Lab Final (Fall'23)

Integral Calculus & Differential Equations (MAT120)

Time: 1 hour 20 minutes **Total: 20 marks**

Answer any 4 questions.

For all the questions:

$$p = \text{Second digit of your ID}$$

$$q = \text{Last digit of your ID}$$

1. (a) Find the First and second derivatives of the function: (2)

$$\frac{x^p \ln(q + p^x)}{1 + px + qx^2}$$

   (b) If $\phi(x, y, z) = 3x^{2p} \sin(y^q z) - 5y \cos^p(3xz^2)$, then upon finding $\frac{\partial^2 \phi}{\partial x^2}$, $\frac{\partial^2 \phi}{\partial y^2}$, and $\frac{\partial^2 \phi}{\partial z^2}$ evaluate the Laplacian: (3)

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

2. If $y = q \cos x - 2p \sin x$, then find
   (a) the expression: $f = y_2 - 4y_1 + 2y$; where, y_1 and y_2 are the 1^{st} & 2^{nd} derivative of y respectively. (2)
   (b) all the maxima for the expression $f = y_2 - 4y_1 + 2y$ in the interval $x \in [-\pi/2, 7\pi/2]$. (2)
   (c) Plot f and f' in the same graph for a given interval of x. (1)

3. (a) Integrate using sympy: (3)

$$\int_1^{e^2} \frac{dx}{x(1 + \ln x)^2} \qquad \text{and} \qquad \int_0^1 \frac{(\cos^{-1} x)^p}{\sqrt{1 - x^2}} dx$$

   (b) Integrate using any numerical method: (2)

$$\int_0^\infty \left[1 - 2\sin^2(px^2)\right] dx$$

4. Consider a circuit with resistor and inductor (RL) in series. The voltage drop in resistor and inductor is given by $V_R = IR$ and $V_L = L\frac{dI}{dt}$ respectively. If the battery voltage is $V(t)$, we then have from Kirchhoff's voltage rule: $V_L + V_R = V(t)$. Or,

$$L\frac{dI}{dt} + RI = V(t)$$

Solve the equation to find the current $I(t)$ with the initial condition: $I(0) = 0$

   (a) for $V(t) = 2p$ and (2)

   (b) for $V(t) = 2p\cos(0.5t)$ (2)

   (c) Also plot the current vs. time graph for the first 50 seconds. Use $L = 2p$ and $R = 10 + 5q$. (1)

5. If one takes $I = \frac{dq}{dt}$ in the RL series circuit, the equation in question 4 becomes a second-order ODE for electric charge Q. The equation for Q:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} = V(t)$$

   (a) Solve the equation when $V(t) = 2p$ and the initial conditions: $Q(0) = 10$ and $\dot{Q}(0) = 0$ (3)

   (b) Also plot the charge (Q) and current ($I = \frac{dQ}{dt}$) vs. time graph for the first 50 seconds. Use $L = 2p$ and $R = 10 + 5q$. (1+1)

6. A freely falling ball with mass m is under quadratic drag ($F_D = \lambda v^2$). Newton's equation of motion:

$$m\frac{dv}{dt} = mg - \lambda v^2$$

   (a) Applying Euler's method, solve the equation and plot $v(t)$ for the first five seconds with the initial condition: $v(0) = 0$. Take $m = p$, $\lambda = 0.5$, and $g = 9.8$. (4)

   (b) What feature of the graph exhibits the presence of air resistance? (1)

NB: Copied code will receive half marks.