EXERCISE IV

TRANSPORMATION OF CO-ORDINATIES

1. (a) Put $x = x_1 + 1$, $y = y_1 - 2$ in the eq. Then $2(x_1 + 1)^2 - (y_2 - 2)^2 - 4(x_1 + 1) + (y_2 - 2) = 0$

or, $2x_1^2 + y_1^2 = 6$. On revoming dashes, the result will follow.

(b) As in (a)

2. (i) By (3) Art 33, $x = (x_1 - y_1)/\sqrt{2}$; $y = (x_1 + y_1)/\sqrt{2}$ Put then in $x^2 - y^2 = a^2$ and similarly $2x_1y_1 = a^2$. Remove subscripts : the result follows.

(ii) As in (i)

3. (a) Proceed as in worked our Ex. 3. $\therefore 3(x-1)^2 - 4(y+1) = 9$

Put $x - 1 = x_1$, $y + 1 = y_1$ Then $3x_1^2 - 4y_1^2 = 9$

(b) and (c) As in 3(a)

4. Put $x = x_1\cos\theta - y_1\sin\theta$. $y_1 = x_1\sin\theta + y_1\cos\theta$, in the given eq. This becomes $a(x_1\cos\theta - y_1\sin\theta)^2 + 2h(x_1\cos\theta - y_1\sin\theta)(x_1\sin\theta + y_1\cos\theta) + b(x_1\sin\theta + y_1\cos\theta)^2 + 2x_1(g\cos\theta + f\sin\theta) + 2y_1(f\cos\theta - g\sin\theta) + c = 0$. Now $(g\cos\theta + f\sin\theta)^2 (f\cos\theta - g\sin\theta)^2 = g^2 + f^2$ which remains the same as in given eq.

5. For the 1st transformation. We have $11(x_1 + 2)^2 + 24(x_1 + 2)(y_1 - 1) + 4(y_1 - 1)^2 - 20(x_1 + 2) - 40(y_1 - 1) - 5 = 0$

or, $11x_1^2 + 24x_1y_1 + 4y_1^2 - 5 = 0$. Remove subscripts. $11x^2 + 24xy + 4y^2 - 5 = 0$ (1) For the 2nd transformation, $\tan\theta = -4/3$

$\therefore \sin\theta = -4/5$, $\cos\theta = 8/5$. (2) Now put $x = x_1\cos\theta - y_1\sin\theta$, and$y_1 = x_1\sin\theta + y_1\cos\theta$ in (i). Then by (2); (1) becomes $11(3/5x_1 + 4/y_1)^2 + 2\angle(3/5x_1 + 4/5y_1)(-4/5x_1 + {}^1/5y_1) + 4(-4/5x_1 + 3/5y_1)^2 - 5 = 0$

or, $x^2 - 4y^2 + 1 = 0$

6. $7(x_1\cos\theta - y_1\sin\theta)^2 - 6\sqrt{3}(x_1\cos\theta - y_1\sin\theta)(x_1\sin\theta + y_1\cos\theta) + 13(x\sin\theta + y_1\cos\theta)^2 = 16$.... (1) Make the co-efficient of x_1y_1 term zero.

6. $\sin\theta\cos\theta - 6\sqrt{3}(\cos^2\theta - \sin^2\theta) = 0$. or, $\tan 2\theta = \sqrt{3}$

or, $2\theta = 60°$ or, $\theta = 30°$. Now put the value of $\theta = 30°$ in (1) and simplify. The result will follow. $21x_1^2 + 7y_1^2 + 13x_1^2 + 18y_1^2 + 39y_1^2 - 18x_1^2 = 64$. or, $x_1^2 + 4y_1^2 = 4$ Revove subscripts.

7. Put $x = x_1 + h$, $y = y_1 + k$ in the eq & simplify.

$2x_1^2 - x_1y_1 + y_1^2 + (4h-k-5)x_1 + (-h+2k-4)y_1 + (2h^2 - hk + k^2 - 5h - 4k + 11) = 0$ .... (i). Make the co-efficients of x and y zero separately.

$4h-k-5=0$, $-h+2k-4=0$, solve. $h=2$, $k=3$. These values make the last term of (1) vanish. Hence the point is (2, 3) & $2x^2-xy+y^2=0$

8. Let the new origin be (α, β). Then $5 = h+\alpha$ .... (1)

$-13 = k+\beta$......... (2), $-3 = -h+\alpha$........ (3) and $11 = -k+\beta$......... (4)

Adding (1) & (3) $\alpha = 1$: adding (2) (4), $\beta = -1$

Now origin is $(1, -1)$

9. 1st transformation $x = x_1 + h$, $y = y_1 + k$ in the eq. Make the co-efficients of x_1 & y_1 zero seperately.

$17h + 9k - 8 = 0$, $9h - 7k - 16 = 0$, Solve

$h = 1$, $k = -1$. Now the eq is $17x^2 - 7y^2 + 18xy = 10$...... (2)

Put $x = x_1\cos\theta - y_1\sin\theta$, $y = x_1\sin\theta + y_1\cos\theta$) in (2).

$\therefore 17(x_1\cos - y_1\sin\theta)^2 - 7(x_1\sin\theta + y_1\cos\theta)^2 + 18(x_1\cos\theta - y_1\sin\theta)(x_1\sin\theta + y_1\cos\theta) = 10$ .... (3) .Make the co-eff of xy zero. Then $\cos 2\theta = 4/5$, $\tan 2\theta = 3/4$. or, $\sin\theta = 1/\sqrt{(10)}$, $\cos\theta = 3/\sqrt{(10)}$. Put them in (3) and simplify. The result will follow.

10. Put $x = x_1\cos 30° - y_1\sin 30°$, $y = x_1 \sin\cos 30° + y_1$ $30°$, i. e., $x = \frac{1}{2}(\sqrt{3}x_1 - y_1)$, $y = \frac{1}{2}(x_1 + \sqrt{3}y_1)$.

$\therefore (x_1\sqrt{3} - y_1)^2 + 2\sqrt{3}(\sqrt{3}x_1 - y_1)(x_1 + 3y_1) - (x_1 + \sqrt{3}y_1)^2 = 8a^2$

or, $8x_1^2 - 8y_1^2 = 8a^2$ or, $x_1^2 - y_1^2 = a^2$.

11. Put $x = x' + 2$, $y' = y' + 3$ in the eq.

then simplify, $\therefore 3x'^2 + 2x'y' + 3y'^2 - 1 = 0$

Removing dashes, $3x^2 + 2xy + 3y^2 - 1 = 0$ .... (1)

Now let the axes be turned through an angle θ.

$\therefore x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

Put them in (i). The eq. is now $3(x'\cos\theta - y'\sin\theta)^2 + 2(x'\cos\theta - y'\sin\theta)(x'\sin\theta + y'\cos\theta) + 3(x'\sin\theta + y'\cos\theta)^2 - 1 = 0$ .... (ii)

Equate co-efficent of $x'y'$ to zero. Then $2(\cos^2\theta - \sin^2\theta) = 0$

$\therefore \cos^2\theta - \sin^2\theta = 0$ or, $\tan^2\theta = 1$ $\therefore \theta = \pi/4$

Put the value of θ in (ii)

Then $4x'^2 + 2y'^2 = 1$ Removing dashes.

$4x^2 + 2y^2 = 1$