

## discrete previous solve

Autumn-22:-

Q1) a) A set is an unordered collection of objects.

cardinality of a set:-

The cardinality of a set is the total number of unique elements in a set. Example:-

$A = \{1, 2, 3, 4, 5, 6\} \rightarrow$  The cardinality of the set A is equal to 6 because set A has six elements.

complement of a set:-

The complement of a set A is a set of all those elements of the universal set which do not belong to A and is denoted by  $A^c$  or  $A'$  or  $\bar{A}$ .

Example:- Let,  $A = \{1, 2\}$

$$U = \{1, 2, 3, 4, 5\}$$

$$A^c / A' / \bar{A} = U - A$$

$$= \{1, 2, 3, 4, 5\} - \{1, 2\}$$

$$= \{3, 4, 5\}$$

or



(a) Cartesian product:-

Let A and B be sets. Cartesian product of A and B denoted by  $A \times B$  is defined as-

$$A \times B = \{ (a, b) : a \in A \text{ and } b \in B \}$$

$(A \times B) \times (C \times D)$  and  $A \times (B \times C) \times D$  are not same

(b) prove that,  $(A \cup B)' = A' \cap B'$

$$\Rightarrow A \cup A' \cap B'$$

$$\Rightarrow \{ x | x \notin (A \cap B) \}$$

$$\Rightarrow \{ x | \neg (x \in (A \cap B)) \}$$

$$\Rightarrow \{ x | \neg (x \in A \wedge x \in B) \}$$

$$\Rightarrow \{ x | \neg (x \in A) \vee \neg (x \in B) \}$$

$$\Rightarrow \{ x | x \notin A \vee x \notin B \}$$

$$\Rightarrow \{ x | x \in A' \vee x \in B' \}$$

$$\Rightarrow \{ x | x \in A' \cup B' \}$$

$$\Rightarrow \{ x | x \in (A \cup B)' \} \text{ [proved]}$$



(b)

proper subsets-

Any subset A is said to be proper subset of B, if there is at least one element of B which does not belong to A, if  $A \subseteq B$  but  $A \neq B$ . It is written as  $A \subset B$ .

$$\text{if, } A - B = \{1, 5, 7, 8, 15\}$$

$$B - A = \{3, 6, 10\}$$

$$A \cap B = \{2, 9, 11, 15\}$$

$$\text{set } A = \{1, 2, 5, 7, 8, 9, 11, 15\}$$

$$\text{set } B = \{3, 6, 10, 2, 9, 11\} = \{2, 3, 6, 9, 10, 11\}$$

(c) Given,  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

a)  $\{3, 4, 5\}$

$$\text{bit string} = 0011100000$$

(b)  $\{1, 3, 6, 10\}$

$$\text{bit string} = 1010010001$$

(c)  $\{2, 3, 4, 7, 8, 9\}$

$$\text{bit string} = 0111001110$$



④ given,  $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

a)  ~~$\{2, 4, 5\}$~~

⑤  $1111001111$  of bit  $2^i$  A  $1000000000$  B

set =  $\{1, 2, 3, 4, 7, 8, 9, 10\}$

⑥  $0101111000$

set =  $\{2, 4, 5, 6, 7\}$

⑦  $10000000001$

set =  $\{1, 10\}$



2(b) : Given:

$I(x)$  =  $x$  has an internet connection

$C(x, y)$  =  $x$  and  $y$  have chat over the internet.

$U$  = All student of the class.

i)  $\neg C(x, y)$

ii)  $\neg \forall x (I(x))$

iii)  $\forall x (I(x) \rightarrow \exists y C(x, y))$



Autumn-22-2(c2)

$$\textcircled{i} \quad \exists x \forall y (x - y = y)$$

= There is a real number  $x$  for every real number  $y$ , such that  $x - y = y$

$$\textcircled{ii} \quad \forall x \forall y (x > 0 \vee (y < 0 \wedge x - y > 0))$$

= For every real number  $x$  and for every real number  $y$  if  $x > 0$  and  $y < 0$ , then  $x - y > 0$ .

This statement says that, for real numbers  $x$  and  $y$ , if  $x$  is positive and  $y$  is negative then  $(x - y)$  is positive.

Then the difference between  $x$  and  $y$  is positive.



Autum-22 - 2(d)

①

| $P$ | $Q$ | $\neg P$ | $P \vee Q$ | $\neg P \wedge (P \vee Q)$ | $\neg P \wedge (P \vee Q) \rightarrow Q$ |
|-----|-----|----------|------------|----------------------------|--|
| T   | T   | F        | T          | F                          | T  |
| T   | F   | F        | T          | F                          | T  |
| F   | T   | T        | T          | T                          | T  |
| F   | F   | T        | F          | F                          | T  |

is a tautology.



11

| P | Q | R | $P \rightarrow Q$ | $P \rightarrow R$ |  | $(P \rightarrow Q) \wedge (P \rightarrow R)$ | $(P \rightarrow Q) \vee (P \rightarrow R)$ |
|---|---|---|-------------------|-------------------|--|--|--|
| T | T | T | T                 | T                 |  | T  | T  |
| T | T | F | T                 | F                 |  | F  | T  |
| T | F | T | F                 | T                 |  | F  | T  |
| T | F | F | F                 | F                 |  | F  | T  |
| F | T | T | T                 | T                 |  | T  | T  |
| F | T | F | T                 | T                 |  | T  | T  |
| F | F | T | T                 | T                 |  | T  | T  |
| F | F | F | T                 | T                 |  | T  | T  |

is a tautology -



4(d)

①  $p$ : I bought a lottery ticket this week

$q$ : I won the million dollar jackpot

②  $\neg p$

③  $p \vee q$

Ans: I bought a lottery ticket this week

or I won the million dollar jackpot

④  $p \rightarrow q$

Ans: If I bought a lottery ticket this week

then I won the million dollar jackpot

⑤  $p \wedge q$

Ans: I bought a lottery ticket this week

and I won the million dollar jackpot

⑥  $p \leftrightarrow q$

⑦  $\neg p \rightarrow \neg q$

Ans: If I did not buy a lottery ticket

this week then I will not win the

million dollar jackpot

⑧  $\neg p \vee (p \wedge q)$

I did not buy a lottery ticket this week

or I bought a ticket this week and



4(d)OT:

$$f(x) = 2x^2 + 3$$

$$g(x) = 3x + 5$$

$$\begin{aligned} f \circ g &= f(g(x)) = f(3x + 5) \\ &= 2 \cdot (3x + 5)^2 + 3 \\ &= 2 \cdot (9x^2 + 25) + 3 \\ &= 18x^2 + 50 + 3 \\ &= 18x^2 + 53 \end{aligned}$$

$$\begin{aligned} g \circ f &= g(f(x)) = g(2x^2 + 3) \\ &= 3 \cdot (2x^2 + 3) + 5 \\ &= 6x^2 + 9 + 5 \\ &= 6x^2 + 14 \end{aligned}$$