Chapter 5 : Linear Discriminant Functions

Reference:
Pattern Classification
Second Edition
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Generative vs Discriminant Approach

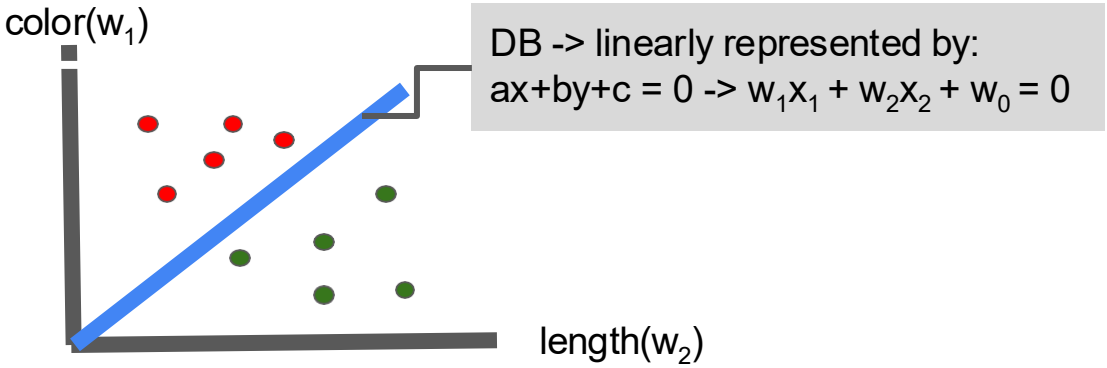
- Generative approaches estimate the discriminant function by first estimating the probability distribution of the patterns belonging to each class.
- Discriminant approaches estimate the discriminant function explicitly, without assuming a probability distribution.

Generative Approach (case of two categories)

$$g(x) = g1(x) - g2(x)$$

- More common to use a single discriminant function instead of two.
- Example: If g(x)=0, then x lies on the decision boundary and can be assigned to either class.

Generative Approach (case of two categories)



DB -> linearly represented by:
$ax+by+c = 0$ -> $w_1x_1 + w_2x_2 + w_0 = 0$

Generative Approach (case of two categories)

Decision boundary , g(x) = ax+by+c

= a.length + b.color + c

Decision Rule:

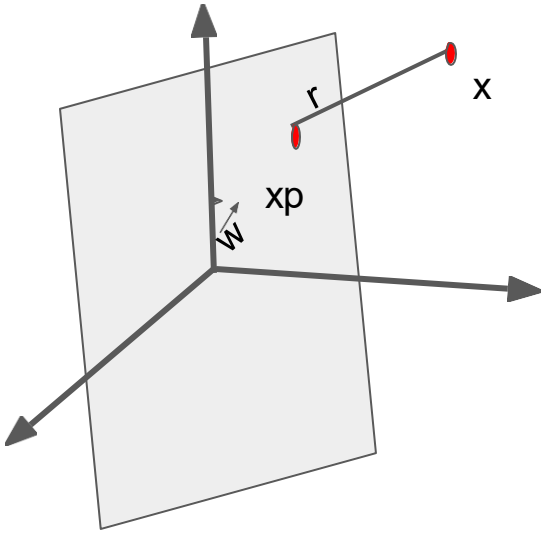
- if g(x)>0 then x ε $\mathbf{w_1}$
- if g(x)<0 then x ε $\mathbf{w_2}$
- if g(x) == 0 then x lies on DB

Decision boundary , $g(x) = w_1x_1 + w_2x_2 + w_0$

Geometric Interpretation of $g(\mathbf{x})$

Hyperplane : Decision Boundary in higher dimensions(>1)

g(x) : gives algebraic measure of distance from X to hyperplane



xp = x projected on hyperplane
x = point outside hyperplane

$H = w^T x + w_0 = 0$
r is a perpendicular on H.

So, $\vec{w} \parallel r$.

$\vec{w}$ lies along the direction of H.

$\vec{r}$ is the distance between x and xp.

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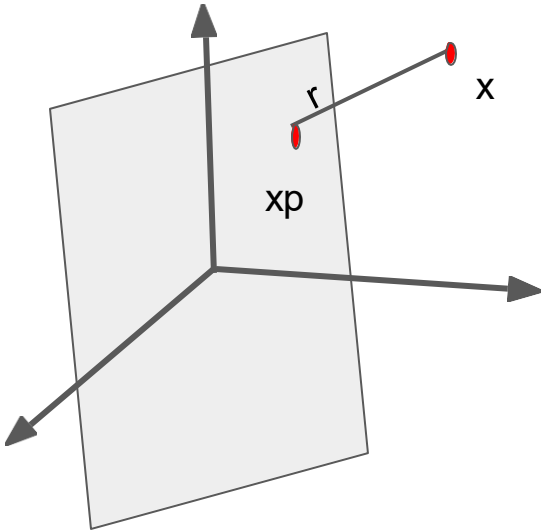
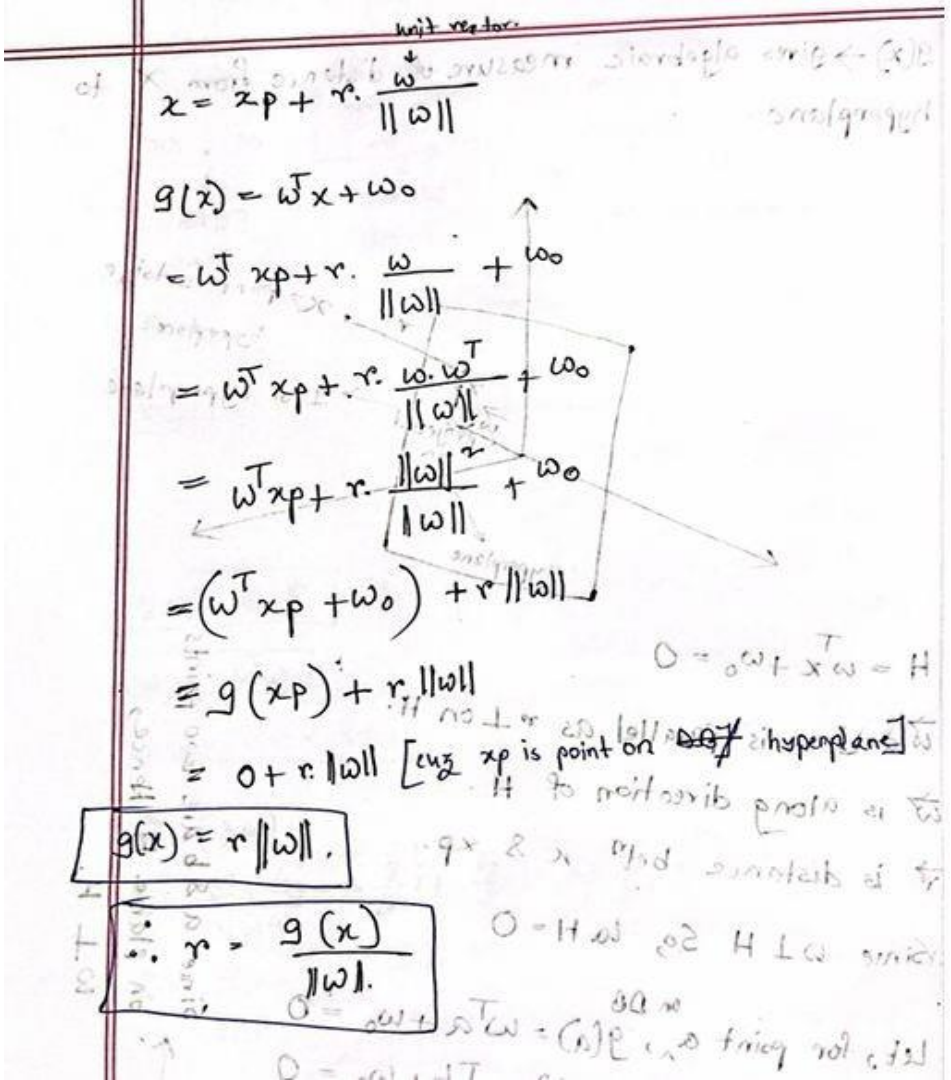


- Prove that r = g(x)/||w||
- $g(x) = 5x_1 - 2x_2 + 3$, $w = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ .Now find r for a new point (1,1)

Solution


unit vector
$x = x_p + r \cdot \frac{\vec{w}}{\|w\|}$
$g(x) = w^T x + w_0$
$= w^T x_p + r \cdot \frac{w}{\|w\|} + w_0$
$= w^T x_p + r \cdot \frac{w \cdot w^T}{\|w\|} + w_0$
$= w^T x_p + r \cdot \frac{\|w\|^2}{\|w\|} + w_0$
$= (w^T x_p + w_0) + r\|w\|$
$= g(x_p) + r\|w\|$
$= 0 + r\|w\|$ [x_p is point on hyperplane]
$g(x) = r\|w\|$.
$\therefore r = \frac{g(x)}{\|w\|}$.

Thank You