The Plane

A plane is a surface such that if any two points are taken on it, the straight line joining them lies wholly on the surface.

The general equation of first degree in $x, y, z \Rightarrow ax + by + cz + d = 0$ represents a plane.

General equation of a plane $ax + by + cz + d = 0$ through one given point (x_1, y_1, z_1) is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

Equation of a plane through three points $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$ is

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

☐ Equation of a plane through the line of intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is $a_1x + b_1y + c_1z + d_1 + \lambda(a_2x + b_2y + c_2z + d_2) = 0$

☐ If a plane $a_1x + b_1y + c_1z + d_1 = 0$ is perpendicular to another plane $a_2x + b_2y + c_2z + d_2 = 0$, then

$$a_1a_2 + b_1b_2 + c_1c_2 = 0.$$

☐ Distance of (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is $D = \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$

Find the equation of the plane passing through the intersection of the planes $x+2y+3z+4=0$ and $4x+3y+2z+1=0$ and the point $(1,2,3)$.

$\Rightarrow$ Any plane through the intersection of the two planes is $x+2y+3z+4+K(4x+3y+2z+1)=0$ ——— (1)

Since it passes through $(1,2,3)$ we can write,

$$1+2\times2+3\times3+4+K(4\times1+3\times2+2\times3+1)=0$$

$$\Rightarrow 1+4+9+4+K(4+6+6+1)=0$$

$$\Rightarrow 18+17K=0$$

$$\Rightarrow K=-\frac{18}{17}$$

Putting the values of K in equation (1) we get,

$$x+2y+3z+4+\left(-\frac{18}{17}\right)(4x+3y+2z+1)=0$$

$$\Rightarrow x+2y+3z+4-\frac{72x}{17}-\frac{54y}{17}-\frac{36z}{17}-\frac{18}{17}=0$$

$$\Rightarrow \frac{17x+34y+51z+68-72x-54y-36z-18}{17}=0$$

$\Rightarrow 55x + 20y - 15z - 50 = 0$

$\Rightarrow 11x + 4y - 3z - 10 = 0$

This is the required plane.

Ans: $11x + 4y - 3z - 10 = 0$

Q. Find the equation of the plane through the point $(4, 0, 1)$ and parallel to the plane $4x + 3y - 12z + 6 = 0$.

$\Rightarrow$ The equation of the plane parallel to $4x + 3y - 12z + 6 = 0$ is

$$4x + 3y - 12z + K = 0 \quad \text{——— (1)}$$

Equation (1) passes through $(4, 0, 1)$.

$\therefore (4 \times 4) + (3 \times 0) - (12 \times 1) + K = 0$

$\Rightarrow 16 - 12 + K = 0$

$\Rightarrow 4 + K = 0$

$\Rightarrow K = -4$

(1) becomes $4x + 3y - 12z - 4 = 0$

Ans:

☐ The equation of the plane passing through the lines of intersection of the planes $2x-y=0$ and $3z-y=0$ and perpendicular to the plane $4x+5y-3z+1=0$

$\Rightarrow$ The equation of the plane passes through the lines of intersection of the two given plannes is $2x-y+K(3z-y)=0$

$$\Rightarrow 2x-(1+K)y+3Kz=0 \quad \text{——— (1)}$$

Equation (1) will be perpendicular to $4x+5y-3z+1=0$

So, we can write, $(2\times4)-(1+K)\times5+3K\times(-3)=0$

$$\Rightarrow 8-5-5K-9K=0$$

$$\Rightarrow K=3/14$$

$\therefore$ The required plane is $2x-y+\frac{3(3z-y)}{14}=0$

$$\Rightarrow 28x-17y+9z=0$$

Ans:

$a_1a_2+b_1b_2+c_1c_2=0$

Find the equation of the plane which is perpendicular to the plane $2x+6y+6z-9=0$ and passing through the points $(2,2,1)$ and $(9,3,6)$

$\Rightarrow$ Given,

$$2x+6y+6z-9=0 \quad \text{——— (1)}$$

We know, Equation of the plane which passes through the point $(2,2,1)$ is

$$a(x-2)+b(y-2)+c(z-1)=0 \quad \text{——— (2)}$$

Equation (2) also passes through $(9,3,6)$.

$\therefore$ $a(9-2)+b(3-2)+c(6-1)=0$

$$\Rightarrow 7a+b+5c=0 \quad \text{——— (3)}$$

Since, equation (2) is perpendicular to equation (1) we can write $2a+6b+6c=0$ ——— (4)

From equation (3) and (4) we get.

$$\frac{a}{6-30} = \frac{b}{10-42} = \frac{c}{42-2} = K$$

$$\Rightarrow \frac{a}{-24} = \frac{b}{-32} = \frac{c}{40} = K$$

$\therefore \frac{a}{-24} = K \quad \Big| \quad \frac{b}{-32} = K \quad \Big| \quad \frac{c}{40} = K$

$\Rightarrow a = -24K \quad \Big| \quad \Rightarrow b = -32K \quad \Big| \quad \Rightarrow c = 40K$

Putting the values of a, b, c in equation

(2) we get, $-24K(x-2) - 32K(y-2) + 40K(z-1) = 0$

$$\Rightarrow K(-24x + 48 - 32y + 64 + 40z - 40) = 0$$

$$\Rightarrow -24x - 32y + 40z + 8 = 0$$

$$\Rightarrow 3x + 4y - 5z + 1 = 0$$

This is the required plane. Ans:

Find the equation of the plane through the line of intersection of the planes $x+2y+3z-4=0$, $2x+y-z+5=0$ and perpendicular to the plane $5x+3y+6z+8=0$

⇒ The equation of the plane which passes through the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$

is $x+2y+3z-4+K(2x+y-z+5)=0$

$$\Rightarrow (1+2K)x+(2+K)y+(3-K)z+(5K-4)=0 \quad \text{——— (1)}$$

Again,

$$5x+3y+6z+8=0 \quad \text{——— (2)}$$

Equation (1) and (2) are perpendicular.

$$\therefore 5(1+2K)+3(2+K)+6(3-K)=0$$

$$\Rightarrow 5+10K+6+3K+18-6K=0$$

$\Rightarrow 7K + 29 = 0$

$\Rightarrow K = -\frac{29}{7}$

Putting the value of K in equation (1)

we get, $7x + 14y + 21z - 28 - 58x - 29y + 29z - 145 = 0$

$\Rightarrow 51x + 15y - 50z + 173 = 0$

which is the required plane.

Find the equation of the plane through the points (2,3,1), (1,1,3) and (2,2,3). Find also the perpendicular distance from the point (5,6,7) to this plane.

$\Rightarrow$ The equation of any plane through (2,3,1) is $a(x-2) + b(y-3) + c(z-1) = 0$ — (*)

As it passes through (1,1,3) and (2,2,3) we can write, $a(1-2) + b(1-3) + c(3-1) = 0$

$$\Rightarrow a+2b-2c=0 \quad \text{——} \quad (1)$$

and $a(2-2)+b(2-3)+c(3-1)=0$

$$\Rightarrow b-2c=0 \quad \text{——} \quad (2)$$

From (2) and (1) we get,

$$\frac{a}{2(-2)-(-2)} = \frac{b}{0-(-2)} = \frac{c}{1-0} = K$$

$$\Rightarrow \frac{a}{-2} = \frac{b}{2} = c = K$$

$\therefore \frac{a}{-2} = K$ | $\frac{b}{2} = K.$ | $\frac{c}{1} = K$

$\Rightarrow a = -2K$ | $\Rightarrow b = 2K$ | $\Rightarrow c = K$

Putting these values in (1) we get,

$$-2(x-2)+2(y-3)+1(z-1)=0$$

$$\Rightarrow 2x-2y-z+3=0 \quad \text{——} \quad (4)$$

The perpendicular distance from $(5,6,7)$ to the equation (4) is

$$D = \frac{(2\times5)-(2\times6)-(7\times1)+3}{\sqrt{2^2+(-2)^2+(-1)^2}}$$

$$= \frac{10-12-7+3}{\sqrt{4+4+1}}$$

$$= \frac{-6}{\sqrt{9}}$$

$$= \frac{-6}{3}$$

$$= -2$$

$= 2$ [As distance can never be negative]

Ans: