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Bessel's Equation

1. Define Bessel's equation.

⇒ The definetion equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-n^2)y = 0$$

is called Bessel's equation.

2. Bessel's function.

⇒ The solution of Bessel's equation.

$$J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{x}{2}\right)^{n+2r}}{\lfloor r\, \Gamma(n+r+1)}$$

3. Prove that i) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$

ii) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$

Soln: We have,

$$J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{x}{2}\right)^{n+2r}}{\lfloor r\, \Gamma(n+r+1)}$$

$$\therefore J_{\frac{1}{2}}(x) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{x}{2}\right)^{\frac{1}{2}+2r}}{\lfloor r \, \Gamma\left(\frac{1}{2}+r+1\right)}$$

$$= \sum_{r=0}^{\infty} \frac{(-1)^r . \left(\frac{x}{2}\right)^{\frac{1}{2}} . \left(\frac{x}{2}\right)^{2r}}{\lfloor r \, . \, \Gamma\left(\frac{3}{2}+r\right)}$$

$$= \left(\frac{x}{2}\right)^{\frac{1}{2}} \left[\frac{1}{\Gamma\frac{3}{2}} - \frac{\left(\frac{x}{2}\right)^2}{\lfloor 1 \, \Gamma\left(\frac{3}{2}+1\right)} + \frac{\left(\frac{x}{2}\right)^4}{\lfloor 2 \, \Gamma\left(\frac{3}{2}+2\right)} - \cdots \right]$$

$$= \frac{\sqrt{x}}{\sqrt{2}} \left[\frac{1}{\frac{1}{2} . \Gamma\frac{1}{2}} - \frac{\left(\frac{x}{2}\right)^2}{\lfloor 1 \, \Gamma\frac{5}{2}} + \frac{\left(\frac{x}{2}\right)^4}{\lfloor 2 \, \Gamma 7/2} - \cdots \right]$$

$$= \frac{\sqrt{x}}{\sqrt{2}} \left[\frac{1}{1/2 . \sqrt{\pi}} - \frac{\left(\frac{x}{2}\right)^2}{\lfloor 1 . 3/2 . 1/2 . \sqrt{\pi}} + \frac{\left(\frac{x}{2}\right)^4}{\lfloor 2 . 5/2 . 3/2 . \frac{1}{2}\sqrt{\pi}} - \cdots \right]$$

$$\left[\because \Gamma 1/2 = \sqrt{\pi} \right]$$

$$= \frac{\sqrt{x}}{\sqrt{2}} . \frac{1}{\frac{1}{2}\sqrt{\pi}} \left[1 - \frac{x^2}{\lfloor 1 . 2^2 . \frac{3}{2}} + \frac{x^4}{\lfloor 2 . 2^4 . \frac{5}{2} . \frac{3}{2}} - \cdots \right]$$

$$= \frac{\sqrt{2} . \sqrt{x}}{\sqrt{\pi}} . \frac{1}{x} \left[x - \frac{x^3}{2.3} + \frac{x^5}{2.3.4.5} - \cdots \right]$$

$$= \frac{\sqrt{2}}{\sqrt{\pi}\sqrt{x}} \left[\frac{x}{\lfloor 1} - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \cdots \right]$$

$$= \sqrt{\frac{2}{\pi x}} \quad \sin x$$

ii) We have

$$J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{x}{2}\right)^{n+2r}}{\lfloor r \, \Gamma(n+r+1)}$$

$$\therefore \; J_{-\frac{1}{2}}(x) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{x}{2}\right)^{-\frac{1}{2}+2r}}{\lfloor r \, \Gamma\left(-\frac{1}{2}+r+1\right)}$$

$$= \left(\frac{x}{2}\right)^{-\frac{1}{2}} \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{x}{2}\right)^{2r}}{\lfloor r \, \Gamma\left(\frac{1}{2}+r\right)}$$

$$= \left(\frac{x}{2}\right)^{-\frac{1}{2}} \left[\frac{1}{\Gamma\frac{1}{2}} - \frac{\left(\frac{x}{2}\right)^2}{\lfloor 1 \, \Gamma 3/2} + \frac{\left(\frac{x}{2}\right)^4}{\lfloor 2 \, \Gamma 5/2} - \cdots \right]$$

$$= \left(\frac{2}{x}\right)^{\frac{1}{2}} \left[\frac{1}{\sqrt{\pi}} - \frac{x^2}{1.2^2.\frac{1}{2}\sqrt{\pi}} + \frac{x^4}{2.2^4.\frac{3}{2}.\frac{1}{2}\sqrt{\pi}} - \cdots \right]$$

$$= \left(\frac{2}{x}\right)^{1/2} \left[\frac{1}{\sqrt{\pi}} - \frac{x^2}{\lfloor 2 . \sqrt{\pi}} + \frac{x^4}{\lfloor 4 . \sqrt{\pi}} - \cdots \right]$$

$$= \sqrt{\frac{2}{x}} \cdot \frac{1}{\sqrt{\pi}} \left[1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \cdots \right]$$

$$= \sqrt{\frac{2}{\pi x}} \cos x$$

proved

Legendre's Equation

1. Define Legendre's equation.

⇒ The differential equation of the form

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$

is called Legendre's d.E, where n is constant.

2. State Rodrigue's formula.

⇒ The differential equation of the form.

$$P_n(x) = \frac{1}{2^n \lfloor n} \cdot \frac{d^n}{dx^n}(x^2-1)^n$$

is called the Rodrigue's formula where n is constant.

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3. Using Rodrigue's formula find the value of

$P_0(x)$, $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$.

Soln:

i) We know the Rodigue's formula,

$$P_n(x) = \frac{1}{2^n \lfloor n} \cdot \frac{d^n}{dx^n} (x^2-1)^n \quad \text{— (1)}$$

$P_0(x)$

Now putting $n=0$ in 1 we get,

$$P_0(x) = \frac{1}{2^0 \lfloor n} \cdot \frac{d^0}{dx^0} (x^2-1)^0$$

$$= 1 \cdot 1$$

$$= 1$$ Ans

$P_1(x)$

Now putting $n=1$ in eqn 1 we get,

$$P_1(x) = \frac{1}{2^1 \lfloor n} \cdot \frac{d^1}{dx^1} (x^2-1)^1$$

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$$= \frac{1}{2} \cdot \frac{d}{dx}(x^2-1)$$

$$= \frac{1}{2} 2x$$

$$= x$$

Ans:

$P_2(x)$

Now, putting $n=2$ in eqn (1) we get,

$$P_2(x) = \frac{1}{2^2 \lfloor 2} \cdot \frac{d^2}{dx^2}(x^2-1)^2$$

$$= \frac{1}{4.2} \cdot \frac{d^2}{dx^2}\{(x^2)^2 - 2x^2+1\}$$

$$= \frac{1}{8} \cdot \frac{d^2}{dx^2}(x^4 - 2x^2+1)$$

$$= \frac{1}{8} \cdot \frac{d}{dx}(4x^3 - 4x)$$

$$= \frac{1}{8}(12x^2 - 4)$$

Ans:

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$P_3(x)$

Now putting $n=3$ in ① we get.

$$P_3(x) = \frac{1}{2^3 . \lfloor 3} . \frac{d^3}{dx^3}(x^2-1)^3$$

$$= \frac{1}{8.6} . \frac{d^3}{dx^3}\{(x^2)^3 - 3(x^2)^2.1 + 3x^2 - 1^3\}$$

$$= \frac{1}{48} . \frac{d^3}{dx^3}(x^6 - 3x^4 + 3x^2 - 1)$$

$$= \frac{1}{48} . \frac{d^2}{dx^2}(6x^5 - 12x^3 + 6x)$$

$$= \frac{1}{48} . \frac{d}{dx}(30x^4 - 36x^2 + 6)$$

$$= \frac{1}{48}(120x^3 - 72x)$$

$$= \frac{12}{48}(10x^3 - 6x)$$

$$= \frac{1}{4}(10x^3 - 6x)$$

Ans

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$P_4(x)$

Now putting $n=4$ in eq^n ① we get.

$$P_4(x) = \frac{1}{2^4 \cdot \underline{L4}} \cdot \frac{d^4}{dx^4}(x^2-1)^4$$

$$= \frac{1}{16 \cdot \underline{L4}} \cdot \frac{d^4}{dx^4}(x^2-1)^3(x^2-1)$$

$$= \frac{1}{384} \cdot \frac{d^4}{dx^4}(x^6-3x^4+3x^2-1)(x^2-1)$$

$$= \frac{1}{384} \cdot \frac{d^4}{dx^4}(x^8-3x^6+3x^4-x^2-x^6+3x^4-3x^2+1)$$

$$= \frac{1}{384} \cdot \frac{d^3}{dx^3}(8x^7-18x^5+12x^3+2x-6x^5+12x^3-6x)$$

$$= \frac{1}{384} \cdot \frac{d^2}{dx^2}(56x^6-90x^4+36x^2+2-30x^4+36x^2-6)$$

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$$= \frac{1}{384} \frac{d}{dx}\left(336x^5 - 360x^3 + 72x - 120x^3 + 72x\right)$$

$$= \frac{1}{384}\left(1680x^4 - 1080x^2 + 72 - 360x^2 + 72\right)$$

$$= \frac{1}{384}\left(1680x^4 - 1440x^2 + 144\right)$$

$$= \frac{1}{384}\left(420x^4 - 360x^2 + 36\right)$$

$$= \frac{2}{96}\left(210x^4 - 180x^2 + 18\right)$$

$$= \frac{1}{48}\left(210x^4 - 180x^2 + 18\right)$$

Ans

4. Using Rodrigue's formula,

show that, $\int_{-1}^{1} [P_n(x)]\, dx = 0$, when $n \neq 0$.

$= 2$, when $n = 0$.

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Soln:

We have Rodrigue's formula,

$$P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n} (x^2-1)^n$$

Now integrating both side w.r.to x between -1 to 1, we get.

$$\int_{-1}^{1} [P_n(x)]\, dx = \int_{-1}^{1} \left[\frac{1}{2^n \lfloor n} \cdot \frac{d^n}{dx^n} (x^2-1)^n\right] dx \quad \text{--- (1)}$$

When $n \neq 0$ then from (1),

$$\int_{-1}^{1} [P_n(x)]\, dx = 0$$

When $n = 0$, then from (1),

$$\int_{-1}^{1} [P_0(x)]\, dx = \int_{-1}^{1} \left[\frac{1}{2^0 \lfloor 0} \cdot \frac{d^0}{dx^0} (x^2-1)^0\right] dx$$

$$= \int_{-1}^{1} dx$$

$$= [x]_{-1}^{1}$$

$$= 1+1 = 2$$

Showed

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Varation of parameter

1. Solve, $y'' + y = \operatorname{cosec} x$

Soln:

Given that, $y'' + y = \operatorname{cosec} x$ —— (1)

Let, $y = e^{mx}$ be the trial solⁿ. So the auxiliary eqⁿ is,

$$m^2 + 1 = 0$$

$$\Rightarrow m^2 = -1$$

$$\therefore m = \pm i$$

Therefore, C.F $= y_c = c_1 \cos x + c_2 \sin x$

Let, $y = A \cos x + B \sin x$ be the G.S. of (1)

where A and B are the function of x.

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Now y_1 and y_2 of waronskian method we have.

$$W = W(y_1, y_2) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x$$

$$= 1$$

Here, $A = -\int \frac{y_2 R(x)}{W} dx = -\int \sin x \csc x \, dx$

$$= -\int dx$$

$$= -x + c_1$$

$B = \int \frac{y_1 R(x)}{W} dx = \int \cos x \csc x \, dx = \int \cot x \, dx$

$$= \log(\sin x) + c_2$$

so, the general solⁿ is,

$$y = (-x + c_1)\cos x + (\log \sin x + c_2)\sin x$$

$$\therefore y = c_1 \cos x + c_2 \sin x - x\cos x + \sin x \log \sin x$$

Ans

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2. Solve $(D^2-1) = \frac{2}{1+e^x}$.

<u>Soln:</u> Given that.

$$(D^2-1)y = \frac{2}{1+e^x} \quad -(1)$$

Let, $y=e^{mx}$ be the trial soln so the auxiliary eqn is, $m^2-1=0$

$\therefore m = \pm 1$

So, $y_c = C.F = c_1e^x + c_2e^{-x}$

Let, $y = Ae^x + Be^{-x}$ be the complete soln of (1).

where A and B are function of x.

By wronskian method.

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

So, $W = W(y_1, y_2) = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$

$= -e^x e^{-x} - e^x e^{-x} = -1-1 = -2$

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Here, $A = -\int \frac{y_2 R(x)}{W} dx = \frac{1}{2}\int e^{-x} \frac{2}{1+e^x} dx = \int \frac{e^{-x}}{1+e^x} dx$

Let, $e^{-x} = z$, or, $-e^{-x} dx = dz$

$$\therefore A = -\int \frac{z\,dz}{1+z} = -\int \left(1 - \frac{1}{1+z}\right) dz$$

$$= -z + \log(1+z) + c_1$$

$$= -e^{-x} + \log(1+e^{-x}) + c_1$$

$B = \int \frac{y_1 R(x)}{W} dx = -\int \frac{e^x}{2} \cdot \frac{2}{1+e^x} dx = -\log(1+e^x) + c_2$

So, the general solution is,

$$y = [-e^{-x} + \log(1+e^{-x}) + c_1]e^x + [-\log(1+e^x) + c_2]e^{-x}$$

$$= -1 + e^x \log(1+e^{-x}) + c_1 e^x - e^{-x}\log(1+e^x) + c_2 e^{-x}$$

$$= c_1 e^x + c_2 e^{-x} + e^x \log(1+e^{-x}) - e^{-x}\log(1+e^x) - 1$$

Ans

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3. Solve $\frac{d^2y}{dx^2} + n^2 y = \sec nx$

<u>Soln:</u> Given eqn is, $\frac{d^2y}{dx^2} + n^2 y = \sec nx$ —— (1)

Let, $y = e^{mx}$ be the trial soln so the auxiliary eqn is $m^2 + n^2 = 0$

$\therefore m = \pm ni$

So, $y_c = c_1 \cos nx + c_2 \sin nx$

Let us suppose that $y = A\cos nx + B\sin nx$ be a soln of (1) where A and B function of x.

By wronskian method of y_1 and y_2.

$$W = \begin{vmatrix} \cos nx & \sin nx \\ -n\sin nx & n\cos x \end{vmatrix} = n(\cos^2 nx + \sin^2 nx) = n$$

Here, $A = -\int \frac{y_2 F(x)}{W} dx = \frac{1}{n} \int \sin nx \sec nx \, dx$

$= -\frac{1}{n} \int \tan nx \, dx$

$= \frac{1}{n^2} \log \cos nx + c_1$

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$$B = \int \frac{y_1 R(x)}{W} = \frac{1}{h}\int \cosh hx \operatorname{sech} hx \, dx$$

$$= \frac{1}{h}\int dx = \frac{1}{h}x + c_2$$

So the general soln is $y = A\cosh hx + B\sinh hx$

$$= \left(\frac{1}{h^2}\log \cosh hx + c_1\right)\cosh hx$$

$$+\left(\quad + c_2\right) \sinh x$$

$$= c_1\cosh hx + c_2 \sinh hx + \frac{1}{h^2}\cosh hx$$

$$\log \cosh hx + \frac{h}{h}\sinh hx.$$

Ans

4. Solve $(D^2+4) = 4\tan 2x$

Soln: Given eqn is $(D^2+4)y = 4\tan 2x$

Let, $y = e^{mx}$ be the trial soln. So the auxiliary eqn is $m^2+4=0$

$$\therefore m = \pm 2i$$

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$\therefore y_c = C.F = C_1 \cos 2x + C_2 \sin 2x$

suppose that $y = Ay_1 + By_2$ be the soln of this eqn where A and B are function of x and where $y_1 = \cos 2x$ $y_2 = \sin 2x$. By wronskian method of y_1 and y_2

$$W = W(y_1, y_2) = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$$= 2(\cos^2 2x + \sin^2 2x)$$

$$= 2$$

Now, $A = -\int \frac{y_2 f(x)}{W} dx = -\int \frac{\sin 2x . 4\tan 2x}{2} dx = -2\int \frac{\sin^2 2x}{\cos 2x} dx$

$$= -2\int \frac{(1-\cos^2 2x)}{\cos 2x} dx = 2\int (\cos 2x - \sec 2x) dx$$

$$= \sin 2x - \log(\sec 2x + \tan 2x) + C_1$$

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$$y_2 = \int \frac{y_1 (R(x))}{w} dx = \int \frac{\cos 2x}{2} 4\tan^2 x$$

$$= 2\int \sin 2x \, dx$$

$$= -2 \cdot \frac{1}{2} \cos 2x + C_2$$

$$= -\cos 2x + C_2$$

So, the general eqn is,

$$y = [\sin 2x - \log(\sec 2x + \tan 2x) + C_1]$$

$$\cos 2x + (-\cos 2x + C_2) \sin 2x$$

$$= C_1 \cos 2x + C_2 \sin 2x + \cos 2x \sin 2x - \cos 2x \log$$

$$(\sec 2x + \tan 2x) - \sin 2x \cos 2x$$

$$= C_1 \cos 2x + C_2 \sin 2x - \cos 2x \log(\sec 2x + \tan 2x)$$

Ans

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1. Solve $xzp + yzq = z^2$

Soln:

Given that.

$$xzp + yzq = z^2 \quad \text{---} (i)$$

Comparing (i) with $Pp + Qq = R$ we get.

$P = xz$

$Q = yz$

$R = z^2$

Now by Rodrigue's auxiliary equation we have.

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

or, $$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{z^2}$$

or, $$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

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Now taking 1st and 2nd ratio we have,

$$\frac{dx}{x} = \frac{dy}{y}$$

Integrating,

$$\ln x = \ln y + \ln c_1$$

or, $\ln\left(x/y\right) = \ln c_1$

$\therefore\ x/y = c_1 = u$ (say)

Again taking 2nd & 3rd ratio,

$$\frac{dy}{y} = \frac{dz}{z}$$

or, $\ln y = \ln z + \ln c_2$

or, $\ln(y/z) = \ln c_2$

$\therefore\ y/z = c_2 = v$ (say)

Hence, the general soln $\phi(x/y, y/z) = 0$

Ans

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2. solve $(x^2 - yz)\, p + (y^2 - zx)\, q = (z^2 - xy)$

<u>Soln:</u> Auxiliary eqn are,

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$$

This gives

$$\frac{dx - dy}{x^2 - yz - y^2 + zx} = \frac{dy - dz}{y^2 - zx - z^2 + xy} = \frac{dz - dx}{z^2 - xy - x^2 + yz}$$

$$\Rightarrow \frac{dx - dy}{(x+y)(x-y) + z(x-y)} = \frac{dy - dz}{(y+z)(y-z) + x(y-z)} = \frac{dz - dx}{(z+x)(z-x) + y(z-x)}$$

$$\Rightarrow \frac{dx - dy}{(x-y)(x+y+z)} = \frac{dy - dz}{(y-z)(x+y+z)} = \frac{dz - dx}{(z-x)(x+y+z)}$$

$$\Rightarrow \frac{dx - dy}{x-y} = \frac{dy - dz}{y-z} = \frac{dz - dx}{z-x}$$

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Now from first two.

$$\frac{dx - dy}{x - y} = \frac{dy - dz}{y - z}$$

$$\Rightarrow \frac{d(x-y)}{(x-y)} = \frac{d(y-z)}{(y-z)}$$

$$\Rightarrow \log(x-y) = \log(y-z) + \log c_1$$

$$\Rightarrow \frac{x-y}{y-z} = c_1 = u \text{ (say)}$$

and from last two we get.

$$\frac{d(y-z)}{y-z} = \frac{d(z-x)}{z-x}$$

$$\Rightarrow \log(y-z) = \log(z-x) + \log c_2$$

$$\Rightarrow \frac{y-z}{z-x} = c_2 = v \text{ (say)}$$

The soln $\phi\left(\frac{x-y}{y-z}, \frac{y-z}{z-x}\right) = 0$

3. Solve $(y+z)p + (z+x)q = x+y$

Sol^n

A.E are $\dfrac{dx}{y+z} = \dfrac{dy}{z+x} = \dfrac{dz}{x+y}$

$$\Rightarrow \frac{dx-dy}{y-x} = \frac{dy-dz}{z-y} = \frac{dz-dx}{x-z}$$

$$\Rightarrow \frac{dx-dy}{-(x-y)} = \frac{dy-dz}{-(y-z)} = \frac{dz-dx}{-(z-x)}$$

$$\Rightarrow \frac{dx-dy}{x-y} = \frac{dy-dz}{y-z} = \frac{dz-dx}{z-x}$$

from first two,

$$\log(x-y) = \log(y-z) + \log c_1$$

$$\frac{x-y}{y-z} = c_1 = u \text{ (say)}$$

then from last two

$$\log(y-z) = \log(z-x) + \log c_2$$

$$\frac{y-z}{z-x} = c_2 = v \text{ (say)}$$

The sol^n $\phi\left(\dfrac{x-y}{y-z}, \dfrac{y-z}{z-x}\right) = 0$

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4. Solve $(y^2+z^2-x^2)p - 2xyq + 2xz = 0$

Soln

A.E are

$$\frac{dx}{y^2+z^2-x^2} = \frac{dy}{-2xy} = \frac{dz}{-2xz}$$

from last two,

$$\frac{dy}{y} = \frac{dz}{z}$$

on integration one soln of auxiliary eqn is

$$y/z = c_1$$

Next using x, y, z as multiplier, we get

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz} = \frac{x\,dx+y\,dy+z\,dz}{x(x^2+y^2+z^2)}$$

$\therefore$ from the last two

$$\frac{dz}{z} = \frac{2(x\,dx+y\,dy+z\,dz)}{x^2+y^2+z^2}$$

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$\log z + \log c_2 = \log(x^2+y^2+z^2)$

$\Rightarrow \quad z c_2 = x^2+y^2+z^2$

$\therefore \quad c_2 = x^2+y^2+z^2/z$

$\phi\left(y/z,\ \dfrac{x^2+y^2+z^2}{z}\right) = 0$ is the solⁿ of the eqⁿ.

5. solve $\dfrac{y^2z}{x}p + xzq = y^2$

Solⁿ: Given that,

$$\frac{y^2z}{x}p + xzq = y^2$$

or, $y^2zp + x^2zq = xy^2$

Comparing ① with $Pp+Qq=R$ we get,

$P = y^2z$

$Q = x^2z$

$R = xy^2$

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Now Legger's auxiliary eqⁿ. we have,

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\Rightarrow \frac{dx}{y^2z} = \frac{dy}{x^2z} = \frac{dz}{xy^2}$$

Now taking first and second ratio we get,

$$\frac{dx}{y^2z} = \frac{dy}{x^2z}$$

or, $x^2dx = y^2dy$

Integrating

$$\frac{x^3}{3} = \frac{y^3}{3} + c$$

$\Rightarrow x^3 - y^3 = c = u$ (say)

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Application

Again taking 1st and 3rd Ratio we get

$$\frac{dx}{y^2 z} = \frac{dz}{xy^2}$$

$\Rightarrow x\,dx = z\,dz$

Integrating,

$x^2 = z^2 + c$

$\Rightarrow x^2 - z^2 = c = v$ (say)

Hence, the general soln $\varphi(x^2 - y^2, x^2 - z^2) = 0$

Ans