2

Circular Convolution, Signal Arithmetic & Signal Drawing

2.1 Theory

Circular convolution, also known as cyclic convolution, is a special case of periodic convolution, which is the convolution of two periodic functions that have the same period [3]. For any vectors $x = (x_1, x_2, \ldots, x_T)^\top \in \mathbb{R}^T$ and $y = (y_1, y_2, \ldots, y_\tau)^\top \in \mathbb{R}^\tau$ with $\tau \leq T$, the circular convolution of two vectors is

$$z = x \star y \in \mathbb{R}^T$$

Equation 2.1: Circular Convolution Equation

donating the operator with symbol $\star$ element-wise, we have

$$z_t = \Sigma_{k=1}^{\tau} x_{t-k+1} y_k, \forall t \in 1, 2, \ldots, T$$

Equation 2.2: Circular Convolution for t-th Element Equation

where z_t is the t-th entry of z and $x_{t-k+1} = x_{t-k+1+T}$ for $t + 1 \leq k$ [4].

2.2 Matlab Code

2.2.1 Circular Convolution

```
x = [1 2 3 4];
h = [1; 1; 1; 1];

tab = zeros(4,4);

for i = 1:1:4
        for j = 1:1:4
                tab(j,i) = x(j);
        end
        x = circshift(x, 1);
end

answ = tab * h;
```

2.2.2 Arithmetic Operation on Signals

```
t = -5:1:20;

fun1 = t>=0 & t<=10;
fun2 = t>=5 & t<=15;

addition = fun1 + fun2;
subtraction = fun1 - fun2;
```

2.2.3 Signal Drawing

```
t = -2:0.001:8;

unit = t>=1 & t<6;
unit2 = t>=3 & t<4;
ramp = (t>=0 & t<1) .* t;
ramp2 = (t>=6 & t<7) .* (7-t);
ramp3 = (t>=2 & t<3) .* (t-2);
ramp4 = (t>=4 & t<5) .* (5-t);

answ1 = unit + ramp+ramp2;
answ2 = 2*unit + 2*ramp+2*ramp2+unit2*2+2*ramp3+2*ramp4;
```

2.3 Output

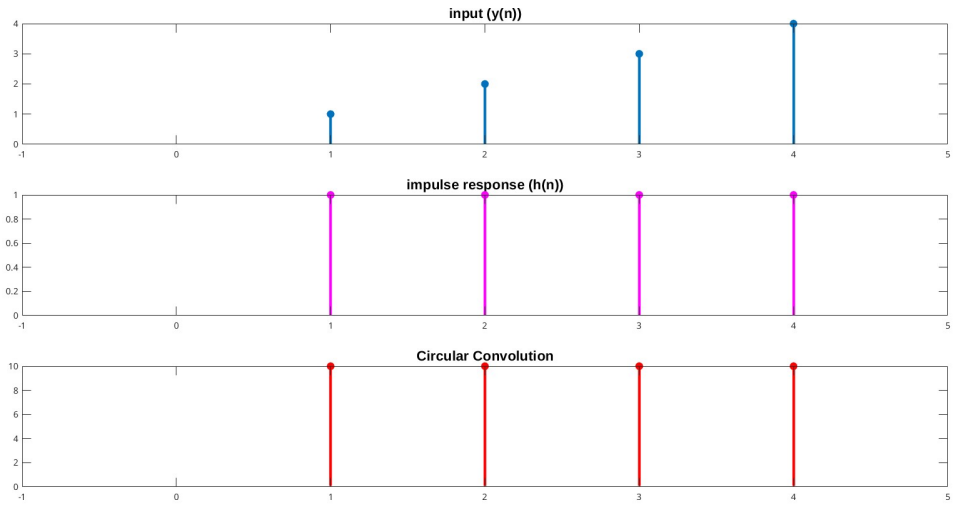

Figure 2.1: Circular Convolution

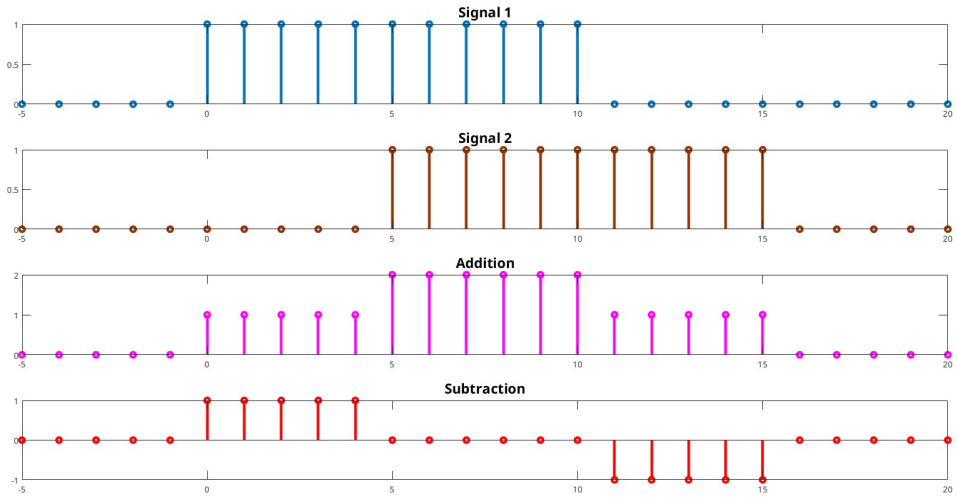

Figure 2.2: Arithmetic Operation on Signals

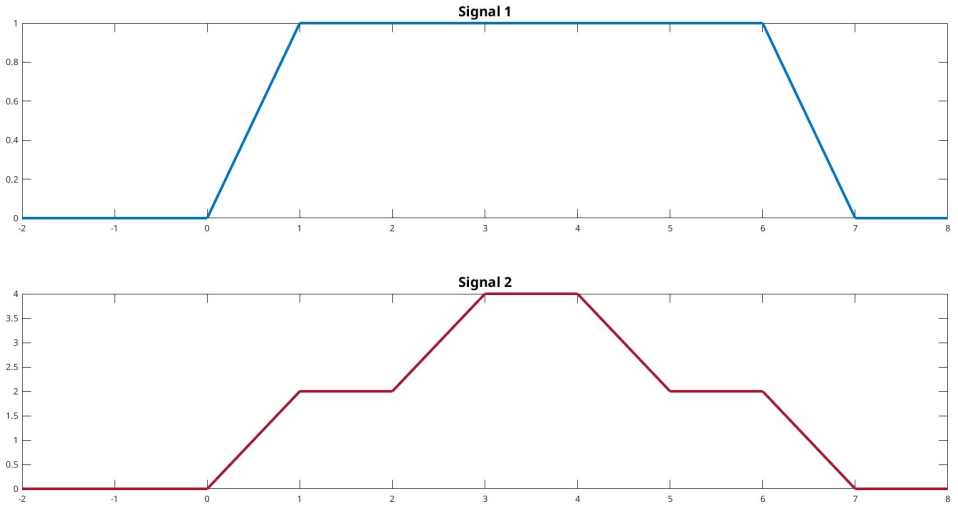

Figure 2.3: Signal Drawing

2.4 Conclusion

The input, impulse and circular convolution signal is shown in figure 2.1. Figure 2.2 has two discrete signal and addition, subtraction operation between them. To draw the output figure 2.3, we used unit step, unit ramp and with conditions.

Bibliography

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[4] X. Chen. "Discrete Convolution and Fast Fourier Transform Explained and Implemented Step by Step", Medium. (Nov. 10, 2022), [Online]. Available: https://medium.com/@xinyu.chen/discrete-convolution-and-fast-fourier-transform-explained-and-implemented-step-by-step-83ff1809378d (visited on 05/03/2023).