Lab Final (Fall'23)

Integral Calculus & Differential Equations (MAT120)

Time: 1 hour 20 minutes **Total: 20 marks**

Answer any 4 questions.

For all the questions:

$$p = \text{Second digit of your ID}$$

$$q = \text{Last digit of your ID}$$

1. (a) Find the First and second derivative of the following functions: (3)

$$f(x) = p^x e^{\cos^{-1}(x^p)} \qquad \text{and} \qquad g(x) = \frac{\ln\{q + \tan^p(x)\}}{\cos(x) p^{\sin(x)}}$$

   (b) See if the function $px^3 - 2x^2 + (2+q)x + 4$ has any extremum. (2)

2. If $y = \sin(px)$, then find

   (a) the expression: $f = y_2 + 3y_1 - qy$; where, y_1 and y_2 are the 1^{st} & 2^{nd} derivative of y respectively. (2)

   (b) all the extrema of the expression f in the interval $x \in [-3\pi/2, 5\pi/2]$. Also plot f and f' in the same graph for given range of x. (2+1)

3. (a) Find the factorial of the number 13.7. (1)

   (b) Integrate using sympy: (2)

$$\int_0^{\pi/2} \frac{dx}{p^2 \cos^2 x + (1+q)^2 \sin^2 x}$$

   (c) Integrate using any numerical method: (2)

$$\int_0^{\infty} \left[2\cos^2(px^2) - 1\right] dx$$

4. Consider a circuit with resistor and inductor (RL) in series. The voltage drop in resistor and inductor is given by $V_R = IR$ and $V_L = L\frac{dI}{dt}$ respectively. If the battery voltage is $V(t)$, we then have from Kirchhoff's voltage rule: $V_L + V_R = V(t)$. Or,

$$L\frac{dI}{dt} + RI = V(t)$$

Solve the equation to find the current $I(t)$ with the initial condition: $I(0) = 0$

(a) for $V(t) = 5p$ and (2)

(b) for $V(t) = 5p\sin(2t)$ (2)

(c) Also plot the current vs. time graph for the first 50 seconds. Use $L = p$ and $R = (4 + q)^2$. (1)

5. Motion of a small ball bearing of mass m dropped in a viscous oil can be modeled by the equation:

$$m\frac{d^2y}{dt^2} + r\frac{dy}{dx} - mg = 0 \quad \text{where} \quad g = 9.8m/s^2$$

(a) Solve the equation with initial condition: $y(0) = 0$, and $\dot{y}(0) = 0$. (2)

(b) Plot depth y and velocity $v = \frac{dy}{dt}$ in the same graph. Use $m = p$ and $r = p + q$. (1+1)

(c) Observing the velocity curve, comment on what happens to velocity as t becomes very large. (1)

6. A freely falling ball with mass m is under linear drag (drag force, $F_D = \lambda v$) and quadratic drag ($F_D = \lambda v^2$). Equation of motion: ()

$$\begin{aligned} \text{Linear Drag:} \quad m\frac{dv}{dt} &= mg - \lambda v \\ \text{Quadratic Drag} \quad m\frac{dv}{dt} &= mg - \lambda v^2 \end{aligned}$$

(a) Applying Euler's method, solve both equations and plot $v(t)$ for the first three seconds with the initial condition: $v(0) = 0$. Take $m = 1$, $\lambda = 0.5$, and $g = 9.8$. (4)

(b) Identify which drag force is more likely to cause air resistance by looking at the two graphs. (1)

NB: Copied code will receive half marks.