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CSE 1223 – Discrete Mathematics

Time: 90 Minutes **Total Marks: 30**

Answer any *three* of the following questions.
Figures in the right hand margin indicate full marks.

1.

a) What is a set? List the elements of the following sets if the universal set is all positive integers less than 31. 2

i. $A = \{x \mid x \text{ is a multiple of } 3\}$
ii. $B = \{x \mid x \text{ is a multiple of } 100\}$
iii. $C = \{x \mid \text{digits of } x \text{ is } 1 \text{ and } 2\}$ 3

b) Let A and B be two sets. Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$. 2

c) Define *Cartesian product*. Let $A = \{\alpha, \beta, \gamma\}$, $B = \{m, n\}$ and $C = \{0, 1\}$. Find $|P(B \times C \times A)|$. 3

d) In a survey of 30 students, it was found that –
15 study Mathematics
12 study Physics
11 study Chemistry
5 study both Mathematics and Chemistry
9 study both Mathematics and Physics
4 study both Physics and Chemistry
3 study all three subjects.
i) Find the number of students who do *not study* any subject.
ii) Find the number of students who read *exactly two* subjects.

2.

a) What is a *proposition*? Let p, q and r be the propositions : 2
p: Rakib reads Daily Nebula.
q: Rakib reads Daily Quasar.
r: Rakib reads Supernova.
Express each of the following propositions as an English sentence-
i) $(p \wedge q) \vee \neg (p \wedge r)$
ii) $\neg (p \wedge \neg r)$
iii) $\neg [(r \vee q) \wedge \neg p]$

b) Define the *converse* and *contrapositive* of an implication. State the converse and contrapositive of the implication "If it is raining today, then I will not go for hunting". 2

c) What do you mean by *universal quantifiers and existential quantifiers*? Use quantifiers to express the following statements- 4
i) Every computer science student needs a course in discrete mathematics.
ii) There is a student in this class who owns a personal computer.
iii) Every student in this class has taken at least one computer science course.
iv) There is a student in this class who has taken at least one course in computer science.

d) Verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction. 2

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1.a

Set :- A set is an unordered collection of distinct objects.

According to provided information, given sets are listed below :-

$$A = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$$

$$B = \{\}$$

$$C = \{12, 21\}$$

1.B

Prove that – $\overline{A \cup B} = \overline{A} \cap \overline{B}$

We prove this identity by showing that -

$\overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$

These steps show that - $\overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$

$= x \in \overline{A \cup B}$ by assumption.

$= x \notin A \cup B$ defn. of complement.

$= \neg((x \in A) \vee (x \in B))$ defn. of union.

$= \neg(x \in A) \wedge \neg(x \in B)$ 2nd De Morgan law

$= x \notin A \wedge x \notin B$ def. of negation

$= x \in \bar{A} \wedge x \in \bar{B}$ def of complement

$= x \in \bar{A} \cap \bar{B}$ def of intersection

Hence, $\overline{A \cup B} = \bar{A} \cap \bar{B}$

1.c

Cartesian Product:- The cartesian product of two sets A and B, denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

Given three sets -

$$A = \{\alpha, \beta, \gamma\}$$

$$B = \{m, n\}$$

$$C = \{0, 1\}$$

~~$B \times C \times A = \{(\alpha, m, 0), (\alpha, m, 1), ($~~

$B \times C \times A = \{(m, 0, \alpha), (m, 0, \beta), (m, 0, \gamma), (m, 1, \alpha), (m, 1, \beta), (m, 1, \gamma), (n, 0, \alpha), (n, 0, \beta), (n, 0, \gamma), (n, 1, \alpha), (n, 1, \beta), (n, 1, \gamma)\}$

The number of element of $B \times C \times A$ is 12.

So, the number of total subsets of $B \times C \times A$ is $2^{12} = 4096$.

Hence, the cardinality of the power set of $B \times C \times A$; $|P(B \times C \times A)| = 4096$.

1.D

Let, M, P and C be the sets for ~~French, for~~ Mathematics, Physics & German respectively.

Here according to the question -

$n(U) = 38$

$n(M) = 15$

$n(P) = 12$

$n(C) = 11$

$n(M \cap P) = 9$

$n(M \cap C) = 5$

$n(P \cap C) = 4$

and $n(M \cap P \cap C) = 3$

(i)

Number of student who do not study any subject.

$$n(U) - n(M) - n(P) - n(C) + n(M \cap P) + n(P \cap C) + n(M \cap C) - n(M \cap P \cap C)$$

$$= 30 - 15 - 12 - 11 + 5 + 9 + 4 - 3$$

$$= 7$$

(11)

Number of students who exactly study one subjects :-

$$n(U) - n(M \cap C) - n(M \cap P) - n(P \cap C) + 2 \times (M \cap P \cap C) - 7$$

$$= 30 - 5 - 9 - 4 + 2 \times 3 - 7$$

$$= 11$$

Number of students who exactly study two subjects :-

$$= 30 - 7 - 11 - 3$$

$$= 9$$

2.a

Proposition:- A proposition is a declarative sentence that is either true or false.

(i) Rakib reads daily Nebula and Rakib reads daily Quasar or it is not the case that Rakib reads daily Nebula and Rakib reads supernova.

(ii) It is not the case that Rakib reads daily Nebula and Rakib does not read Supernova.

(iii) It is not the case that Rakib reads Supernova or Rakib reads daily Quasar and Rakib does not read daily Nebula.

2.B

Converse:- For $P \rightarrow q$, the converse of the conditional statement is "If q then P.

Contrapositive:- For $P \rightarrow q$, the contrapositive of the conditional statement is "If not q then not P."

"If it is raining today, then I will not go for hunting."

The converse of given implication is -

"If I do not go for hunting then it is raining."

The contrapositive of give implication! -

"If I go for hunting then it is not raining."

2.C

(i) $\forall x \; S(x)$

(ii) $\exists x \left(S(x) \right)$

(iii) $\forall x \left(S(x) \longrightarrow C(x) \right)$

(iv) $\exists x \left(S(x) \wedge C(x) \right)$

2D

Contradiction:- A contradiction is a proposition which is always false.

Given that –

$$(p \wedge q) \wedge \neg (p \vee q)$$

The truth table of given equation :-

p	q	$p \wedge q$	$p \vee q$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

From the truth table, it is clearly seen that value of $(p \wedge q) \wedge \neg (p \vee q)$ is always false.

Hence, it is a contradiction.

3.a

Function:- A function is a relationship from elements of one set to elements of another set. Functions are sometimes called mappings or transformation.