International Islamic University Chittagong

Department of Computer Science and Engineering

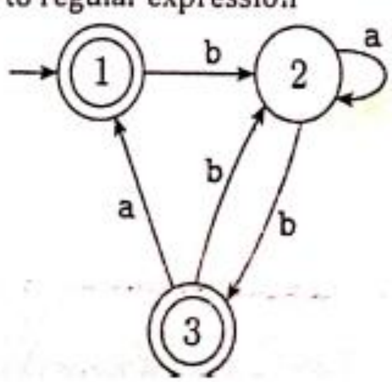
B. Sc. in CSE Midterm Examination, Autumn 2022

Course Code: CSE 2425 Course Title: Theory of Computing

Total marks: ***30*** *Time:* ***90*** *minutes*

[Answer all the questions; in some questions, there might be options; Figures in the right hand margin indicate full marks.]

		Marks	CO	DL
1. a)	Why do you think that the study of the theory of computation is important? Mention any two reasons in brief.	2	CO1	E
b)	Construct a DFA for recognizing decimal integers which are divisible by 2 and starts with Y. Here, Y is the last digit of your ID. **OR** Construct a DFA for recognizing binary numbers which are divisible by 2 and whose length is odd.	2	CO1	C
c)	Construct DFA for the following languages where alphabet is {0, 1}. i. $\{w \mid$ every 0 in w is followed by a 1$\}$ ii. $\{w \mid w$ has exactly two 0's and at most three 1's$\}$ iii. $\{w \mid w$ does not contain the substring 0110$\}$	3	CO1	C
d)	Write regular expressions for the languages described in **1(c)**.	3	CO2	C
2. a)	What are the languages described by the following regular expressions? Write a one sentence description for each language. (**Any two**) i. $0^*(0 \cup 11)^*$ ii. $(00 \cup 1)^*(11)^*$ iii. $((1(11)^*00) \cup (11)^*0)^*$ iv. $101((11)^* \cup (00)^*)$	2	CO2	N
b)	Prove that every nondeterministic finite automaton has an equivalent deterministic finite automaton.	4	CO2	N
c)	Prove that the regular language is closed under the concatenation operation. **OR** Prove that the regular language is closed under the star operation.	4	CO2	N
3. a)	Convert the following regular expressions to NFA. (**Any two**) i. $(1^* \cup 0(00)^*)^*111$ ii. $101((11)^* \cup (00)^*)$ iii. $((1(11)^*00) \cup (11)^*0)^*$ iv. $(00 \cup 1)^*(11)^*$	2	CO1	A
b)	Give an NFA recognizing the language $(11 \cup 00)^*$. Convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.	4	CO1	A
c)	Convert the following DFA to regular expression	4	CO1	A



ANSWER TO THE QUESTION NO. 01
(a)

1. Autumn- 2022

a) Why do you think that the study of the theory of computation is important? Mention any two reasons in brief.

Answer: Theory of computation plays an important role in compiler design as well as analysis of complex software and hardware systems.

1. Understanding the limits and capabilities of computation: It allows to determine whether a problem is computable or not to measure the time and space complexity of algorithms. This knowledge helps computer scientists design better algorithms, optimize programs, and build efficient systems.

2. Building strong foundations for computer science: The theory of computation provides a strong theoretical foundation for computer science. It helps computer scientists reason about the correctness and efficiency of algorithms and data structures. It also provides a framework for studying the behavior of complex system, such as networks distributed systems and databases.

ANSWER TO THE QUESTION NO. 01

(b)

b) Construct a DFA for recognizing decimal integers which are divisible by 2 and starts with Y. Here Y is the last digit of your ID.

$Y = 2$

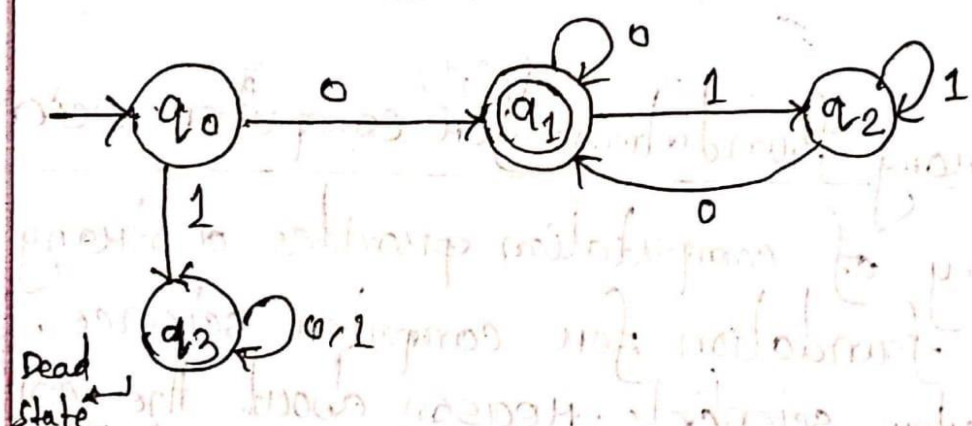
<u>Answer:</u> $L = \{2, 22, 24, 26 \ldots\ldots\}$

if $n = 0$ to 9.

$n \% 2 = 0 = 0$ (say)

$n \% 2 != 0 = 1$ (say)

$\therefore \Sigma = \{0, 1\}$



Or

Construct a DFA for recongnizing binary numbers which are divisible by 2 and whose length is odd.

length is odd.

so, $|W| = 1 \pmod 2$ $[|W| \% 2 == 1]$

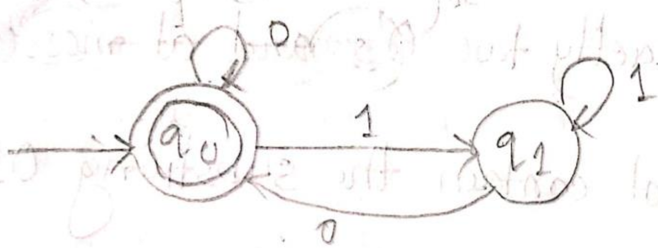
$(3)_{10} = (11)_2$

$(4)_{10} = (100)_2 = 001$

$(10)_{10} = (1010)_2$

last bit is = 0.

$(6)_{10} = (110)_2$



ANSWER TO THE QUESTION NO. 01

(c)

(c) Construct a DFA for ~~the~~ the following languages where alphabets is $\{0,1\}$.

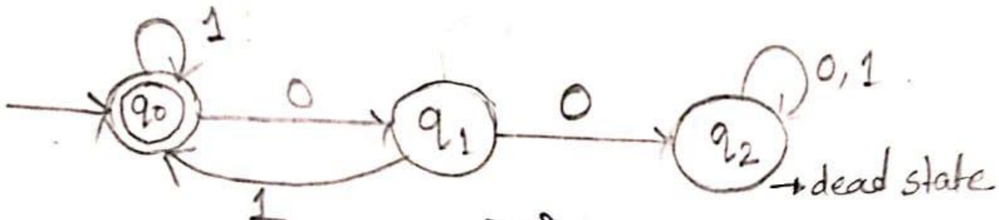
i. $\{w \mid$ every 0 in w is followed by a 1$\}$

ii. $\{w \mid w$ has exactly two 0's and at most three 1's$\}$

iii. $\{w \mid w$ does not contain the substring 0110$\}$.

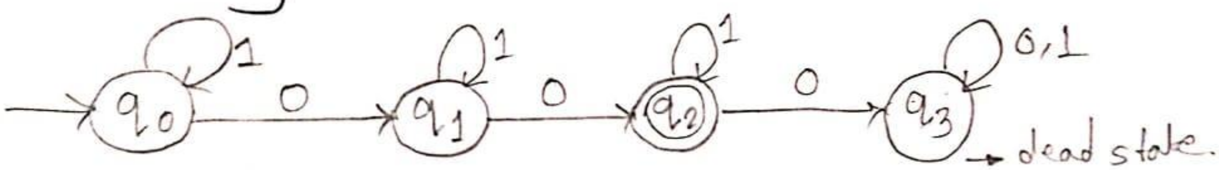
(i)

$L = \{\epsilon, 01, 1011, 0111, 0001, 1, 11, 1\}$.

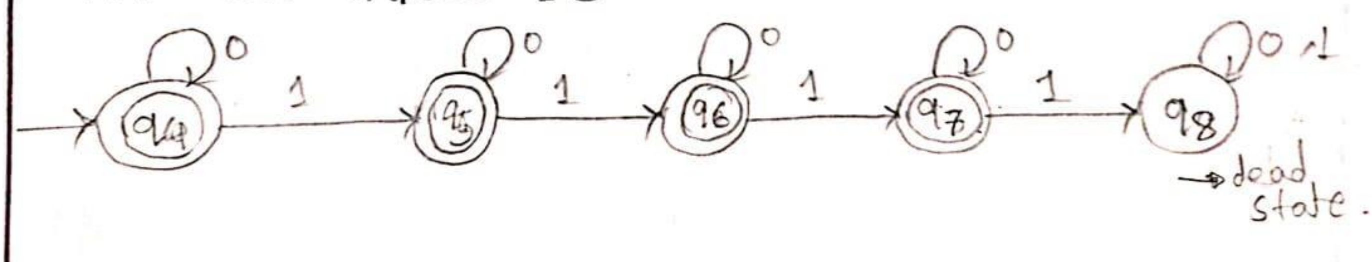


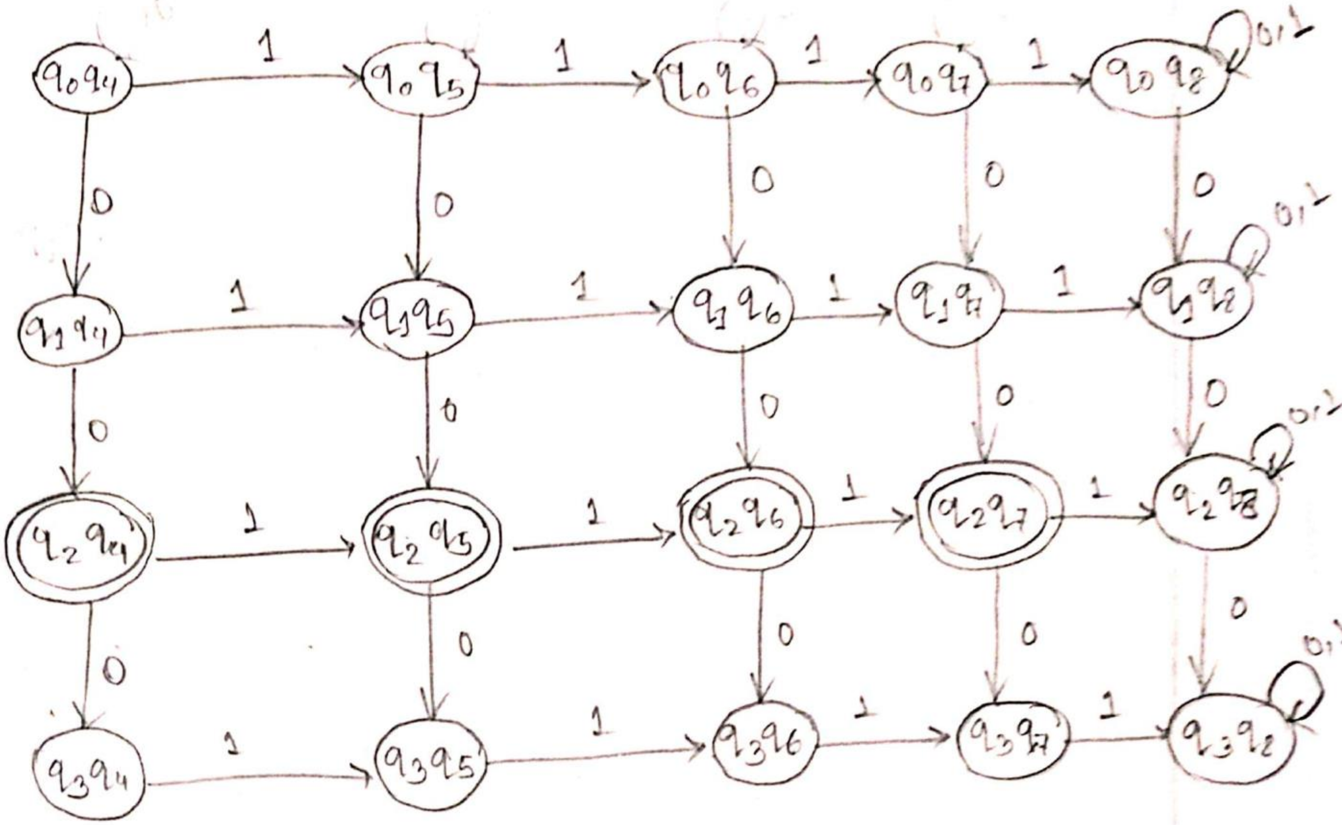
(ii)

Exactly two 0's —

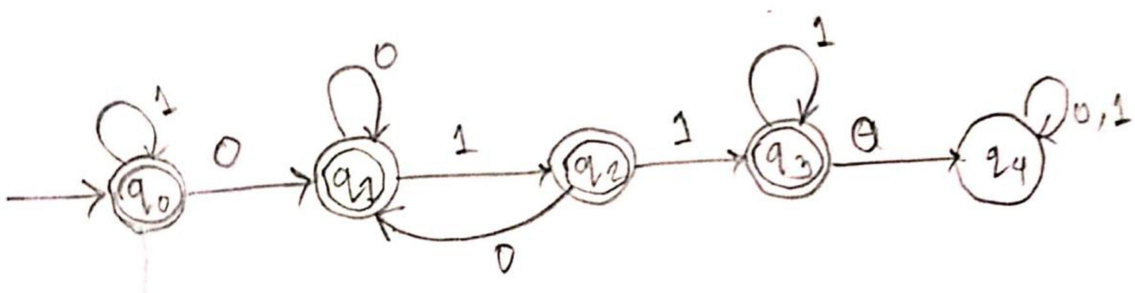


At most three 1's




q0q4
q0q5
q0q6
q0q7
q0q8
q1q4
q1q5
q1q6
q1q7
q1q8
q2q4
q2q5
q2q6
q2q7
q2q8
q3q4
q3q5
q3q6
q3q7
q3q8
0,1

(iii)


q0
q1
q2
q3
q4
0,1

ANSWER TO THE QUESTION NO. 01

(d)

Q : Write regular expressions for the language s described in 1(c).

Answer:

i. {w | every 0 in w is followed by a 1}

   Regular expression : 1*(01)*1*

ii. {w | w has exactly two 0's and at most three 1's}

   Regular expression : (1*01*01*).(0*(1∪∈)0*(1∪∈)0*)

iii. {w | w does not contain the substring 0110}

   Regular expression : ε ∪ 0 ∪ 1 ∪ 0(0∪1)* ∪ 1(0∪1)* ∪ (00∪01)(0∪1)* ∪ 1(0∪1)(0∪1) ∪ 0(0∪1)(0∪1)

ANSWER TO THE QUESTION NO. 02

(b)

Q: Prove that every nondeterministic finite automaton has an equivalent deterministic finite automaton.

<u>Proof:</u>

PROOF Let $N = (Q, \Sigma, \delta, q_0, F)$ be the NFA recognizing some language A. We construct a DFA $M = (Q', \Sigma, \delta', q_0', F')$ recognizing A. Before doing the full construction, let's first consider the easier case wherein N has no ε arrows. Later we take the ε arrows into account.

1. $Q' = \mathcal{P}(Q)$.
   Every state of M is a set of states of N. Recall that $\mathcal{P}(Q)$ is the set of subsets of Q.
2. For $R \in Q'$ and $a \in \Sigma$, let $\delta'(R, a) = \{q \in Q |\ q \in \delta(r, a) \text{ for some } r \in R\}$. If R is a state of M, it is also a set of states of N. When M reads a symbol a in state R, it shows where a takes each state in R. Because each state may go to a set of states, we take the union of all these sets. Another way to write this expression is
   $$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a).^4$$
3. $q_0' = \{q_0\}$.
   M starts in the state corresponding to the collection containing just the start state of N.
4. $F' = \{R \in Q' |\ R \text{ contains an accept state of } N\}$.
   The machine M accepts if one of the possible states that N could be in at this point is an accept state.

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[4]The notation $\bigcup_{r \in R} \delta(r, a)$ means: the union of the sets $\delta(r, a)$ for each possible r in R.

Now we need to consider the ε arrows. To do so, we set up an extra bit of notation. For any state R of M, we define $E(R)$ to be the collection of states that can be reached from members of R by going only along ε arrows, including the members of R themselves. Formally, for $R \subseteq Q$ let

$$E(R) = \{q |\ q \text{ can be reached from } R \text{ by traveling along 0 or more } \varepsilon \text{ arrows}\}.$$

Then we modify the transition function of M to place additional fingers on all states that can be reached by going along ε arrows after every step. Replacing $\delta(r, a)$ by $E(\delta(r, a))$ achieves this effect. Thus

$$\delta'(R, a) = \{q \in Q |\ q \in E(\delta(r, a)) \text{ for some } r \in R\}.$$

Additionally, we need to modify the start state of M to move the fingers initially to all possible states that can be reached from the start state of N along the ε arrows. Changing q_0' to be $E(\{q_0\})$ achieves this effect. We have now completed the construction of the DFA M that simulates the NFA N.

The construction of M obviously works correctly. At every step in the computation of M on an input, it clearly enters a state that corresponds to the subset of states that N could be in at that point. Thus our proof is complete.

ANSWER TO THE QUESTION NO. 02

(c)

Q: Prove that the class of regular languages is closed under the concatenation operation.

Proof: Solved in Spring'18

or

Q: Prove that the class of regular languages is closed under the star operation.

Proof

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize A_1^*

1. $Q = \{q_0\} \cup Q_1$.
   The states of N are the states of N_1 plus a new state.
2. The state q_0 is the new start state.
3. $F = \{q_0\} \cup F_1$
   The accept states are the old accept states plus the new state.
4. Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$.

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon. \\ \{q_1\} & q = q_0 \text{ and } a = \varepsilon. \\ \emptyset & q = q_0 \text{ and } a \neq \varepsilon. \end{cases}$$

ANSWER TO THE QUESTION NO. 03

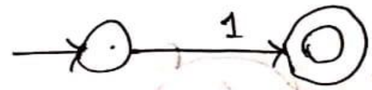
(a)

Only 2 of them are abled to solve

3(a) Convert the following regular expressions to NFA.

i. $(1^* \cup 0(00)^*)^* 111$

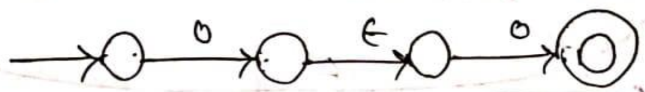
1



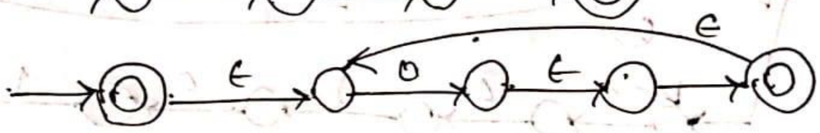
1^*

0

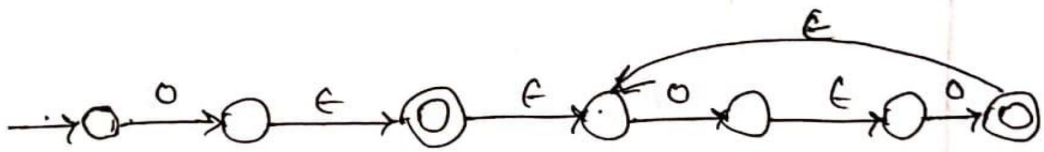
00



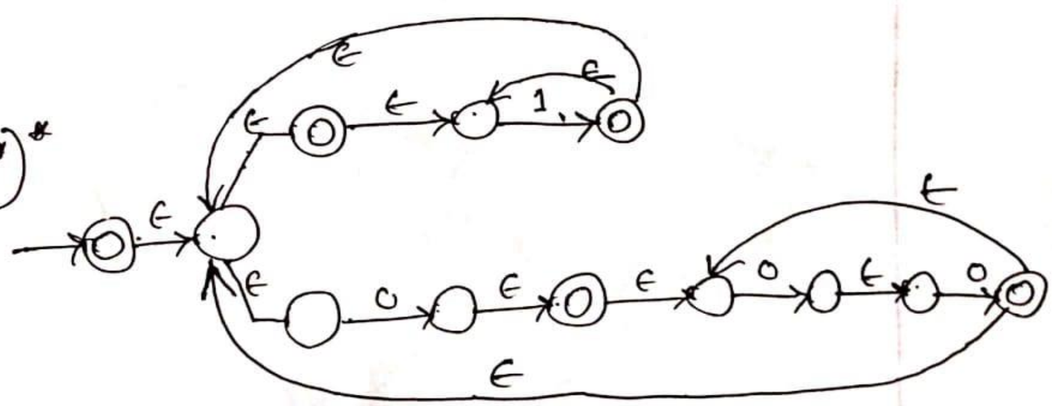
$(00)^*$



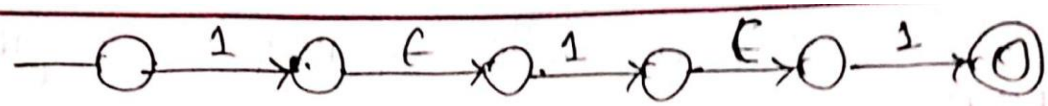
$0(00)^*$



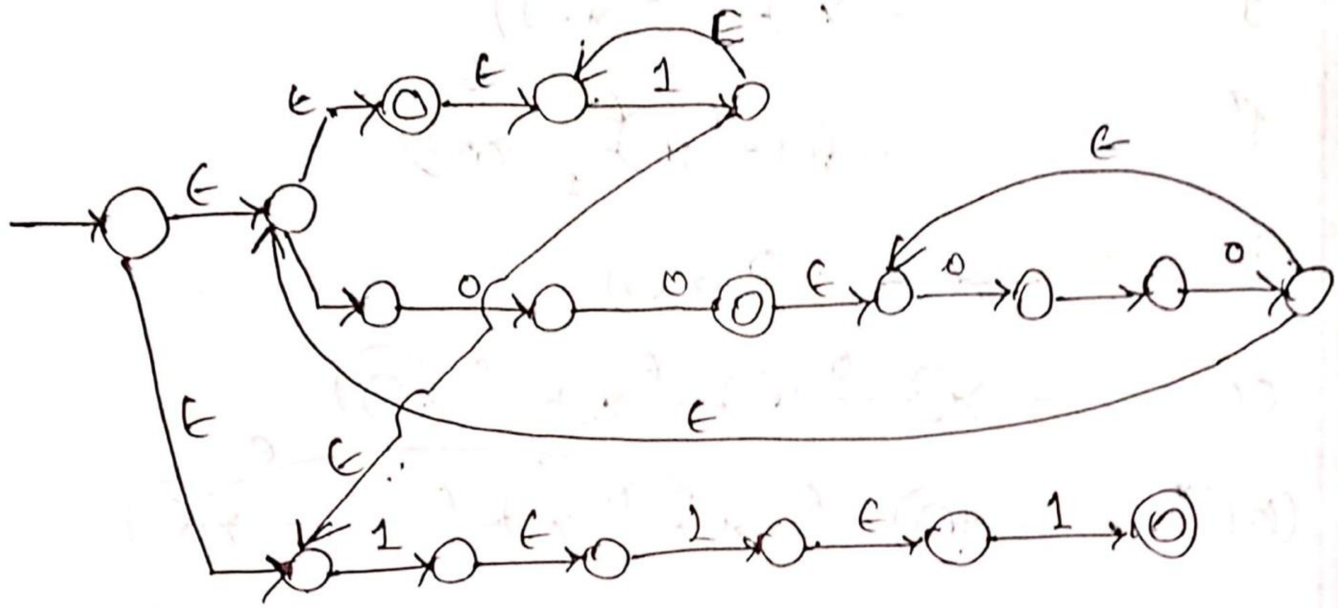
$(1^* \cup 0(00)^*)^*$



111.

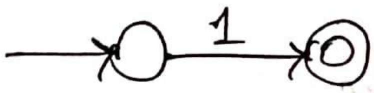


$(1^* \cup 0(00)^*)^* 111$



(iv) $(00 \cup 1)^* 11^*$

1



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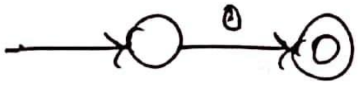
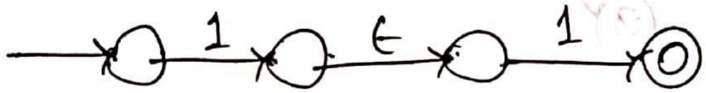
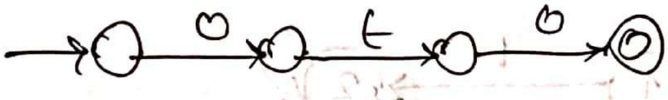


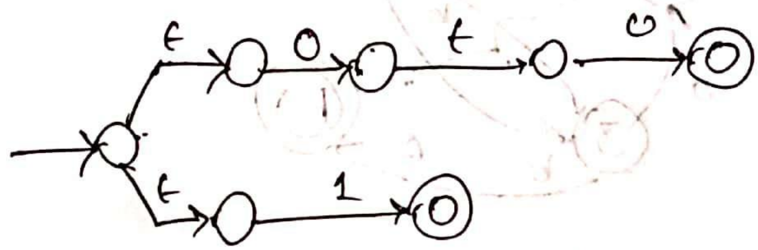
11



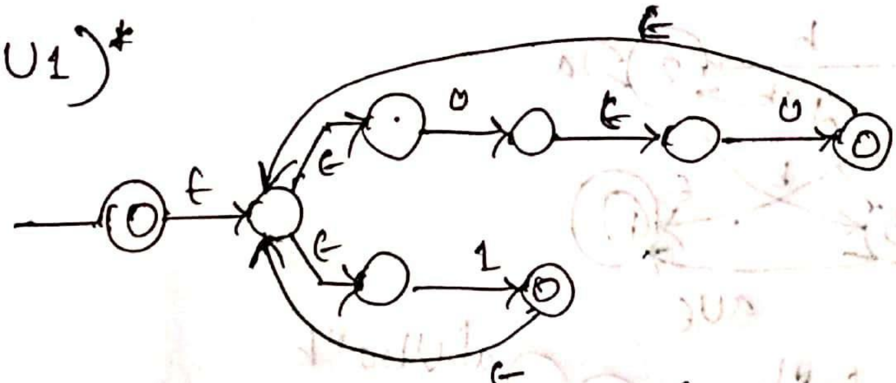
00



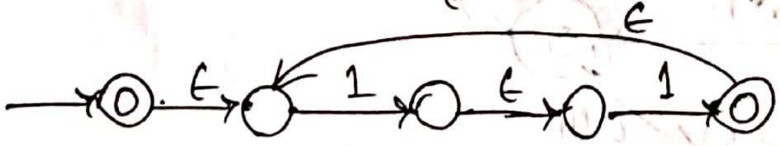
$00 \cup 1$



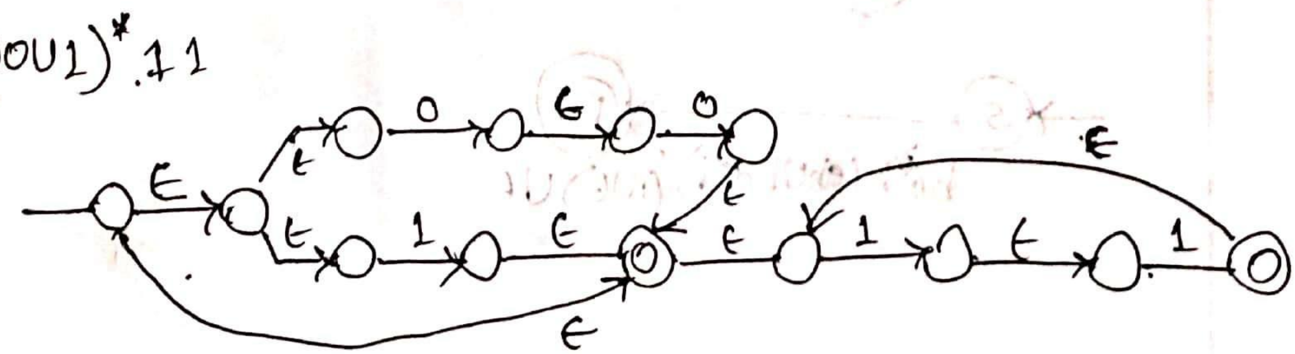
$(00 \cup 1)^*$



$(11)^*$



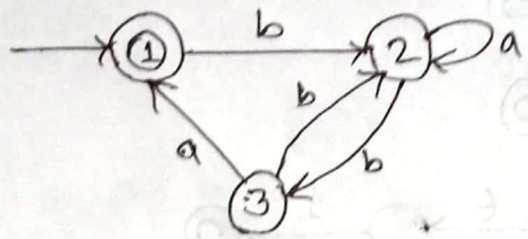
$(00 \cup 1)^* . 11$



ANSWER TO THE QUESTION NO. 03

(c)

3(c) Convert the following DFA to regular expression.



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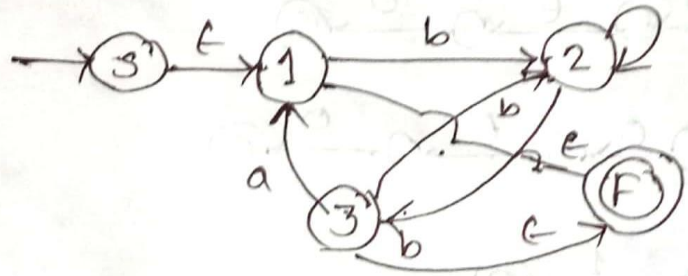

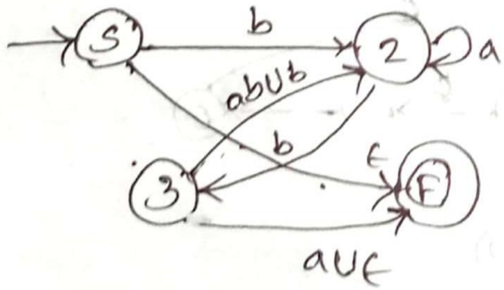

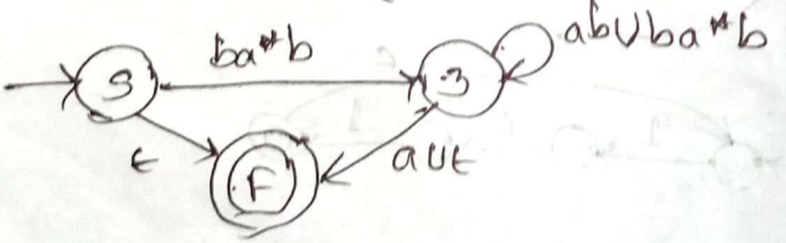

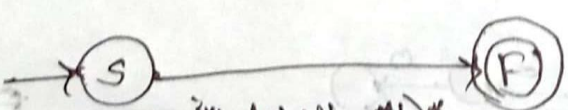

$ba^*b(ab \cup ba^*b)^*(a \cup \epsilon) \cup \epsilon$

PS: Some questions are not solved here. If you have any confusion regarding some solutions please have it under consideration and search about its accuracy.