International Islamic University Chittagong (IIUC)
Department of Computer Science and Engineering (CSE)
B. Sc. in CSE, Final Examination, Autumn-2018
Course Code: MATH-3501, Course Title: Mathematics-V
Marks: 50 Time: 2:30 hours

[Answer any **two** questions from **group A** and any **three** questions from **Group B.**
Separate Answer script must be used for Group A and Group B]

Group-A

1. a) Define Fourier series of $f(t)$ for the interval $(-L, L)$. Sketch the following function for three cycles and hence find its Fourier series: 7

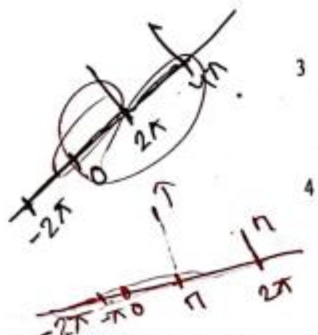
$$f(t) = \begin{cases} -t; & -\pi \le t < 0 \\ t; & 0 \le t < \pi \end{cases}$$

b) Plot the line spectrum for the following Fourier series: 3

$$f(t) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{1}{2n-1} \cos n\omega t + \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \sin n\omega t$$

2. a) Find Harmonic analysis of the given Fourier series 3

$$f(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n} \sin \frac{n\pi t}{6}$$



b) 4

$$y = f(t) = t\ ;\ 0 \le t \le \pi$$
$$= \pi\ ;\ \pi \le t \le 2\pi$$

$f(t) = f(t + 2\pi)$ Here, $T = 2L = 2\pi$ $\therefore L = \pi$

Sketch a graph of $f(t)$ in the interval $-2\pi < t < 2\pi$

c) Plot the line (at least 6) spectrum (discrete frequency spectra) for the Fourier series 3

$$\underbrace{f(t)}_{\text{Complex wave}} = \underbrace{10}_{\text{DC value}} + \underbrace{2\sum_{n=1}^{\infty} \frac{3}{n} \sin n\pi t}_{\text{AC value}}$$

3. a) State and prove the convolution theorem for Fourier transform. 5

b) Define convolution sum and convolution integral. If $x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$ and $h[n] = u[n+2]$, find the convolution sum $x[n] * h[n]$. 5

Group-B

4. a) Define unit step function. Express the following function in terms of unit step functions and hence find its Laplace transform: 4

$$f(t) = \begin{cases} 8; & t < 2 \\ 6; & t > 2 \end{cases}$$

$\frac{1}{2}$ 3. Default Error 77

$t=90$

b) Define ramp function. Sketch the wave form of the following signal: 3
$$x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$
c) Define Laplace transform of a function $f(t)$. Evaluate $\mathcal{L}\{t^2 e^{-2t}\}$ 3

5. a) Find the inverse Laplace transform of $\frac{3s+7}{s^2-2s-3}$ 3

b) Solve the following IVP by Laplace transform: 7
$$Y'' + Y = t;\ Y(0) = 1, Y'(0) = -2$$

6. a) Sketch the waveforms of the following signals 5
$$x(t) = 4u(t-1) - 8u(t-4) + 4u(t-6)$$
b) Find Fourier Transform of 5
$$f(t) = \begin{cases} 2 & ;0 \le t < 1 \\ -2 & ;-1 \le t < 0 \\ 0 & ;|t| > 1 \end{cases}$$

7. a) Write a MATLAB function to construct a complex wave $f(t)$ in the time interval of[-4, 20] for the following Fourier series: 5
$$\underbrace{f(t)}_{\text{Complex wave}} = \underbrace{4}_{\text{DC value}} + \underbrace{\sum_{n=1}^{\infty} \frac{2}{n\pi}(1-\cos n\pi)\sin nt}_{\text{AC value}}$$

b) Write MATLAB code to sketch line spectrum (at least 6)for the following complex wave 3
$$\underbrace{f(t)}_{\text{Complex wave}} = \underbrace{2.5}_{\text{DC value}} + \underbrace{[-\frac{5}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}(\cos n\pi - 1)\sin \frac{n\pi t}{4}]}_{\text{AC value}}$$

c) If 2

$x[n] = 3\ ;\ n = 0$ and $h[n] = 2\ ;\ n = 0$

$= 2\ ;\ n = 1$ $\quad = -2\ ;\ n = 1$

Write MATLAB code to find the convolution sumfor the above signals.

MATHMATICS SULN AU18

@ALFAZ_EMON

@TANVIR_MAHTAB

1A.

$$y = f(t) = -t\ ; \quad -\pi \le t \le 0$$

$$y = t\ ; \quad 0 \le t \le \pi \text{-----------------------(i)}$$

$\mathbf{f(t) = f(t+2\pi)}$ Here, $\mathbf{T = 2L = 2\pi}$ $\quad \mathbf{\therefore L = \pi}$

a) Sketch the function for 3 cycles:

b) Find the Fourier series for the function

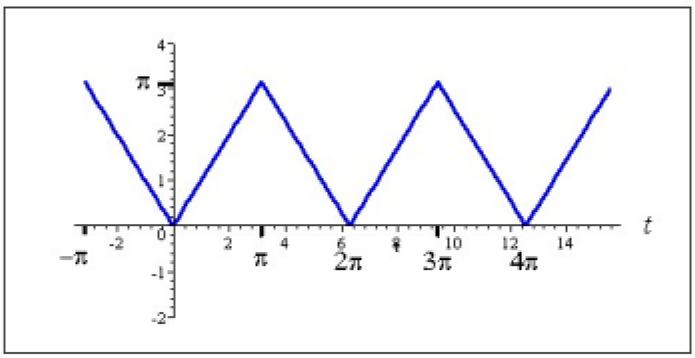


Figure 57: A periodic signal with period $\mathbf{T = 2L = 2\pi}$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)dt$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(t)dt + \frac{1}{\pi}\int_{0}^{\pi} f(t)dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (-t)dt + \frac{1}{\pi}\int_{0}^{\pi} tdt \qquad \text{[From (i)]}$$

$$= -\frac{1}{\pi}\left[\frac{t^2}{2}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{t^2}{2}\right]_{0}^{\pi} \qquad [\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\ ; n \ne -1]$$

$$= -\frac{1}{\pi}\left[0 - \frac{(-\pi)^2}{2}\right] + \frac{1}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$= -\frac{1}{\pi}\left[-\frac{\pi^2}{2}\right] + \frac{1}{\pi}\left[\frac{\pi^2}{2}\right]$$

$$= \frac{1}{\pi}\times\frac{\pi^2}{2} + \frac{1}{\pi}\times\frac{\pi^2}{2}$$

$$= \frac{\pi}{2} + \frac{\pi}{2}$$

$$=\frac{2\pi}{2}$$
$$=\pi$$
$$a_n=\frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}dt$$
$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos\frac{n\pi t}{\pi}dt$$
$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)dt$$
$$=\frac{1}{\pi}\int_{-\pi}^{0} f(t)\cos(nt)dt+\frac{1}{\pi}\int_{0}^{\pi} f(t)\cos(nt)dt$$
$$=\frac{1}{\pi}\int_{-\pi}^{0}(-t)\cos(nt)dt+\frac{1}{\pi}\int_{0}^{\pi} t\cos(nt)dt \qquad \text{[From (i)]}$$
$$=-\frac{1}{\pi}\int_{-\pi}^{0} t\cos(nt)dt+\frac{1}{\pi}\int_{0}^{\pi} t\cos(nt)dt \text{ ------------------(ii)}$$

Now, Let, $I=\int t\cos(nt)dt$

$$=t\int\cos(nt)dt-\int[\frac{d}{dt}(t)\int\cos(nt)dt]dt \quad [\because \int uvdx=u\int vdx-\int\{\frac{d}{dx}(u)\int vdx\}dx]$$
$$=t\frac{\sin(nt)}{n}-\int 1.\frac{\sin(nt)}{n}dt \qquad [\int\cos mx\,dx=\frac{1}{m}\sin mx]$$
$$=\frac{t}{n}\sin(nt)-\frac{1}{n}\int\sin(nt)dt$$
$$=\frac{t}{n}\sin(nt)-\frac{1}{n}\frac{(-\cos(nt))}{n} \qquad [\because \int\sin mx\,dx=\frac{-1}{m}\cos mx]$$
$$=\frac{t}{n}\sin(nt)+\frac{1}{n^2}\cos(nt) \text{ --------------------------(iii)}$$

Putting the value of (iii) in (ii)

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)dt=-\frac{1}{\pi}\int_{-\pi}^{0} t\cos(nt)dt+\frac{1}{\pi}\int_{0}^{\pi} t\cos(nt)dt$$
$$=-\frac{1}{\pi}\left[\frac{t}{n}\sin(nt)+\frac{1}{n^2}\cos(nt)\right]_{-\pi}^{0}+\frac{1}{\pi}\left[\frac{t}{n}\sin(nt)+\frac{1}{n^2}\cos(nt)\right]_{0}^{\pi}$$
$$=-\frac{1}{\pi}[\frac{0}{n}\sin(0)+\frac{1}{n^2}\cos(0)-\frac{(-\pi)}{n}\sin(-n\pi)-\frac{1}{n^2}\cos(-n\pi)]+\frac{1}{\pi}[\frac{\pi}{n}\sin(n\pi)+\frac{1}{n^2}\cos(n\pi)$$
$$-\frac{0}{n}\sin(0)-\frac{1}{n^2}\cos(0)]$$
$$=\frac{-1}{\pi}[0+\frac{1}{n^2}\times 1+\frac{\pi}{n}\sin(-n\pi)-\frac{1}{n^2}\cos(-n\pi)]+\frac{1}{\pi}[\frac{\pi}{n}\times 0+\frac{1}{n^2}\cos(n\pi)-0-\frac{1}{n^2}\times 1]$$

$$=\frac{-1}{\pi}\left[\frac{1}{n^2}-\frac{\pi}{n}\sin(n\pi)-\frac{1}{n^2}\cos(n\pi)\right]+\frac{1}{\pi}\left[\frac{1}{n^2}\cos(n\pi)-\frac{1}{n^2}\right]$$

$[\sin(-\theta)=-\sin\theta\,;\ \cos(-\theta)=\cos\theta;\ \sin n\pi=0 \text{ for } n=1,2,3\ldots\ldots\ldots]$

$$=\frac{-1}{\pi}\left[\frac{1}{n^2}-\frac{\pi}{n}\times 0-\frac{1}{n^2}\cos(n\pi)\right]+\frac{1}{\pi}\left[\frac{1}{n^2}\cos(n\pi)-\frac{1}{n^2}\right]$$

$$=\frac{-1}{\pi n^2}+\frac{1}{\pi n^2}\cos(n\pi)+\frac{1}{\pi n^2}\cos(n\pi)-\frac{1}{\pi n^2}$$

$$=\frac{2}{\pi n^2}\cos n\pi-\frac{2}{\pi n^2}$$

$$=\frac{2}{\pi n^2}(\cos n\pi-1) \qquad \left[\text{Taking Common } \frac{2}{\pi n^2}\right]$$

$$b_n=\frac{1}{L}\int_{-\pi}^{\pi} f(t)\sin\frac{n\pi t}{L}dt$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin\frac{n\pi t}{\pi}dt$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)dt$$

$$=\frac{1}{\pi}\int_{-\pi}^{0} f(t)\sin(nt)dt+\frac{1}{\pi}\int_{0}^{\pi} f(t)\sin(nt)dt$$

$$=\frac{1}{\pi}\int_{-\pi}^{0} (-t)\sin(nt)dt+\frac{1}{\pi}\int_{0}^{\pi} t\sin(nt)dt$$

$$=-\frac{1}{\pi}\int_{-\pi}^{0} t\sin(nt)dt+\frac{1}{\pi}\int_{0}^{\pi} t\sin(nt)dt \text{ ---------------------(iv)}$$

Now $\int t\sin(nt)dt$

$$=t\int\sin(nt)dt-\int\left\{\frac{d}{dt}(t)\int\sin(nt)dt\right\}dt \quad \left[\because \int uv\,dx=u\int v\,dx-\int\left\{\frac{d}{dx}(u)\int v\,dx\right\}dx\right]$$

$$=t\left(\frac{-\cos(nt)}{n}\right)-\int 1.\frac{-\cos(nt)}{n}dt$$

$$=-\frac{t}{n}\cos(nt)+\frac{1}{n}\int\cos(nt)dt$$

$$=\frac{-t}{n}\cos(nt)+\frac{1}{n}\frac{\sin(nt)}{n} \qquad \left[\int\cos mx\,dx=\frac{1}{m}\sin mx\right]$$

$$=-\frac{t}{n}\cos(nt)+\frac{1}{n^2}\sin(nt) \text{ ------------------------------(v)}$$

Putting the value of (v) in (iv)

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)dt=-\frac{1}{\pi}\int_{-\pi}^{0} t\sin(nt)dt+\frac{1}{\pi}\int_{0}^{\pi} t\sin(nt)dt$$

$$=-\frac{1}{\pi}\left[\frac{-t}{n}\cos(nt)+\frac{1}{n^2}\sin(nt)\right]_{-\pi}^{0}+\frac{1}{\pi}\left[\frac{-t}{n}\cos(nt)+\frac{1}{n^2}\sin(nt)\right]_{0}^{\pi}$$

$$=-\frac{1}{\pi}[\frac{-0}{n}\cos(0)+\frac{1}{n^2}\sin(0)-\frac{-(-\pi)}{n}\cos(-n\pi)-\frac{1}{n^2}\sin(-n\pi)]+\frac{1}{\pi}[-\frac{\pi}{n}\cos(n\pi)$$

$$+\frac{1}{n^2}\sin(n\pi)-\frac{-(0)}{n}\cos(0)-\frac{1}{n^2}\sin(0)]$$

$$=-\frac{1}{\pi}[0+0-\frac{\pi}{n}\cos(n\pi)+\frac{1}{n^2}\sin(n\pi)]+\frac{1}{\pi}[-\frac{\pi}{n}\cos(n\pi)+\frac{1}{n^2}\sin(n\pi)+0-0]$$

$$=-\frac{1}{\pi}[-\frac{\pi}{n}\cos(n\pi)+\frac{1}{n^2}\times 0]+\frac{1}{\pi}[-\frac{\pi}{n}\cos(n\pi)+\frac{1}{n^2}\times 0]$$

$$[\sin(-\theta)=-\sin\theta\ ;\ \cos(-\theta)=\cos\theta;\ \sin n\pi=0 \text{ for } n=1,2,3..........]$$

$$=\frac{\pi}{\pi n}\cancel{\cos(n\pi)}-\frac{\pi}{\pi n}\cancel{\cos(n\pi)}$$

$$=0$$

The Fourier series for the above function is:

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(\frac{n\pi}{L})t+\sum_{n=1}^{\infty}b_n\sin(\frac{n\pi}{L})t$$

$$=\frac{\pi}{2}+\sum_{n=1}^{\infty}\frac{2}{n^2\pi}(\cos n\pi-1)\cos(\frac{n\pi}{L})t+0$$

$$=\frac{\pi}{2}+\sum_{n=1}^{\infty}\frac{2}{n^2\pi}(\cos n\pi-1)\cos(\frac{n\pi}{\pi})t$$

$$\underbrace{f(t)}_{\text{Complex wave}}=\underbrace{\frac{\pi}{2}}_{\text{DC value}}+\underbrace{\sum_{n=1}^{\infty}\frac{2}{n^2\pi}(\cos n\pi-1)\cos nt}_{\text{AC value}}$$

1.B

$$f(t)=\frac{\pi}{2}+\sum_{n=1}^{\infty}\frac{1}{2n-1}\cos nt+\sum_{n=1}^{\infty}\frac{(-1)^n}{2n}\sin nt \text{ ------------------------------------(i)}$$

This series has an interesting graph for the above function $f(t)$

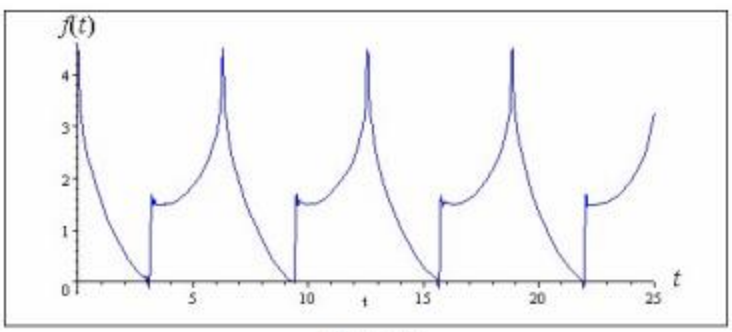


Figure 77

We have the Fourier series is $f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos(n\omega t)+b_n\sin(n\omega t))$

We can see from the series (i) that

$a_n = \frac{1}{2n-1} \qquad b_n = \frac{(-1)^n}{2n}$

Now, using $\Rightarrow R=\sqrt{a^2+b^2}$ [From (viii), page no 66]

Let $R_n = C_n = \sqrt{a_n^{\ 2}+b_n^{\ 2}}$

$a_n = \frac{1}{2n-1}$	$b_n = \frac{(-1)^n}{2n}$	$C_n = \sqrt{a_n^2+b_n^2}$
$a_1 = 1$	$b_1 = -\frac{1}{2}$	$C_1 = \sqrt{1^2+\left(-\frac{1}{2}\right)^2} = 1.118$
$a_2 = \frac{1}{3}$	$b_2 = \frac{1}{4}$	$C_2 = \sqrt{\left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2} = 0.4167$
$a_3 = \frac{1}{5}$	$b_3 = -\frac{1}{6}$	$C_3 = \sqrt{\left(\frac{1}{5}\right)^2+\left(-\frac{1}{6}\right)^2} = 0.260$
$a_4 = \frac{1}{7}$	$b_4 = \frac{1}{8}$	$C_4 = \sqrt{\left(\frac{1}{7}\right)^2+\left(\frac{1}{8}\right)^2} = 0.190$

Here, from (i),

Here, $n\omega = n$

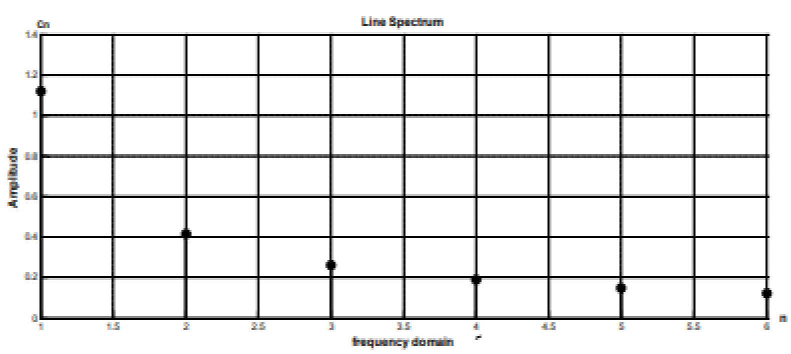
For $n=1$; **Fundamental Frequency = 1st Harmonic = $\omega=1$**

For $n=2$; **2nd Harmonic** $=2\omega=2$

For $n=3$; **3rd Harmonic $=3\omega=3$**

For $n=4$; **4th Harmonic $=4\omega=4$**

The resulting line spectrum is:



2A

2.a. here given fourier series are

$$f(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n} \sin \frac{n\pi t}{6} \quad \text{--- (2)}$$

we know fourier series is

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right) \quad \text{--- (1)}$$

here $n\omega = \frac{n\pi}{6}$ [find by compare 1 and 2]

for $n=1$, 1st hermonic is $1\omega = \frac{1\pi}{6} = \frac{\pi}{6}$

for $n=2$, 2nd hermonic is $2\omega = \frac{2\pi}{6} = \frac{\pi}{3}$

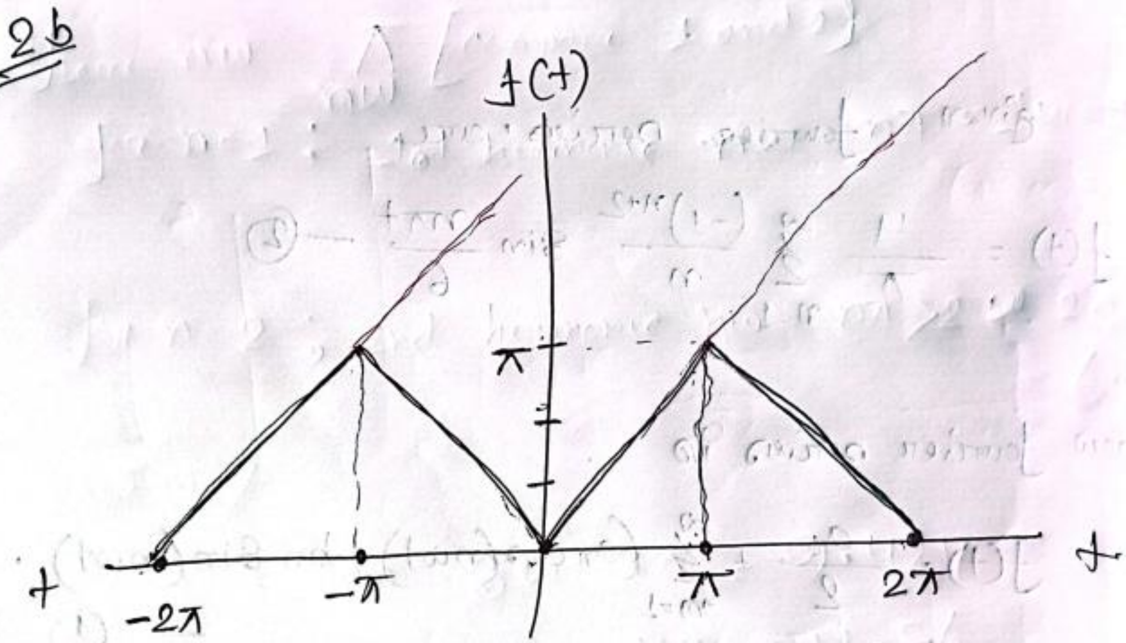
for $n=3$, 3rd hermonic is $3\omega = \frac{3\pi}{6} = \frac{\pi}{2}$

for $n=4$, 4th hermonic is $4\omega = \frac{4\pi}{6} = \frac{2\pi}{3}$

Ans

2B

2b



$$y = f(t) = \begin{cases} t \;; & 0 \leq t \leq \pi \\ \pi \;; & \pi \leq t \leq 2\pi \end{cases}$$

interval $-2\pi < t < 2\pi$

2C

Au-18

2a $f(t) = 10 + 2 \sum_{n=1} \frac{3}{n} \sin n\pi t$ — (1)

we kno the fourier series are.

$$f(t) = \frac{a_n}{2} + \sum_{n=1} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$ — (ii)

then here,

$$a_n = 0 \quad b_n = 2 \cdot \frac{3}{n} = \frac{6}{n}$$

we know that $R_n = C_n = \sqrt{a_n^2 + b_n^2}$

So.

$a_n = 0$, find $b_n = \frac{6}{n}$

$b_n = b_1 = \frac{6}{1} = 6$ then $C_1 = \sqrt{a_1^2 + b_1^2}$
$= \sqrt{0^2 + 6^2} = \sqrt{36} = 6$

$b_n = b_2 = \frac{6}{2} = 3$ then $C_2 = \sqrt{0_2^2 + 3_2^2} = \sqrt{9} = 3$

$b_n = b_3 = \frac{6}{3} = 2$ then $C_3 = \sqrt{0_3^2 + 2_3^2} = \sqrt{4} = 2$

$b_n = b_4 = \frac{6}{4} = \frac{3}{2}$ then $C_4 = \sqrt{0_4^2 + \left(\frac{3}{2}\right)^2} = \sqrt{2.25}$
$= 1.5$

here $n\omega = n\pi$ [compare 1 and 2]

for $n=1$; 1st harmonic $n\omega = n\pi = 1\omega = 1\pi$
$= 3.141$

for $n=2$; 2nd harmonic $n\omega = n\pi \Rightarrow 2\pi = 2\pi$
$= 6.2832$

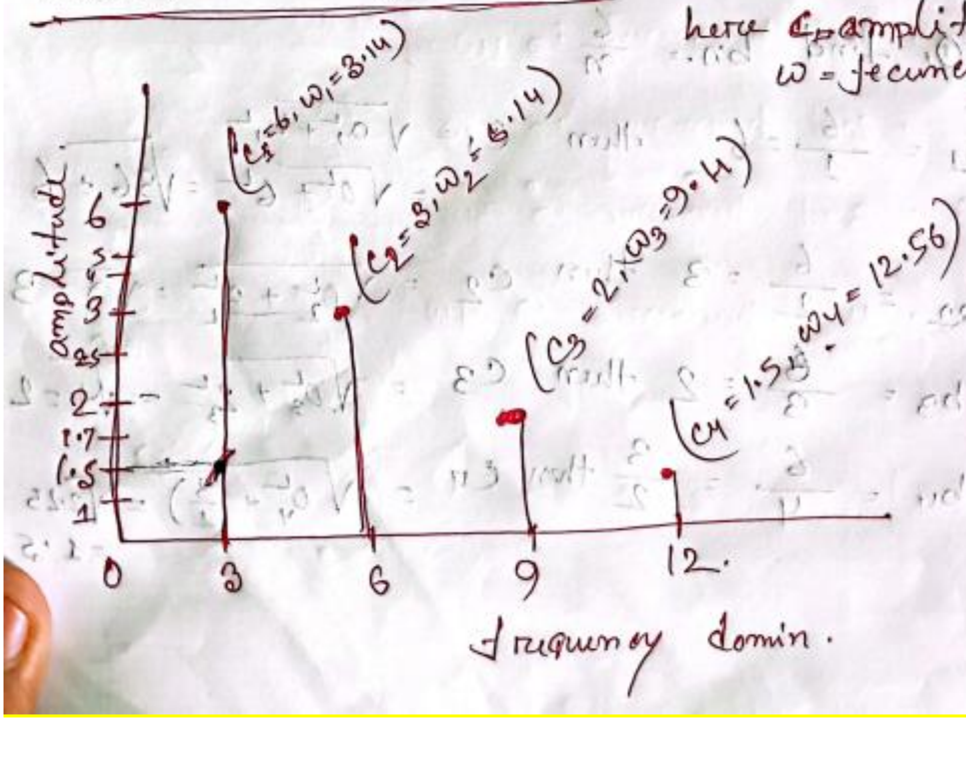
Same way

$n=3$ then $3\omega = 3\pi = 9.424$

$n=4$ then $4\omega = 4\pi = 12.56$

result Line spectrum is.

here C = amplitude
ω = frequency



3B

Convolution Sum

The following steps are to be taken

i. Folding
ii. Shifting
iii. Multiplication
iv. Summation

1st times:

i. Folding
ii. Multiplication
iii. Summation

2nd times and more

i. Shifting
ii. Multiplication
iii. Summation

Example 102:

Evaluate the convolution sums of y [n] = x [n]*h [n]

Where,

$$x[n] = \begin{cases} 1, & n = 0 \\ 3, & n = 1 \end{cases} \quad \text{and} \quad h[n] = \begin{cases} 2, & n = 0 \\ 1, & n = 1 \end{cases}$$; ***n represents the time index***

MATLAB

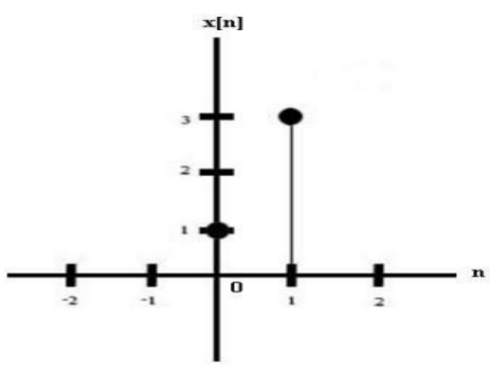
x=[1 3]

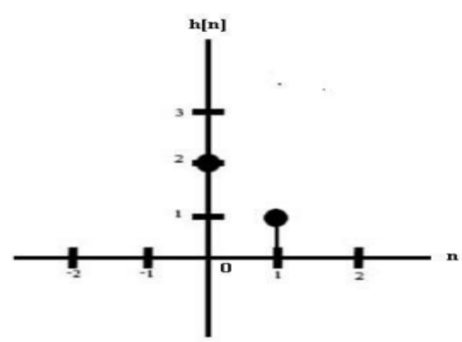
h=[2 1]

y =conv(x,h)

y= [2 7 3]

Solution:





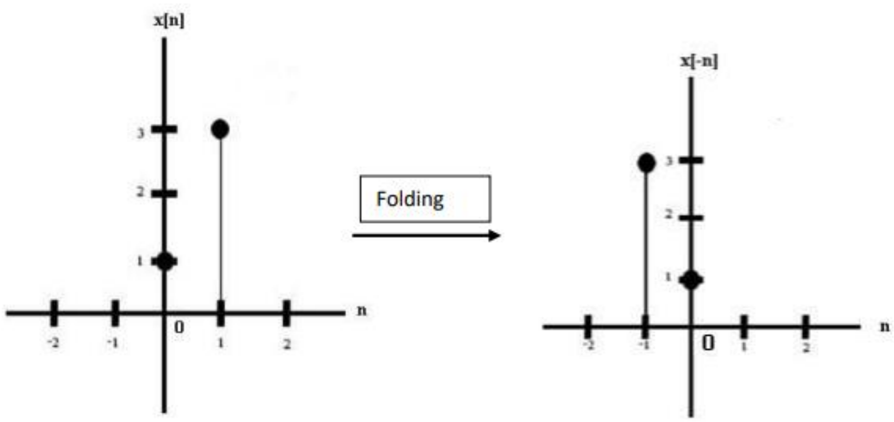
1st time:

(i). Folding:

$$x[n] = x[-n]$$

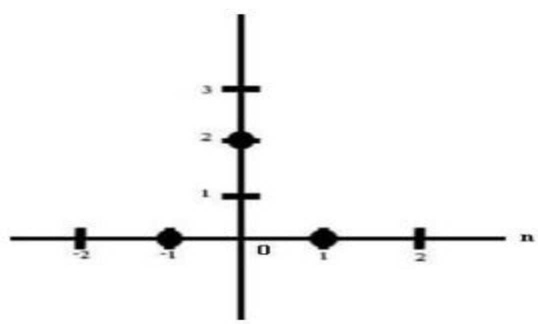
i.e. $x[0] = x[0]$

$x[1] = x[-1]$



(ii). Multiplication:

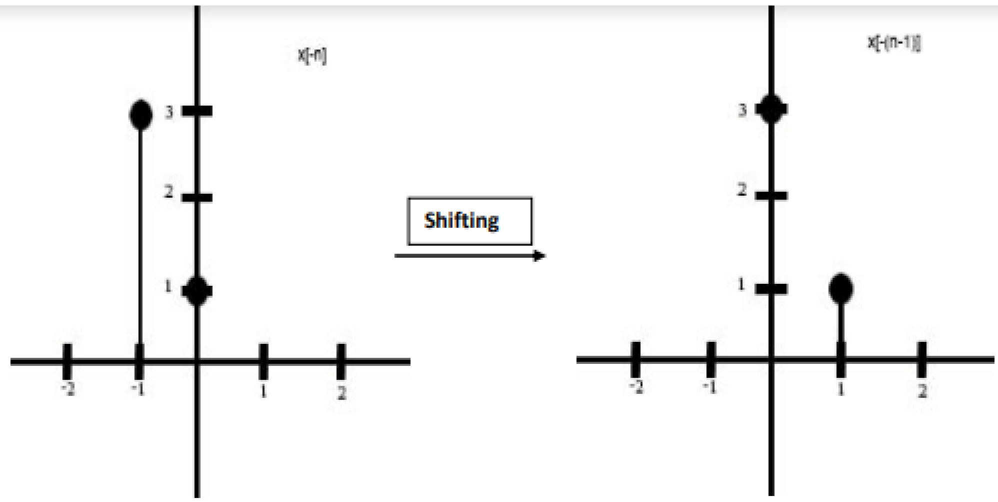
$x[-n] * h[n]$



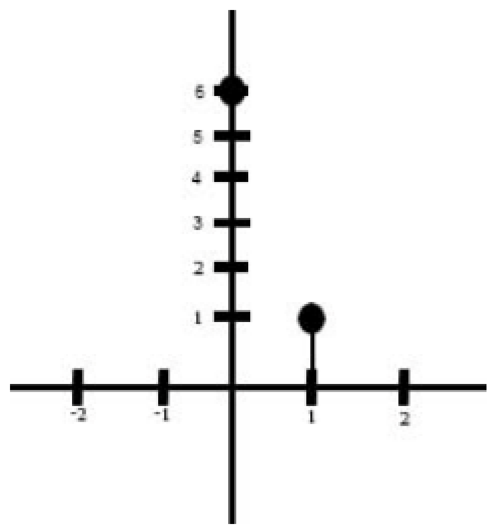
(iii). Summation: $y[0] = 0 + 2 + 0 = 2$

2nd time:

(i). Shifting:



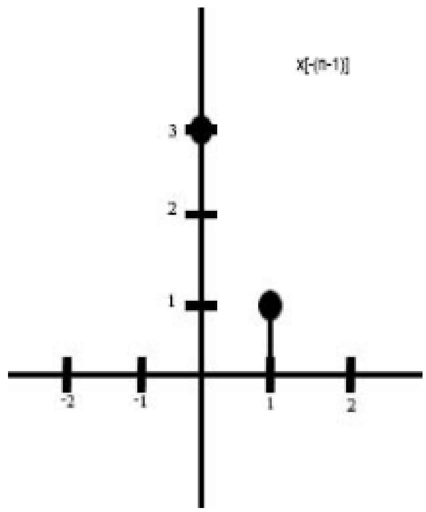
(ii) Multiplication: x[-(n-1)] * h[n]

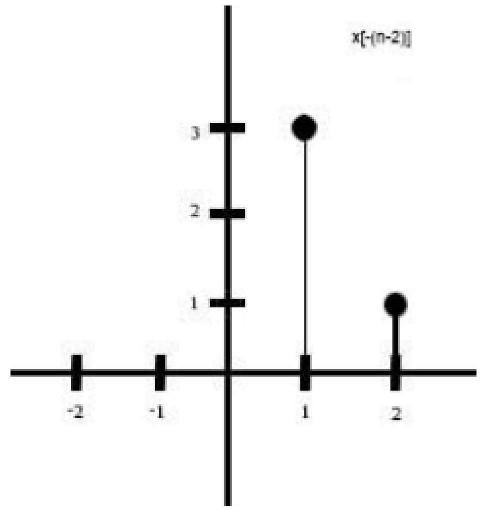


(iii). Summation: y[1] = 0 + 6 + 1 + 0 = 7

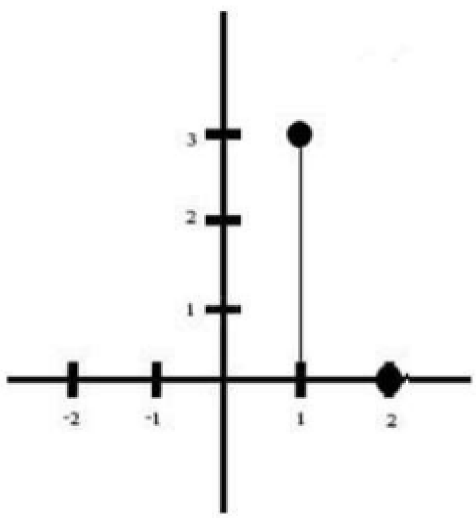
3rd time:

(i). Shifting:





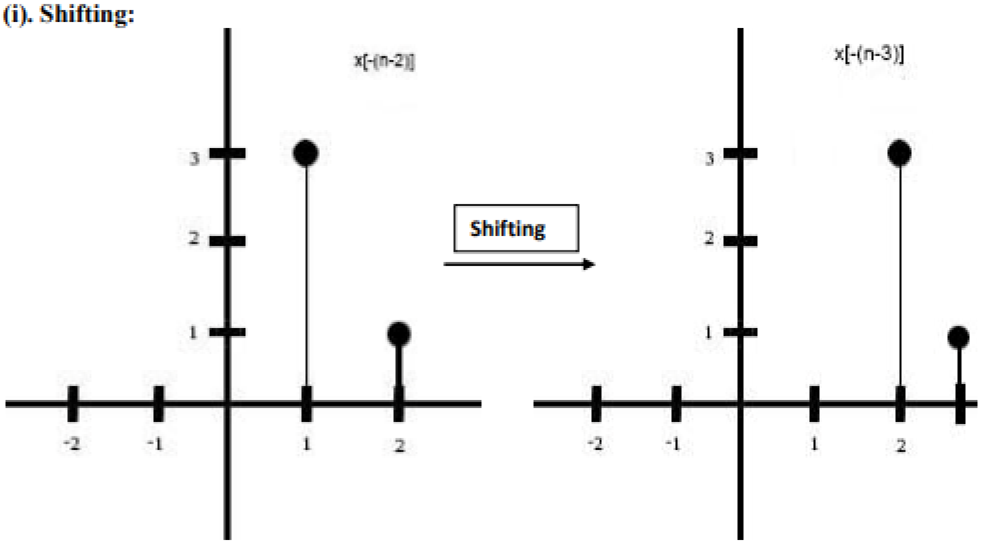
(ii). Multiplication: x[-(n-2)] * h[n]



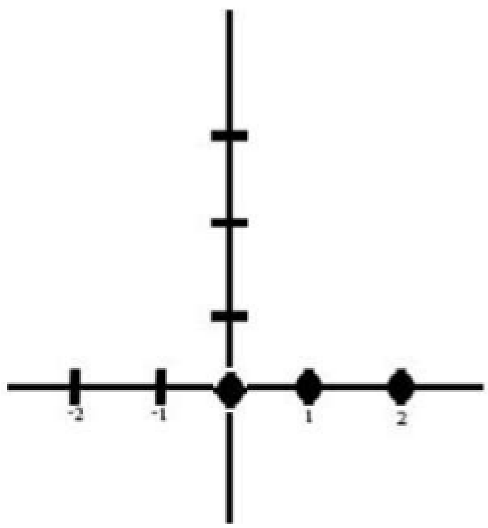
(iii). Summation: y [2] = 0 + 0 + 3 + 0 = 3

4^{th} time:

(i). Shifting:



(ii). Multiplication: x[-(n-3)] * h[n]



iii) Summation: y[3] = 0+0+0+0=0

Finally we get,

∴ y[0] = 2

y[1] = 7

y[2] = 3

∴ **Convolution Sum: y[n] = [2 7 3]**

4A

Example 92: Express the following function in terms of unit step functions and find its Laplace transform:

$$f(t)=\begin{cases}8\ ; & t<2\\ 6\ ; & t>2\end{cases}$$

Solution:

We have

$$L(u(t-a))=\frac{e^{-as}}{s}$$

$$\therefore L(u(t-2))=\frac{e^{-2s}}{s}\text{-------------}(i)$$

Given,

$$f(t)=\begin{cases}8\ ; & t<2\\ 6\ ; & t>2\end{cases}$$

$$f(t)=\begin{cases}8+0; & t<2\\ 8-2\ ; & t>2\end{cases}$$

$$f(t)=8+\begin{cases}0\ ; & t<2\\ -2\ ; & t>2\end{cases}$$

$$f(t)=8+(-2)\begin{cases}0\ ; & t<2\\ 1\ ; & t>2\end{cases}$$

$$f(t)=8+(-2)\begin{cases}1\ ; & t>2\\ 0\ ; & t<2\end{cases}$$

$$f(t)=8+(-2)u(t-2)$$
$$f(t)=8-2u(t-2)$$
$$L\{f(t)\}=L\{8-2u(t-2)\}$$
$$L\{f(t)\}=L\{8\}-2Lu(t-2)\}$$
$$L\{f(t)\}=8L\{1\}-2Lu(t-2)\}$$
$$L\{f(t)\}=8\times\frac{1}{s}-2\times\frac{e^{-2s}}{s}$$

$[\because L(1)=\frac{1}{S}(\textit{from example } 55)]$ & $L[u(t-2)=\frac{e^{-2s}}{s}$(from example 91)]

4B

Example 90.

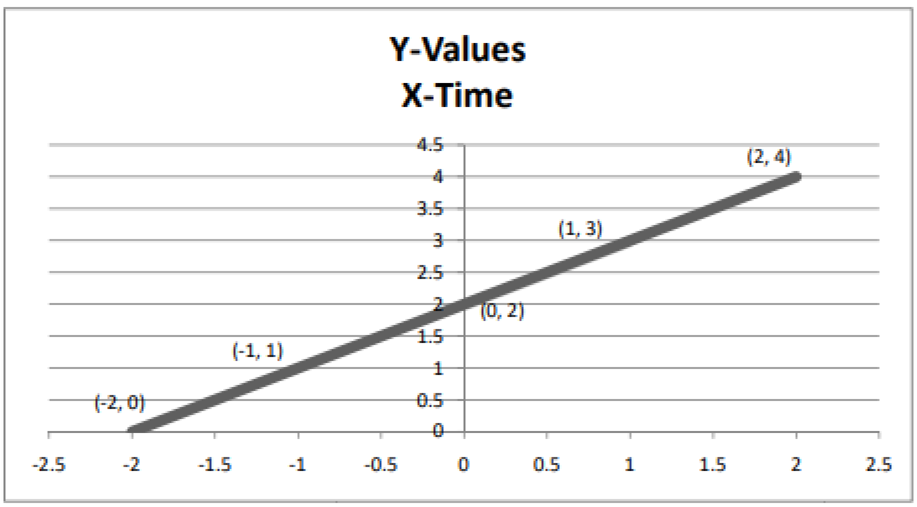
$$x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$

<u>*Solve:*</u>

$r(t+2) = t+2;\ t \geq -2$ $\qquad$ *Here,* $t+2=0$

$\qquad = 0;\ \ t < -2$ $\qquad$ $\therefore t = -2$

t	-2	-1	0	1	2	3
$r(t+2) = t+2$	0	1	2	3	4	5

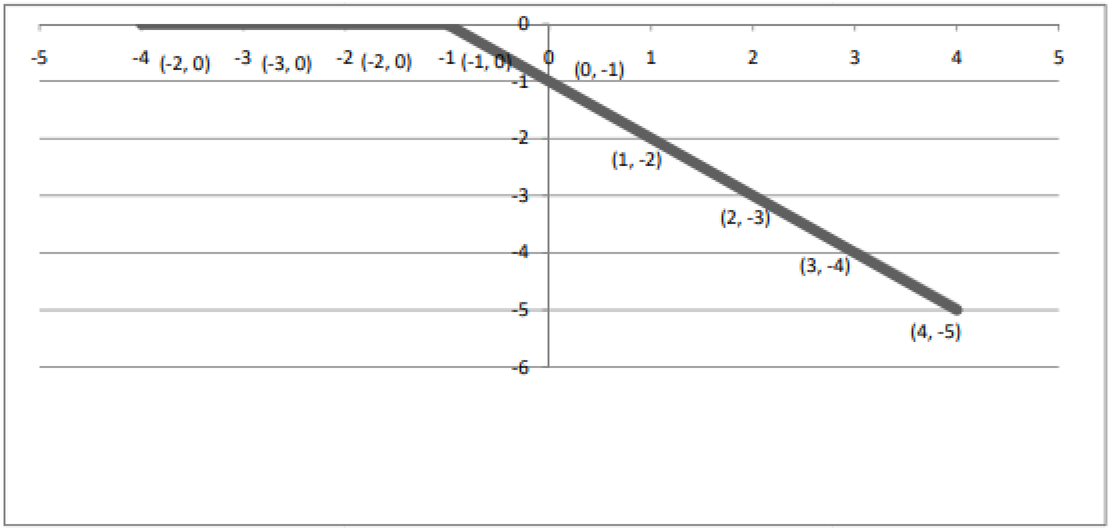


Again,

$-r(t+1) = -(t+1); t \geq -1$ $\qquad$ *Here,* $t+1=0$

$\qquad = 0;\ \ t < -1$ $\qquad$ $\therefore t = -1$

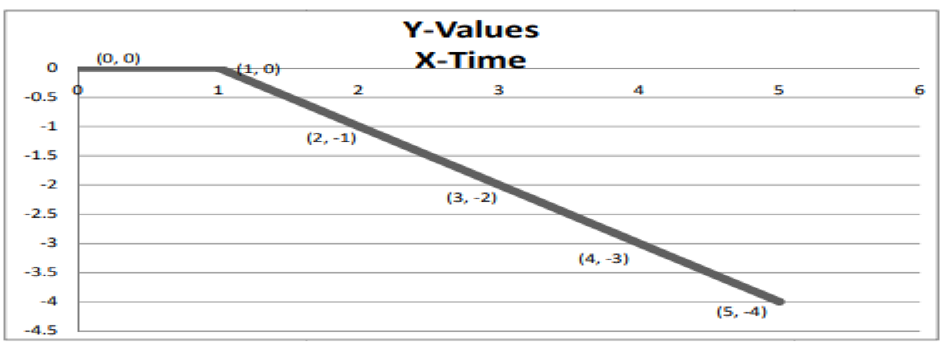
t	-1	0	1	2	3	4
$-r(t+1) = -(t+1)$	0	-1	-2	-3	-4	-5



$$\therefore -r(t-1) = -(t-1) \quad ; t \geq 1 \qquad\qquad Here,\ t-1=0$$

$$= 0 \quad ; t < 1 \qquad\qquad \therefore t = 1$$

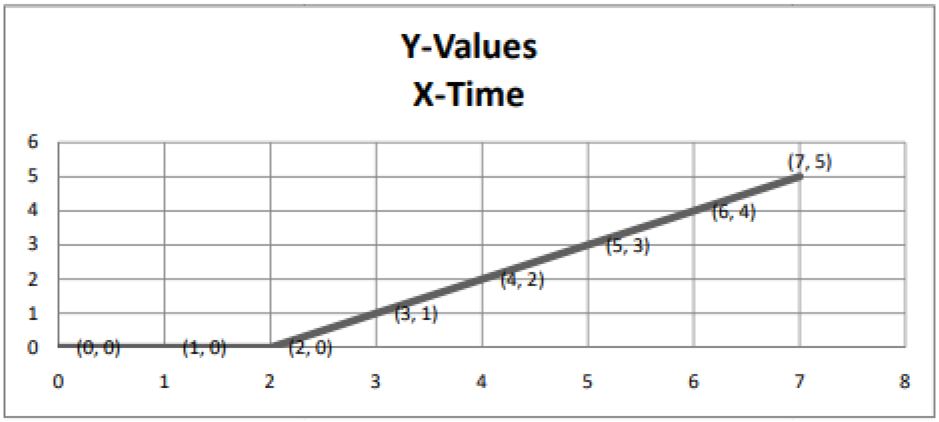
t	1	2	3	4	5
$-r(t-1) = -(t-1)$	0	-1	-2	-3	-4



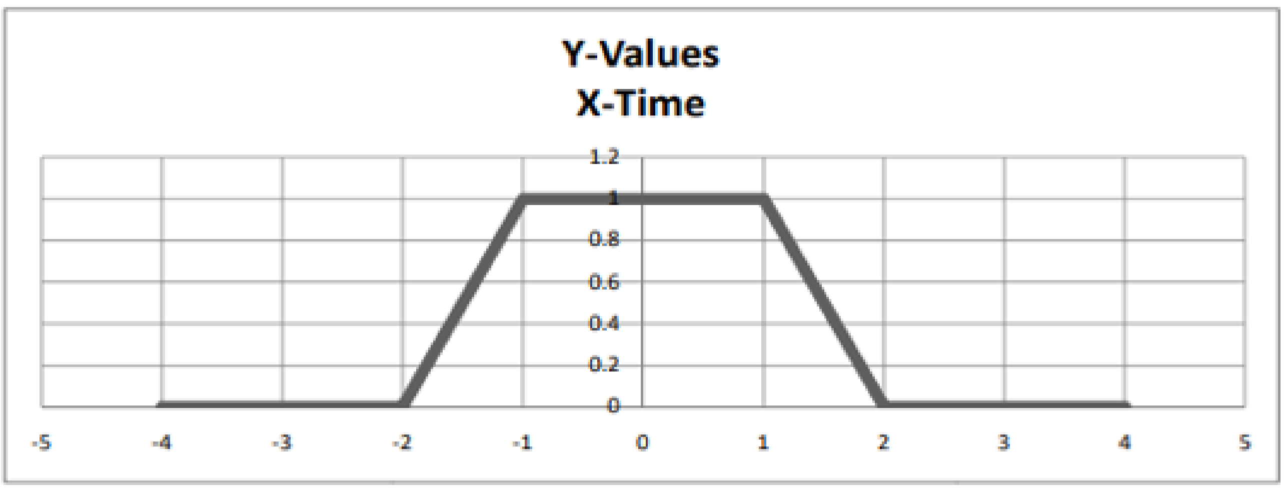
$$r(t-2) = (t-2) \quad ; t \geq 2 \qquad\qquad Here, t-2=0$$

$$= 0 \quad ; t < 2 \qquad\qquad \therefore t = 2$$

t	2	3	4	5	6	7
$r(t-2) = t-2$	0	1	2	3	4	5



$$\therefore x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$



4C

$L(f(t)) = L(e^{-at}) = \frac{1}{s+a}$

$e^{-t} = L^{-1}\frac{1}{s+1}$

$L^{-1}\frac{1}{s+1} = e^{-t}$

$L^{-1}\left(\frac{3s+7}{(s-3)(s+1)}\right) = 4e^{3t} - e^{-t}$ Ans

c) Evaluate $L\{t^2 e^{-2t}\}$

Here, $L\{e^{-2t} \times t^2\}$

$f(t) = t^2$

The 1st shift theorem state that

if $L\{f(t)\} = f(s)$ —— (i)

Then $L\{e^{-2t} \times t^2\} = f(s+2)$ —— (ii)

we have, according to eqn (i) $L\{f(t)\} = L(t^2) = \frac{2!}{s^{2+1}} = \frac{2!}{s^3}$

if $f(s) = \frac{2!}{s^3}$

$\therefore f(s+2) = \frac{2!}{(s+2)^3}$

Hence According to eqn (ii) we can write,

$L\{e^{-2t} \times t^2\} = f(s+2) = \frac{2!}{\quad}$ = Ans

5A

(5) a) $\frac{3s+7}{s^2-2s-3}$

$= \frac{3s+7}{s^2-3s+s-3}$

$= \frac{3s+7}{s(s-3)+1(s-3)}$

$= \frac{3s+7}{(s+1)(s-3)}$

Let $\frac{3s+7}{(s-3)(s+1)} \equiv \frac{A}{s-3} + \frac{B}{s+1}$ — (i)

Multiplying $(s-3)(s+1)$ both side

$3s+7 \equiv A(s+1) + B(s-3)$ — (ii)

$s = -1$ in (ii)

$3(-1)+7 = A(-1+1) + B(-1-3)$

$4 = -4B$

$\Rightarrow B = -1$

$s = 3$ in (ii)

$3.3+7 = A(3+1) + B.0$

$\Rightarrow 16 = A4$

$A = 4$

$\frac{3s+7}{(s-3)(s+1)} = \frac{4}{s-3} - \frac{1}{s+1}$

$\left(\frac{3s+7}{(s-3)(s+1)}\right) = L^{-1}\left(\frac{4}{s-3}\right) - L^{-1}\left(\frac{1}{s+1}\right)$

$= 4L^{-1}\left(\frac{1}{s-3}\right) - L^{-1}\left(\frac{1}{s+1}\right)$

$f(t) = L(e^{at}) = \frac{1}{s-a}$

$e^{at} = L^{-1}\frac{1}{s-a}$

$\Rightarrow e^{3t} = L^{-1}\frac{1}{s-3}$

$L^{-1}\frac{1}{s-3} = e^{3t}$

5B

Q-103: Solve the following Initial Value Problem (IVP) by Laplace Transform: $Y'' + Y = t$, $Y(0) = 1$, $Y'(0) = -2$

Solution

Let $Y = f(t)$

$Y' = f'(t)$

$Y'' = f''(t)$

Given,

$Y'' + Y = t$

That is $\frac{d^2Y}{dt^2} + Y = t$

Taking the Laplace transform of both sides of the differential equation and using the given conditions, we have

$Y'' + Y = t$

$L\{Y''\} + L\{Y\} = L\{t\}$

We have,

$L(f''(t)) = s^2 L\{f(t)\} - s\,f(0) - f'(0)$

$L\{Y''\} + L\{Y\} = L\{t\}$

$s^2 L\{Y\} - s f(0) - f'(0) + L\{Y\} = L\{t\}$

$s^2 y - sf(0) - f'(0) + y = L\{t\}$ [let, $L\{Y\} = y$]

$s^2 y - sf(0) - f'(0) + y = \frac{1}{s^2}$ [let, $L\{t\} = \frac{1}{s^2}$]

$s^2 y - s.1 - (-2) + y = \frac{1}{s^2}$ [Given, $Y(0) = f(0) = 1$ $Y'(0) = f'(0) = -2$]

$s^2 y + y - s.1 + 2 = \frac{1}{s^2}$

$s^2 y + y - s.1 + 2 - \frac{1}{s^2} = 0$

$y(s^2 + 1) - s + 2 - \frac{1}{s^2} = 0$

$y(s^2 + 1) = s - 2 + \frac{1}{s^2}$

$y = \frac{s - 2}{s^2 + 1} + \frac{1}{s^2(s^2 + 1)}$

$y = \frac{s}{s^2 + 1} - \frac{2}{s^2 + 1} + \frac{1}{s^2} - \frac{1}{(s^2 + 1)}$

$y = \frac{s}{s^2 + 1} + \frac{1}{s^2} - \frac{3}{s^2 + 1}$

$\therefore L\{Y\} = y = \frac{s}{s^2+1} + \frac{1}{s^2} - \frac{3}{s^2+1}$

$\therefore Y = L^{-1}(y) = L^{-1}[\frac{s}{s^2+1} + \frac{1}{s^2} - \frac{3}{s^2+1}]$

$\therefore Y = L^{-1}(y) = L^{-1}[\frac{s}{s^2+1}] + L^{-1}[\frac{1}{s^2}] - 3L^{-1}[\frac{1}{s^2+1}]$

$\therefore Y = L^{-1}(y) = \cos t + t - 3\sin t$ Answer

Proof:

$\therefore Y = \cos t + t - 3\sin t$

$\therefore Y' = -\sin t + 1 - 3\cos t$

$\therefore Y'' = -\cos t + 0 + 3\sin t$

$\therefore Y'' + Y = -\cos t + 0 + 3\sin t + \cos t + t - 3\sin t$

$\therefore Y'' + Y = t$

Again,

$\therefore Y = \cos t + t - 3\sin t$

$\therefore Y(0) = \cos 0 + 0 - 3\sin 0$

$\therefore Y(0) = 1$

Again

$\therefore Y' = -\sin t + 1 - 3\cos t$

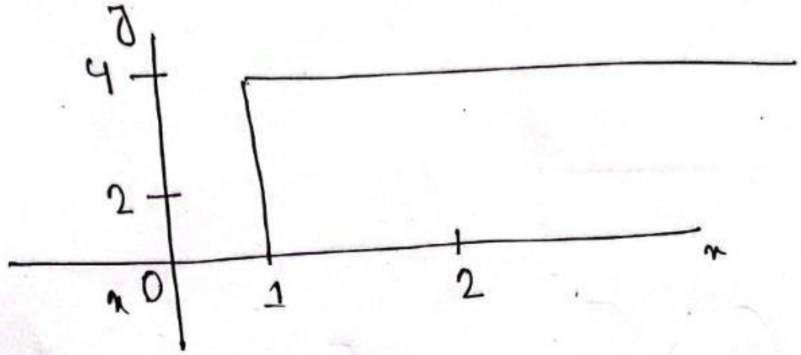
$\therefore Y'(0) = -\sin 0 + 1 - 3\cos 0$

$\therefore Y'(0) = 0 + 1 - 3.1$

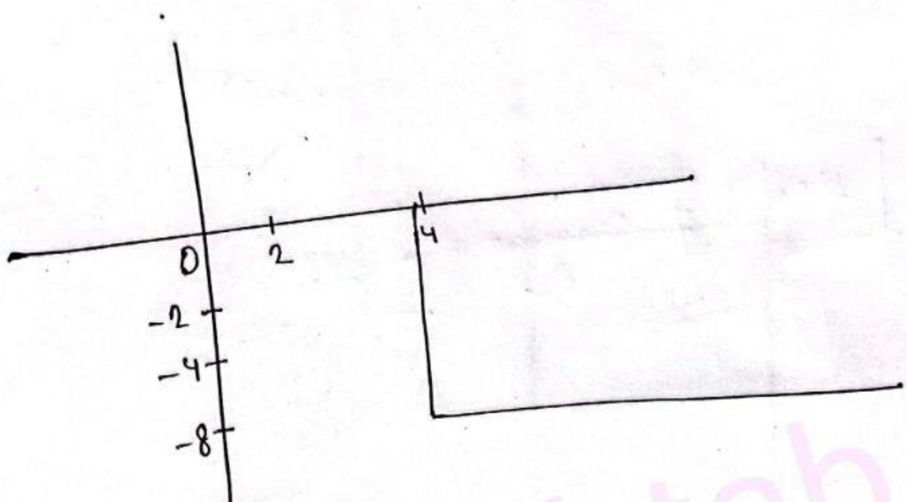
$\therefore Y'(0) = -2$

⑥ a) $x(t) = 4u(t-1) - 8u(t-4) + 4u(t-6)$

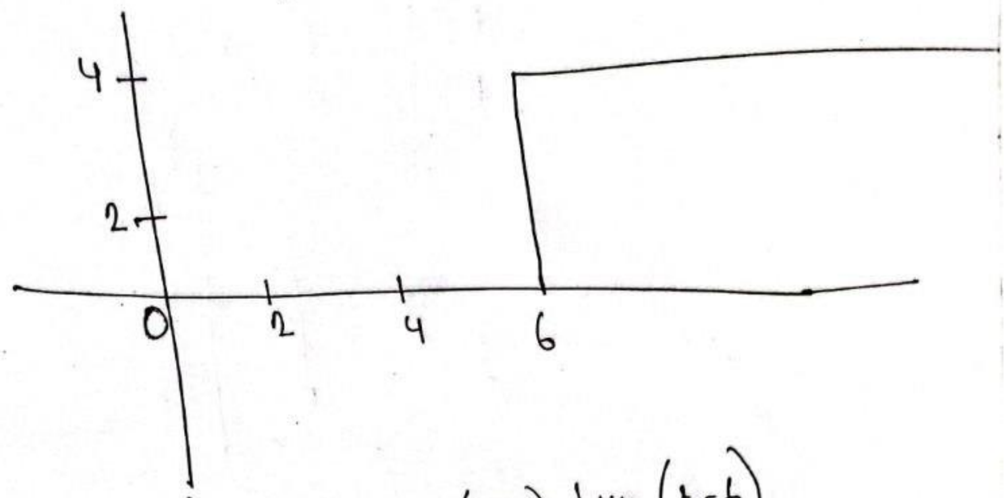
$$4u(t-1) = 4 ; \quad t \geq 1$$
$$= 0 ; \quad t < 1$$



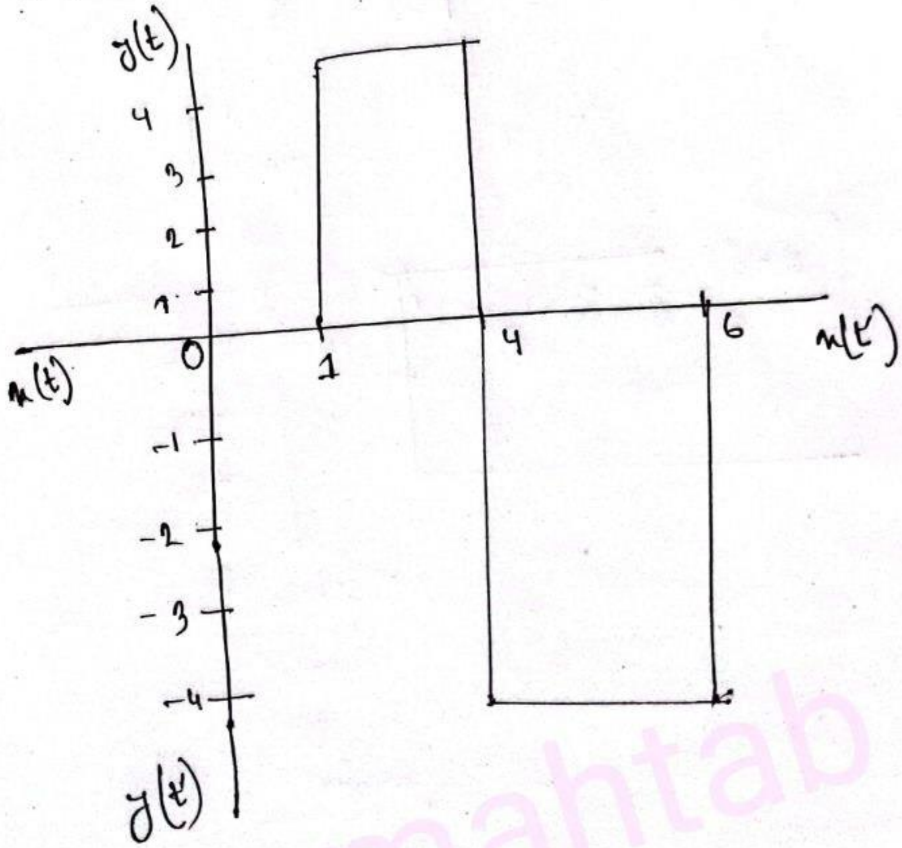
$$-8u(t-4) = -8 ; \quad t \geq 4$$
$$= 0 ; \quad t < 4$$



$$4u(t-6) = 4 ; \quad t \geq 6$$
$$= 0 ; \quad t < 6$$



$$x(t) = 4u(t-1) - 8u(t-4) + 4u(t-6)$$



ANS 7A

7a

$$f(t) = 4 + \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1-\cos n\pi\right)\sin n t$$

$$f(t) = \left(4+\frac{2}{1\pi}\left(1-\cos\pi\right)\sin t\right) + \left(4+\frac{2}{2\pi}\left(1-\cos 2\pi\right)\sin 2\pi t\right) + \left(4+\frac{2}{3\pi}\left(1-\cos 3\pi\right)\sin 3\pi t\right)$$

$$f(t) = \left(4+\frac{2}{\pi}\left(1-\cos\pi\right)\sin t\right) + \left(4+\frac{2}{2\pi}\left(1-\cos 2\pi\right)\sin 2\pi t\right) + 4 + \frac{2}{3\pi}\left(1-\cos 3\pi\right)\sin 3\pi t).$$

f(n) MathLab (n)

```
t= -4 : . 0.001 : 20 ;
y = 4 ; sum: y ;
for (i=1 : 1: n) ;
sum = sum + (2/pi)*(1/i)* sum (i*pi*t)
end ;
figure ;
plot (t,sum) ;
```

7B

7b

here $f(t) = 2.5 + \left[-\frac{5}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\cos n\pi - 1)\sin\frac{n\pi t}{4}\right]$

$\cancel{f(t) = \left(2.5 + -\frac{5}{\pi}\frac{1}{1}(\cos\pi - 1)\sin\frac{1\pi t}{4}\right.}$

here

$a_n = 0$

$b_n = \frac{1}{n}(\cos n\pi - 1)$

```
n = 1:6
w = nπ/4
an = 0
bn = 1/n (cosnπ-1) = .1/n (cos .nπ -1)
r =. sqrt (a.^2 + b.^2)

stem (w,r);
```

7C

```
X=[3,2];
Y=[2,-2];
Y=CONV(X,H);
```

Thank you!!

|^|ASSALAMUALAIKUM|^|