Lab Final (Fall'23)

Integral Calculus & Differential Equations (MAT120)

Time: 1 hour 20 minutes **Total: 20 marks**

Answer any 4 questions.

For all the questions:

$$p = \text{Second digit of your ID}$$

$$q = \text{Last digit of your ID}$$

1. If

$$y = \frac{\sin(px) + \cos(qx)}{p + x^2 \ln x}$$

   (a) Find the expression: $f = 2y'' - y' + 3y$ (3)

   (b) Plot y and f in sympy subplot for the interval $x \in [0, 10]$. (2)

2. (a) If $\phi(x, y, z) = 5xz\ln(p + y^q) - 2py^2e^{4xz}$, then upon finding $\frac{\partial^2\phi}{\partial x^2}$, $\frac{\partial^2\phi}{\partial y^2}$, and $\frac{\partial^2\phi}{\partial z^2}$ evaluate the Laplacian: (3)

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

   (b) Integrate using sympy: (2)

$$\int_0^1 \frac{(\sin^{-1} x)^p}{\sqrt{1 - x^2}} dx$$

3. Find the following integrals using any numerical method (2+3)

$$\int_0^{\pi/2} e^{p\cos(q+x)} dx \qquad \text{and} \qquad \int_0^{\infty} \frac{1}{x^2} \cos\left(\frac{x^3 - px}{x^2 - q}\right) dx$$

4. Consider a circuit with resistor and inductor (RL) in series. The voltage drop in resistor and inductor is given by $V_R = IR$ and $V_L = L\frac{dI}{dt}$ respectively. If the battery voltage is $V(t)$, we then have from Kirchhoff's voltage rule: $V_L + V_R = V(t)$. Or,

$$L\frac{dI}{dt} + RI = V(t)$$

   (a) Solve the equation to find the current $I(t)$ with the initial condition: $I(0) = 0$ for $V(t) = 2e^{-pt} + 5p\cos(4t)$ (3)

   (b) Plot the current vs. time graph for the first 10 seconds. Use $L = 2p$ and $R = 1 + q$. Observing the graph can you comment on whether the current becomes AC or not? (2)

5. If one takes $I = \frac{dq}{dt}$ in the RL series circuit, the equation in question 4 becomes a second-order ODE for electric charge Q. The equation for Q:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} = V(t)$$

   (a) Solve the equation when $V(t) = 2p$ and the initial conditions: $Q(0) = 0$ and $\dot{Q}(0) = 10$ (3)

   (b) Plot current ($I = \frac{dQ}{dt}$) vs. time graph for the first 50 seconds. Use $L = 2p$ and $R = 10 + 5q$. (2)

6. A freely falling ball with mass m is under quadratic drag ($F_D = \lambda v^2$). Newton's equation of motion:

$$m\frac{dv}{dt} = mg - \lambda v^2$$

   (a) Applying Euler's method, solve the equation and plot $v(t)$ for the first 10 seconds with the initial condition: $v(0) = 0$. Take $m = p$, $\lambda = 0.5$, and $g = 9.8$. (4)

   (b) Observing the graph, comment on what happens to velocity as t becomes very large. (1)

NB: Copied code will receive half marks.