Lab Final (Fall'23)

Integral Calculus & Differential Equations (MAT120)

Time: 1 hour 20 minutes **Total: 20 marks**

Answer any 4 questions.

For all the questions:

$$p = \text{Second digit of your ID}$$

$$q = \text{Last digit of your ID}$$

1. (a) Find the value of the first and second derivative of the following function at $x = \pi/2$ and $x = 2\pi$. (2)

$$y = \frac{\sin^p(x) + \tan(qx)}{1 + px + qx^2}$$

   (b) If $\phi(x, y, z) = 3x^2 z \sin^{-1}(py)$, then upon finding $\frac{\partial^2\phi}{\partial x^2}$, $\frac{\partial^2\phi}{\partial y^2}$, and $\frac{\partial^2\phi}{\partial z^2}$ evaluate the Laplacian: (3)

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z}$$

2. (a) If $y = \sin(px)$, then write a code to count all the extrema of the expression: $f = y_2 + 3y_1 - qy$ in the interval $x \in [-3\pi/2, 5\pi/2]$. (3)

   (b) If $y = 4x^p - x^3$, then find all the minima for $f = y_2 - 3y_1 + 2y$. (2)

3. (a) Integrate using sympy: (2)

$$\int_1^3 \frac{\cos(\ln x)}{x} dx$$

   (b) Find the following integrals using any numerical method (3)

$$\int_0^\infty \frac{\sin x \cos x}{x} dx$$

4. Consider a circuit with resistor and inductor (RL) in series. The voltage drop in resistor and inductor is given by $V_R = IR$ and $V_L = L\frac{dI}{dt}$ respectively. If the battery voltage is $V(t)$, we then have from Kirchhoff's voltage rule: $V_L + V_R = V(t)$. Or,

$$L\frac{dI}{dt} + RI = V(t)$$

   (a) Solve the equation to find the current $I(t)$ with the initial condition: $I(0) = 0$ for $V(t) = 5e^{-qt} + 10\sin(pt)$ (3)

   (b) Plot the current vs. time graph for the first 10 seconds. Use $L = p$ and $R = 10 + q$. (2)

5. If one takes $I = \frac{dq}{dt}$ in the RLC series circuit, the equation in question 4 becomes a second-order ODE for electric charge Q. The equation for Q:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V(t)$$

   (a) Solve the equation when $V(t) = 10$ and the initial conditions: $Q(0) = 0$ and $\dot{Q}(0) = 10$ (3)

   (b) Plot current ($I = \frac{dQ}{dt}$) vs. time graph for the first 50 seconds. Use $L = p$, $R = 10+q$, and $C = 0.1p$. (2)

6. Motion of a small ball bearing of mass m dropped in a viscous oil can be modeled by the equation:

$$m\frac{d^2y}{dt^2} + r\frac{dy}{dt} - mg = 0 \quad \text{where} \quad g = 9.8m/s^2$$

   (a) Solve the equation with initial condition: $y(0) = 0$, and $\dot{y}(0) = 0$. (2)

   (b) Plot velocity $v = \frac{dy}{dt}$ in for first 20 seconds. Use $m = 1$ and $r = 0.5p$. (2)

   (c) Observing the velocity curve, comment on what happens to velocity as t becomes very large. (1)

NB: Copied code will receive half marks.