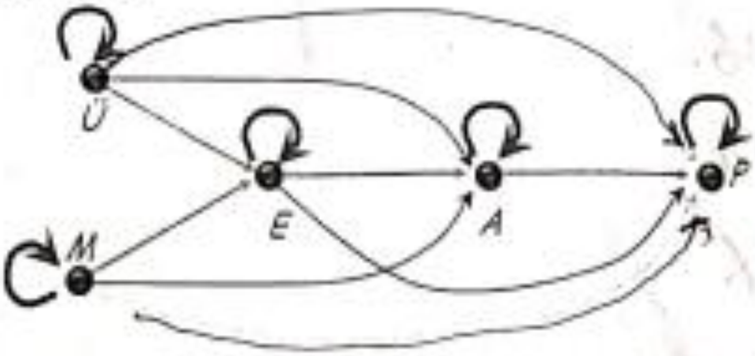
International Islamic University Chittagong (IIUC)
Department of Computer Science and Engineering (CSE)
B. Sc. in CSE, Mid Term Examination, Autumn-2022
Course Code: MATH-2407, Course Title: Mathematics-IV

Time: 1:30 Hours — Marks: 30

[N.B. Please answer the questions sequentially, Figures in the right margin indicates full marks]

		Marks	CLO	DL
1.	a) Let the function $f: R^* \to R^*$ be defined by $y = f(x) = x^2 + x - 2$ then find the value of $f^{-1}(10)$	3	CLO1	C2
	b)	3	CLO1	C2



Determine whether the above relation is Transitive, Anti-symmetric and Transitive Reflexive

c) Using Demoivres theorem find the quadratic equation whose roots are the nth power of the roots of the equation, $x^2 - 2x\cos\theta + 1 = 0$ — 4 CLO1 C2

Or

c) If $(1+i\frac{x}{a})(1+i\frac{x}{b})(1+i\frac{x}{c})\ldots\ldots = A + iB$, Then prove that — 4 CLO1 C2

$$(1+\frac{x^2}{a^2})(1+\frac{x^2}{b^2})(1+\frac{x^2}{c^2})\ldots\ldots = A^2 + B^2$$

2. $\boxed{a)}$ A circle $|z-3| = 2$ in the z-plane. Determine its image in the w-plane when transformation by $w = \frac{1}{z}$ — 8 CLO1

b) Test the function $f(x,y,z) = x^2y + y^2z + z^2y$ is harmonic or not. — 2 CLO2

Or

b) Determine the function, $w = e^z$ is regular (analytic) or not. — 2 CLO2

3. a) Evaluate the integral $\int_c \bar{z}\,dz$ from $z = 0$ to $z = 1 + i$ along the curve c. — 6 CLO2

$\boxed{b)}$ Using Cauchy's Integral Formula evaluate $\int_c \frac{z}{z^2 - 3z + 2}dz$ where c is the circle $|z-1| = \frac{1}{2}$ — 4 CLO2

Or

b) Evaluate $\int_c \frac{2z+1}{z^2+z}dz$ Where c is the circle $|z| = \frac{1}{2}$ — 4 CLO2

Answer to the Q.no-1(a)

$$f^{-1}(10) = \{x \in R^{\#};\ x^2 + x - 2 = 10\}$$

$$= \{x \in R^{\#};\ x^2 + x - 12 = 0\}$$

$$= \left\{x \in R^{\#};\ x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-12)}}{2.1}\right\}$$

$$= \left\{x \in R^{\#};\ x = \frac{-1 \pm \sqrt{1+48}}{2}\right\}$$

$$= \left\{x \in R^{\#};\ x = \frac{-1 \pm 7}{2}\right\}$$

$$= \{x \in R^{\#};\ x = -4, 3\}$$

$$= \{-4, 3\}$$ (Ans).

Answer to the Q.no - 1(b)

$R = \{(U,U), (U,E), (U,P), (E,A), (E,E), (E,P), (M,M), (M,A), (M,E), (M,P), (A,A), (A,P), (P,P)\}$

The relation is reflexive because every element is related to itself.

The relation is anti-symmetric.

The relation is not transitive because $(U,U) \in R$ and $(U,E) \in R$ but $(U,E) \notin R$.

Answer to the Q. no - 1(c)

Given,

$$x^2 - 2x\cos\theta + 1 = 0$$

$$x = \frac{-(-2\cos\theta) \pm \sqrt{(-2\cos\theta)^2 - 4.1.1}}{2.1}$$

or, $$x = \frac{2\cos\theta \pm \sqrt{-4 + 4\cos^2\theta}}{2}$$

or, $$x = \frac{2\cos\theta \pm \sqrt{4(1+\cos^2\theta)}}{2}$$

or, $$x = \frac{2\cos\theta \pm \sqrt{4i^2\sin^2\theta}}{2}$$

or, $$x = \frac{2\cos\theta \pm 2i\sin\theta}{2}$$

or, $x = \cos\theta \pm \sin\theta$ —— (i)

Let α and β be the root of eq^n (i)

$\therefore \alpha = \cos\theta + \sin\theta$

$\beta = \cos\theta - \sin\theta$

We have to form a new eq^n whose roots are α^n and β^n.

Any eq^n is x^2 - (sum of roots) x + product of roots = 0

$$x^2-(\alpha^n+\beta^n)+\alpha^n\beta^n=0$$

or, $x^2-[(\cos\theta+i\sin\theta)^n+(\cos\theta-i\sin\theta)^n]x+(\cos\theta+i\sin\theta)^n(\cos\theta\cancel{+}-i\sin\theta)^n=0$

or, $x^2-[(\cos n\theta+i\sin\theta)+(\cos n\theta-i\sin nn\theta)]x$
$+(\cos n\theta+i\sin nn\theta)(\cos n\theta-i\sin nn\theta)=0$

$[\because (\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$
$(\cos\theta-i\sin\theta)^n=\cos n\theta-i\sin n\theta\,]$

or $x^2-[\cos n\theta+i\sin n\theta-\cos n\theta-i\sin n\theta]x+[\cos^2 n\theta-i^2\sin nn\theta]=0$

or, $x^2-[2\cos n\theta]x+1=0$

or, $x^2-2x\cos n\theta+1=0$ (Ans.).

Or

$(1+i\frac{x}{a})(1+i\frac{x}{b})(1+i\frac{x}{c})=A+iB$ ——— (i)

Let, $1=r\cos\alpha$ $\quad$ $1=r\cos\cancel{\alpha}\beta$ $\quad$ $1=r\cos\cancel{\alpha}\gamma$

$\frac{x}{a}=r\sin\alpha$ $\quad$ $\frac{x}{b}=r\sin\cancel{\alpha}\beta$ $\quad$ $\frac{x}{c}=r\sin\cancel{\alpha}\gamma$

$\therefore \frac{x}{a}=\tan\alpha$ $\quad$ $\therefore \beta\cancel{\alpha}=\tan^{-1}\frac{x}{b}$ $\quad$ $\therefore \gamma\cancel{\alpha}=\tan^{-1}\frac{x}{c}$

$\therefore \alpha=\tan^{-1}\frac{x}{a}$

From (i)

$$(1+i\tan\alpha)(1+i\tan\beta)(1+i\tan\gamma)\ .\ .\ .\ = A+iB$$

or, $$\left(1+i\frac{\sin\alpha}{\cos\alpha}\right)\left(1+i\frac{\sin\beta}{\cos\beta}\right)\left(1+i\frac{\sin\gamma}{\cos\gamma}\right)\ .\ .\ .\ = A+iB$$

or, $$\left(\frac{\cos\alpha+i\sin\alpha}{\cos\alpha}\right)\left(\frac{\cos\beta+i\sin\beta}{\cos\beta}\right)\left(\frac{\cos\gamma+i\sin\gamma}{\cos\gamma}\right)\ .\ .\ .\ = A+iB$$

or, $$\left(\frac{1}{\cos\alpha}\right)\left(\frac{1}{\cos\beta}\right)\left(\frac{1}{\cos\gamma}\right)\ .\ .\ .\ (\cos\alpha+i\sin\alpha)(\cos\beta+i\sin\beta)(\cos\gamma+i\sin\gamma)\ .\ .\ .\ = A+iB$$

or $$(\sec\alpha)(\sec\beta)(\sec\gamma)\ .\ .\ .\ [\cos(\alpha+\beta+\gamma+\ .\ .\ .)+i\sin(\alpha+\beta+\gamma+\ .\ .\ .)\ .\ .\ .] = A+iB$$

Equating real and imagenary part,

$$(\sec\alpha)(\sec\beta)(\sec\gamma)\ .\ .\ .\ [\cos(\alpha+\beta+\gamma+\ .\ .\ .)] = A\ .\ .\ .\ \text{(ii)}$$

$$(\sec\alpha)(\sec\beta)(\sec\gamma)\ .\ .\ .\ [\sin(\alpha+\beta+\gamma+\ .\ .\ .)] = B\ .\ .\ .\ \text{(iii)}$$

Squaring (ii) and (iii)

$$(\sec^2\alpha)(\sec^2\beta)(\sec^2\gamma)\ .\ .\ .\ [\cos^2(\alpha+\beta+\gamma)+\ .\ .\ .] = A^2\ .\ .\ .\ \text{(iv)}$$

$$(\sec^2\alpha)(\sec^2\beta)(\sec^2\gamma)\ .\ .\ .\ [\sin^2(\alpha+\beta+\gamma+\ .\ .\ .] = B^2\ .\ .\ .\ \text{(v)}$$

Adding ⓘⓥ and ⓥ

$(\sec^2\alpha \sec^2\beta \sec^2\gamma)\left[\cos^2(\alpha+\beta+\gamma+\ldots) + \sec^2\alpha\sec^2\beta\sec^2\gamma \cdot \sin^2(\alpha+\beta+\gamma+\ldots)\right] = A^2+B^2$

or, $(\sec^2\alpha \sec^2\beta \sec^2\gamma)\{\cos^2(\alpha+\beta+\gamma+\ldots) + \sin^2(\alpha+\beta+\gamma+\ldots)\} = A^2+B^2$

or, $(\sec^2\alpha \sec^2\beta \sec^2\gamma) . 1 = A^2+B^2$

or, $(1+\tan^2\alpha)(1+\tan^2\beta)(1+\tan^2\gamma) \ldots = A^2+B^2$

$\therefore (1+\frac{x^2}{a^2})(1+\frac{x^2}{b^2})(1+\frac{x^2}{c^2}) \ldots = A^2+B^2$

[Proved].

Answer to the Q.no-2

(a)

Given,

$|z-3|=2$

or, $\sqrt{(x-3)^2+y^2}=2$

or, $(x-3)^2+y^2=4$

or $(x-3)^2+(y-0)^2=2^2$

Center of Circle $(3,0)$ and radius $=2$. Hence we can say $(x,y)=(3,0)$.

or, $(x-3)^2+y^2=4$

or $x^2-6x+9+y^2=4$

or, $x^2+y^2-6x+5=0$ . . . . ⓘ

Again,

$W=\frac{1}{z}$

or, $W=\frac{1}{x+jy}$

or, $W=\frac{x-jy}{x^2+y^2}$

or, $u+jv=\frac{x}{x^2+y^2}-j\frac{y}{x^2+y^2}$

Equating $U=\frac{x}{x^2+y^2}$, $V=\frac{-y}{x^2+y^2}$

putting the value of x and y in ①

$$x^2+y^2-6x+5=0$$

or, $\left(\frac{u}{u^2+v^2}\right)^2+\left(\frac{-v}{u^2+v^2}\right)^2-6\left(\frac{u}{u^2+v^2}\right)+5=0$

or, $\frac{u^2}{(u^2+v^2)^2}+\frac{v^2}{(u^2+v^2)^2}-\frac{6u}{u^2+v^2}+5=0$

or, $\frac{u^2+v^2}{(u^2+v^2)^2}-\frac{6u}{u^2+v^2}+5=0$

or, $\frac{1}{u^2+v^2}-\frac{6u}{u^2+v^2}+5=0$

or, $5(u^2+v^2)-6u+1=0$

or, $u^2+v^2-\frac{6}{5}u+\frac{1}{5}=0$

or, $u^2+v^2-2.\frac{\cancel{6}3}{5}u+2.0.v+\frac{1}{5}=0$

or, $u^2+v^2+2.\left(-\frac{3}{5}\right)u+2.0.v+\frac{1}{5}=0$ . . . . (ii)

From (ii)

$g=-\frac{3}{5}$ $f=0$ and $c=\frac{1}{5}$

The new center is $(-g,-f)=\left(\frac{3}{5},0\right)$ (Ans.)

Radius $=\sqrt{\left(-\frac{3}{5}\right)^2+0^2-\frac{1}{5}}=\sqrt{\frac{9}{25}-\frac{1}{5}}=\sqrt{\frac{9-5}{25}}=\sqrt{\frac{4}{25}}=\frac{2}{5}$ (Ans.)

Answer to the Q.no - 2(a)

Given,

$$f(x, y, z) = x^2y + y^2z + z^2y$$

Differentiating with respect to x.

$$\frac{\partial u}{\partial x} = 2xy$$

Again differentiating,

$$\frac{\partial^2 u}{\partial x^2} = 2y$$

Differentiating with respect to y.

$$\frac{\partial u}{\partial y} = x^2 + 2yz + z^2$$

Again differentiating,

$$\frac{\partial^2 u}{\partial y^2} = 2z$$

As $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2y + 2z \neq 0$

The function is not harmonic.

Or

Given,

$$W = e^{z} = e^{x+iy}$$

Or, $W = e^{x} \cdot e^{iy}$

Or, $W = e^{x}(\cos y + i\sin y)$ [∵ from eular's formula $e^{iy} = \cos y + i\sin y$]

Or, $W = U + iV = e^{x}\cos y + i e^{x}\sin y$

Equating,

$u = e^{x}\cos y$ . . . (i)

$V = e^{x}\sin y$ . . . (ii)

Differentiating (i) with respect to x.

$$\frac{\partial u}{\partial x} = e^{x}\cos y$$

" (i) " " " y

$$\frac{\partial u}{\partial y} = -e^{x}\sin y$$

Differentiating (ii) with respect to x.

$$\frac{\partial V}{\partial x} = e^{x}\sin y$$

" " " " " y

$$\frac{\partial V}{\partial y} = e^{x}\cos y$$

Here,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\Rightarrow e^{x} \cos y = e^{x} \cos y$$

L.H.S = R.H.S

$$\frac{\partial u}{\partial y} = - \frac{\partial v}{\partial x}$$

$$\Rightarrow -e^{x} \sin y = -e^{x} \sin y$$

L.H.S = R.H.S

Hence, the function is analytic.

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3.no

(a)

Evaluate the integral $\int_c \bar{z} dz$ from $z=0$ to $z=1+i$ along the curve c.

Given,

$$z=0$$

or, $x+iy = 0+i.0$

$\therefore x=0$ and $y=0$

Again,

$$z=1+i$$

or, $x+iy = 1+i.1$

$\therefore x=1$ and $y=1$

The equation is,

$$\frac{y-0}{0-1} = \frac{x-0}{0-1}$$

or, $\frac{y}{-1} = \frac{x}{-1}$

or, $y = x$

We have,

$$z = x + iy$$

or, $z = x + ix \quad [y = x]$

or, $\frac{dz}{dx} = 1 + i$

or, $dz = (1+i)dx$

Now,

$$\int_0^1 (x + iy)(1+i)\,dx$$

or, $(1+i)\int_0^1 x(1+i)\,dx$

or, $(1+i)^2 \cdot \left[\frac{x^2}{2}\right]_0^1$

or, $(1+i)^2\left[\frac{1}{2} - \frac{0}{2}\right]$

$= \frac{1}{2}(1+i)^2$ (Ans.).

3(b)

Using Cauchy's Integral formula evaluate $\int_c \frac{z}{z^2-3z+2} dz$ where c is the circle. $|z-1| = \frac{1}{2}$

Given,

$$|z-1| = \frac{1}{2}$$

or, $\sqrt{(x-1)^2+y^2} = \frac{1}{2}$

or, $(x-1)^2 + y^2 = \frac{1}{4}$

or, $(x-1)^2 + (y-0)^2 = \left(\frac{1}{2}\right)^2$

Center $(1,0)$ and radius $= \frac{1}{2}$

Poles: $z^2 - 3z + 2 = 0$

or, $z^2 - 2z - z + 2 = 0$

or, $(z-2)(z-1) = 0$

$\therefore z = 2, 1$

Only one pole at $z=1$ is inside the circle.

$$\int \frac{z}{z^2-3z+2} dz$$

$$= \int \frac{z}{(z-2)(z-1)} dz$$

Here $f(z) = \frac{z}{z-2} dz$

$\therefore f(1) = \frac{1}{1-2} = \frac{1}{-1} = 1$

Hence from Cauchy's Integral formula,

$$\int \frac{f(z)}{z-a} dz = 2\pi i \times f(a)$$

or, $\int \frac{f(z)}{z-1} dz = 2\pi i \times f(1)$

or, $\int \frac{\frac{z}{z-2}}{z-1} dz = 2\pi i \times 1$

or, $\int \frac{\frac{z}{z-2}}{z-1} dz = 2\pi i$

(Ans.).

3(b) (or)

Evaluate $\int \frac{2z-1}{z^2+z}\,dz$ where c is the circle $|z| = \frac{1}{2}$

Given,

$$|z| = \frac{1}{2}$$

or, $\sqrt{x^2 + iy^2} = \frac{1}{2}$

or, $x^2 + iy^2 = \frac{1}{4}$

or, $(x-0)^2 + i(y-0)^2 = \left(\frac{1}{2}\right)^2$

Center (0,0) and radius $= \frac{1}{2}$

Poles: $z^2 + z = 0$

or, $z(z+1) = 0$

$\therefore z = 0, -1$

Only one pole at $z=0$ inside the circle.

$$\int \frac{2z-1}{z^2+z}\,dz$$

or, $\int \frac{2z-1}{z(z+1)}\,dz$

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Here, $f(z) = \frac{2z+1}{z+1} dz$

$\therefore f(0) = \frac{1}{1} = 1$

From Cauchy's Integral formula,

$$\int \frac{f(z)}{z-a} dz = 2\pi i \times f(a)$$

or, $$\int \frac{\frac{2z+1}{z+1}}{z} dz = 2\pi i \times 1$$

$$\therefore \int \frac{\frac{2z+1}{z+1}}{z} = 2\pi i$$ (Ans).

IF THERE IS ANY MISTAKE, PROVIDE THE CORRECT ANSWER IN COMMENT