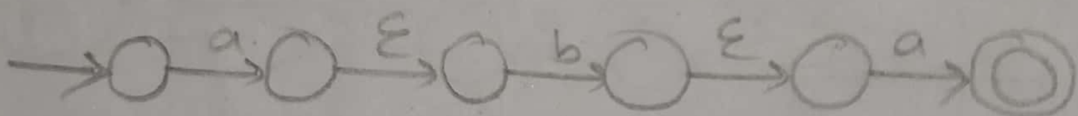


ii) $(a \cup b)^* aba$

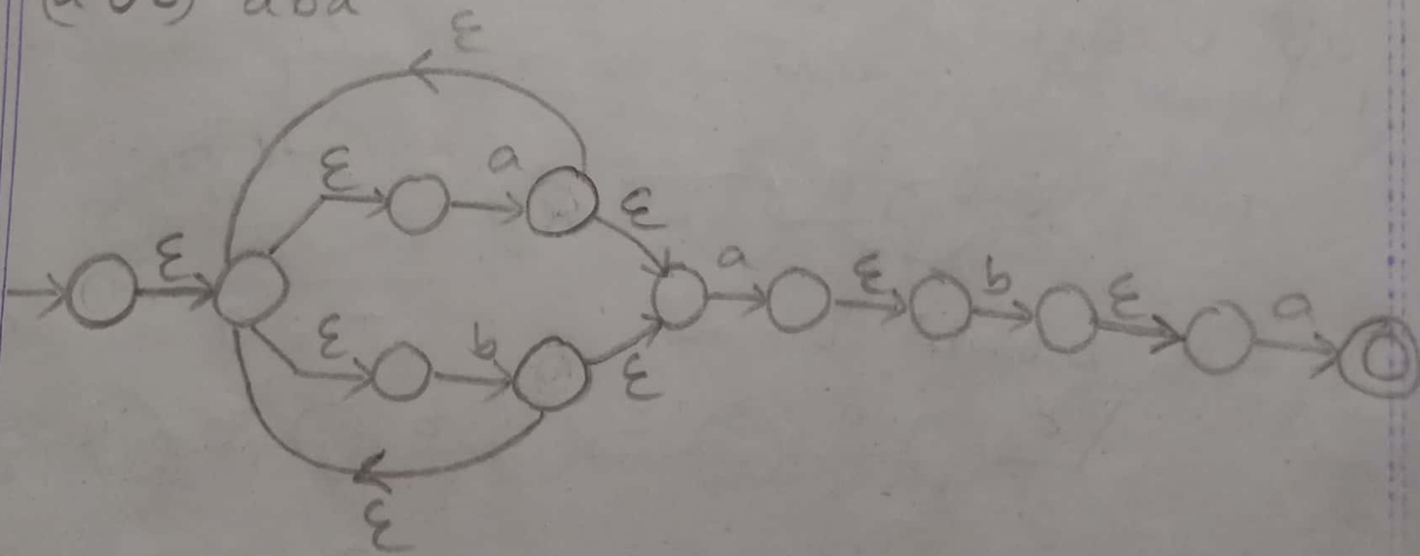
$(a \cup b)^*$



aba

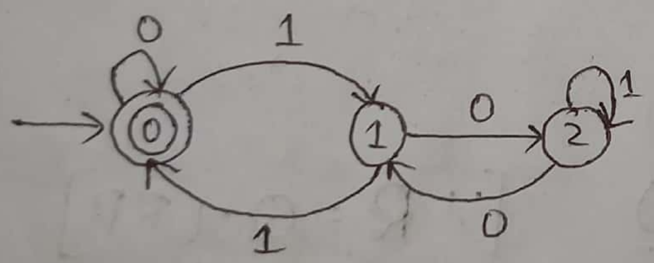


$(a \cup b)^* aba$



c) GNFA - It stands for "Generalized Non-Deterministic Finite Automaton". It is a theoretical model of computation that extends the standard ~~Non-Deterministic~~ NFA by allowing transitions between states to include regular expressions as labels.

Convert the following DFA to regular expressions:



Let the states 0, 1, 2 be a, b, c respectively

$a = \varepsilon + a0 + b1 \longrightarrow$ (i)

$b = a1 + c0 \longrightarrow$ (ii)

$c = b0 + c1 \longrightarrow$ (iii)

Eq (iii)

$c = b0 + c1$

Putting values of b,

$c = a01 + c00 + c1$

$\underbrace{c}_{R} = \underbrace{a01}_{Q} + \underbrace{c}_{R}\underbrace{(00 + 1)}_{P}$ $\left[\because R = Q + RP \quad R = QP^*\right]$

$c = a01(00+1)^* \longrightarrow$ (iv)

Putting values of ~~and~~ b and c in 1

$$a = \varepsilon + a0 + b1$$
$$= \varepsilon + a0 + a11 + c01$$
$$= \varepsilon + a0 + a11 + a01(00+1)^{*}01$$
$$a = \varepsilon + a[0 + 11 + 01(00+1)^{*}01]$$
$$= \varepsilon[0 + 11 + 01(00+1)^{*}01]^{*} \qquad [\because \varepsilon R = R]$$
$$\therefore a = [0 + 11 + 01(00+1)^{*}01]^{*}$$ ~~Ans~~

Ans:- R.E : $[0 + 11 + 01(00+1)^{*}01]^{*}$

4a) Give the formal definition of context free grammar.

Ans:- A context-free grammar is a 4-tuple (V, Σ, R, S) where

1. V is a finite set called the variables,
2. Σ is a finite set, disjoint from V, called the terminals.
3. R is a finite set of rules,
4. $S \in V$ is the start variable.

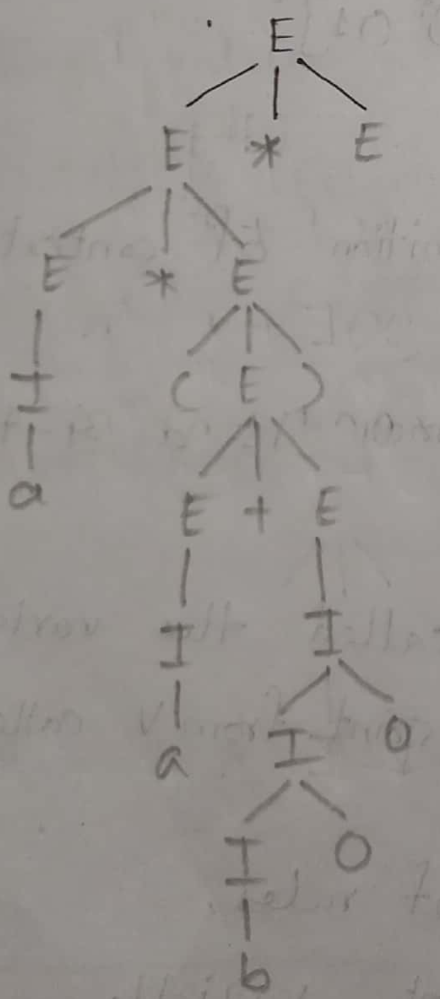
4b) Given the grammar:

$$E \to I \mid E+E \mid E*E \mid (E)$$

$$I \to a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$

Compute a leftmost derivation and a corresponding parse tree for the input string $a*(a+b00)$.

Ans: Leftmost derivation:



$E \to E*E$

$E \to I*(E)$

$E \to a*(E+E)$

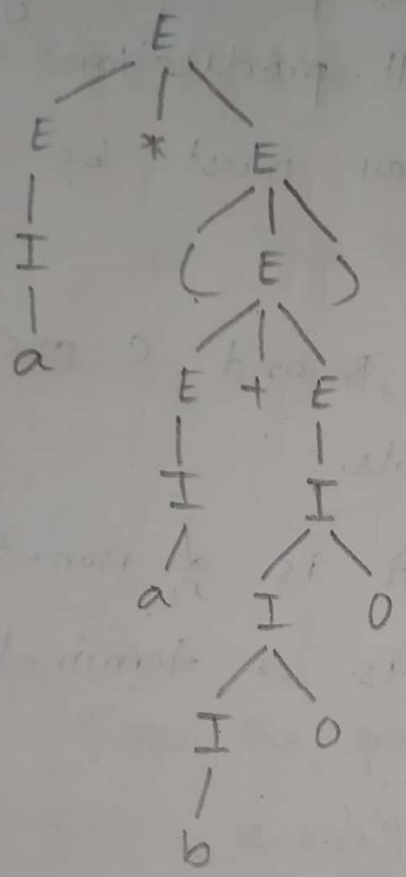
$E \to a*(I+I)$

$E \to a*(\cancel{I}a+I0)$

$E \to a*(a+I00)$

$E \to a*(a+b00)$

parse tree:



4c) Define Chomsky Normal Form. Convert the following grammar in Chomsky Normal Form:

$S \rightarrow aXbX$

$X \rightarrow aY \mid bY \mid \varepsilon$

$Y \rightarrow X \mid c$

Ans:- Chomsky Normal Form (CNF) is a way to represent a context-free grammar in a specific form that allows for simpler parsing algorithms. In CNF, all production rules of the grammar must be on of two forms:

1. $A \rightarrow BC$, where A, B and C are non-terminal symbols.

2. $A \rightarrow a$, where A is a non-terminal symbol and a is a terminal symbol.

$S \rightarrow aX \mid bX$
$X \rightarrow aY \mid bY \mid \varepsilon$
$Y \rightarrow X \mid c$

⇒ Remove the Null Productions:

After removing $X \rightarrow \varepsilon$: $S \rightarrow aX \mid bX \mid a \mid b$
$X \rightarrow aY \mid bY$
$Y \rightarrow X \mid c \mid \varepsilon$

After removing $Y \rightarrow \varepsilon$: $S \rightarrow aX \mid bX \mid a \mid b$
$X \rightarrow aY \mid bY \mid a \mid b$
$Y \rightarrow X \mid c$

⇒ Remove the Unit Productions : $Y \rightarrow X$

After removing $Y \rightarrow X$: $S \rightarrow aX \mid bX \mid a \mid b$

$X \rightarrow aY \mid bY \mid a \mid b$

$Y \rightarrow aY \mid bY \mid a \mid b \mid c$

⇒ Now change the productions

$S \rightarrow aX \mid bX$ $X \rightarrow aY \mid bY$ $Y \rightarrow aY \mid bY$

Finall we get: $S \rightarrow WX \mid ZX \mid a \mid b$

$X \rightarrow WY \mid ZY \mid a \mid b$

$Y \rightarrow WY \mid ZY \mid a \mid b \mid c$

$W \rightarrow a$

$Z \rightarrow b$

which the CNF for the given CFG