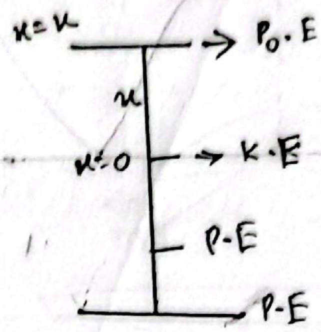
Waves and Oscillations

① Energy in Simple Harmonic Oscillation:



Let, a particle of simple harmonic oscillation has amplitude A; angular frequency ω and phase constant δ, After time, t, if the displacement of the particle is x,

$$x = A\sin(\omega t + \delta)$$

$$F = -Kx \quad , \quad (F' = Kx)$$

$$\therefore W = U = \int_0^x F'\,dx = \int_0^x Kx\,dx = K\int_0^x x\,dx = K\left[\frac{x^2}{2}\right]_0^x$$

$$= \frac{1}{2}Kx^2$$

Potential energy, $U = \frac{1}{2}KA^2\sin^2(\omega t + \delta)$

$= \frac{1}{2}KA^2$ [$\sin^2(\omega t+\delta)$ highest value is 1]

$$V = \frac{dx}{dt} = \frac{d}{dt}\left[A\sin(\omega t + \delta)\right]$$

$$= \omega A\cos(\omega t + \delta)$$

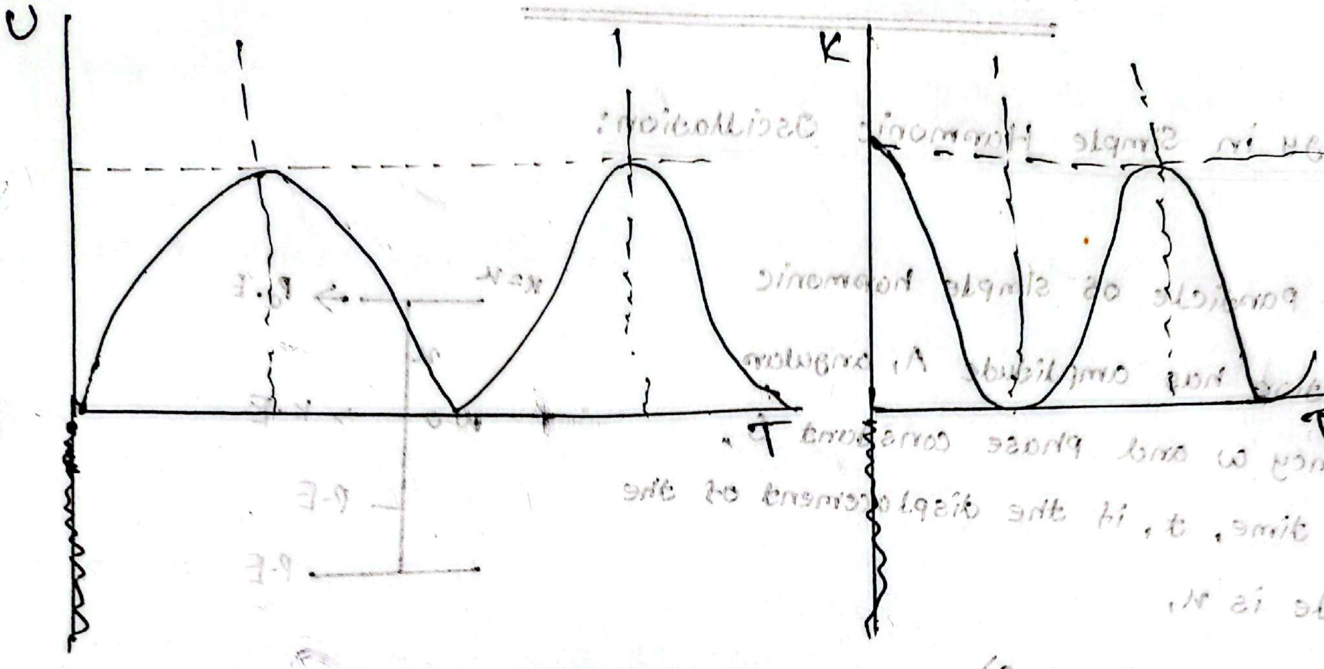
Kinetic energy, $K = \frac{1}{2}mv^2$

$$= \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \delta)$$

Again, $\omega^2 = \frac{K}{m}$

$\therefore K = \frac{1}{2}KA^2\cos^2(\omega t + \delta) = \frac{1}{2}KA^2$ [$\cos^2(\omega t+\delta)$ highest value is 1]

CamScanner

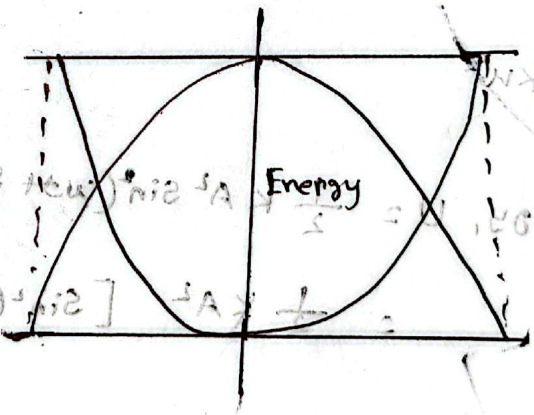


Total energy at any point, $E = K + U$

$$= \frac{1}{2} KA^2 \cos^2(\omega t + \delta) + \frac{1}{2} KA^2 \sin^2(\omega t + \delta)$$

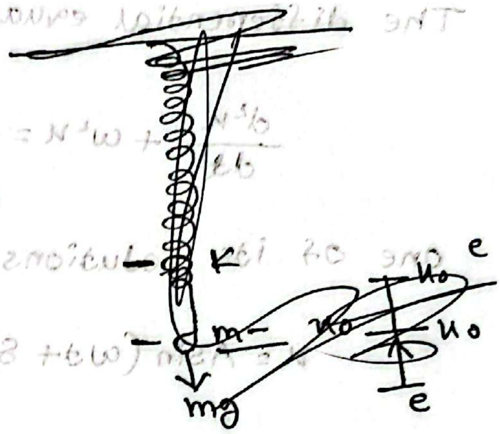
$$= \frac{1}{2} KA^2$$

$\therefore E \propto A^2$



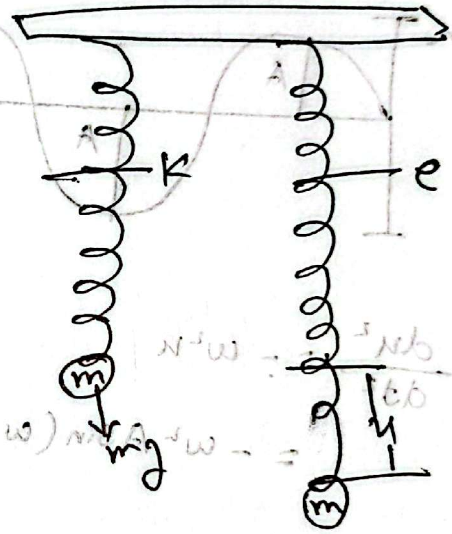
2| Oscillation of a Spring:

A spring suspended from a rigid position and loaded at the other end with a weight will oscillate in simple harmonic oscillation when pulled off.



$$mg = Ke \quad \text{—(1)}$$

$$[T_0 = ke]$$



$$T_1 = K(e+x)$$

$$\Sigma F = ma$$

$$\Rightarrow mg - T = ma$$

$$\Rightarrow mg - Ke - Kx = ma$$

$$\Rightarrow -Kx = ma$$

$$\Rightarrow a = -\frac{K}{m}x$$

$$\therefore a = -\omega^2 x$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{K}{m}}} = 2\pi\sqrt{\frac{m}{K}}$$

$$\therefore T = 2\pi\sqrt{\frac{e}{g}} \qquad [mg = Ke]$$

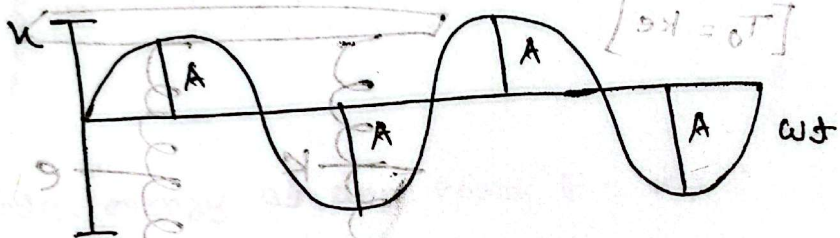
31 Solution to the Differential Equation of SHO:

The differential equation of simple harmonic oscillation,

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{--- (1)}$$

one of its solutions is,

$$x = A\sin(\omega t + \delta) \quad \text{--- (1a)}$$



$$\frac{dx^2}{dt^2} = -\omega^2 x$$

$$= -\omega^2 A\sin(\omega t + \delta)$$

$$\frac{d^2x}{dt^2} = a$$

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$= \frac{d}{dt}(v)$$

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[A\sin(\omega t + \delta)\right]$$

$$= \omega A\cos(\omega t + \delta)$$

$$\therefore \frac{d^2x}{dt^2} = \frac{d}{dx}\left(\frac{dx}{dt}\right) = \frac{d}{dx}\left[\omega A\cos(\omega t + \delta)\right]$$

$$= -\omega^2 A\sin(\omega t + \delta)$$