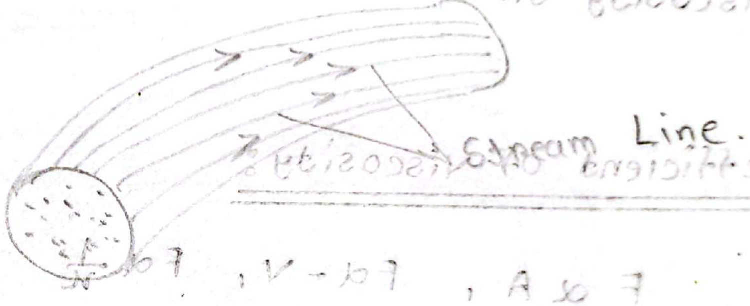
Dynamics of Fluids

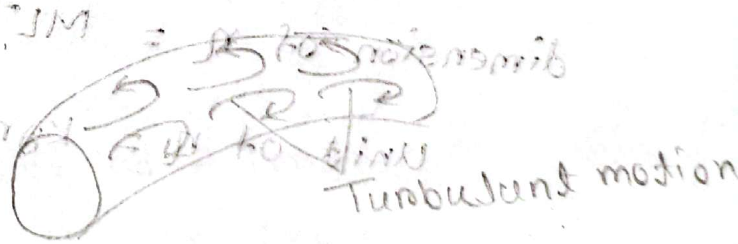
1 Stream line :

A Sream line may be defined as a curve, the tangent to which at any point gives the direction of flow of the fluid at that point. for, it may be straight or curved, according as the lateral pressure on it is the same throughout or different, in the latter case, the pressure being greater on the convex side than on the concave one. Two streamline can never cross one another.

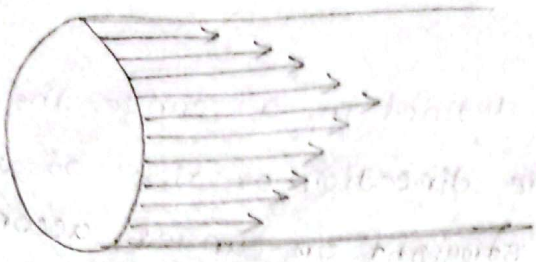


2 Turbulent motion :

The flow of the liquid loses all its steadiness and orderliness and becomes zigzag or sinnus, acquiring what is called turbulent motion.



3) Viscosity :



This property of a liquid by virtue of which it opposes relative motion between its different layers is known as viscosity or internal friction of the liquid.

4) Coefficient of viscosity :

$F \propto A , \quad F \propto -v , \quad F \propto \frac{1}{x}$

$F \propto -\frac{Av}{x}$

$\therefore F = -\eta \frac{Av}{x}$ [η = coefficient of viscosity]

dimension of $\eta = ML^{-1}T^{-1}$

Unit of $\eta = kg m^{-1} sec^{-1}$

$\therefore F = -\eta \frac{A \cdot dv}{dx}$ → Newton's law of viscous flow in streamline motion.

$\therefore \eta = -\frac{Fx}{Av}$

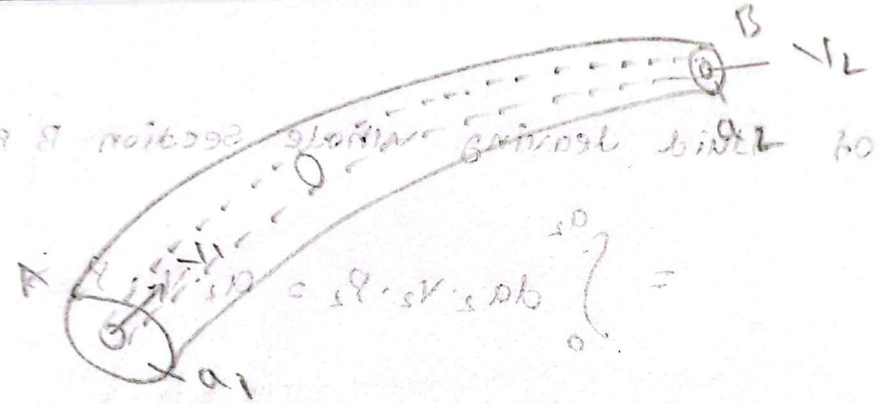
5| Critical Velocity :

It was Osborne Reynolds who first showed by direct experiment that the critical velocity V_c of a liquid is given by the relation,

$$V_c = \frac{K \cdot \eta}{\rho r}$$

[K being called Reynold's number
K = 1000]

6| Equation of continuity of flow :



Imagine the fluid to be flowing through a pipe AB with a_1 and a_2 as its areas of cross-section at sections A and B.

Mass of Fluid entering the tube at A per unit time per unit volume $= da_1 \cdot ds_1 \cdot \rho_1 / dt$

$$= da_1 \cdot v_1 \cdot \rho_1 \qquad \left[\text{as } ds_1/dt = v_1\right]$$

Mass of fluid leaving the tube at B per unit time per unit volume $= da_2 \cdot ds_2 \cdot \rho_2 / dt$

$= da_2 \cdot V_2 \cdot \rho_2$ $\quad [\text{as } ds_2/dt = V_2]$

$\therefore$ total mass of fluid entering whole section A per sec

$$= \int_0^{a_1} da_1 \cdot V_1 \cdot \rho_1 = a_1 \cdot V_1 \cdot \rho_1$$

total mass of fluid leaving whole section B per sec.

$$= \int_0^{a_2} da_2 \cdot V_2 \cdot \rho_2 = a_2 \cdot V_2 \cdot \rho_2$$

Since the fluid is compressible,

$$\rho_1 = \rho_2$$

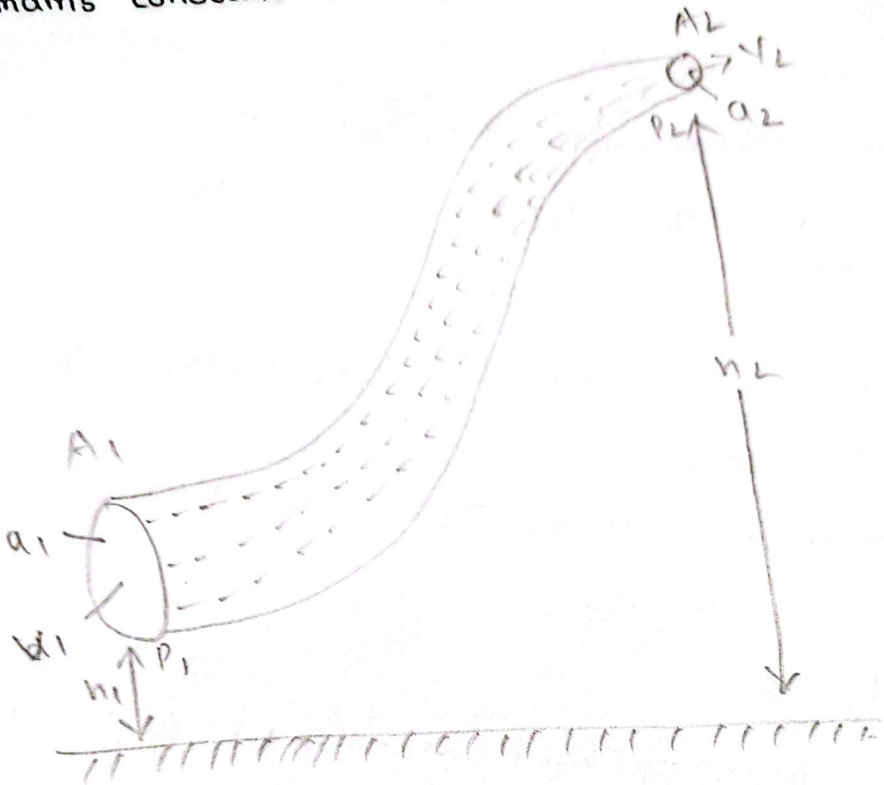
we have from the law of conservation of matter,

$$a_1 V_1 = a_2 V_2$$

$\therefore$ the rate of flow at A = rate of flow at B.

7| Bernoulli's Theorem:

This theorem states that, "the total energy per unit mass of a liquid flowing from one point to another, without any friction, remains constant throughout the displacement.".



Potential energy + Pressure energy + kinetic energy = a constant.

or, $hg + \frac{P}{\rho} + \frac{1}{2}V^2 = C$

This relation is known as Bernoulli's Equation.

Work done per second on the fluid entering A_1,

$W_1 = a_1 V_1 P_1$

Work done per second on the fluid leaving A_2,

$$W_2 = a_2 v_2 p_2$$

$\therefore$ Total work done on the fluid,

$$W = W_1 - W_2$$
$$= p_1 a_1 v_1 - p_2 a_2 v_2$$
$$= (p_1 - p_2) a_1 v_1 \qquad [\because a_2 v_2 = a_1 v_1]$$

$\therefore$ Change in Potential energy, $E_1 = (a_1 v_1) \rho g (h_2 - h_1)$

$\therefore$ Change in kinetic energy, $E_2 = \frac{1}{2} (a_1 v_1) \rho (v_2^2 - v_1^2)$

$\therefore$ $W = E_1 + E_2$

or, $(p_1 - p_2) a_1 v_1 = (a_1 v_1) \rho g (h_2 - h_1) + \frac{1}{2}(a_1 v_1) \rho (v_2^2 - v_1^2)$

or, $p_1 - p_2 = \rho g (h_2 - h_1) + \frac{1}{2} \rho (v_2^2 - v_1^2)$

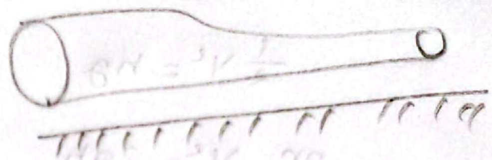
or, $p_1 - p_2 = \rho g h_2 - \rho g h_1 + \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2$

or, $p_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$

$\Rightarrow$ or, $p + \rho g h + \frac{1}{2} \rho v^2 = C$

or, $\frac{p}{\rho} + g h + \frac{1}{2} v^2 = C$

or, $\frac{p}{\rho} + \frac{1}{2}v^2 = C$ [when h is constant]

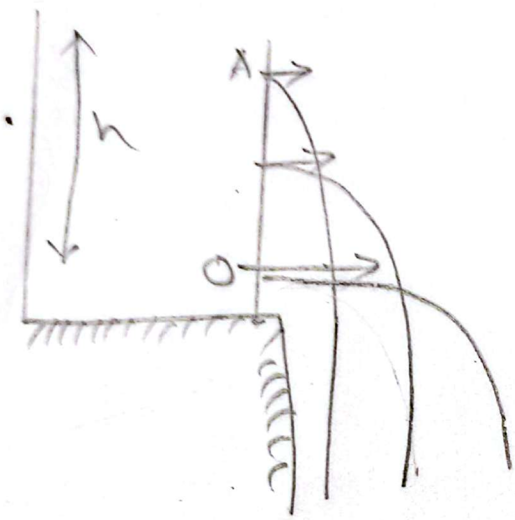


or, $p + \frac{1}{2}\rho v^2 = C$

Static Pressure

dynamic Pressure.

8) Torricelli's Theorem :



Let the surface of the liquid be at a height h above the level of the circular and sharp edged orifice O in a tank.

total energy at A

$= $ Pressure energy + Potential energy + Kinetic energy

$= 0 + hg + 0$

total energy at O,

the level of the orifice $= 0 + 0 + \frac{1}{2}v^2$

total energy remains the same,

$$\frac{1}{2}v^2 = hg$$

or, $v^2 = 2gh$

or, $v = \sqrt{2gh}$.

this is the velocity of efflux of the liquid at the orifice O.