Rolle's Theorem

Q. State Rolle's theorem. Verify the truth of Rolle's theorem for the function $f(x) = x^2 - 3x + 2$, in the interval $(1, 2)$.

Solution:—

Statement: If $f(x)$ be a any function where

(i) $f(x)$ is continuous in the close interval $[a, b]$ i.e, $a \leq x \leq b$

(ii) $f(x)$ is differentiable i.e; $f'(x)$ exists in the open interval (a, b) i.e $a < x < b$.

and (iii) $f(a) = f(b)$.

Then there exists at least one value of x (say c) between a and b i.e, $a < c < b$ such that $f'(c) = 0$.

2nd Part:— Given the function,

$$f(x) = x^2 - 3x + 2 \longrightarrow (1)$$

As the given function (1) is a polynomial function, So the function is continuous in the close interval ~~(0)~~ $[1,2]$ also the function is differentiable in the open interval $(1,2)$.

Also according to the question $a=1$ and $b=2$

Now, $f(a) = a^2-3a+2$ [by ①]

$= 1^2-3.1+2$

$= 0$

and $f(b) = b^2-3b+2$ [by ①]

$= 2^2-3.2+2$

$= 0$

$\therefore f(a) = f(b)$.

Again, diff. ① w. r. to x we get,

$f'(x) = 2x-3$

$\therefore f'(c) = 2c-3$

Now by Rolle's theorem we have,

$f'(c) = 0$

or, $2c-3=0$.

or, $2c = 3$

or, $c = 3/2$

$= 1.5$

Which is within the interval $(1, 2)$.

i.e; $a < c < b$ or, $1 < 1.5 < 2$

Hence, the Rolle's theorem is verified

H.W

Same type of all questions. Changing the function and interval.

Mean-Value Theorem

Q. State Mean-Value theorem. Verify the Mean-Value theorem for the function $f(x) = 3+2x-x^2$ in the interval $(0, 1)$.

Solution:—

Statement:— If any function be $f(x)$, where

(i) $f(x)$ is continuous in the close interval $[a, b]$ i.e, $a \le x \le b$.

(ii) $f(x)$ is differentiable i.e; $f'(x)$ exist in the open interval (a, b) i.e, $a < x < b$

then there exists at least one value of x (say c) between a and b i.e, $a < c < b$, such that,

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

2nd Part:— Given the function,

$$f(x) = 3+2x-x^2 \longrightarrow (1)$$

As the given function (1) is polynomial function, So the ~~g~~ function is continuous in the close interval $[0, 1]$ also the function is differentiable in the open interval $(0, 1)$.

Also according to the question $a=0$ and $b=1$

Now, $f(a) = 3+2a-a^2$ [by ①]

$= 3+2.0-0^2$

$= 3$

and $f(b) = 3+2b-b^2$

$= 3+2.1-1^2$

$= 4$

Again, diff. ① w. r. to x we get,

$f'(x) = 2-2x$

$\therefore f'(c) = 2-2c$

Now by Mean Value theorem, We have

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

or, $2-2c = \dfrac{4-3}{1-0}$

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or, $2-2c = 1$

or, $2c = 2-1$

or, $2c = 1$

or, $c = \frac{1}{2}$

$= 0.5$

Which is ~~the~~ within the interval $(0, 1)$.

i.e, $a < c < b$ or, $0 < 0.5 < 1$.

Hence, the Mean-Value theorem verified

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H.W

Same type of all question, changing the function and interval.

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8. ## Maclaurin's Theorem

State Maclaurin's theorem. Expand $\ln(1+x)$ in ascending powers of x using Maclaurin's theorem.

Statement:— If $f(x)$ be any function which can be expand an infinite series of ascending power of x, ~~each~~ and each term of that expansion are differentiable then,

$$f(x) = f(0) + xf'(0) + \frac{x^2}{\lfloor 2} f''(0) + \frac{x^3}{\lfloor 3} f'''(0) + \cdots + \frac{x^n}{\lfloor n} f^n(0) + \cdots$$

Which is known as Maclaurin's theorem.

2nd Part:— let the given function,

$$f(x) = \ln(1+x)$$

Now diff. successively w. r. to x we get,

$$f'(x) = \frac{1}{1+x}$$

$$= (1+x)^{-1}$$

$$= 1 - x + x^2 - x^3 + \cdots$$

$$\therefore f''(x) = -1 + 2x - 3x^2 + \cdots$$

$$\therefore f'''(x) = 2 - 6x + \cdots$$

$$\cdots$$

$$\cdots$$

$$\cdots$$

$$\cdots$$

Now putting $x=0$ in the aboves equations.

We have,

$f(0)=0$

$f'(0)=1$

$f''(0)=-1$

$f'''(0)=2$

Now by Maclaurin's theorem We have,

$$f(x)=f(0)+xf'(0)+\frac{x^2}{\lfloor 2}f''(0)+\frac{x^3}{\lfloor 3}f'''(0)+\cdots$$

$$\text{or, } \ln(1+x)=0+x.1+\frac{x^2}{\lfloor 2}(-1)+\frac{x^3}{\lfloor 3}.2+\cdots$$

$$\therefore \ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots$$

Answer

HoW :

$\sin x$, $\cos x$, e^{x}, e^{mx}, e^{-x}, a^{x}, $\tan^{-1}x$, $\ln(1-x)$,

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Taylor's Theorem

Q. State Taylor's theorem. Expand $\frac{1}{x}$ in ascending power of $x-2$ by Taylor's theorem.

Statement:— If $f(x)$ be any function which has derivatives of all order in an open interval I, containing a, then for each positive integer n and for each x in I,

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{\lfloor 2}(x-a)^2 + \frac{f'''(a)}{\lfloor 3}(x-a)^3 + \cdots\cdots$$

which is known as Taylor's theorem.

2nd Part:— Let the given function,

$$f(x) = \frac{1}{x}$$

Now diff. successively w.r. to x we get

$$f'(x) = -\frac{1}{x^2} = -x^{-2}$$

$$\therefore f''(x) = -(-2)x^{-3} = 2x^{-3} = \frac{2}{x^3}$$

$$\therefore f'''(x) = 2(-3)x^{-4} = -6x^{-4} = \frac{-6}{x^4}$$

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Now putting $x=2$ in the above equations

We have, $f(2) = \frac{1}{2}$

$f'(2) = -\frac{1}{4}$

$f''(2) = \frac{2}{2^3} = \frac{1}{4}$

$f'''(2) = -\frac{6}{2^4} = -\frac{3}{8}$.

Now by Taylor's theorem we have,

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{\lfloor 2} f''(a) + \frac{(x-a)^3}{\lfloor 3} f'''(a) + \cdots$$

$\therefore$ According to question here, $a = 2$

$$\therefore f(x) = f(2) + (x-2) f'(2) + \frac{(x-2)^2}{\lfloor 2} f''(2) + \frac{(x-2)^3}{\lfloor 3} f'''(2) + \cdots$$

$$\text{or, } \frac{1}{x} = \frac{1}{2} + (x-2)\left(-\frac{1}{4}\right) + \frac{(x-2)^2}{\lfloor 2}\left(\frac{1}{4}\right) + \frac{(x-2)^3}{\lfloor 3}\left(-\frac{3}{8}\right) + \cdots$$

$$\therefore \frac{1}{x} = \frac{1}{2} - \frac{(x-2)}{\lfloor 1}\left(\frac{1}{4}\right) + \frac{(x-2)^2}{\lfloor 2}\left(\frac{1}{4}\right) - \frac{(x-2)^3}{\lfloor 3}\left(\frac{3}{8}\right) + \cdots$$

(Answer).

* "যদি জীবনকে ভালোবাসো তাহলে সময়ের অপচয় করোনা, কেননা জীবনতো সময়ের সমষ্টি ছবি"

H.W. Expand by Taylor's theorem,

* $\ln x$ in ascending power of x
* $\ln(1+x)$ in ascending power of x.
* $\ln x$ in ascending power of $x-3$.
* $\sin x$ in ascending power of $x-\pi/2$.
* $\cos x$ in ascending power of $x-\pi/2$.
* e^x in ascending power of $x-1$.

"End of Mid Term Exam Syllabus.
Wish you all the best."

M. Islam