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International Islamic University Chittagong
Department of Computer Science & Engineering
B.Sc. in CSE., Mid Term Examination, Autumn-2019
Course Code: MATH-2407 Course Title: Mathematics-IV

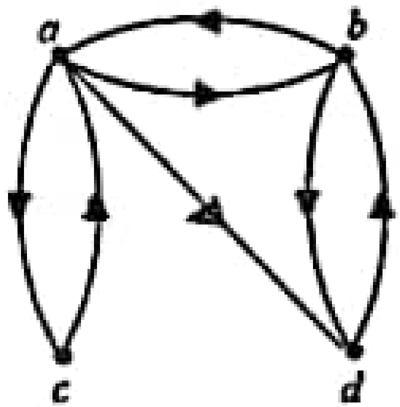
Total Marks: 30 Time: 1 hour & 30 minutes

[Answer any *three* questions]

1. a) What will be the result if you multiply the n-number of complex numbers in polar coordinates where $r_1, r_2, r_3, \ldots, r_n$ are the distance from the origin of complex numbers and $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are the angles with x-axis respectively. Also show the result if $\theta_1 = \theta_2 = \theta_3 = \ldots = \theta_n = \theta$ 7

b) Using Demoivre's theorem, If $x_r = \cos\frac{\pi}{3^r} + i\sin\frac{\pi}{3^r}$, prove that $x_1 x_2 x_3 \ldots \infty = i$ 3

2. a) 4

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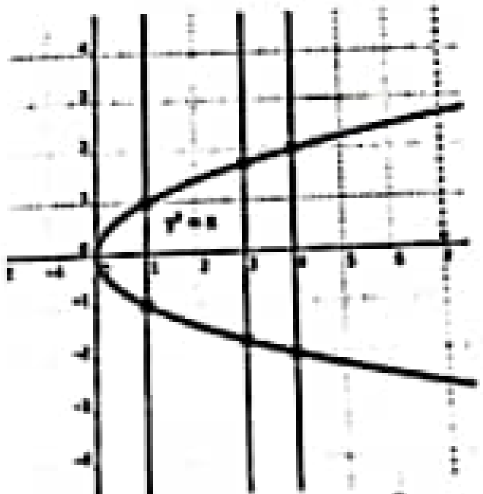
Check whether the above relation is reflexive, symmetric, anti-symmetric and transitive or not.

b) 1



Comment on the above graph? Is it a function or not?

c) Determine whether $f(z) = f(x+iy) = (x^3 - 3xy^2 - 2x) + i(3x^2y - y^3 - 2y)$ is analytic or not. 5

3. a) Is $f(x,y,z) = x^2 + y^2 - 2z^2$ harmonic?

b) A straight line joining $A(-j)$ and $B(2+j)$ in the z-plane is mapped onto the w-plane by the transformation equation $w = \frac{1}{z}$. Justify your mapping.

4. a) If $f(z) = \frac{z}{(z-1)(z+1)^2}$, find the residues of $f(z)$ at the poles

b) Evaluate $\int_c \frac{z}{z^2 - 3z + 2} dz$ by Cauchy's Integral Formula

Where c is the circle $|z-2| = \frac{1}{2}$

c) Evaluate $\int_C (x + jy)dz$ along the contour C defined by the line from $z = 0$ to $z = 3 + j$

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Autumn - 2019

1.a

Complex Number in polar coordinates $(x+iy) = (r\cos\theta + ir\sin\theta)$
$$(x+iy) = r(\cos\theta + i\sin\theta)$$

Let another complex number.

$$x_1+iy_1 = r_1(\cos\theta_1 + i\sin\theta_1)$$
$$x_2+iy_2 = r_2(\cos\theta_2 + i\sin\theta_2)$$
$$x_3+iy_3 = r_3(\cos\theta_3 + i\sin\theta_3)$$
$$\cdots$$
$$\cdots$$
$$\cdots$$
$$\therefore x_n+iy_n = r_n(\cos\theta_n + i\sin\theta_n)$$

$$(x_1+iy_1)(x_2+iy_2) = r_1(\cos\theta_1+i\sin\theta_1)\, r_2(\cos\theta_2 + i\sin\theta_2)$$
$$= r_1r_2(\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\cos\theta_2\sin\theta_1 + i^2\sin\theta_1\sin\theta_2)$$
$$= r_1r_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i\cos\theta_1\sin\theta_2 + i\cos\theta_2\sin\theta_1)$$
$$= r_1r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

$$\therefore r_1r_2(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2) = r_1r_2\{\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\}$$

$$(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2) = \{\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\} \quad (1)$$

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Now,

$$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_3) =$$

$$\{\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\}(\cos\theta_3 + i\sin\theta_3)$$

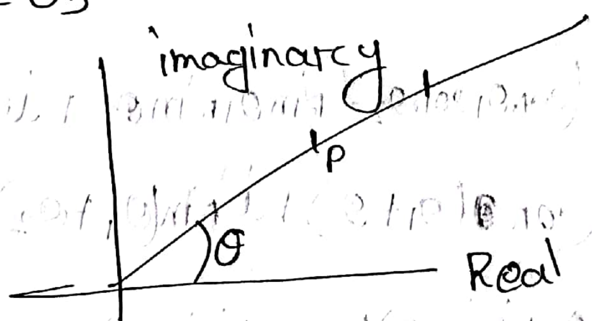
$$= \{\cos(\theta_1+\theta_2)\cos\theta_3 + i\sin\theta_3\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\cos\theta_3 + i^2\sin(\theta_1+\theta_2)\sin\theta_3\}$$

$$= \{\cos(\theta_1+\theta_2+\theta_3) + i\sin(\theta_1+\theta_2+\theta_3)\}$$

So,

$$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_3) \cdots (\cos\theta_n + i\sin\theta_n) = \cos(\theta_1+\theta_2+\theta_3+\cdots+\theta_n) + i\sin(\theta_1+\theta_2+\theta_3+\cdots+\theta_n)$$

if $\theta_1 = \theta_2 = \theta_3 = \cdots = \theta_n = \theta$




$$(\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta) \cdots (\cos\theta + i\sin\theta)$$

$$= \cos(\theta+\theta+\theta+\cdots+\theta) + i\sin(\theta+\theta+\cdots)$$

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

Ans.

1 (b)

$x_r = \cos\frac{\pi}{3^r} + i\sin\frac{\pi}{3^r}$, prove that $x_1 x_2 x_3 \cdots \infty = i$

Given,

$$x_r = \cos\frac{\pi}{3^r} + i\sin\frac{\pi}{3^r}$$

putting $r = 1, 2, 3, 4, 5, \ldots$

$$x_1 = \cos\frac{\pi}{3^1} + i\sin\frac{\pi}{3^1}$$

$$x_2 = \cos\frac{\pi}{3^2} + i\sin\frac{\pi}{3^2}$$

$$x_3 = \cos\frac{\pi}{3^3} + i\sin\frac{\pi}{3^3}$$

$\therefore x_1 x_2 x_3 \cdots$

$$\left(\cos\frac{\pi}{3^1} + i\sin\frac{\pi}{3^1}\right)\left(\cos\frac{\pi}{3^2} + i\sin\frac{\pi}{3^2}\right)\left(\cos\frac{\pi}{3^3} + i\sin\frac{\pi}{3^3}\right)\cdots$$

$$= \cos\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \cdots\right) + i\sin\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \cdots\right)\cdots$$

$$= \cos\left\{\frac{\pi}{3}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots\right)\right\} + i\sin\left\{\frac{\pi}{3}\left(1 + \frac{1}{3^2} + \frac{1}{3^3} + \cdots\right)\right\}\cdots$$

$$= \cos\left\{\frac{\pi}{3}\left(1 - \frac{1}{3}\right)^{-1}\right\} + i\sin\left\{\frac{\pi}{3}\left(1 - \frac{1}{3}\right)^{-1}\right\}$$

$$= \cos\left\{\frac{\pi}{3}\left(\frac{3-1}{3}\right)^{-1}\right\} + i\sin\left\{\pi/3\left(\frac{3-1}{3}\right)^{-1}\right\}$$

$$= \cos\left\{\frac{\pi}{3} \times \frac{3}{2}\right\} + i\sin\left(\frac{\pi}{3} \times \frac{3}{2}\right)$$

$$= \cos\frac{\pi}{2} + i\sin\pi/2$$

[redacted]

$= 0 + \hat{i}.1$

$= \hat{i}$

$\boxed{\text{Proved}}$

2(b)

2(a)

- here,

  relation is not reflexive because.

  $\{a,a\} \notin R$ ex: $\{a,a\},\{b,b\} \notin R$

- here the relation is symmetric because.

  $\{a,b\} \in R$ implies $\{b,a\} \in R$

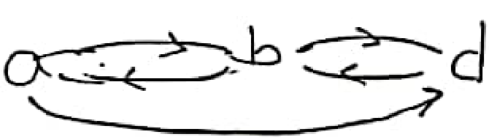
  ex: $\{a,b\}\ \{b,a\} \in R$.

- here the relation is not anti symmetric bcz.

  $\{a,b\} \in R$ and $(b,a) \in$ but $a=b \notin R$ so that it is not anti-symmtric.

- here the relation is ~~to~~ transitive bcz.

  $(a,b) \in R$ and ~~$(b,d)\in$~~ $(b,d) \in R$ and implies. $(a,d) \in R$.



2(b)

the graph is not function.

Any number of vertical lines will..
interect this oval twice. for instance
the y-axis intersects twice.

## 2(c)

$f(z) = f(x+iy) = (x^3 - 3xy^2 - 2x) + i(3x^2y - y^3 - 2y)$

$u + iv = (x^3 - 3xy^2 - 2x) + i(3x^2y - y^3 - 2y)$

Equating real and imaginary part.

$u = x^3 - 3xy^2 - 2x$ ——— (1)

$v = 3x^2y - y^3 - 2y$ ——— (11)

from (1)

$\frac{\delta u}{\delta x} = 3x^2 - 3y^2 - 2$

$\frac{\delta u}{\delta y} = -6xy$

from (11)

$\frac{\delta v}{\delta x} = 6xy$

$\frac{\delta v}{\delta y} = 3x^2 - 3y^2 - 2$

$\frac{\delta u}{\delta x} = \frac{\delta v}{\delta y}$

$\therefore 3x^2 - 3y^2 - 2 = 3x^2 - 3y^2 - 2$

$\therefore$ L.H.S = R.H.S

$\frac{\delta v}{\delta x} = -\frac{\delta u}{\delta y}$

$= 6xy = -(-6xy)$

$\therefore 6xy = 6xy.$

L.H.S = R.H.S

The eqn is analytic.

**Example 11**

A straight line joining $A(-j)$ and $B(2+j)$ in the z-plane is mapped onto the w-plane by the transformation equation $w=\frac{1}{z}$

Ans To The Q 3B

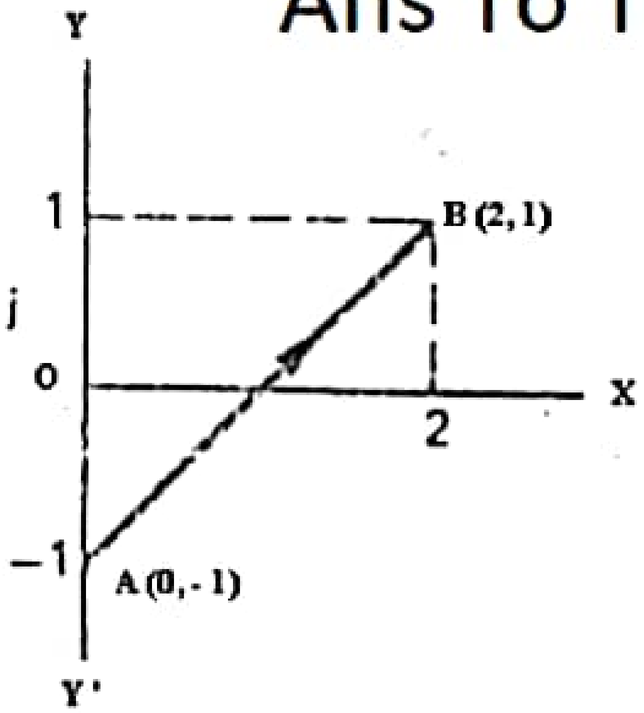



**Figure 27**

**Solution:**

Given,

$$w=\frac{1}{z}$$

$$w=\frac{1}{x+jy} \qquad [z=x+jy]$$

$$w=\frac{x-jy}{(x+jy)(x-jy)} \qquad [\text{Multiplying by } x-jy]$$

$$w=\frac{x-jy}{x^2-jxy+jxy-j^2y^2}$$

$$w=\frac{x-jy}{x^2+y^2} \qquad [\because j^2=-1]$$

$$u+jv=\frac{x-jy}{x^2+y^2} \qquad [w=u+jv]$$

$$u+jv=\frac{x}{x^2+y^2}-j\frac{y}{x^2+y^2} \qquad \text{------------------------(i)}$$

Equating the coefficient of real and imaginary part, we get,

$$u=\frac{x}{x^2+y^2} \qquad \text{-----------(ii)}$$

$$v=\frac{-y}{x^2+y^2} \qquad \text{----------(iii)}$$

Given, $A(0-j.1)$

That is, $A(0,-1)$ ------------------------(iv)

Here,

$x=0, y=-1$

Putting the value of x and y in (ii) and (iii)

$$u=\frac{x}{x^2+y^2} \qquad v=\frac{-y}{x^2+y^2}$$

$$u=\frac{0}{0+(-1)^2} \qquad v=\frac{-(-1)}{0^2+(-1)^2}$$

$$u=\frac{0}{0+1} \qquad v=\frac{1}{1}$$

$$u=\frac{0}{1} \qquad v=1$$

$$u=0 \qquad v=1$$

$$\therefore w=u+jv=0+j.1$$

The image of A is $A'(w=0+j.1)$

That is $A'(0,1)$ ------------------------(v)

Again,

$B(z=2+j.1)$

That is, $B(2,1)$ ------------------------(vi)

Here, $x=2, y=1$

Putting the value of x and y in (ii) and (iii),

$$u=\frac{x}{x^2+y^2} \qquad v=\frac{-y}{x^2+y^2}$$

$$u=\frac{2}{2^2+1^2} \qquad v=\frac{-1}{2^2+1^2}$$

$$u=\frac{2}{5} \qquad v=\frac{-1}{5}$$

$$\therefore w=u+jv=\frac{2}{5}-j\frac{1}{5}$$

The image of B is $B'(w=\frac{2}{5}-j\frac{1}{5})$

That is $B'(\frac{2}{5},-\frac{1}{5})$ ------------------------(vii)

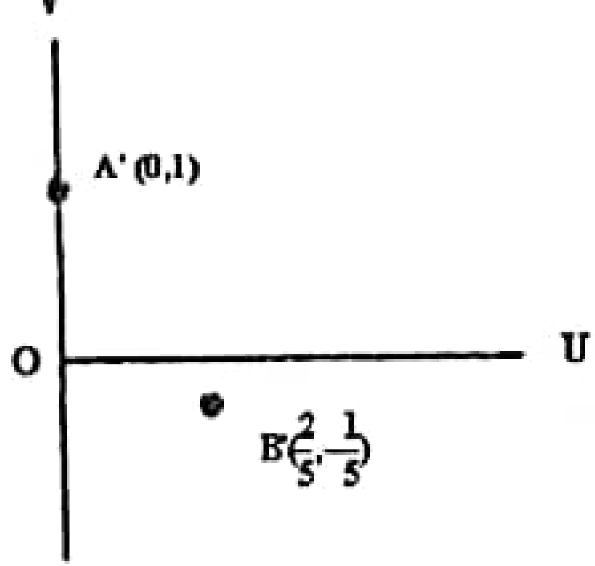



Figure 28

From Figure 27:
Given A(0,-1) and B(2,1)
The equation of the line AB is,

$$\frac{y-y_1}{y_1-y_2}=\frac{x-x_1}{x_1-x_2}$$

$$\Rightarrow \frac{y-(-1)}{-1-1}=\frac{x-0}{0-2}$$

$$\Rightarrow \frac{y+1}{-1-1}=\frac{x-0}{0-2}$$

$$\Rightarrow \frac{y+1}{-2}=\frac{x}{-2}$$

$$\Rightarrow y+1=x$$

$$\therefore y=x-1 \qquad \text{------------}(viii)$$

Again, Given

$$w=\frac{1}{z}$$

$$\therefore z=\frac{1}{w}$$

$$z=\frac{1}{u+jv} \qquad [w=u+jv]$$

$$z=\frac{u-jv}{(u+jv)(u-jv)}$$

$$z=\frac{u-jv}{u^2-(jv)^2}$$

$$z=\frac{u-jv}{u^2+v^2} \qquad [\because j^2=-1]$$

$$x+jy=\frac{u-jv}{u^2+v^2} \qquad [z=x+jy]$$

$$i.e.\ x+jy=\frac{u}{u^2+v^2}-j\frac{v}{u^2+v^2} \text{ ---------}(ix)$$

Equating the coefficient of real and imaginary part, we get,

$$x=\frac{u}{u^2+v^2}\ ; \qquad y=\frac{-v}{u^2+v^2} \text{ ----------}(x)$$

Putting the value of x and y in (viii),

$$y=x-1$$

$$\Rightarrow \frac{-v}{u^2+v^2}=\frac{u}{u^2+v^2}-1$$

$$\Rightarrow \frac{-v}{u^2+v^2}=\frac{u-u^2-v^2}{u^2+v^2}$$

$$\Rightarrow -v=u-u^2-v^2$$

$$\Rightarrow u-u^2-v^2+v=0$$

$$\Rightarrow -u+u^2+v^2-v=0$$

$$\Rightarrow u^2-u+v^2-v=0$$

$$\Rightarrow (u^2-u)+(v^2-v)=0$$

$$\Rightarrow u^2-2\cdot u\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+v^2-2\cdot v\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2=0$$

$$\Rightarrow u^2-2\cdot u\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2+v^2-2\cdot v\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\frac{1}{4}-\frac{1}{4}=0$$

$$\Rightarrow \left(u-\frac{1}{2}\right)^2+\left(v-\frac{1}{2}\right)^2-\frac{2}{4}=0$$

$$\Rightarrow \left(u-\frac{1}{2}\right)^2+\left(v-\frac{1}{2}\right)^2-\frac{1}{2}=0$$

$$\Rightarrow \left(u-\frac{1}{2}\right)^2+\left(v-\frac{1}{2}\right)^2=\frac{1}{2}$$

$$\Rightarrow \left(u-\frac{1}{2}\right)^2+\left(v-\frac{1}{2}\right)^2=\left(\frac{1}{\sqrt{2}}\right)^2 \text{ -----------}(xi)$$

The equation (xi) represents an equation of a circle whose centre $C\left(\frac{1}{2},\frac{1}{2}\right)$ and

Radius $=\frac{1}{\sqrt{2}}$

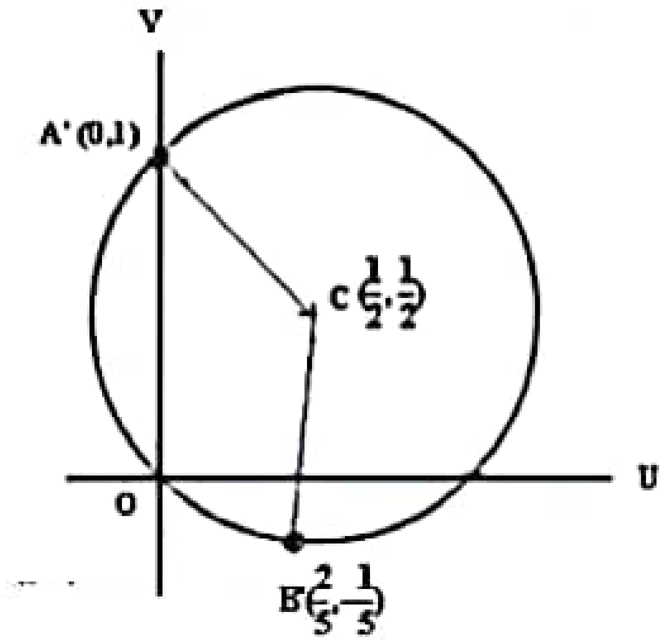
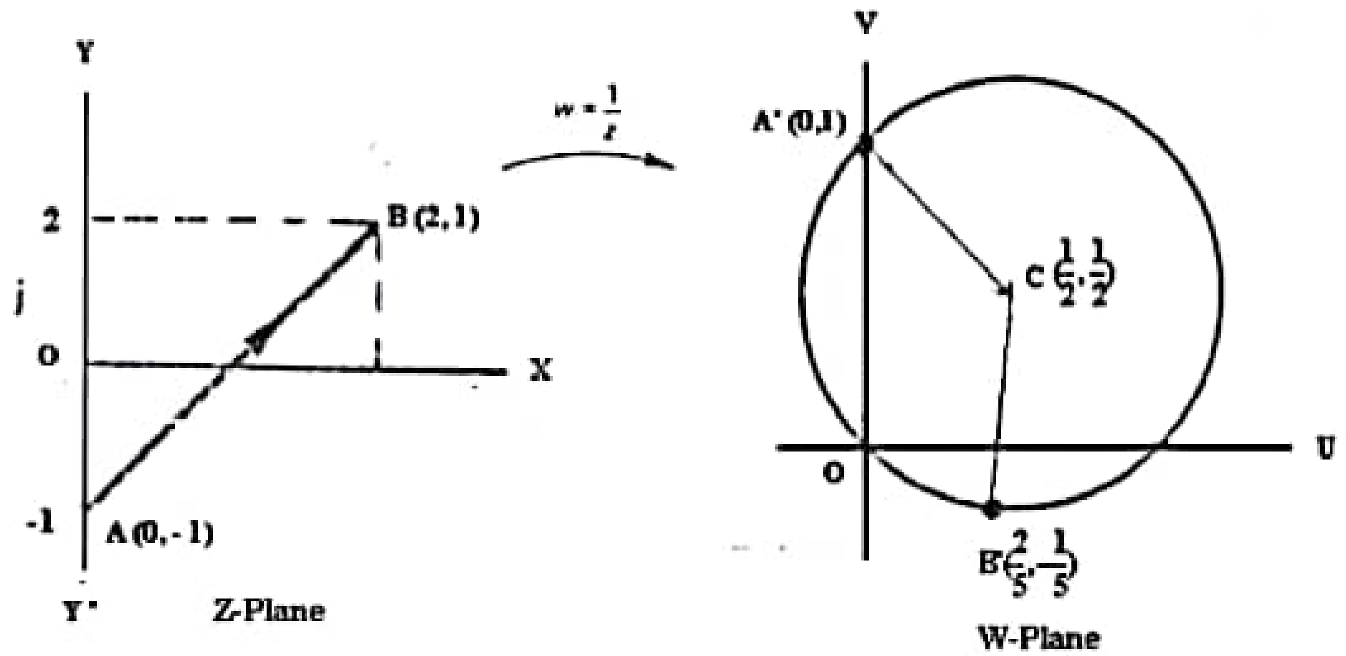



**Figure 29**




**Justification of radius of the circle:**

We have $A'(w = 0 + j.1)$ that is the coordinate of $A'(0,1)$ and the center $C\left(\frac{1}{2},\frac{1}{2}\right)$

$$\therefore A'C = \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2}$$

$$\therefore A'C = \sqrt{(0 - \frac{1}{2})^2 + (1 - \frac{1}{2})^2}$$

$$\therefore A'C = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2}$$

$$\therefore A'C = \sqrt{\frac{1}{4} + \frac{1}{4}}$$

$$\therefore A'C = \sqrt{\frac{2}{4}}$$

$$\therefore A'C = \sqrt{\frac{1}{2}}$$

$$\therefore A'C = \frac{1}{\sqrt{2}} \text{ (Proved)}$$

$$\therefore \text{Radius} = \frac{1}{\sqrt{2}}$$

We have $B'(\frac{2}{5}, -\frac{1}{5})$ and $C(\frac{1}{2}, \frac{1}{2})$

$$\therefore B'C = \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2}$$

$$B'C = \sqrt{(\frac{2}{5} - \frac{1}{2})^2 + (-\frac{1}{5} - \frac{1}{2})^2}$$

$$B'C = \sqrt{(\frac{4-5}{10})^2 + (\frac{-2-5}{10})^2}$$

$$B'C = \sqrt{(\frac{-1}{10})^2 + (\frac{-7}{10})^2}$$

$$B'C = \sqrt{\frac{1}{100} + \frac{49}{100}}$$

$$B'C = \sqrt{\frac{50}{100}}$$

$$B'C = \sqrt{\frac{1}{2}}$$

$$\therefore B'C = \frac{1}{\sqrt{2}} \text{ (Proved)}$$

$$\therefore \text{Radius} = \frac{1}{\sqrt{2}}$$

Q-2: Evaluate $\int_c \frac{z}{z^2-3z+2}dz$

Ans To The Question No 4(b)

Where c is the circle $|z-2| = \frac{1}{2}$

We have,

$z = x + jy$

$z - 2 = x + jy - 2$

$z - 2 = x - 2 + jy$

$\therefore |z - 2| = \sqrt{(x-2)^2 + y^2}$

Given,

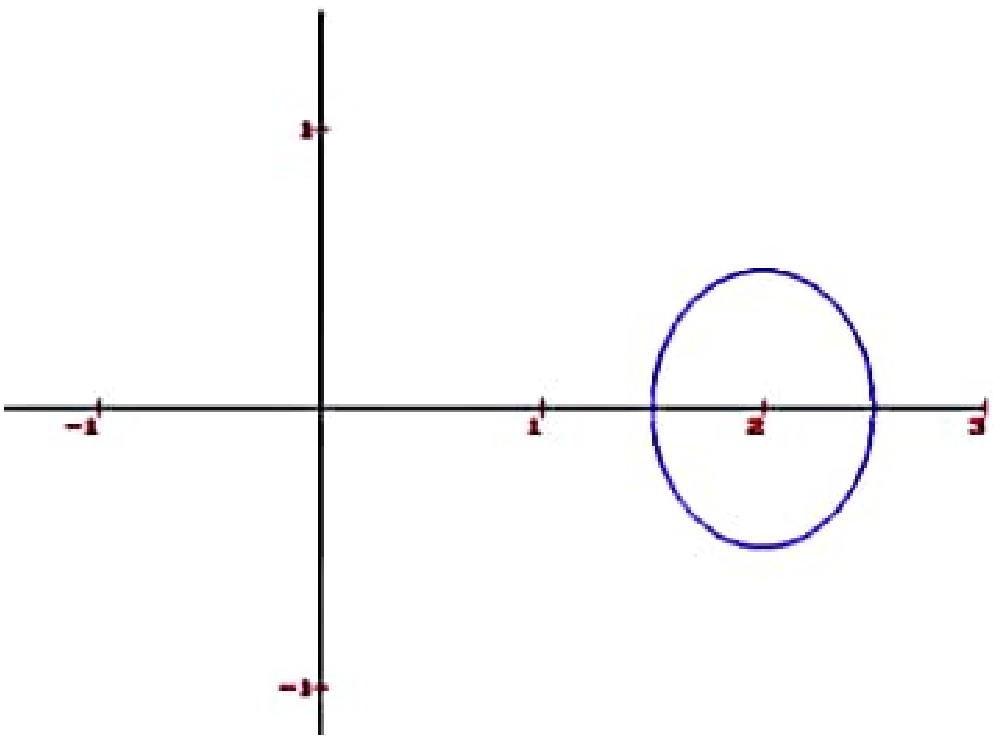
$|z - 2| = \frac{1}{2}$

$\therefore |z - 2| = \sqrt{(x-2)^2 + y^2} = \frac{1}{2}$

$\therefore (x-2)^2 + y^2 = \frac{1}{4}$

$\therefore (x-2)^2 + (y-0)^2 = (\frac{1}{2})^2$

Which is the equation of a circle whose Center (2, 0), Radius $= \frac{1}{2}$



Poles: $z^2 - 3z + 2 = 0$

That is $z = 1,2$

There is only one pole at $z = 2$ inside the given circle.

$$\int_c \frac{z}{z^2 - 3z + 2} dz$$

$$= \int_c \frac{z}{z^2 - 2z - z + 2} dz$$

$$= \int_c \frac{z}{z(z-2) - 1(z-2)} dz$$

$$= \int_c \frac{z}{(z-1)(z-2)} dz$$

$$= \int_c \frac{\frac{z}{z-1}}{z-2} dz$$

Here, $f(z) = \frac{z}{z-1}$

$\therefore f(2) = \frac{2}{2-1} = 2$

Hence, from Cauchy's Integral Formula:

$$\int_c \frac{f(z)}{z-a} dz = 2\pi i \times f(a)$$

$$\int_c \frac{f(z)}{z-2} dz = 2\pi i \times f(2) \qquad [a = 2]$$

$$\int_c \frac{\frac{z}{z-1}}{z-2} dz = 2\pi i \times f(2)$$

$$\int_c \frac{\frac{z}{z-1}}{z-2}\,dz = 2\pi i \times 2$$

$$\int_c \frac{\frac{z}{z-1}}{z-2}\,dz = 4\pi i$$

No, the function f(x,y,z) = x^2 y^2 2z is not harmonic.

To check whether a function is harmonic or not, we need to verify that it satisfies the Laplace equation, which states that the sum of the second partial derivatives with respect to each variable should be equal to zero:

$$\partial^2 f/\partial x^2 + \partial^2 f/\partial y^2 + \partial^2 f/\partial z^2 = 0$$

3A

Let's calculate the second partial derivatives of f(x,y,z):

$\partial^2 f/\partial x^2$ = 2y^2 2z
$\partial^2 f/\partial y^2$ = 2x^2 2z
$\partial^2 f/\partial z^2$ = 4x^2 y^2

Now, let's substitute these partial derivatives into the Laplace equation:

$\partial^2 f/\partial x^2 + \partial^2 f/\partial y^2 + \partial^2 f/\partial z^2$ = (2y^2 2z) + (2x^2 2z) + (4x^2 y^2) = 4x^2 y^2 + 4x^2 y^2 + 8x^2 y^2 = 16x^2 y^2 z

Since this is not equal to zero, the function f(x,y,z) is not harmonic.