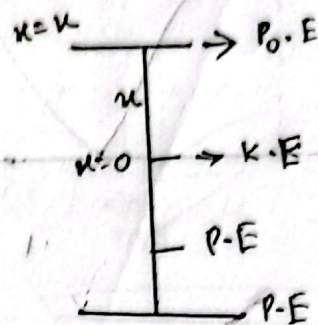


Waves and Oscillations

① Energy in Simple Harmonic Oscillation:

Let, a particle of simple harmonic oscillation has amplitude A , angular frequency ω and phase constant δ .
At time, t , if the displacement of the particle is x ,

$$x = A \sin(\omega t + \delta)$$



$$F = -Kx \quad , \quad (F' = -Kx) \quad \frac{1}{2} = \frac{1}{2} K A^2 \sin^2(\omega t + \delta)$$

$$\therefore W = U = \int_0^x F' dx = \int_0^x Kx dx = K \left[\frac{x^2}{2} \right]_0^x = \frac{1}{2} Kx^2$$

Potential energy, $U = \frac{1}{2} K A^2 \sin^2(\omega t + \delta)$

$$= \frac{1}{2} K A^2 [\sin^2(\omega t + \delta) \text{ highest value is } 1]$$

$$v = \frac{dx}{dt} = \frac{d}{dt} [A \sin(\omega t + \delta)] = \omega A \cos(\omega t + \delta)$$

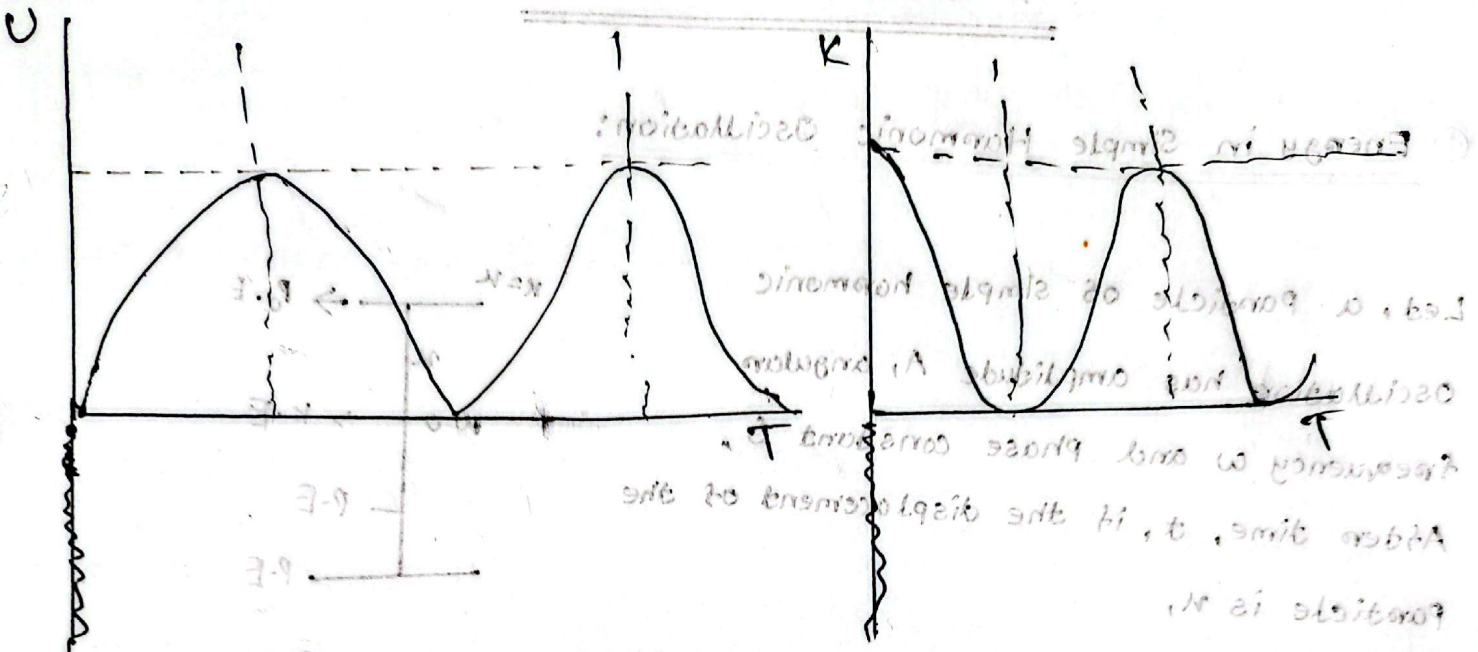
Kinetic energy, $K = \frac{1}{2} mv^2$

$$= \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \delta)$$

Again, $\omega^2 = \frac{K}{m}$

$$\therefore K = \frac{1}{2} K A^2 \cos^2(\omega t + \delta) = \frac{1}{2} K A^2 [\cos^2(\omega t + \delta) \text{ highest value is } 1]$$

Values and oscillations



Let a particle of simple harmonic oscillation has amplitude A, oscillates with frequency ω and phase constant ϕ . At any time, t , if the displacement of the particle is x , then

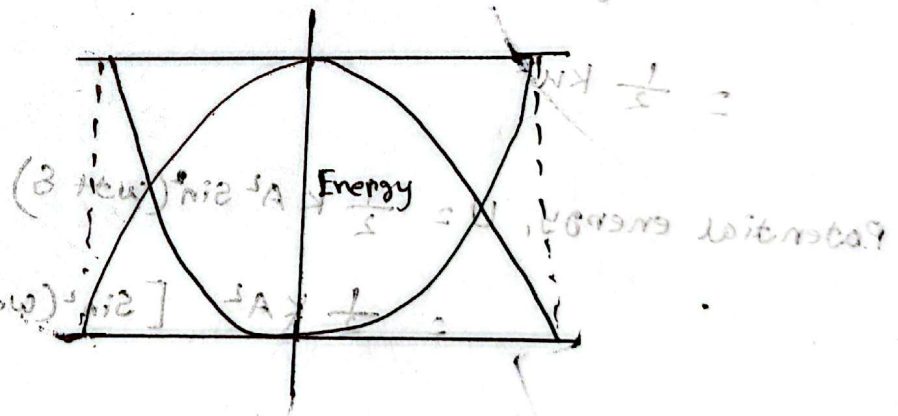
$$x = A \sin(\omega t + \phi)$$

Total energy at any point, $E = K + U$

$$= \frac{1}{2} K A^2 \cos^2(\omega t + \phi) + \frac{1}{2} K A^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2} K A^2 [\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)] = \frac{1}{2} K A^2$$

$$E = \frac{1}{2} K A^2 = \frac{1}{2} m \omega^2 A^2$$



$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \phi)$$

$$U = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$$

21 Oscillation of a Spring:

A spring suspended from a rigid position and loaded at the other end with a weight will oscillate in simple harmonic oscillation when pulled off.

$$mg = ke \quad \text{--- (1)}$$

$$T_1 = k(e + u)$$

$$\Sigma F = ma$$

$$\Rightarrow mg - T = ma$$

$$\Rightarrow mg - ke - ku = ma$$

$$\Rightarrow -ku = ma$$

$$\Rightarrow \left[a = -\frac{k}{m} u \right] \frac{b}{eb} = \frac{nb}{eb} \text{ s/w}$$

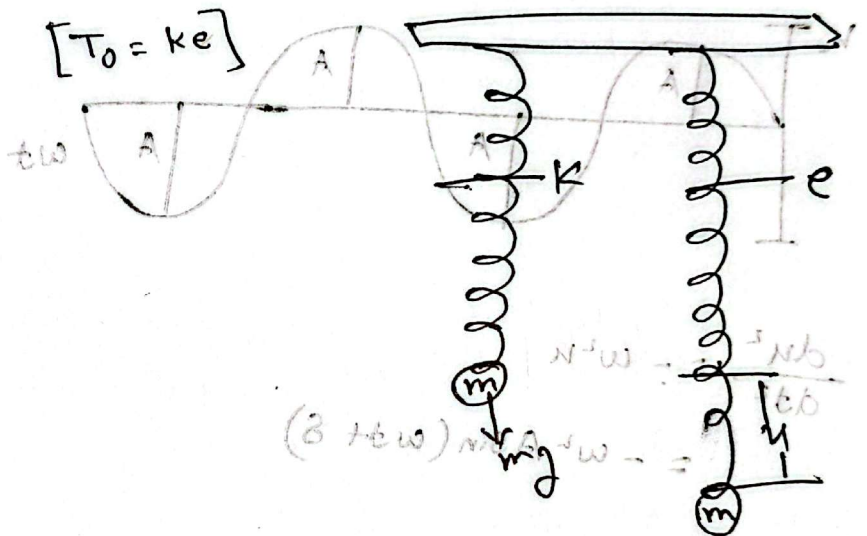
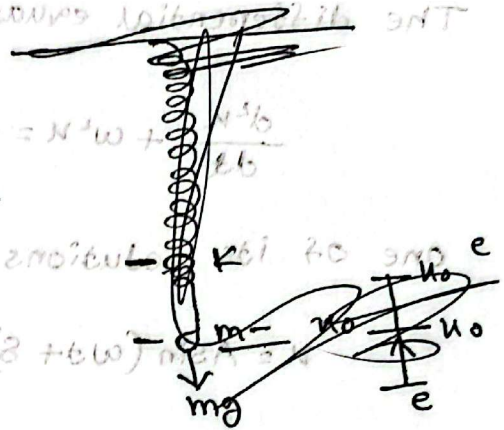
$$\therefore a = -\omega^2 u$$

$$\Rightarrow \frac{d^2 u}{dt^2} + \omega^2 u = 0$$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi \sqrt{\frac{m}{k}}$$

$$\therefore T = 2\pi \sqrt{\frac{e}{g}}$$

$$[mg = ke] (g + ew) \text{ m/s}^2 \text{ w} \dots$$



$$0 = \frac{N^3 b}{eb}$$

$$\left(\frac{nb}{eb} \right) \frac{b}{eb} =$$

$$(v) \frac{b}{eb} =$$

$$\left(\frac{nb}{eb} \right) \frac{b}{nb} = \frac{N^3 b}{eb}$$

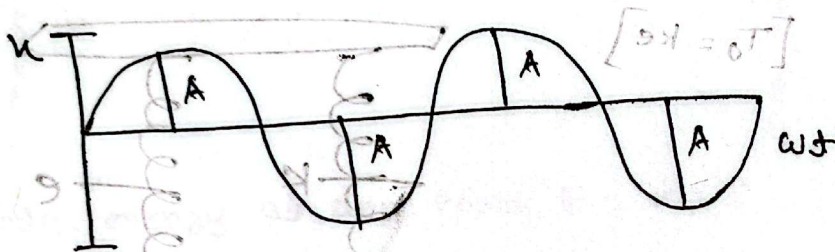
31 Solution to the Differential Equation of SHO:

The differential equation of simple harmonic oscillation,

$$\frac{d^2 u}{dt^2} + \omega^2 u = 0 \quad \text{--- (1)}$$

one of its solutions is,

$$u = A \sin(\omega t + \delta) \quad \text{--- (2)}$$



$$\begin{aligned} \frac{d^2 u}{dt^2} &= -\omega^2 u \\ &= -\omega^2 A \sin(\omega t + \delta) \end{aligned}$$

$$\begin{aligned} \frac{d^2 u}{dt^2} &= a \\ &= \frac{d}{dt} \left(\frac{du}{dt} \right) \\ &= \frac{d}{dt} (v) \end{aligned}$$

$$\begin{aligned} \therefore \frac{d^2 u}{dt^2} &= \frac{d}{dt} \left(\frac{du}{dt} \right) = \frac{d}{dt} [\omega A \cos(\omega t + \delta)] \\ &= -\omega^2 A \sin(\omega t + \delta) \end{aligned}$$

$$\begin{aligned} v &= \frac{du}{dt} = \frac{d}{dt} [A \sin(\omega t + \delta)] \\ &= \omega A \cos(\omega t + \delta) \end{aligned}$$